FLAVOR (MODELS) IN FINITE UNIFIED THEORIES

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+ CDM…

STANDARD MODE

 $+ \not\vdash$ \forall : \forall \circ $\not\vdash$ \forall \circ ϕ + \vdash \vdash

 $+ |D_{\alpha}\varphi|^2 - \vee(\varphi)$

LAGRANGIAN

 $J = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ $+ i \nabla \mathcal{B} \psi + h.c.$

DO YOU NOT UNDERSTAND?

 $\bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu G^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \tfrac{1}{2} \partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c^2} M^2 Z^0_\mu$ $\frac{1}{2}\partial_{\mu}A_{\nu}\partial_{\mu}A_{\nu}-\frac{1}{2}\partial_{\mu}H\partial_{\mu}H-\frac{1}{2}m_h^2H^2-\partial_{\mu}\phi^+\partial_{\mu}\phi^--M^2\phi^+\phi^--\frac{1}{2}\partial_{\mu}\phi^0\partial_{\mu}\phi^0 \frac{1}{2c^2}M\oint \oint \Phi -\beta_h[2\frac{M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2 + \oint \oint \Phi + 2\phi^+ \phi^-)] + \frac{2M}{g^2}\alpha_h - i g c_w [\partial_v Z_\mu^0(W_\mu^+ W_\nu^- W_{\nu}^+W_{\mu}^-)-Z_{\nu}^0(W_{\mu}^+\partial_{\nu}W_{\mu}^- -W_{\mu}^-\partial_{\nu}W_{\mu}^+)+Z_{\mu}^0(W_{\nu}^+\partial_{\nu}W_{\mu}^- -W_{\nu}^-\partial_{\nu}W_{\mu}^+)]-ig s_w \ \partial_{\nu}A_{\mu}(W_{\mu}^-W_{\nu}^- W^+_\nu W^-_\mu$ $- A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$ $\frac{1}{2}g^2W_u^+W_v^-W_u^+W_v^-+g^2c_w^2(Z_u^0W_u^+Z_u^0W_v^- - Z_u^0Z_u^0W_v^+W_v^-)+g^2s_w^2(A_\mu W_u^+A_\nu W_v^ A_{\mu}A_{\mu}W_{\nu}^{+}W_{\nu}^{-}$) + $g^{2}s_{\omega}c_{\omega}A_{\mu}Z_{\nu}^{0}(W_{\mu}^{+}W_{\nu}^{-}-W_{\nu}W_{\mu}^{-})-2A_{\mu}Z_{\mu}^{0}W_{\nu}^{+}W_{\nu}^{-}]-g\alpha[H^{3}+$ $H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac{1}{2}g^2\alpha_h H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi$ $2(\phi^0)^2H^2$] –9 $MW_t^+W_u^-H$ – $\frac{1}{2}g\frac{M}{2}Z_u^0Z_u^0H$ – $\frac{1}{2}ig[W_u^+(\phi^0\partial_\mu\phi^- - \phi^- \partial_\mu\phi^0)$ – $W_u^-(\phi^0\partial_\mu\phi)$ $\phi^0 \partial_\mu H$) $ig \frac{\pi^0}{\epsilon_u} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + ig s_\omega M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - ig \frac{1-2\epsilon^2_\mu}{2\epsilon_\omega} Z^0_\mu (\phi^+ \partial_\mu \phi^- \partial_{\mu}\phi^{\text{F}}$) + igsw $A_{\mu}(\phi^{\dagger}\partial_{\mu}\phi^- - \phi^{\dagger}\partial_{\mu}\phi^{\dagger}) - \frac{1}{2}g^2W_{\mu}^+W_{\mu}^+H^2 + (\phi^0)^2 + 2\phi^{\dagger}\phi^$ $x^2 + 2(2s_{\omega}^2 - 1)^2 \phi^+ \phi^- \right] - \frac{1}{2}g^2 \frac{\sigma_{\omega}}{c} Z_{\mu}^0 \phi^0(W^+_{\mu} \phi^-)$ $\frac{1}{2}g^2 s_w A_\mu \phi^0 (W^+_n \phi^- + W^-_\mu \phi^+) + \frac{1}{2} i g^2 s_w A_\mu H (W^+_n \phi^-)$ $-\bar{u}_j^{\lambda}(\gamma\partial + m_{\mu}^{\lambda})u_j^{\lambda} - d_j^{\lambda}(\gamma\partial + m_d^{\lambda} d_j^{\lambda} + i g s_w A_{\mu}[-(e^{\lambda}\gamma^{\mu}e^{\lambda}) + \frac{2}{3}(\bar{u}_j^{\lambda}\gamma^{\mu}e^{\lambda})]$ $Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda)+(\bar{e}^\lambda\gamma^\mu(4s_\omega^2-1-\gamma^5)e^\lambda)$ $\mu^2(1-\frac{8}{3}s_w^2-\gamma^5)d_j^2\big)\big]+ \frac{49}{2\sqrt{2}}W^+_\mu[(\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda)-(u_j^\lambda\gamma^\mu(1+\gamma^5))]$ $\gamma^5)C_{\lambda\kappa}d_2^{\kappa}\big)\big]+{}\frac{49}{2}\mathcal{W}_\mu^{-}\big[(\bar{e}^\lambda\gamma^{\mu}(1+\gamma^5)\nu^{\bar{\lambda}})+(\bar{d}_3^{\bar{\kappa}}\tilde{C}_{\lambda\kappa}^{\bar{\kappa}}\gamma^{\mu}(1+\gamma^5)\bar{u}_3^{\lambda})\big]+\frac{49}{2\sqrt{2}}\frac{m_e}{M}\big[-\phi^+(\bar{\nu}^\lambda(1+\gamma^5)\bar{u}_3^{\bar{\kappa}})\big]$ $(\gamma^5)e^{\lambda}$ + $\phi^-(e^{\lambda}(1+\gamma^5)\nu^{\lambda})$ - $\frac{9}{M}$ $\frac{m_e^2}{M}$ $[H(e^{\lambda}e^{\lambda})+i\phi^0(e^{\lambda}\gamma^5e^{\lambda})]$ + $\frac{49}{2M\sqrt{6}}\phi^+$ - $m_d^2(\bar{u}_j^{\lambda}C_{\lambda\kappa}(1-\gamma^5)\nu^{\lambda})$ $\gamma^5) d_j^{\kappa}$) + $m_u^{\lambda} (\bar{u}_j^{\lambda} C_{\lambda \kappa} (1 + \gamma^5) d_j^{\kappa}) + \frac{i g}{2 M \sqrt{2}} \phi^{-} [m_d^{\lambda} (\bar{d}_j^{\lambda} C_{\lambda \kappa}^{\dagger} (1 + \gamma^5) u_j^{\kappa}) - m_u^{\kappa} (\bar{d}_j^{\lambda} C_{\lambda \kappa}^{\dagger} (1 \gamma^5)u_j^{\kappa}]-\frac{9}{2} \frac{m_j^{\kappa}}{M}H(u_j^{\lambda}u_j^{\lambda})-\frac{9}{2} \frac{m_j^{\kappa}}{M}H(d_j^{\lambda}d_j^{\lambda})+\frac{16}{2} \frac{m_j^{\lambda}}{M}\phi^0(u_j^{\lambda}\gamma^5u_j^{\lambda})$ $\frac{16}{3}$ $\frac{m_0^2}{M}$ ϕ^0 $\left(\frac{\partial^2}{\partial x^2}\gamma^5 d_3^2\right)$ + $X^+(\partial^2 - M^2)X^+ + X^-(\partial^2 - M^2)X^- + X^0(\partial^2 - M^2)X^0 + Y\partial^2 Y + i q c_w W^+_u(\partial_u X^0 X^- \partial_{u} X^{+} X^{0}$ + $ig s_{w} W_{\mu}^{+} (\partial_{\mu} \bar{Y} X^{-} - \partial_{\mu} X^{+} Y) + ig c_{w} W_{\mu}^{-} (\partial_{\mu} X X^{0} - \partial_{\mu} \bar{X}^{0} X^{+}) +$ igs_w $W_{\mu}^{-}(\partial_{\mu}X-Y-\partial_{\mu}YX^{+})+igc_{w}Z_{\mu}^{0}(\partial_{\mu}X^{+}X^{+}\partial_{\mu}X^{-}X^{-})+igs_{w}A_{\mu}(\partial_{\mu}X^{+}X^{+} \partial_u \bar{X} - X^-$) – $\frac{1}{2} g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{q_u^2} \bar{X}^0 X^0 H] + \frac{1-2c_0^2}{2c_0} ig M [\bar{X}^+ X^0 \phi^+ -$

WHAT PART OF

 $-\tfrac{1}{2}\partial_\nu g_\mu^a\partial_\nu g_\mu^a-g_sf^{abc}\partial_\mu g_\nu^a g_\nu^b\,g_\nu^c-\tfrac{1}{4}g_s^2f^{abc}f^{ade}g_\mu^b g_\nu^c g_\mu^d g_\nu^e+\tfrac{1}{2}ig_s^2(\bar q_i^{\sigma}\gamma^\mu q^{\sigma})g_\mu$

- ➤ What happens as we approach the Planck scale?
- What happened at the early Universe?
- ➤ How do we go from an effective theory like the SM to a more fundamental one?
- ➤ How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- ➤ Why/how are the elementary particle masses so different?

