Quantum Tomography and Entanglement of Top Quarks at the LHC with the CMS Experiment

On behalf of the CMS Collaboration,

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December 5th, 2024

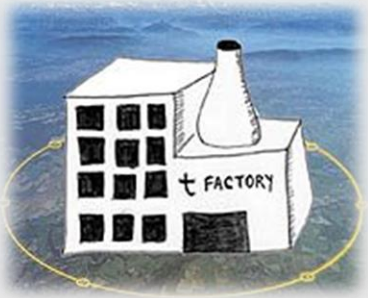
Top Quark Physics

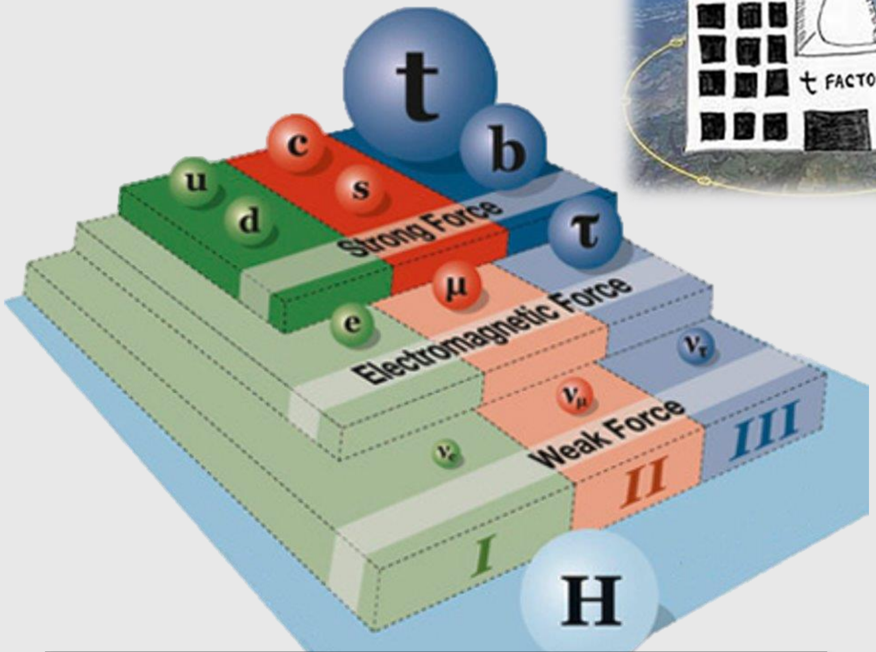
- Heaviest fundamental particle discovered thus far

  $\boldsymbol{m_t = 172.52 \pm 0.33}$ **GeV** [PRL 132 261902]

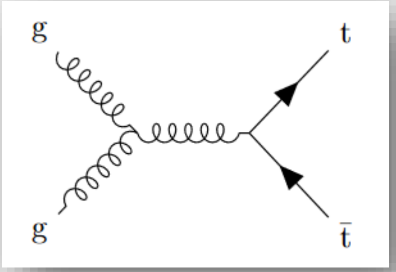
- LHC is a top quark factory with more than 100M pairs produced thus far

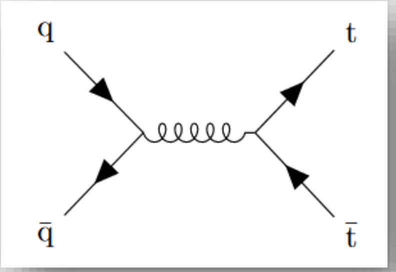
- Spin information is preserved best in the leptonic decays of the top quark

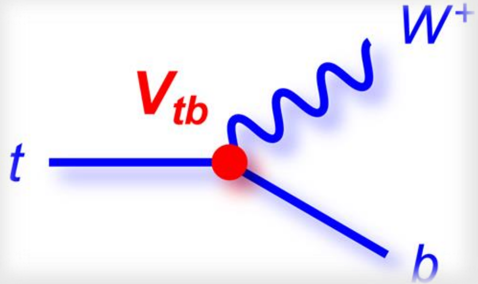




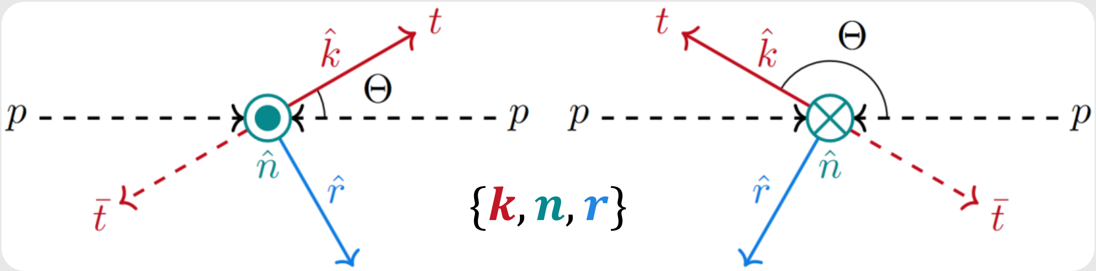
$$\underbrace{\frac{1}{m_t}}_{\substack{\text{production} \\ 10^{-27}\ \text{s}}} < \underbrace{\frac{1}{\Gamma_t}}_{\substack{\text{lifetime} \\ 10^{-25}\ \text{s}}} < \underbrace{\frac{1}{\Lambda_{QCD}}}_{\substack{\text{hadronization} \\ 10^{-24}\ \text{s}}} < \underbrace{\frac{m_t}{\Lambda^2}}_{\substack{\text{spin-flip} \\ 10^{-21}\ \text{s}}}$$







Top Spin Correlations



$$\frac{1}{\sigma}\frac{d^4\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\left(1 + \boldsymbol{B}_1 \cdot \hat{l}_1 + \boldsymbol{B}_2 \cdot \hat{l}_2 - \hat{l}_1 \cdot \boldsymbol{C} \cdot \hat{l}_2\right)$$

$\Omega_{1,2}$: Lepton solid angle in parent top rest frame

- Top quark spin cannot be measured directly

Spin Polarization $\boldsymbol{B}_{1,2}$

Spin Correlation $\boldsymbol{C}$

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{1}{2}(1 + \lambda_x x) f(x)$$

λ_x extracted from asymmetry

- Typically measured in the helicity basis $\{\boldsymbol{k}, \boldsymbol{n}, \boldsymbol{r}\}$
- Measurement of spin correlations can be used to perform quantum state tomography

Spin Density Matrix

$$\rho = 1_4 + B_i^+ \sigma^i \otimes 1_2 + B_i^- 1_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j$$

- Spin correlations depend on higher orbital momenta ($m_{t\bar{t}}$, $\cos\Theta$) and initial state

Quantum Correlations

- Top pairs are produced in mixed states

- A whole hierarchy of quantum correlations exists for mixed states

- All can be studied from the measurement of $\boldsymbol{B}$ and $\boldsymbol{C}$.

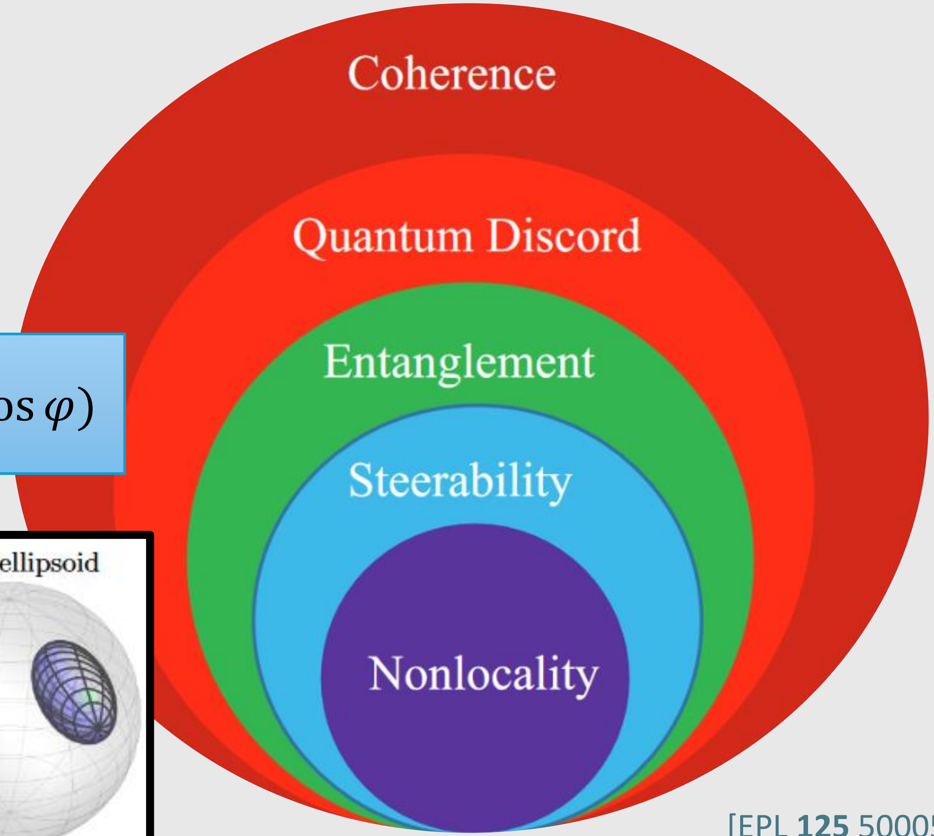
- $\cos\varphi = \hat{l}^{+} \cdot \hat{l}^{-}$ offers a proxy for entanglement ($D = -\mathrm{Tr}(\boldsymbol{C})/3$)

- Quantum steering offers a geometric representation.

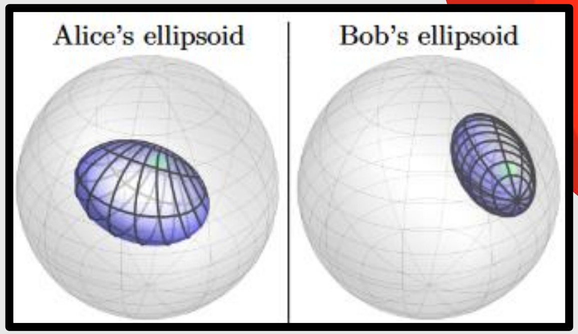
Spin Density Matrix

$$\rho = 1_4 + B_i^{+}\sigma^i \otimes 1_2 + B_i^{-} 1_2 \otimes \sigma^i + C_{ij}\sigma^i \otimes \sigma^j$$

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$



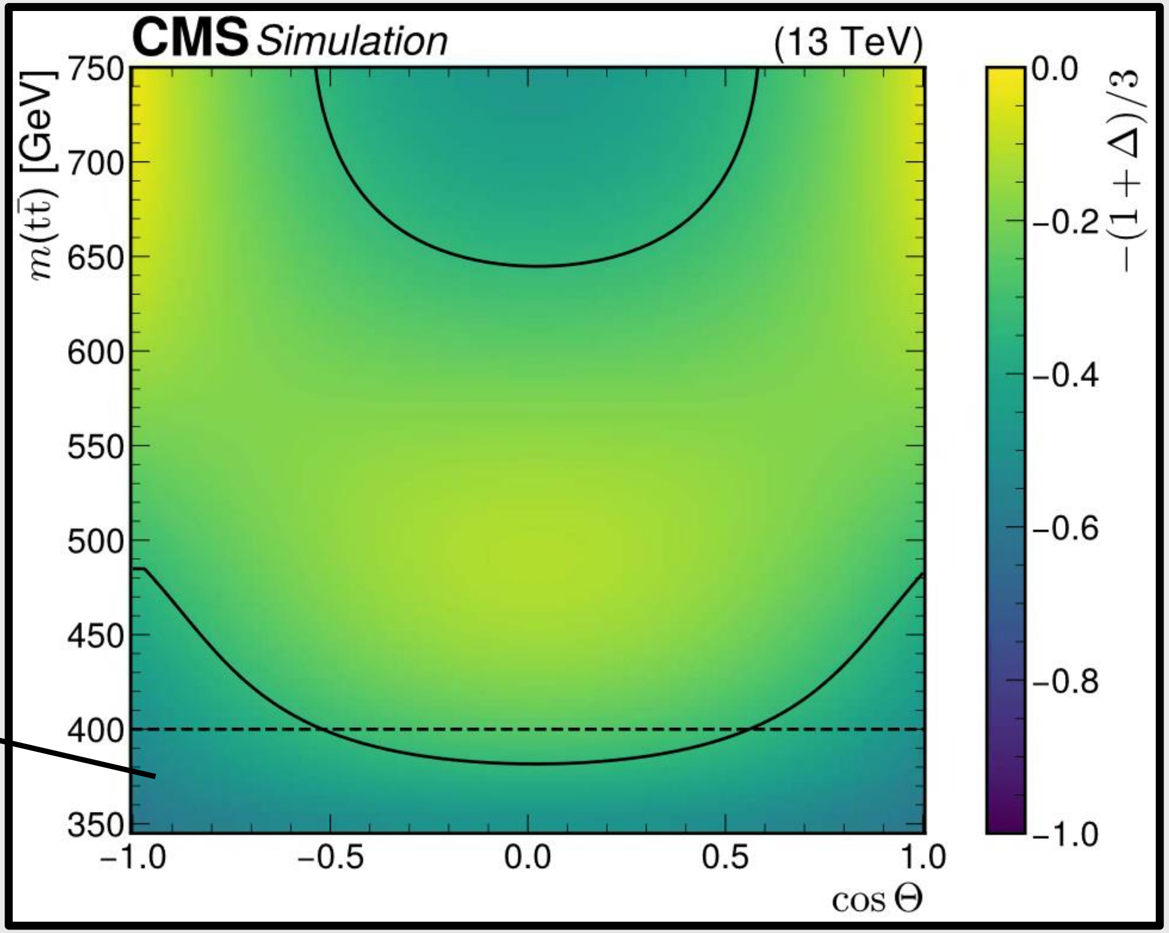
[EPL **125** 50005]



