Thermal and Magnetic Fluctuations in QED: Implications for Symmetry, Particle Mass, and Heavy-Ion Collisions

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In collaboration with

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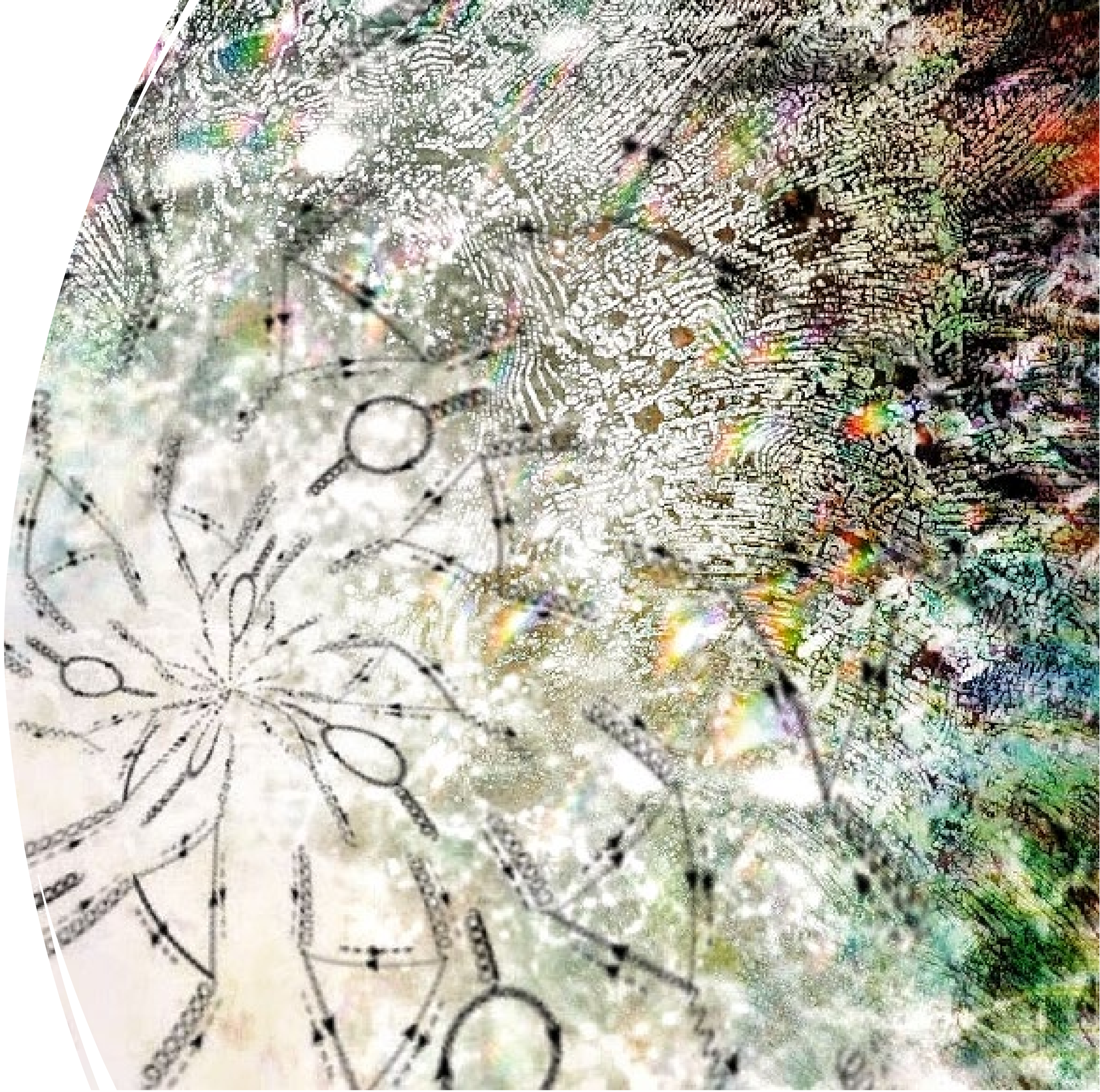
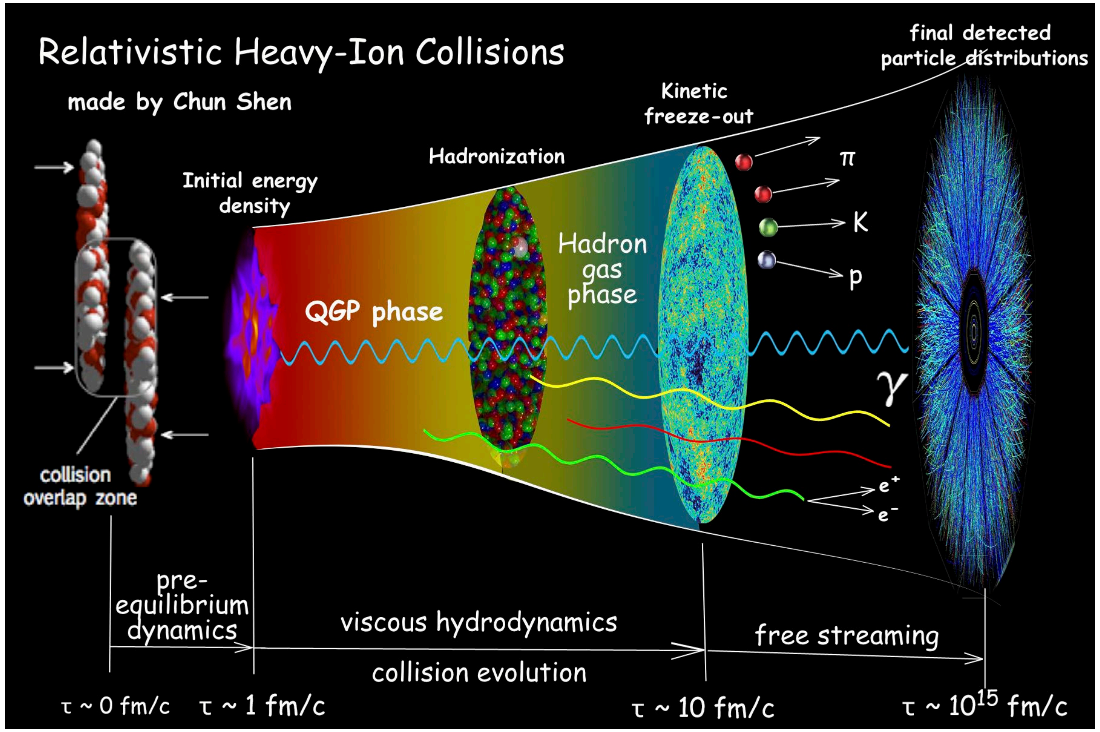


