Analysis of the muon anomalous magnetic moment in an $U(1)_d$ model in addition to the Standard Model

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CONTENT

STANDARD MODEL

The **elementary particles** are the fundamental constituents of all matter, they are considered to be **point-like** and **structureless**. That is, they do not occupy a volume in space. In the **Standard Model** they are characterized in two large groups: **bosons** and **fermions**.

Some of the **most important properties** of leptons are:

Lepton	Mass $[MeV/c^2]$	Mean lifetime $[s]$
Electron e^-	0.511	∞
Electron neutrino ν_e	0	∞
Muon μ^-	105.658	2.197×10^{-6}
Muon neutrino ν_μ	0	∞
Tau τ^-	1777	$(291.0 \pm 1.5) \times 10^{-15}$
Tau neutrino ν_τ	0	∞

Table 1: Leptons of the Standard Model.

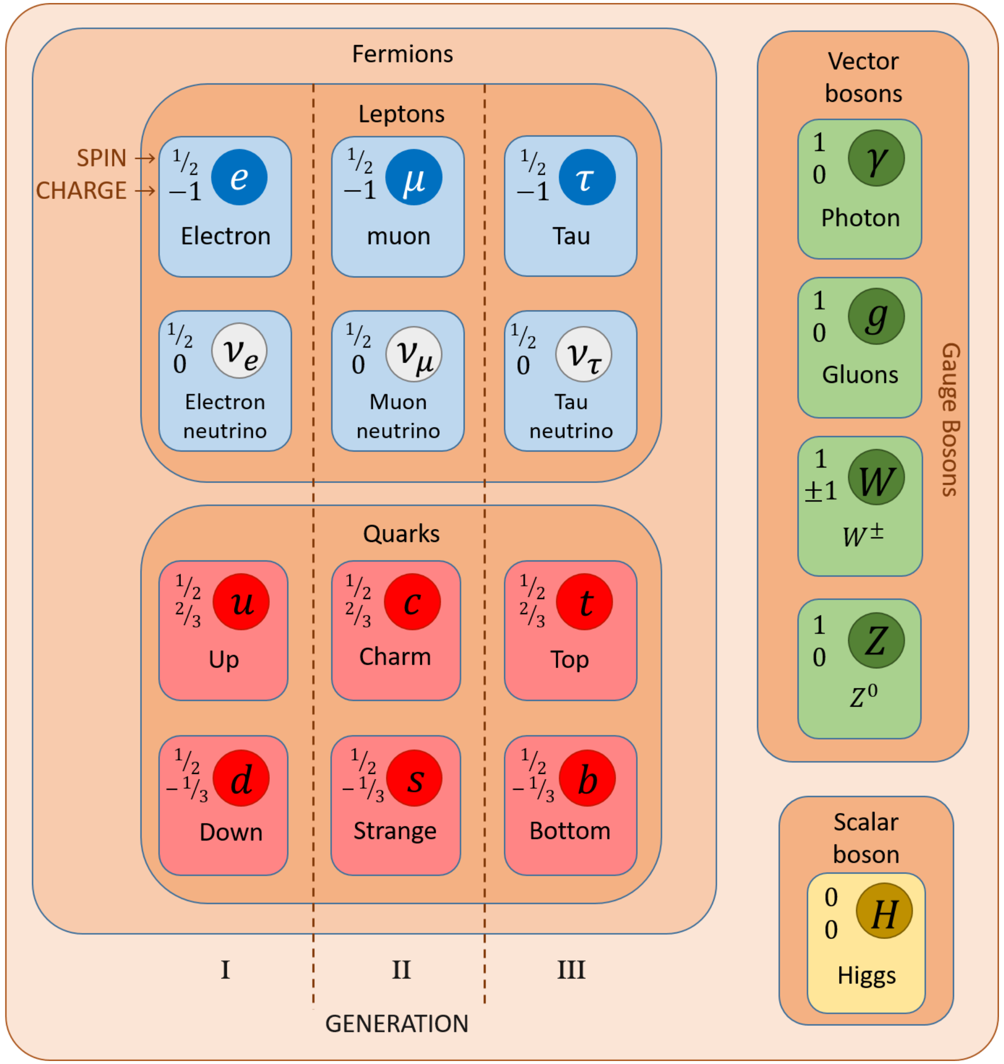

Figure 1: Standard Model of Particle physics.

STANDARD MODEL

The **Standard Model** is a local norm theory; therefore, all its interactions are described by **local gauge symmetries**.

The **gauge transformations** are given by unitary matrices belonging to a certain Lie group $U(N)$ or $\mathrm{SU}(N)$. Because of this, the transformation matrices can be represented as:

$$U_\theta(x) = e^{-i\theta^a(x)T_a}, \qquad (1)$$

where $\theta^a(x)$ are the **parameters of the transformation** and T_a are the **generators** of the group associated to that representation. Which obey the following Lie algebra:

$$[T_a, T_b] = if_{ab}{}^{c}T_c. \qquad (2)$$

Figure 2: Yang & Mills in 1999.

STANDARD MODEL

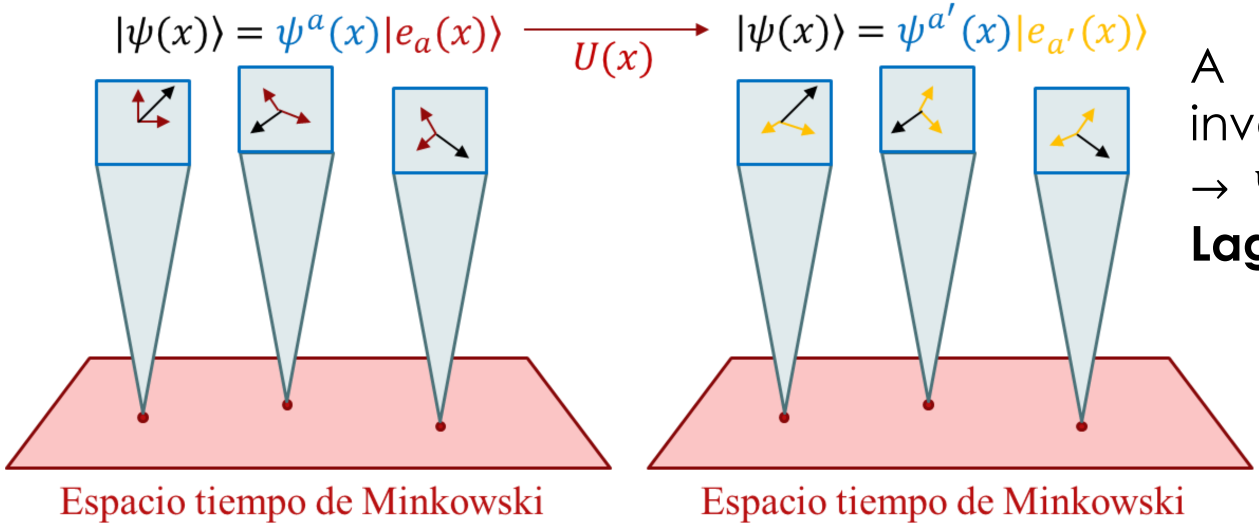

Figure 3: Local gauge transformation.

