# Complex $S_3$-symmetric 3HDM

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## Motivation for three Higgs doublets

New sources of CP violation in the scalar sector

Possibility of having a discrete symmetry and still have CP violation, explicit or spontaneous

Rich phenomenology, including DM candidates

Why not more? Three fermion generations may suggest three doublets

## Motivation for imposing discrete symmetries

Symmetries reduce the number of free parameters leading to (testable) predictions

Symmetries help control HFCNC (e.g. NFC or MFV suppression in BGL models)

Symmetries are needed to stabilise DM

# Our work

We discuss a three-Higgs-doublet model with an underlying $S_3$ symmetry allowing in principle for complex couplings

We list all possible vacuum structures allowing for CP violation in the scalar sector specifying whether it can be explicit or spontaneous

This classification is based strictly on the exact $S_3$ -symmetric scalar potential without soft symmetry breaking terms

Different regions of parameter space correspond to different vacua with implications that are outlined in our work

In a previous work the  scalar potential with real couplings was studied. In that case CP was explicitly conserved and could only be violated spontaneously for special vacua, which we identified

Emmanuel-Costa, Ogreid, Osland, M. N. R, 2016

# The Scalar potential

**$S_3$ is the permutation group involving three objects,** $\phi_1, \phi_2, \phi_3$

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \mathrm{hc}]$$

$$V_4 = A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{ C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \mathrm{hc}]\}$$
$$+ \frac{1}{2}E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \mathrm{hc}] + \sum_{i \neq j \neq k \neq i, j<k} \{\frac{1}{2} E_2[(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \mathrm{hc}]$$
$$+ \frac{1}{2} E_3[(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \mathrm{hc}] + \frac{1}{2} E_4[(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \mathrm{hc}]\}$$

Derman, 1979

**here all fields appear on equal footing**

**this representation is not irreducible, for instance, the combination**

$$\phi_1 + \phi_2 + \phi_3$$

**remains invariant, it splits into two irreducible representations,**

**doublet and singlet:** $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad h_S$ of $S_3$

# Decomposition into these two irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

This definition does not treat equally $\phi_1, \phi_2, \phi_3$ they could be interchanged

Notice similarity with tribimaximal mixing in the leptonic sector

Harrison, Perkins and Scott, 1999

In our analysis we adopt the singlet-doublet representation of $S_3$

# The scalar potential in the singlet-doublet representation

$$
\begin{aligned}
V_2 =& \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \\
V_4 =& \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\
&+ \left\{ \lambda_4 \left[ (h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) \right] + \text{h.c.} \right\} \\
&+ \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] \\
&+ \left\{ \lambda_7 \left[ (h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) \right] + \text{h.c.} \right\} + \lambda_8 (h_S^\dagger h_S)^2 .
\end{aligned}
$$

Das and Dey, 2014

No symmetry for the interchange of $h_1$ and $h_2$; symmetry for $h_1 \to -h_1$

$\lambda_4$ plays a special rôle

There are two couplings, $\lambda_4$ and $\lambda_7$, that could be complex. Hence, CP symmetry can be broken explicitly. All other couplings have to be real due to the hermiticity of the potential.

Here we are interested in expanding the set of solutions identified and classified previously for the real potential by allowing for complex coefficients.

# Choice of a suitable basis for the analysis of the complex scalar potential

The most general approach of allowing for $\lambda_4$ and $\lambda_7$ to be complex together with two vacuum phases would yield redundant solutions

In principle we could consider a basis with real vevs and complex couplings through:

$$h_i = e^{i\theta_i} h_i', \quad i = \{1, 2\}.$$

however, in this case $(\lambda_2 + \lambda_3)$ would get a phase and the potential would change form

This can be avoided by choosing $\theta_1 = \theta_2 \equiv \theta$ in any rephasing of the Higgs doublets

This phase can be chosen in such a way that either $\lambda_4$ or $\lambda_7$ become real

so that, in general, we are left with two vacuum phases and one complex coupling

We are only interested in cases with non-vanishing phases in the couplings since the cases with spontaneous CP violation were already analysed

It is convenient to choose a basis with $\lambda_4$ the only complex coefficient rather than $\lambda_7$