Outlook

- **Motivation**
- **The Replica Trick**
- **Classical Magnetic Fluctuations**
- **Thermal Fluctuations**
- **Conclusions**

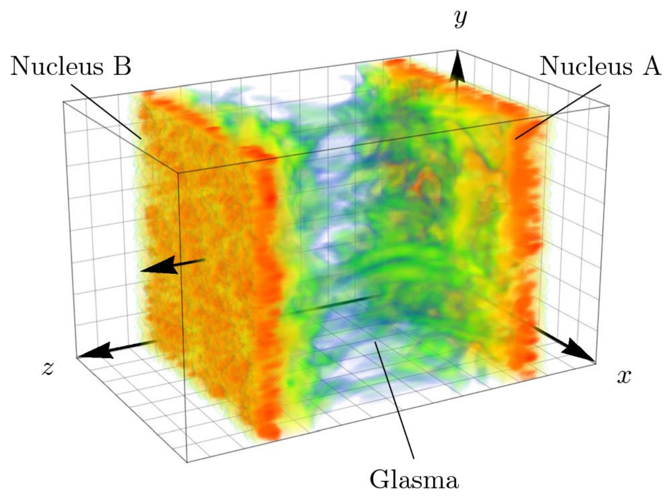
Motivation



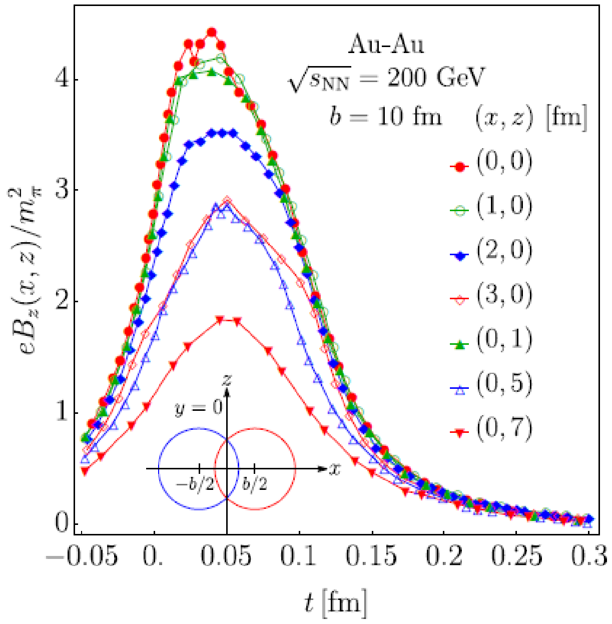
[https://u.osu.edu/vishnu/category/visualization/]

While heavy-ion collision experiments are highly precise and carefully controlled, the resulting medium from these collisions is 'messy,' chaotic, and filled with fluctuations.

