FLAVOR (MODELS) IN FINITE UNIFIED THEORIES

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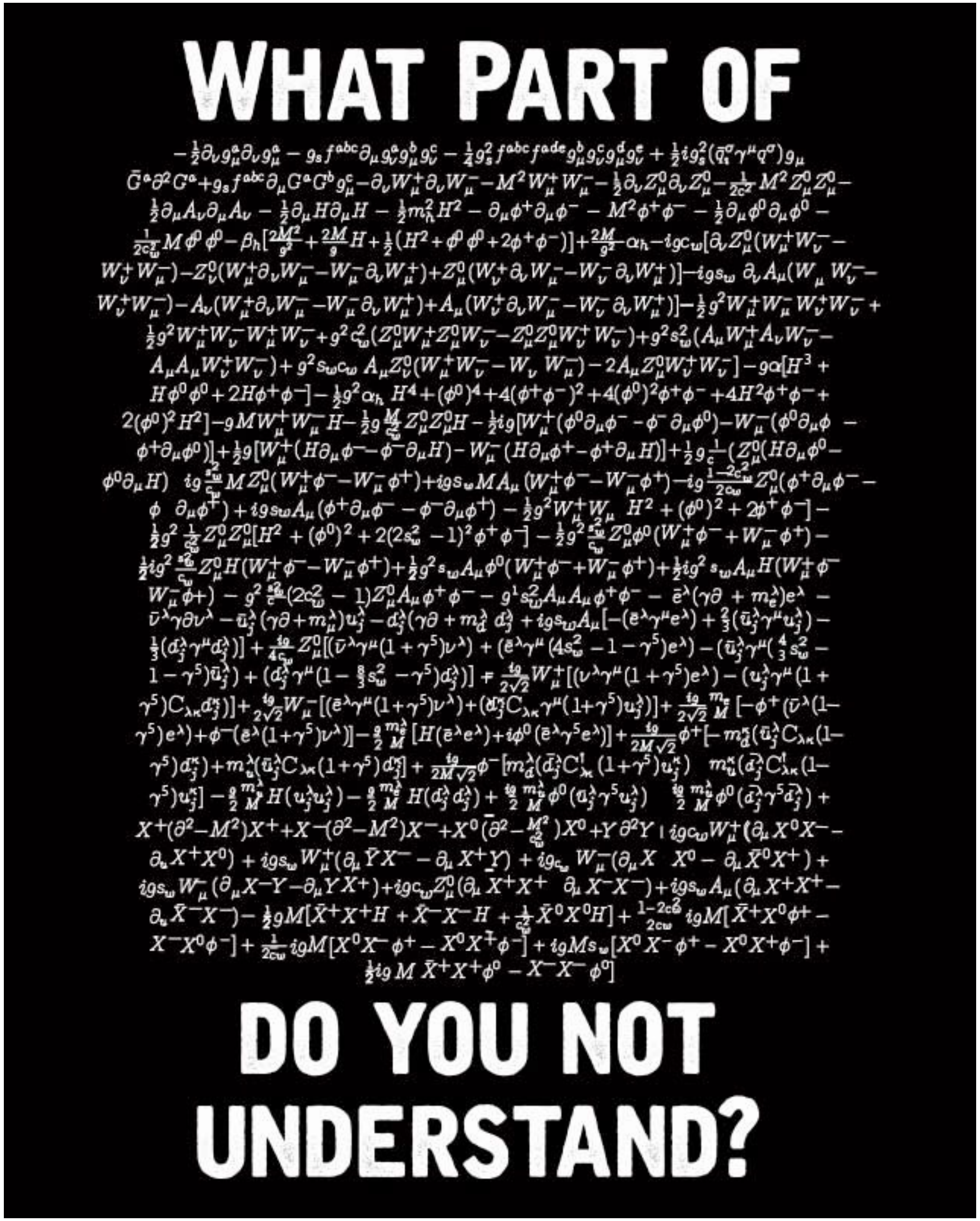
CONAHCYT
CONSEJO NACIONAL DE HUMANIDADES CIENCIAS Y TECNOLOGÍAS



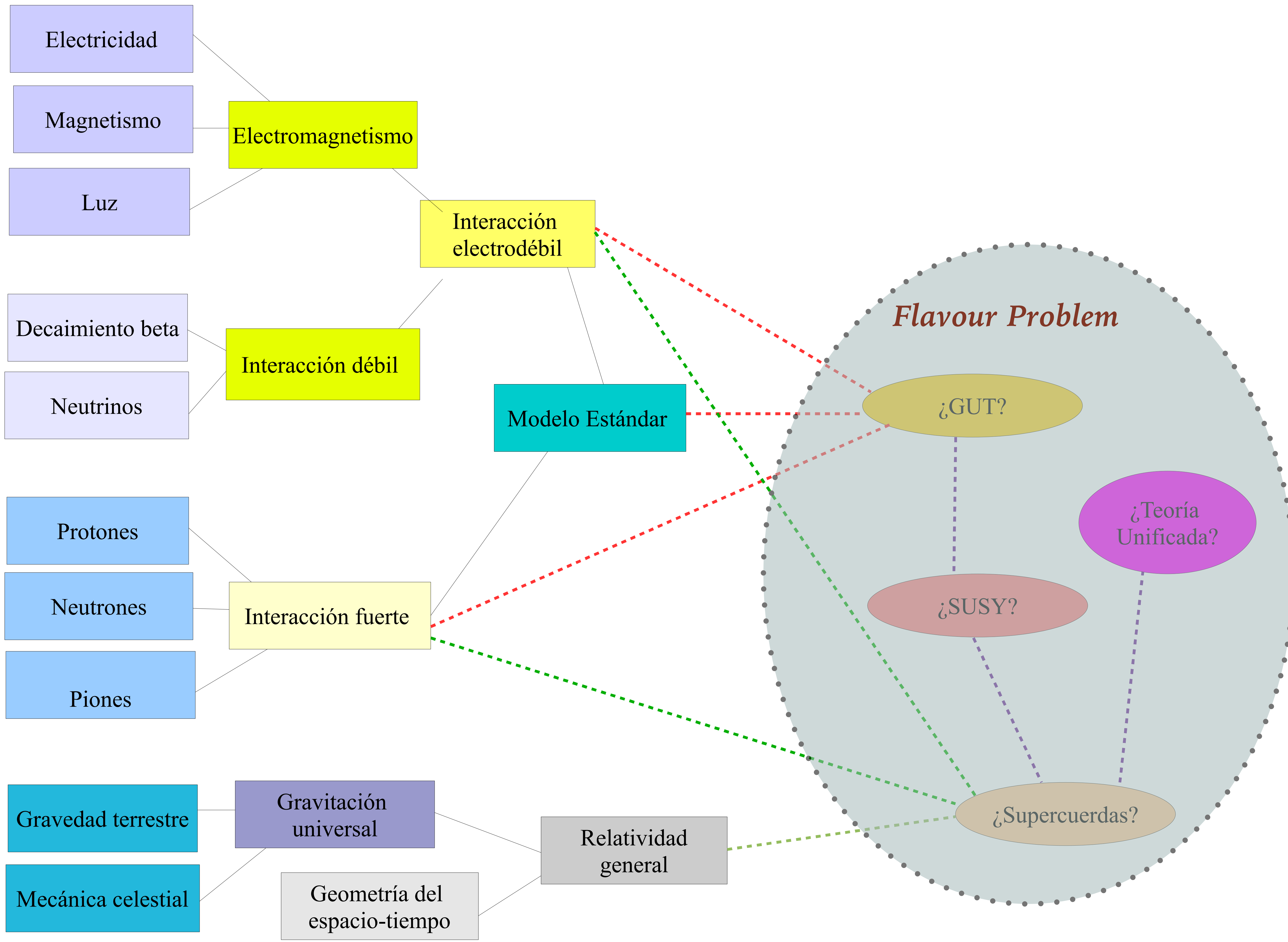
$+ \Lambda CDM\ldots$

WHAT'S GOING ON?

- What happens as we approach the Planck scale?
- What happened at the early Universe?
- How do we go from an effective theory like the SM to a more fundamental one?

- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- Why/how are the elementary particle masses so different?
- Is there more than one Higgs, more scalars?
- What about flavor?

- **Where is the new physics?**


Electricidad
Magnetismo
Luz
Electromagnetismo
Interacción electrodébil
Decaimiento beta
Neutrinos
Interacción débil
Modelo Estándar
Protones
Neutrones
Piones
Interacción fuerte
Gravedad terrestre
Mecánica celestial
Gravitación universal
Geometría del espacio-tiempo
Relatividad general
Flavour Problem
¿GUT?
¿Teoría Unificada?
¿SUSY?
¿Supercuerdas?

FLAVOUR

➤ Interactions that distinguish between different flavours

- why 3 generations?
- why those masses?
- why the gap between neutral and charged fermions
- why the difference between mixing matrices?
- why that amount of CP violation?
- …

- *Fermion masses*
- *Mixing*
- *CP violation*

Lead to discoveries

- $\Gamma(KL\to\mu+\mu-)/\Gamma(K+\to\mu+\nu)$ → *charm quark*
- Δm_K → *charm mass*
- Δm_B → *top mass*
- ε_K → *third generation*
- *ν oscillation* → *ν mass*

Connections to new/unknown physics

- *Dark matter*
- *Baryogenesis*
- *Leptogenesis*
- *EW phase transition*
- *??*

Nir, CERN–LATAM School HEP (2015)

SOME ASPECTS OF THE FLAVOUR PROBLEM

- Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- Neutrino masses unknown, only difference of squared masses.
- Type of hierarchy (normal or inverted) also unknown
- Higgs sector under study

- Quark mixing angles

$$\theta_{12} \approx 13.0^o$$
$$\theta_{23} \approx 2.4^o$$
$$\theta_{13} \approx 0.2^o$$

