Two texture zeros for Dirac Neutrinos in a Diagonal charged lepton basis

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Outline

- Motivation
- The model
- Algebra Polar Theorem
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- One texture zero
- Two Texture zeros
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Motivation

Neutrino Oscillations

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Neutrino Masses

Neutrino Oscillations

The model:

SM + three Right-Handed Neutrinos (SMRHN)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_Q$$

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The Dirac Lagrangian mass term for the lepton sector is given by

$$-\mathcal{L} = \bar{\nu}_L M_n \nu_R + \bar{l}_L M_l l_R + h.c.$$

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The Dirac Lagrangian mass term for the lepton sector is given by

$$-\mathcal{L} = \bar{\nu}_L M_n \nu_R + \bar{l}_L M_l l_R + h.c.$$

M_l is in the diagonal form; this implies: $U_{PMNS} = 1_{3\times3} U_\nu$ is the oscillation matrix

Algebra Polar theorem: C = H U

Counting of Parameters:

$$18 \rightarrow 9 \begin{cases} 6 \text{ Reals P.} \rightarrow TTZ \rightarrow 4 \text{ Real P.} \\ 3 \text{ Phases} \rightarrow WBT \rightarrow 1 \text{ Phase.} \end{cases}$$

One physical prediction, the lightest neutrino mass.

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Weak Basis Transformation

$$M_\nu = \begin{pmatrix} |m_{\nu_e\nu_e}| & |m_{\nu_e\nu_\mu}|e^{i\phi_{e\mu}} & |m_{\nu_e\nu_\tau}|e^{i\phi_{e\tau}} \\ |m_{\nu_e\nu_\mu}|e^{-i\phi_{e\mu}} & |m_{\nu_\mu\nu_\mu}| & |m_{\nu_\mu\nu_\tau}|e^{i\phi_{\mu\tau}} \\ |m_{\nu_e\nu_\tau}|e^{-i\phi_{e\tau}} & |m_{\nu_\mu\nu_\tau}|e^{-i\phi_{\mu\tau}} & |m_{\nu_\tau\nu_\tau}| \end{pmatrix}; \quad M_\phi = \text{Diag}(e^{i\phi_1}, 1, e^{i\phi_2}), \quad M'_\nu = M_\phi^\dagger M_\nu M_\phi$$

Case A: $\phi_1 = \phi_{e\mu}$ and $\phi_2 = \phi_1 - \phi_{e\tau} = \phi_{e\mu} - \phi_{e\tau}$. Producing

$$M'_\nu = \begin{pmatrix} |m_{\nu_e\nu_e}| & |m_{\nu_e\nu_\mu}| & |m_{\nu_e\nu_\tau}| \\ |m_{\nu_e\nu_\mu}| & |m_{\nu_\mu\nu_\mu}| & |m_{\nu_\mu\nu_\tau}|e^{i\psi} \\ |m_{\nu_e\nu_\tau}| & |m_{\nu_\mu\nu_\tau}|e^{-i\psi} & |m_{\nu_\tau\nu_\tau}| \end{pmatrix};$$

with $\psi = \phi_{\mu\tau} + \phi_2 = \phi_{\mu\tau} + \phi_{e\mu} - \phi_{e\tau}$.

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Case B: $\phi_1 = \phi_{e\mu}$ and $\phi_2 = -\phi_{\mu\tau}$. Producing

$$M'_\nu = \begin{pmatrix} |m_{\nu_e\nu_e}| & |m_{\nu_e\nu_\mu}| & |m_{\nu_e\nu_\tau}|e^{-i\psi} \\ |m_{\nu_e\nu_\mu}| & |m_{\nu_\mu\nu_\mu}| & |m_{\nu_\mu\nu_\tau}| \\ |m_{\nu_e\nu_\tau}|e^{i\psi} & |m_{\nu_\mu\nu_\tau}| & |m_{\nu_\tau\nu_\tau}| \end{pmatrix}.$$

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Case C: $\phi_2 = -\phi_{\mu\tau}$ and $\phi_1 = \phi_2 + \phi_{e\tau} = \phi_{e\tau} - \phi_{\mu\tau}$. Producing

