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The Standard Model lifetime is slightly shorter

Yutaro Shoji (IJS, Ljubljana)
with P. Baratella, M. Nemevšek, K. Trailović and L. Ubaldi

2406.05180/hep-ph

Discrete 2024, IJS Ljubljana, 2-6 Dec. 2024

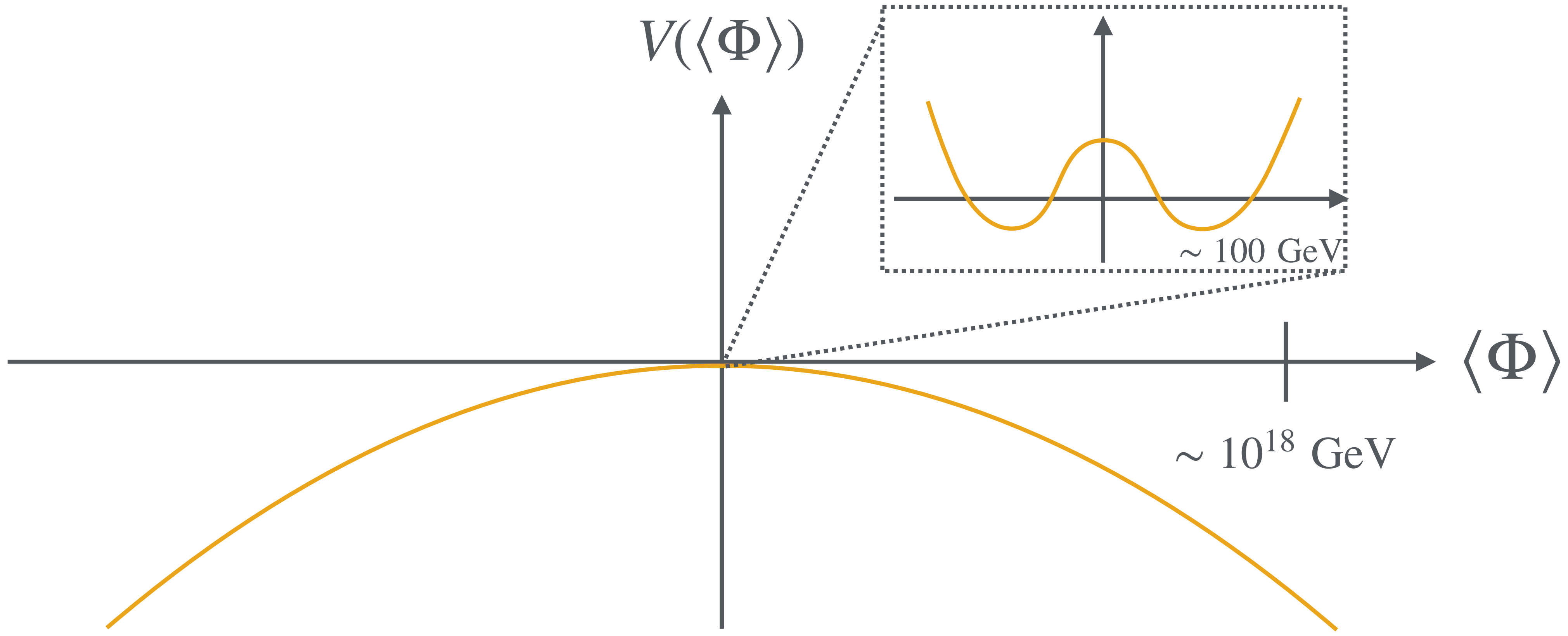
Standard Model at very high energy

$$V(\Phi) = m^2|\Phi|^2 + \lambda|\Phi|^4$$

$$16\pi^2 \frac{d\lambda}{d\ln\mu} \simeq 12 y_t^2 \lambda - 6 y_t^4$$

$|\Phi| \gg v$

$$\simeq \lambda|\Phi|^4 \ (\lambda < 0)$$



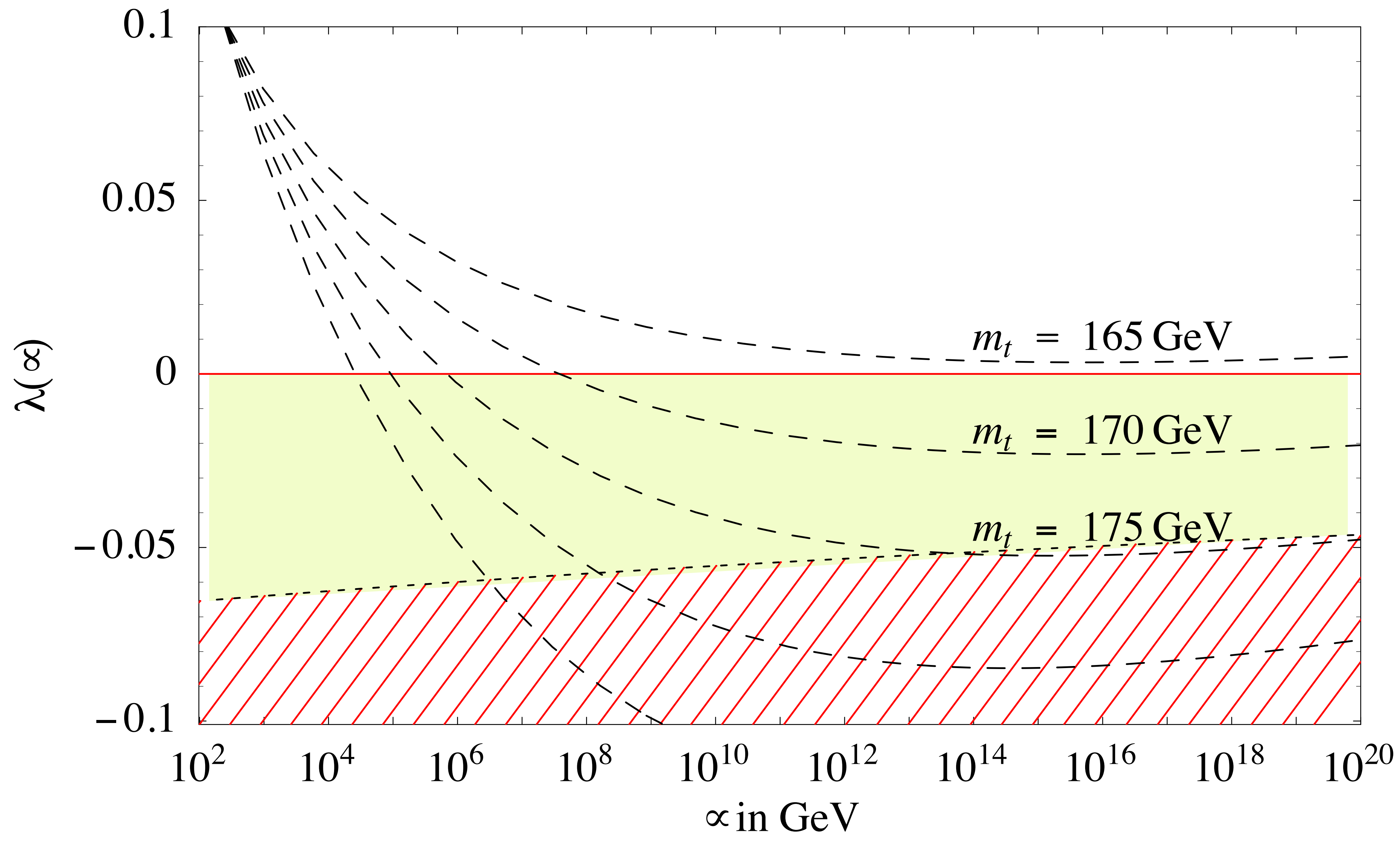
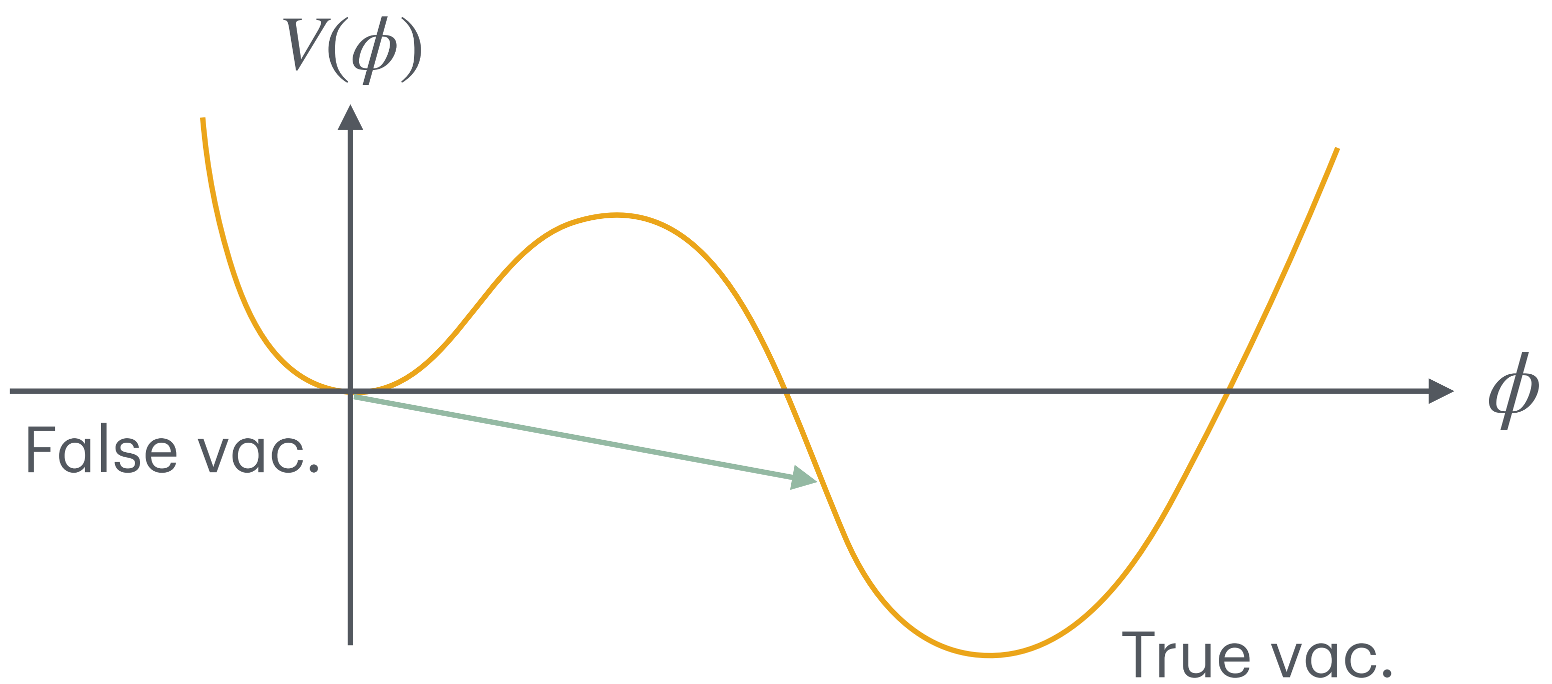