[PRL 113 020402]

SM Predictions

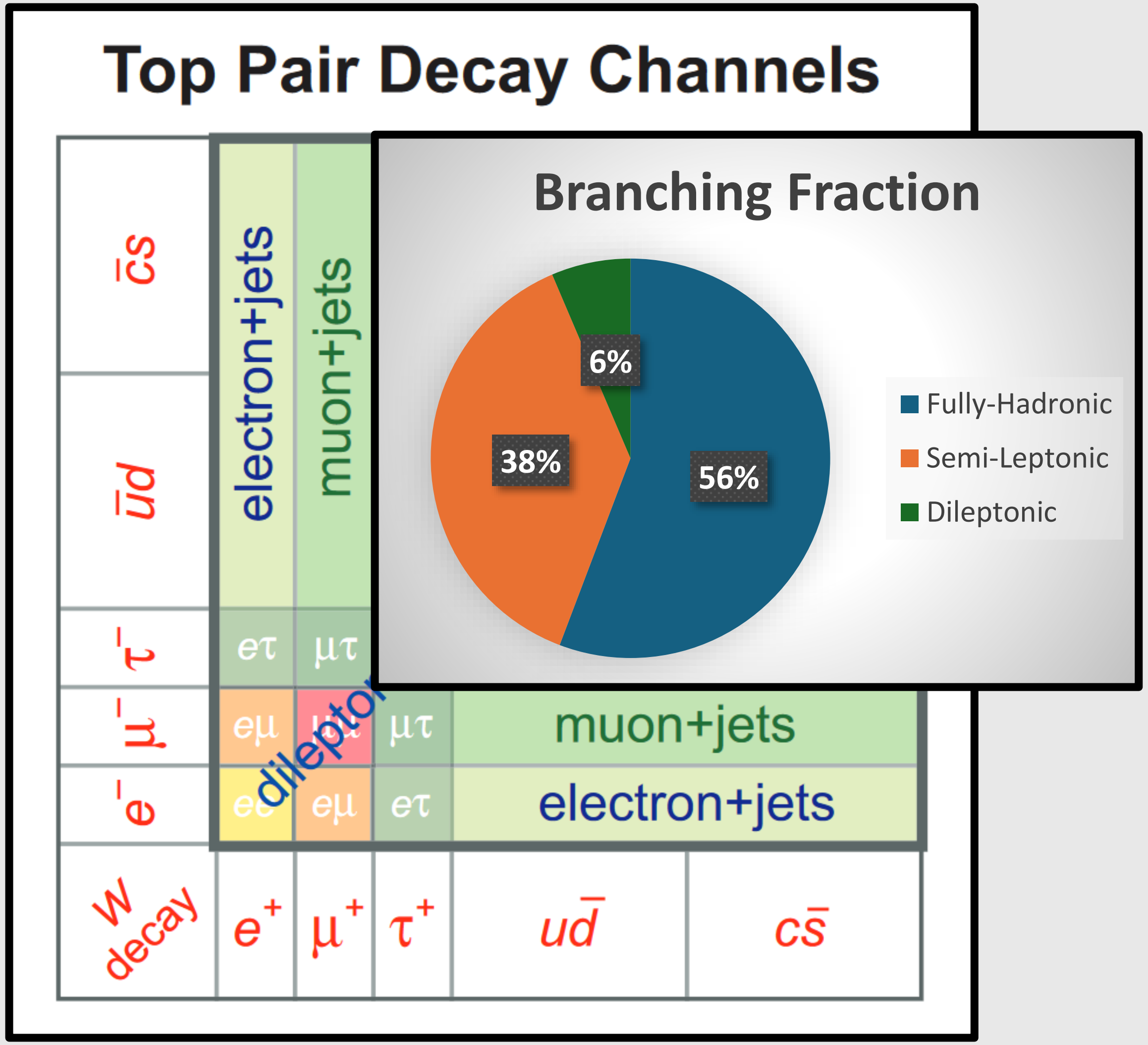
- Entanglement is observed by using the Peres-Horodecki criterion which for top pairs simplifies to
  $$\Delta = C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$
- At low $m(\mathrm{t\bar{t}})$, $C_{kk}, C_{rr} > 0$ then
  $$\Delta = \mathrm{Tr}(\boldsymbol{C}) - 1$$
  $$\Delta + 1 = -3D > 1$$



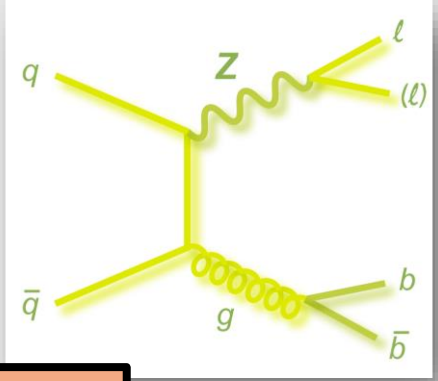
[ROPP 87 (2024) 117801]

Signal Process

- Signal: $t\bar{t}$ pairs decaying into $(W^+b)(W^-\bar{b})$, and W decaying into $e\nu$ or $\mu\nu$, "Prompt" signal
- τ channels are unstable, when it decays to $e\nu$ or $\mu\nu$, "viaTau" signal
- Sample produced at NLO with POWHEGv2
- Alternative samples produced with MG5_aMC@NLO at NLO using MADSPIN
- Parton shower & Hadronization: PYTHIA8



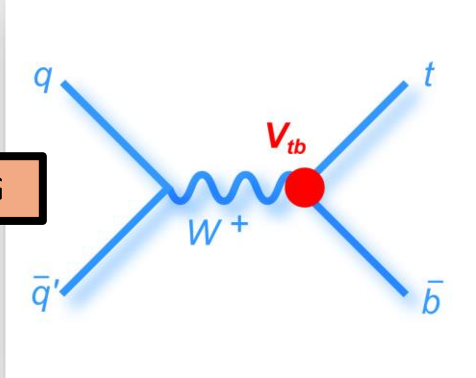
Background Processes



MG5_AMC@NLO

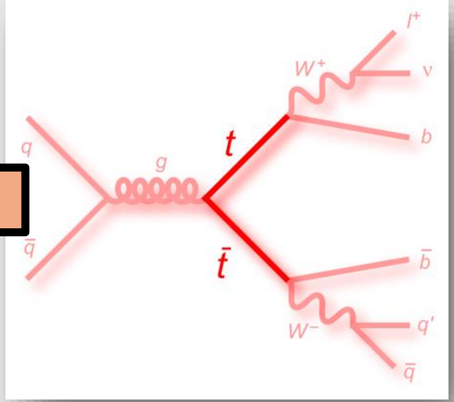
$Z + \text{Jets}$

POWHEG

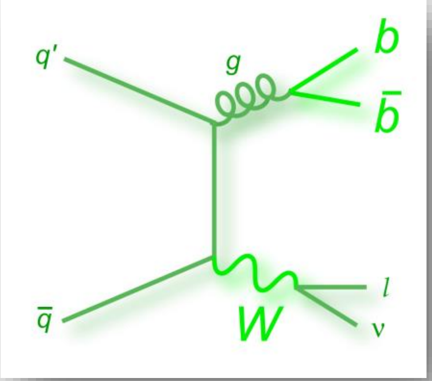


Single Top

POWHEG

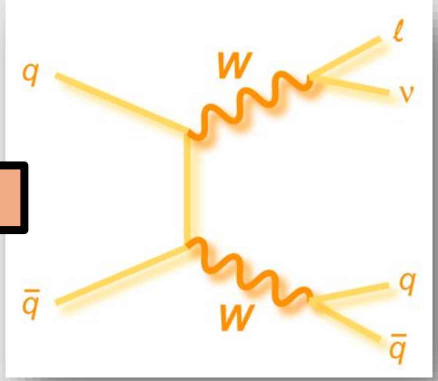


Other $t\bar{t}$

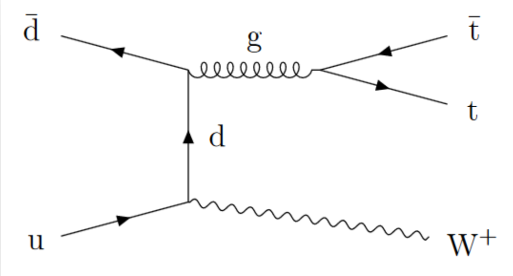


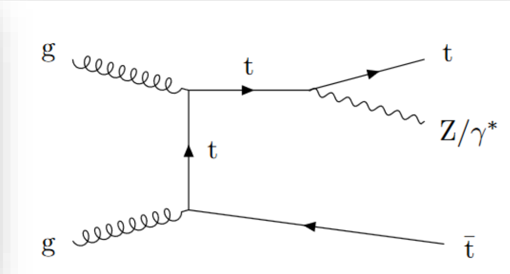
$W + \text{Jets}$

PYTHIA8



Diboson





MG5_AMC@NLO

$t\bar{t} + Z/W$

[JHEP08(2018)011]

Event Selection

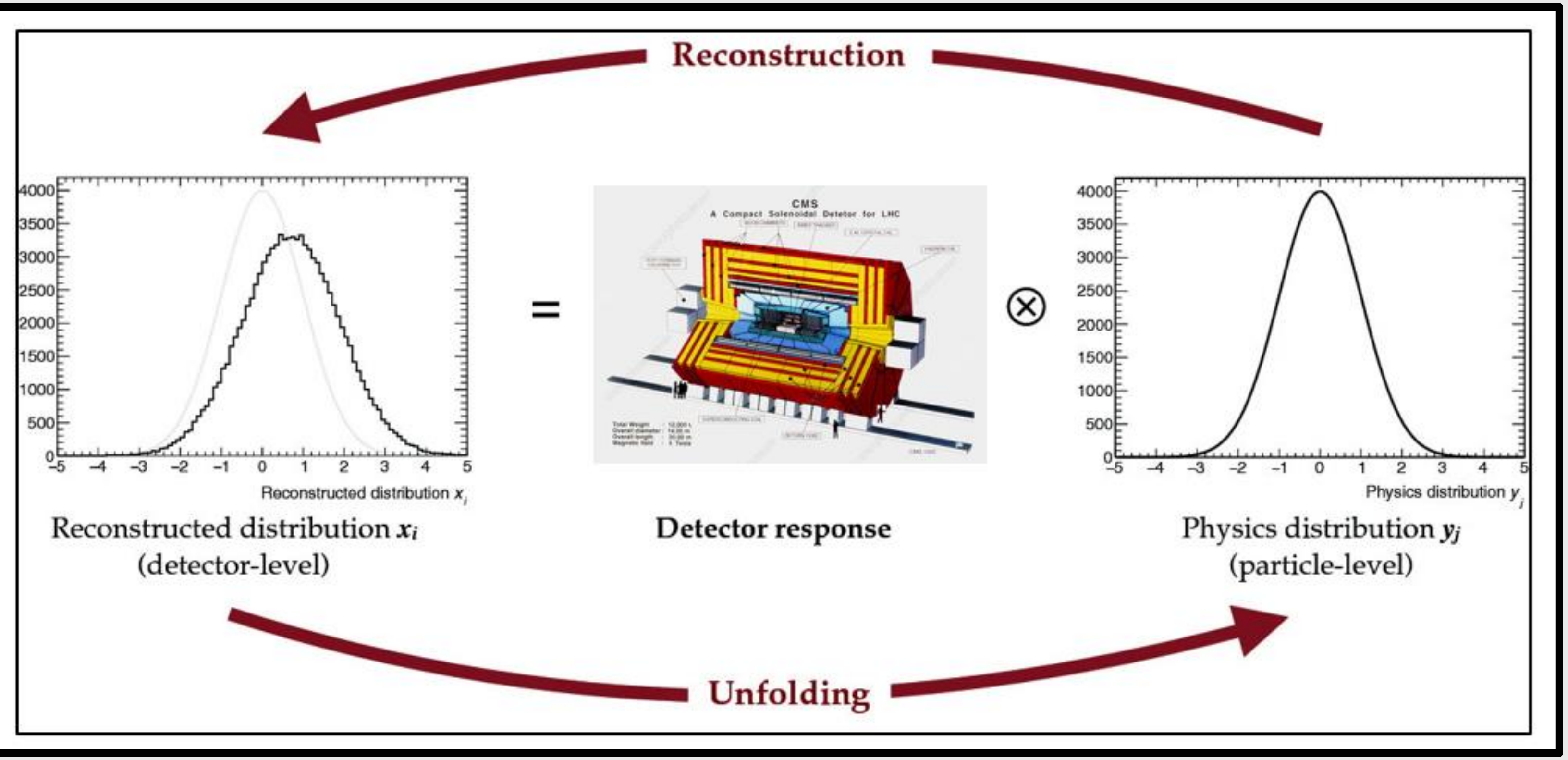
- 2 isolated, oppositely charged leptons (e^+e^-, $\mu^+\mu^-$,$e^\pm\mu^\mp$)
- $n_{\text{jets}} \geq 2$
- $n_{\text{bjets}} \geq 1$
- $m_{l\bar{l}} > 20$ GeV
- $Z +$ jets suppression (e^+e^-, $\mu^+\mu^-$):
  - $m_{l\bar{l}} < 76$ GeV or $m_{l\bar{l}} > 106$ GeV
  - $p_T^{\text{miss}} > 40$ GeV

[PRD 73 054015]

- Top kinematic reconstruction algorithm
  - Assumptions:
    - $p_T^{\text{miss}} \rightarrow$ coming from the two neutrinos only
    - $m_\nu = 0$ GeV, $m_W = 80.4$ GeV, $m_{\text{t}} = 172.5$ GeV
  - Solve analytical on-shell equations for each interaction vertex
  - Smear jet/lepton energies and directions for resolution effects
  - Final solution is a weighted average from 100 smearings
  - Events with no solution are rejected