A clear example of global gauge invariance under the transformation $\Psi \to \Psi' = e^{i\alpha^a T_a}\Psi$, is present in the **Dirac Lagrangian**:

$$\mathcal{L}_D = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - \bar{\Psi}M\Psi. \qquad (3)$$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix} \quad M = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & m_n \end{bmatrix}$$

But for the **local gauge transformation** $\Psi \to \Psi' = e^{-iq\theta^a(x)T_a}\Psi$:

$$\mathcal{L}' = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi + q\left(\partial_\mu\theta^a\right)\bar{\Psi}\gamma^\mu T_a\Psi - \bar{\Psi}M\Psi. \qquad (4)$$

In order to keep the **Lagrangian invariant** it is necessary to introduce the **covariant derivative** $D_\mu = \partial_\mu + iqT_aA_\mu^a$:

$$\boxed{F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + qf_{ab}^c A_\mu^a A_\nu^b}$$

$$\mathcal{L} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + i\bar{\Psi}\gamma^\mu D_\mu\Psi - \bar{\Psi}M\Psi. \qquad (5)$$

$U(1)_d$ MODEL

The $\boldsymbol{U(1)_d}$ **model** is a simple extension of the Standard Model by the addition of a new **gauge field** associated with a new **gauge symmetry**. The Lagrangian underlying the addition of the new $U(1)_d$ symmetry group to the Standard Model is **[1]**:

$$\mathcal{L}_{SM+U(1)_d} = \mathcal{L}_{SM} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}\kappa B^{\mu\nu}F^{Y}_{\mu\nu} + \mathcal{L}_{Higgs'} + \cdots, \qquad (6)$$

$$B^{\mu\nu} = \partial^{\mu}V^{\nu} - \partial^{\nu}V^{\mu}. \qquad (7)$$

Assuming **spontaneous** $\boldsymbol{U(1)_d}$ **symmetry breaking**:

$$\mathcal{L}_{SM+U(1)_d} = \mathcal{L}_{SM} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}\kappa B^{\mu\nu}F^{Y}_{\mu\nu} + \frac{1}{2}m_V^2 V^{\mu}V_{\mu} + \mathcal{L}_{higgs'} + \cdots, \qquad (8)$$

so as to, the **kinetic Lagrangian** is given by:

$$\mathcal{L}_{Kinetic} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}\kappa B^{\mu\nu}F^{Y}_{\mu\nu}. \qquad (9)$$

$U(1)_d$ MODEL

Note: A scenario is assumed for a dark photon in which the known quarks and leptons have no $U(1)_d$ charge.

This Lagrangian can also be obtained by redefining the electromagnetic 4-potential:

$$A^\mu \rightarrow A^\mu - \kappa V^\mu,$$
$$F^{\mu\nu} \rightarrow F^{\mu\nu} - \kappa B^{\mu\nu}. \qquad (10)$$

Which allows me to find an **effective interaction Lagrangian** of the Model:

$$\mathcal{L}_{int}^{eff} = g_e \bar{\psi} \gamma^\mu (A_\mu - \kappa V_\mu) \psi = g_e \bar{\psi} \gamma^\mu A_\mu \psi - g_e \kappa \bar{\psi} \gamma^\mu V_\mu \psi, \qquad (11)$$

The first term continues to describe an **fermionic interaction** by Standard Model's photon. Furthermore, **the second term** corresponds to an **effective interaction** between the Standard Model's fermions and the dark photon.

#SOMOSUA

$U(1)_d$ MODEL

Analyzing the free Lagrangian associated with the V^μ field of the model's Lagrangian (equation 12), a **Proca-type Lagrangian** is found:

$$\mathcal{L}_{U(1)_d}^{Proca} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}m^2 V^\mu V_\mu. \qquad (12)$$

Whose plane wave solution associated with its equation of motion is:

$$V_\mu(x) = \int \frac{dq^3}{(2\pi)^3}\frac{1}{\sqrt{2E_q}}\sum_{j=1}^{3}\left(\epsilon_\mu^j(q)a_{q,j}e^{-iqx} + \epsilon_\mu^{j*}(q)a_{q,j}^\dagger e^{iqx}\right). \qquad (13)$$

With which, **the Feynman propagator for the dark photon** is obtained:

$$\langle 0|T\{V_\mu(y)V_\nu(x)\}|0\rangle = \frac{-i}{q^2 - m_V^2 + i\epsilon}\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{m_V^2}\right]. \qquad (14)$$

The **magnetic moment** is an intrinsic property of particles that depends on their **spin** $\vec{\boldsymbol{S}}$ and their **angular momentum** $\vec{\boldsymbol{L}}$, and tells how sensitive these are to an **external magnetic field** $\vec{\boldsymbol{B}}$. In the non-relativistic limit, the Dirac equation in the presence of an external magnetic field has the following Hamiltonian:

$$H = \frac{\vec{p}^2}{2m_l} + V(r) + \frac{e}{2m_l}\vec{B}\cdot\left(\vec{L} + g_l\vec{S}\right), \qquad (15)$$

where $\vec{S} = \frac{\vec{\sigma}}{2}$, m is the particle's mass and g_l is its **gyromagnetic factor**.

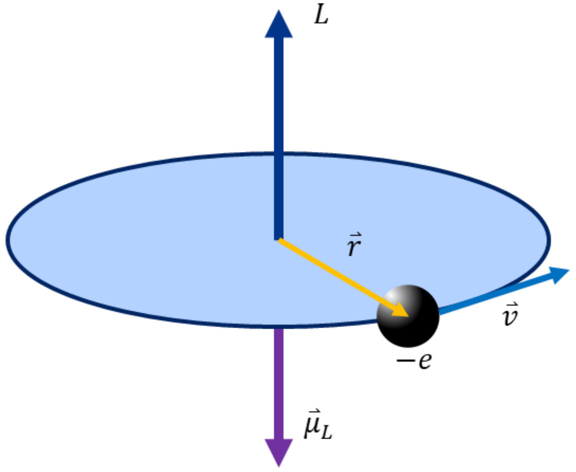

Figure 5: Angular magnetic moment.

$$\boxed{\overrightarrow{\mu_L} = \frac{-e}{2m_l}\vec{L}} \xrightarrow{\text{analogously}} \boxed{\vec{\mu} = g_l\frac{-e}{2m_l}\vec{S}}$$

The g_l factor is adimensional.