- ➤ Is there more than one Higgs, more scalars?
- What about flavor?

➤ **Where is the new physics?**

FLAVOUR

➤ Interactions that distinguish between different flavours

- ➤ **why 3 generations?**
- ➤ **why those masses?**
- ➤ **why the gap between neutral and charged fermions**
- ➤ **why the difference between mixing matrices?**
- ➤ **why that amount of CP violation?**
- ➤ **…**
- *•Fermion masses*
- *•Mixing*
- *•CP violation*

Connections to new/unknown physics

- *•Dark matter*
- *•Baryogenesis*
- *•Leptogenesis*
- *•EW phase transition*

Lead to discoveries

- *• ^Γ(KL*→*µ +µ−)/ Γ(K+*→*µ+ν)* → *charm quark*
- \bullet $\Delta m_K \rightarrow$ *charm mass*
- \bullet $\Delta m_B \rightarrow$ top mass
- $\bullet \varepsilon_K \rightarrow \text{third generation}$
- *oscillation* → *mass*

Nir, CERN–LATAM School HEP (2015)

•??

SOME ASPECTS OF THE FLAVOUR PROBLEM

➤ Quark and charged lepton masses very different, very hierarchical

 $m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$

```
m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1
```
 $m_e: m_\mu: m_\tau \sim 10^{-5}: 10^{-2}: 1$

- ➤ Neutrino masses unknown, only difference of squared masses.
- ➤ Type of hierarchy (normal or inverted) also unknown
- ➤ Higgs sector under study

➤ Quark mixing angles

 $\theta_{12} \approx 13.0^{\circ}$ $\theta_{23} \approx 2.4^{\circ}$ $\theta_{13} \approx 0.2^{\circ}$

➤ Neutrino mixing angles

 $\Theta_{12} \approx 33.8^\circ$ $\Theta_{23} \approx 48.6^\circ$ $\Theta_{13} \approx 8.6^\circ$

- ➤ Small mixing in quarks, large mixing in neutrinos. Very different ?
- ► Is there an underlying symmetry?

PMNS VS CKM

QUARKS, CHARGED LEPTONS AND HIGGS INTERRELATED ya que todos tienen *Y* 6= 0. Por lo tanto, *^L*ME = 0 (7.12) \overline{F} D LEPTONS AND HIGGS INTERRELATED En la base de la base de la base de la economiente de la economiente de la economiente de la economie de la ec

➤ Yukawa couplings: several orders of magnitude of difference, strong hierarchy 7.1.3 Sector de Yukawa *L*Yuk La parte de Yukawa del lagrangiano se da por ria base definive definition of the base definition of the Vukawa couplings: several orders of magnitude of difference, *SULOS SURGER SURGER SURGER SURGER SURGER* f magn *,* lde of c *,* ϵ rence ; *dR, sR, b^R* (7.25)

$$
\mathcal{L}_Y^{\text{ME}} = \widehat{Y_{ij}^d Q_{Li}} \phi D_{Rj} + \widehat{Y_{ij}^d Q_{Li}} \tilde{\phi} U_{Rj} + \widehat{Y_{ij}^e L_{Li}} \phi E_{Rj} + \text{h.c.}
$$

Also neutrinos, but they could acquire mass other ways. Also neutrinos, but they could acquire mass other ways.

➤ Higgs sector: \blacktriangleright Hioos sector[.] ρ unggo succur. El potencial escalar se da por

$$
\mathcal{L}_{\phi}^{\text{ME}} = \left(\mu^2 \phi^{\dagger} \phi - \mathcal{O}(\phi^{\dagger} \phi)\right)^2 \qquad v^2 = -\frac{\mu^2}{\lambda}
$$

SU(2), y los singletes de quarks down *SU*(2), de la siguiente manera:

➤ hierarchy problem (quadratic radiative corrections) para cambiar la base a una donde *Y ^e* es diagonal y real: Esta parte del *problem* (quadratic radiative corrections) podemos reescaribir la ecuación (7.26) de la siguiente manera (hasta un término constante): en la siguiente manera (hasta un término constante): en la siguiente manera (hasta un término constante): en la siguiente de la s

- ➤ limits to perturbative unitarity *Y*ˆ *^e* = diag (*ye, yµ, y*⌧) (7.15) \sum limits to perturbativ
	- \blacktriangleright Why M_{Higgs} \sim 125 GeV? *SU*(2) y los tres singletes de leptones *SU*(2) de la siguiente manera: ⁼ *µ*² $CaV₂$ *[|]*hi*[|]* ⁼ *v/*p2. Debemos hacer una elección de la dirección de ^hi, y la elegimos en la dirección real

$$
V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{33} & s_{23} \\ 0 & -s_{31} & c_{33} \end{pmatrix} \begin{pmatrix} c_{33} & 0 & s_{33}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{12}c_{33} & s_{12}c_{33} \\ -s_{13}c_{33} & s_{12}c_{33} \\ s_{13}c_{33} & s_{14}c_{33} \end{pmatrix}
$$
\n
$$
= \begin{pmatrix} c_{12}c_{13} & 0 & s_{13}e^{-i\delta} \\ -s_{12}c_{23}c_{33} - c_{12}c_{33}s_{33}e^{i\delta} & c_{12}c_{33} - s_{12}c_{33}s_{33}e^{i\delta} \\ s_{13}c_{33} & -c_{12}c_{33}s_{33}e^{i\delta} & -c_{12}s_{33} - s_{12}c_{33}s_{33}e^{i\delta} \\ 0.027435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.022486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182_{-0.00074}^{+0.00031} \\ 0.000857_{-0.00018}^{+0.00020} & 0.04110_{-0.00072}^{+0.00033} & 0.999118_{-0.000036}^{+0.000031} \end{pmatrix}
$$
\n
$$
\begin{vmatrix} V_{CKM} & V_{ds} & V_{ds} \\ V_{ds}V_{ds} & V_{ds} \\ V_{ds}V_{ds} & V_{ds} \end{vmatrix}
$$
\n
$$
= \begin{pmatrix} V_{CKM} & V_{ds} & V_{ds} \\ V_{ck}V_{ds} & V_{ds} \\ V_{ds}V_{ds} & V_{ds} \end{pmatrix}
$$
\n
$$
= \begin{pmatrix} V_{CKM} & V_{ds} & V_{ds} \\ V_{ck}V_{ds} & V_{ds} \\ V_{ds}V_{ds} & V_{ds} \end{pmatrix}
$$

PMNS MATRIX PONTECORVO-MAKI-NAKAGAWA-SAKATA $t_{\rm max}$ is the mass hierarchy could be the inverse of that shown, with Δt being the lightest of the lightest Δt \sim 3 m in the general 3 \sim 3 μ mixing matrix \sim 3 μ m.