# Results obtained previously for the real potential

| Vacuum | $\rho_1, \rho_2, \rho_3$ | $w_1, w_2, w_S$ | Comment |
|---|---|---|---|
| R-0 | $0, 0, 0$ | $0, 0, 0$ | Not interesting |
| R-I-1 | $x, x, x$ | $0, 0, w_S$ | $\mu_0^2 = -\lambda_8 w_S^2$ |
| R-I-2a | $x, -x, 0$ | $w, 0, 0$ | $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_1^2$ |
| R-I-2b | $x, 0, -x$ | $w, \sqrt{3}w, 0$ | $\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$ |
| R-I-2c | $0, x, -x$ | $w, -\sqrt{3}w, 0$ | $\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$ |
| R-II-1a | $x, x, y$ | $0, w, w_S$ | $\mu_0^2 = \frac{1}{2}\lambda_4 \frac{w_2^3}{w_S} - \frac{1}{2}\lambda_a w_2^2 - \lambda_8 w_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$ |
| R-II-1b | $x, y, x$ | $w, -w/\sqrt{3}, w_S$ | $\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$<br>$\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$ |
| R-II-1c | $y, x, x$ | $w, w/\sqrt{3}, w_S$ | $\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2,$<br>$\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$ |
| R-II-2 | $x, x, -2x$ | $0, w, 0$ | $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2,\ \lambda_4 = 0$ |
| R-II-3 | $x, y, -x-y$ | $w_1, w_2, 0$ | $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right), \lambda_4 = 0$ |
| R-III | $\rho_1, \rho_2, \rho_3$ | $w_1, w_2, w_S$ | $\mu_0^2 = -\frac{1}{2}\lambda_a(w_1^2 + w_2^2) - \lambda_8 w_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right) - \frac{1}{2}\lambda_a w_S^2,$<br>$\lambda_4 = 0$ |

$$\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7,$$
$$\lambda_b = \lambda_5 + \lambda_6 - 2\lambda_7.$$

# Complex vacua

Table 2: Complex vacua. Notation: $\epsilon = 1$ and $-1$ for C-III-d and C-III-e, respectively; $\xi = \sqrt{-3\sin 2\rho_1/\sin 2\rho_2}$, $\psi = \sqrt{[3+3\cos(\rho_2 - 2\rho_1)]/(2\cos\rho_2)}$. With the constraints of Table 4 the vacua labelled with an asterisk (*) are in fact real.

| | IRF (Irreducible Rep.) | RRF (Reducible Rep.) |
|---|---|---|
| | $w_1, w_2, w_S$ | $\rho_1, \rho_2, \rho_3$ |
| C-I-a | $\hat{w}_1, \pm i\hat{w}_1, 0$ | $x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$ |
| C-III-a | $0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$ | $y, y, xe^{i\tau}$ |
| C-III-b | $\pm i\hat{w}_1, 0, \hat{w}_S$ | $x+iy, x-iy, x$ |
| C-III-c | $\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$ | $xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$ |
| C-III-d,e | $\pm i\hat{w}_1, \epsilon\hat{w}_2, \hat{w}_S$ | $xe^{i\tau}, xe^{-i\tau}, y$ |
| C-III-f | $\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$ | $re^{i\rho} \pm ix, re^{i\rho} \mp ix, \frac{3}{2}re^{-i\rho} - \frac{1}{2}re^{i\rho}$ |
| C-III-g | $\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$ | $re^{-i\rho} \pm ix, re^{-i\rho} \mp ix, \frac{3}{2}re^{i\rho} - \frac{1}{2}re^{-i\rho}$ |
| C-III-h | $\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$ | $xe^{i\tau}, y, y$<br>$y, xe^{i\tau}, y$ |
| C-III-i | $\sqrt{\frac{3(1+\tan^2\sigma_1)}{1+9\tan^2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$,<br>$\pm\hat{w}_2 e^{-i\arctan(3\tan\sigma_1)}, \hat{w}_S$ | $x, ye^{i\tau}, ye^{-i\tau}$<br>$ye^{i\tau}, x, ye^{-i\tau}$ |
| C-IV-a* | $\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$ | $re^{i\rho} + x, -re^{i\rho} + x, x$ |
| C-IV-b | $\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$ | $re^{i\rho} + x, -re^{-i\rho} + x, -re^{i\rho} + re^{-i\rho} + x$ |
| C-IV-c | $\sqrt{1+2\cos^2\sigma_2}\hat{w}_2$,<br>$\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$ | $re^{i\rho} + r\sqrt{3(1+2\cos^2\rho)} + x$,<br>$re^{i\rho} - r\sqrt{3(1+2\cos^2\rho)} + x, -2re^{i\rho} + x$ |
| C-IV-d* | $\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$ | $r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$ |
| C-IV-e | $\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$,<br>$\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$ | $re^{i\rho_2} + re^{i\rho_1}\xi + x, re^{i\rho_2} - re^{i\rho_1}\xi + x$,<br>$-2re^{i\rho_2} + x$ |
| C-IV-f | $\sqrt{2 + \frac{\cos(\sigma_1 - 2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1}$,<br>$\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$ | $re^{i\rho_1} + re^{i\rho_2}\psi + x$,<br>$re^{i\rho_1} - re^{i\rho_2}\psi + x, -2re^{i\rho_1} + x$ |
| C-V* | $\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$ | $xe^{i\tau_1}, ye^{i\tau_2}, z$ |