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For the case in which the process corresponds to the **interaction** between a Dirac fermion and a **"classical" electromagnetic field** (Figure 6), the gyromagnetic factor g_l corresponds exactly to 2.

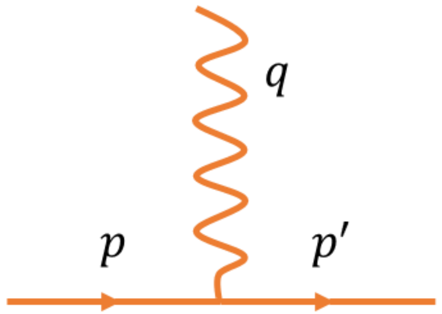


Figure 6: Tree-order interaction between a charged lepton and an electromagnetic field.

For this process with **momentum transfer** corresponding to $q = p' - p$, the calculation of the scattering amplitude using the Feynman rules for QED is:

$$-i\mathcal{M}_0^{\mu} = i\mathcal{g}_e\overline{\boldsymbol{u}}(\boldsymbol{p}')\gamma^{\mu}\boldsymbol{u}(\boldsymbol{p}). \qquad (16)$$

However, by using the **quantum field theory treatment** new contributions to the gyromagnetic factor g_l appear. Therefore, the **anomaly** is defined:

$$a_l = \frac{g_l - 2}{2}. \qquad (17)$$

These corrections to the g_l value can be obtained from expanding **the vertex correction function $\Gamma^\mu(p, p')$**. These contributions correspond to the diagrams in Figure 7.

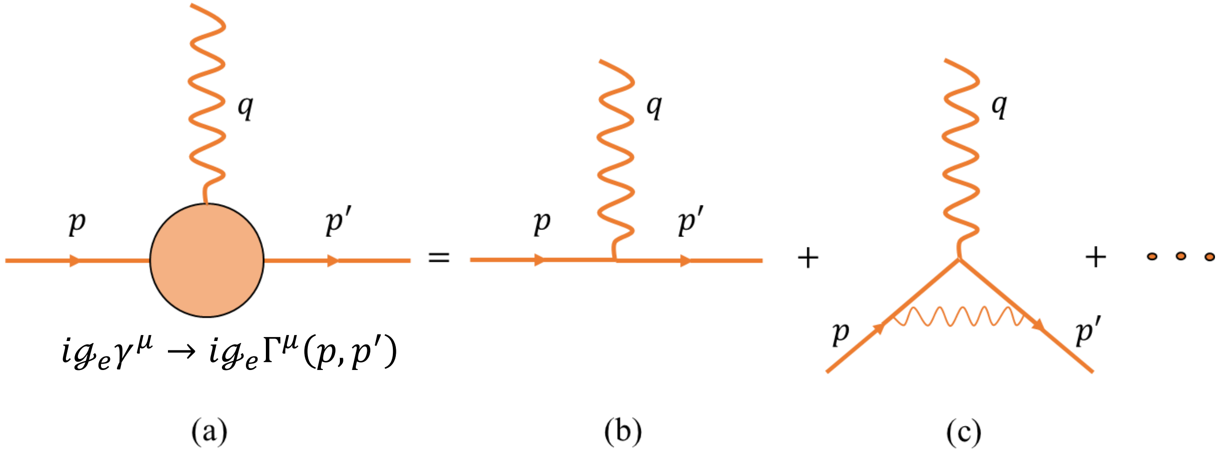


Figure 7: Expansion of the vertex correction function $\Gamma^\mu(p, p')$. **(a)** Total contribution. **(b)** Tree-order contribution. **(c)** 1-loop contribution.

so as to, the total contribution will be the superposition of all contributions of all possible processes:

$$-i\mathcal{M}^{\mu} = i g_e[\boldsymbol{\bar{u}}(\boldsymbol{p'})\,\Gamma^{\mu}(p,p')\,\boldsymbol{u}(\boldsymbol{p})] \tag{18}$$

Thus, for a process of specific $\boldsymbol{n}$**-order** one has:

$$-i\mathcal{M}_n^{\mu} = i g_e\left[\boldsymbol{\bar{u}}(\boldsymbol{p'})\,\Gamma^{\mu(n)}(p,p')\,\boldsymbol{u}(\boldsymbol{p})\right] \tag{19}$$

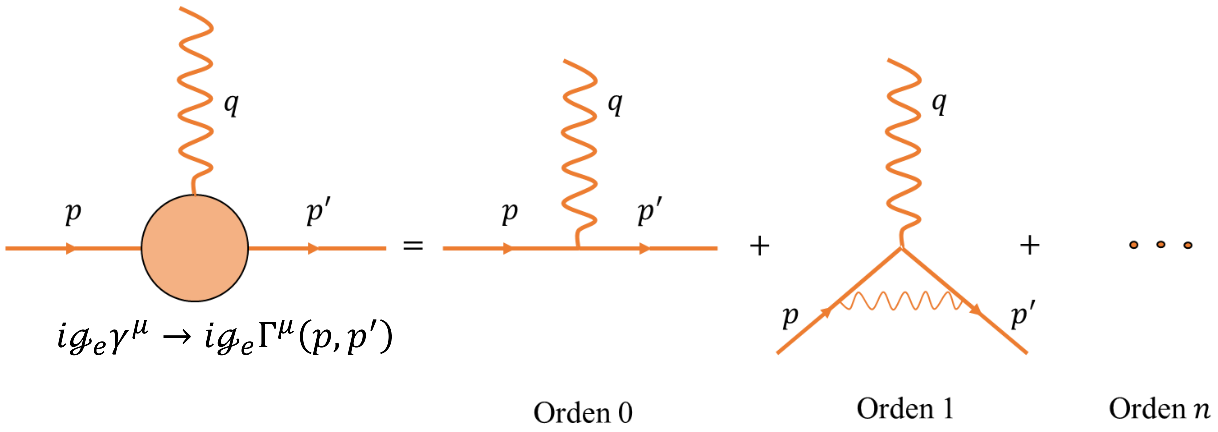


Figure 8: Expansion of the correction to the vertex $\Gamma^{\mu(n)}(p,p')$ of n-order.