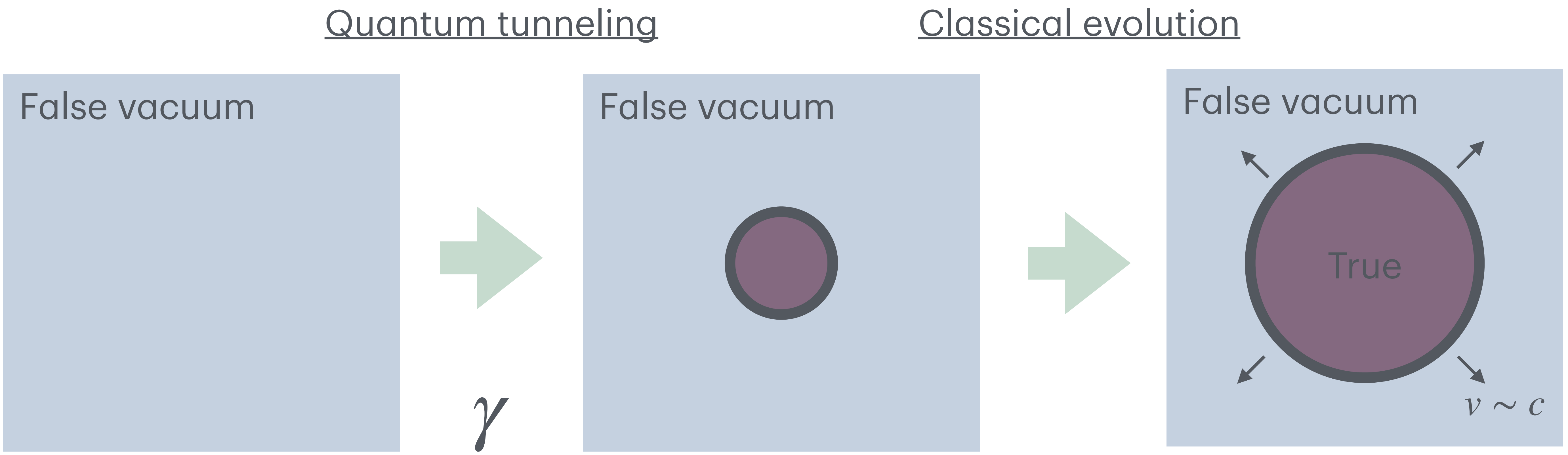
Fig. From [G. Isidori, G. Ridolfi, A. Strumia, '01]

Meta-stability

(Zero temperature)

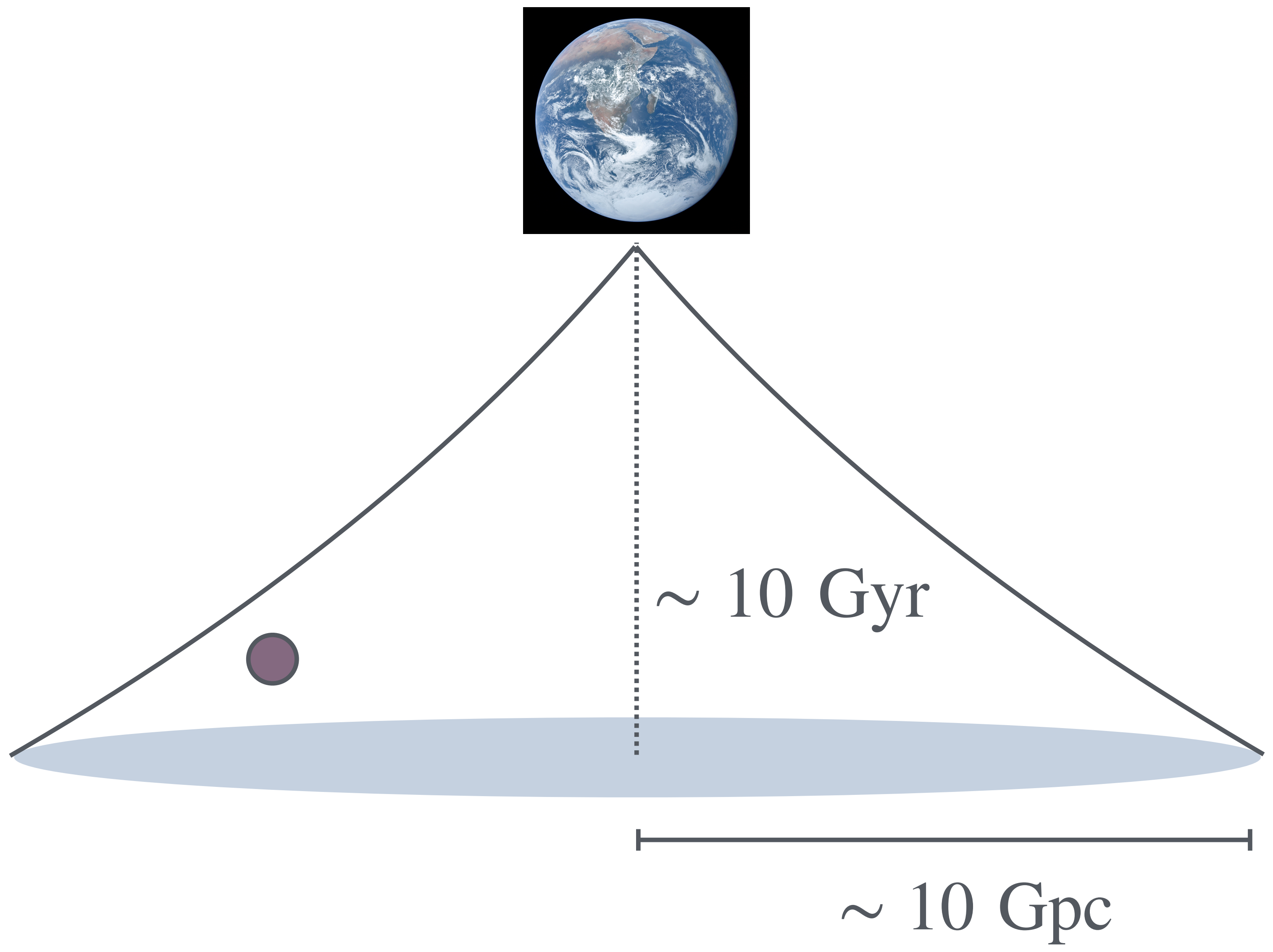
Vacuum decay





Bubble nucleation rate per unit volume

Meta-stability



Our Universe is stable enough if

$$\gamma \ll H_0^4 \sim 10^{-3}\,\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$$