$$
U = \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{vmatrix} \begin{vmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{vmatrix} \begin{vmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{vmatrix}
$$

atmospheric
reactor
solar

is somewhat clumsy, and can be more easily expressed as the product of three

- ► Neutrinos also mix → neutrino oscillations
- ➤ Dirac or Majorana
- \triangleright Three mixing angles and a phase: atmospheric Θ_{23} , solar Θ_{12} and reactor Θ_{13} . Possible also Majorana phases
- ➤ Only determined squared mass differences

$$
\Delta m_{ij}^2 = m_i^2 - m_j^2
$$

FERMION AND SCALAR SECTORS

- ➤ Free parameters in quarks: 6 masses ->Yukawa couplings 3 mixing angles CP violating phase
- ➤ Unitarity —> Jarlskog invariants
- ➤ Free parameters in neutrinos: 6 masses 3 mixing angles CP violating phase 2 Majorana phases
- ➤ Unitarity? —> Also Jarlskog invariants

 Plus Higgs vev and self coupling

FLAVOUR SYMMETRIES

➤ Flavour symmetries: continuous or discrete?

discrete could lead to domain walls

continuous breaking may give massless Goldstone bosons

- ➤ At low energies now discrete preferred. Could be:
	- ➤ Residual symmetry from breaking from continuous one
	- ➤ From the breaking of a larger discrete group
	- ➤ Discrete from the "beginning"

All the particles we have discovered so far…

MULTI-HIGGS MODELS AND FLAVOUR SYMMETRIES

- ➤ 2HDM widely studied, several studies on 3HDM (**Branco et al,; King et al,** *JHEP* **01 (2014) 052 al, 2014)**
- Extra Higgs doublets and discrete symmetries \rightarrow continuous symmetries
- After minimization of the potential there might be residual symmetries \rightarrow unphysical quark sector, degenerate masses/zero masses/zeroes in VCKM. e.g. S_3 , S_4 , A_4 , $\Delta(54)$ all have residual symmetries in 3HDM
- Z_N Abelian symmetries very popular, easier to implement

Complicated potential, many new parameters, many "exotic" scalars

Ivanov, Prog.Part.Nucl.Phys. **95 (2017)** • H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, 1003.3552

• S. F. K., A. Merle, S. Morisi, Y. Shimizu, and M. Tanimoto, 1402.4271

Non-Abelian Family Symmetry

from Steve King's talk at Modular Invariance Approach to the lepton and quark flavor problem, Mainz, May 2024

Cyclic Symmetries

Wikipedia, from Juan Camilo's Acosta poster, 9th COMHEP

MASS MATRICES TEXTURES — TEXTURE ZEROES *6.3. Texture zeros of quark mass matrices 6.3.1. Where do texture zeros come from?* If some elements of the fermion mass matrices are vanishing, the number of their free parammass matrices has already been excluded by today's experimental data. ICAIURES TEAIURE EERUES

➤ Zeroes in the mass matrices —> less parameters, underlying symmetries: Fritzsch τ \sim 201003 in the mass matrices. \sim \sim NNI $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{array}{cc} \text{Sylillidutes: riltZsCI} \end{array}$ it is always possible to simultaneously transform two arbitrary 3 ⇥ 3 quark mass matrices *M*^u mass matrices \longrightarrow and \longrightarrow $\frac{1}{2}$

This version excluded already

$$
M_{\mathbf{q}} = \begin{pmatrix} 0 & C_{\mathbf{q}} & 0 \\ C_{\mathbf{q}}^* & 0 & B_{\mathbf{q}} \\ 0 & B_{\mathbf{q}}^* & A_{\mathbf{q}} \end{pmatrix}
$$

 $hierarchical A \gg |B| \gg |C|$

➤ In SM and extensions (no FC right-handed currents) is always structure coming from possible to simultaneously the Mu and *Md* to Hermitian or NNI textures \sum In CM and ovtoncions (no Γ features and calculations (*II*O 1 C down sector sam s_{residual} to cimultanoought the M_{H} ratios (i.e., *mu*/*mc*, *mc*/*mt* E lo simultaneously the will same dynamics \overline{a} **i** xing Phys.Rept. 854 (2020) *Xing Phys.Rept.* 854 (2020) and (*n*,*m*) as *one* texture zero instead of two texture zeros. That is why the Fritzsch form of *M*^u and *M*^d is also referred $\frac{1}{\sqrt{2}}$

➤ NNI

$$
M'_{\mathbf{q}} = \begin{pmatrix} 0 & C_{\mathbf{q}} & 0 \\ C'_{\mathbf{q}} & 0 & B_{\mathbf{q}} \\ 0 & B'_{\mathbf{q}} & A_{\mathbf{q}} \end{pmatrix}
$$

 $B' \neq B, C' \neq C$

 \triangleright What works? up and ensions (no FC down sector same structure, coming from $\mathbb{E}|\mathbb{E}|\gg|\mathbb{C}|$ is a purely empirical assumption. On the other other on the other other on the other other

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ALLOWED TEXTURES analytical approximations for the predictions of the Fritzsch ansatz and thus obtain a few more

Table 14: The five phenomenologically viable five-zero textures of Hermitian quark mass matrices.

		\prod	$\mathop{\rm III}$	IV	$\overline{\mathbf{V}}$
$M_{\rm u}$ =	$\begin{array}{c} C_\mathrm{u} \ B_\mathrm{u}' \ 0 \end{array}$ $\mathbf{0}$ $\begin{pmatrix} 0 \\ C_u^* \end{pmatrix}$ $\begin{matrix} 0 \end{matrix}$ $A_{\rm u}$ $\begin{array}{c} 0 \end{array}$	$\begin{array}{c} C_\mathrm{u} \\ 0 \\ B_\mathrm{u}^* \end{array}$ $\left(0 \right)$ $\vert 0 \rangle$ $B_{\rm u}$ $\Bigg \begin{smallmatrix} C^*_{\mathrm{u}} \ 0 \end{smallmatrix}\Bigg $ $A_{\rm u}$	$D_{\rm u}$ ^{0} $\vert 0 \vert$ $\begin{bmatrix} 0 \end{bmatrix}$ B'_{u} 0 $\overline{0}$ $(D_{\mathsf{u}}^*$ $A_{\rm u}$	$\begin{array}{c} C_\mathrm{u} \\ B_\mathrm{u}' \\ B_\mathrm{u}^* \end{array}$ $\vert 0 \vert$ $\ket{0}$ $\Bigg \begin{matrix} C_\mathrm{u}^* \ 0 \end{matrix}\Bigg $ $B_{\rm u}$ $A_{\rm u}$	$D_{\rm u}$ $\begin{array}{c} 0 \\ B_\mathrm{u}^\prime \\ B_\mathrm{u}^* \end{array}$ $\left(0 \right)$ $\begin{pmatrix} 0 \ D^*_\mathrm{u} \end{pmatrix}$ $B_{\rm u}$ $A_{\rm u}$
$M_{\rm d}$ =	$\begin{matrix} C_{\rm d} \ B_{\rm d}' \ B_{\rm d}^* \end{matrix}$ $\overline{0}$ $\begin{pmatrix} 0 \ C_d^* \ 0 \end{pmatrix}$ $\frac{B_{\rm d}}{A_{\rm d}}$	$\begin{matrix} C_{\rm d} \\ B_{\rm d}' \\ B_{\rm d}^* \end{matrix}$ $\begin{pmatrix} 0 \ C_{\rm d}^* \ 0 \end{pmatrix}$ $\ket{0}$ $\begin{bmatrix} B_d \\ A_d \end{bmatrix}$	C_d B_d' B_d^* $\begin{pmatrix} 0 \\ C_\text{d}^* \\ 0 \end{pmatrix}$ 0 ² $\begin{bmatrix} B_\mathrm{d} \\ A_\mathrm{d} \end{bmatrix}$	$\begin{matrix} C_{\rm d} \ B_{\rm d}^\prime \ 0 \end{matrix}$ $\vert 0 \rangle$ $\begin{pmatrix} 0 \ C_{\rm d}^* \ 0 \end{pmatrix}$ $\begin{bmatrix} 0 \\ A_{d} \end{bmatrix}$	$\begin{matrix} C_{\rm d} \\ B_{\rm d}^\prime \\ 0 \end{matrix}$ $\vert 0 \rangle$ $\begin{pmatrix} 0 \\ C_\text{d}^* \\ 0 \end{pmatrix}$ $\overline{0}$ A_{d}

strongly hierarchical. Above textures first found by Ramond et al (1993), still work today if not Above textures inst found by Ramond et al (1993), still work today if not
strongly hierarchical. \mathcal{C}