Ward's identity: If $\mathcal{M}(q) = \epsilon_\mu(q)\mathcal{M}^\mu(q)$ is the amplitude of some QED process involving an external photon of momentum q, then:

$$q_\mu \mathcal{M}^\mu(q) = 0.$$

Gordon's decomposition: For any solution $\boldsymbol{u(p)}$ of the massive Dirac equation, it is satisfied that:

$$\bar{\boldsymbol{u}}(\boldsymbol{p}')\,\gamma^\mu\,\boldsymbol{u}(\boldsymbol{p}) = \bar{\boldsymbol{u}}(\boldsymbol{p}')\left[\frac{p'^\mu + p^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]\boldsymbol{u}(\boldsymbol{p}).$$

Considering **Ward's identity** and **Gordon's decomposition**, the **most general vertex correction function** associated with this process is:

$$\Gamma^\mu(p,p') = F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{q_\nu\sigma^{\mu\nu}}{2m_l}, \qquad (20)$$

where $\boldsymbol{F_1(q^2)}$ and $\boldsymbol{F_2(q^2)}$ are called **form factors**. A sum of contributions for each n can be associated with the form factors:

$$\Gamma^\mu(p,p') = \sum_{n=0}^{\infty}\Gamma^{\mu(n)}(p,p') = \sum_{n=0}^{\infty}\left[F_1^{(n)}(q^2)\gamma^\mu + iF_2^{(n)}(q^2)\frac{q_\nu\sigma^{\mu\nu}}{2m_l}\right]. \qquad (21)$$

Using the **Born approximation** for an **electrostatic potential** it is found that the first form factor $F_1^{(n)}$ corresponds to a modification of the **electric charge** and must fulfill that $F_1^{(0)}(q^2=0)=1$, therefore, $F_1^{(n)}(q^2=0)=0$ for any $n \geq 1$.

On the other hand, using the **Born approximation** only for a **magnetostatic potential**, we obtain that:

$$\vec{\mu} = \frac{-e}{m_l}[F_1(0)+F_2(0)]\vec{S}, \qquad (22)$$

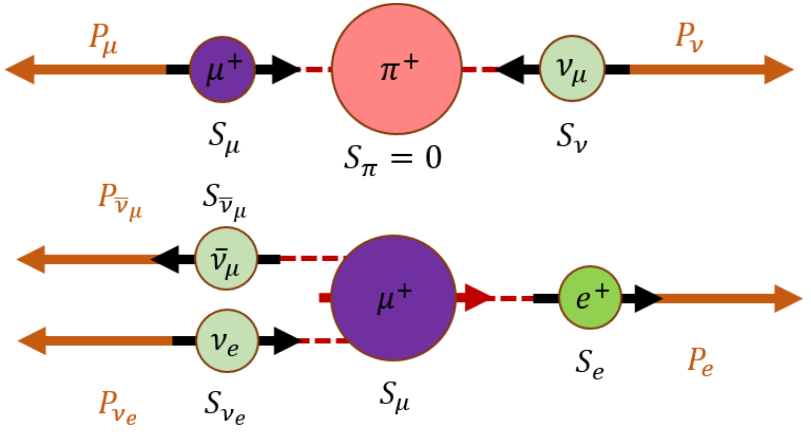
$$\Rightarrow g_l = 2[F_1(0)+F_2(0)] = 2 + 2F_2(0). \qquad (23)$$

So, for the value of the **anomalous magnetic moment** of a charged lepton we have:

$$a_l = F_2(0) = \sum_{n=0}^{\infty} F_2^{(n)}(0)\,. \qquad (24)$$

ANOMALOUS MAGNETIC MOMENT OF THE MUON

- **g-2 Collaboration:**



- **White Paper:** Theoretical calculation of the muon anomalous magnetic moment by performing a perturbative expansion.

- **BMW:** Calculation of the contribution of the hadronic polarization of the leading-order vacuum.

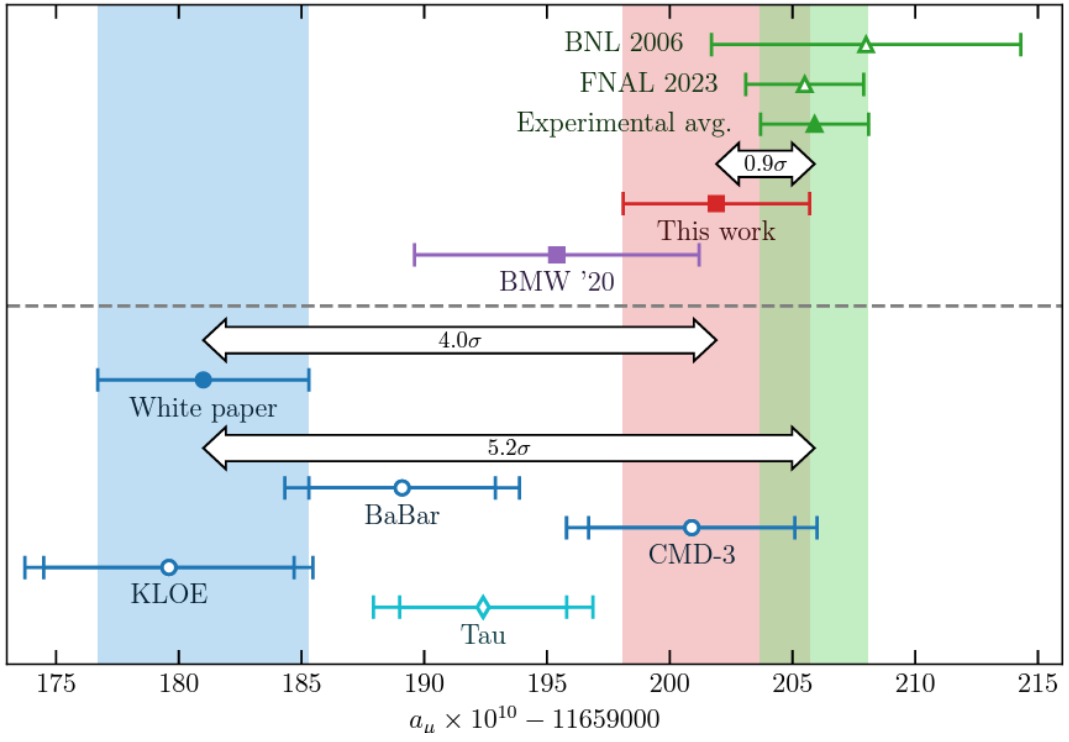


Figure 9: Experimental and theoretical predictions for muon g-2.
Source: BMW Collaboration(2024).

- **CMD-3:** Experimental measurement of the cross section of the $e^- + e^+ \rightarrow \pi^+ + \pi^-$ process at energies of 1.2 GeV at the VEPP-2000 electron-positron collider.

PARTIAL RESULTS

The **Feynman diagram** associated with the **1-loop contribution** of the effective interaction between Z' boson and muon is:

Note: The vertexes in the process will correspond to $i\mathcal{L}_{int}$.