# Constraints

| Vacuum | Constraints |
|---|---|
| C-I-a | $\mu_1^2 = -2\left(\lambda_1 - \lambda_2\right)\hat{w}_1^2$ |
| C-III-a | $\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$<br>$\lambda_4 = \frac{4\cos\sigma_2 \hat{w}_S}{\hat{w}_2}\lambda_7$ |
| C-III-b | $\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\lambda_b \hat{w}_S^2,$<br>$\lambda_4 = 0$ |
| C-III-c | $\mu_1^2 = -(\lambda_1 + \lambda_3)(\hat{w}_1^2 + \hat{w}_2^2),$<br>$\lambda_2 + \lambda_3 = 0, \lambda_4 = 0$ |
| C-III-d,e | $\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{(\hat{w}_1^2 - \hat{w}_2^2)^2}{\hat{w}_S^2} - \epsilon\lambda_4 \frac{(\hat{w}_1^2 - \hat{w}_2^2)(\hat{w}_1^2 - 3\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}$<br>$-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \epsilon\lambda_4 \frac{\hat{w}_S(\hat{w}_1^2 - \hat{w}_2^2)}{4\hat{w}_2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_7 = \frac{\hat{w}_1^2 - \hat{w}_2^2}{\hat{w}_S^2}(\lambda_2 + \lambda_3) - \epsilon\frac{(\hat{w}_1^2 - 5\hat{w}_2^2)}{4\hat{w}_2\hat{w}_S}\lambda_4$ |
| C-III-f,g | $\mu_0^2 = -\frac{1}{2}\lambda_b\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\lambda_b \hat{w}_S^2, \lambda_4 = 0$ |
| C-III-h | $\mu_0^2 = -2\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$<br>$\lambda_4 = \mp\frac{2\cos\sigma_2 \hat{w}_S}{\hat{w}_2}\lambda_7$ |
| C-III-i | $\mu_0^2 = \frac{16\left(1 - 3\tan^2\sigma_1\right)^2}{(1 + 9\tan^2\sigma_1)^2}(\lambda_2 + \lambda_3)\frac{\hat{w}_2^4}{\hat{w}_S^2} \pm \frac{6\left(1 - \tan^2\sigma_1\right)(1 - 3\tan^2\sigma_1)}{(1 + 9\tan^2\sigma_1)^{\frac{3}{2}}}\lambda_4 \frac{\hat{w}_2^3}{\hat{w}_S}$<br>$-\frac{2(1 + 3\tan^2\sigma_1)}{1 + 9\tan^2\sigma_1}(\lambda_5 + \lambda_6)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\frac{4(1 + 3\tan^2\sigma_1)}{1 + 9\tan^2\sigma_1}(\lambda_1 - \lambda_2)\hat{w}_2^2 \mp \frac{\left(1 - 3\tan^2\sigma_1\right)}{2\sqrt{1 + 9\tan^2\sigma_1}}\lambda_4 \hat{w}_2 \hat{w}_S$<br>$-\frac{1}{2}(\lambda_5 + \lambda_6)\hat{w}_S^2,$<br>$\lambda_7 = -\frac{4\left(1 - 3\tan^2\sigma_1\right)\hat{w}_2^2}{(1 + 9\tan^2\sigma_1)\hat{w}_S^2}(\lambda_2 + \lambda_3) \mp \frac{\left(5 - 3\tan^2\sigma_1\right)\hat{w}_2}{2\sqrt{1 + 9\tan^2\sigma_1}\hat{w}_S}\lambda_4$ |

