# Dynamical Instabilities in the Generalized SU(2) Proca Theory: Challenges for Cosmic Acceleration

Santiago García-Serna

GRECO

Facultad de Ciencias Naturales y Exactas
Universidad del Valle

# Generalized SU(2) Proca Theories

$$S \equiv \int \mathrm{d}x^4 \sqrt{-g}\left(\mathcal{L}_{\rm EH} + \mathcal{L}_{YM} + \sum_{i=1}^{2} \chi_i \mathcal{L}_2^i + \sum_{i=3}^{7} \frac{\chi_i}{m_{\rm P}^2}\mathcal{L}_2^i + \sum_{i=1}^{6} \frac{\alpha_i}{m_{\rm P}^2}\mathcal{L}_{4,2}^i + \mathcal{L}_m + \mathcal{L}_r\right)$$

$$\mathcal{L}_{\rm EH} \equiv \frac{m_{\rm P}^2}{2} R, \quad \mathcal{L}_{\rm YM} \equiv -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}, \quad \mathcal{L}_2^1 \equiv (B^a \cdot B_a)\left(B^b \cdot B_b\right),$$

$$\mathcal{L}_2^2 \equiv \left(B^a \cdot B^b\right)(B_a \cdot B_b), \quad \mathcal{L}_2^3 \equiv A_a^{\mu\nu} A^\rho{}_\nu{}^a B_\mu^b B_{\rho b}, \quad \mathcal{L}_2^4 \equiv A_a^{\mu\nu} A^\rho{}_\nu{}^b B_{\mu b} B_\rho^a,$$

$$\mathcal{L}_2^5 \equiv A_a^{\mu\nu} A^\rho{}_\nu{}^b B_\mu^a B_{\rho b}, \quad \mathcal{L}_2^6 \equiv A_a^{\mu\nu} A_{\mu\nu}^a \left(B_b \cdot B^b\right), \quad \mathcal{L}_2^7 \equiv A_a^{\mu\nu} A_{\mu\nu}^b \left(B^a \cdot B_b\right),$$

$$\mathcal{L}_{4,2}^1 \equiv \left(B_b \cdot B^b\right)\left[S_\mu^{\mu a} S_{\nu a}^\nu - S_\nu^{\mu a} S_{\mu a}^\nu\right] + 2(B_a \cdot B_b)\left[S_\mu^{\mu a} S_\nu^{\nu b} - S_\nu^{\mu a} S_\mu^{\nu b}\right],$$

$$\mathcal{L}_{4,2}^2 \equiv A_{\mu\nu}^a S_\sigma^{\mu b} B_a^\nu B_b^\sigma - A_{\mu\nu}^a S_\sigma^{\mu b} B_b^\nu B_a^\sigma + A_{\mu\nu}^a S_\rho^{\rho b} B_a^\mu B_b^\nu,$$

$$\mathcal{L}_{4,2}^3 \equiv B^{\mu a} R^\alpha{}_{\sigma\rho\mu} B_{\alpha a} B^{\rho b} B_b^\sigma + \frac{3}{4}\left(B_b \cdot B^b\right)(B^a \cdot B_a) R,$$

$$\mathcal{L}_{4,2}^4 \equiv \left[\left(B_b \cdot B^b\right)(B^a \cdot B_a) + 2(B_a \cdot B_b)\left(B^a \cdot B^b\right)\right] R,$$

$$\mathcal{L}_{4,2}^5 \equiv G_{\mu\nu} B^{\mu a} B_a^\nu \left(B^b \cdot B_b\right), \quad \mathcal{L}_{4,2}^6 \equiv G_{\mu\nu} B^{\mu a} B^{\nu b} (B_a \cdot B_b).$$

$$\begin{aligned} \alpha_2 &= 2\alpha_3, \\ \alpha_6 &= -20\alpha_1 + 6\alpha_3 - 3\alpha_5, \\ \chi_3 &= 0, \\ \alpha_4 &= -2\alpha_1 + \frac{7}{20}\alpha_3, \\ \alpha_5 &= -\frac{20\alpha_1 - 14\alpha_3}{3}, \\ \chi_7 &= 5\alpha_1 + \alpha_3 - \frac{\chi_4}{2} - 3\chi_6, \end{aligned}$$

Tensor perturbations:

**Constant-Roll Inflation in the Generalized SU(2) Proca Theory**

Juan C. Garnica (Santander Industrial U.), L. Gabriel Gomez (Santiago de Chile U.), Andres A. Navarro (Santo Tomas U.), Yeinzon Rodriguez (Santander Industrial U. and Antonio Narino U.)
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We consider the FLRW background $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j$, and the configuration for the vector field $B_{0a}(t) = 0, \quad B_{ia}(t) = a(t)\psi(t)\delta_{ia}$ and $\hat{g} \equiv \sqrt{\tilde{g}^2 - 6\chi_1 - 2\chi_2}$.