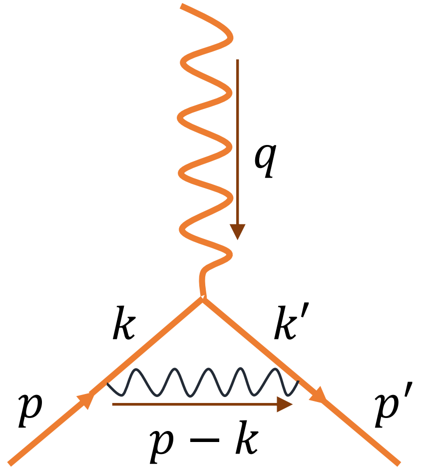


Figure 10: 1-loop contribution with the Z' boson for the anomaly.

Factor	Contribución
QED vertex	$i g_e \gamma^\mu$
Effective vertex	$-i\kappa g_e \gamma^\mu$
Muon propagator	$\dfrac{i(\gamma^\mu k_\mu + m_\mu)}{k^2 - m_\mu^2 + i\epsilon}$
Z' boson propagator	$\dfrac{-i g_{\mu\nu}}{q^2 - m_V^2 + i\epsilon}$

Figure 11: Propagators and vertexes for the process.

$$-i g_e \bar{u}(p')\Gamma^{\mu(1)}(p,p')u(p) = \int \frac{d^4k}{(2\pi)^4} \bar{\boldsymbol{u}}(\boldsymbol{p}')(-i g_e \kappa \gamma^\alpha) \frac{i(\not{k}' + m_\mu)}{k'^2 - m_\mu^2 + i\epsilon} (i g_e \gamma^\mu) \frac{i(\not{k} + m_\mu)}{k^2 - m_\mu^2 + i\epsilon} (-i g_e \kappa \gamma^\sigma)\boldsymbol{u}(\boldsymbol{p}) \frac{-i g_{\alpha\sigma}}{(k-p)^2 - m_V^2 + i\epsilon}. \quad (25)$$

PARTIAL RESULTS

Using the properties of the **gamma matrices**:

$$\gamma^\alpha\gamma^\mu\gamma_\alpha = -2\gamma^\mu, \qquad \gamma^\alpha\gamma^\mu\gamma^\nu\gamma_\alpha = 4g^{\mu\nu}, \qquad \gamma^\alpha\gamma^\nu\gamma^\mu\gamma^\sigma\gamma_\alpha = -2\gamma^\sigma\gamma^\mu\gamma^\nu.$$

Therefore:

$$\bar{u}(p')\Gamma^{\mu(1)}(p,p')u(p) = 2ig_e^2\kappa^2\int\frac{d^4k}{(2\pi)^4}\bar{\boldsymbol{u}}(\boldsymbol{p}')\frac{\not{k}\gamma^\mu\not{k}' - 2m_\mu(k'^\mu + k^\mu) + m_\mu^2\gamma^\mu}{(k'^2 - m_\mu^2 + i\epsilon)(k^2 - m_\mu^2 + i\epsilon)[(k-p)^2 - m_V^2 + i\epsilon]}\boldsymbol{u}(\boldsymbol{p}). \quad (26)$$

The Feynman Parameters: They are a tool that allows us to evaluate loop integrals in quantum field theory in an easier way.

$$\frac{1}{A_1A_2\cdots A_n} = \int_0^1 dx_1dx_2\cdots dx_n\,\delta\left(\sum_{i=1}^n x_i - 1\right)\frac{(n-1)!}{[A_1x_1 + A_2x_2 + \cdots + A_nx_n]^n}. \quad (27)$$

For $n = 3$:

$$\frac{1}{A_1A_2A_3} = \int_0^1 dx_1dx_2dx_3\,\delta(x_1 + x_2 + x_3 - 1)\frac{2}{[A_1x_1 + A_2x_2 + A_3x_3]^3}. \quad (28)$$

PARTIAL RESULTS

In this case:

$$\begin{aligned} A_1 &= k'^2 - m_\mu^2 + i\epsilon \\ A_2 &= k^2 - m_\mu^2 + i\epsilon \\ A_3 &= (k-p)^2 - m_V^2 + i\epsilon. \end{aligned} \quad (29)$$

Furthermore, considering **4-momentum conservation** at the vertex ($k' = k + q$) and the **Dirac delta condition** ($x_1 + x_2 + x_3 = 1$), we have:

$$4ig_e^2\kappa^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx_1 dx_2 dx_3\, \bar{\boldsymbol{u}}(\boldsymbol{p}') \frac{\delta(x_1+x_2+x_3-1)\left[\not{k}\gamma^\mu \not{k}' - 2m_\mu(k'^\mu + k^\mu) + m_\mu^2\gamma^\mu\right]}{\left[k^2 + 2k(x_1 q - x_3 p) + x_1 q^2 + x_3 p^2 - m_\mu^2(x_1 + x_2) - m_V^2 x_3 + i\epsilon\right]^3} \boldsymbol{u}(\boldsymbol{p}). \quad (30)$$

Developing the denominator:

$$4ig_e^2\kappa^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx_1 dx_2 dx_3\, \bar{\boldsymbol{u}}(\boldsymbol{p}') \frac{\delta(x_1+x_2+x_3-1)\left[\not{k}\gamma^\mu \not{k}' - 2m_\mu(k'^\mu + k^\mu) + m_\mu^2\gamma^\mu\right]}{\left[(k + x_1 q - x_3 p)^2 - m_\mu^2(1-x_3)^2 - m_V^2 x_3 + x_1 x_2 q^2 + i\epsilon\right]^3} \boldsymbol{u}(\boldsymbol{p}). \quad (31)$$

And, defining the following parameters:

$$l = k + x_1 q - x_3 p, \qquad \Delta = m_\mu^2(1-x_3)^2 + m_V^2 x_3 - x_1 x_2 q^2.$$

PARTIAL RESULTS

First, the numerator must be written in terms of the new variable l:

$$N = \not{k}\gamma^{\mu}\not{k}' - 2m_{\mu}(k'^{\mu} + k^{\mu}) + m_{\mu}^{2}\gamma^{\mu}. \qquad (32)$$

For the **first term**:

$$\not{k}\gamma^{\mu}\not{k}' = l\gamma^{\mu}l + l[\gamma^{\mu}x_3p + \gamma^{\mu}(1-x_1)q] + [x_3p + x_1q]\gamma^{\mu}l + [x_3p + x_1q]\gamma^{\mu}x_3p + [x_3p + x_1q]\gamma^{\mu}(1-x_1)q.$$

For the **second term**:

$$-2m_{\mu}(k'^{\mu} + k^{\mu}) = -4m_{\mu}l^{\mu} - 2m_{\mu}[(1-2x_1)q^{\mu} + 2x_3p^{\mu}]. \qquad (34)$$