► But so far the best one is:

$$
M_{\rm q} = \begin{pmatrix} 0 & C_{\rm q} & 0 \\ C_{\rm q}^* & B_{\rm q}' & B_{\rm q} \\ 0 & B_{\rm q}^* & A_{\rm q} \end{pmatrix}
$$

TEXTURES AT HIGH ENERGIES u, dina .
........... Z

 \blacktriangleright Usually express mass matrices as mass ratios \rightarrow they remain stable below eW scale, but renormalize above it, depending on model $\frac{1}{2}$ when the expressions of the expressions of the expressions of the equations of the expressions of the expres Gepenant_o on moder

↵u,d(*t*) d*t*

(281)

➤ From high to low energies they get renormalized as, in Eq. (275), one may use Eqs. (280b) to get the corresponding quark may use \mathbb{I}^* and (280b) to get the corresponding quark matrices at \mathbb{I}^* at \mathbb{I}^* FIOIII HIGH to low elle

$$
M_{\rm u}(\Lambda_{\rm EW}) \simeq \gamma_{\rm u} \left[\begin{pmatrix} 0 & C_{\rm u} & 0 \\ C_{\rm u}^* & B_{\rm u}' & B_{\rm u} I_{\rm r}^{C_{\rm u}^{\rm u}} \\ 0 & B_{\rm u}^* I_{\rm r}^{C_{\rm u}^{\rm u}} & A_{\rm u} I_{\rm r}^{C_{\rm u}^{\rm u}} \end{pmatrix} + \frac{I_{\rm r}^{C_{\rm u}^{\rm u}} - 1}{A_{\rm u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & |B_{\rm u}|^2 & B_{\rm u} B_{\rm u}' \\ 0 & B_{\rm u}^* B_{\rm u}' & 0 \end{pmatrix} \right]
$$

$$
M_{\rm d}(\Lambda_{\rm EW}) \simeq \gamma_{\rm d} \left[\begin{pmatrix} 0 & C_{\rm d} & 0 \\ C_{\rm d}^* & B_{\rm d}^* & B_{\rm d} \\ 0 & B_{\rm d}^* I_{\rm r}^{C_{\rm u}^{\rm u}} & A_{\rm d} I_{\rm r}^{C_{\rm u}^{\rm u}} \end{pmatrix} + \frac{I_{\rm r}^{C_{\rm u}^{\rm u}} - 1}{A_{\rm u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & B_{\rm u} B_{\rm d}^* & A_{\rm d} B_{\rm u} \\ 0 & B_{\rm u} B_{\rm d}^* & A_{\rm d} B_{\rm u} \\ 0 & B_{\rm u}^* B_{\rm d}^* & B_{\rm u}^* B_{\rm d} \end{pmatrix} \right]
$$

We see that the texture zeros of both *M*^u and *M*^d keep unchanged in this approximation, but the *I*'s are the one-loop corrections,γ anomalous dimensions, C's coefficients in the running

Textures remain, coefficients change, for MSSM there is dependence on soft breaking terms 138

HOW DO WE GO BEYOND THE SM?

HOW DO WE MOVE UP (OR DOWN) IN ENERGY?

- ➤ We know how a QFT behaves at different scales through the renormalization group RG
- ➤ The theory has the same structure at different energy scales, but the parameters — couplings and masses — change with energy
- ➤ Related to scale invariance and conformal invariance

HOW TO GO BEYOND THE STANDARD MODEL (BSM)?

 \triangleright Traditional way \Rightarrow addition of symmetries

 $N=1$ SUSY

➤ Very effective, but too many free parameters

 Can get messy…

➤ Complementary approach Look for renormalization group invariant relations at high energies

 $GUT \Rightarrow Planck$

➤ Resulting theory has few free parameters ∴ very predictive

Relates gauge and Yukawa sector Predictions for 3rd generation masses

RENORMALIZATION GROUP INVARIANTS RGI

➤ Search for more fundamental theory ⇒ less parameters Renormalization Group Invariants (RGI)

$$
\Phi(g_1, \dots, g_N) = 0
$$

$$
\mu d\Phi/d\mu = \sum_{i=1}^N \beta_i \partial \Phi/\partial g_i = 0
$$

➤ Equivalent to solve reduction equations

$$
\beta_g\left(dg_i/dg\right)=\beta_i
$$

 $i = 1, \ldots, N$

- **Reduced theory has only one coupling and its beta function**
- **Reduction** ➝ **power series solution**
- **Uniqueness of solution can be studied at one-loop**

```
 Zimmermann (1985); Zimmermann, Oehme, Sibold (1984-1985)
```
REDUCTION OF COUPLINGS

- ➤ Couplings related to a primary coupling totally reduced — all couplings depend on one partially reduced — some couplings depend on one
- Can be applied to SUSY and non-SUSY models
- SM analyzed results now ruled out, still impressive Kubo, Sibold, Zimmermann (1984-1987)
- 2HDM analyzed Denner (1990) now re-analysed: possible to have one-loop reduced equations in type II 2HDM at a high-scale boundary May Pech, MM, Patellis, Zoupanos (2023)

➤ Under some conditions SUSY unification models **finite = absence of ∞ renormalizations**

- ➤ Many solutions imply SUSY
- ➤ SUSY indispensable for finiteness
- ➤ And no… SUSY not excluded experimentally but some low energy models are indeed excluded

FINITENESS = SCALE/CONFORMAL INVARIANCE

 \triangleright All-loop finiteness $\Rightarrow \beta = 0$ to all orders in perturbation theory

- ➤ Scale/conformal invariance Conformal and scale invariant $=$ Yukawa couplings Scale invariant $=$ Soft breaking terms Do not depend on energy scale Based on RGI and reduction of couplings
- ➤ Gives UV completion of the QFT
- ➤ Reduces greatly the number of free parameters \Rightarrow new symmetries
- \triangleright Partial reduction \Rightarrow predictions for 3rd generation masses

FINITE SU(5) THEORIES — THIRD GENERATION

▶ Prediction for top mass — very clean

 M_{top} th $\sim 178 \text{ GeV}$ 1993 Kapetanakis, M.M., Zoupanos m_bot also predicted, large tan beta M_{top} ^{exp} = 176 \pm 18 GeV 1995 M_{top} th \sim 172.5 GeV 2007 *Heinemeyer, M.M.,Zoupanos* M_{top} ^{exp} = 173.1 \pm .09 GeV 2013

➤ Prediction for Higgs mass — depends on soft breaking terms, also very restricted

> M_{Higgs} th ~ 121 – 126 GeV 2008, 2013 Heinemeyer, M.M., Zoupanos $M_{\text{Higgs}}^{\text{exp}} = 126 \pm 1 \text{ GeV}$ 2013

FINITESS ⇒ **GAUGE YUKAWA UNIFICATION**

Grand Unified SUSY $N=1$, no gauge anomalies:

$$
W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k
$$

$$
\beta_g^{(1)} = 0 = \gamma_i^{j(1)}
$$

$$
\sum_i T(R_i) = 3C_2(G),
$$

$$
\frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R_i)
$$

T Dynkin index of irrep, C2 Casimir invariant of group Cijk Yukawa couplings, g gauge coupling

- **Restricts the gauge group** \bigcirc
- **Relates gauge and Yukawa couplings**
- **If finite to all orders** \Rightarrow **Conformal invariance**
- **May imply extra symmetries, in this case discrete** \bigcirc
- **Just analyze one-loop solution** \bigcirc
- **One-loop finite ⇒ two-loop finite**
- **Isolated and non-degenerate solution** \Rightarrow \bigcirc **all-loop finite Lucchesi, Piguet, Sibold**

β= 0 non-renormalization of coupling constants, not complete UV finiteness where field renormalization is absent

SUSY BREAKING SSB

► Explicit/soft breaking >100 new free parameters

$$
-\mathcal{L}_{\rm SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)_i^j \phi^{*i} \phi_j + \frac{1}{2} M \lambda \lambda + \text{H.c.}
$$