# Considering the tensor perturbations and other simplifications

Tensor Perturbations

$$
\begin{aligned}
\alpha_2 &= 2\alpha_3\,,\\
\alpha_6 &= -20\alpha_1 + 6\alpha_3 - 3\alpha_5\,,\\
\chi_3 &= 0\,,\\
\alpha_4 &= -2\alpha_1 + \frac{7}{20}\alpha_3\,,\\
\alpha_5 &= -\frac{20\alpha_1 - 14\alpha_3}{3}\,,\\
\chi_7 &= 5\alpha_1 + \alpha_3 - \frac{\chi_4}{2} - 3\chi_6\,,
\end{aligned}
$$

Useful definitions

$$
\alpha_3 \to \frac{-c_1 + c_2 + 20\alpha_1}{20}, \qquad \chi_5 \to \frac{c_1 + 9c_2 - 20\alpha_1}{10}
$$

The Equation of Motion

$$
m_{\rm P}^2 G_{\mu\nu} = T^{(B)}_{\mu\nu} + T^{(m)}_{\mu\nu},
$$

$$
\begin{aligned}
3m_{\rm P}^2 H^2 &= \rho_B + \rho_m,\\
-2m_{\rm P}^2 \dot{H} &= p_B + \rho_B + \rho_m,
\end{aligned}
$$

Where the density and the pressure are

$$
\begin{aligned}
\rho_B \equiv& \left(\dot\psi + H\psi\right)^2 \left[\frac{3}{2} - 9c_2\frac{\psi^2}{m_{\rm P}^2}\right] + \frac{3}{2}\hat{g}^2\psi^4\\
&+ 6H(c_1 - c_2)\frac{\psi^3\dot\psi}{m_{\rm P}^2},\\
p_B \equiv& \left(\dot\psi + H\psi\right)^2 \left[\frac{1}{2} + 3c_2\frac{\psi^2}{m_{\rm P}^2}\right] + \frac{1}{2}\hat{g}^2\psi^4\\
&+ 6\frac{\psi^2}{m_{\rm P}^2}(c_2 - c_1)\left\{\dot\psi^2 - \psi^2\left(H^2 + \frac{\dot H}{3}\right) + \frac{1}{3}\psi\ddot\psi\right\},
\end{aligned}
$$

The EOM of the field is

The Lagrangian

$$
\begin{aligned}
0 =& \ddot\psi + 3H\dot\psi + \psi\left(2H^2 + \dot H - 6c_2\frac{\dot\psi^2}{m_{\rm P}^2}\right)\\
&+ 2\psi^3\left[\hat{g}^2 + 3\left(c_1 - 2c_2\right)\frac{H^2}{m_{\rm P}^2} + \left(c_1 - 4c_2\right)\frac{\dot H}{m_{\rm P}^2}\right]\\
&- 6c_2\frac{\psi^2}{m_{\rm P}^2}\left(\ddot\psi + 3H\dot\psi\right).
\end{aligned}
$$

$$
\frac{\chi_4}{m_{\rm P}^2}\left(\mathcal{L}_2^4 - \frac{\mathcal{L}_2^7}{2}\right) \quad \ldots - \frac{20}{3}\mathcal{L}_{4,2}^5 + 5\mathcal{L}_2^7\Big) \quad \ldots + \mathcal{L}_2^7\Big)\Big].
$$

# Dynamical system approach

Dynamical variables

$$x \equiv \frac{\dot{\psi}}{\sqrt{2}m_{\rm P}H}, \quad y \equiv \frac{\psi}{\sqrt{2}m_{\rm P}}, \quad z \equiv \sqrt{\frac{\hat{g}}{2m_{\rm P}H}}\psi. \qquad \hat{g} \equiv \sqrt{\tilde{g}^2 - 6\chi_1 - 2\chi_2}. \qquad p \equiv \frac{\ddot{\psi}}{m_{\rm P}H}, \quad \epsilon \equiv -\frac{\dot{H}}{H^2}$$

The EOM

$$1 = (x+y)^2(1-12c_2y^2) + 8(c_1-c_2)xy^3 + 2z^4,$$

$$\epsilon = 2 + 12c_1y^4 - 4y^3(c_1-c_2)\left(\frac{p}{\sqrt{2}} + \epsilon y\right) - 4(c_1-7c_2)xy^3 - 12(c_1-2c_2)x^2y^2,$$

$$\frac{p}{\sqrt{2}} = 2y^2\left(2x(4c_1-7c_2) + 3\sqrt{2}c_2p\right) + \frac{2(x^2-1)}{y} + y\left(\epsilon - 12c_2x^2\right) + x + 4y^3(c_1\epsilon - 3c_1 - 4c_2\epsilon).$$

Dynamical system

$$x' = \frac{p}{\sqrt{2}} + x\epsilon, \quad y' = x$$

We neglect the matter contribution in order to study the epochs where the universe experiments accelerated expansion.