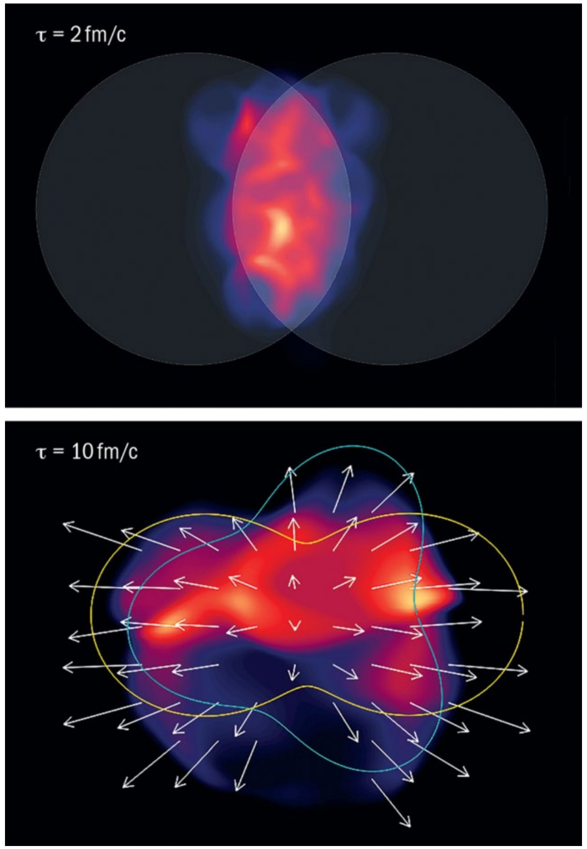
Motivation



[Eur. Phys. J. A **56**, 243 (2020)]



[V. Voronyuk, PRC 83, 054911 (2011)]



In the initial stage of a heavy-ion collision, there are significant fluctuations in both the density and the magnetic fields

Similarly, the early and final stages of the QGP phase are also characterized by thermal fluctuations.

The Replica Trick

We have proposed a method, based on Parisi's findings, to incorporate magnetic and thermal fluctuations into QED/QCD systems

“For the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales”

- We average over the noise/fluctuation configurations.
- By averaging the replicated partition function, we can access the effective interactions

$$\overline{\ln Z[A]} = \lim_{n\to 0} \frac{\overline{Z^n[A]} - 1}{n}$$

Originally, this was a 'trick' (or conjecture) that was proven approximately 30 years later.

Used in condensed matter systems:

- **Spin Glasses:** The replica method was initially developed to study the complex energy landscapes and disorder in spin glass systems.
- **Disordered Systems:** It is widely used in statistical physics to analyze systems with quenched disorder, such as random field models and neural networks.

Classical Magnetic Fluctuations

PHYSICAL REVIEW D **107,** 096014 (2023)

QED fermions in a noisy magnetic field background

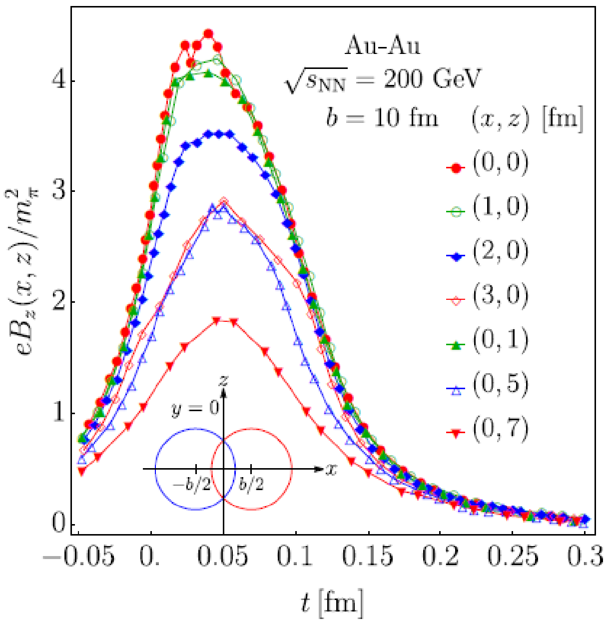
Jorge David Castaño-Yepes,[1,*] Marcelo Loewe,[1,2,3,4,†] Enrique Muñoz,[1,5,‡]
Juan Cristóbal Rojas,[6,§] and Renato Zamora[7,8,‖]

PHYSICAL REVIEW D **108,** 116013 (2023)