| Vacuum | Constraints |
|---|---|
| C-IV-a* | $\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_4 = 0, \lambda_7 = 0$ |
| C-IV-b | $\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)^2}{\hat{w}_S^2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_4 = 0, \lambda_7 = -\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$ |
| C-IV-c | $\mu_0^2 = 2\cos^2\sigma_2\left(1 + \cos^2\sigma_2\right)\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$<br>$-\left(1 + \cos^2\sigma_2\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left[2\left(1 + \cos^2\sigma_2\right)\lambda_1 - \left(2 + 3\cos^2\sigma_2\right)\lambda_2 - \cos^2\sigma_2\lambda_3\right]\hat{w}_2^2$<br>$-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_4 = -\frac{2\cos\sigma_2 \hat{w}_2}{\hat{w}_S}\left(\lambda_2 + \lambda_3\right), \lambda_7 = \frac{\cos^2\sigma_2 \hat{w}_2^2}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$ |
| C-IV-d* | $\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_4 = 0, \lambda_7 = 0$ |
| C-IV-e | $\mu_0^2 = \frac{\sin^2(2(\sigma_1 - \sigma_2))}{\sin^2(2\sigma_1)}\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$<br>$-\frac{1}{2}\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_1 - \lambda_2\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_4 = 0, \lambda_7 = -\frac{\sin(2(\sigma_1 - \sigma_2))\hat{w}_2^2}{\sin 2\sigma_1 \hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$ |
| C-IV-f | $\mu_0^2 = -\frac{(\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1)\cos(\sigma_2 - \sigma_1)}{2\cos^2\sigma_1}\lambda_4 \frac{\hat{w}_2^3}{\hat{w}_S}$<br>$-\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{2\cos\sigma_1}\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{\cos\sigma_1}\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2$<br>$-\frac{3\cos 2\sigma_1 + 2\cos(2(\sigma_1 - \sigma_2)) + \cos 2\sigma_2 + 4}{4\cos(\sigma_1 - \sigma_2)\cos\sigma_1}\lambda_4 \hat{w}_2 \hat{w}_S - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_2 + \lambda_3 = -\frac{\cos\sigma_1 \hat{w}_S}{2\cos(\sigma_2 - \sigma_1)\hat{w}_2}\lambda_4, \lambda_7 = -\frac{\cos(\sigma_2 - \sigma_1)\hat{w}_2}{2\cos\sigma_1 \hat{w}_S}\lambda_4$ |
| C-V* | $\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$<br>$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$<br>$\lambda_2 + \lambda_3 = 0, \lambda_4 = 0, \lambda_7 = 0$ |

# Complex vacua, Spontaneous CP Violation

Table 1: Spontaneous CP violation

| Vacuum | $\lambda_4$ | SCPV | Vacuum | $\lambda_4$ | SCPV | Vacuum | $\lambda_4$ | SCPV |
|---|---|---|---|---|---|---|---|---|
| C-I-a | X | no | C-III-f,g | 0 | no | C-IV-c | X | yes |
| C-III-a | X | yes | C-III-h | X | yes | C-IV-d | 0 | no |
| C-III-b | 0 | no | C-III-i | X | no | C-IV-e | 0 | no |
| C-III-c | 0 | no | C-IV-a | 0 | no | C-IV-f | X | yes |
| C-III-d,e | X | no | C-IV-b | 0 | no | C-V | 0 | no |

- C-III-a $(0,\ \hat{w}_2 e^{i\sigma_2},\ \hat{w}_S)$;
- C-III-h $(\sqrt{3}\hat{w}_2 e^{i\sigma_2},\ \pm\hat{w}_2 e^{i\sigma_2},\ \hat{w}_S)$;
- C-IV-c $\left(\sqrt{1+2\cos^2\sigma_2}\,\hat{w}_2,\ \hat{w}_2 e^{i\sigma_2},\ \hat{w}_S\right)$;
- C-IV-f $\left(\sqrt{2+\frac{\cos(\sigma_1-2\sigma_2)}{\cos\sigma_1}}\,\hat{w}_2 e^{i\sigma_1},\ \hat{w}_2 e^{i\sigma_2},\ \hat{w}_S\right)$;