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As a summary:

$$M_\nu = \begin{pmatrix} m_{\nu_e\nu_e} & m_{\nu_e\nu_\mu} & m_{\nu_e\nu_\tau} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu\nu_\mu} & m_{\nu_\mu\nu_\tau} \\ m_{\nu_\tau\nu_e} & m_{\nu_\tau\nu_\mu} & m_{\nu_\tau\nu_\tau} \end{pmatrix} = U_{PMNS} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{PMNS}^\dagger$$

$$= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} U_{e1}^* & U_{\mu1}^* & U_{\tau1}^* \\ U_{e2}^* & U_{\mu2}^* & U_{\tau2}^* \\ U_{e3}^* & U_{\mu3}^* & U_{\tau3}^* \end{bmatrix}$$

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$$U_{PMNS} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{bmatrix};$$

One texture zero in the diagonal

$$m_{\nu_e \nu_e} = m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 + m_3 |U_{e3}|^2 = 0,$$

Using the constrain $|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$

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We find that $$|U_{e2}|^2 = \frac{m_3}{m_3 - m_2} - \frac{m_3 - m_1}{m_3 - m_2}|U_{e1}|^2.$$

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In general
$$|U_{\alpha 2}|^2 = \frac{m_3}{m_3 - m_2} - \frac{m_3 - m_1}{m_3 - m_2}|U_{\alpha 1}|^2,$$

for $\alpha = e$ if $m_{\nu_e\nu_e} = 0$; $\alpha = \mu$ if $m_{\nu_\mu\nu_\mu} = 0$, and $\alpha = \tau$ if $m_{\nu_\tau\nu_\tau} = 0$.

Numerical Analysis: $m_{\nu_e \nu_e} = 0$

$$\frac{s_{12}^2 c_{13}^2 \Delta m_{32}^2}{(M_T - m_1)} = m_3 - (m_3 - m_1) c_{12}^2 c_{13}^2 .$$

$$\chi^2(m_1) = \left(\frac{\sin^2\theta_{12} - \sin^2\tilde{\theta}_{12}}{\sigma(\sin^2\theta_{12})} \right)^2$$

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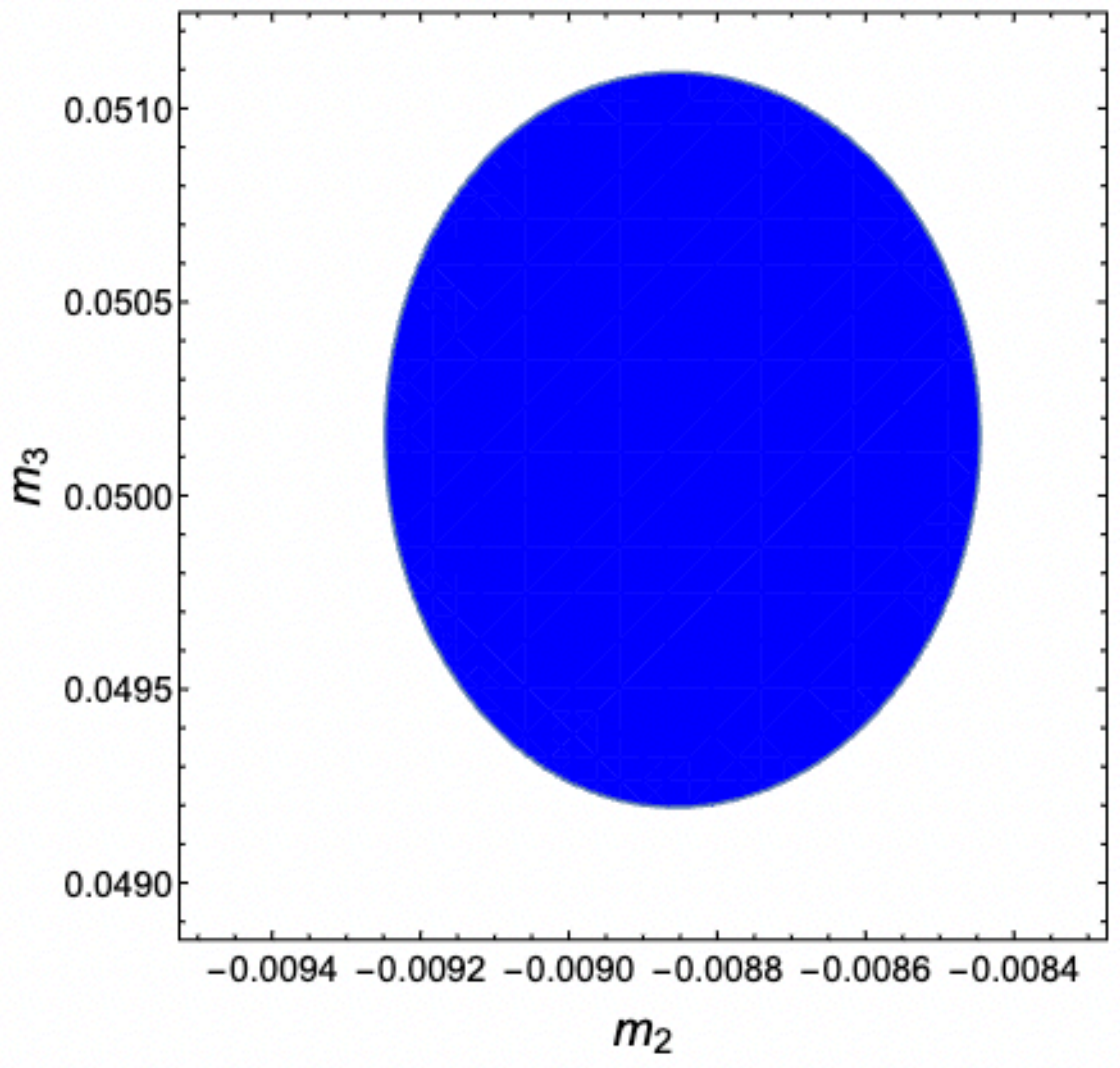
$$\frac{s_{12}^2 c_{13}^2 \Delta m_{32}^2}{(M_T - m_1)} = m_3 - (m_3 - m_1) c_{12}^2 c_{13}^2. \qquad \chi^2(m_1) = \left(\frac{\sin^2\theta_{12} - \sin^2\tilde{\theta}_{12}}{\sigma(\sin^2\theta_{12})} \right)^2$$