# Fixed points and stability

$$x' = \frac{p}{\sqrt{2}} + x\epsilon, \quad y' = x$$

The fix

$A_\pm = \Big\{ x$ ... $\frac{1}{2}\sqrt{\frac{1}{6c_2} - }$ ... $\frac{\sqrt{1-48c_2}}{6c_2} \Big\}.$

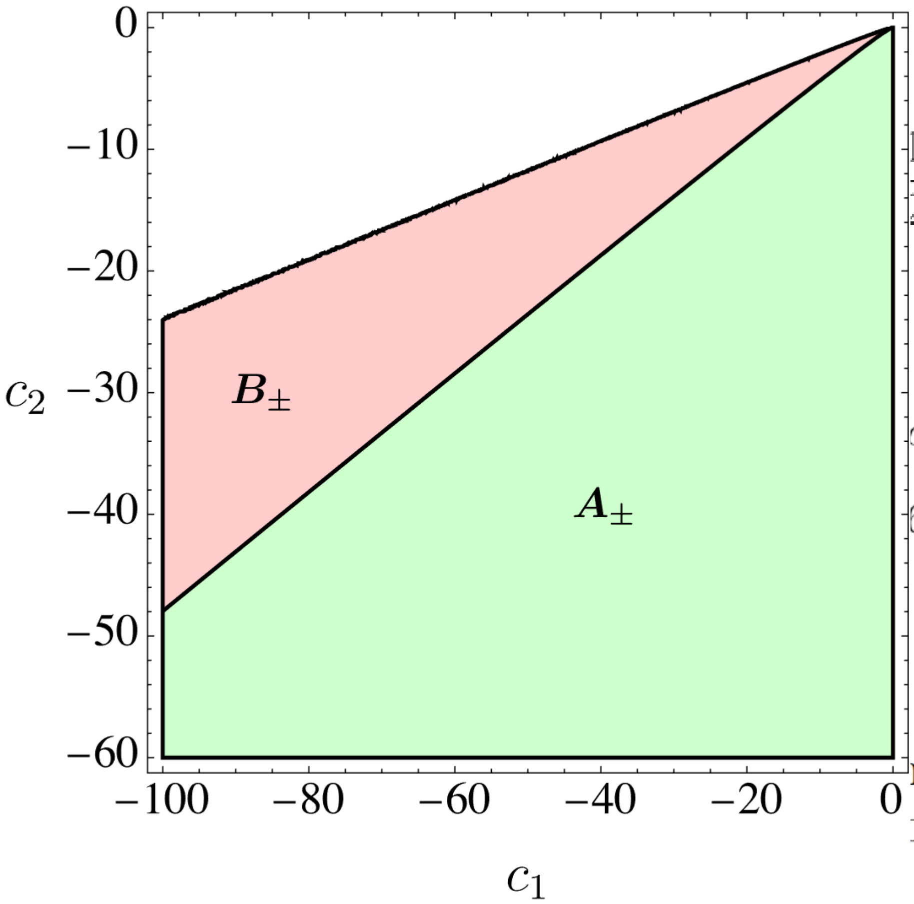
$A_\pm \in$ ... $0 < c_2 \leq$

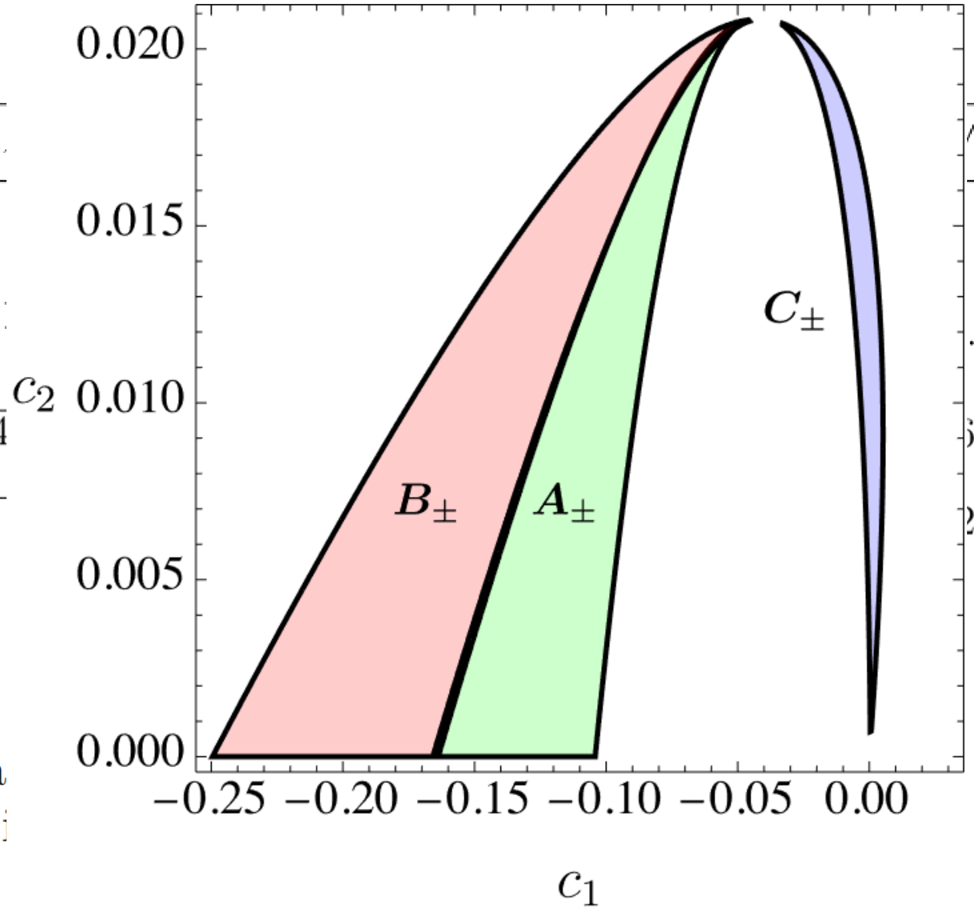