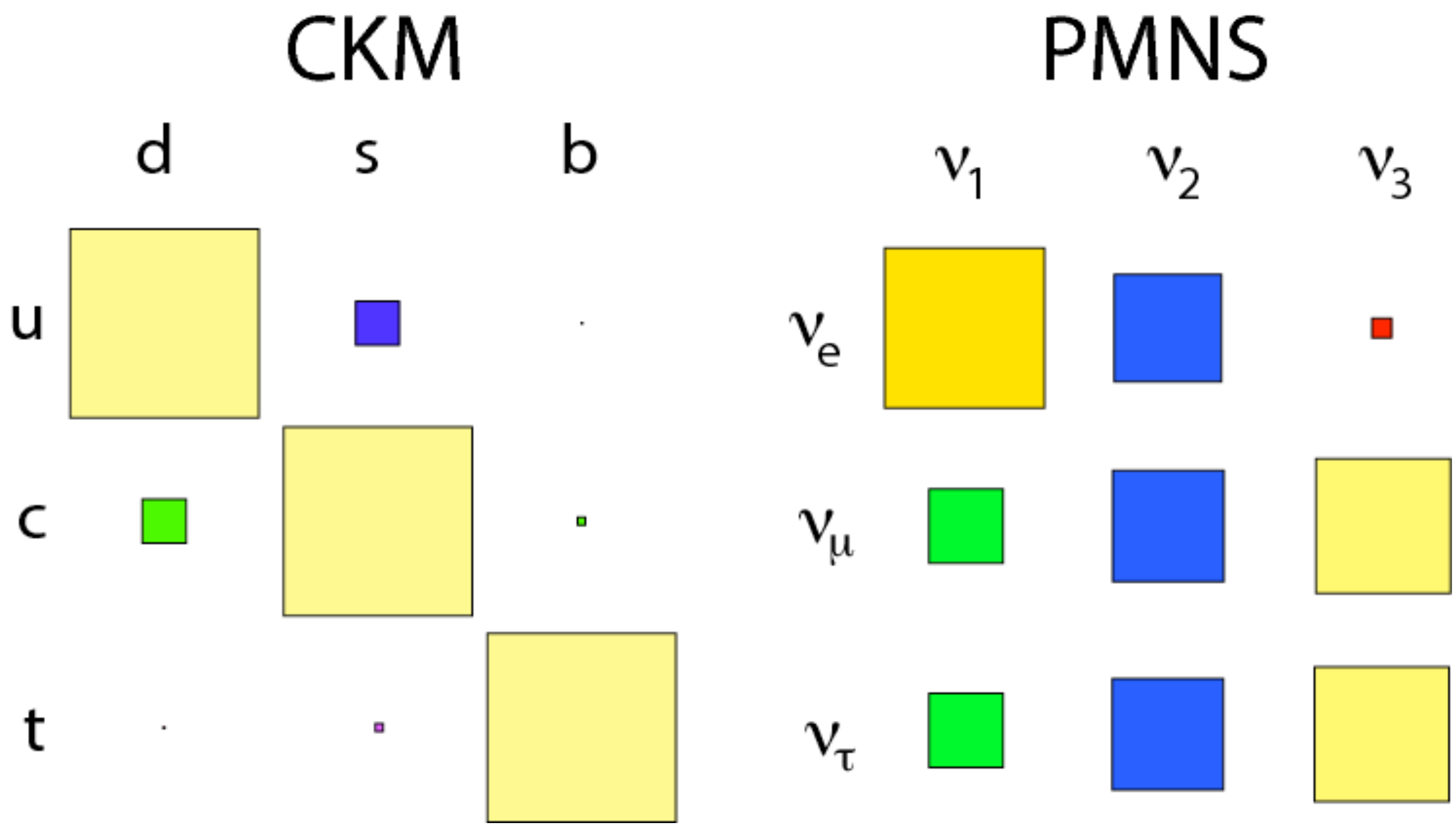
- Neutrino mixing angles

$$\Theta_{12} \approx 33.8^\circ$$
$$\Theta_{23} \approx 48.6^\circ$$
$$\Theta_{13} \approx 8.6^\circ$$

- Small mixing in quarks, large mixing in neutrinos. Very different
- Is there an underlying symmetry?

?

PMNS VS CKM



hierarchical

democratic

QUARKS, CHARGED LEPTONS AND HIGGS INTERRELATED

- Yukawa couplings: several orders of magnitude of difference, strong hierarchy

$$\mathcal{L}_{\mathrm{Y}}^{\mathrm{ME}} = Y_{ij}^{d}\overline{Q_{Li}}\phi D_{Rj} + Y_{ij}^{u}\overline{Q_{Li}}\tilde{\phi} U_{Rj} + Y_{ij}^{e}\overline{L_{Li}}\phi E_{Rj} + \text{h.c.}.$$

Also neutrinos, but they could acquire mass other ways.

- Higgs sector:

$$\mathcal{L}_{\phi}^{\mathrm{ME}} = -\mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2 \qquad v^2 = -\frac{\mu^2}{\lambda}$$

  - hierarchy problem (quadratic radiative corrections)
  - limits to perturbative unitarity
  - Why $M_{\mathrm{Higgs}} \sim 125$ GeV?

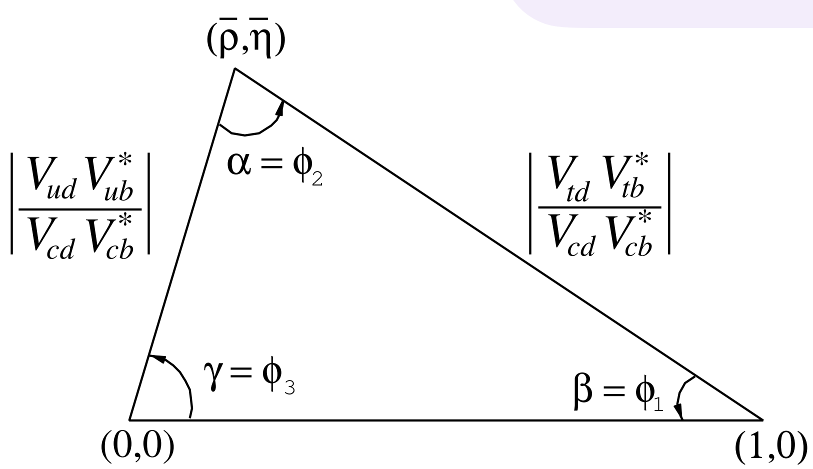
$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$V_{\rm CKM}$ very well determined

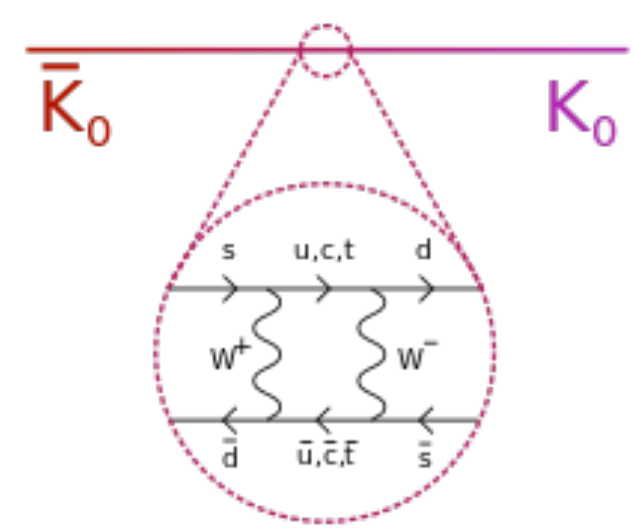
PDG 2023

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$



$$J = (3.08^{+0.15}_{-0.13}) \times 10^{-5}$$

$$\sin\theta_{12} = 0.22500 \pm 0.00067\,, \qquad \sin\theta_{13} = 0.00369 \pm 0.00011\,,$$
$$\sin\theta_{23} = 0.04182^{+0.00085}_{-0.00074}\,, \qquad \delta = 1.144 \pm 0.027\,.$$



K, B, B_S, D processes can be used to study new physics

FCNCs very sensitive to BSM

PMNS MATRIX PONTECORVO-MAKI-NAKAGAWA-SAKATA

$$U = \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{vmatrix} \begin{vmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{vmatrix} \begin{vmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

↑ atmospheric ↑ reactor ↑ solar

- Neutrinos also mix → neutrino oscillations
- Dirac or Majorana
- Three mixing angles and a phase: atmospheric Θ_{23}, solar Θ_{12} and reactor Θ_{13}. Possible also Majorana phases
- Only determined squared mass differences

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

FERMION AND SCALAR SECTORS

- Free parameters in quarks:
  6 masses ->Yukawa couplings
  3 mixing angles
  CP violating phase
- Unitarity —> Jarlskog invariants

- Free parameters in neutrinos:
  6 masses
  3 mixing angles
  CP violating phase
  2 Majorana phases
- Unitarity? —> Also Jarlskog invariants

Plus Higgs vev and self coupling

FLAVOUR SYMMETRIES

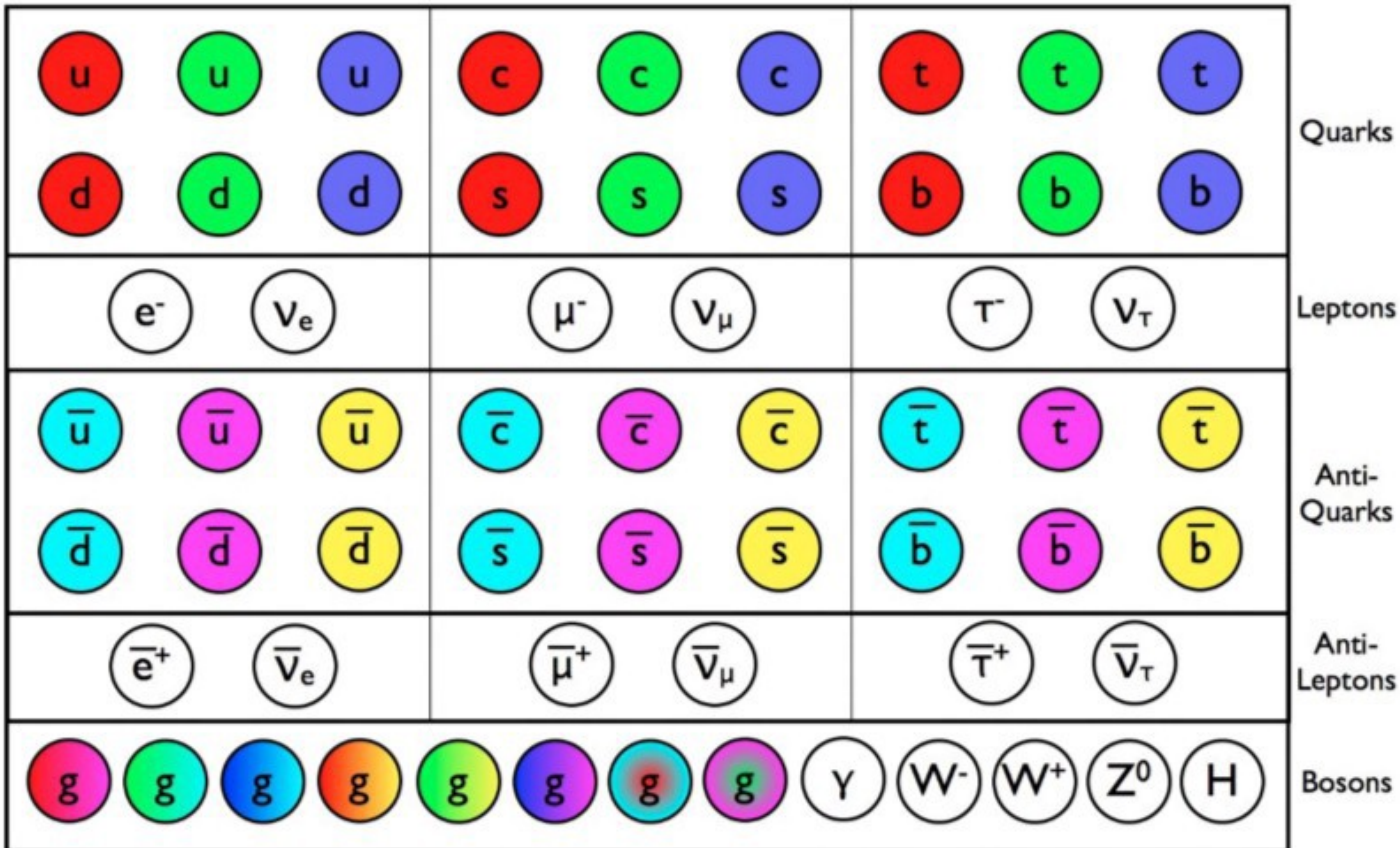
- Flavour symmetries: continuous or discrete?