Then:

$$N = \not{l}\gamma^{\mu}\not{l} + \not{l}[\gamma^{\mu}x_3\not{p} + \gamma^{\mu}(1-x_1)\not{q}] + [x_3\not{p} + x_1\not{q}]\gamma^{\mu}\not{l} + [x_3\not{p} + x_1\not{q}]\gamma^{\mu}x_3\not{p} + [x_3\not{p} + x_1\not{q}]\gamma^{\mu}(1-x_1)\not{q}$$
$$-4m_{\mu}l^{\mu} - 2m_{\mu}[(1-2x_1)q^{\mu} + 2x_3p^{\mu}] + m_{\mu}^{2}\gamma^{\mu}. \qquad (35)$$

Now, considering the following identities:

$$\int \frac{d^4l}{(2\pi)^4}\frac{l^{\mu}}{(l^2-\Delta+i\epsilon)^3} = 0, \qquad \int \frac{d^4l}{(2\pi)^4}\frac{l^{\mu}l^{\nu}}{(l^2-\Delta+i\epsilon)^3} = \int \frac{d^4l}{(2\pi)^4}\frac{\frac{1}{4}g^{\mu\nu}l^2}{(l^2-\Delta+i\epsilon)^3}.$$

PARTIAL RESULTS

The **general expression** can be simplified to:

$$\bar{\boldsymbol{u}}(\boldsymbol{p}')\Gamma^{\mu(1)}(p,p')\boldsymbol{u}(\boldsymbol{p}) = 4ig_e^2\kappa^2 \int \frac{d^4l}{(2\pi)^4}\int_0^1 dx_1 dx_2 dx_3\, \bar{\boldsymbol{u}}(\boldsymbol{p}')\frac{\delta(x_1+x_2+x_3-1)}{[l^2-\Delta+i\epsilon]^3}$$
$$\times\left\{\left[-\frac{l^2}{2}+m_\mu^2(1-x_3^2-2x_3)+(1-x_2)(1-x_1)q^\mu\right]\gamma^\mu + x_3 m_\mu(x_3-1)(p^\mu+p'^\mu)\right\}\boldsymbol{u}(\boldsymbol{p}). \quad (36)$$

Or, with the **Gordon's decomposition**:

$$\bar{\boldsymbol{u}}(\boldsymbol{p}')\Gamma^{\mu(1)}(p,p')\boldsymbol{u}(\boldsymbol{p}) = 4ig_e^2\kappa^2 \int \frac{d^4l}{(2\pi)^4}\int_0^1 dx_1 dx_2 dx_3\, \bar{\boldsymbol{u}}(\boldsymbol{p}')\frac{\delta(x_1+x_2+x_3-1)}{[l^2-\Delta+i\epsilon]^3}$$
$$\times\left\{\left[-\frac{l^2}{2}+m_\mu^2(1-x_3^2-2x_3)+(1-x_2)(1-x_1)q^\mu\right]\gamma^\mu - 2x_3 m_\mu^2(x_3-1)\frac{i\sigma^{\mu\nu}q_\nu}{2m_\mu}\right\}\boldsymbol{u}(\boldsymbol{p}). \quad (37)$$

Comparing expression (37) with the expression of the vertex correction function (20):

$$F_2^{(1)}(q^2) = 4ig_e^2\kappa^2\int\frac{d^4l}{(2\pi)^4}\int_0^1 dx_1dx_2dx_3\,\delta(x_1+x_2+x_3-1)\frac{2x_3m_\mu^2(1-x_3)}{[l^2-\Delta+i\epsilon]^3}. \quad (38)$$

Note: $\Gamma^{\mu(1)}(p,p') = F_1^{(1)}(q^2)\gamma^\mu + iF_2^{(1)}(q^2)\frac{q_\nu\sigma^{\mu\nu}}{2m_\mu}.$ (20)

Wick's rotation: This procedure involves a complex rotation in the time component of the momentum l ($l^0 \to i l_E^0$), changing the space-time metric from Lorentzian to Euclidean signature. This makes it easier to integrate:

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{[l^2 - \Delta + i\epsilon]^n} = \frac{i(-1)^n}{(4\pi)^2} \frac{1}{(n-1)(n-2)} \frac{1}{\Delta^{n-2}}.$$

For this case ($n = 3$):

$$\begin{aligned} F_2^{(1)}(q^2) &= 4 g_e^2 \kappa^2 \int_0^1 dx_1 dx_2 dx_3\, \delta(x_1 + x_2 + x_3 - 1) \frac{2 x_3 m_\mu^2 (1 - x_3)}{2(4\pi)^2 \Delta} \\ &= \frac{\alpha}{2\pi} \times \kappa^2 \int_0^1 dx_1 dx_2 dx_3\, \delta(x_1 + x_2 + x_3 - 1) \frac{x_3 m_\mu^2 (1 - x_3)}{m_\mu^2 (1 - x_3)^2 + m_V^2 x_3 - x_1 x_2 q^2}. \end{aligned} \tag{39}$$

Evaluating $F_2^{(1)}(q^2 = 0) = a_\mu^V$:

$$\begin{aligned} a_\mu^V &= \frac{\alpha}{2\pi} \times \kappa^2 \int_0^1 dx_1 dx_2 dx_3\, \delta(x_1 + x_2 + x_3 - 1) \frac{x_3 m_\mu^2 (1 - x_3)}{m_\mu^2 (1 - x_3)^2 + m_V^2 x_3} \\ &= \frac{\alpha}{2\pi} \times \kappa^2 \int_0^1 dx_3 \int_0^{1 - x_3} dx_1 \frac{x_3 m_\mu^2 (1 - x_3)}{m_\mu^2 (1 - x_3)^2 + m_V^2 x_3}. \end{aligned} \tag{40}$$

PARTIAL RESULTS

$$\Rightarrow \boldsymbol{a_\mu^V(m_V,\kappa^2)} = \frac{\alpha}{2\pi}\kappa^2\int_0^1 dx_3\,\frac{2x_3m_\mu^2(1-x_3)^2}{m_\mu^2(1-x_3)^2+m_V^2x_3}$$

The $\boldsymbol{\kappa^2}$ **factor** in the contribution expression affects the **order of magnitude** of $\boldsymbol{a_\mu^V}$.