- SSB can also be restricted through $RGI \Rightarrow \beta = 0$
- ➤ Leads to a sum rule among scalars and gauging masses

$$
(m_i^2 + m_j^2 + m_k^2)/MM^{\dagger} = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)} + O(g^4)
$$

▶ Breaks conformal invariance BUT remains scale invariant!

one- and two-loop finiteness conditions known all-loop finiteness possible

 Kazakov, Jack, Jones, Pickering…

- **Depends on the gaugino mass M**
- **Scale invariant but not conformal**

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman; Kobayashi, Kubo, Zoupanos

SU(5) FINITE UNIFIED MODELS

The one- and two-loop finiteness conditions imply following matter content:

3 **5** + 3 **10** + 4 (**5** + **5**) + **24**

3 generations, 4 pairs of Higgs doublets one field in the adjoint

- ➤ Soft scalar masses obey sum rule
- ➤ No proton decay
- ➤ At GUT scale finiteness is broken ⇒ MSSM finiteness broken
- ➤ Rotation of FUT Higgs sector ⇒ 2 Higgs doublets of MSSM maximally coupled to third generations

SU(5) FUT THIRD GENERATION

- ➤ Restricted matter spectrum, in particular lots of Higgses
- ➤ Relationship between gauge and Yukawa couplings
- ➤ Sum rule relating mass of Higgs doublets, soft scalars and unified gaugino Mass

$$
g_t^2 = \frac{4}{5} g^2 ; \qquad \qquad g_{b,\tau}^2 = \frac{3}{5} g^2 ;
$$

$$
m_{H_u}^2 + 2m_{10}^2 = M^2 ; \qquad m_{H_d}^2 - 2m_{10}^2 = -\frac{M^2}{3} ; \qquad m_{\overline{5}}^2 + 3m_{10}^2 = \frac{4M^2}{3}
$$

 \blacktriangleright Yukawa couplings determined in terms of g^2 , soft breaking terms depend on M and m₁₀

INTERPLAY HIGH-LOW ENERGIES: SEARCHES AT FUTURE COLLIDERS

Heinemeyer, Kalinowski, Kotlarski , Mondragon, Patellis, Tracas, Zoupanos (2021)

FUTB — 3rd generation

1 free parameter in gauge-Yukawa sector 2 free parameters in soft SUSY breaking

Higgs mass range determined by finiteness, sum rule, B physics constraints and radiative top contributions to Higgs mass \Rightarrow heavy spectrum

MANY ASPECTS OF FINITENESS STUDIED

➤ SU(5) models extensively studied **Rabi et al; Kazakov et al; Quirós et al; MM, Zoupanos et a** ➤ One coincides with a non-standard Calabi-Yau **MM, Zoupanos** ➤ Finite string theories and criteria for branes **Ibáñez** ➤ Models with three generations **Babu, Enkhbat, Gogoladze; MM & Jiménez; Estrada, MM, Patellis, Zoupanos** \triangleright SU(N)^k models finite \Longleftrightarrow 3 generations only SU(3)³ compatible with phenomenology MM, Ma, Zoupanos ➤ Relations non-commutative theories and finiteness **Jack, Jones** ➤ Proof of conformal invariance (dimensionless part) **Kazakov, Bork; MM & Reyes** ➤ Relation between finiteness and QFT in curved space-time & inflation **Elizalde, Odintsov, et al** ➤ Recent reviews **Heinemeyer, M.M, Tracas, Zoupanos, Phys.Rept. 814(2019); Fortsch.Phys. ⁶⁸ (2020)**

SU(N)K — SU(3)3 *5.4. Finite SU*(*N*) of the supersymmetric multiplets (*N, N*⇤ coefficient in the renormalization-group equation of each *SU*(*N*) gauge coupling is simply given by $N_{\rm h} = 20(3)$ of the supersymmetric multiplets (*N, N*⇤

► SU(N)^k models finite \Leftrightarrow three generations! $SU(N)_1 \times SU(N)_2 \times \cdots \times SU(N)_k$ ³ *unification* $SU(N)_1 \times SU(N)_2 \times \cdots \times SU(N)_k$ $SU(1)$ $1 \times SU(1)$ $2 \times$ $1 \times SU(1)$ $1 \times SU(1)$ $\frac{1}{\sqrt{2}}$ \Leftrightarrow three $\left\{\begin{array}{c}$ *N* + *nf* $J(N)^k$ models finite \Leftrightarrow three generations! *,* ¹*,...,* 1) + (1*, ^N, ^N*⇤ $\frac{1}{2}$ V_1 *N* + *nf* \overline{N} $\frac{1}{2}$ $\cdots \times SU$ $V(Y)$

 \blacktriangleright Trinification model beta function SU(3)³ *n* ification model beta function SU(3)³ contraction in the renormalization of each $\mathcal{C}(0)$ gauge coupling is simply given by $\mathcal{C}(0)$ $1 \t1 \t1$ is $\begin{pmatrix} 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{pmatrix}$ b $\begin{pmatrix} 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{pmatrix}$ of $\begin{pmatrix} 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{pmatrix}$ $\text{model beta function } SU(3)^3$ This means the only solution of Eq. (3) the only solution $\mathcal{L}(\mathcal{S})$

$$
b = \left(-\frac{11}{3} + \frac{2}{3}\right)N + n_f\left(\frac{2}{3} + \frac{1}{3}\right)\left(\frac{1}{2}\right)2N = -3N + n_fN.
$$

 \blacktriangleright Finite \iff 3 generations field theory, the existence of three families of quarks and leptons is natural in such models, provided the matter content $t = \frac{1}{2}$ *duh* transformation as a strange of the contract of

$$
q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3),
$$

$$
\lambda = \begin{pmatrix} N & E^c & v \\ E & N^c & e \\ v^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*).
$$

▶ Only SU(5) and SU(3)³ seem to have phenomenological possibilities so far *E N^c e* In order for all the gauge couplings to be equal at an energy scale, *M*GUT, the cyclic symmetry *Z*³ must be imposed, i.e. nly SU(5) and SU(3)³ seem to have phenomenological model in Ref. **in Ref. [137] in Fig. 2003** in Ref. [137] in the context of superstring-inspired **E6.**

MM, Ma, Zoupanos (2004);
Heinemeyer, MM, Ma, Zoupanos (2010) MM, Ma, Zoupanos (2004);

d^c dc d^c

!!
!!

,..., 1) +···+ (*N*⇤

, 1*,* 1*,..., N*). The one-loop -function

h^c h^c h^c

2-LOOP SU(3)3 FINITE MODEL

➤ 2-loop finite SU(3)3 trinification model, parametric solution of reduction equations

$$
f^2 = r\left(\frac{16}{9}\right)g^2
$$
, $f^2 = (1 - r)\left(\frac{8}{3}\right)g^2$

r parameterizes different solutions, f and f' are Yukawa couplings for quarks and leptons

- Good top and bottom masses with one parameter
- Large tan , heavy SUSY spectrum
- Possibility of neutrino masses, consistent with seesaw
- At high energies vector-like down type quarks
- Split-SUSY possible

GYU FROM REDUCTION OF COUPLINGS AT WORK

WHAT NOW? FLAVOR…

- ➤ So far detailed analysis only for third generation
- ➤ As mentioned, some 3 generation finite models exist
- ➤ SU(5) models some textures given
- \blacktriangleright SU(3)³ naturally have 3 generations
- ➤ How to do it more systematically?