discrete
could lead to domain walls

continuous
breaking may give massless
Goldstone bosons

- At low energies now discrete preferred. Could be:
    - Residual symmetry from breaking from continuous one
    - From the breaking of a larger discrete group
    - Discrete from the "beginning"

All the particles we have discovered so far…



E. Siegel / Beyond the Galaxy, World Scientific

Only one Higgs boson…

MULTI-HIGGS MODELS AND FLAVOUR SYMMETRIES

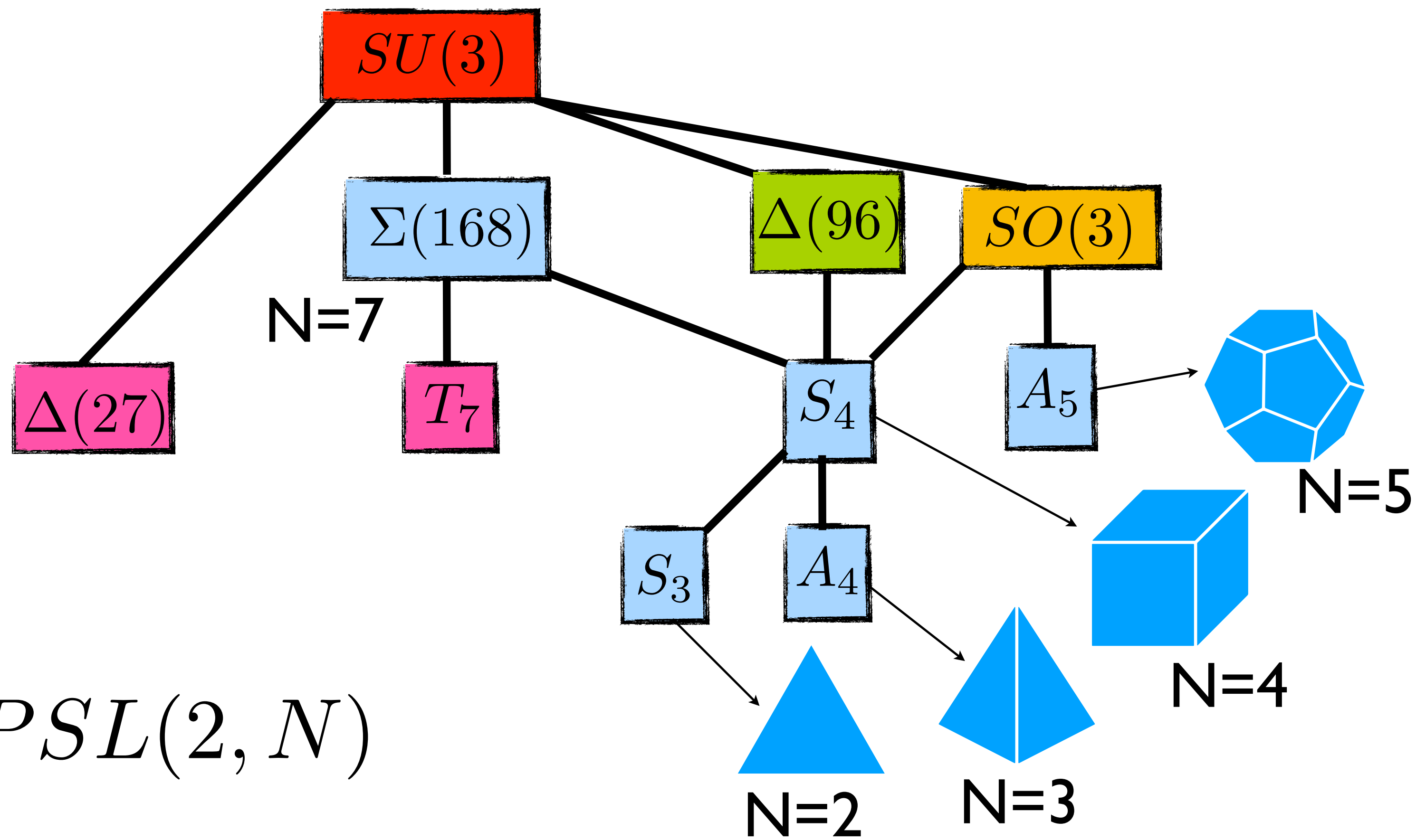
- 2HDM widely studied, several studies on 3HDM **(Branco et al,; King et al, *JHEP* 01 (2014) 052 al, 2014)**
- Extra Higgs doublets and discrete symmetries → continuous symmetries
- After minimization of the potential there might be residual symmetries → unphysical quark sector, degenerate masses/zero masses/zeroes in V_{CKM}, e.g. S_3, S_4, A_4, $\Delta(54)$ all have residual symmetries in 3HDM
- Z_N Abelian symmetries very popular, easier to implement

Complicated potential, many new parameters, many "exotic" scalars

Ivanov, Prog.Part.Nucl.Phys. 95 (2017)

- H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, 1003.3552
- S. F. K., A. Merle, S. Morisi, Y. Shimizu, and M. Tanimoto, 1402.4271

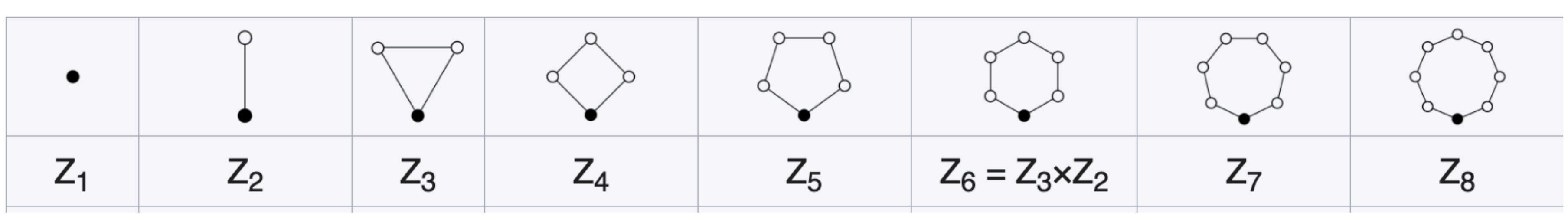
Non-Abelian Family Symmetry



$$PSL(2, N)$$

from Steve King's talk at Modular Invariance Approach to the lepton and quark flavor problem, Mainz, May 2024

Cyclic Symmetries



Wikipedia, from Juan Camilo's Acosta poster, 9th COMHEP

MASS MATRICES TEXTURES — TEXTURE ZEROES

- Zeroes in the mass matrices —> less parameters, underlying symmetries: Fritzsch

This version excluded already

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & 0 & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

hierarchical A»|B|»|C|

- In SM and extensions (no FC right-handed currents) is always possible to simultaneously the Mu and Md to Hermitian or NNI textures

- NNI

$$M'_q = \begin{pmatrix} 0 & C_q & 0 \\ C'_q & 0 & B_q \\ 0 & B'_q & A_q \end{pmatrix}$$

B' ≠ B, C' ≠ C

- What works? up and down sector same structure, coming from same dynamics

Xing Phys.Rept. 854 (2020)

ALLOWED TEXTURES

Table 14: The five phenomenologically viable five-zero textures of Hermitian quark mass matrices.

	I	II	III	IV	V
$M_u =$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B_u' & 0 \\ 0 & 0 & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & 0 & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & D_u \\ 0 & B_u' & 0 \\ D_u^* & 0 & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B_u' & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & D_u \\ 0 & B_u' & B_u \\ D_u^* & B_u^* & A_u \end{pmatrix}$
$M_d =$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B_d' & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B_d' & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B_d' & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B_d' & 0 \\ 0 & 0 & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B_d' & 0 \\ 0 & 0 & A_d \end{pmatrix}$

Above textures first found by Ramond et al (1993), still work today if not strongly hierarchical.

- But so far the best one is:

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & B_q' & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

From Z. Xing, Phys.Rept. 854 (2020)

TEXTURES AT HIGH ENERGIES

- Usually express mass matrices as mass ratios →
  they remain stable below eW scale, but renormalize above it, depending on model

- From high to low energies they get renormalized as,

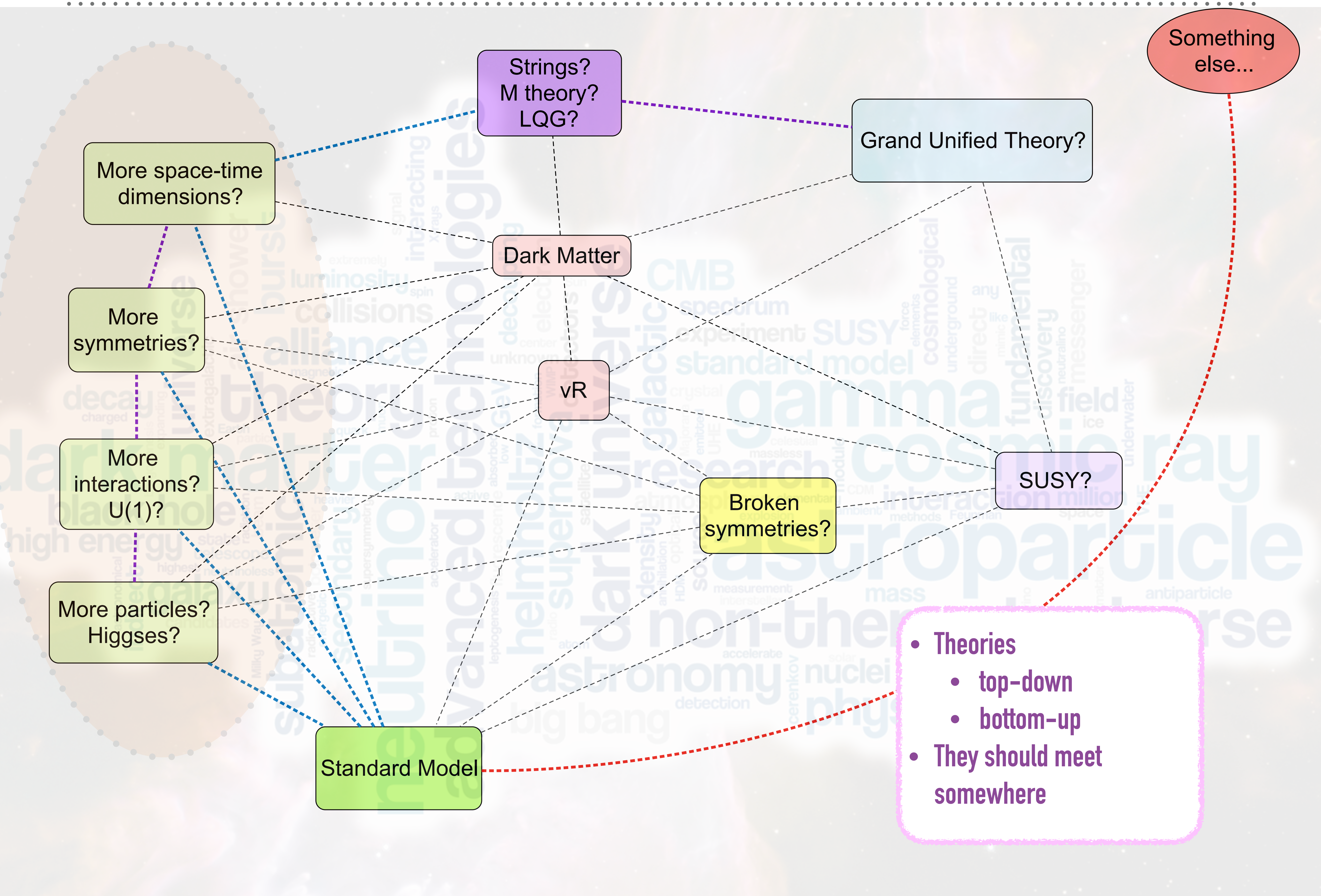
$$M_\mathrm{u}(\Lambda_\mathrm{EW}) \simeq \gamma_\mathrm{u}\left[\begin{pmatrix} 0 & C_\mathrm{u} & 0 \\ C_\mathrm{u}^* & B'_\mathrm{u} & B_\mathrm{u} I_t^{C_\mathrm{u}^\mathrm{u}} \\ 0 & B_\mathrm{u}^* I_t^{C_\mathrm{u}^\mathrm{u}} & A_\mathrm{u} I_t^{C_\mathrm{u}^\mathrm{u}} \end{pmatrix} + \frac{I_t^{C_\mathrm{u}^\mathrm{u}} - 1}{A_\mathrm{u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & |B_\mathrm{u}|^2 & B_\mathrm{u} B'_\mathrm{u} \\ 0 & B_\mathrm{u}^* B'_\mathrm{u} & 0 \end{pmatrix}\right]$$