$w_B$ ... $c_1 + \sqrt{1-4}$ ... $6c_2 < 1 + 16c_1$

$6c_1 + \sqrt{1-}$ ... $2c_2 > 1 + 8c_1.$

Stabil ... $_{1,2}) > 0.$

$J_{ij} \equiv$ ... he eigenva ... other one i



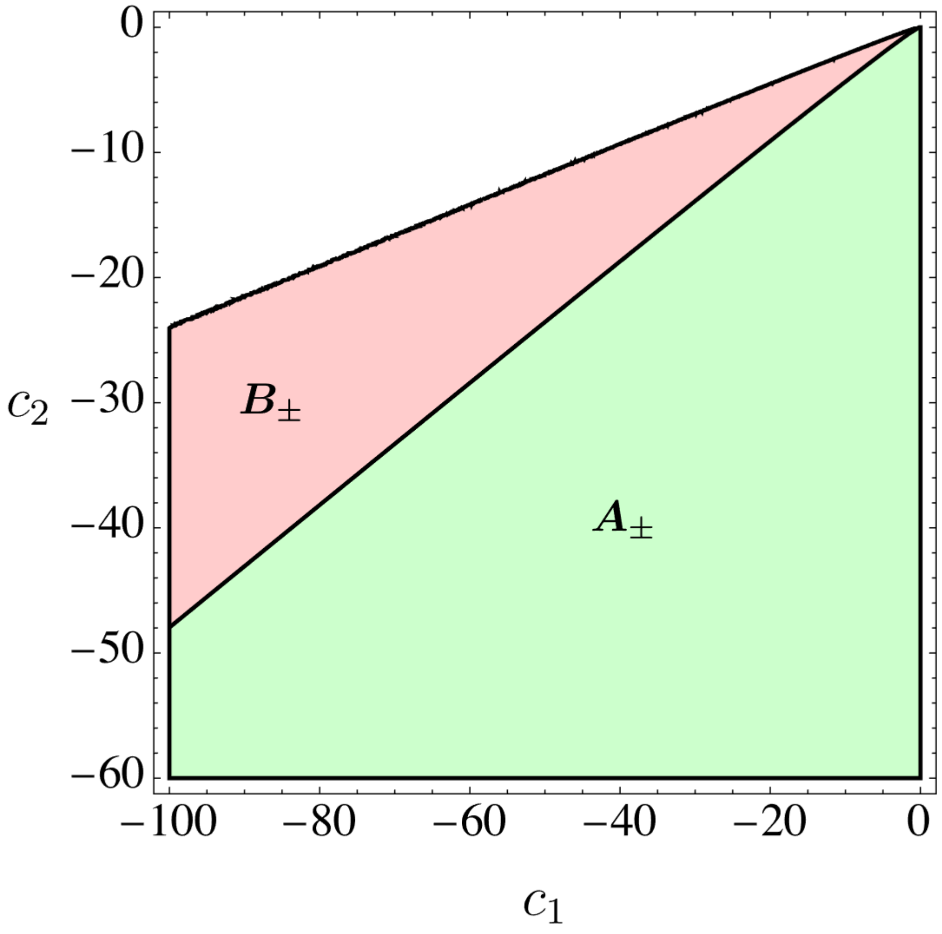




- An attractor (or sink) if $\mathrm{Re}(\lambda_{1,2}) < 0$.