Vacuum decay rate

[T. Banks, C. M. Bender, T. T. Wu, ’73; S. R. Coleman, ’77; C. G. Callan, S. R. Coleman, ’77]

Bubble nucleation rate

$$\gamma = Ae^{-B}$$

$$B = S_E(\bar{\phi}) - S_E(v_{\mathrm{FV}})$$

$\bar{\phi}$: Bounce

Non-trivial O(4) symmetric solution of Euclidean EoM

overshoot-undershoot, polygonal bounce,
asymptotic expansions from thin-wall, gradient flow, …

$$A = \left(\frac{B}{2\pi}\right)^2 \left(\frac{\det' S_E''(\bar{\phi})}{\det S_E''(v_{\mathrm{FV}})}\right)^{-1/2}$$

One-loop corrections to the action

Scalar

$$\det S_E^{''(\phi)} = \det[-\partial^2 + m_\phi^2]$$

Expand with hyperspherical harmonics

$$= \prod_{l=0}^{\infty} \det\left[-\partial_\rho^2 - \frac{3}{\rho}\partial_\rho + \frac{l(l+2)}{\rho^2} + m_\phi^2(\rho)\right]^{(l+1)^2}$$

1-dim functional determinant

→ “Gelfand-Yaglom theorem”

Gauge boson + NG boson

$$\det S_E^{''(Aa)} = \det\begin{pmatrix} (-\partial^2 + m_A^2)\delta_{\mu\nu} & 2g\partial_\mu\bar{\phi} \\ 2g\partial_\nu\bar{\phi} & -\partial^2 + m_a^2 \end{pmatrix}$$

$$= ?$$

We need vector hyperspherical harmonics

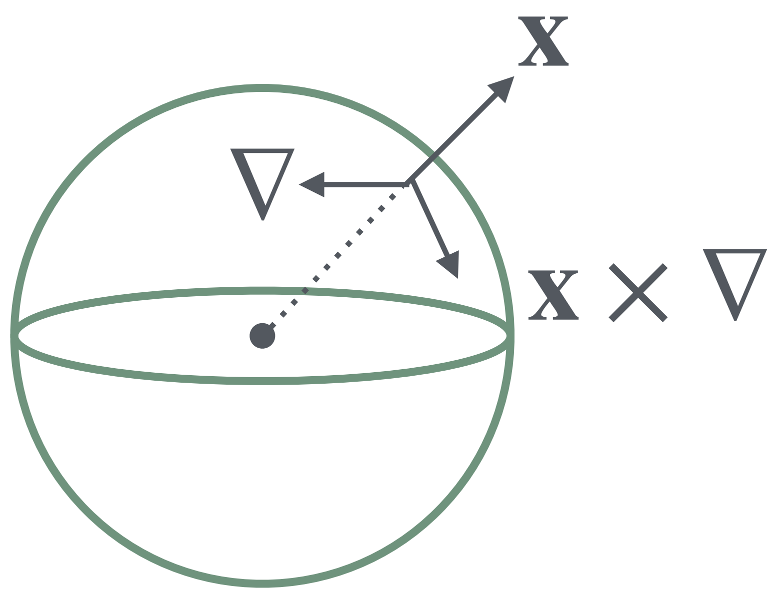
Vector spherical harmonics (?)

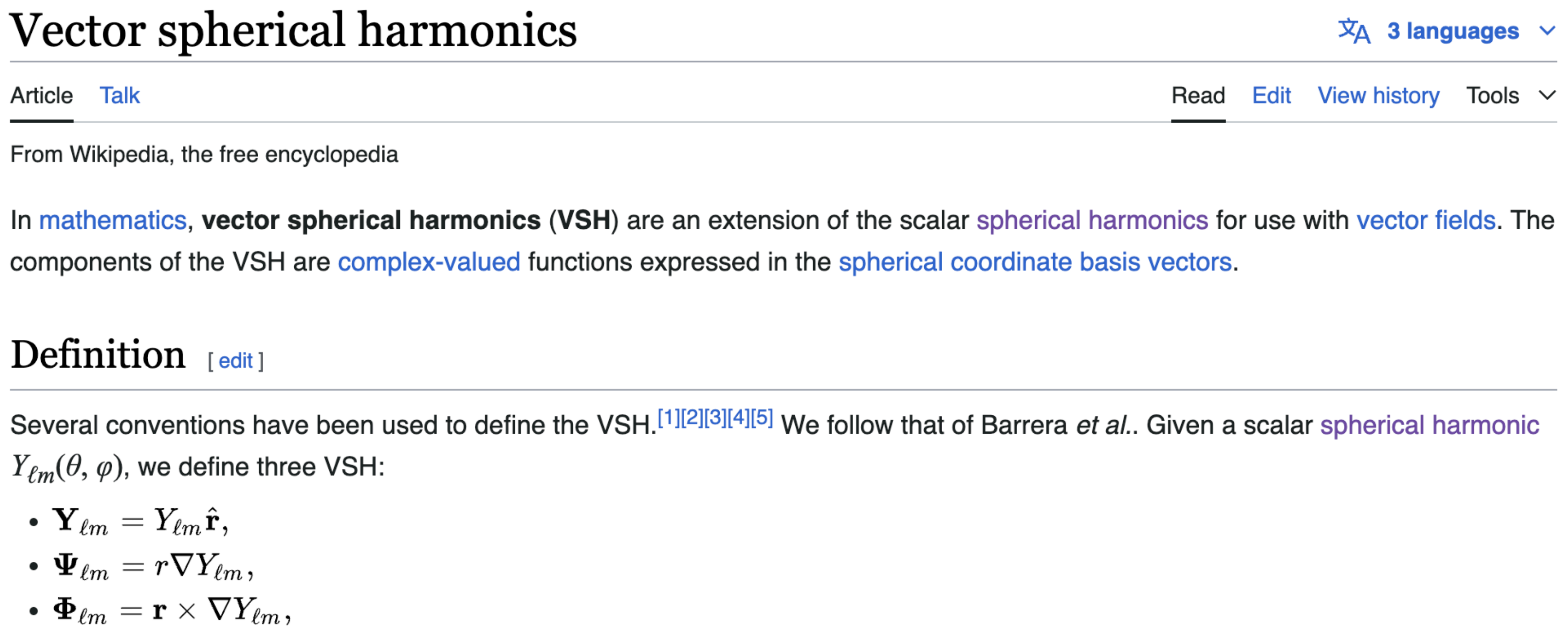
3D

$$\hat{x}^i Y_{lm} \qquad \deg = (2l+1)$$

$$|x|\,\partial_i Y_{lm} \qquad \deg = (2l+1) \quad (l>0)$$

$$\epsilon_{ijk} x^j \partial^k Y_{lm} \qquad \deg = (2l+1) \quad (l>0)$$





Vector spherical harmonics

3 languages

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From Wikipedia, the free encyclopedia

In mathematics, **vector spherical harmonics** (**VSH**) are an extension of the scalar spherical harmonics for use with vector fields. The components of the VSH are complex-valued functions expressed in the spherical coordinate basis vectors.

Definition [edit]

Several conventions have been used to define the VSH.[1][2][3][4][5] We follow that of Barrera *et al.*. Given a scalar spherical harmonic $Y_{\ell m}(\theta, \varphi)$, we define three VSH:

- $\mathbf{Y}_{\ell m} = Y_{\ell m}\hat{\mathbf{r}}$,
- $\mathbf{\Psi}_{\ell m} = r\nabla Y_{\ell m}$,
- $\mathbf{\Phi}_{\ell m} = \mathbf{r} \times \nabla Y_{\ell m}$,

4D

[G. Isidori, G. Ridolfi, A. Strumia, '01]

$$\hat{x}^\mu Y_{l m_A m_B} \qquad \deg = (l+1)^2$$

$$|x|\,\partial_\mu Y_{l m_A m_B} \qquad \deg = (l+1)^2 \quad (l>0)$$

$$\epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{l m_A m_B} \qquad \deg = (l+1)^2 \quad (l>0)$$

$$\epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l m_A m_B} \qquad \deg = (l+1)^2 \quad (l>0)$$

with $V_1^\mu V_{2\mu} = 0$

This basis has been used in the previous one-loop calculations of SM vacuum decay rate

G. Isidori, G. Ridolfi, A. Strumia, '01
A. Andreassen, W. Frost, M. D. Schwartz, '18
S. Chigusa, T. Moroi, YS, '17&'18

Tensor product states

Eigenvalues

$$so(4) \simeq su(2)_A \times su(2)_B$$

$$J_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu} \qquad A_J^i \qquad B_J^i$$

Total
$$\begin{cases} A_J^2 : j_A(j_A+1) & A_{J3} : M_A \\ B_J^2 : j_B(j_B+1) & B_{J3} : M_B \\ J^2 = \frac{1}{2} J^{\mu\nu} J_{\mu\nu} = 2(A_J^2 + B_J^2) & \end{cases}$$