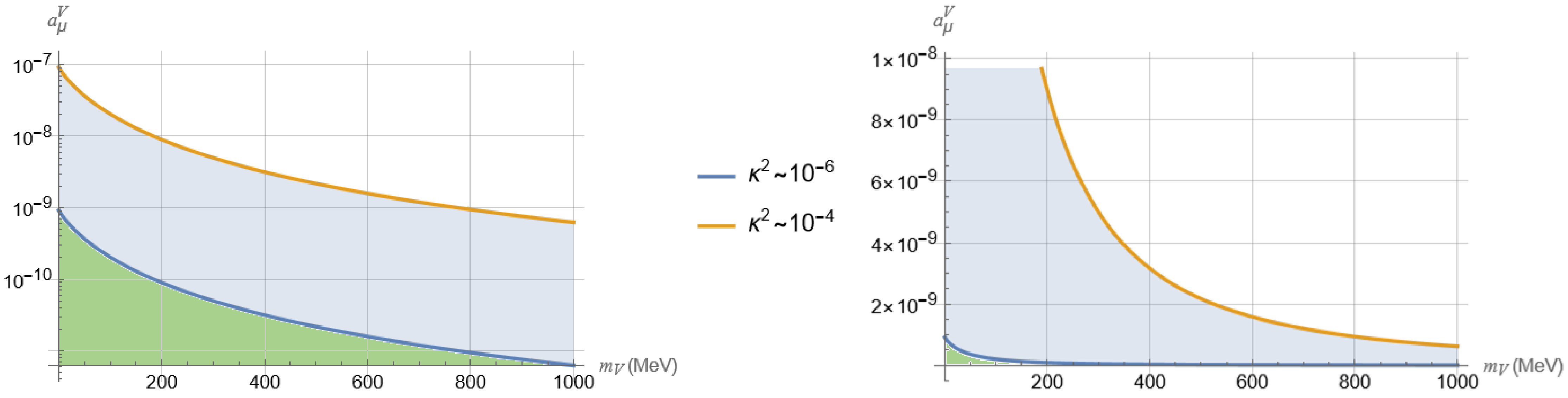


Figure 12: Z' boson contribution for the anomaly.

The $\boldsymbol{a_\mu^V}$ value is inversely proportional to $\boldsymbol{Z'}$ **boson mass** $(\boldsymbol{m_V})$.

Upper limits $\kappa \sim \mathcal{O}(10^{-2}-10^{-3})$ **[4]**

$m_V \gtrsim 100\ MeV$ **[5]**

PARTIAL RESULTS

This result will be compared with the theoretical predictions of **WP**, **CMD-3** and **BMW'24**.

$a_\mu^{Exp} = 116\,592\,055(24) \times 10^{-11}$

Collaboration	a_μ^{SM}	$a_\mu^{NP} = a_\mu^{Exp} - a_\mu^{SM}$
White Paper	$116\,591\,810(43) \times 10^{-11}$	$245(49) \times 10^{-11}$
CMD-3	$116\,592\,006(49) \times 10^{-11}$	$49(55) \times 10^{-11}$
BMW'24	$116\,592\,019(38) \times 10^{-11}$	$36(45) \times 10^{-11}$

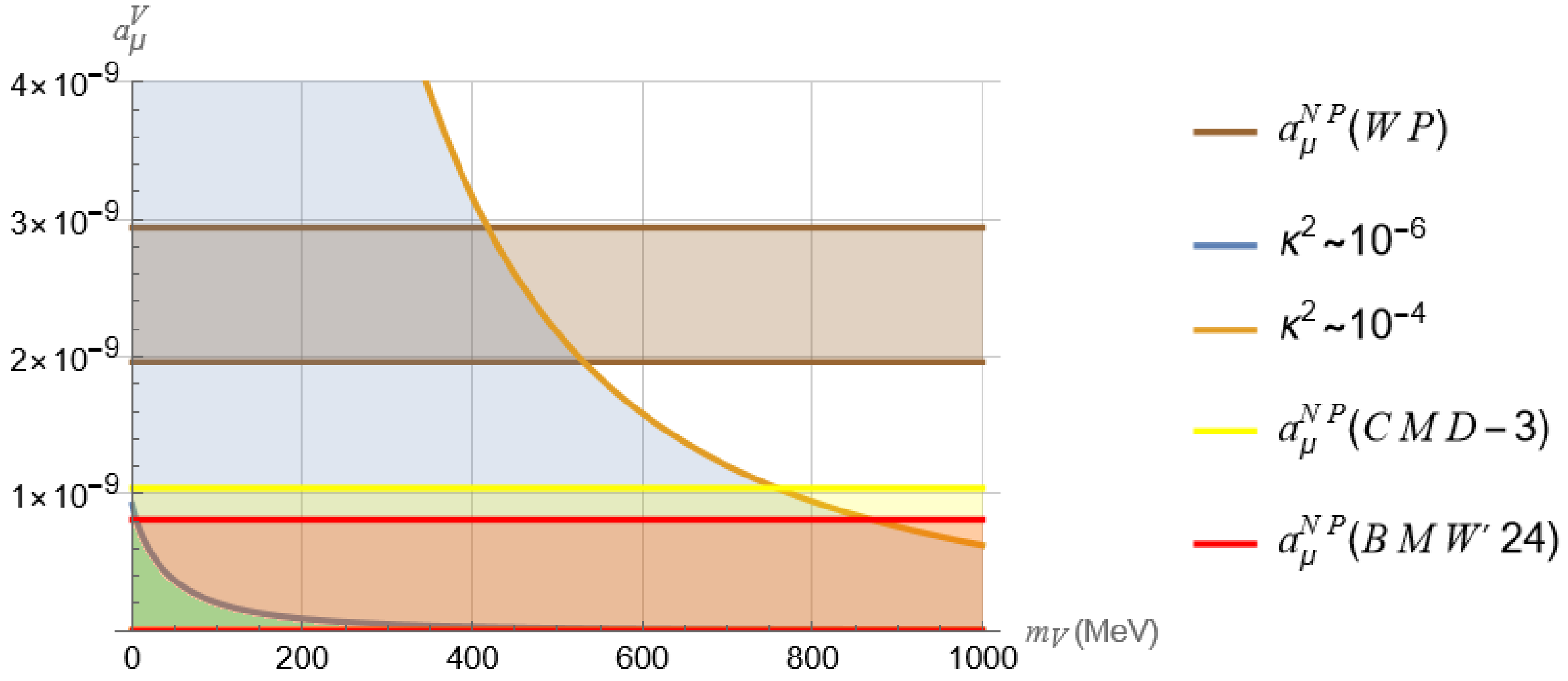
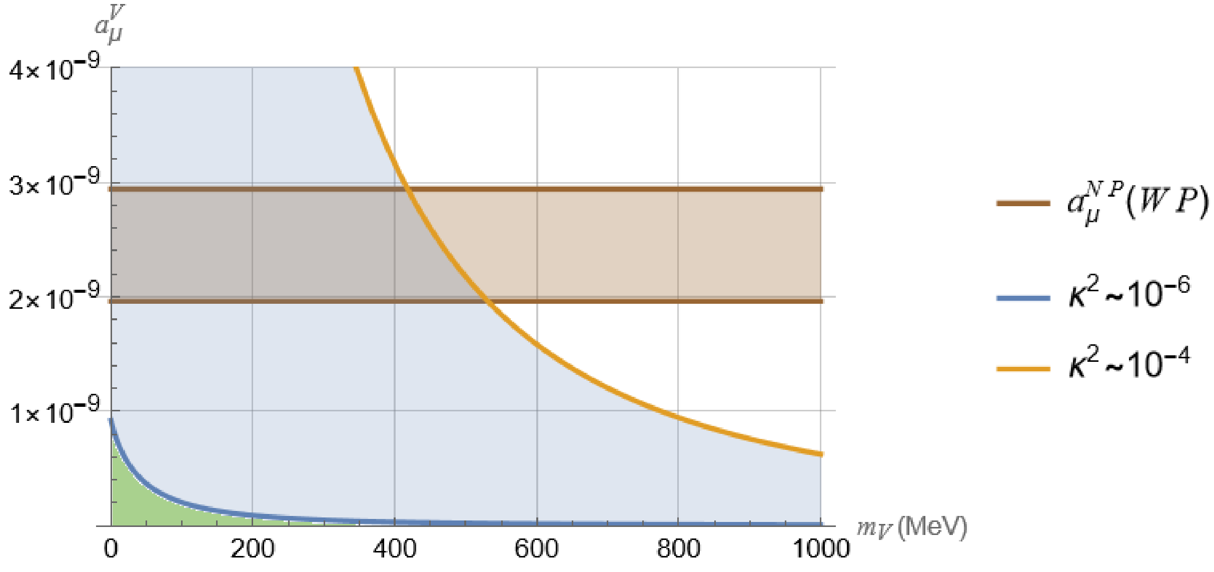
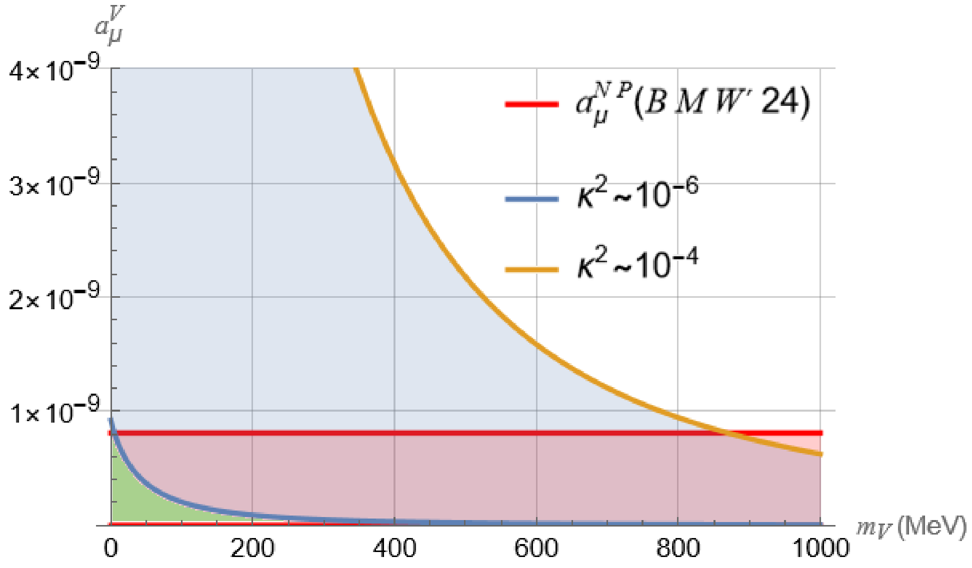