$$M_\mathrm{d}(\Lambda_\mathrm{EW}) \simeq \gamma_\mathrm{d}\left[\begin{pmatrix} 0 & C_\mathrm{d} & 0 \\ C_\mathrm{d}^* & B'_\mathrm{d} & B_\mathrm{d} \\ 0 & B_\mathrm{d}^* I_t^{C_\mathrm{d}^\mathrm{u}} & A_\mathrm{d} I_t^{C_\mathrm{d}^\mathrm{u}} \end{pmatrix} + \frac{I_t^{C_\mathrm{d}^\mathrm{u}} - 1}{A_\mathrm{u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & B_\mathrm{u} B_\mathrm{d}^* & A_\mathrm{d} B_\mathrm{u} \\ 0 & B_\mathrm{u}^* B'_\mathrm{d} & B_\mathrm{u}^* B_\mathrm{d} \end{pmatrix}\right]$$

I's are the one-loop corrections, γ anomalous dimensions, C's coefficients in the running

- Textures remain, coefficients change, for MSSM there is dependence on soft breaking terms

HOW DO WE GO BEYOND THE SM?

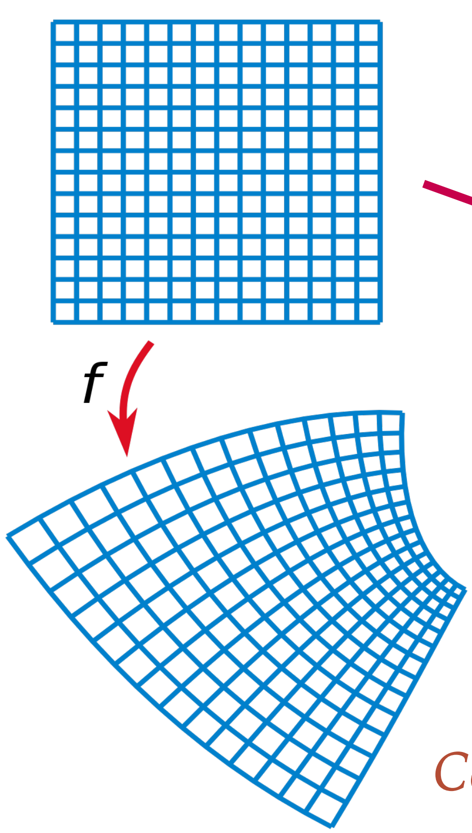


- Theories
  - top-down
  - bottom-up
- They should meet somewhere

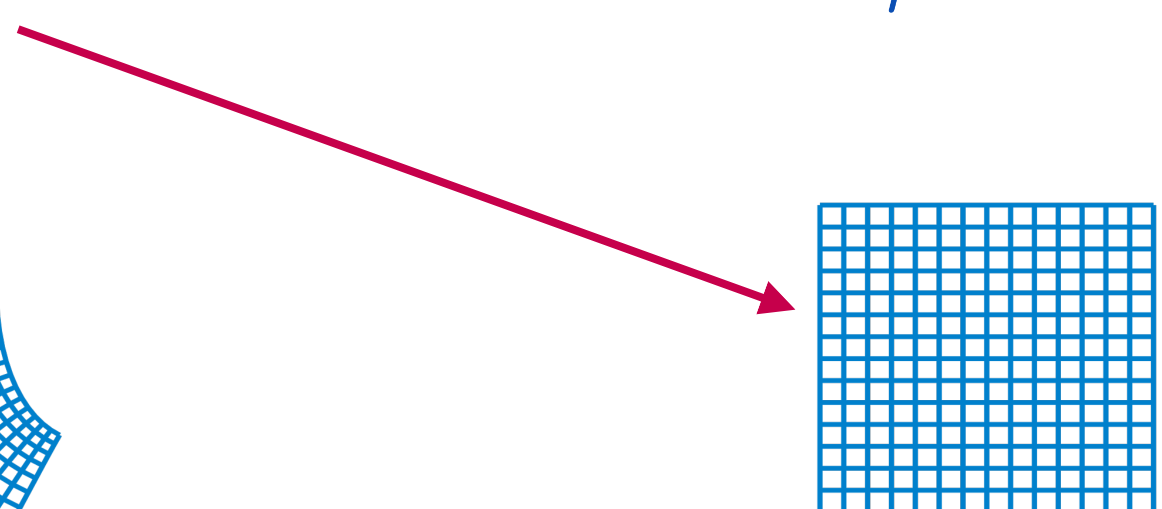
HOW DO WE MOVE UP (OR DOWN) IN ENERGY?

- We know how a QFT behaves at different scales through the renormalization group RG
- The theory has the same structure at different energy scales, but the parameters — couplings and masses — change with energy
- Related to scale invariance and conformal invariance

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} \qquad \gamma(\phi) = \mu \frac{\partial \ln Z}{\partial \mu}$$



Conformally invariant



Scale invariant

Scale invariance appears in many physical systems

HOW TO GO BEYOND THE STANDARD MODEL (BSM)?

- Traditional way $\Rightarrow$ addition of symmetries

  N=1 SUSY

- Very effective, but too many free parameters

  Can get messy…

- Complementary approach
  Look for renormalization group invariant relations
  at high energies

  GUT $\Rightarrow$ Planck

- Resulting theory has few free parameters $\therefore$ very predictive

Relates gauge and Yukawa sector
Predictions for 3rd generation masses

RENORMALIZATION GROUP INVARIANTS RGI

- Search for more fundamental theory $\Rightarrow$ less parameters
  Renormalization Group Invariants (RGI)

$$\Phi(g_1, \ldots, g_N) = 0$$

$$\mu\, d\Phi/d\mu = \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i = 0$$

- Equivalent to solve reduction equations

$$\beta_g\,(dg_i/dg) = \beta_i$$

$$i = 1, \ldots, N$$

- Reduced theory has only one coupling and its beta function
- Reduction $\longrightarrow$ power series solution
- Uniqueness of solution can be studied at one-loop

Zimmermann (1985); Zimmermann, Oehme, Sibold (1984-1985)

REDUCTION OF COUPLINGS

- Couplings related to a primary coupling
  totally reduced — all couplings depend on one
  partially reduced — some couplings depend on one

- Can be applied to SUSY and non-SUSY models

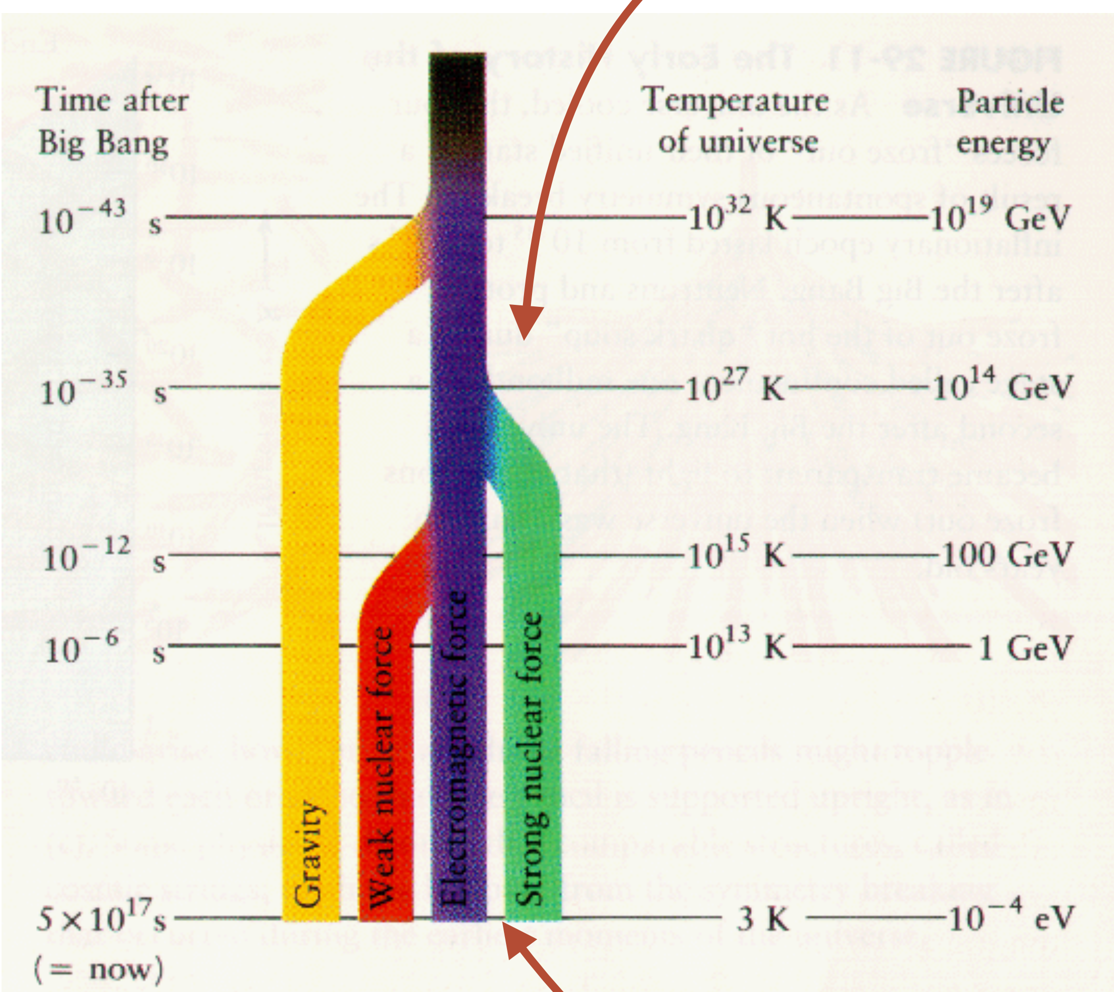
- SM analyzed — results now ruled out, still impressive
  Kubo, Sibold, Zimmermann (1984-1987)

- 2HDM analyzed Denner (1990) — now re-analysed:
  possible to have one-loop reduced equations in type II 2HDM at a high-scale boundary May Pech, MM, Patellis, Zoupanos (2023)