Orbital $\quad L^2 : l(l+2)$

Labeling convention

$$(j_A, j_B)_l$$

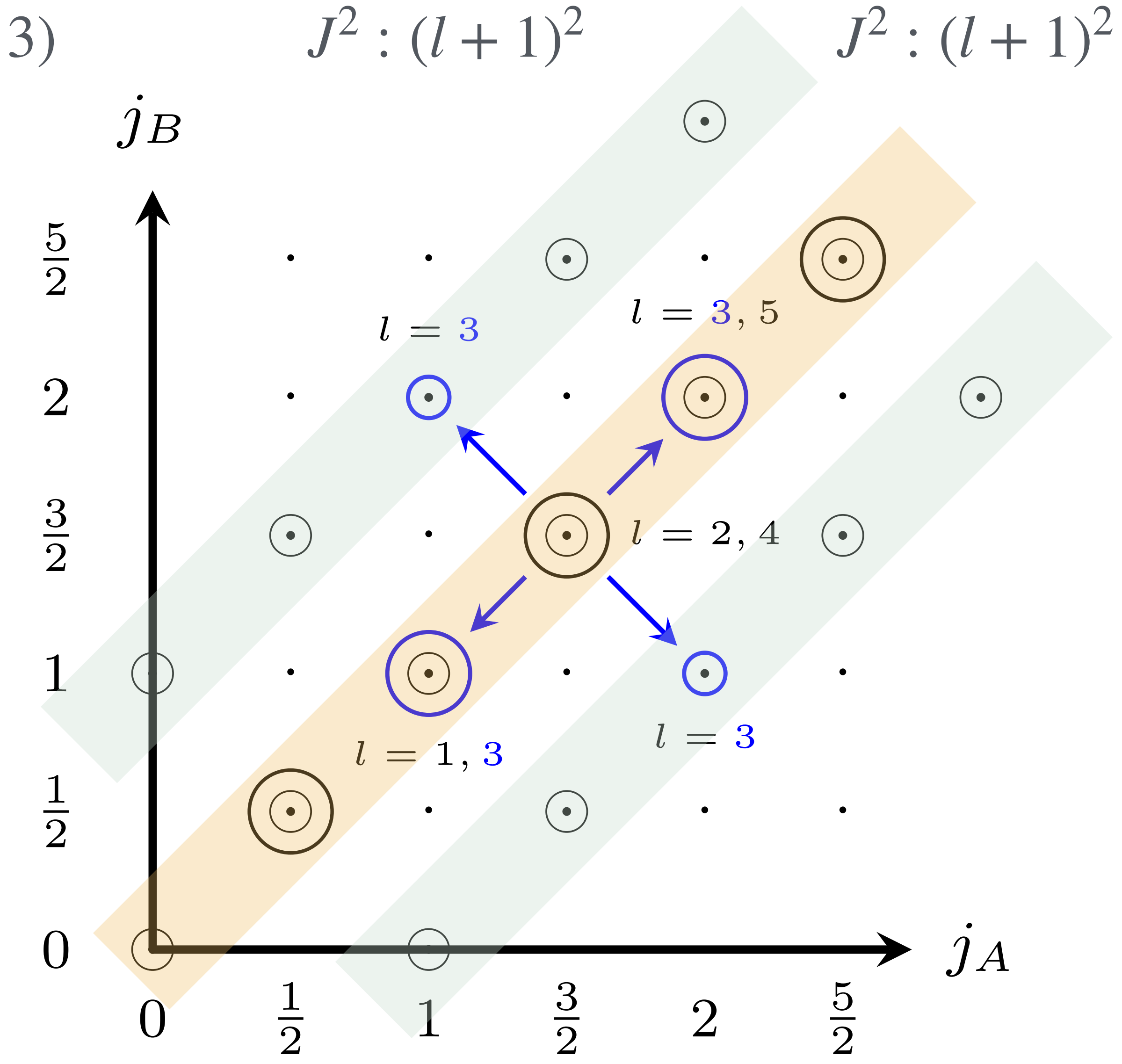
Constant 4D vector with two spinor indices

$$Y_{l m_A m_B} \times V_{\chi_A \chi_B}$$

$$\left(\frac{l}{2}, \frac{l}{2}\right)_l \otimes \left(\frac{1}{2}, \frac{1}{2}\right)_0$$

$$= \left(\frac{l+1}{2}, \frac{l+1}{2}\right)_l \oplus \left(\frac{l+1}{2}, \frac{l-1}{2}\right)_l \oplus \left(\frac{l-1}{2}, \frac{l+1}{2}\right)_l \oplus \left(\frac{l-1}{2}, \frac{l-1}{2}\right)_l$$

$$J^2 : (l+1)(l+3) \qquad J^2 : (l+1)^2 \qquad J^2 : (l+1)^2 \qquad J^2 : (l-1)(l+1)$$



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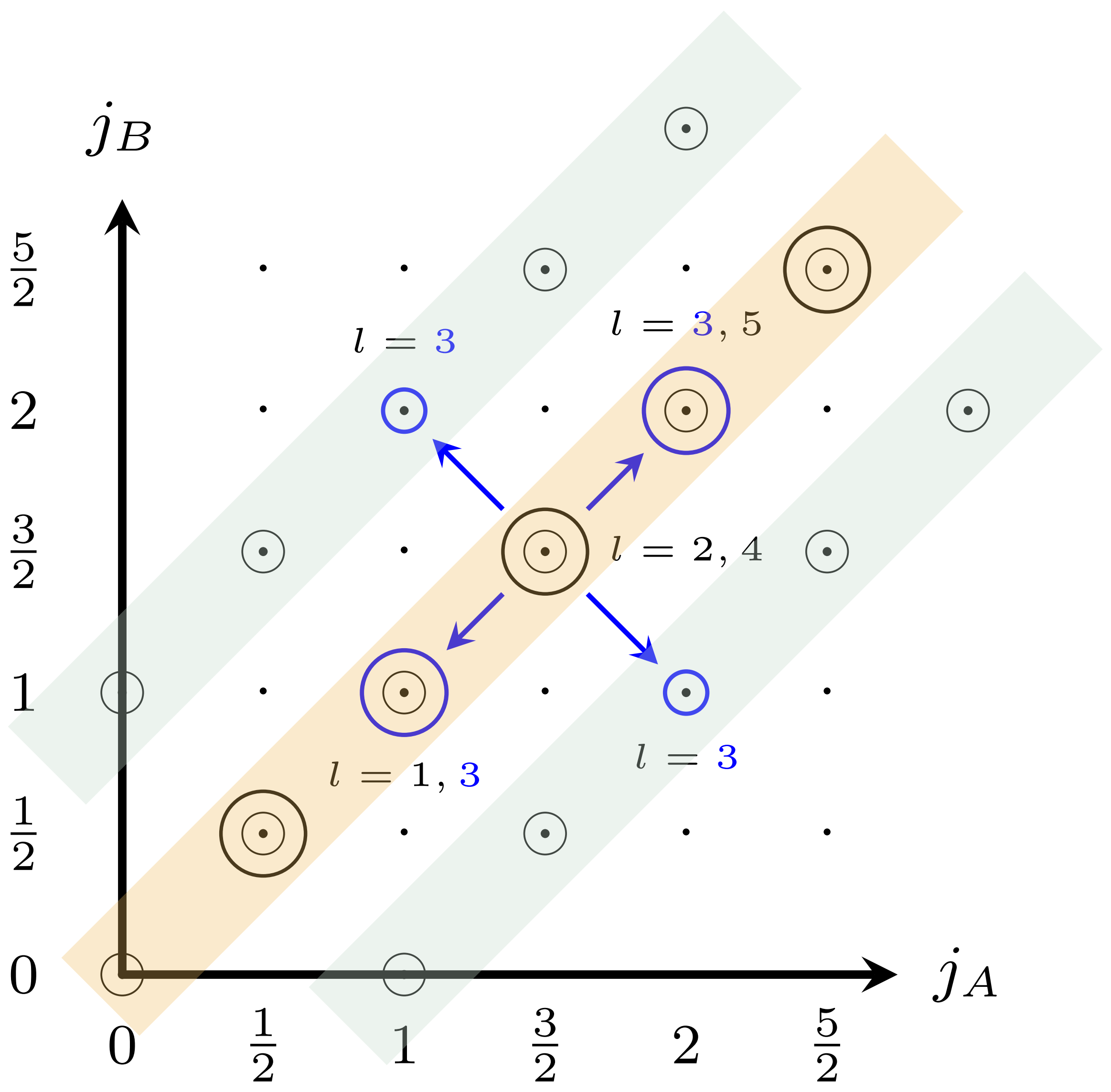