Unfolding Procedure

$$\vec{y}^{\mathrm{MC}} = \mathbf{M} \cdot \vec{x}^{\mathrm{MC}}$$



Naively:
$$\vec{x} = \mathbf{M}^{-1}\vec{y}$$

- Regularization to suppress stat fluctuations
- χ^2 fit to apply both regularization and unfolding simultaneously

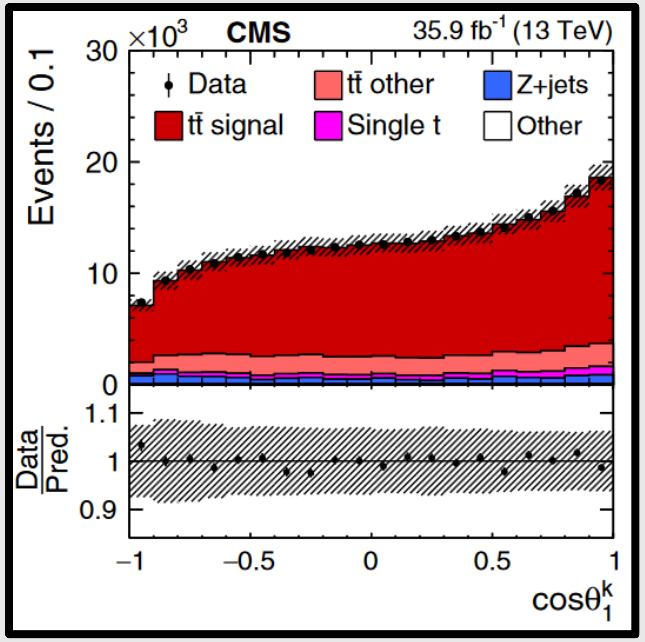
$$\chi^2_{\mathrm{unf}} = \chi^2_{\mathbf{M}} + \tau^2\chi^2_{\mathbf{L}} + \lambda\sum_i(\mathbf{M}\vec{x} - \vec{y})_i$$

Measurement of Top Quark Spin Density Matrix

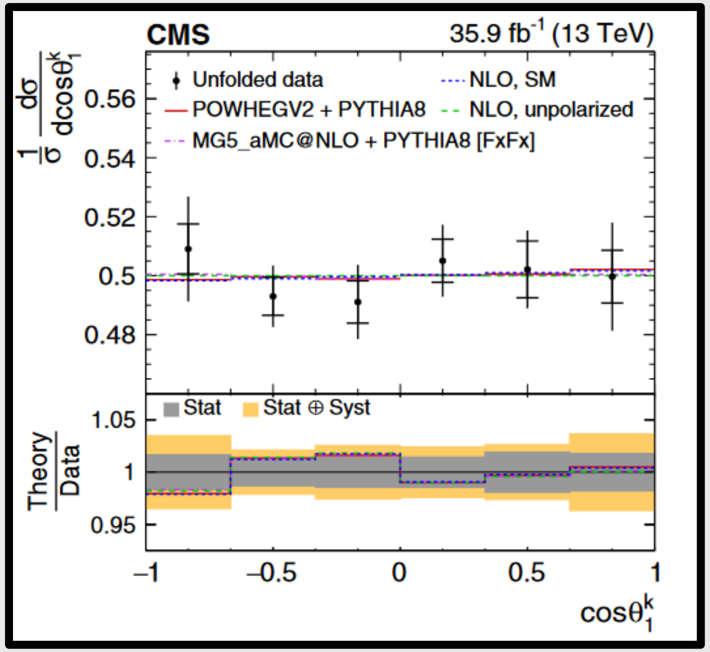
[PRD 100 (2019) 072002]

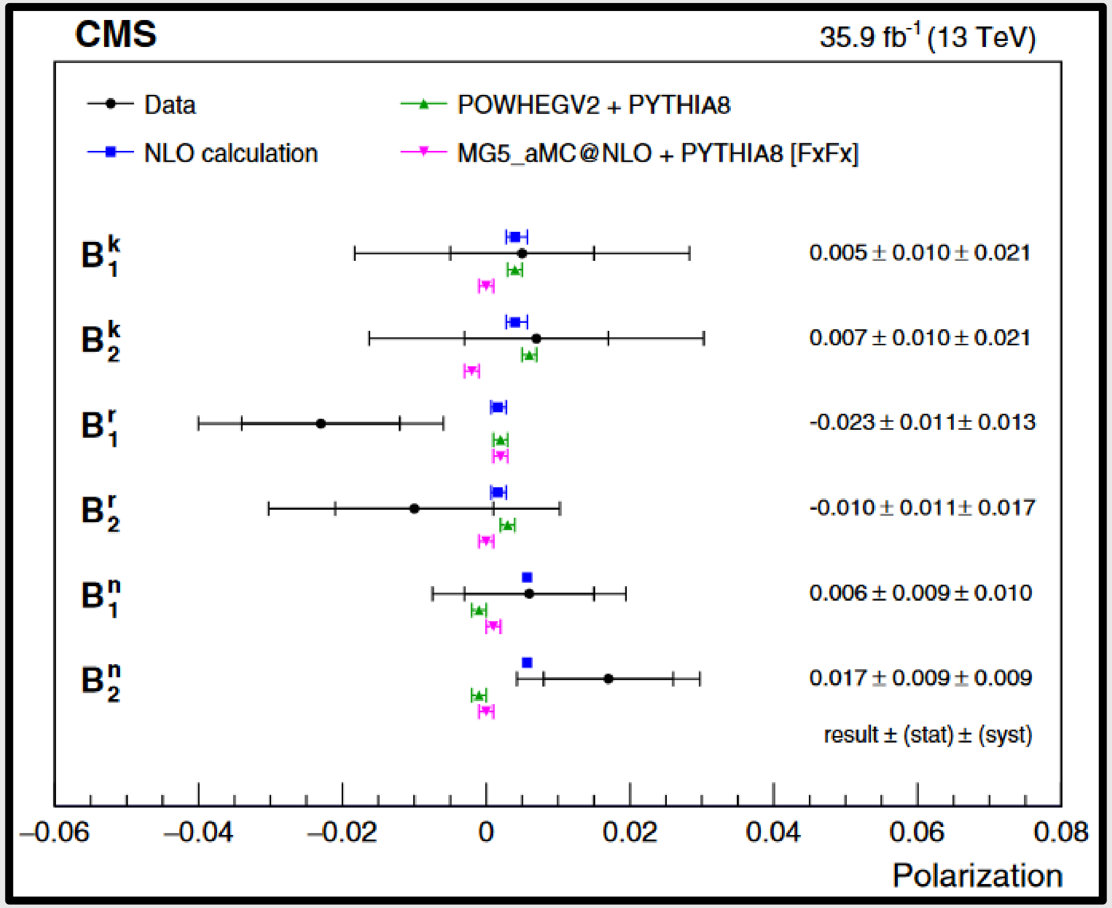
- Zero polarization observed at parton level

Reconstruction Level

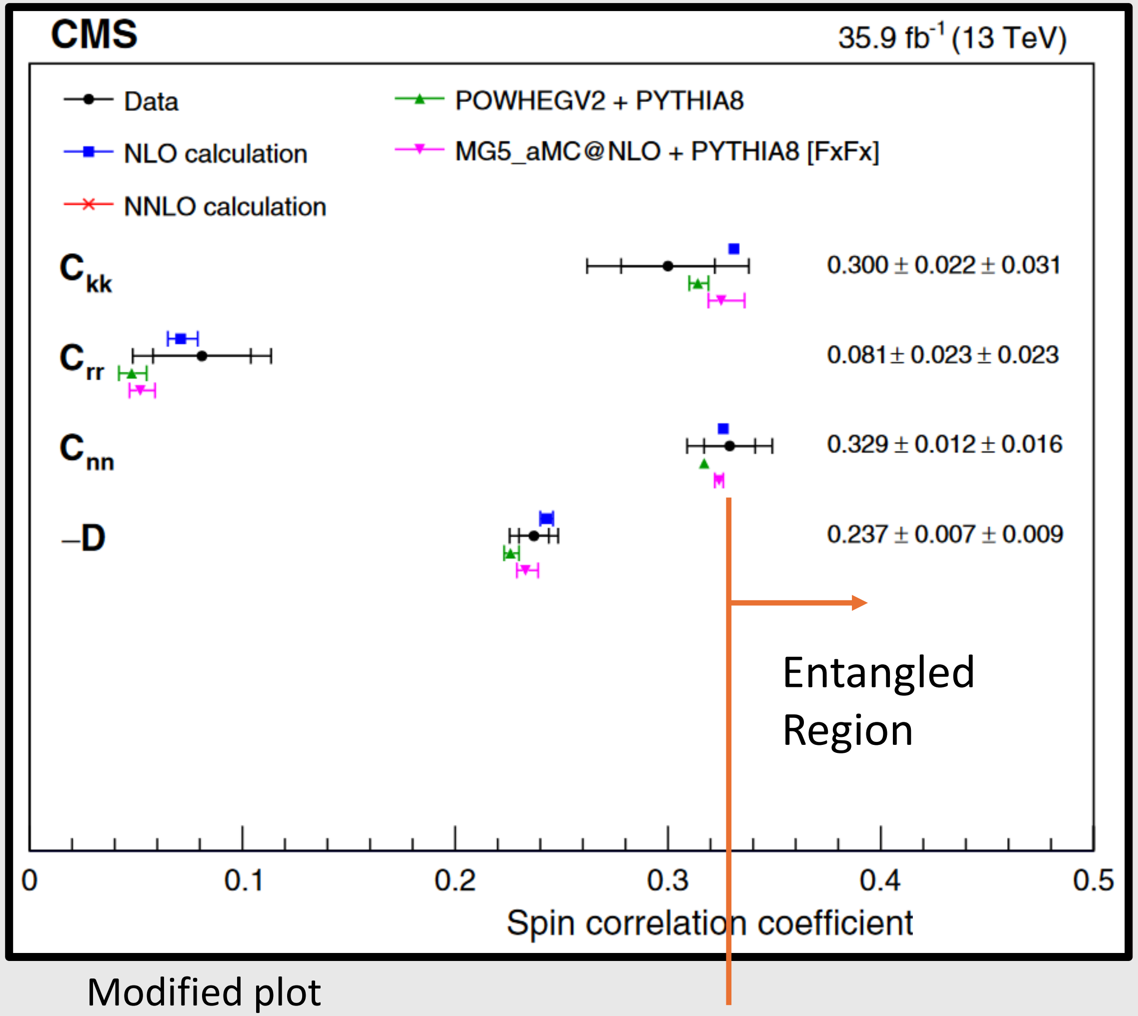


Unfolded Distribution





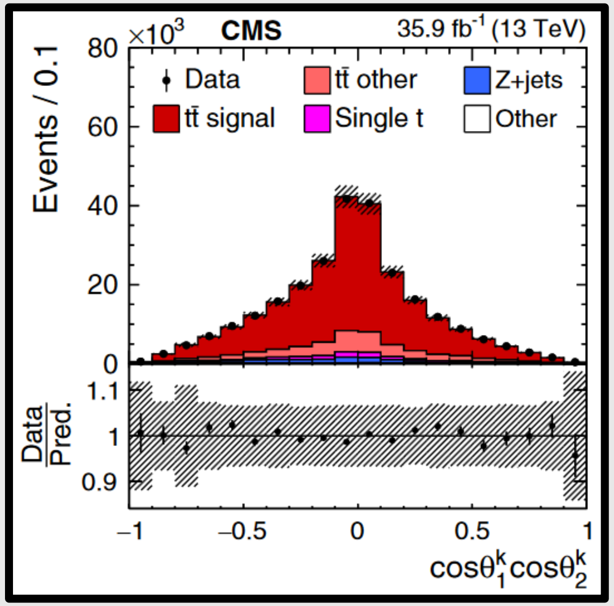
Measurement of Top Quark Spin Density Matrix