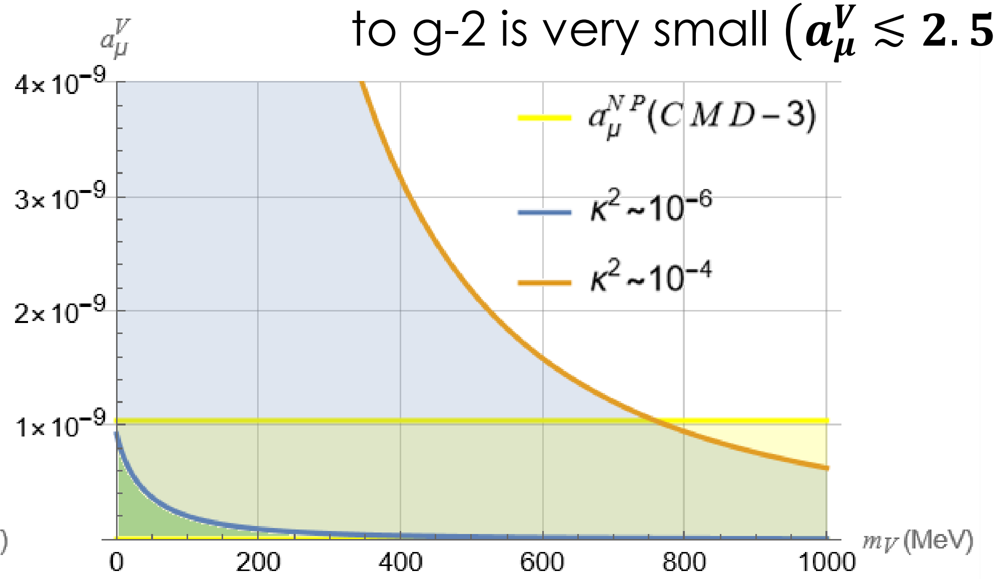
Figure 13: Comparison between the Z' boson contribution for the anomaly and theoretical predictions.

PARTIAL RESULTS



- For $\mathbf{100\ MeV} < \boldsymbol{m_V} < \mathbf{550}\ \boldsymbol{MeV}$ and $\mathbf{10^{-6}} \lesssim \boldsymbol{\kappa^2} \lesssim \mathbf{10^{-4}}$ there is agreement with the **WP** prediction.
- For $\boldsymbol{\kappa^2} \lesssim \mathbf{10^{-6}}$ the discrepancy presented by **WP** cannot be explained.
- For $\boldsymbol{m_V} \gtrsim \mathbf{100\ MeV}$ and $\boldsymbol{\kappa^2} \lesssim \mathbf{10^{-6}}$ there is agreement with the predictions of **CMD-3** and **BMW'24**. However, their contribution to g-2 is very small $\left(\boldsymbol{a_\mu^V} \lesssim \mathbf{2.5} \times \mathbf{10^{-10}}\right)$.





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