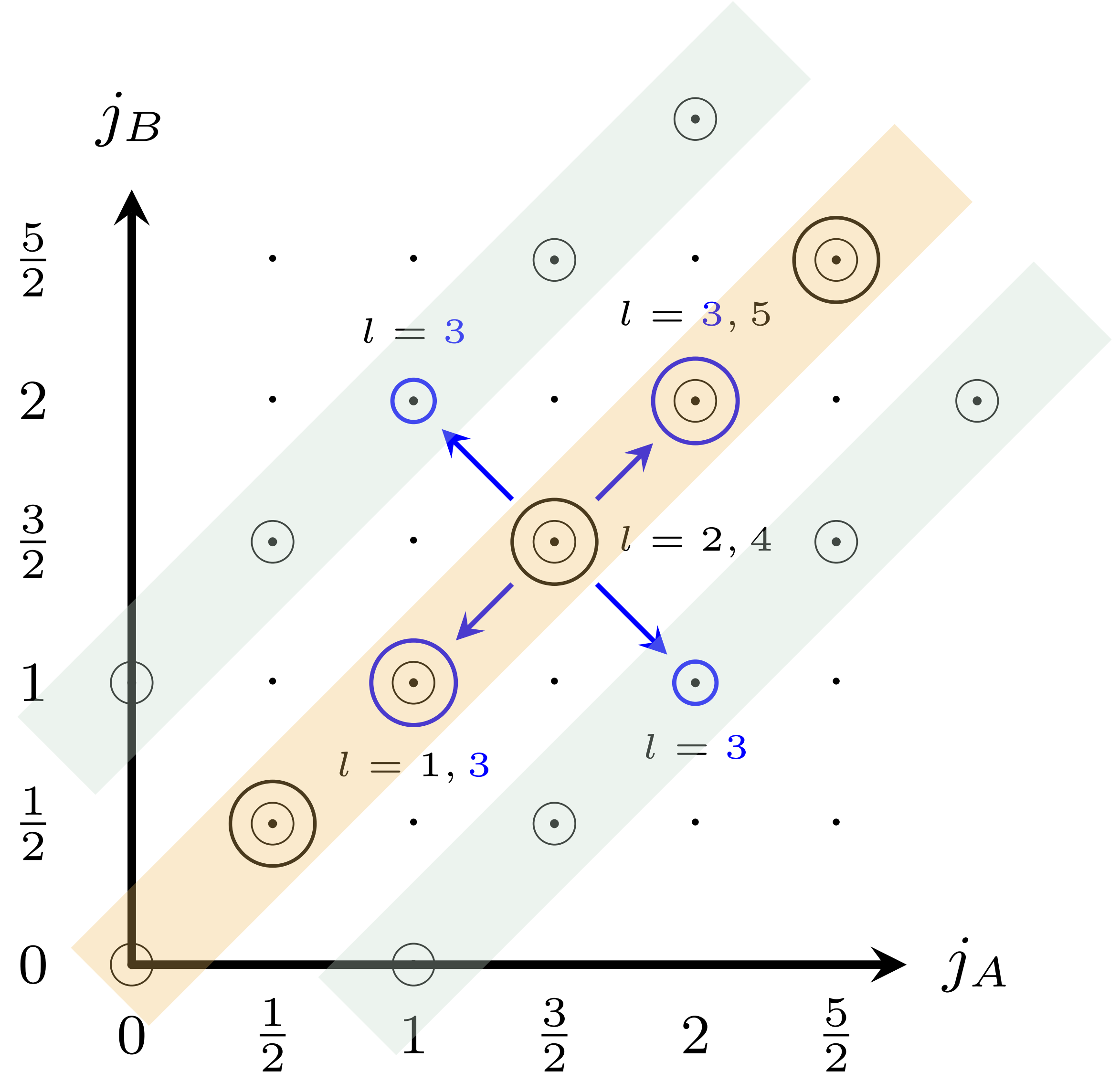
$$(l+1)\hat{x}^\mu Y_{(l+1)m_A m_B} + |x|\partial^\mu Y_{(l+1)m_A m_B} \qquad \left(\frac{l+1}{2},\frac{l+1}{2}\right)_l \qquad \deg = (l+2)^2$$

$J^2 : (l+1)(l+3)$

$$(l+1)\hat{x}^\mu Y_{(l-1)m_A m_B} - |x|\partial^\mu Y_{(l-1)m_A m_B} \qquad \left(\frac{l-1}{2},\frac{l-1}{2}\right)_l \qquad \deg = l^2$$

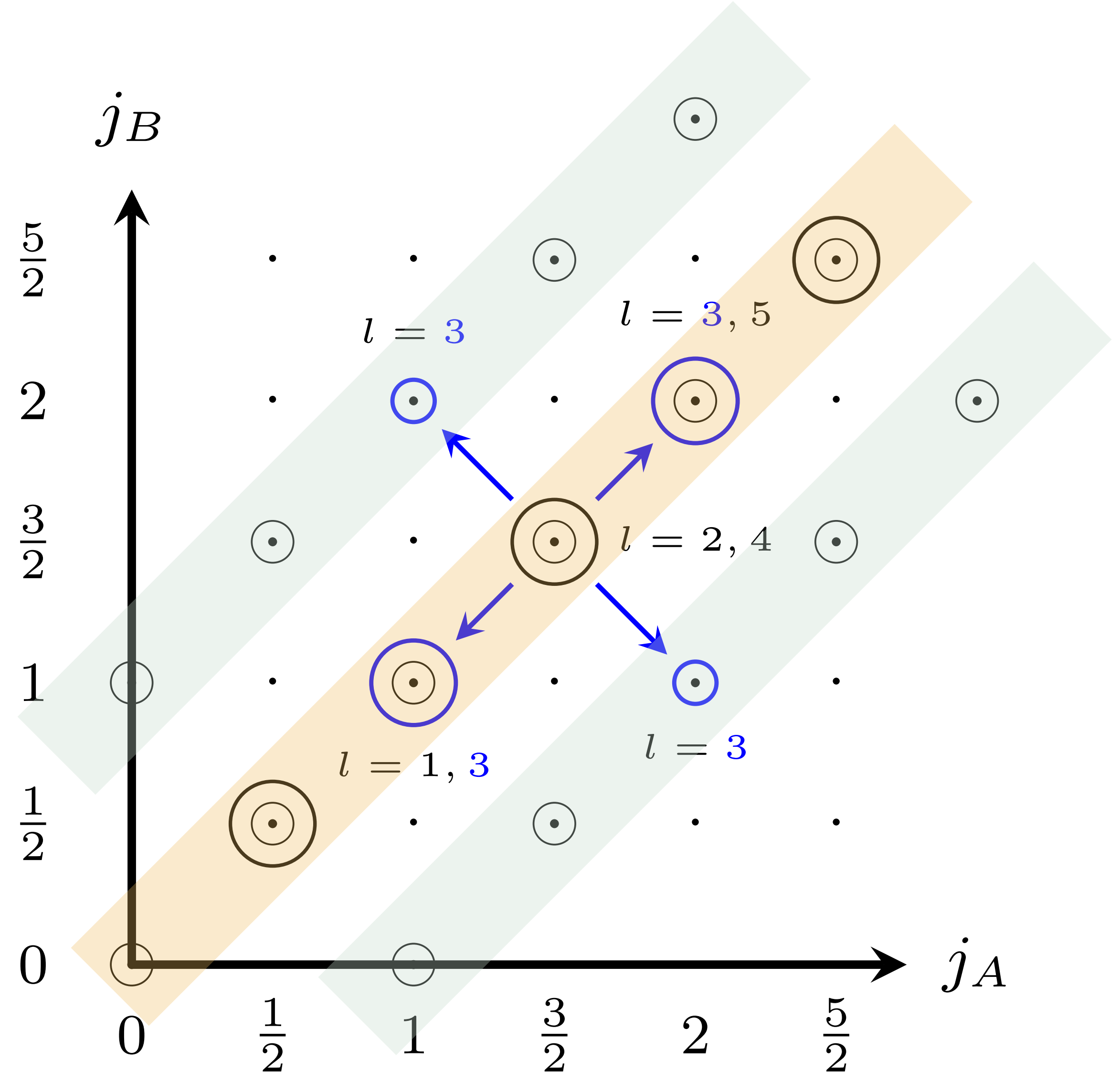
$J^2 : (l-1)(l+1)$

Correspondence



$(l+1)\hat{x}^\mu Y_{(l+1)m_A m_B} + \lvert x \rvert \partial^\mu Y_{(l+1)m_A m_B}$	$\left(\frac{l+1}{2}, \frac{l+1}{2}\right)_l$ $J^2 : (l+1)(l+3)$	$\deg = (l+2)^2$
$(l+1)\hat{x}^\mu Y_{(l-1)m_A m_B} - \lvert x \rvert \partial^\mu Y_{(l-1)m_A m_B}$	$\left(\frac{l-1}{2}, \frac{l-1}{2}\right)_l$ $J^2 : (l-1)(l+1)$	$\deg = l^2$
$\epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{l m_A m_B}$	$J^2 : (l+1)^2$	$\deg = (l+1)^2$
$\epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l m_A m_B}$	$J^2 : (l+1)^2$	$\deg = (l+1)^2$