$$m_1 = 0.00208 \text{ eV}, \quad m_2 = -0.00886 \text{ eV}, \quad m_3 = 0.0501 \text{ eV}$$



(a) Plot evaluated at $m_1 = 0.00208$ eV.

One texture zero outside the diagonal

$$m_{\nu_e\nu_\mu} = m_1 U_{e1} U^*_{\mu 1} + m_2 U_{e2} U^*_{\mu 2} + m_3 U_{e3} U^*_{\mu 3} = 0,$$

Using the constrain $U_{e1} U^*_{\mu 1} + U_{e2} U^*_{\mu 2} + U_{e3} U^*_{\mu 3} = 0$

$$U_{e1} U^*_{\mu 1} U^*_{e2} U_{\mu 2} + \left(\frac{m_3 - m_2}{m_3 - m_1}\right) |U_{e2}|^2 |U_{\mu 2}|^2 = 0,$$

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Which implies $Im.(U_{e1} U^*_{\mu 1} U^*_{e2} U_{\mu 2}) = J = 0;$ CP conserving

Two texture zeros

$$A_1 = \begin{pmatrix} 0 & 0 & b \\ 0 & x_2 & c \\ b^* & c^* & x_3 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & a & 0 \\ a^* & x_2 & c \\ 0 & c^* & x_3 \end{pmatrix}; \quad A_3 = \begin{pmatrix} 0 & a & b \\ a^* & x_2 & 0 \\ b^* & 0 & x_3 \end{pmatrix},$$

$$A_4 = \begin{pmatrix} x_1 & 0 & b \\ 0 & 0 & c \\ b^* & c^* & x_3 \end{pmatrix}, \quad A_5 = \begin{pmatrix} x_1 & a & 0 \\ a^* & 0 & c \\ 0 & c^* & x_3 \end{pmatrix}; \quad A_6 = \begin{pmatrix} x_1 & a & b \\ a^* & 0 & 0 \\ b^* & 0 & x_3 \end{pmatrix},$$

$$A_7 = \begin{pmatrix} x_1 & 0 & b \\ 0 & x_2 & c \\ b^* & c^* & 0 \end{pmatrix}, \quad A_8 = \begin{pmatrix} x_1 & a & 0 \\ a^* & x_2 & c \\ 0 & c^* & 0 \end{pmatrix}; \quad A_9 = \begin{pmatrix} x_1 & a & b \\ a^* & x_2 & 0 \\ b^* & 0 & 0 \end{pmatrix},$$