- Under some conditions SUSY unification models
  finite =
  absence of ∞ renormalizations

> - Many solutions imply SUSY
> - SUSY indispensable for finiteness
> - And no… SUSY not excluded experimentally but some low energy models are indeed excluded

FINITENESS = SCALE/CONFORMAL INVARIANCE



- All-loop finiteness ⇒ $\beta = 0$
  to all orders in perturbation theory
- Scale/conformal invariance
  Conformal and scale invariant = Yukawa couplings
  Scale invariant = Soft breaking terms
  Do not depend on energy scale
  Based on RGI and reduction of couplings
- Gives UV completion of the QFT
- Reduces greatly the number of free parameters
  ⇒ new symmetries
- Partial reduction ⇒ predictions for 3rd generation masses

FINITE SU(5) THEORIES — THIRD GENERATION

- Prediction for top mass — very clean

$M_{top}^{th} \sim 178$ GeV 1993 Kapetanakis, M.M., Zoupanos

m_bot also predicted, large tan beta

$M_{top}^{exp} = 176 \pm 18$ GeV 1995

$M_{top}^{th} \sim 172.5$ GeV 2007 Heinemeyer, M.M.,Zoupanos

$M_{top}^{exp} = 173.1 \pm .09$ GeV 2013

- Prediction for Higgs mass — depends on soft breaking terms, also very restricted

$M_{Higgs}^{th} \sim 121 - 126$ GeV 2008, 2013 Heinemeyer, M.M., Zoupanos

$M_{Higgs}^{exp} = 126 \pm 1$ GeV 2013

FINITESS $\Longrightarrow$ GAUGE YUKAWA UNIFICATION

Grand Unified SUSY N=1, no gauge anomalies:

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k$$

$$\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$$

$$\sum_i T(R_i) = 3C_2(G) \,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i)$$

T Dynkin index of irrep, C_2 Casimir invariant of group *C_{ijk} Yukawa couplings, g gauge coupling*

- Restricts the gauge group
- Relates gauge and Yukawa couplings
- If finite to all orders $\Rightarrow$ Conformal invariance
- May imply extra symmetries, in this case discrete

- Just analyze one-loop solution
- One-loop finite $\Rightarrow$ two-loop finite
- Isolated and non-degenerate solution $\Rightarrow$ all-loop finite Lucchesi, Piguet, Sibold

β= 0 non-renormalization of coupling constants, not complete UV finiteness where field renormalization is absent

SUSY BREAKING SSB

- Explicit/soft breaking >100 new free parameters 😫

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

- SSB can also be restricted through RGI ⇒ β = 0
- Leads to a sum rule among scalars and gauging masses

$$(\ m_i^2 + m_j^2 + m_k^2\ )/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(2)} + O(g^4)$$

- Breaks conformal invariance BUT remains scale invariant!

- one- and two-loop finiteness conditions known
- all-loop finiteness possible

Kazakov, Jack, Jones, Pickering…

- Depends on the gaugino mass M
- Scale invariant but not conformal

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman; Kobayashi, Kubo, Zoupanos

SU(5) FINITE UNIFIED MODELS

The one- and two-loop finiteness conditions imply following matter content:

$$3\ \overline{\mathbf{5}} + 3\ \overline{\mathbf{10}} + 4\ (\mathbf{5} + \overline{\mathbf{5}}) + \overline{\mathbf{24}}$$

3 generations, 4 pairs of Higgs doublets one field in the adjoint

- Soft scalar masses obey sum rule
- No proton decay
- At GUT scale finiteness is broken $\Rightarrow$ MSSM finiteness broken
- Rotation of FUT Higgs sector $\Rightarrow$ 2 Higgs doublets of MSSM maximally coupled to third generations

SU(5) FUT THIRD GENERATION

- Restricted matter spectrum, in particular lots of Higgses
- Relationship between gauge and Yukawa couplings
- Sum rule relating mass of Higgs doublets, soft scalars and unified gaugino Mass

$$g_t^2 = \frac{4}{5}\, g^2 \;\; ; \qquad g_{b,\tau}^2 = \frac{3}{5}\, g^2 \; ;$$

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2 \;\; ; \quad m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} \;\; ; \quad m_{\bar{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$$

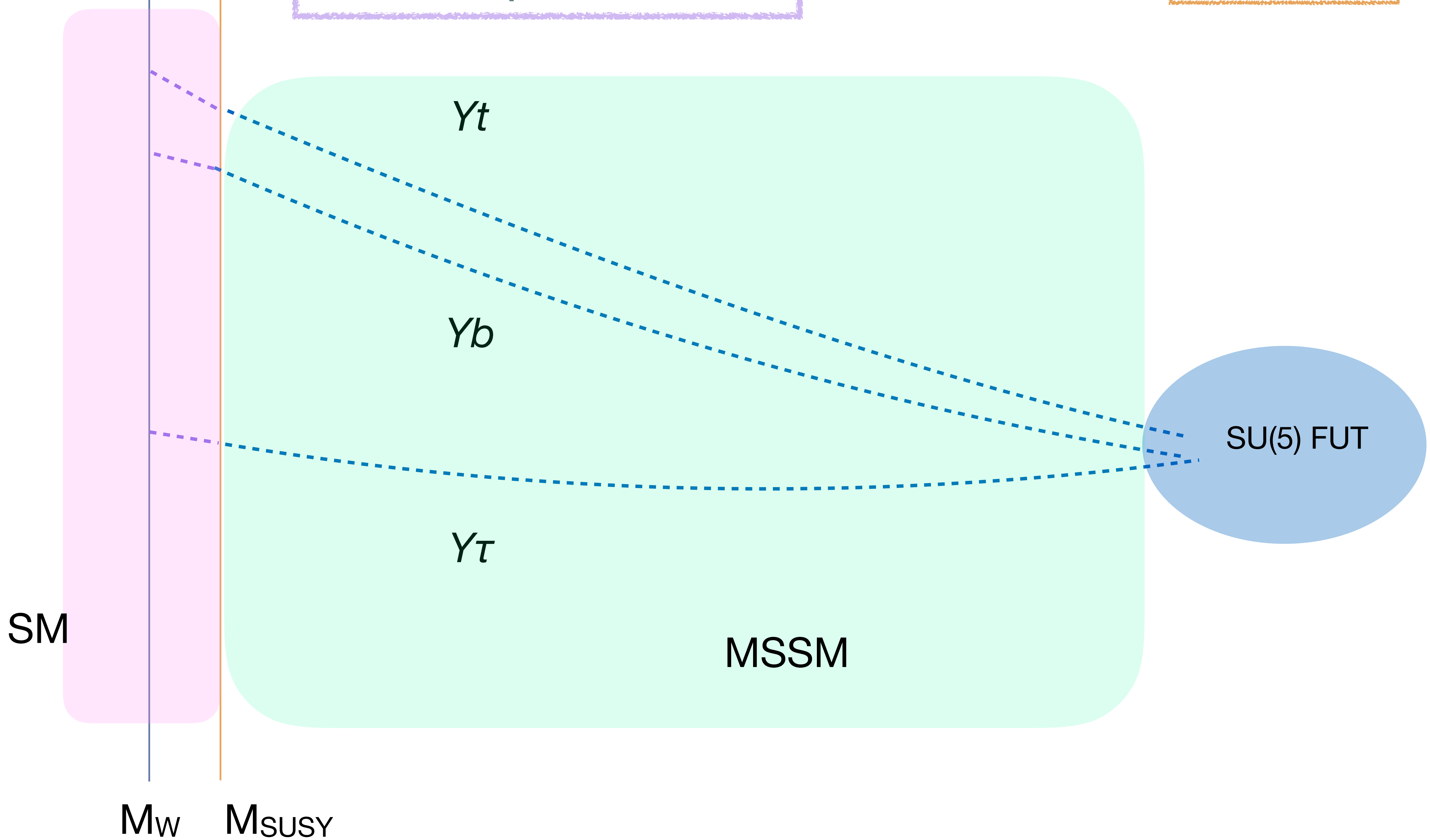
- Yukawa couplings determined in terms of g^2, soft breaking terms depend on M and m_{10}

Finite soft breaking terms included

⇒ SUSY corrections to Yb and Ytau

⇒ soft SUSY spectrum

$$Y^2_t = k_t\, g^2$$
$$Y^2_{b,\tau} = k_b\, g^2$$



Results confronted to experimental constraints ⇒

gives available parameter space

$m_t = Y_t\, v_u \qquad v_u / v_d = \tan\beta$

$m_{b,\tau} = Y_{b,\tau}\, v_d \qquad v_d = m_\tau^{exp} / Y_\tau$

INTERPLAY HIGH-LOW ENERGIES: SEARCHES AT FUTURE COLLIDERS

GUT scale, Finiteness gives:

- Relations between gauge-Yukawa couplings
- Sum rule for soft breaking terms
- ⇒ Very few free parameters

Require:

- Absence of proton decay
- Proper unification of gauge couplings
- MSSM

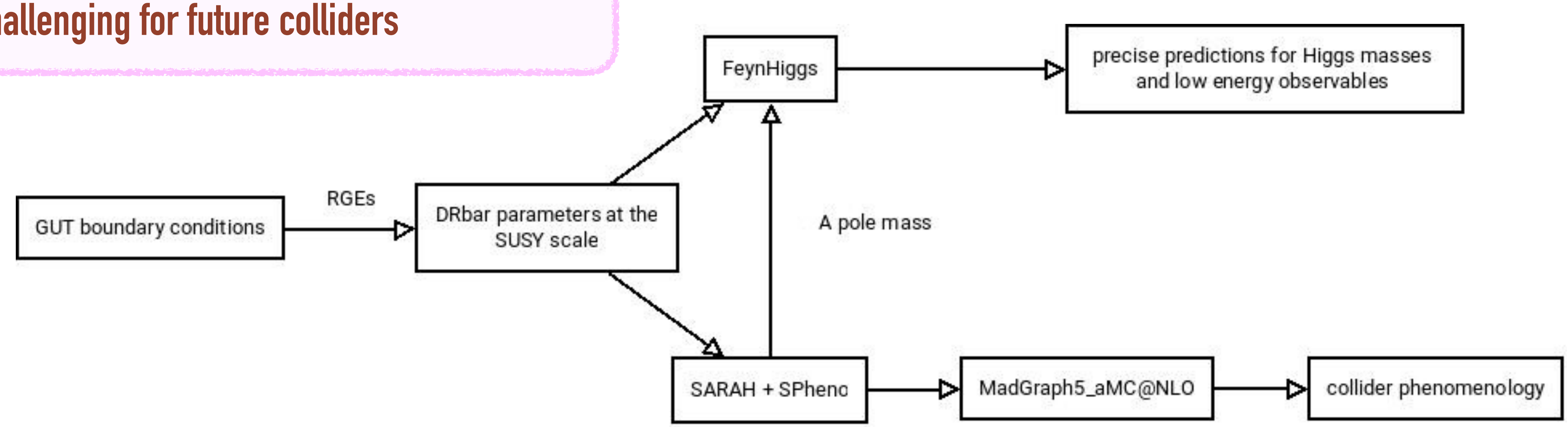
Low energies:

- Radiative eW symmetry breaking
- Include SUSY radiative corrections
- Quark and Higgs masses in experimental range
- Compliance with B physics (not trivial)