Correspondence



$$(l+1)\hat{x}^\mu Y_{(l+1)m_A m_B} + |x|\partial^\mu Y_{(l+1)m_A m_B} \quad \left(\frac{l+1}{2}, \frac{l+1}{2}\right)_l \quad \text{deg} = (l+2)^2$$

$J^2 : (l+1)(l+3)$

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$$\epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{l m_A m_B} \quad J^2 : (l+1)^2 \quad \text{deg} = (l+1)^2$$

$$\epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l m_A m_B} \quad J^2 : (l+1)^2 \quad \text{deg} = (l+1)^2$$

$$\left(\frac{l+1}{2}, \frac{l-1}{2}\right)_l \quad J^2 : (l+1)^2 \quad \text{deg} = l(l+2)$$

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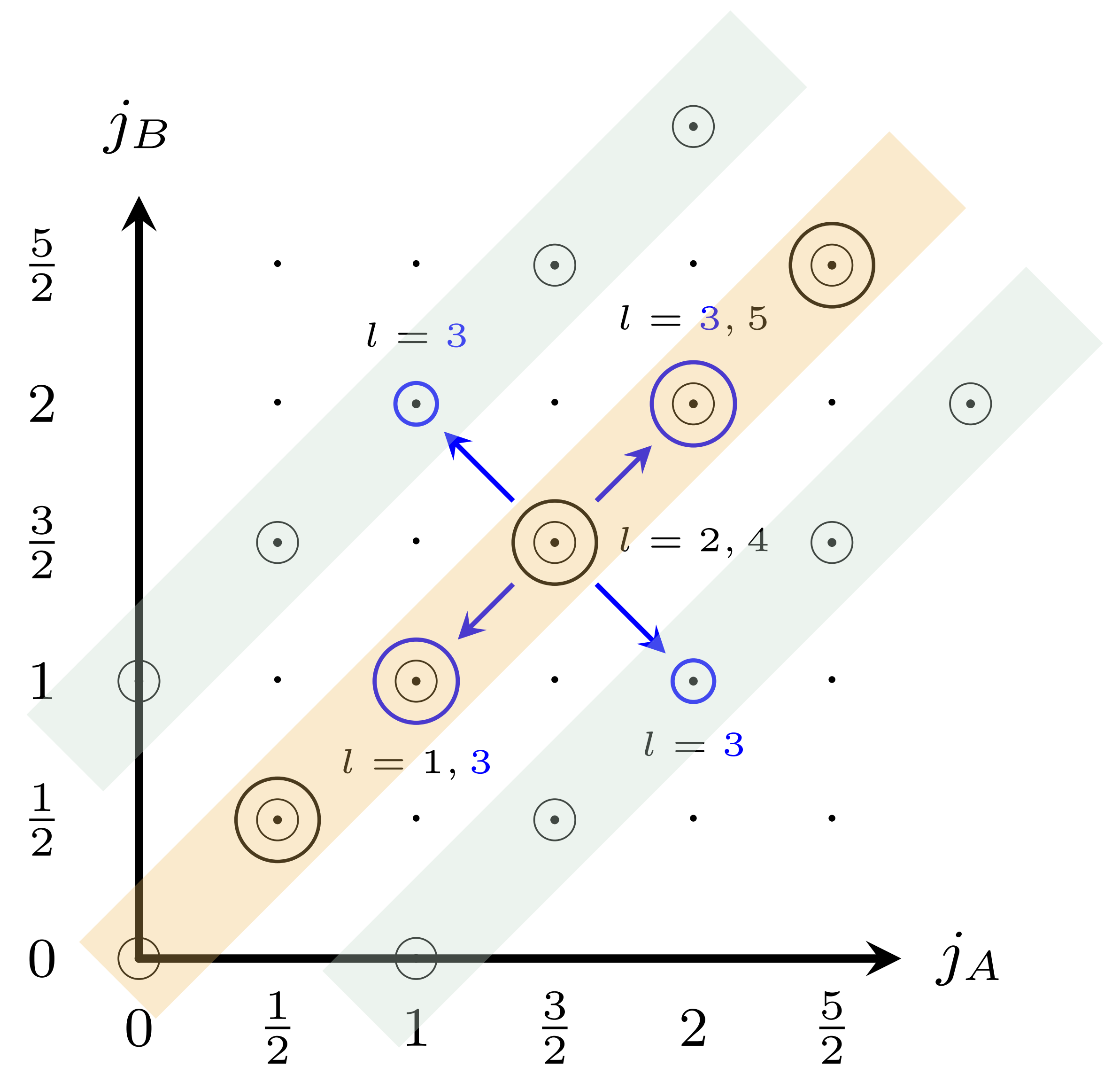

$(l+1)\hat{x}^\mu Y_{(l+1)m_A m_B} + \lvert x \rvert \partial^\mu Y_{(l+1)m_A m_B}$	$\left(\frac{l+1}{2}, \frac{l+1}{2}\right)_l$ $J^2 : (l+1)(l+3)$	$\deg = (l+2)^2$
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$\left(\frac{l+1}{2}, \frac{l-1}{2}\right)_l$	$J^2 : (l+1)^2$	$\deg = l(l+2)$
$\left(\frac{l-1}{2}, \frac{l+1}{2}\right)_l$	$J^2 : (l+1)^2$	$\deg = l(l+2)$

Something is going wrong with the transverse modes

$$2(l+1)^2 = 2l(l+2) + 2$$

Dependence and incompleteness

Collapsed space

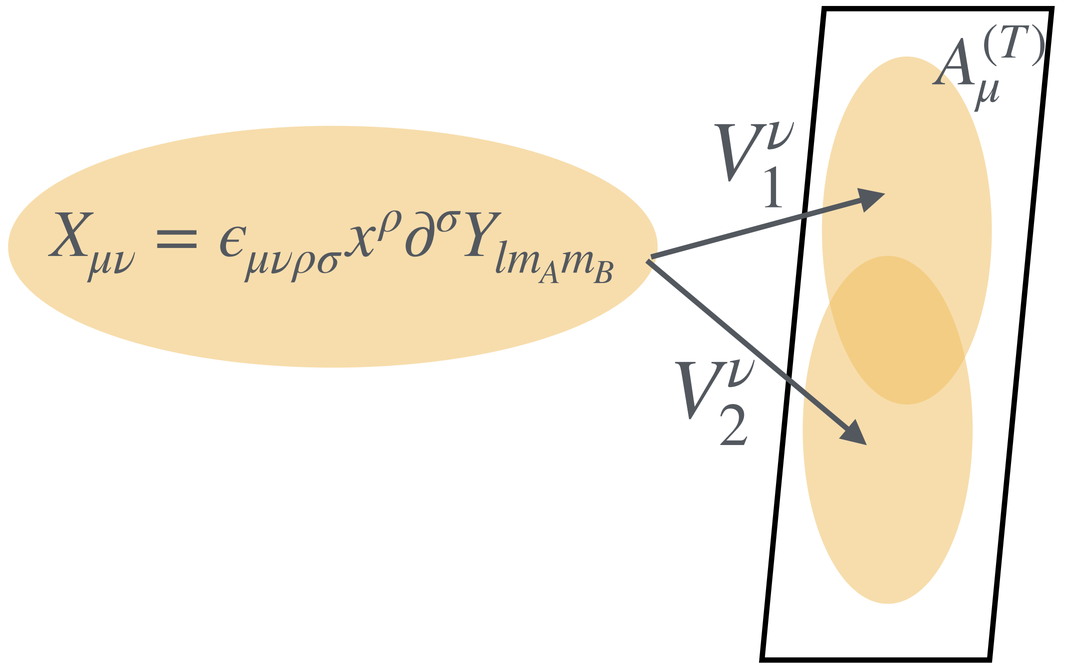
$V_1 = (0,0,0,1),\ V_2 = (0,0,1,0)$ without loss of generality

$$0 = \sum_{m=-l/2}^{l/2} (-1)^m \epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{lm(-m)}$$