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$$A_{10} = \begin{pmatrix} x_1 & a & 0 \\ a^* & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}, \quad A_{11} = \begin{pmatrix} x_1 & 0 & b \\ 0 & x_2 & 0 \\ b^* & 0 & x_3 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & c \\ 0 & c^* & x_3 \end{pmatrix}.$$

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$$D_1 = \begin{pmatrix} 0 & a & b \\ a^* & 0 & c \\ b^* & c^* & x_3 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0 & a & b \\ a^* & x_2 & c \\ b^* & c^* & 0 \end{pmatrix}, \quad D_3 = \begin{pmatrix} x_1 & a & b \\ a^* & 0 & c \\ b^* & c^* & 0 \end{pmatrix}$$

Two texture zeros

For example A7 form:

$$A_7 = \begin{pmatrix} x_1 & 0 & b \\ 0 & x_2 & c \\ b^* & c^* & 0 \end{pmatrix}. \qquad Diag(m_1, -m_2, m_3)$$

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Using the invariants $\mathrm{tr}[M]$, $\mathrm{tr}[M^2]$, and $\det[M]$.

$$\begin{pmatrix} x_1 & 0 & \sqrt{\frac{(x_1-m_1)(x_1+m_2)(-x_1+m_3)}{2x_1-m_1+m_2-m_3}} \\ 0 & -x_1+m_1-m_2+m_3 & \sqrt{\frac{(x_1-m_1+m_2)(-x_1+m_1+m_3)(x_1+m_2-m_3)}{2x_1-m_1+m_2-m_3}} \\ \sqrt{\frac{(x_1-m_1)(x_1+m_2)(-x_1+m_3)}{2x_1-m_1+m_2-m_3}} & \sqrt{\frac{(x_1-m_1+m_2)(-x_1+m_1+m_3)(x_1+m_2-m_3)}{2x_1-m_1+m_2-m_3}} & 0 \end{pmatrix}$$

After diagonalizing this texture, we obtained the UPMNS in terms of the free parameter.

$$U_{\text{PMNS}} = \begin{pmatrix} -\sqrt{\frac{(x_1+m_2)(x_1+m_2-m_3)(m_3-x_1)}{(m_1+m_2)(m_3-m_1)(2x_1-m_1+m_2-m_3)}} & \sqrt{\frac{(x_1-m_1)(x_1-m_1+m_2)(-x_1+m_1+m_3)}{(m_1+m_2)(m_1-m_3)(-2x_1+m_1-m_2+m_3)}} & \sqrt{\frac{(x_1-m_1)(x_1+m_2-m_3)}{(m_1+m_2)(-m_1+m_3)}} \\ -\sqrt{\frac{(m_1-x_1)(m_3-x_1)(-x_1+m_1+m_3)}{(m_1+m_2)(m_2+m_3)(-2x_1+m_1-m_2+m_3)}} & -\sqrt{\frac{(x_1+m_2)(x_1-m_1+m_2)(x_1+m_2-m_3)}{(m_1+m_2)(m_2+m_3)(2x_1-m_1+m_2-m_3)}} & \sqrt{\frac{(x_1+m_2)(-x_1+m_1+m_3)}{(m_1+m_2)(m_2+m_3)}} \\ \sqrt{\frac{(x_1-m_1)(x_1+m_2)(x_1-m_1+m_2)}{(m_1-m_3)(m_2+m_3)(-2x_1+m_1-m_2+m_3)}} & \sqrt{\frac{(x_1-m_3)(x_1-m_1-m_3)(x_1+m_2-m_3)}{(m_1-m_3)(m_2+m_3)(-2x_1+m_1-m_2+m_3)}} & \sqrt{\frac{(x_1-m_1+m_2)(x_1-m_3)}{(m_1-m_3)(m_2+m_3)}} \end{pmatrix}$$

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However, this particular matrix is obtained from the neutral sector, we call this type of matrix an oscillation matrix.