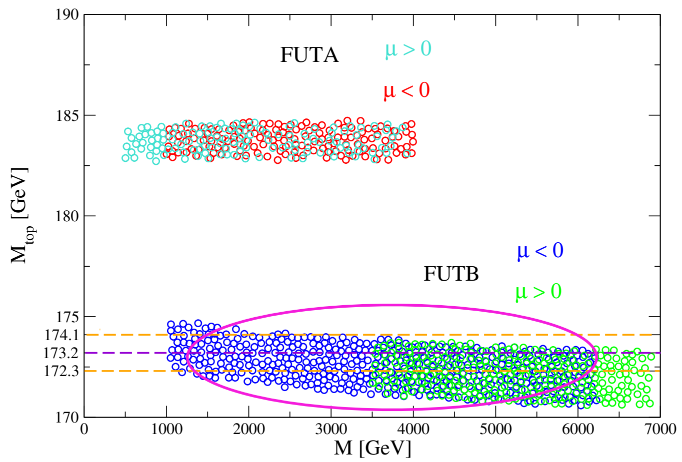
Large tan β
High SUSY spectrum > 1 TeV
Challenging for future colliders

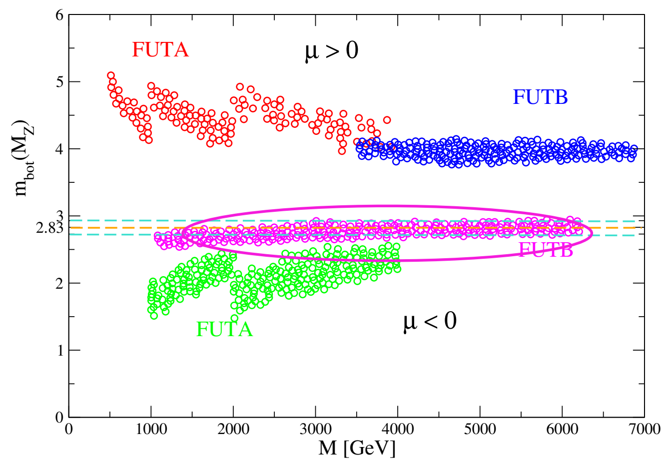
B constraints:

- BR (b → sγ)
- BR (Bs → μ+μ-)
- BR (Bu → $\tau\upsilon$) Bs
- Δ M Bs SM/MSSM



Heinemeyer, Kalinowski, Kotlarski , Mondragon, Patellis, Tracas, Zoupanos (2021)

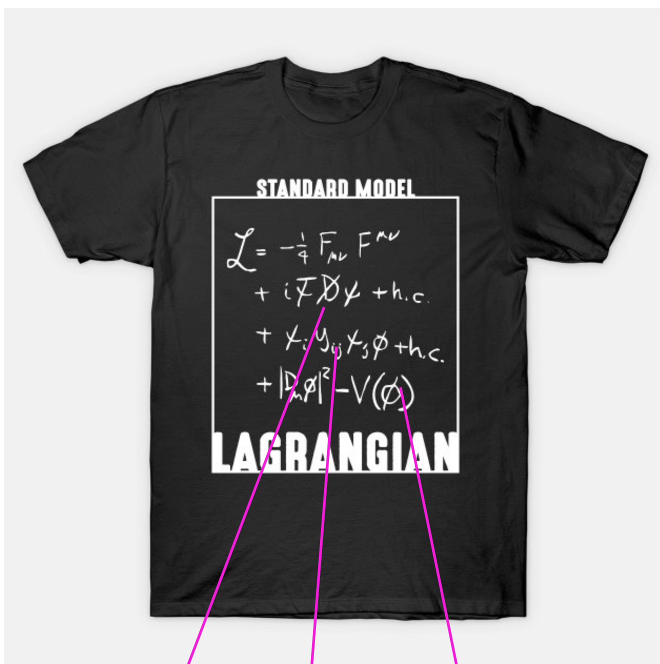




FUTB — 3rd generation

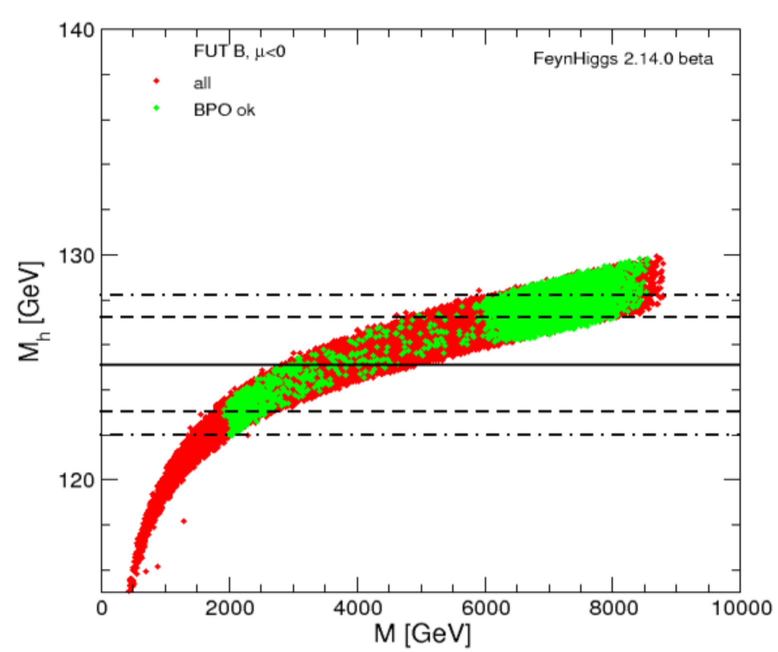
1 free parameter in gauge-Yukawa sector

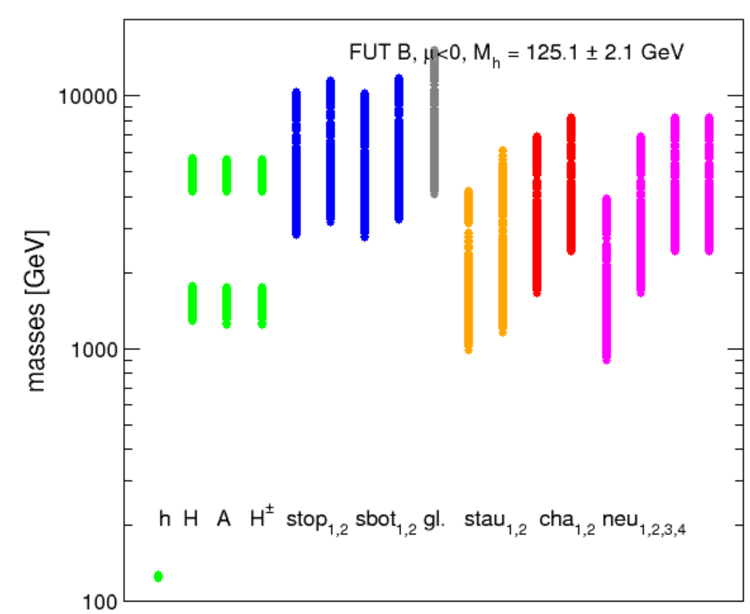
2 free parameters in soft SUSY breaking



Higgs mass range determined by finiteness, sum rule, B physics constraints and radiative top contributions to Higgs mass ⇒ heavy spectrum

These are now related!





MANY ASPECTS OF FINITENESS STUDIED

- $SU(5)$ models extensively studied Rabi et al; Kazakov et al; Quirós et al; MM, Zoupanos et a
- One coincides with a non-standard Calabi-Yau MM, Zoupanos
- Finite string theories and criteria for branes Ibáñez
- Models with three generations Babu, Enkhbat, Gogoladze; MM & Jiménez; Estrada, MM, Patellis, Zoupanos
- $SU(N)^k$ models finite $\Longleftrightarrow$ 3 generations
  only $SU(3)^3$ compatible with phenomenology MM, Ma, Zoupanos
- Relations non-commutative theories and finiteness Jack, Jones
- Proof of conformal invariance (dimensionless part) Kazakov, Bork; MM & Reyes
- Relation between finiteness and QFT in curved space-time & inflation
  Elizalde, Odintsov, et al
- Recent reviews Heinemeyer, M.M, Tracas, Zoupanos, Phys.Rept. 814 (2019); Fortsch.Phys. 68 (2020)

SU(N)^k — SU(3)^3

- SU(N)^k models finite $\Leftrightarrow$ three generations!

$$SU(N)_1 \times SU(N)_2 \times \cdots \times SU(N)_k$$

- Trinification model beta function SU(3)^3

$$b = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right)\left(\frac{1}{2}\right) 2N = -3N + n_f N \,.$$

- Finite $\Longleftrightarrow$ 3 generations

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3),$$

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*).$$

- Only SU(5) and SU(3)^3 seem to have phenomenological possibilities so far

MM, Ma, Zoupanos (2004);
Heinemeyer, MM, Ma, Zoupanos (2010)

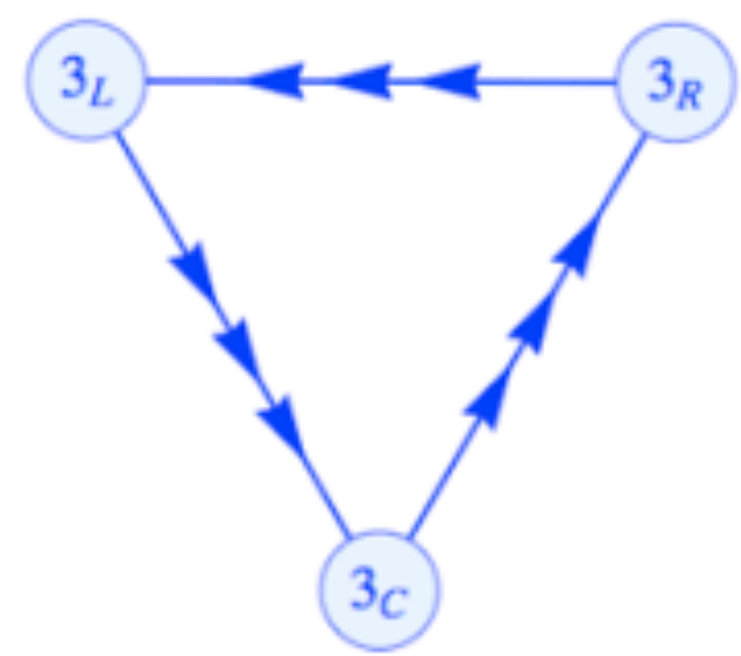
2-LOOP SU(3)3 FINITE MODEL

- 2-loop finite SU(3)3 trinification model, parametric solution of reduction equations

$$f^2 = r\left(\frac{16}{9}\right)g^2, \quad f'^2 = (1-r)\left(\frac{8}{3}\right)g^2$$