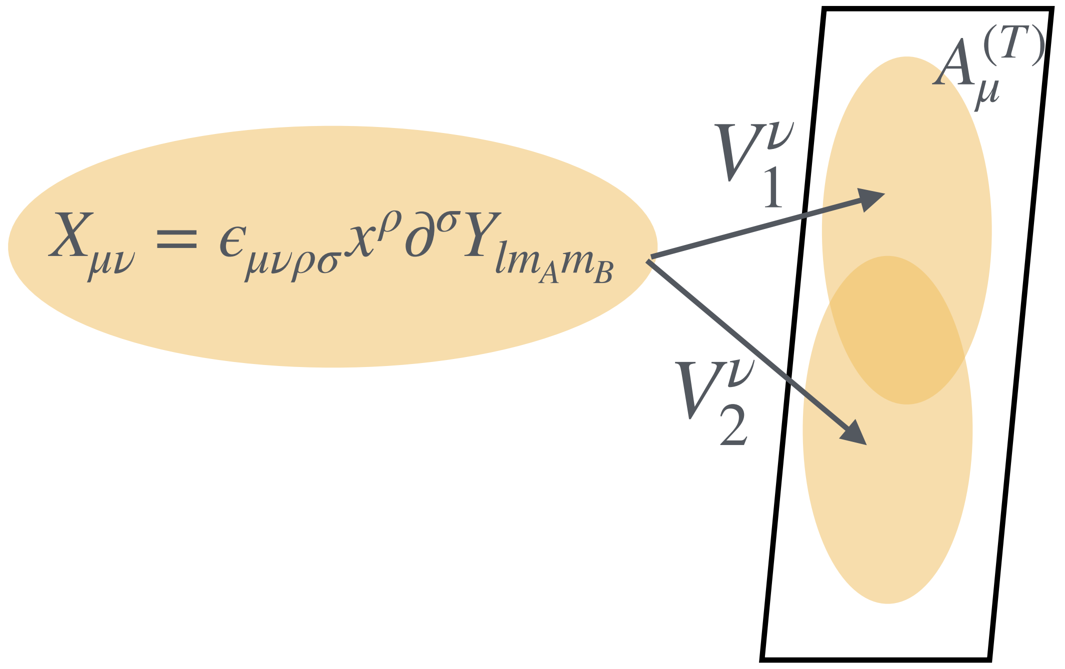
$\deg : (l+1)^2 - 1$

$$0 = \sum_{m=-l/2}^{l/2} \epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{lm(-m)}$$

$\deg : (l+1)^2 - 1$



Dependence and incompleteness



Collapsed space

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$$0 = \sum_{m=-l/2}^{l/2} (-1)^m \epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{lm(-m)} \qquad 0 = \sum_{m=-l/2}^{l/2} \epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{lm(-m)}$$

$\deg : (l+1)^2 - 1$ $\qquad$ $\deg : (l+1)^2 - 1$

Overlapping space

$$\epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{lm(-m)} = i \sum_{\sigma=-l/2}^{m} \epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l\sigma(-\sigma)} + i \sum_{\sigma=-l/2}^{m-1} \epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l\sigma(-\sigma)}$$

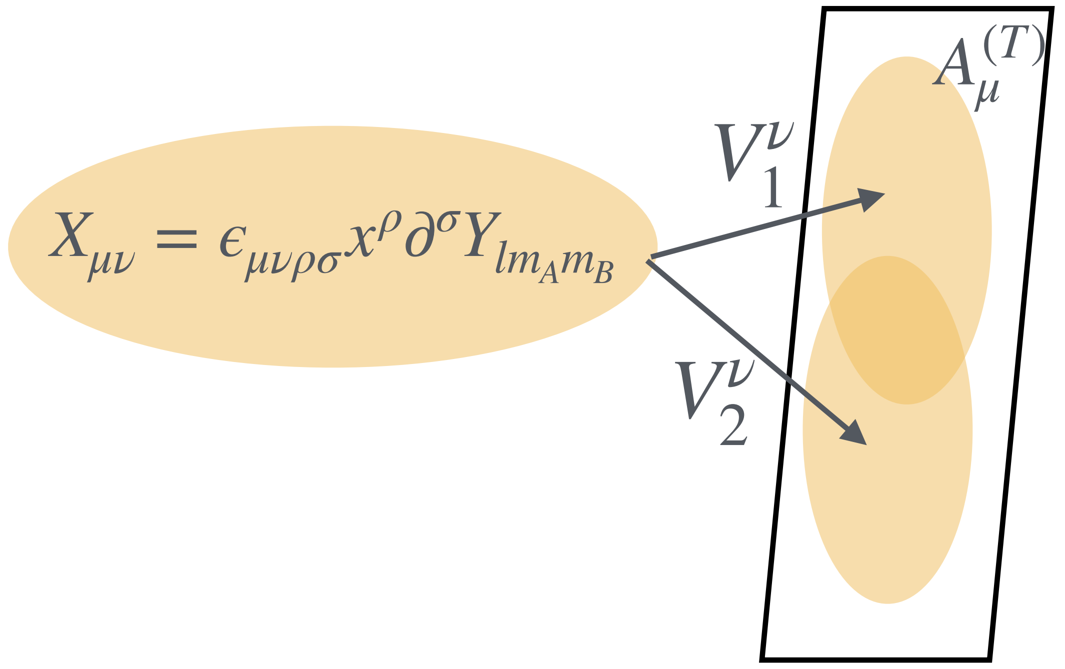
: l independent identities

(The others are linearly independent)

Dependence and incompleteness

Collapsed space

$V_1 = (0,0,0,1),\ V_2 = (0,0,1,0)$ without loss of generality



$$0 = \sum_{m=-l/2}^{l/2} (-1)^m \epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{lm(-m)} \qquad 0 = \sum_{m=-l/2}^{l/2} \epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{lm(-m)}$$

$\deg : (l+1)^2 - 1$ $\qquad$ $\deg : (l+1)^2 - 1$

Overlapping space

$$\epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{lm(-m)} = i \sum_{\sigma=-l/2}^{m} \epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l\sigma(-\sigma)} + i \sum_{\sigma=-l/2}^{m-1} \epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l\sigma(-\sigma)}$$

: l independent identities

(The others are linearly independent)

$2(l+1)^2 - 2 - l$ independent functions

$\epsilon_{\mu\nu\rho\sigma} V_1^\nu x^\rho \partial^\sigma Y_{l m_A m_B}$
$\epsilon_{\mu\nu\rho\sigma} V_2^\nu x^\rho \partial^\sigma Y_{l m_A m_B}$ is incomplete