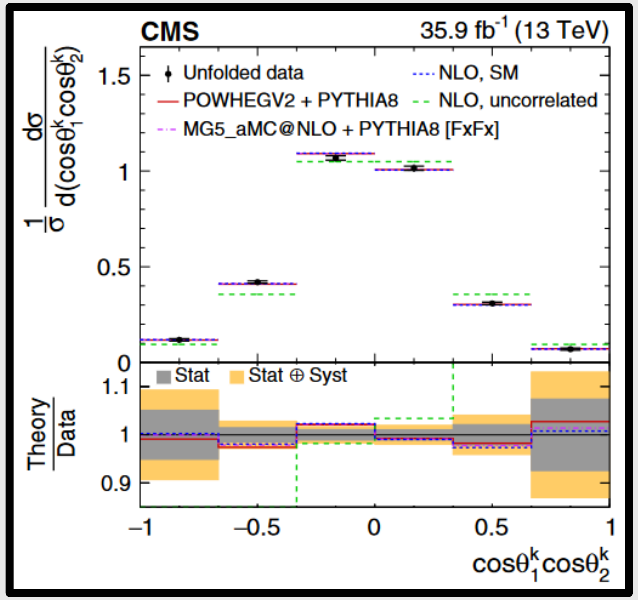
Modified plot

- Non-zero correlations observed

Reconstruction Level



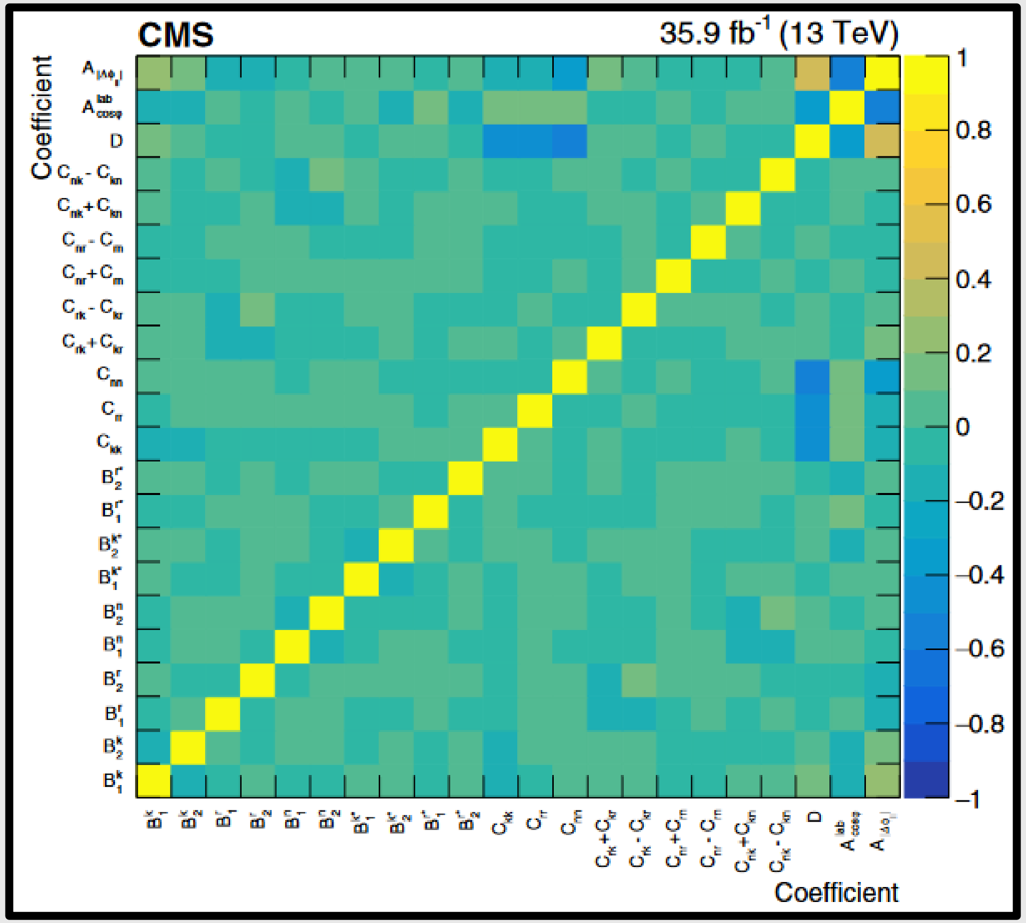
Unfolded Distribution



[PRD 100 (2019) 072002]

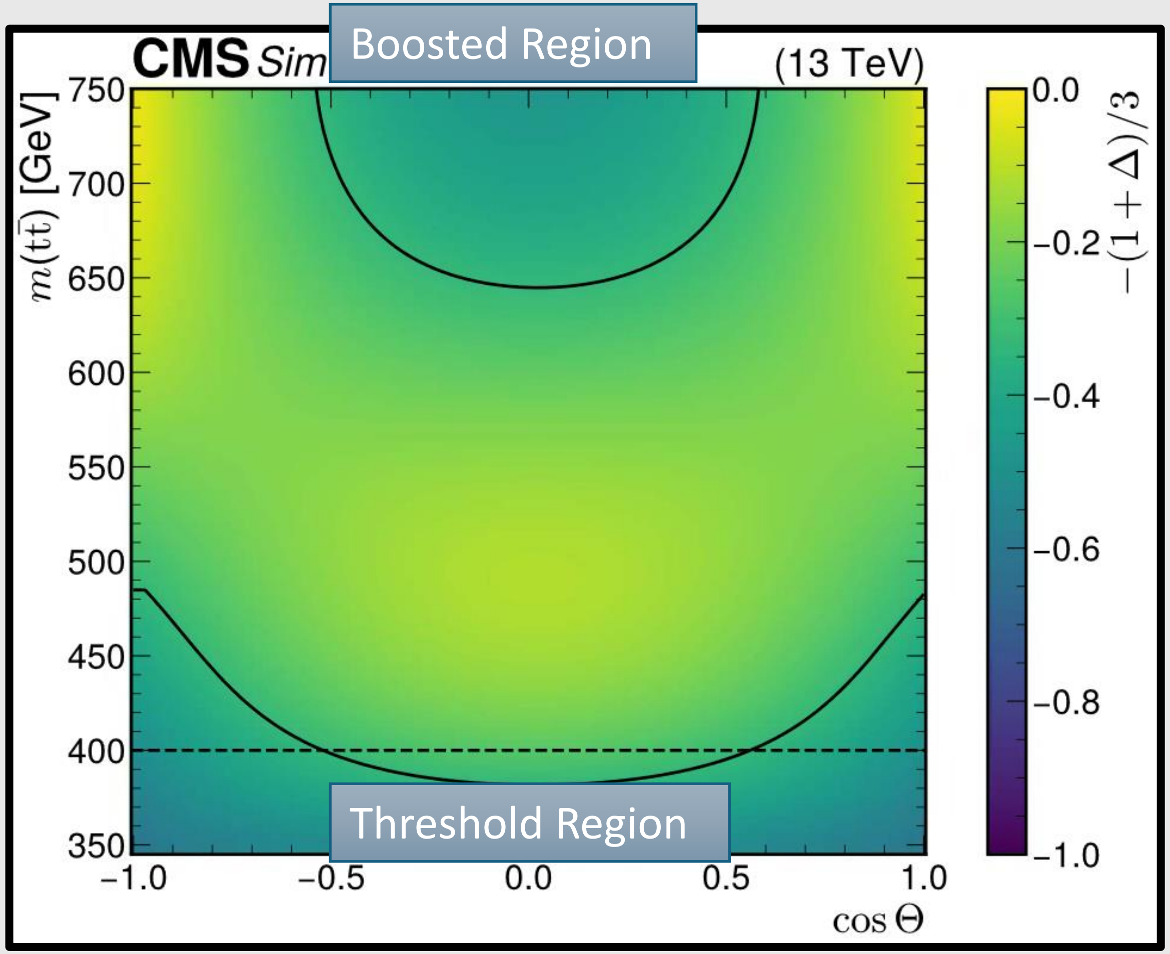
Measurement of Top Quark Spin Density Matrix

- Correlation coefficients and covariance matrices publicly available via [HepData]
- They give a spin density matrix ρ
- From ρ, other quantum correlations can be studied



[PRD 100 (2019) 072002]

Probing Entanglement



- Perform a binned profile likelihood fit of $\cos\varphi$ in:
  - $345 < m(\mathrm{t\bar{t}}) < 400$ GeV &
  - $\beta_z < 0.9$

$$\beta_z = \left|\frac{p_z^{\mathrm{t}} + p_z^{\bar{\mathrm{t}}}}{E^{\mathrm{t}} + E^{\bar{\mathrm{t}}}}\right|$$

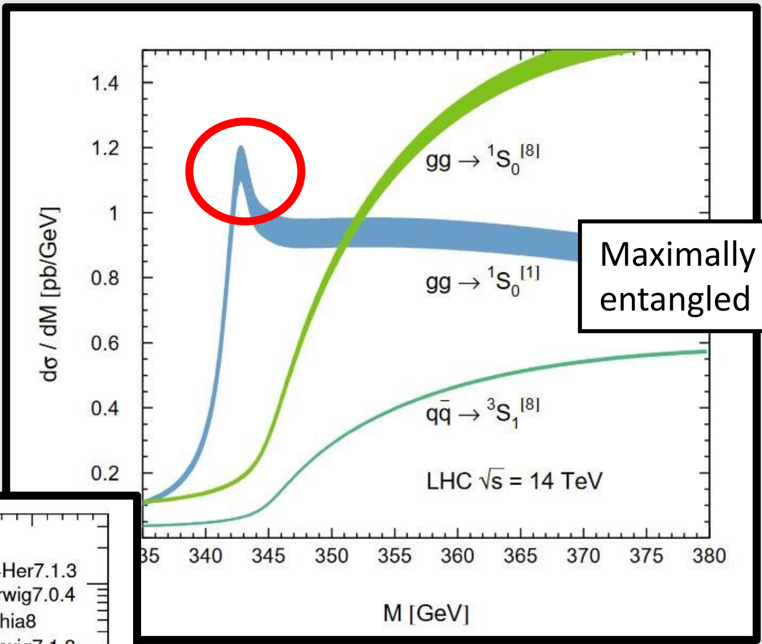
- Entanglement proxy $D = \mathrm{Tr}(\boldsymbol{C})/3$

$$\Delta + 1 = \mathrm{Tr}(\boldsymbol{C}) = -3D > 1$$

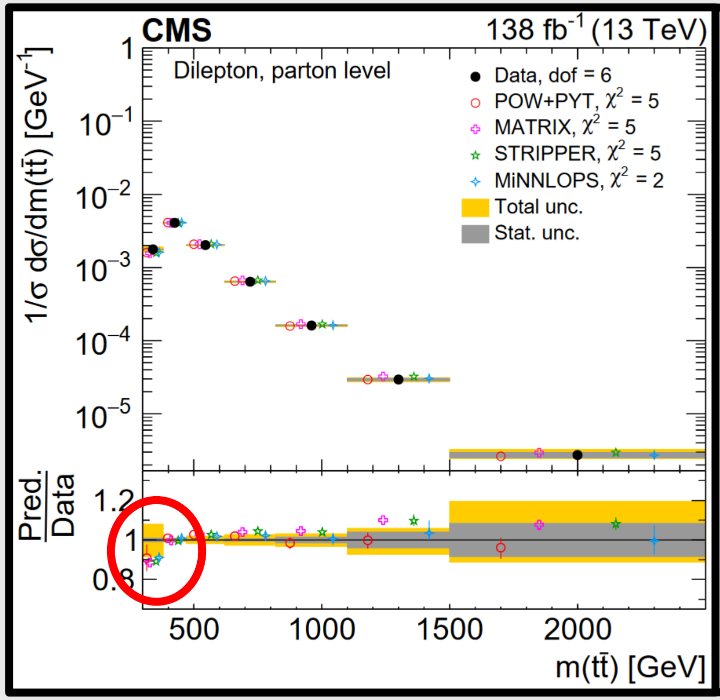
Singlet Bound State

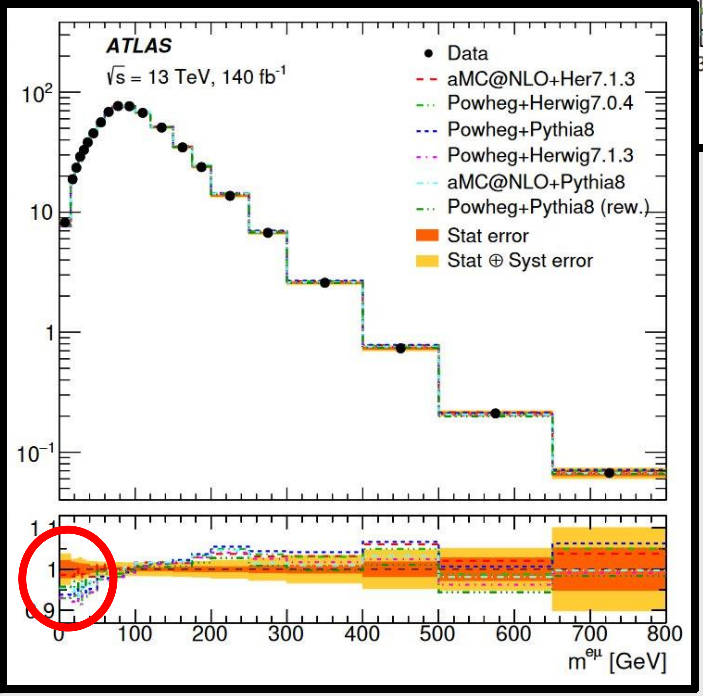
- Theory predicts a color singlet near threshold
- Large mismodeling seen for $m(t\bar{t}) \approx 345$ GeV
- Signal model includes a singlet toy model η_t

[JHEP 06 (2020), 158]



Maximally entangled state





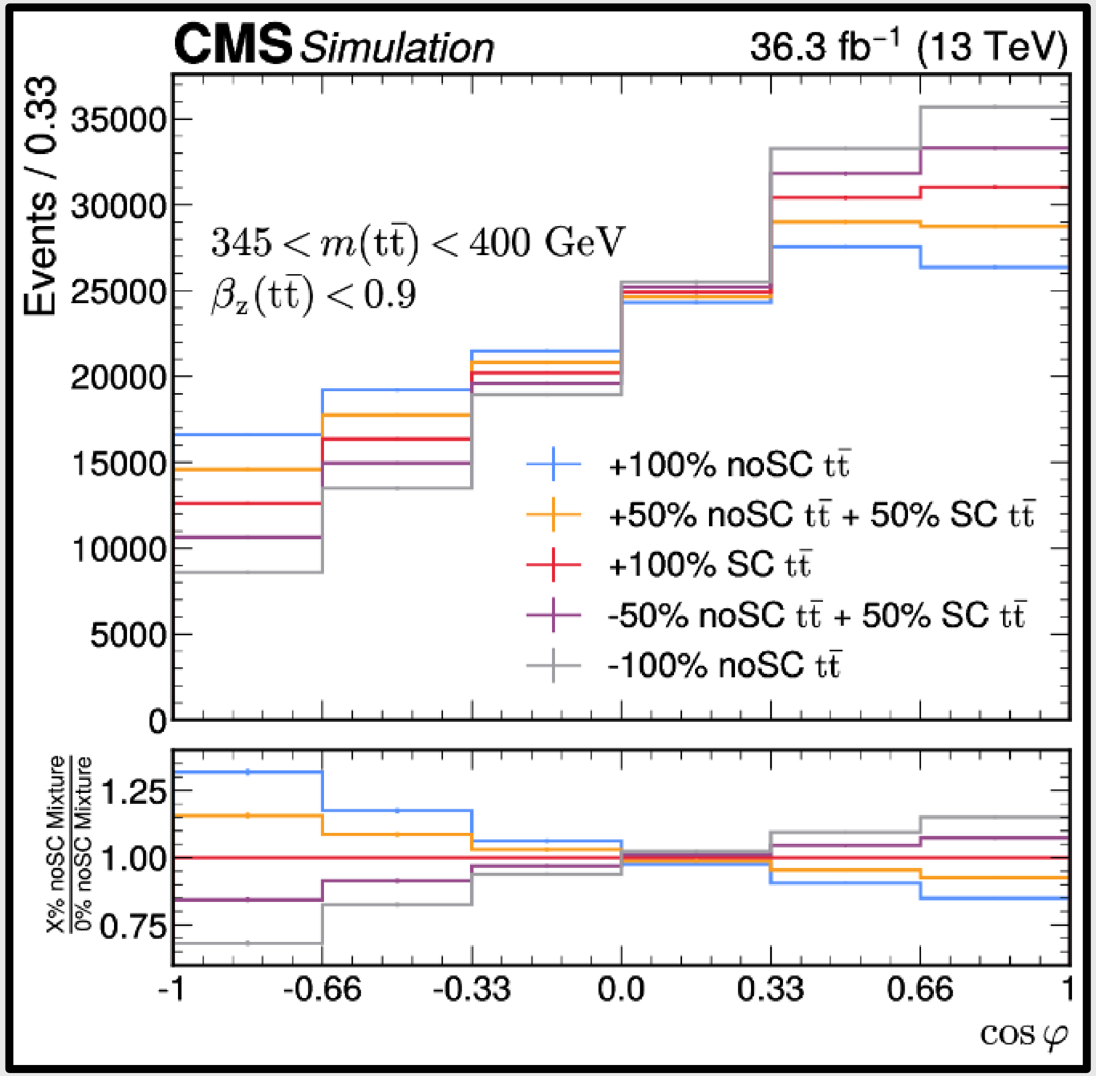
[JHEP 07 (2023), 141]