GENERAL SUPERPOTENTIAL FOR SU(5) FUTS For the above-mentioned matter content we will use the following notation: 3 ¯ *^a*0*ⁱ* neutrinos, 3 *Xij ^a*⁰ superfields in the 10 represent the up-type quarks and antiquarks,

▶ The SU(5) superpotential of possible finite models is $\bar{\mathcal{H}}_{ai} = \bar{\mathbf{5}}$, $\mathcal{H}_{a}^{i} = \mathbf{5}$, $\bar{\Psi}_{a'i} = \bar{\mathbf{5}}$, $X_{a'}^{ij}$ $= 10$, $\Sigma^{i}_{j} = 24$ \mathbf{r} and \mathbf{r} and the charged and the charged and the charged anti-leptons, 4 \mathbf{r} *^a* and 4 *^H*¯*ai* in the ⁵ $\bar{\mathcal{H}}_{\cdot} = \bar{\mathbf{5}}$ $\mathcal{H}^{i} = 5$ $\bar{\Psi}_{\cdot} = \bar{\mathbf{5}}$ $X^{ij} = 10$ $\Sigma^{i} = 24$ for the gauge group a ⁰ a ⁰ a ⁰ *a*⁰ *j*

3 generations, 4 pairs of Higgs doublets and one field in the adjoint fermion generations and the non-primed in the adjoint and one field in the adjoint

3 **5** + 3 **10** + 4 (**5** + **5**) + **24** $3\overline{5}+3\overline{10}+4(5+\overline{5})+\overline{24}$

$$
\mathcal{W}_{SU(5)-R} = \bar{g}_{a'b'a} \bar{\Psi}_{b'i} X_{a'}^{ij} \bar{\mathcal{H}}_{aj} + \frac{1}{2} g_{a'b'a} \epsilon_{ijklm} X_{a'}^{ij} X_{b'}^{kl} \mathcal{H}_a^m + f_{ab} \bar{\mathcal{H}}_{ai} \Sigma^i{}_j \mathcal{H}_b^j
$$

+
$$
\frac{1}{3!} p \Sigma^i{}_j \ \Sigma^j{}_k \ \Sigma^k{}_i + \frac{1}{2} \lambda^{(\Sigma)} \Sigma^i{}_j \Sigma^j{}_i + m_{ab} \bar{\mathcal{H}}_{ai} \mathcal{H}_b^i.
$$

 $\overline{g}_{ijk} = down Yukawa couplings, g_{ijk} = up Yukawa couplings$

WHAT ABOUT FLAVOR? 3 GENERATIONS

- ➤ Look for FUT 3 generation models all-loops 2-loops
- ➤ Solutions for Yukawa couplings
- ➤ Sum rule in SSB
- ➤ Check absence of proton decay
- ➤ Mass matrices

Everything OK? then…

- ▶ Rotate to MSSM
- ➤ Look again for mass matrices
- ▶ Good textures?

FINITE S3 MODEL

- ➤ Solutions to the RE may imply extra symmetries, so far discrete
- \triangleright There are models with A_4 and Q_6 , Babu et al; MM & Jiménez
- \triangleright S₃, smallest non-Abelian discrete group, successful at low energies
- ➤ Irreps: **2, 1, 1A** →Two generations in doublet, third in singlet

➤ Look for all-loop finite model rat for all loop finite model

S3 MASS MATRICES essays to solve the doublet-triplet splitting. The selection of the finite splitting. The finite splitting splitting. The finite splitting splitting. The finite splitting splitting. The finite splitting On the other hand, the terms *^mabH*¯*aH^b* break the cyclic symmetries, and are necconditions, which depend on the trilinear terms. The solutions terms. The solutions to the solutions to the finite *m*² = *m*² = e1 e2 **1** \mathbf{r} *MM† ^m*²

2

The up and down mass matrices for this model are:

.
.

$$
|g_{113}|^2 = \frac{4}{5}g_5^2 \ , \ |g_{131}|^2 = \frac{4}{5}g_5^2 \ , \ |\bar{g}_{113}|^2 = \frac{3}{5}g_5^2 \ , \ |\bar{g}_{131}|^2 = \frac{3}{5}g_5^2 \ , \ |\bar{g}_{311}|^2 = \frac{3}{5}g_5^2 \ ,
$$

$$
|f_{11}|^2 = |f_{33}|^2 = 0 \ , \ |f_{44}|^2 = g_5^2 \ , \ |p|^2 = \frac{15}{7}g_5^2 \ ,
$$

e
e e d

2

MM† ²*m*²

 $\frac{1}{\sqrt{7}}$ 1 $\frac{1}{\sqrt{7}}$ I disawa couplings completely determined; $\sqrt{\frac{1}{4.1}}$ Yukawa couplings completely determined!

$$
M_u = \begin{pmatrix} g_{113} \langle \mathcal{H}_3^5 \rangle & 0 & g_{131} \langle \mathcal{H}_1^5 \rangle \\ 0 & g_{113} \langle \mathcal{H}_3^5 \rangle & g_{131} \langle \mathcal{H}_2^5 \rangle \\ g_{131} \langle \mathcal{H}_1^5 \rangle & g_{131} \langle \mathcal{H}_2^5 \rangle & 0 \end{pmatrix},
$$

$$
M_d = \begin{pmatrix} \bar{g}_{113} \langle \bar{\mathcal{H}}_{35} \rangle & 0 & \bar{g}_{131} \langle \bar{\mathcal{H}}_{15} \rangle \\ 0 & \bar{g}_{113} \langle \bar{\mathcal{H}}_{35} \rangle & \bar{g}_{131} \langle \bar{\mathcal{H}}_{25} \rangle \\ \bar{g}_{311} \langle \bar{\mathcal{H}}_{15} \rangle & \bar{g}_{311} \langle \bar{\mathcal{H}}_{25} \rangle & 0 \end{pmatrix}
$$

real, to avoid extra sources of CP violations of CP violations and CP violation of CP violation.
The contract of CP violation of CP violation.
The contract of CP violation of CP violation. BUL... LOO TESLITCLIVE, TWO HIDSES dIHIOST UEGENETALE s_{max} spectrum at low the mass as a second degenerate. PUU....COO I COLICLIVC, CWO IIICOOCO CIIIIOOL QCSCIICICICICI But…too restrictive, two masses almost degenerate

CYCLIC SYMMETRIES — 3 GENERATIONS models that express cyclic symmetries. This classification is divided into two block IC SYMMETRIES - 3 GENERATIONS and another four matrices *V* (*i*) , where they couple to the four Higgs pairs. The

subscripts 3 and 4 in *V* (*i*) and *V* (*i*) Classification of SU(5) FUT with off-diagonal γ done already For the case of three pairs of Higgs coupled to the fermions, the matrices are:

Coupled to 3 Higgs doublets

$$
V_3^{(1)} = \begin{pmatrix} g_{111} \langle \mathcal{H}_1^5 \rangle & g_{123} \langle \mathcal{H}_2^5 \rangle & g_{132} \langle \mathcal{H}_2^5 \rangle \\ g_{213} \langle \mathcal{H}_3^5 \rangle & g_{222} \langle \mathcal{H}_2^5 \rangle & g_{231} \langle \mathcal{H}_1^5 \rangle \\ g_{312} \langle \mathcal{H}_2^5 \rangle & g_{321} \langle \mathcal{H}_1^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} , V_3^{(2)} = \begin{pmatrix} g_{112} \langle \mathcal{H}_2^5 \rangle & g_{121} \langle \mathcal{H}_1^5 \rangle & 0 \\ g_{211} \langle \mathcal{H}_1^5 \rangle & g_{223} \langle \mathcal{H}_2^5 \rangle \\ 0 & g_{322} \langle \mathcal{H}_2^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} V_3^{(3)} = \begin{pmatrix} g_{113} \langle \mathcal{H}_3^5 \rangle & g_{121} \langle \mathcal{H}_1^5 \rangle & 0 \\ g_{211} \langle \mathcal{H}_1^5 \rangle & g_{223} \langle \mathcal{H}_2^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} , V_3^{(4)} = \begin{pmatrix} g_{111} \langle \mathcal{H}_1^5 \rangle & 0 & 0 \\ 0 & g_{223} \langle \mathcal{H}_2^5 \rangle & g_{232} \langle \mathcal{H}_2^5 \rangle \\ 0 & g_{322} \langle \mathcal{H}_2^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix}
$$

while for the case where the four pairs of Higgs couplets they are: \sim 2000 μ are: \sim 2000 μ are: \sim 2000 μ are: μ and μ are:

$$
V_4^{(1)} = \begin{pmatrix} g_{111} \langle \mathcal{H}_1^5 \rangle & g_{124} \langle \mathcal{H}_4^5 \rangle & g_{132} \langle \mathcal{H}_2^5 \rangle \\ g_{214} \langle \mathcal{H}_4^5 \rangle & g_{222} \langle \mathcal{H}_2^5 \rangle & g_{231} \langle \mathcal{H}_1^5 \rangle \\ g_{312} \langle \mathcal{H}_2^5 \rangle & g_{321} \langle \mathcal{H}_1^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} , \quad V_4^{(2)} = \begin{pmatrix} g_{112} \langle \mathcal{H}_2^5 \rangle & g_{121} \langle \mathcal{H}_1^5 \rangle & 0 \\ g_{211} \langle \mathcal{H}_1^5 \rangle & g_{222} \langle \mathcal{H}_2^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} V_4^{(3)} = \begin{pmatrix} g_{113} \langle \mathcal{H}_3^5 \rangle & g_{121} \langle \mathcal{H}_1^5 \rangle & g_{132} \langle \mathcal{H}_2^5 \rangle \\ g_{211} \langle \mathcal{H}_1^5 \rangle & g_{222} \langle \mathcal{H}_2^5 \rangle & g_{234} \langle \mathcal{H}_4^5 \rangle \\ g_{312} \langle \mathcal{H}_2^5 \rangle & g_{324} \langle \mathcal{H}_4^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} , \quad V_4^{(4)} = \begin{pmatrix} g_{113} \langle \mathcal{H}_3^5 \rangle & g_{121} \langle \mathcal{H}_1^5 \rangle & g_{132} \langle \mathcal{H}_2^5 \rangle \\ g_{211} \langle \mathcal{H}_1^5 \rangle & g_{223} \langle \mathcal{H}_3^5 \rangle & g_{234} \langle \mathcal{H}_4^5 \rangle \\ g_{312} \langle \mathcal{H}_2^5 \rangle & g_{324} \langle \mathcal{H}_4^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix}
$$

These matrices with same structure and down mass matrices with same structure and h ^{*Babu*} Enkhbat Gogoladze (2003) *M*^{*u*} *M*^{*u} Top and down mass matrices with same structure* Babu, Enkhbat, Gogoladze (2003)

parametric relations among couplings \Rightarrow 2-loop solution We find the following symmetries \Rightarrow

$$
up-type
$$

\nYukawa
\nYukawa
\n
$$
|g_{124}|^2 = |g_{214}|^2 = \frac{4}{5}g_5^2
$$
, $|g_{222}|^2 = \frac{2}{5}g_5^2$, $|g_{231}|^2 = |g_{321}|^2 = \frac{1}{10}(8g_5^2 - 5|g_{111}|^2)$,
\n
$$
|g_{333}|^2 = \frac{6}{5}g_5^2
$$
, $|\bar{g}_{111}|^2 = |\bar{g}_{124}|^2 = \frac{3}{20}(8g_5^2 - 5|g_{111}|^2)$,
\n
$$
down-type
$$

\n
$$
|\bar{g}_{214}|^2 = \frac{3}{4}|g_{111}|^2
$$
, $|\bar{g}_{222}|^2 = |\bar{g}_{231}|^2 = \frac{3}{10}g_5^2$, $|\bar{g}_{321}|^2 = -\frac{3}{20}(2g_5^2 - 5|g_{111}|^2)$,
\nYukawa
\n
$$
|\bar{g}_{333}|^2 = \frac{9}{10}g_5^2
$$
, $|f_{22}|^2 = \frac{3}{4}g_5^2$, $|f_{33}|^2 = \frac{g_5^2}{4}$, $|p|^2 = \frac{15}{7}g_5^2$,
\n
$$
|g_{132}|^2 = |g_{312}|^2 = |\bar{g}_{132}|^2 = |\bar{g}_{312}|^2 = |f_{11}|^2 = |f_{44}|^2 = 0
$$
.

By imposing the positivity conditon to the squared norm of the couplings, we find the following constraint for $|g_{111}|^2$:

$$
\frac{2}{5}g_5^2 \le |g_{111}|^2 \le \frac{8}{5}g_5^2
$$

 . (81) *implies more symmetry = more zeroes evaluating at the end points*

EXAMPLE OF SOLUTIONS Solutions with extra zero textures are found in the limiting values \mathbf{r} are found in the coupling values for the coupling *|g*111*|* 2 . For instance, when *|g*111*|* \mathbf{B} is the positivity condition to the squared norm of the squared norm of the couplings, we find the coupling system of the coupling system of the coupling system of the couplings, we find the coupling system of the the following constraint for *|g*111*|* 2

- Many solutions, depend on the free parameter | g_{111} |² zero, leading to the following mass textures. The following mass textures is a steading mass textures is a ste
. $\left[1\right]$ \sim $\overline{1}$ *g*2 ⁵ *|g*111*|* 2 $\overline{1}$ *g*2 ⁵ *.* (81) Solutions, append on the lice parameter [8111]
- ➤ Taking the Yukawa values at extreme points in inequality → more zeroes, more symmetry? $\frac{1}{2}$ are the Yukawa values at ext 0 awa values at extrem α *noints in inequality* q uality \rightarrow clearly incompatible with phenomenology. Another solution is found when *|g*¯321*|* Taking the Yukawa values at extreme points in inequality \rightarrow zero, and there we readed as the s

8

$$
|g_{111}|^2 = \frac{2}{5}g_5^2
$$

leads to leads to 0, which leads to *|g*111*|*

$$
M_u = \begin{pmatrix} g_{111} \langle \mathcal{H}_1^5 \rangle & g_{124} \langle \mathcal{H}_4^5 \rangle & 0 \\ g_{214} \langle \mathcal{H}_4^5 \rangle & g_{222} \langle \mathcal{H}_2^5 \rangle & g_{231} \langle \mathcal{H}_1^5 \rangle \\ 0 & g_{321} \langle \mathcal{H}_1^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} , \quad M_d = \begin{pmatrix} \overline{g}_{111} \langle \overline{\mathcal{H}}_{15} \rangle & \overline{g}_{124} \langle \overline{\mathcal{H}}_{45} \rangle & 0 \\ \overline{g}_{214} \langle \overline{\mathcal{H}}_{45} \rangle & \overline{g}_{222} \langle \overline{\mathcal{H}}_{25} \rangle & \overline{g}_{231} \langle \overline{\mathcal{H}}_{15} \rangle \\ 0 & 0 & \overline{g}_{333} \langle \overline{\mathcal{H}}_{15} \rangle \end{pmatrix}
$$

- \blacktriangleright M_u compatible with phenomenology, but M_d?
- ➤ RGE analysis and sum rule might change a bad structure into a good/bad one Cakir, Solmaz Xin (2008); Zhang, Zhou (2008) \mathbf{d}

ALL-LOOP FINITE MODEL — V4 2 ¹ *Xkl* ² *^H^m* ¹ + *g*232✏*ijklmXij* ² *Xkl* ⁺ *^f*33*H*¯3*j*⌃*^j iHi* ³ ⁺ *^f*44*H*¯4*j*⌃*^j iHi* ⁴ + 1

 Estrad a, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

 \triangleright We find the following symmetries \Rightarrow isolated solution unique relation among couplings ⇒ all-loop finite solution Using the allowed couplings in (87), we obtain the following solutions: *|f*33*|* ² ⁼ *[|]f*44*[|]* ² ⁼ ¹ decay. This breaking does not affect the finiteness conditions, which apply only to $\frac{1}{2}$ the following symmetries \rightarrow isolated <u>2</u> ⁵ , Table 6: Cyclic discrete symmetries of model 4*.*1 to obtain isolated, non-degenerate \cup $\frac{1}{2}$ plings \Rightarrow all all-loop finite solution we find the following sym

$$
|g_{114}|^2 = |g_{121}|^2 = |g_{211}|^2 = |g_{232}|^2 = |g_{322}|^2 = |g_{333}|^2 = \frac{4}{5}g_5^2
$$

$$
|\bar{g}_{114}|^2 = |\bar{g}_{121}|^2 = |\bar{g}_{211}|^2 = |\bar{g}_{232}|^2 = |\bar{g}_{322}|^2 = |\bar{g}_{333}|^2 = \frac{3}{5}g_5^2
$$

$$
|f_{33}|^2 = |f_{44}|^2 = \frac{1}{2}g_5^2 \qquad |p|^2 = \frac{15}{7}g_5^2
$$