# Coming back to the complex potential

# Explicit CP violation

Compact notation:

$$V_2 = Y_{ab}\left(h_a^\dagger h_b\right),$$
$$V_4 = \frac{1}{2} Z_{abcd}\left(h_a^\dagger h_b\right)\left(h_c^\dagger h_d\right),$$

Branco,Lavoura, Silva 1999

$$
\begin{aligned}
Y_{11} &= Y_{22} = \mu_1^2, & Y_{33} &= \mu_0^2,\\
Z_{1111} &= Z_{2222} = 2\lambda_1 + 2\lambda_3, & Z_{3333} &= 2\lambda_8,\\
Z_{1122} &= Z_{2211} = 2\lambda_1 - 2\lambda_3, & Z_{1133} &= Z_{2233} = Z_{3311} = Z_{3322} = \lambda_5,\\
Z_{1221} &= Z_{2112} = -2\lambda_2 + 2\lambda_3, & Z_{1331} &= Z_{2332} = Z_{3113} = Z_{3223} = \lambda_6,\\
Z_{1212} &= Z_{2121} = 2\lambda_2 + 2\lambda_3, & Z_{1313} &= Z_{2323} = Z_{3131} = Z_{3232} = 2\lambda_7,
\end{aligned}
$$

$$Z_{1123} = Z_{1213} = Z_{1312} = Z_{1321} = Z_{2113} = Z_{2311} = -Z_{2223} = -Z_{2322} = \lambda_4^{\mathrm{R}} - i\lambda_4^{\mathrm{I}},$$
$$Z_{1132} = Z_{1231} = Z_{2131} = Z_{3112} = Z_{3121} = Z_{3211} = -Z_{2232} = -Z_{3222} = \lambda_4^{\mathrm{R}} + i\lambda_4^{\mathrm{I}}.$$

# Explicit CP violation

**Powerful and elegant tool: CP odd Higgs basis invariants built from Y- and Z- tensors**

See references [65-71] in our paper

$$\mathrm{I}_{5Z}^{(1)} = \mathbb{I}\mathrm{m}\left[Z_{aabc}Z_{dbef}Z_{cghe}Z_{idgh}Z_{fijj}\right],$$
$$\mathrm{I}_{5Z}^{(2)} = \mathbb{I}\mathrm{m}\left[Z_{abbc}Z_{daef}Z_{cghe}Z_{idgh}Z_{fjji}\right],$$
$$\mathrm{I}_{6Z}^{(1)} = \mathbb{I}\mathrm{m}\left[Z_{abcd}Z_{baef}Z_{gchi}Z_{djke}Z_{fkil}Z_{jglh}\right],$$
$$\mathrm{I}_{6Z}^{(2)} = \mathbb{I}\mathrm{m}\left[Z_{abcd}Z_{baef}Z_{gchi}Z_{dejk}Z_{fhkl}Z_{lgij}\right],$$
$$\mathrm{I}_{7Z} = \mathbb{I}\mathrm{m}\left[Z_{abcd}Z_{eafc}Z_{bgdh}Z_{iejk}Z_{gflm}Z_{hlkn}Z_{minj}\right],$$

$$\mathrm{I}_{2Y3Z} = \mathbb{I}\mathrm{m}\left[Z_{abcd}Z_{befg}Z_{dchf}Y_{ga}Y_{eh}\right].$$

Complex computation due to high number of contraction of indices requiring special simplification techniques!

Anton Kunčinas Odd Magne Ogreid

# Explicit CP violation

**Theorem 1.** *The quadrilinear part of the $S_3$-symmetric 3HDM potential, $V_4$, explicitly conserves CP if and only if* $\mathrm{I}_{5Z}^{(1)} = \mathrm{I}_{5Z}^{(2)} = \mathrm{I}_{6Z}^{(1)} = \mathrm{I}_{6Z}^{(2)} = \mathrm{I}_{7Z} = 0$.