Complete basis set for T-modes

Complete orthogonal basis set

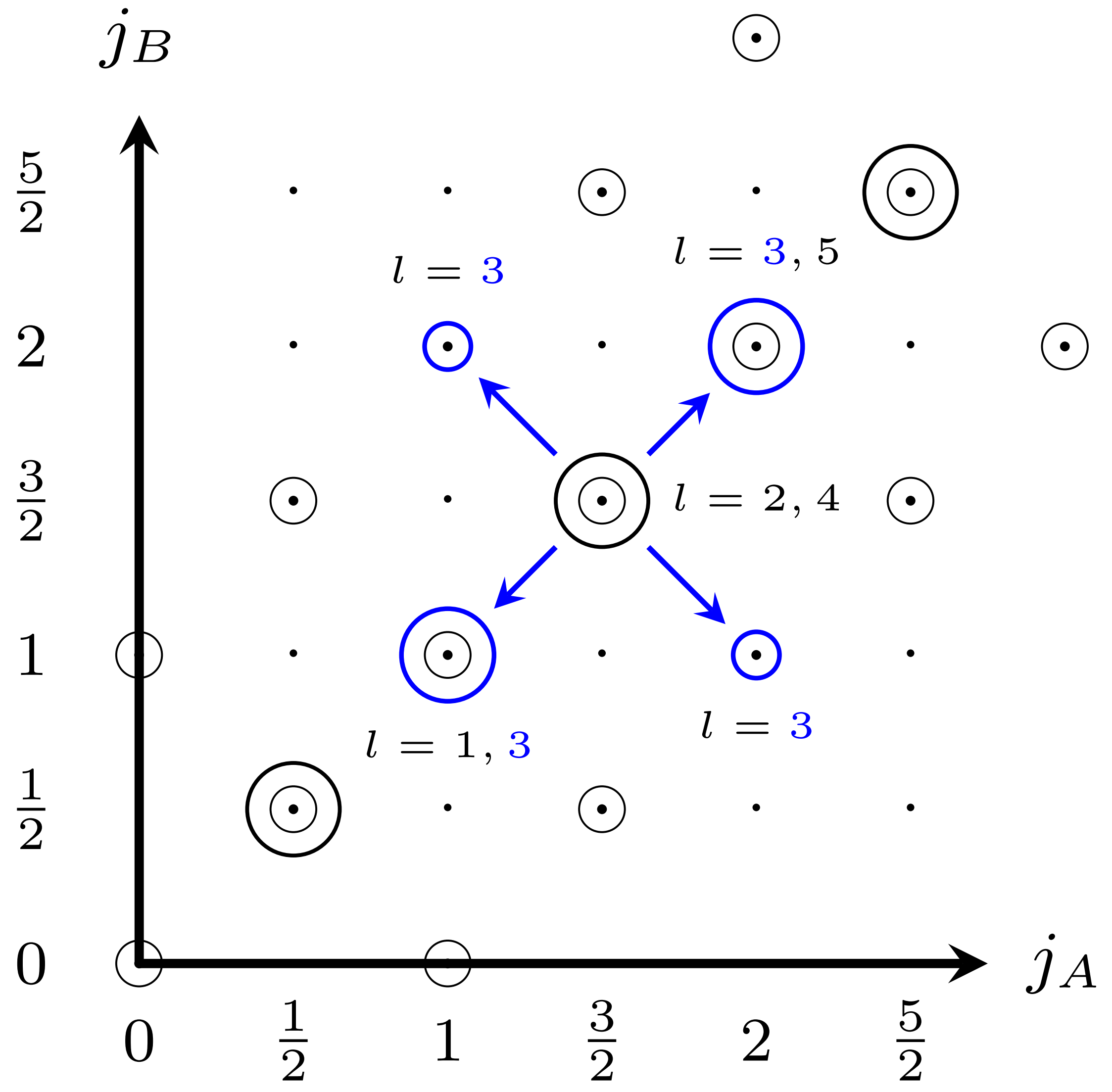
Clebsch-Gordan coefficients

$$\left(\frac{l+1}{2},\frac{l-1}{2}\right)_l \qquad C^{\frac{l+1}{2},M_A}_{\frac{l}{2}m_A\frac{1}{2}\chi_A}\, C^{\frac{l-1}{2},M_B}_{\frac{l}{2}m_B\frac{1}{2}\chi_B}\, Y_{lm_Am_B} V_{\chi_A\chi_B}$$

$$\left(\frac{l-1}{2},\frac{l+1}{2}\right)_l \qquad C^{\frac{l-1}{2},M_A}_{\frac{l}{2}m_A\frac{1}{2}\chi_A}\, C^{\frac{l+1}{2},M_B}_{\frac{l}{2}m_B\frac{1}{2}\chi_B}\, Y_{lm_Am_B} V_{\chi_A\chi_B}$$

$$V^{\mu}_{\chi_A\chi_B} = [\epsilon\sigma^{\mu}]_{\chi_A\chi_B}$$

$$\epsilon = i\sigma^2,\ \sigma^{\mu} = (\boldsymbol{\sigma}, i)$$



Or

$$\left(\frac{l+1}{2},\frac{l-1}{2}\right)_l \qquad \epsilon_{\mu\nu\rho\sigma} W^{(1)\nu}_{lM_Am_A} x^{\rho}\partial^{\sigma} Y_{lm_Am_B} \qquad \begin{array}{l} M_A = -\frac{l+1}{2}, \cdots, \frac{l+1}{2} \\ m_B = -\frac{l}{2}, \cdots, \frac{l-2}{2} \end{array} \quad M_B = m_B + \frac{1}{2}$$

$$W^{(1)\nu}_{lM_Am_A} = C^{\frac{l+1}{2},M_A}_{\frac{l}{2}m_A\frac{1}{2}\chi_A} V^{\nu}_{\chi_A\frac{1}{2}}$$

$$\left(\frac{l-1}{2},\frac{l+1}{2}\right)_l \qquad \epsilon_{\mu\nu\rho\sigma} W^{(2)\nu}_{lM_Bm_B} x^{\rho}\partial^{\sigma} Y_{lm_Am_B} \qquad \begin{array}{l} M_B = -\frac{l+1}{2}, \cdots, \frac{l+1}{2} \\ m_A = -\frac{l}{2}, \cdots, \frac{l-2}{2} \end{array} \quad M_A = m_A + \frac{1}{2}$$

$$W^{(2)\nu}_{lM_Bm_B} = C^{\frac{l+1}{2},M_B}_{\frac{l}{2}m_B\frac{1}{2}\chi_B} V^{\nu}_{\frac{1}{2}\chi_B}$$

Results

[P. Baratella, M. Nemevšek, YS, K. Trailović, L. Ubaldi, '24]

The decay rate is computed using ELVAS

[S. Chigusa, T. Moroi, YS, '17 & '18]

with the correct T-mode degeneracy

Results

$$\log_{10}[\gamma \times \mathrm{Gyr\ Gpc}^3] = -871^{+35+175+209}_{-37-253-330}$$

(Δm_h Δm_t $\Delta \alpha_s$)

$\times 10^6$

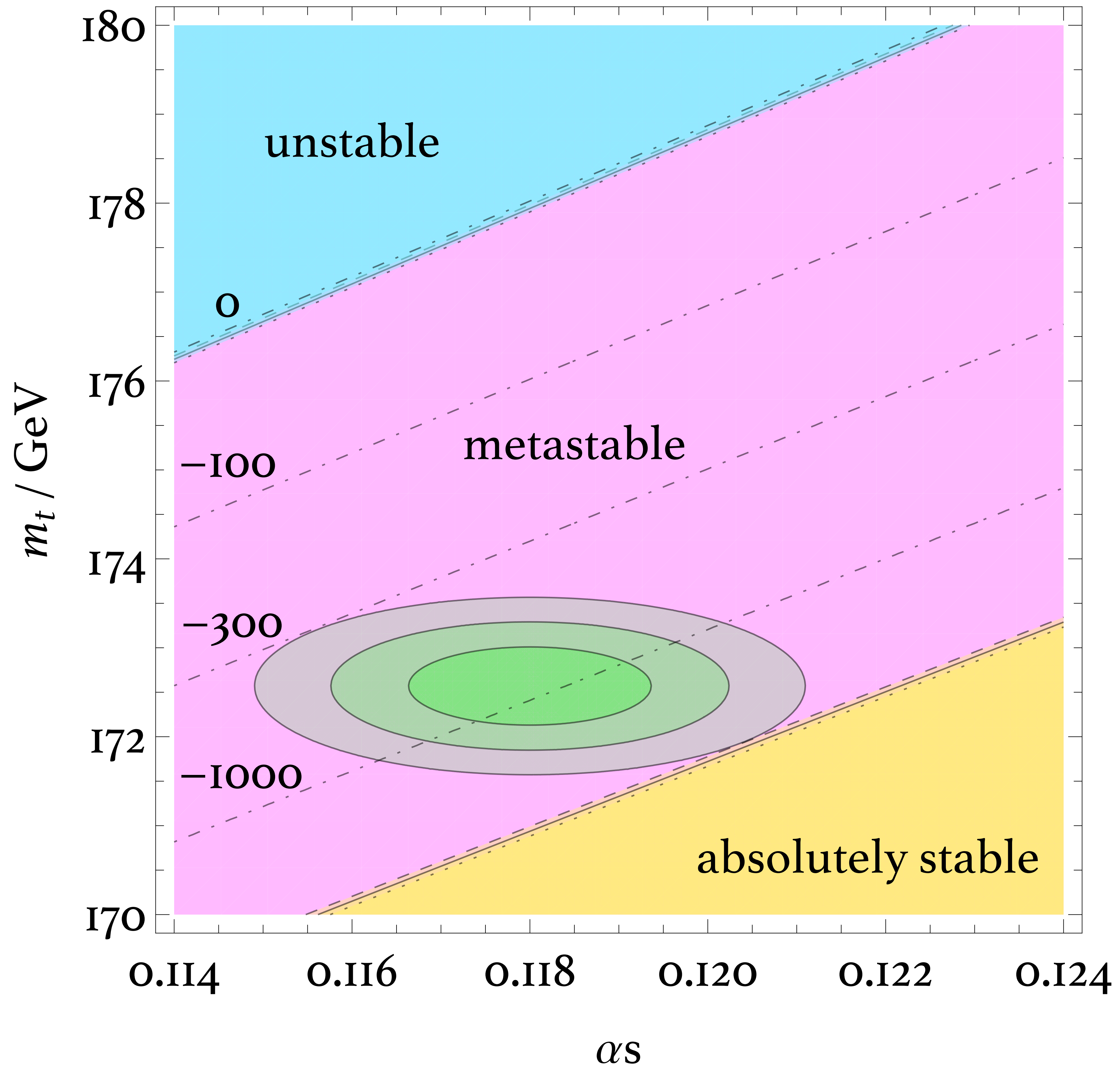
$$\log_{10}[\gamma_{\mathrm{prev}} \times \mathrm{Gyr\ Gpc}^3] = -877$$

SM parameters [PDG 2024]

$$m_h = 125.20 \pm 0.11 \ \mathrm{GeV}$$

$$m_t = 172.57 \pm 0.29 \ \mathrm{GeV}$$

$$\alpha_s = 0.1180 \pm 0.0009$$



Our correction =~6% of gauge boson contribution

Summary

- The computation of the prefactor is important for the precise determination of vacuum decay rates. (Otherwise we would have exp(~70) of uncertainty)
- In the previous studies, the degeneracy of the gauge transverse modes was wrongly counted. In addition, the original “basis” functions were neither independent or complete.
- We provide the correct degeneracy factor and a complete orthogonal basis set. Our discussion is general and applicable to any 4D gauge determinant.
- The vacuum decay rate in the standard model is updated and the change of the rate is about 10^6.