# Cosmological viability of the fixed points

At the fixed point $A_{\pm}$, the density $\rho_B$ is positive if

$$c_1 \leq 0 \quad \wedge \quad c_2 > \frac{c_1}{2} + \sqrt{-\frac{c_1}{24}}.$$




$$\frac{\rho_B}{m_{\mathrm{P}}^4} = 3\hat{g}^2 \left(\frac{y}{z}\right)^4 \left[y^2 - 12c_2 y^4 + x^2\left(1 - 12c_2 y^2\right) + 2x\left[y + 4(c_1 - 4c_2)y^3\right] + 2z^4\right].$$

- At the fixed point $A_{\pm}$, the density $\rho_B$ becomes negative.
- At the fixed points $B_{\pm}$ and $C_{\pm}$ the density $\rho_B$ goes to infinity.

# Pseudo stationary states or nullclines

$$x' = \frac{p}{\sqrt{2}} + x\epsilon, \quad y' = x$$

- The straight lines and constant roll condition.

$$y = \beta x, \quad \ddot{\psi} = \frac{H}{\beta}\dot{\psi} \qquad \lim_{x\to\infty}\left(\frac{1}{\beta} - \frac{x'}{x}\right) = 0$$

When we consider the limit of the $x$ and $y$ large we got the equation on $\beta$

$$0 = \left(-\frac{4}{3} + \frac{7}{3}\frac{c_2}{c_1}\right) + \left(-\frac{37}{9} + \frac{8}{9}\frac{c_1}{c_2} + \frac{56}{9}\frac{c_2}{c_1}\right)\beta + \left(\frac{4}{3} - \frac{2}{3}\frac{c_1}{c_2} + \frac{7}{3}\frac{c_2}{c_1}\right)\beta^2 + \beta^3.$$

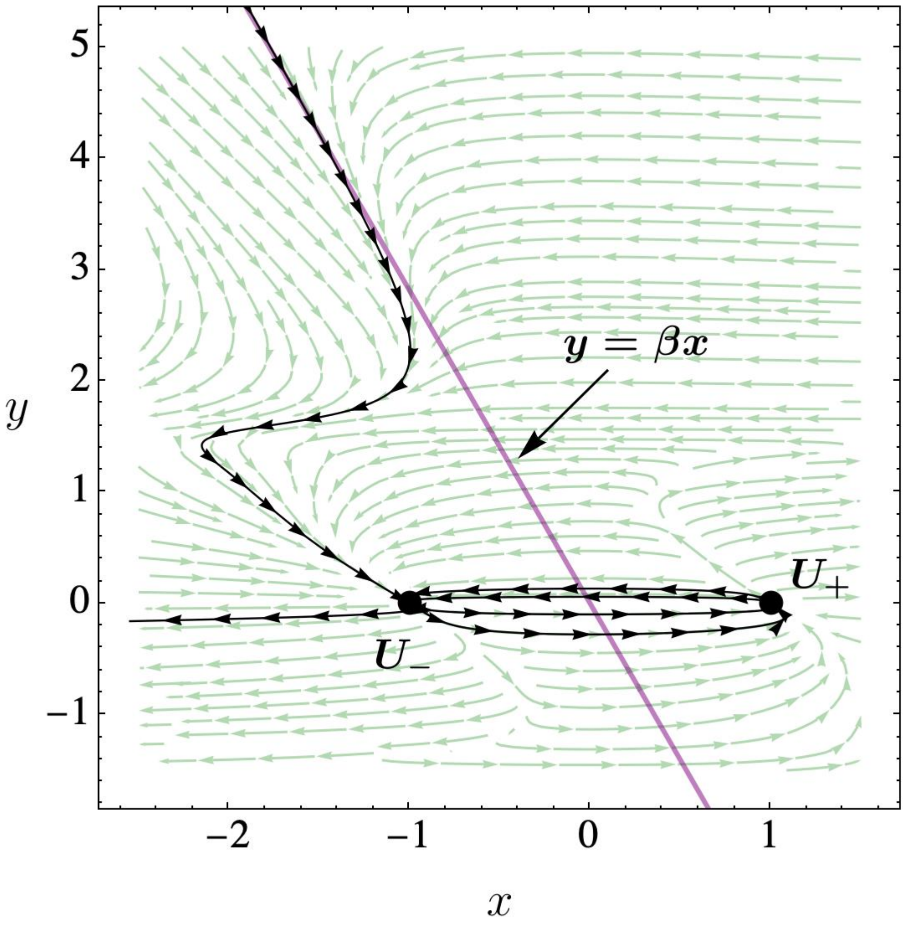
such that the slopes of the straight lines are

$$\beta_0 = \tfrac{4}{3} - \tfrac{7}{3}\tfrac{c_2}{c_1}, \qquad w_B = -1$$

$$\beta_\pm = -\tfrac{4}{3} + \tfrac{c_1}{3c_2} \pm \tfrac{\sqrt{(c_1-c_2)(c_1-7c_2)}}{3c_2}, \quad w_B = -\tfrac{23}{9} + \tfrac{8}{9}\tfrac{c_1}{c_2} \pm \tfrac{8}{9c_2}\sqrt{(c_1-c_2)(c_1-7c_2)}.$$