- **Solution 0:** $\lambda_4^{\mathrm{I}} = 0$;
- **Solution 1:** $\lambda_4^{\mathrm{R}} = 0$;
- **Solution 2:** $\lambda_7 = 0$;
- **Solution 3 ($\boldsymbol{\lambda_4^{\mathrm{R}} \lambda_4^{\mathrm{I}} \lambda_7 \neq 0}$):**

$$\left(\lambda_4^{\mathrm{R}}\right)^2 = -\frac{(\lambda_{23} - \lambda_7)(2\lambda_{23} + \lambda_7)^2}{\lambda_7},$$

$$\left(\lambda_4^{\mathrm{I}}\right)^2 = \frac{(\lambda_{23} + \lambda_7)(2\lambda_{23} - \lambda_7)^2}{\lambda_7},$$

$$\lambda_{23} \equiv \lambda_2 + \lambda_3$$

$$\lambda_5 = 2\left(\lambda_1 + \lambda_2\right),$$

$$\lambda_6 = 4\lambda_3,$$

$$\lambda_8 = \lambda_1 - \lambda_2.$$

For each of these solutions we were able to show that there exists a real basis for $V_4$

# Explicit CP violation

**Theorem 2.** *The $S_3$-symmetric 3HDM potential, $V = V_2 + V_4$, explicitly conserves CP if and only if* $\mathrm{I}^{(1)}_{5Z} = \mathrm{I}^{(2)}_{5Z} = \mathrm{I}^{(1)}_{6Z} = \mathrm{I}^{(2)}_{6Z} = \mathrm{I}_{7Z} = \mathrm{I}_{2Y3Z} = 0$.

- **Solution $3'$ ($\boldsymbol{\lambda_4^{\mathrm{R}}\lambda_4^{\mathrm{I}}\lambda_7 \neq 0}$):**

$$\mu_1^2 = \mu_0^2,$$
$$\left(\lambda_4^{\mathrm{R}}\right)^2 = -\frac{(\lambda_{23}-\lambda_7)(2\lambda_{23}+\lambda_7)^2}{\lambda_7},$$
$$\left(\lambda_4^{\mathrm{I}}\right)^2 = \frac{(\lambda_{23}+\lambda_7)(2\lambda_{23}-\lambda_7)^2}{\lambda_7},$$

$$\lambda_{23} \equiv \lambda_2 + \lambda_3$$
$$\lambda_5 = 2\left(\lambda_1+\lambda_2\right),$$
$$\lambda_6 = 4\lambda_3,$$
$$\lambda_8 = \lambda_1 - \lambda_2.$$

For each of the solutions we were able to show that there exists a real basis for V

No additional continuous symmetries for solution $\mathbf{3}'$

de Medeiros Varzielas, Ivanov 2019

The potential has the structure of the $\Delta(54)$-symmetric

For the general 3HDM, the necessary and sufficient set of CP-odd invariants needed for explicit CP conservation has not yet been identified

# Summary of different CP violating models

| Scalar potential | Vacuum | vevs | CPV | $\mathcal{L}_Y$ |
|---|---|---|---|---|
| complex | R-I-1 | $(0,\, 0,\, w_S)$ | explicit | trivial |
| complex | R-I-2a | $(w_1,\, 0,\, 0)$ | explicit | - |
| complex | R-I-2b,c | $(w_1,\, \pm\sqrt{3}w_1,\, 0)$ | explicit | - |
| complex | C-I-a | $(\hat{w}_1,\, \pm i\hat{w}_1,\, 0)$ | explicit | - |
| complex<br>real | C-III-a | $(0,\, \hat{w}_2 e^{i\sigma_2},\, \hat{w}_S)$ | explicit<br>spontaneous | trivial |
| complex<br>real | C-III-h | $(\sqrt{3}\hat{w}_2 e^{i\sigma_2},\, \pm\hat{w}_2 e^{i\sigma_2},\, \hat{w}_S)$ | explicit<br>spontaneous | trivial |
| real$^{\alpha}$ | C-IV-c | $\left(\sqrt{1+2\cos^2\sigma_2}\hat{w}_2,\, \hat{w}_2 e^{i\sigma_2},\, \hat{w}_S\right)$ | spontaneous | any |
| real$^{\alpha}$ | C-IV-f | $\left(\sqrt{2+\frac{\cos(\sigma_1-2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1},\, \hat{w}_2 e^{i\sigma_2},\, \hat{w}_S\right)$ | spontaneous | any |
| complex$^{\beta}$ | C-IV-g | $(\hat{w}_1 e^{i\sigma_1},\, \pm i\hat{w}_1 e^{i\sigma_1},\, \hat{w}_S)$ | explicit | any |
| complex | C-V | $(\hat{w}_1 e^{i\sigma_1},\, \hat{w}_2 e^{i\sigma_2},\, \hat{w}_S)$ | explicit | any |

$^{\alpha}$ In C-IV-c and C-IV-f there is a massless scalar present. Soft symmetry breaking would remove the massless scalar.