Excess of events observed at threshold [CMS-PAS-HIG-22-013]

[arXiv:2402.08486] (submitted to JHEP)

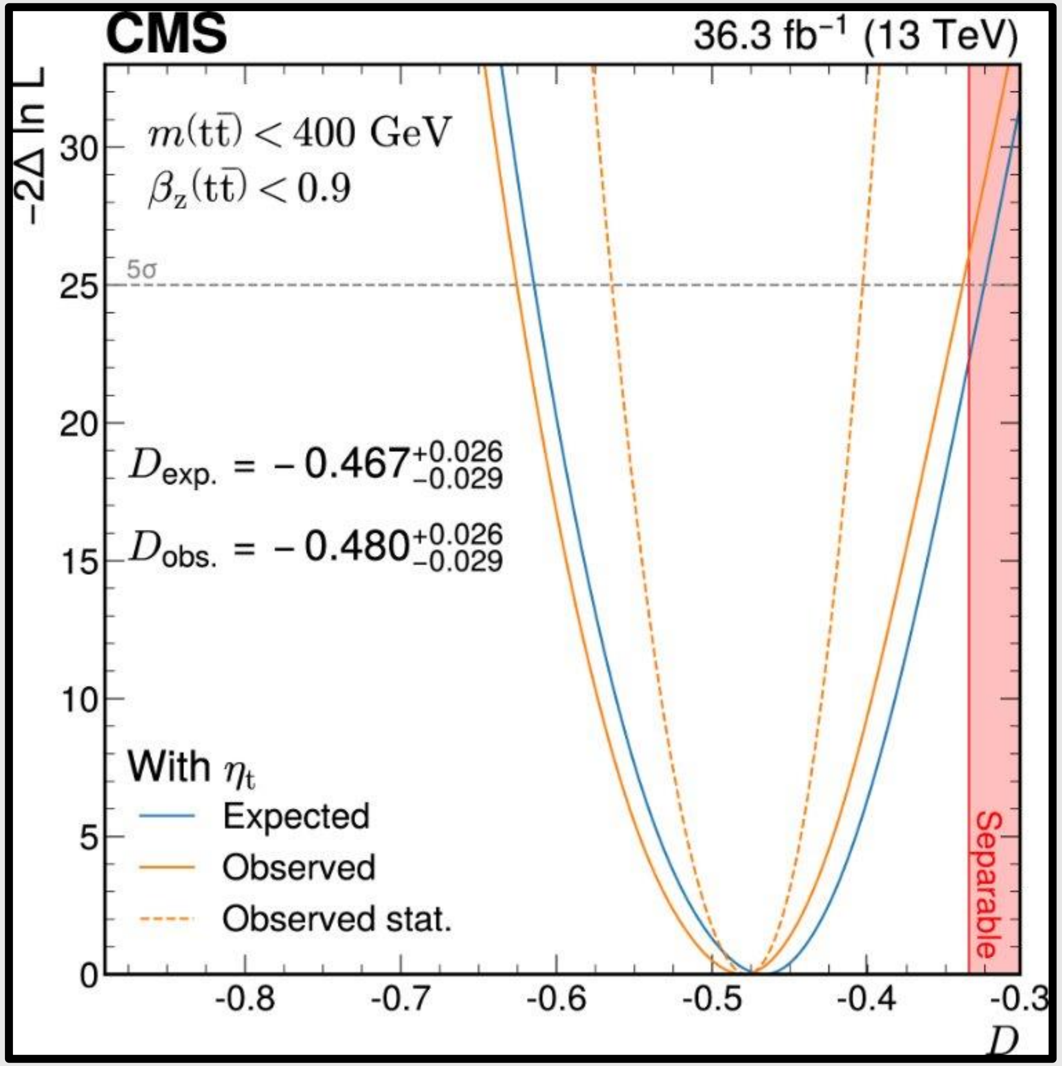
Fit Procedure

- Need to fit POI D
  - Binned profile likelihood fit using CMS Combine tool [Comput. Softw. Big Sci. (2024) 8:19]
  - Create mixtures of SM and no spin corr. to get variations of D
  - Added η_t as signal mixed in with $t\bar{t}$
  - Best D value and uncertainty obtained from the $-2\Delta\ln L$



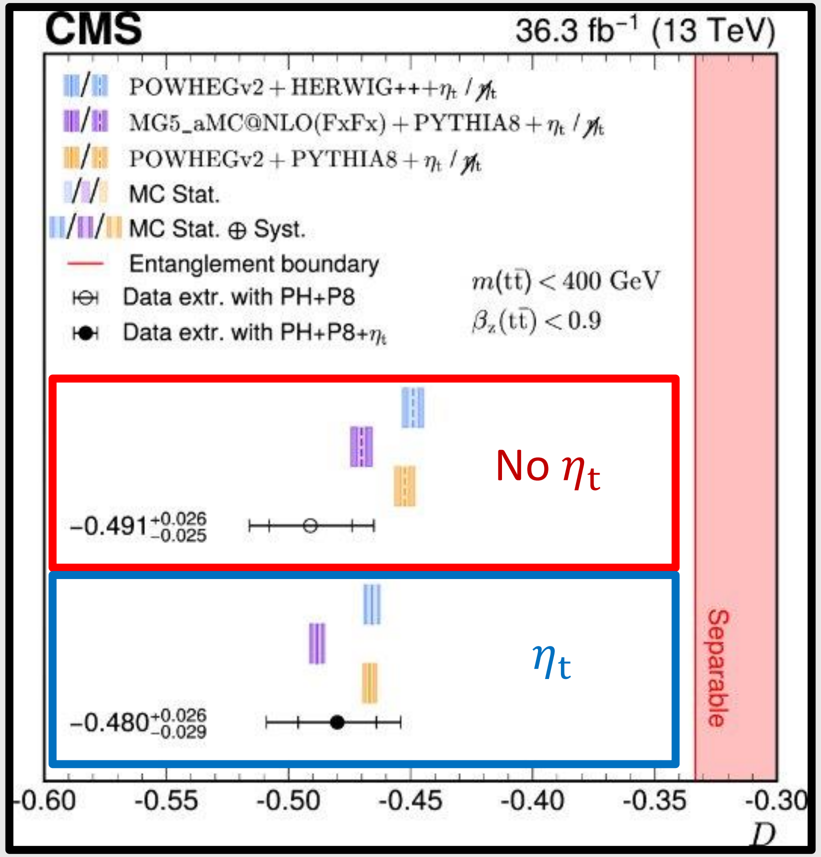
[ROPP 87 (2024) 117801]

Entanglement Observation



- **$> 5\sigma$ observation of entangled top quarks!!!!**

- Agreement with data improved using η_t model

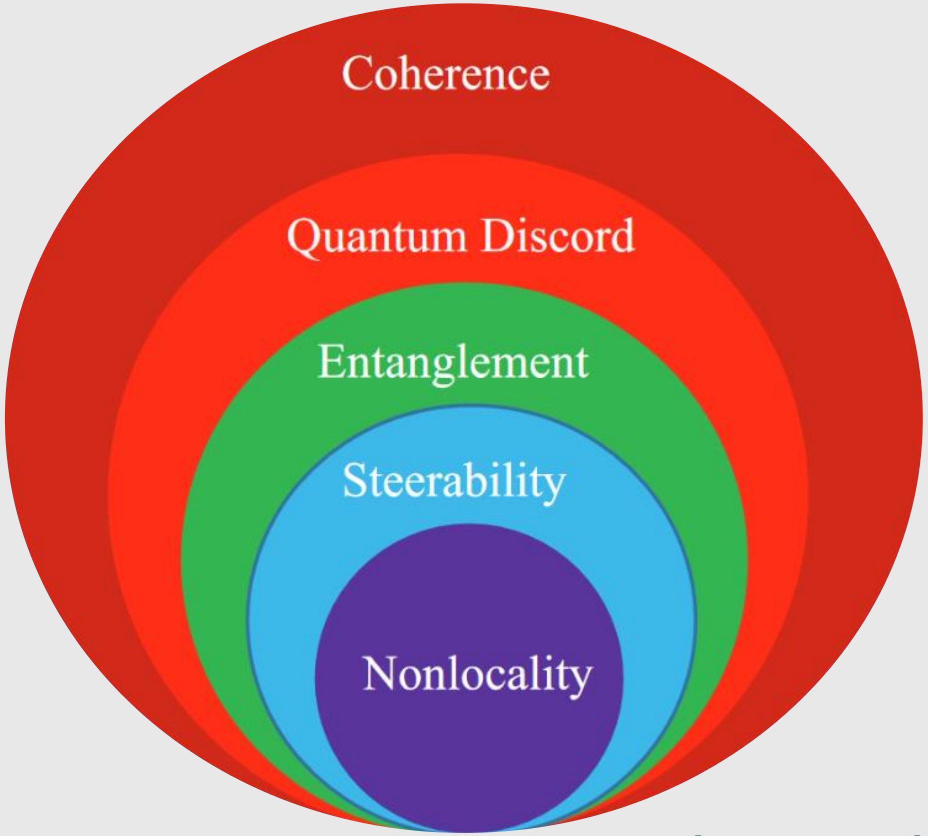


[ROPP 87 (2024) 117801]

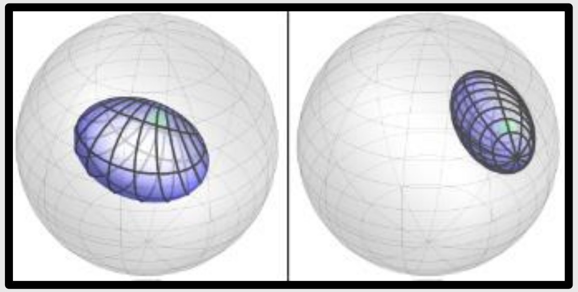
Conclusion

- Top quarks were proven to **be entangled** in both regions:
  - Threshold region [ROPP 87 (2024) 117801]
  - Boosted region [arXiv:2409.11067] (other work)

- **Quantum tomography** of top quark-antiquark pairs is a door for quantum information at high energy physics



[EPL **125** 50005]



Thanks for your time

Contact:

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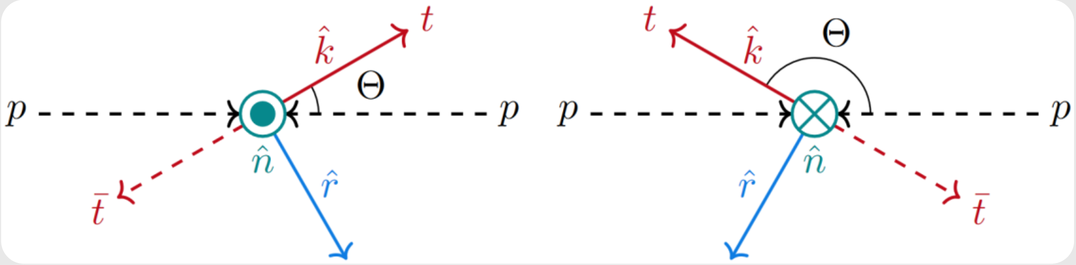
BACKUP

Top Spin Correlations



$$\frac{1}{\sigma}\frac{d^4\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}(1 + \boldsymbol{B}_1 \cdot \hat{l}_1 + \boldsymbol{B}_2 \cdot \hat{l}_2 - \hat{l}_1 \cdot \boldsymbol{C} \cdot \hat{l}_2)$$

$\{\boldsymbol{k}, \boldsymbol{n}, \boldsymbol{r}\}$ $\boldsymbol{n} = \frac{\mathrm{sgn}(\cos\Theta)}{\sin\Theta}(\boldsymbol{p} \times \boldsymbol{k})$ $\boldsymbol{r} = \frac{\mathrm{sgn}(\cos\Theta)}{\sin\Theta}(\boldsymbol{p} - \boldsymbol{k}\cos\Theta)$

Spin Polarization $\boldsymbol{B}_{1,2} = \kappa \boldsymbol{P}_{1,2}$
Spin Correlation $\boldsymbol{C} = \kappa\bar{\kappa}C$

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{1}{2}(1 + \lambda_x x)f(x)$$