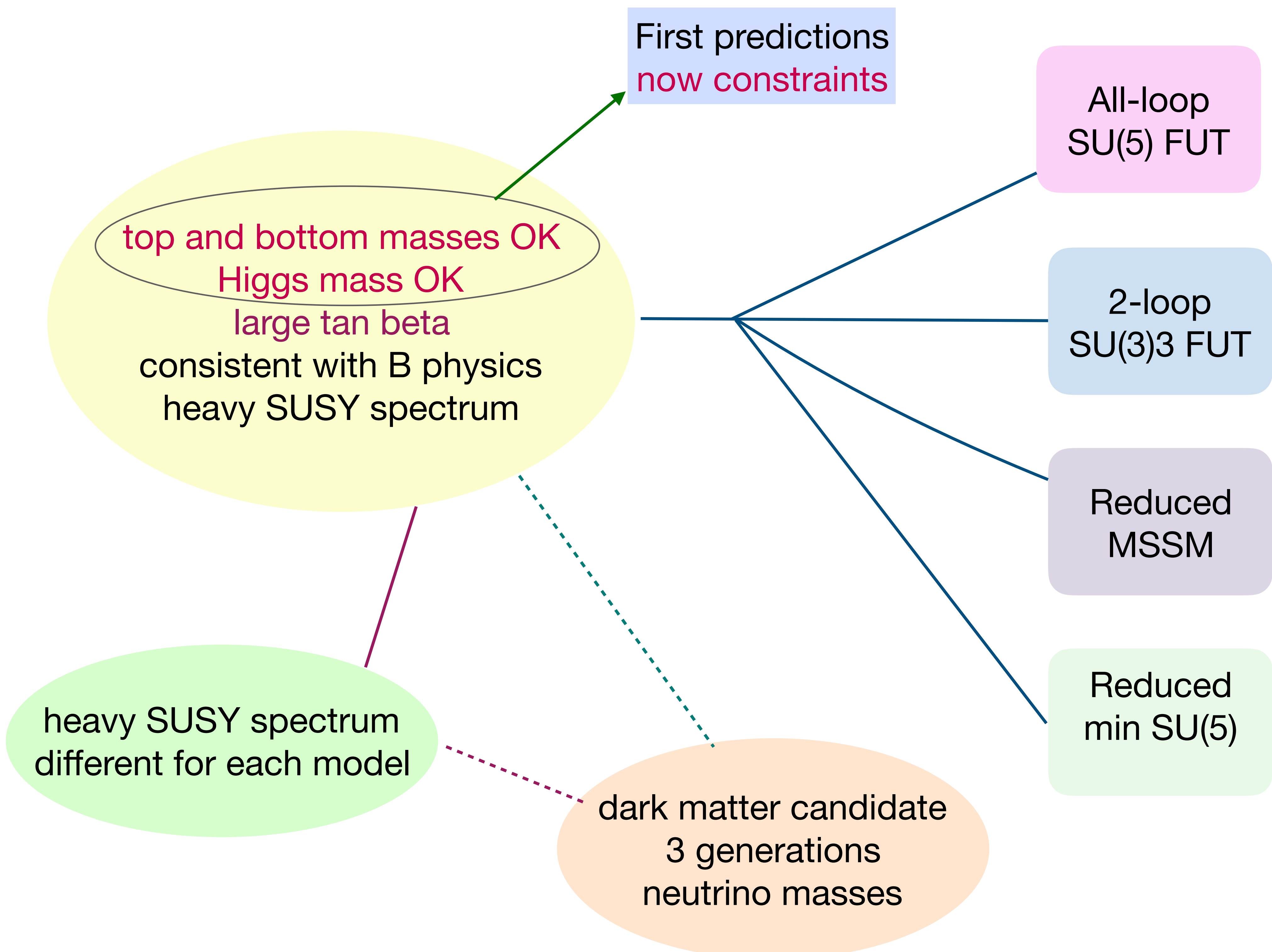
r parameterizes different solutions, f and f' are Yukawa couplings for quarks and leptons

- Good top and bottom masses with one parameter
- Large tan , heavy SUSY spectrum
- Possibility of neutrino masses, consistent with seesaw
- At high energies vector-like down type quarks
- Split-SUSY possible



MM, Reyes, Porod, Zoupanos, work in progress

GYU FROM REDUCTION OF COUPLINGS AT WORK



WHAT NOW? FLAVOR…

- So far detailed analysis only for third generation
- As mentioned, some 3 generation finite models exist
- SU(5) models some textures given
- $SU(3)^3$ naturally have 3 generations
- How to do it more systematically?

GENERAL SUPERPOTENTIAL FOR SU(5) FUTS

- The SU(5) superpotential of possible finite models is

$$\bar{\mathscr{H}}_{ai} = \bar{\mathbf{5}}\ , \quad \mathscr{H}^{i}_{\ a} = \mathbf{5}\ , \quad \bar{\Psi}_{a'i} = \bar{\mathbf{5}}\ , \quad X^{ij}_{a'} = \mathbf{10}\ , \quad \Sigma^{i}_{\ j} = \mathbf{24}$$

3 generations, 4 pairs of Higgs doublets and one field in the adjoint

$$3\ \bar{\mathbf{5}} + 3\ \overline{\mathbf{10}} + 4\ (\mathbf{5} + \bar{\mathbf{5}}) + \overline{\mathbf{24}}$$

$$\mathcal{W}_{SU(5)-R} = \bar{g}_{a'b'a}\bar{\Psi}_{b'i}X^{ij}_{a'}\bar{\mathcal{H}}_{aj} + \frac{1}{2}g_{a'b'a}\epsilon_{ijklm}X^{ij}_{a'}X^{kl}_{b'}\mathcal{H}^{m}_{a} + f_{ab}\bar{\mathcal{H}}_{ai}\Sigma^{i}_{\ j}\mathcal{H}^{j}_{b}$$
$$+ \frac{1}{3!}p\Sigma^{i}_{\ j}\ \Sigma^{j}_{\ k}\ \Sigma^{k}_{\ i} + \frac{1}{2}\lambda^{(\Sigma)}\Sigma^{i}_{\ j}\Sigma^{j}_{\ i} + m_{ab}\bar{\mathcal{H}}_{ai}\mathcal{H}^{i}_{b}\ .$$

$\bar{g}_{ijk}$ = down Yukawa couplings, g_{ijk} = up Yukawa couplings

WHAT ABOUT FLAVOR? 3 GENERATIONS

- Look for FUT 3 generation models
  all-loops
  2-loops
- Solutions for Yukawa couplings
- Sum rule in SSB
- Check absence of proton decay
- Mass matrices

Proton decay →
2 fine tunings

Everything OK? then…

- Rotate to MSSM
- Look again for mass matrices
- Good textures?

FINITE S3 MODEL

- Solutions to the RE may imply extra symmetries, so far discrete
- There are models with A_4 and Q_6, Babu et al; MM & Jiménez
- S_3, smallest non-Abelian discrete group, successful at low energies
- Irreps: **2, 1, 1$_A$** → Two generations in doublet, third in singlet

Superfields	$\begin{pmatrix} \bar{\Psi}_{1i} \\ \bar{\Psi}_{2i} \end{pmatrix}$	$\bar{\Psi}_{3i}$	$\begin{pmatrix} X_1^{ij} \\ X_2^{ij} \end{pmatrix}$	X_3^{ij}	$\begin{pmatrix} \mathcal{H}_1^i \\ \mathcal{H}_2^i \end{pmatrix}$	$\mathcal{H}_3^i$	$\mathcal{H}_4^i$	$\begin{pmatrix} \bar{\mathcal{H}}_{1i} \\ \bar{\mathcal{H}}_{2i} \end{pmatrix}$	$\bar{\mathcal{H}}_{3i}$	$\bar{\mathcal{H}}_{4i}$	$\Sigma^i{}_j$
Irreducible representations of S_3	**2**	$\mathbf{1_S}$	**2**	$\mathbf{1_S}$	**2**	$\mathbf{1_S}$	$\mathbf{1_A}$	**2**	$\mathbf{1_S}$	$\mathbf{1_A}$	$\mathbf{1_S}$

- Look for all-loop finite model

Z_n	$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	X_1	X_2	X_3	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	$\bar{\mathcal{H}}_1$	$\bar{\mathcal{H}}_2$	$\bar{\mathcal{H}}_3$	$\bar{\mathcal{H}}_4$	Σ
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_3	0	0	1	1	1	2	0	0	1	0	1	1	2	0	0

S3 MASS MATRICES

$$|g_{113}|^2 = \frac{4}{5}g_5^2 \quad , \quad |g_{131}|^2 = \frac{4}{5}g_5^2 \quad , \quad |\bar{g}_{113}|^2 = \frac{3}{5}g_5^2 \quad , \quad |\bar{g}_{131}|^2 = \frac{3}{5}g_5^2 \quad , \quad |\bar{g}_{311}|^2 = \frac{3}{5}g_5^2 \, ,$$

$$|f_{11}|^2 = |f_{33}|^2 = 0 \quad , \quad |f_{44}|^2 = g_5^2 \quad , \quad |p|^2 = \frac{15}{7}g_5^2 \, ,$$

Yukawa couplings completely determined!

$$M_u = \begin{pmatrix} g_{113}\langle \mathcal{H}_3^5 \rangle & 0 & g_{131}\langle \mathcal{H}_1^5 \rangle \\ 0 & g_{113}\langle \mathcal{H}_3^5 \rangle & g_{131}\langle \mathcal{H}_2^5 \rangle \\ g_{131}\langle \mathcal{H}_1^5 \rangle & g_{131}\langle \mathcal{H}_2^5 \rangle & 0 \end{pmatrix} \, ,$$

$$M_d = \begin{pmatrix} \bar{g}_{113}\langle \bar{\mathcal{H}}_{35} \rangle & 0 & \bar{g}_{131}\langle \bar{\mathcal{H}}_{15} \rangle \\ 0 & \bar{g}_{113}\langle \bar{\mathcal{H}}_{35} \rangle & \bar{g}_{131}\langle \bar{\mathcal{H}}_{25} \rangle \\ \bar{g}_{311}\langle \bar{\mathcal{H}}_{15} \rangle & \bar{g}_{311}\langle \bar{\mathcal{H}}_{25} \rangle & 0 \end{pmatrix}$$

But…too restrictive, two masses almost degenerate

CYCLIC SYMMETRIES — 3 GENERATIONS

Classification of SU(5) FUT with off-diagonal γ done already

Coupled to 3 Higgs doublets

$$V_3^{(1)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & g_{123}\langle\mathcal{H}_3^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{213}\langle\mathcal{H}_3^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{231}\langle\mathcal{H}_1^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{321}\langle\mathcal{H}_1^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_3^{(2)} = \begin{pmatrix} g_{112}\langle\mathcal{H}_2^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

$$V_3^{(3)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_3^{(4)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & 0 & 0 \\ 0 & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

Coupled to 4 Higgs doublets

$$V_4^{(1)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & g_{124}\langle\mathcal{H}_4^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{214}\langle\mathcal{H}_4^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{231}\langle\mathcal{H}_1^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{321}\langle\mathcal{H}_1^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_4^{(2)} = \begin{pmatrix} g_{112}\langle\mathcal{H}_2^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ 0 & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

$$V_4^{(3)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_4^{(4)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

Top and down mass matrices with same structure

Babu, Enkhbat, Gogoladze (2003)

2-LOOP FINITE MODEL — $V_4{}^1$

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

Z_n	$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	X_1	X_2	X_3	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	$\bar{\mathcal{H}}_1$	$\bar{\mathcal{H}}_2$	$\bar{\mathcal{H}}_3$	$\bar{\mathcal{H}}_4$	Σ
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_8	4	3	5	0	7	1	0	2	6	1	4	6	2	5	0