 \triangleright For the SSB \Rightarrow sum rule \Rightarrow 3 free parameters *H*¯⁴ *For the SSB* \Rightarrow sum rule \Rightarrow 3 free parameters T_{or} the CCD \rightarrow cum rule \rightarrow 2 free parameters $\text{for the odd} \rightarrow \text{with the two points } \theta$

$$
m_{\tilde{\psi}_1}^2 = m_{\tilde{\psi}_3}^2 = \frac{1}{6} \left(-MM^\dagger + 9m_{H_3}^2 \right) , \quad m_{\tilde{\psi}_2}^2 = \frac{1}{6} \left(-MM^\dagger - 6m_{H_1}^2 + 15m_{H_3}^2 \right) ,
$$

\n
$$
m_{\tilde{\chi}_1}^2 = m_{\tilde{\chi}_3}^2 = \frac{1}{2} \left(MM^\dagger - m_{H_3}^2 \right) , \quad m_{\tilde{\chi}_2}^2 = \frac{1}{2} \left(MM^\dagger - 2m_{H_1}^2 + m_{H_3}^2 \right) ,
$$

\n
$$
m_{\tilde{H}_1}^2 = m_{\tilde{H}_2}^2 = \frac{1}{3} \left(2MM^\dagger + 3m_{H_1}^2 - 6m_{H_3}^2 \right) , \quad m_{\tilde{H}_3}^2 = m_{\tilde{H}_4}^2 = \frac{1}{3} \left(2MM^\dagger - 3m_{H_3}^2 \right) ,
$$

\n
$$
m_{H_2}^2 = m_{H_1}^2 ; \quad m_{H_4}^2 = m_{H_3}^2 , \quad m_{\phi_\Sigma}^2 = \frac{1}{3} MM^\dagger .
$$

\n(89)

arg (¯*g*333) = *^C*³ *C*⁴ + arg (*g*114)arg (*g*121)arg (*g*211) + arg (*g*322) + arg (¯*g*232) = ¯³ *.* ALL-LOOP FINITE MASS MATRICES **Estrada, MM, Patellis, Zoupanos, Fortschr.** Physion

the phases in the mass in the mass mass mass matrix was are various ways to distribute the four of the four of
the four states was to distribute the four states was to distribute the four states was to distribute the four *Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001*

- \blacktriangleright It is possible to find the minimum amount of phases Phasing mvariance rephasing invariants
- The mass matrices are them. ➤ The mass matrices are then:

$$
M_u = \begin{pmatrix} g_{114} \langle \mathcal{H}_4^5 \rangle & g_{121} \langle \mathcal{H}_1^5 \rangle & 0 \\ g_{211} \langle \mathcal{H}_1^5 \rangle & 0 & g_{232} \langle \mathcal{H}_2^5 \rangle \\ 0 & g_{322} \langle \mathcal{H}_2^5 \rangle & g_{333} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} = \frac{2}{\sqrt{5}} g_5 \begin{pmatrix} \langle \mathcal{H}_4^5 \rangle & \langle \mathcal{H}_1^5 \rangle & 0 \\ \langle \mathcal{H}_1^5 \rangle & 0 & \langle \mathcal{H}_2^5 \rangle \\ 0 & \langle \mathcal{H}_2^5 \rangle & e^{i\phi_3} \langle \mathcal{H}_3^5 \rangle \end{pmatrix} ,
$$

$$
M_d = \begin{pmatrix} \bar{g}_{114} \langle \bar{\mathcal{H}}_{45} \rangle & \bar{g}_{121} \langle \bar{\mathcal{H}}_{15} \rangle & 0 \\ \bar{g}_{211} \langle \bar{\mathcal{H}}_{15} \rangle & 0 & \bar{g}_{232} \langle \bar{\mathcal{H}}_{25} \rangle \\ 0 & \bar{g}_{322} \langle \bar{\mathcal{H}}_{25} \rangle & \bar{g}_{333} \langle \bar{\mathcal{H}}_{35} \rangle \end{pmatrix} = \sqrt{\frac{3}{5}} g_5 \begin{pmatrix} \langle \bar{\mathcal{H}}_{45} \rangle & \langle \bar{\mathcal{H}}_{15} \rangle & 0 \\ e^{i \bar{\phi}_1} \langle \bar{\mathcal{H}}_{15} \rangle & 0 & \langle \bar{\mathcal{H}}_{25} \rangle \\ 0 & e^{i \bar{\phi}_2} \langle \bar{\mathcal{H}}_{25} \rangle & e^{i \bar{\phi}_3} \langle \bar{\mathcal{H}}_{35} \rangle \end{pmatrix}.
$$

After the rotation in the Higgs sector to the MSSM basis: $\overline{\mathcal{O}}$

Same solution as FUTB for 3rd generation! we know it works…

$$
M_u = \frac{2}{\sqrt{5}} g_5 \begin{pmatrix} \widetilde{\alpha}_4 & \widetilde{\alpha}_1 & 0 \\ \widetilde{\alpha}_1 & 0 & \widetilde{\alpha}_2 \\ 0 & \widetilde{\alpha}_2 & e^{i\phi_3} \widetilde{\alpha}_3 \end{pmatrix} \langle \mathcal{K}_3^5 \rangle ,
$$

\n
$$
M_d = \sqrt{\frac{3}{5}} g_5 \begin{pmatrix} \widetilde{\beta}_4 & \widetilde{\beta}_1 & 0 \\ e^{i\overline{\phi}_1} \widetilde{\beta}_1 & 0 & \widetilde{\beta}_2 \\ 0 & e^{i\overline{\phi}_2} \widetilde{\beta}_2 & e^{i\overline{\phi}_3} \widetilde{\beta}_3 \end{pmatrix} \langle \overline{\mathcal{K}}_{35} \rangle .
$$

where ↵e*ⁱ* and Σ_{ij} , β_i refer to the rotation angles in up and down sectors respectively, $\Sigma \beta_i = \Sigma \alpha_i = 1$

FINALLY, HOW MANY FREE PARAMETERS?

Low energies: radiative electroweak breaking, fix m_τ **^{exp} and SM vev give tan** β \Rightarrow **12 parameters left:**

The soft breaking terms, the phases, and the rotation angles $\phi_1, \phi_2, \phi_3, \phi_4, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, M, \mu$

GUT scale 89 free parameters

Yukawa couplings, soft breaking terms, phases, vev's of the Higgs fields

After Finiteness solutions 33 free parameters

Require doublet-triplet splitting, rotation to MSSM basis with constraints over angles, rephasing invariants

 Only one phase is observable

 \Rightarrow ϕ_{obs} , α_1 , α_2 , α_3 , β_1 , β_2 , β_3 , M, μ **only 9 parameters left to fit masses and mixing angles**

WHAT ABOUT NEUTRINO MASSES, DARK MATTER, ETC?

- ➤ **SU(5) models:** Cold DM LSP is neutralino ⇒ overabundance
- ➤ Neutrino masses may be incorporated by breaking R symmetry ⇒ gravitino Dark Matter
- ➤ Other mechanisms? thermal inflation?
- \ge g-2 like in SM
- ➤ **SU(3)3 models:** ν_R are present
- ➤ Neutrino masses may be generated by seesaw or radiatively
- ➤ Depending on the breaking of $SU(3)³$ DM may be neutralino (or scalar?)
- ➤ Neutralino DM overabundance

Flavor Structure may change the above!

CONCLUSIONS AND OUTLOOK

- ➤ Reduction of couplings finiteness powerful principle implies Gauge Yukawa Unification
- ➤ Conformal or scale invariant theory
- ➤ SSB terms satisfy a sum rule among soft scalars
- ➤ SSB same as anomaly mediated scenario
- ➤ Finiteness reduces greatly number of free parameters completely finite theories SU(5)
- ➤ Very predictive
- ➤ Flavor 3 generation models 2-loops: Yukawa couplings determined within a range All-loops: Yukawa couplings completely determined
- ➤ Leads to viable mass textures
- ➤ Drastic reduction in number of free parameters
- ➤ Free parameters come from Higgs sector, SSB and phases
- ➤ More fundamental theory?

 How can we restrict phases? CP violation? Higgs sector? Flavor processes? Dark matter? Inflation? Bariogenesis?