# Pseudo stationary states or nullclines

- Existence of the central zone




$$x_i = 5 \times 10^9, \quad y_i = 10^{10}, \quad c_1 = 0.0206, \quad c_2 = 0.0366.$$

Solving $x' = 0$ when $y \to 0$, we find the pseudo fixed points $U_{\pm} = \{\pm 1, 0\}$

- The stability of this points are describing by the eigensystem

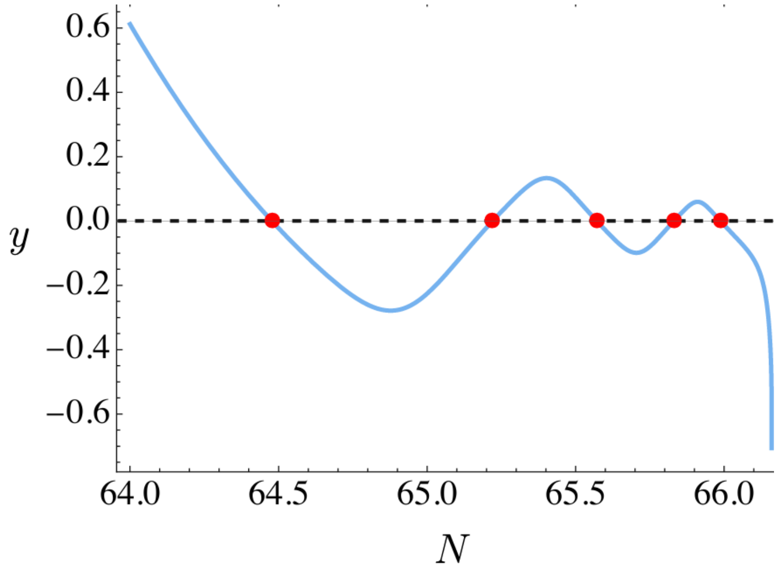
$$U_+ : \quad \lambda_1^+ = -\frac{1}{2}y, \quad \lambda_2^+ = 3 + \frac{4}{y},$$
$$\nu_1^+ = \{0, 1\}, \quad \nu_2^+ = \left\{3 + \frac{4}{y}, 1\right\},$$

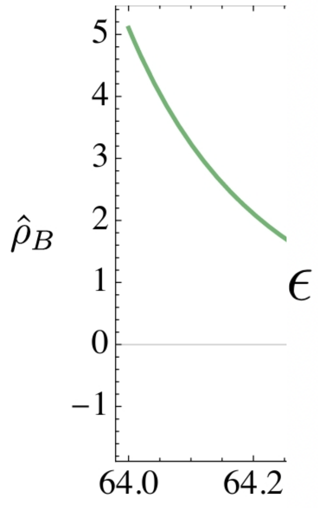
$$U_- : \quad \lambda_1^- = 3 - \frac{4}{y}, \quad \lambda_2^- = \frac{1}{2}y,$$
$$\nu_1^- = \left\{3 - \frac{4}{y}, 1\right\}, \quad \nu_2^- = \{0, 1\}.$$