We find the following symmetries $\Rightarrow$
parametric relations among couplings $\Rightarrow$ 2-loop solution

up-type Yukawa

$$|g_{124}|^2 = |g_{214}|^2 = \frac{4}{5}g_5^2 \ , \ |g_{222}|^2 = \frac{2}{5}g_5^2 \ , \ |g_{231}|^2 = |g_{321}|^2 = \frac{1}{10}\left(8g_5^2 - 5\,|g_{111}|^2\right) \ ,$$

$$|g_{333}|^2 = \frac{6}{5}g_5^2 \ , \ |\bar{g}_{111}|^2 = |\bar{g}_{124}|^2 = \frac{3}{20}\left(8g_5^2 - 5\,|g_{111}|^2\right) \ ,$$

down-type Yukawa

$$|\bar{g}_{214}|^2 = \frac{3}{4}\,|g_{111}|^2 \ , \ |\bar{g}_{222}|^2 = |\bar{g}_{231}|^2 = \frac{3}{10}g_5^2 \ , \ |\bar{g}_{321}|^2 = -\frac{3}{20}\left(2g_5^2 - 5\,|g_{111}|^2\right) \ ,$$

$$|\bar{g}_{333}|^2 = \frac{9}{10}g_5^2 \ , \ |f_{22}|^2 = \frac{3}{4}g_5^2 \ , \ |f_{33}|^2 = \frac{g_5^2}{4} \ , \ |p|^2 = \frac{15}{7}g_5^2 \ ,$$

$$|g_{132}|^2 = |g_{312}|^2 = |\bar{g}_{132}|^2 = |\bar{g}_{312}|^2 = |f_{11}|^2 = |f_{44}|^2 = 0 \ .$$

By imposing the positivity conditon to the squared norm of the couplings, we find the following constraint for $|g_{111}|^2$:

$$\frac{2}{5}g_5^2 \leq |g_{111}|^2 \leq \frac{8}{5}g_5^2 \ .$$

evaluating at the end points implies more symmetry = more zeroes

EXAMPLE OF SOLUTIONS

- Many solutions, depend on the free parameter $|g_{111}|^2$
- Taking the Yukawa values at extreme points in inequality → more zeroes, more symmetry?

$$|g_{111}|^2 = \frac{2}{5} g_5^2$$

leads to

$$M_u = \begin{pmatrix} g_{111}\langle \mathcal{H}_1^5\rangle & g_{124}\langle \mathcal{H}_4^5\rangle & 0 \\ g_{214}\langle \mathcal{H}_4^5\rangle & g_{222}\langle \mathcal{H}_2^5\rangle & g_{231}\langle \mathcal{H}_1^5\rangle \\ 0 & g_{321}\langle \mathcal{H}_1^5\rangle & g_{333}\langle \mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad M_d = \begin{pmatrix} \bar{g}_{111}\langle \bar{\mathcal{H}}_{15}\rangle & \bar{g}_{124}\langle \bar{\mathcal{H}}_{45}\rangle & 0 \\ \bar{g}_{214}\langle \bar{\mathcal{H}}_{45}\rangle & \bar{g}_{222}\langle \bar{\mathcal{H}}_{25}\rangle & \bar{g}_{231}\langle \bar{\mathcal{H}}_{15}\rangle \\ 0 & 0 & \bar{g}_{333}\langle \bar{\mathcal{H}}_{15}\rangle \end{pmatrix}$$

- M_u compatible with phenomenology, but M_d?
- RGE analysis and sum rule might change a bad structure into a good/bad one

Cakir, Solmaz Xin (2008); Zhang, Zhou (2008)

ALL-LOOP FINITE MODEL — $V_4{}^2$

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

Z_n	$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	X_1	X_2	X_3	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	$\bar{\mathcal{H}}_1$	$\bar{\mathcal{H}}_2$	$\bar{\mathcal{H}}_3$	$\bar{\mathcal{H}}_4$	Σ
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_3	0	2	0	0	2	0	1	1	0	0	1	1	0	0	0
Z_4	3	3	2	3	3	2	2	3	0	2	2	3	0	2	0

- We find the following symmetries ⇒ isolated solution unique relation among couplings ⇒ all-loop finite solution

$$|g_{114}|^2 = |g_{121}|^2 = |g_{211}|^2 = |g_{232}|^2 = |g_{322}|^2 = |g_{333}|^2 = \frac{4}{5}g_5^2$$

$$|\bar{g}_{114}|^2 = |\bar{g}_{121}|^2 = |\bar{g}_{211}|^2 = |\bar{g}_{232}|^2 = |\bar{g}_{322}|^2 = |\bar{g}_{333}|^2 = \frac{3}{5}g_5^2 \quad ,$$

$$|f_{33}|^2 = |f_{44}|^2 = \frac{1}{2}g_5^2 \quad , \quad |p|^2 = \frac{15}{7}g_5^2 \quad .$$

- For the SSB ⇒ sum rule ⇒ 3 free parameters

$$m^2_{\tilde{\bar{\psi}}_1} = m^2_{\tilde{\bar{\psi}}_3} = \frac{1}{6}\left(-MM^\dagger + 9m^2_{H_3}\right) \quad , \quad m^2_{\tilde{\bar{\psi}}_2} = \frac{1}{6}\left(-MM^\dagger - 6m^2_{H_1} + 15m^2_{H_3}\right),$$

$$m^2_{\tilde{\chi}_1} = m^2_{\tilde{\chi}_3} = \frac{1}{2}\left(MM^\dagger - m^2_{H_3}\right) \quad , \quad m^2_{\tilde{\chi}_2} = \frac{1}{2}\left(MM^\dagger - 2m^2_{H_1} + m^2_{H_3}\right) \;,$$

$$m^2_{\bar{H}_1} = m^2_{\bar{H}_2} = \frac{1}{3}\left(2MM^\dagger + 3m^2_{H_1} - 6m^2_{H_3}\right) \quad , \quad m^2_{\bar{H}_3} = m^2_{\bar{H}_4} = \frac{1}{3}\left(2MM^\dagger - 3m^2_{H_3}\right) \;,$$

$$m^2_{H_2} = m^2_{H_1} \quad ; \quad m^2_{H_4} = m^2_{H_3} \quad , \quad m^2_{\phi_\Sigma} = \frac{1}{3}MM^\dagger \quad . \tag{89}$$

ALL-LOOP FINITE MASS MATRICES

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

- It is possible to find the minimum amount of phases — rephasing invariants
- The mass matrices are then:

$$M_u = \begin{pmatrix} g_{114}\langle\mathcal{H}_4^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & 0 & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} = \frac{2}{\sqrt{5}}g_5 \begin{pmatrix} \langle\mathcal{H}_4^5\rangle & \langle\mathcal{H}_1^5\rangle & 0 \\ \langle\mathcal{H}_1^5\rangle & 0 & \langle\mathcal{H}_2^5\rangle \\ 0 & \langle\mathcal{H}_2^5\rangle & e^{i\phi_3}\langle\mathcal{H}_3^5\rangle \end{pmatrix} ,$$

$$M_d = \begin{pmatrix} \bar{g}_{114}\langle\bar{\mathcal{H}}_{45}\rangle & \bar{g}_{121}\langle\bar{\mathcal{H}}_{15}\rangle & 0 \\ \bar{g}_{211}\langle\bar{\mathcal{H}}_{15}\rangle & 0 & \bar{g}_{232}\langle\bar{\mathcal{H}}_{25}\rangle \\ 0 & \bar{g}_{322}\langle\bar{\mathcal{H}}_{25}\rangle & \bar{g}_{333}\langle\bar{\mathcal{H}}_{35}\rangle \end{pmatrix} = \sqrt{\tfrac{3}{5}}g_5 \begin{pmatrix} \langle\bar{\mathcal{H}}_{45}\rangle & \langle\bar{\mathcal{H}}_{15}\rangle & 0 \\ e^{i\bar{\phi}_1}\langle\bar{\mathcal{H}}_{15}\rangle & 0 & \langle\bar{\mathcal{H}}_{25}\rangle \\ 0 & e^{i\bar{\phi}_2}\langle\bar{\mathcal{H}}_{25}\rangle & e^{i\bar{\phi}_3}\langle\bar{\mathcal{H}}_{35}\rangle \end{pmatrix} .$$

- After the rotation in the Higgs sector to the MSSM basis:

$$M_u = \frac{2}{\sqrt{5}}g_5 \begin{pmatrix} \widetilde{\alpha}_4 & \widetilde{\alpha}_1 & 0 \\ \widetilde{\alpha}_1 & 0 & \widetilde{\alpha}_2 \\ 0 & \widetilde{\alpha}_2 & e^{i\phi_3}\widetilde{\alpha}_3 \end{pmatrix} \langle\mathcal{K}_3^5\rangle ,$$

$$M_d = \sqrt{\frac{3}{5}}g_5 \begin{pmatrix} \widetilde{\beta}_4 & \widetilde{\beta}_1 & 0 \\ e^{i\bar{\phi}_1}\widetilde{\beta}_1 & 0 & \widetilde{\beta}_2 \\ 0 & e^{i\bar{\phi}_2}\widetilde{\beta}_2 & e^{i\bar{\phi}_3}\widetilde{\beta}_3 \end{pmatrix} \langle\bar{\mathcal{K}}_{35}\rangle .$$

Same solution as FUTB for 3rd generation! we know it works…

α_i, β_i refer to the rotation angles in up and down sectors respectively, $\Sigma\beta_i = \Sigma\alpha_i = 1$

FINALLY, HOW MANY FREE PARAMETERS?

GUT scale 89 free parameters

Yukawa couplings, soft breaking terms, phases, vev's of the Higgs fields

After Finiteness solutions

33 free parameters

Require doublet-triplet splitting, rotation to MSSM basis with constraints over angles, rephasing invariants

Low energies:

radiative electroweak breaking, fix m_τ^{exp} and SM vev give $\tan\beta$

$\Rightarrow$ **12 parameters left:**

The soft breaking terms, the phases, and the rotation angles

$\phi_1, \phi_2, \phi_3, \phi_4, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, M, \mu$

Only one phase is observable

$\Rightarrow \phi_{obs}, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, M, \mu$

only 9 parameters left to fit masses and mixing angles

WHAT ABOUT NEUTRINO MASSES, DARK MATTER, ETC?

- **SU(5) models:**
  Cold DM
  LSP is neutralino
  $\Rightarrow$ overabundance
- Neutrino masses may be incorporated by breaking R symmetry $\Rightarrow$
  gravitino Dark Matter
- Other mechanisms?
  thermal inflation?
- g-2 like in SM

- **$SU(3)^3$ models:**
  ν_R are present
- Neutrino masses may be generated by seesaw or radiatively
- Depending on the breaking of $SU(3)^3$
  DM may be neutralino
  (or scalar?)
- Neutralino DM overabundance

Flavor Structure may change the above!

CONCLUSIONS AND OUTLOOK

- Reduction of couplings finiteness powerful principle implies Gauge Yukawa Unification
- Conformal or scale invariant theory
- SSB terms satisfy a sum rule among soft scalars
- SSB same as anomaly mediated scenario
- Finiteness reduces greatly number of free parameters completely finite theories SU(5)
- Very predictive

- Flavor 3 generation models
  2-loops: Yukawa couplings determined within a range
  All-loops: Yukawa couplings completely determined
- Leads to viable mass textures
- Drastic reduction in number of free parameters
- Free parameters come from Higgs sector, SSB and phases
- More fundamental theory?

How can we restrict phases? CP violation?
Higgs sector? Flavor processes?
Dark matter? Inflation? Bariogenesis?

Thank you!