$^{\beta}$ C-IV-g results in at least two negative mass-squared eigenvalues. Introduction of soft symmetry breaking terms might solve the issue.

It is possible to have CP violation without breaking $S_3$ (see R-I-1)

entries with “-“ indicate that it is not possible to generate realistic masses and mixing

| | |
|---|---|
| R-I-1 | there is a pair of charged mass degenerate states and two pairs of neutral mass-degenerate states |
| C-III-a | realistic masses and mixing require the fermions to transform trivially under the symmetry and require complex Yukawa couplings. Has a viable DM candidate for a real potential |
| C-III-h | realistic masses and mixing require the fermions to transform trivially under the symmetry and require complex Yukawa couplings |
| C-IV-c | possible to fit both fermion masses and the CKM matrix however, there is an accidental massless scalar state in the model |
| C-IV-f | this vacuum is a generalisation of C-IV-c but a massless scalar state is also present |
| C-IV-g | possible to fit both fermion masses and mixing however, there are negative mass-squared scalars |
| C-V | possible to fit both fermion masses and the CKM matrix; can also yield a realistic scalar sector. Remarkable possibility of having light neutral scalars of order a few Mev escaping detection. More details in our paper. |

# Potentially realistic models with real Yukawa couplings

C-IV-c  C-IV-f  C-IV-g  C-V

only C-V survives without the need for soft breaking terms due to unrealistic scalar spectrum

A numerical study of C-V was performed fitting several parameters

- Masses of the up- and down-quarks;
- The absolute values, arguments of the unitarity triangle ($\alpha$, $\sin 2\beta$, $\gamma$) and independent measure of CP violation ($J$) [89, 90] of the CKM matrix;
- Interactions of the SM-like Higgs boson with fermions. We assume the Higgs boson signal strength in the $b$-quark channel [91–93] as a reference point and apply the corresponding limits to other channels;
- Suppressed scalar mediated FCNC [94, 95];
- CP properties of the SM-like Higgs boson [96, 97];
- Upper limit on the decay of the $t$-quark into lighter charged scalars when decays are not kinematically suppressed [98, 99];