$$U_{PMNS} = \begin{pmatrix} 0.801 \rightarrow 0.842 & 0.519 \rightarrow 0.580 & 0.142 \rightarrow 0.155 \\ 0.248 \rightarrow 0.505 & 0.473 \rightarrow 0.682 & 0.649 \rightarrow 0.764 \\ 0.270 \rightarrow 0.521 & 0.483 \rightarrow 0.690 & 0.628 \rightarrow 0.746 \end{pmatrix}$$

- Numerical analysis of the form

In the numerical analyses we use the values of the neutrino squared masses and the values of the mixing angles:

$$\begin{aligned}
\Delta m^2_{atm} &= (2.47 - 2.63) \times 10^{-3}\mathrm{eV}^2, \\
\Delta m^2_{sol} &= (6.94 - 8.14) \times 10^{-5}\mathrm{eV}^2 = \Delta m^2_{21}, \\
\sin^2\theta_{atm} &= (4.34 - 6.10) \times 10^{-1} = \sin^2\theta_{23}, \\
\sin^2\theta_{sol} &= (2.71 - 3.69) \times 10^{-1} = \sin^2\theta_{12}, \\
\sin^2\theta_{Reac} &= (2.00 - 2.41) \times 10^{-2} = \sin^2\theta_{13},
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$$

$$\chi^2(m_1, x_1) = \sum_{i<j} \left(\frac{\sin^2\theta_{ij} - \sin^2\tilde{\theta}_{ij}}{\sigma(\sin^2\theta_{ij})} \right)^2, \quad \text{with } i,j = 1,2,3.$$

And, with the constrains $m_3 > x_1 > m_1,\ 2x_1 > m_3,\ x_1 + m_2 > m_3$ to have real solutions

We obtain

$$|m_1| = 0.0333 \quad |m_2| = 0.0344 \quad |m_3| = 0.0608 \ eV$$

$$\sin^2\theta_{12} = 0.315, \quad \sin^2\theta_{23} = 0.646, \quad \sin^2\theta_{13} = 0.022.$$

Following the same procedure for the other eight textures, the results are:

Texture	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	$\lvert m_1\rvert$ (eV)	$\lvert m_2\rvert$ (eV)	$\lvert m_3\rvert$ (eV)
1: A_1	0.298	0.250	0.022	0.0018	0.0091	0.0512
2: A_2	0.305	0.018	0.014	0.0044	0.0096	0.0503
3: A_3	0.334	0.007	0.022	0.0021	0.0087	0.0501
4: A_4	0.318	0.200	0.022	0.0046	0.0097	0.0512
5: A_5	0.465	0.010	0.032	0.0153	0.0175	0.0534
6: A_6	0.524	0.003	0.014	0.0209	0.0227	0.0546
7: A_7	0.315	0.646	0.022	0.0333	0.0344	0.0608
8: A_8	0.022	0.515	0.023	0.2685	0.2686	0.2731
9: A_9	0.023	0.516	0.023	0.2765	0.2767	0.2810

Two texture zeros off the diagonal

None of the three cases is viable because each one of them is associated with a vanishing oscillation parameter

$A_{10} \rightarrow \theta_{13} = 0\,,$ $A_{11} \rightarrow \theta_{23} = 0$ and $A_{12} \rightarrow \theta_{12} = 0$

Two texture zeros off the diagonal

None of the three cases is viable because each one of them is associated with a vanishing oscillation parameter

$A_{10} \to \theta_{13} = 0\,,\qquad A_{11} \to \theta_{23} = 0$ and $A_{12} \to \theta_{12} = 0$

Two texture zeros in the main diagonal

Our result shows that none of the three different textures with two zeroes in the main diagonal is able to reproduce the three measured mixing angles in the Upmns oscillation matrix.

Conclusions

- The texture zeros diminishing the mathematical parameters in the models.
- The texture zeros give us an alternative to the PDG mixing matrix parametrization, in a function of four parameters for two texture zeros.
- CP-symmetry is conserving when we have at least one zero off the diagonal.
- The forms with two texture zeros only one of them predict a neutrino mixing compatible with the current neutrino oscillation phenomenology.

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Two Texture Zeros for Dirac Neutrinos in a Diagonal charged lepton basis

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