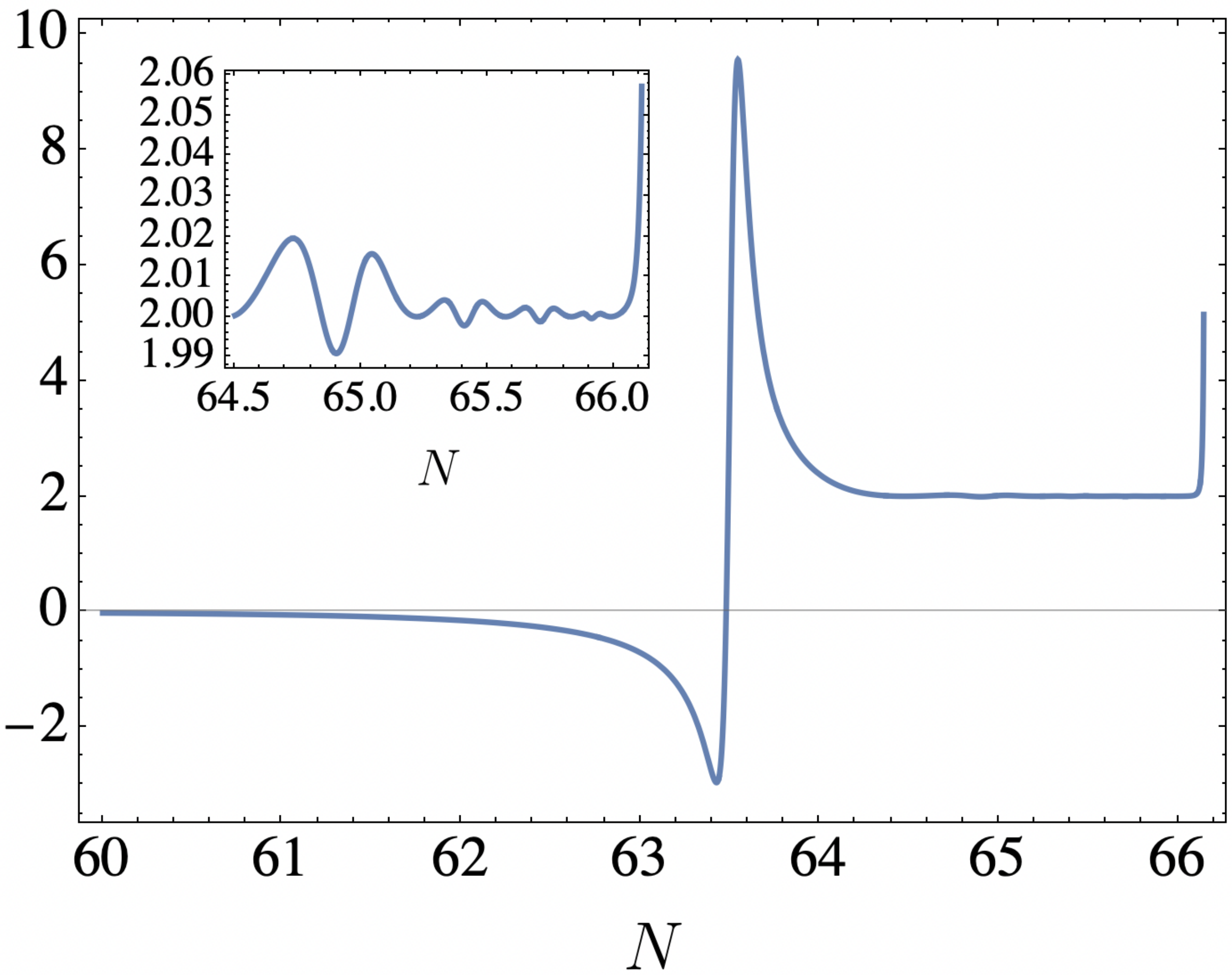
# Instabilities in the central zone

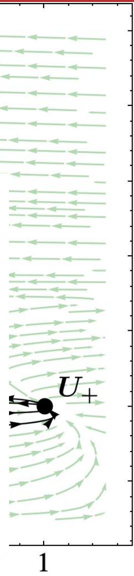
- In the inflation scenario $\dfrac{\rho_B}{\hat{g}^2 m_P^4} \approx 3y^3, \quad \dfrac{p_B}{\hat{g}^2 m_P^4} \approx y^3$

  It violates the weak energy condition.