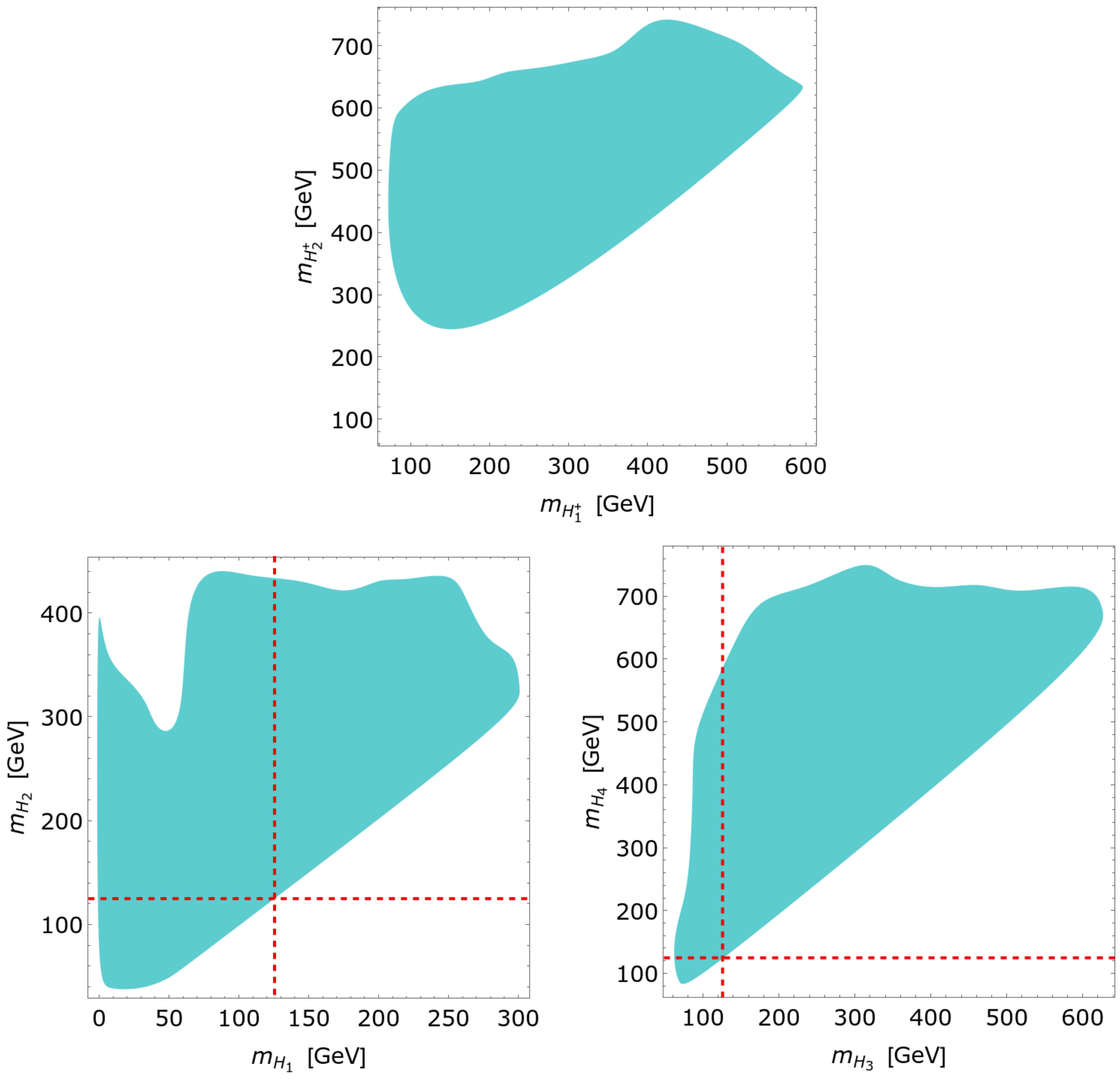


**Figure 2**. Scatter plots of masses that satisfy constraints in the C-V model. Top: the charged sector, $H_i^\pm$. Bottom: the active sector, $H_i$. In the neutral sector the red line indicates a 125 GeV state.

# Conclusions

Many interesting aspects of the models presented here remain to be analysed

Potential DM candidates exist as was shown in previous works of ours

Khater, Kunčinas, Ogreid, Osland, MNR, 2021 Kunčinas, Ogreid, Osland, MNR, 2022

Many important studies of 3HDM have appeared in the literature, and several of them are cited in our paper.

Still many important questions remain open

Multi-Higgs models are at present a fertile ground of research

The LHC may bring important news for this field in the near future

Back-up slide

We have the following $S_3$ doublets:

$$\begin{pmatrix}\bar{Q}_1\\ \bar{Q}_2\end{pmatrix}_L, \quad \begin{pmatrix}u_1\\ u_2\end{pmatrix}_R, \quad \begin{pmatrix}d_1\\ d_2\end{pmatrix}_R, \quad \begin{pmatrix}h_1 & h_2\end{pmatrix}$$

and singlets:

$$\bar{Q}_{3L}, \quad u_{3R}, \quad d_{3R}, \quad h_S,$$

where indices 1,2,3 on quark fields $\bar{Q}, u, d$ label the families. Mass terms arise from the following generic structures: $\bar{Q}_L \phi d_R$ or $\bar{Q}_L \tilde{\phi} u_R$, where $\phi$ and $\tilde{\phi} = -i[\phi^\dagger \sigma_2]^T$ are scalar SU(2) doublets.

As a result, the mass matrix will have the structure

$$\mathcal{M} = \begin{pmatrix} y_1^d w_S + y_2^d w_2 & y_2^d w_1 & y_4^d w_1 \\ y_2^d w_1 & y_1^d w_S - y_2^d w_2 & y_4^d w_2 \\ y_5^d w_1 & y_5^d w_2 & y_3^d w_S \end{pmatrix}$$