λ_x extracted from asymmetry

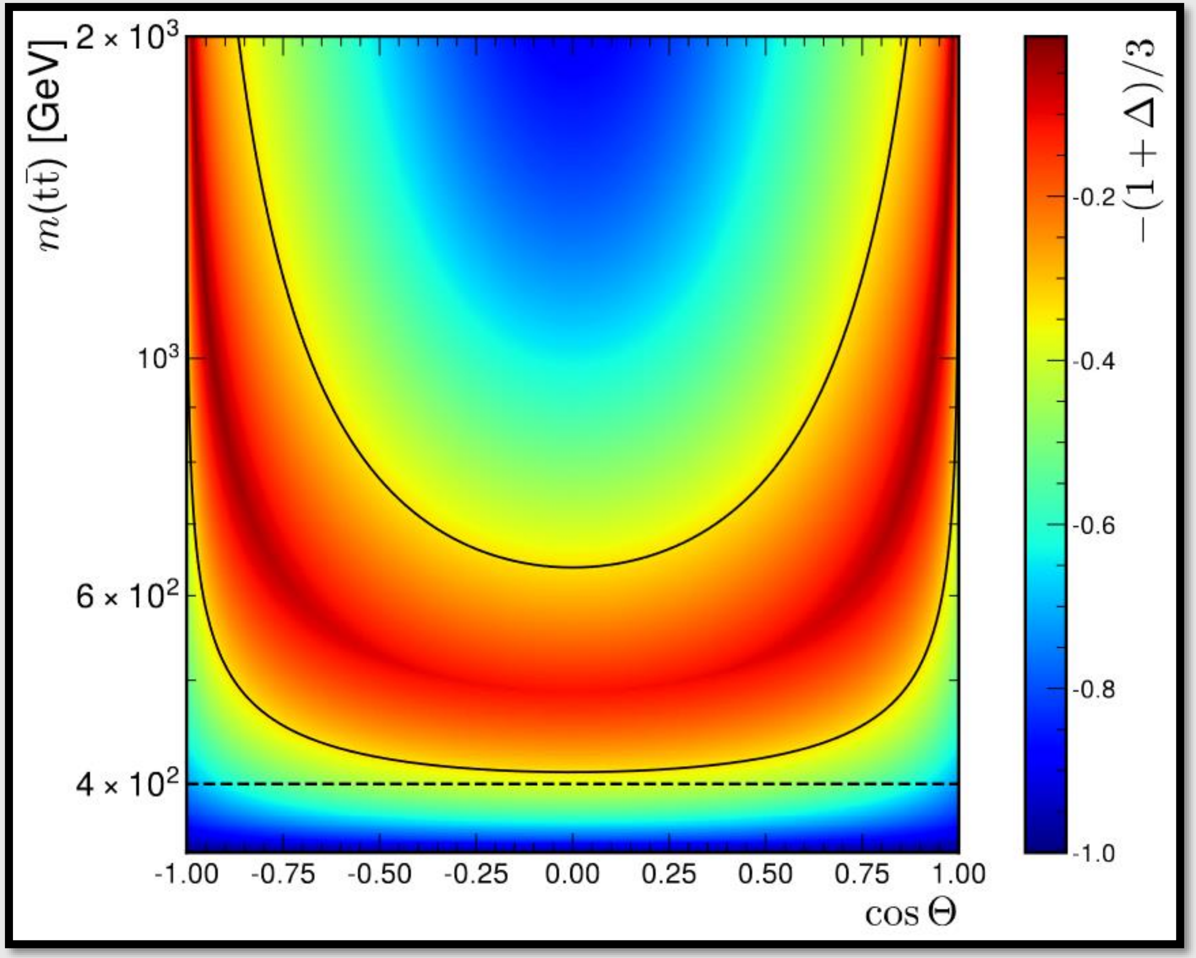
$$\begin{aligned} \kappa_{l^+} &\approx 1{,}00 \\ \kappa_b &\approx -0{,}39 \\ \kappa_{W^+} &\approx 0{,}39 \\ \kappa_{\bar{d},\bar{s}} &\approx 0{,}97 \\ \kappa_{u,c} &\approx -0{,}32 \end{aligned}$$

Spin Density Matrix
$\rho = 1_4 + B_i^+ \sigma^i \otimes 1_2 + B_i^- 1_2 \otimes \sigma^i + C_{ij}\sigma^i \otimes \sigma^j$

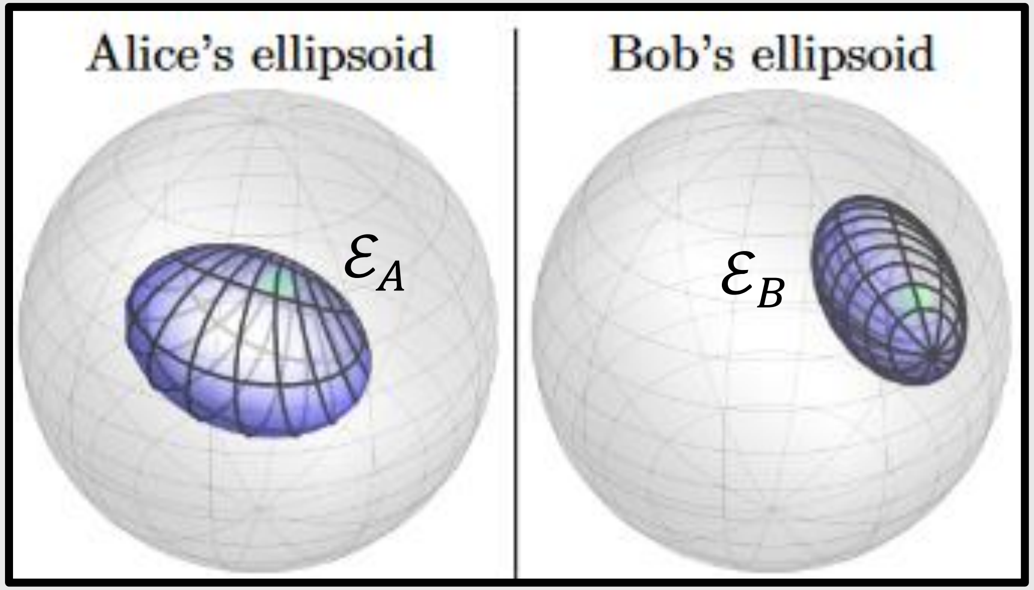
- Top quark spin cannot be measured directly
- Typically measured on the helicity basis
- Measurement of spin correlations can be used to perform quantum state tomography
- Spin correlations depend on higher orbital momenta ($m_{t\bar{t}}$, cos Θ) and initial state

Entanglement

- Observed by using the Peres-Horodecki criterion which gives the condition ($\lambda_i = \mathrm{eig}\{\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}\}$)
  $$C[\rho] = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) > 0$$
- For top pairs, it can be reduced to
  $$\Delta = C_{33} + |C_{11} + C_{22}| - 1 > 0$$
- At low $m(\mathrm{t\bar{t}})$, $C_{11}, C_{22} > 0$ then
  $$D = -\frac{\mathrm{tr}[\boldsymbol{C}]}{3} < -\frac{1}{3}$$
- The distribution of $\cos\varphi = \hat{l}^+ \cdot \hat{l}^-$ can be written as
  $$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$



Steering Ellipsoid



$$Q_A = \tilde{C}_A \tilde{C}_A^T = \gamma^2 \big(C - \vec{B}_A \vec{B}_B^T\big)\big(1 + \gamma^2 \vec{B}_B \vec{B}_B^T\big)\big(C^T - \vec{B}_B \vec{B}_A^T\big)$$

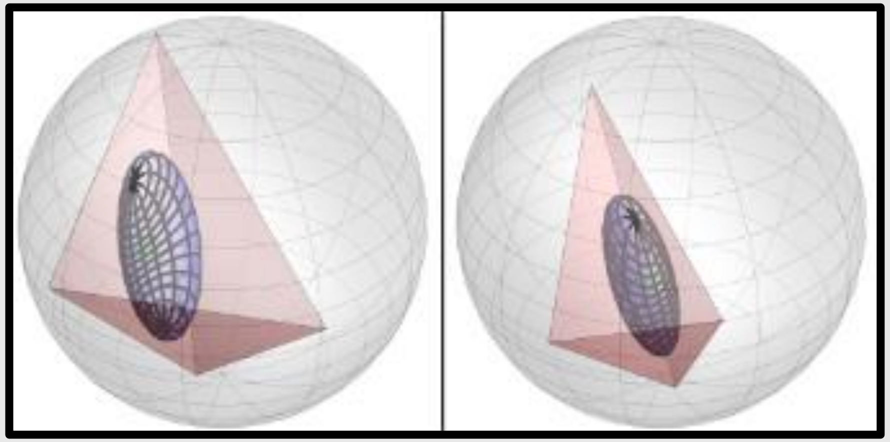
$$(\vec{x} - \vec{c}_A)^T Q_A^{-1} (\vec{x} - \vec{c}_A) = 1 \qquad \vec{c}_A = \frac{\vec{B}_A - C\vec{B}_B}{1 - B_B^2}$$

- Geometric representation of quantum correlations
- Alternative geometric criteria for entanglement:

$$D_{ell} = c^4 - 2c^2\big(1 - tr(Q) + 2\hat{c}^T Q \hat{c}\big) + h(Q) > 0$$
$$V > V_\star = 4\pi/81$$

- Surface condition for limit on Local Hidden State (LHS) models (EPR-Steerability)

$$\hat{S} = \frac{1}{2\pi} \int d^2\hat{n} \sqrt{\hat{n}^T C^T C \hat{n}} > 1$$



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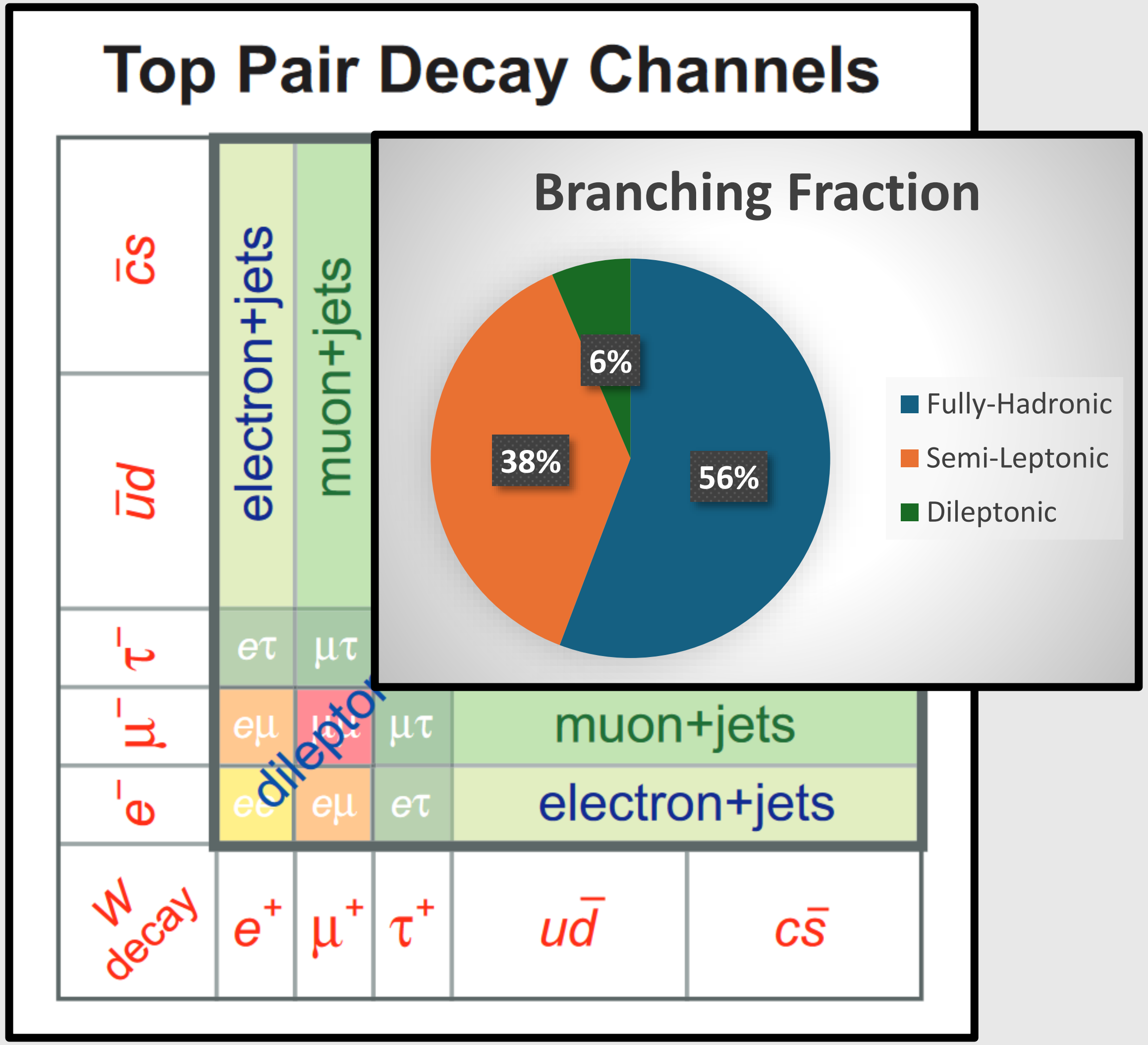
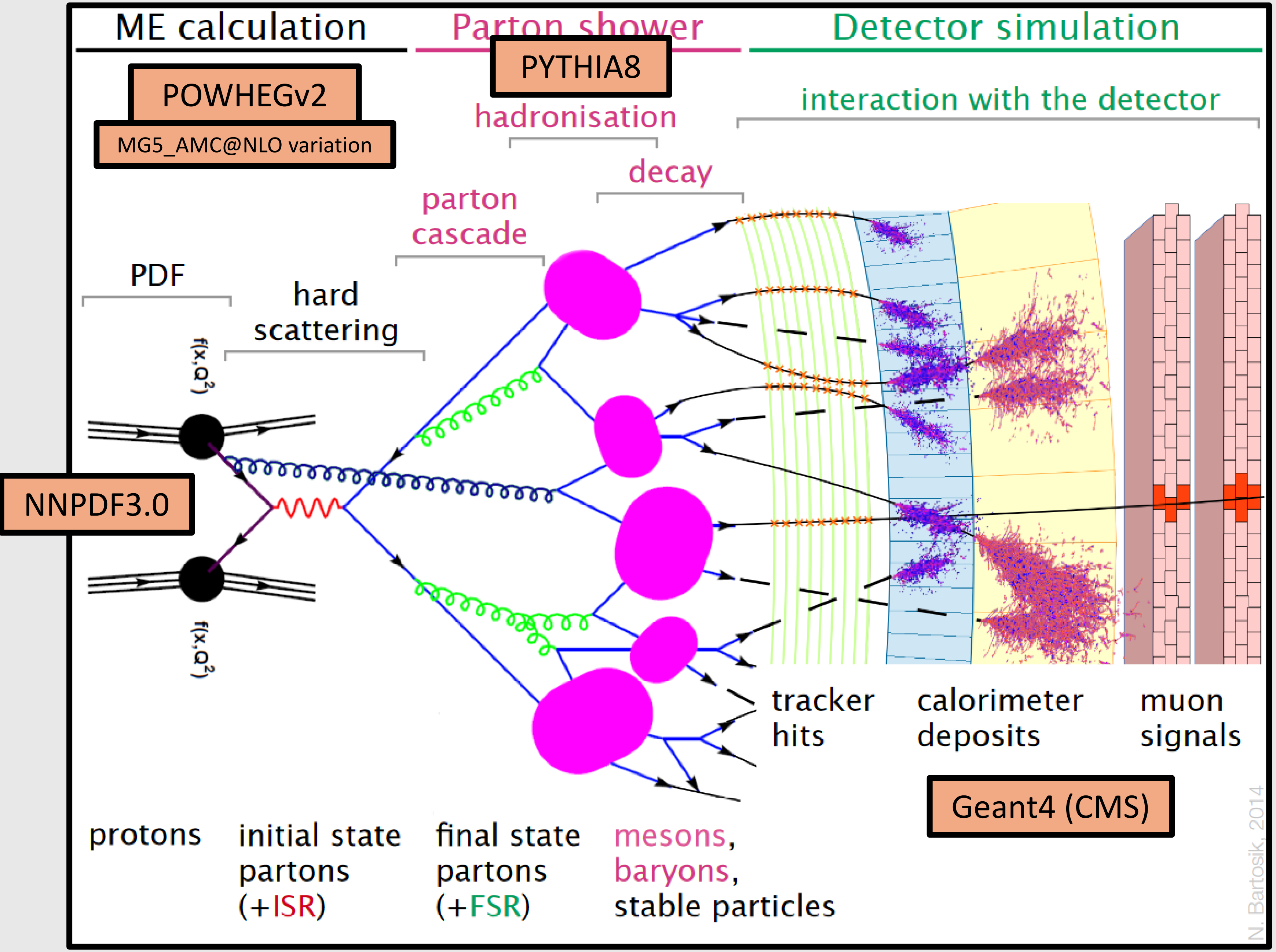