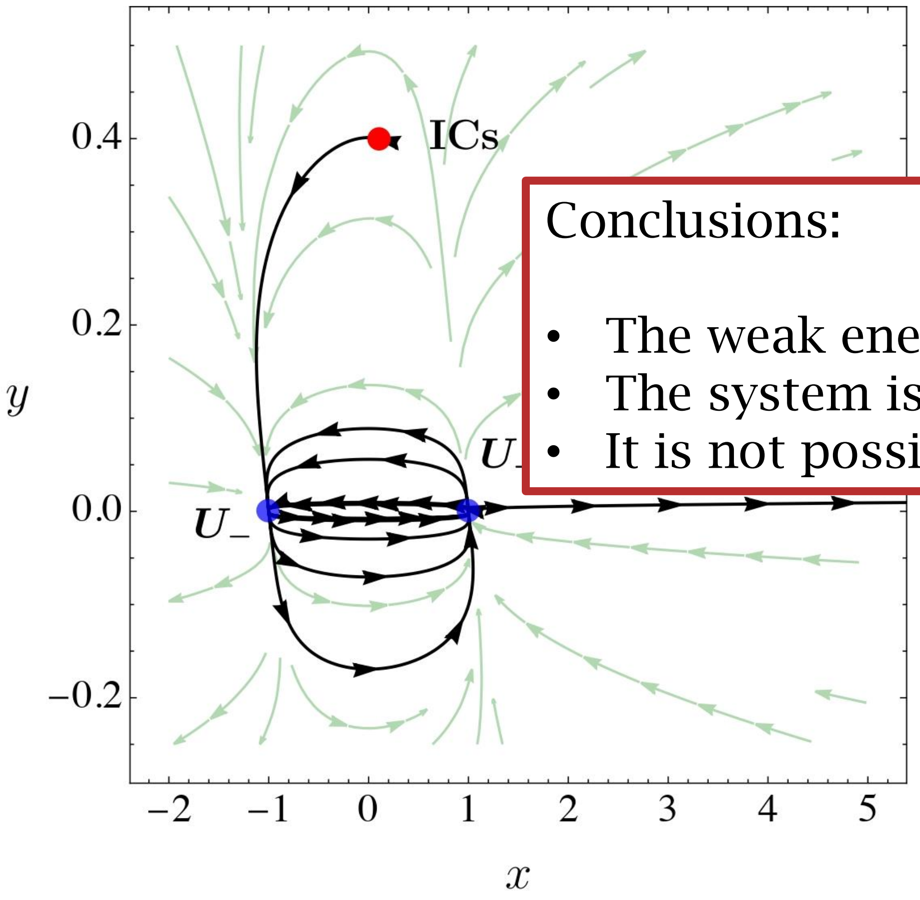




$$x \equiv \frac{\dot{\psi}}{\sqrt{2} m_P H}, \quad y \equiv \frac{\psi}{\sqrt{2} m_P} \to H \equiv \frac{1}{x}\frac{dy}{dy}$$

# Instabilities in the central zone

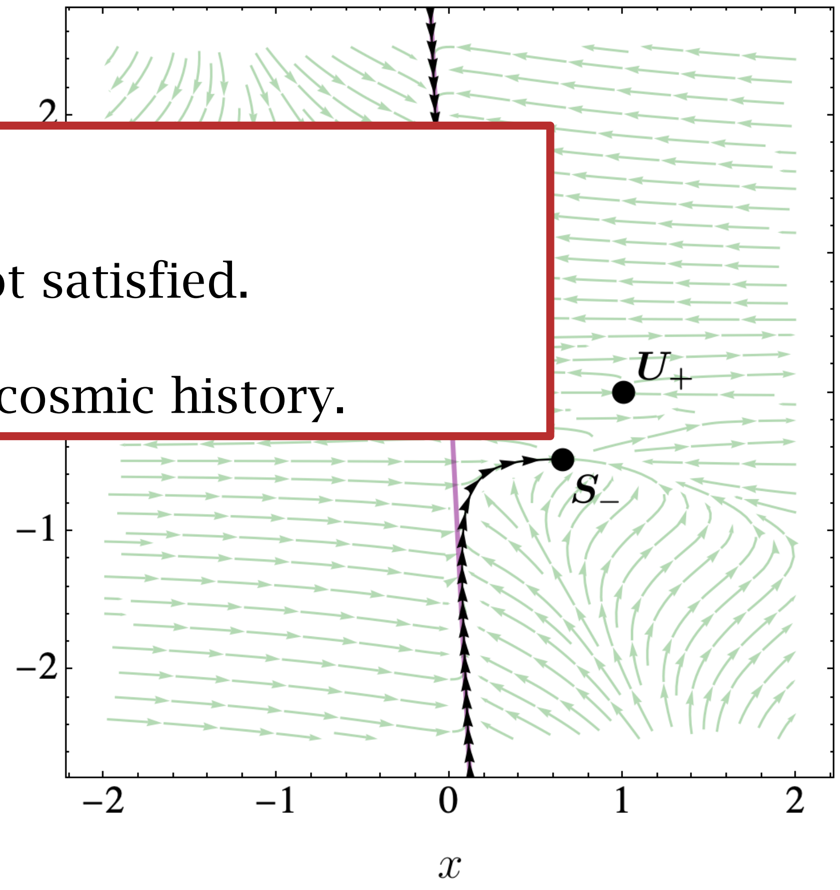
- In the dark energy scenario




$x_i = 0.1, \quad y_i = 0.4, \quad c_1 = 0.0206, \quad c_2 = 0.0366.$

- Renormalization of the autonomous set

$$x' = \frac{\sum_{i=0}^{3} f_i(c_1, c_2, y)x^i}{y(1 - 12c_2y^2 + 8(c_1 - c_2)y^4 + 16(c_1 - c_2)(c_1 - 7c_2)y^6)} = \frac{0}{0}?$$




$x_i = \pm 4 \times 10^9, \quad y_i = \mp 10^{10}, \quad c_1 = 0.2, \quad c_2 = 2.2.$

Conclusions:

- The weak energy conditions is not satisfied.
- The system is higgled unstable.
- It is not possible predict the full cosmic history.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