- $m_{\mathrm{t}} = 172.5$ GeV
- PYTHIA8 CUETP8M2T4 ($t\bar{t}$ and single top) and CUETP8M1 tuning for underlying events.

High Level Trigger (HLT) Selection

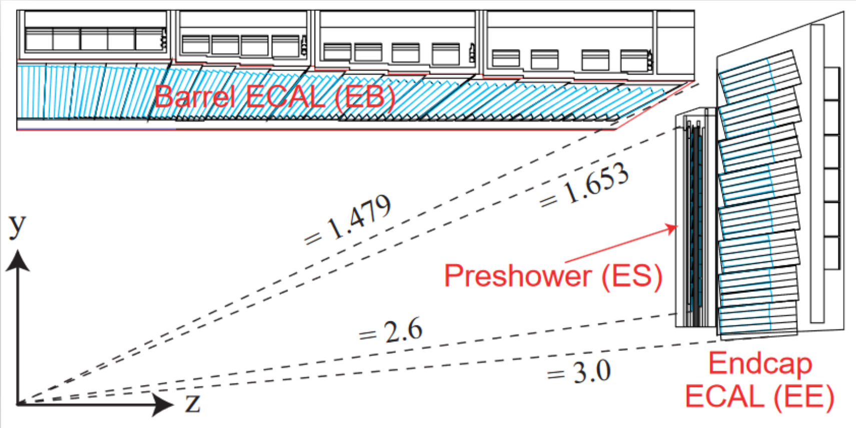
- **Single-lepton trigger:**
  - $p_T(e) > 27$ GeV
  - $p_T(\mu) > 24$ GeV
- **Same-flavor dilepton trigger:**
  - Electron
    - Leading: $p_T > 23$ GeV
    - Trailing: $p_T > 12$ GeV
  - Muon
    - Leading: $p_T > 17$ GeV
    - Trailing: $p_T > 8$ GeV
- **Different-flavor dilepton trigger:**
  - $p_T(e) > 12$ GeV, $p_T(\mu) > 23$ GeV, or
  - $p_T(e) > 23$ GeV, $p_T(\mu) > 8$ GeV

→ **Particle-flow algorithm reconstruction**

3 channels: $e^+e^-, \mu^+\mu^-, e^\pm\mu^\mp$

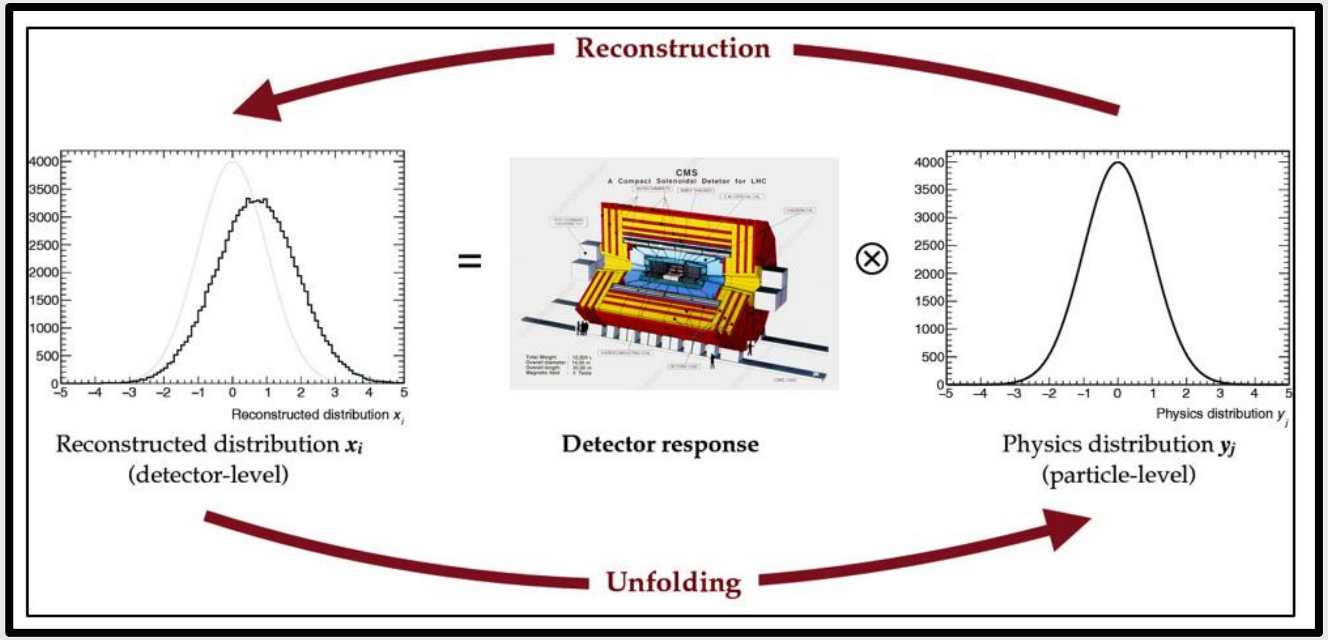
Object Selection

- **Electron candidates**
  - Leading: $p_T > 25$ GeV
  - Trailing: $p_T > 20$ GeV
  - $|\eta| > 2.4$
  - Exclusion zone: $1.44 < |\eta_{\text{cluster}}| < 1.57$
  - Isolation criteria $I_{\text{rel}} = \frac{(\sum_k p_T^k)}{p_T(e)}$, $k \to$ hadrons/photons $\Delta R < 0.3$
    - In the barrel: $I_{\text{rel}} < 0.0588$
    - In the end cap: $I_{\text{rel}} < 0.0571$
  - Other identification requirements



- **Muon candidates**
  - Leading: $p_T > 25$ GeV
  - Trailing: $p_T > 20$ GeV
  - $|\eta| > 2.4$
  - Isolation criteria $I_{\text{rel}} < 0.15$ ($\Delta R < 0.4$)
  - Other identification requirements
- **Jets**
  - Anti-k_T clustering algorithm with $\Delta R < 0.4$.
  - $p_T > 30$ GeV
  - $|\eta| < 2.4$
  - $\Delta R(\text{jet}, \text{lepton}) > 0.4$
  - b jet identification with algorithm CSV(v.2)
    - Efficiency: $\approx 79 - 87\%$
    - Probability of misidentification: $\approx 10\%$

Unfolding Procedure



$$\vec{y}^{\mathrm{MC}} = \mathbf{M} \cdot \vec{x}^{\mathrm{MC}}$$

- Regularization to suppress stat fluctuations
- χ^2 fit to apply both regularization and unfolding simultaneously
- Make linearity test to check for bias on regularization

$$\chi^2_{\mathrm{unf}} = \chi^2_{\mathbf{M}} + \tau^2 \chi^2_{\mathbf{L}} + \lambda \sum_i (\mathbf{M}\vec{x} - \vec{y})_i$$

Matrix Inversion

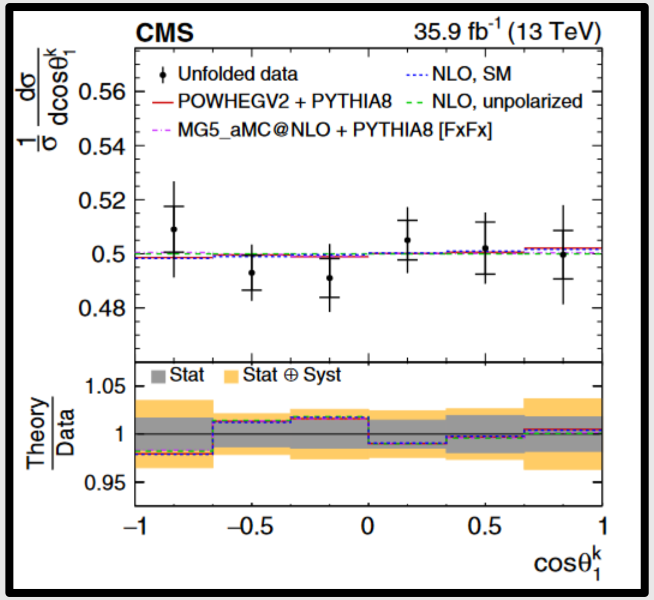
Explicit Regularization

Lagrange Multipliers, constraint on number of events

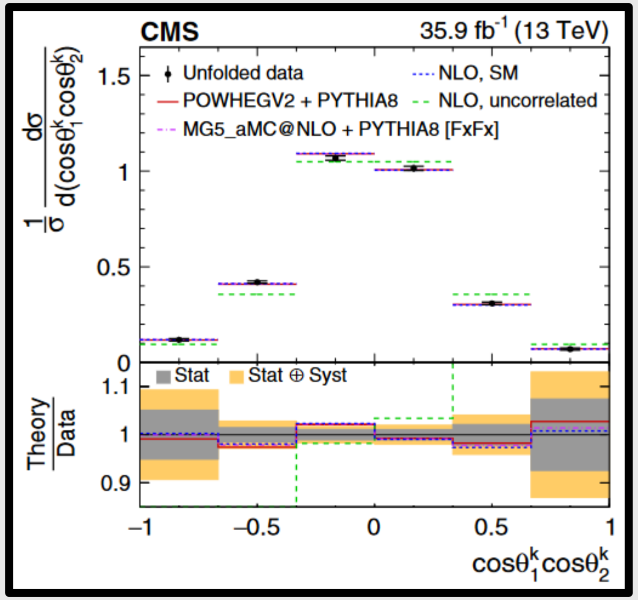
Unfolded Distributions

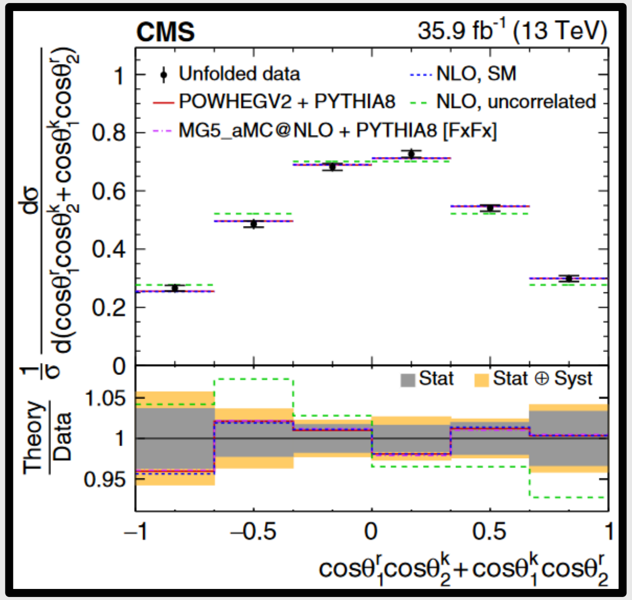
$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}x} = \frac{1}{2}(1+\lambda_x x)f(x)$$

Polarizations



Correlations



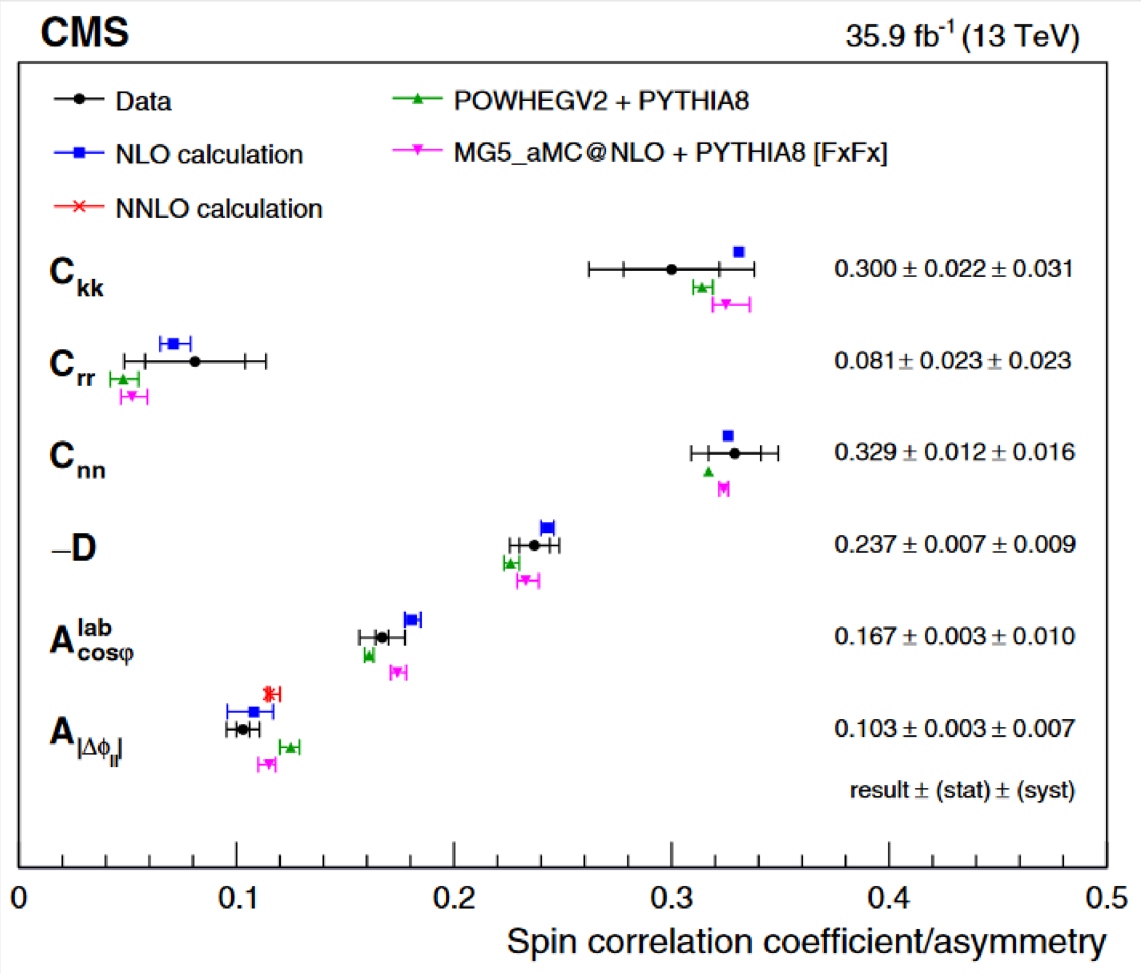


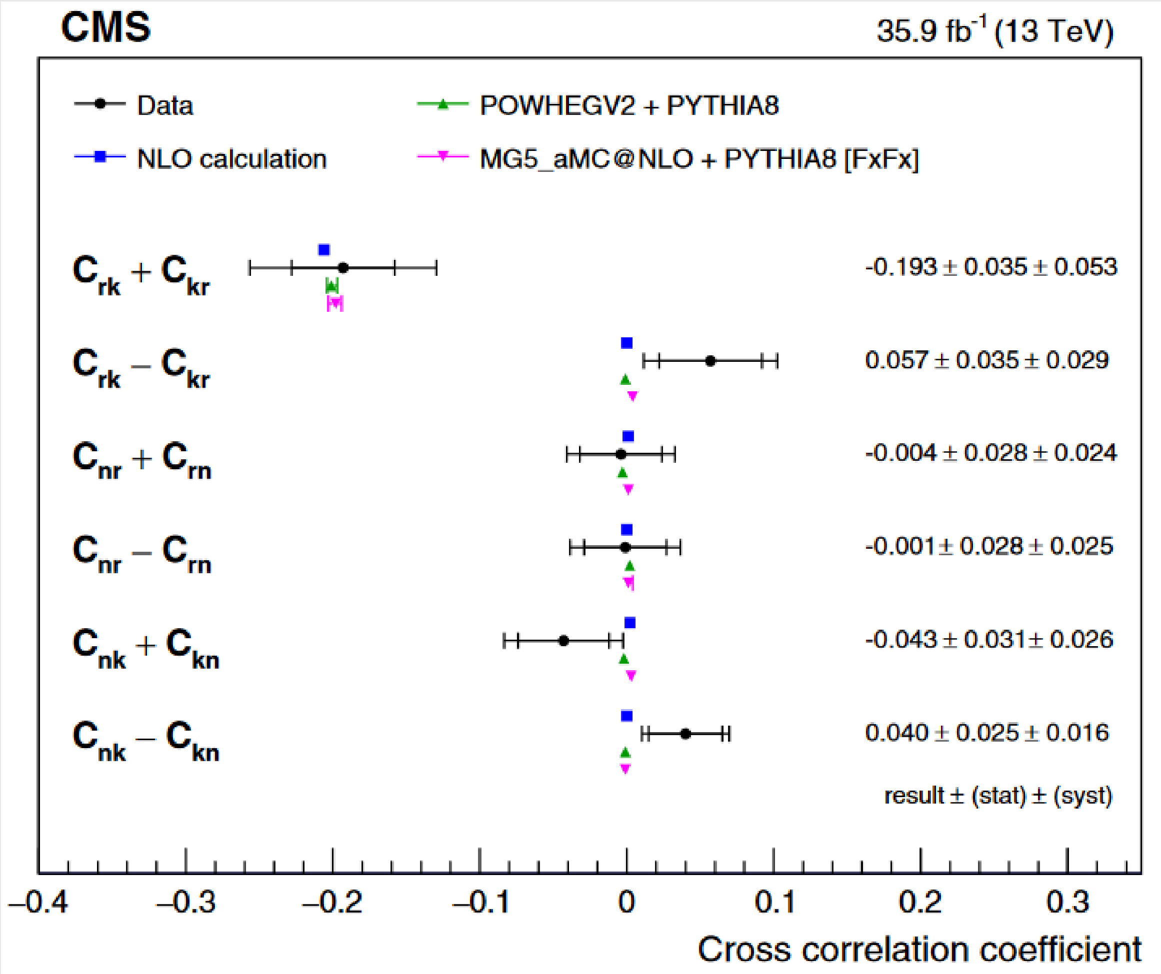
$$f(x) = 1$$
$$\lambda_x = B^i_{1,2}$$
$$x = \cos\theta^i_1$$

$$f(x) = \ln\left(\frac{1}{|x|}\right)$$
$$\lambda_x = -C_{ij}$$
$$x = \cos\theta^i_1 \cos\theta^j_2$$

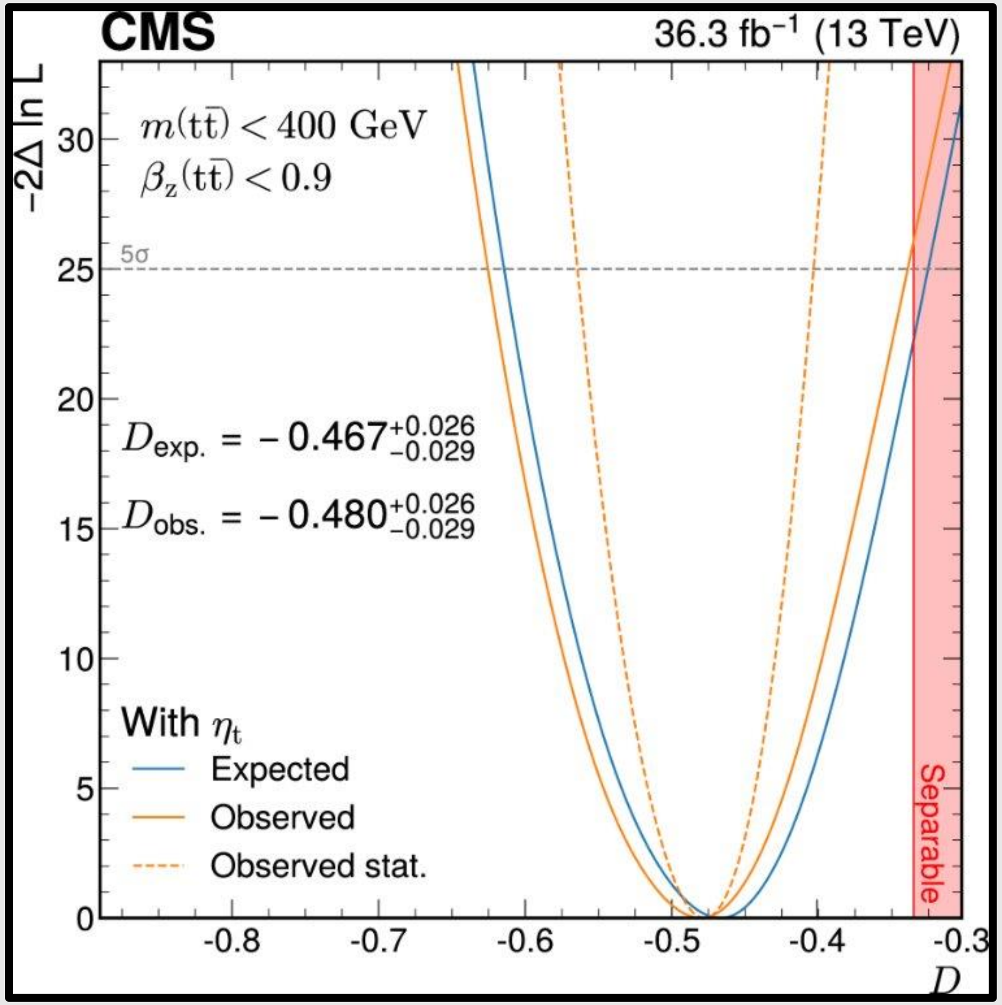
$$f(x) = \cos^{-1}|x|$$
$$\lambda_x = -\frac{C_{ij} \pm C_{ji}}{2}$$
$$x = \cos\theta^i_1 \cos\theta^j_2 \pm \cos\theta^j_1 \cos\theta^i_2$$

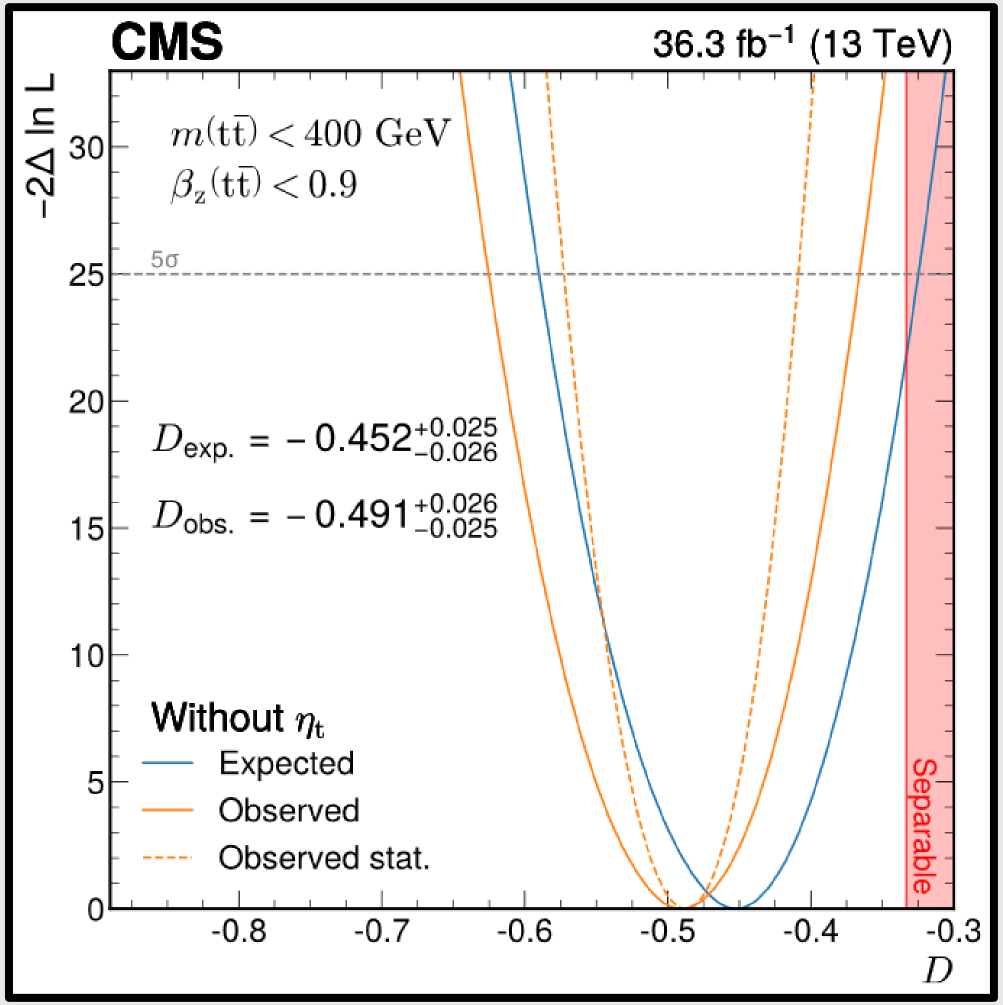
Correlation coefficients





Entanglement Observation





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