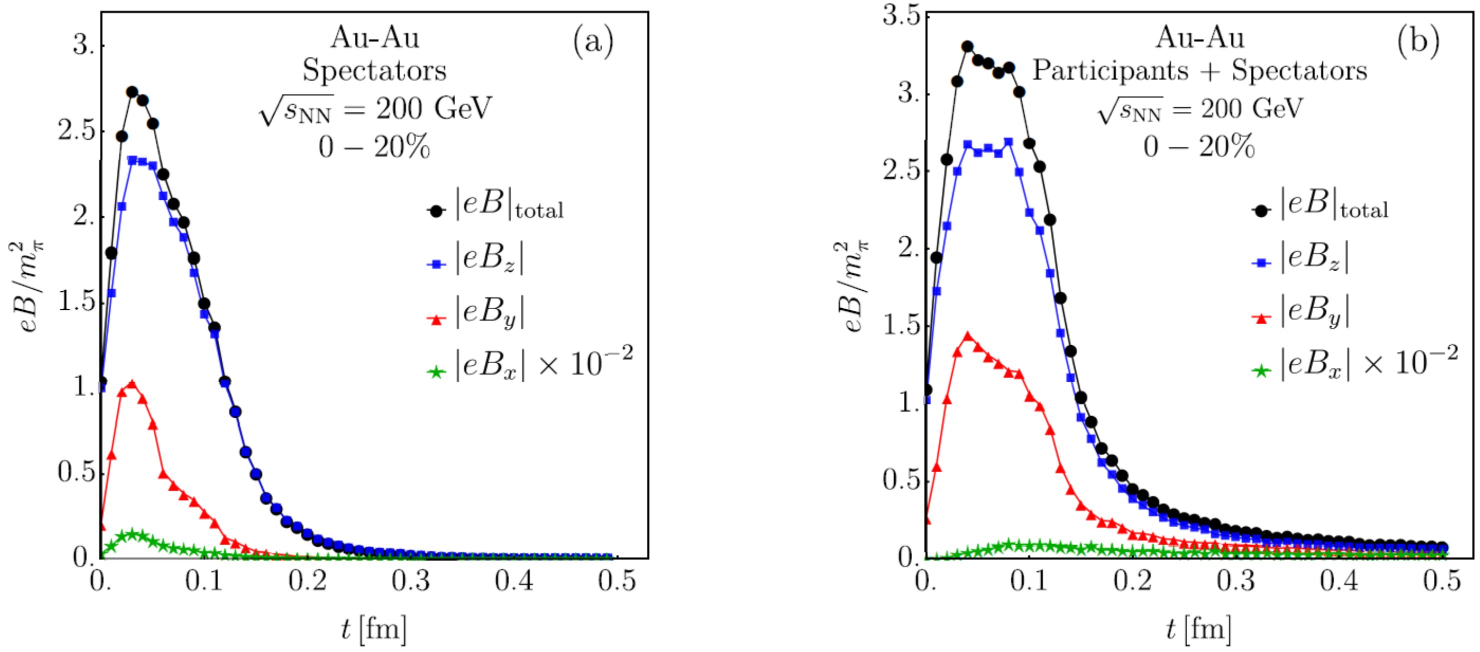
QED fermions in a noisy magnetic field background: The effective action approach

Jorge David Castaño-Yepes,[1,*] Marcelo Loewe,[2,3,†] Enrique Muñoz,[1,4,‡] and Juan Cristóbal Rojas[5,§]

Classical Magnetic Fluctuations



[V. Voronyuk, PRC 83, 054911 (2011)]



[J.D. Castaño-Yepes, arXiv:2103.12898v2 [hep-ph], (2021)]

$$A^\mu(x) \to A^\mu(x) + A^\mu_{\text{BG}}(x) + \delta A^\mu_{\text{BG}}(\mathbf{x})$$

$$\langle \delta A^j_{\text{BG}}(\mathbf{x}) \delta A^k_{\text{BG}}(\mathbf{x}') \rangle = \Delta_B \delta_{j,k} \delta^3(\mathbf{x} - \mathbf{x}'),$$

$$\langle \delta A^\mu_{\text{BG}}(\mathbf{x}) \rangle = 0.$$

$$\Delta_B \sim \pi^{5/2} (\delta B)^2 r_0^5 N^{5/3} \left(\frac{N_{\text{part}}}{2N} \right)^{5/3} \sim 0.26 \text{ MeV}^{-1}$$

Classical Magnetic Fluctuations

$$A^\mu(x) \to A^\mu(x) + A^\mu_{\rm BG}(x) + \delta A^\mu_{\rm BG}(\mathbf{x})$$

$$\langle \delta A^j_{\rm BG}(\mathbf{x}) \delta A^k_{\rm BG}(\mathbf{x}') \rangle = \Delta_B \delta_{j,k} \delta^3(\mathbf{x} - \mathbf{x}'),$$
$$\langle \delta A^\mu_{\rm BG}(\mathbf{x}) \rangle = 0.$$

$$\longrightarrow \quad dP[\delta A^\mu_{\rm BG}] = \mathcal{N} e^{-\int d^3x \frac{[\delta A^\mu_{\rm BG}(\mathbf{x})]^2}{2\Delta_B}} \mathcal{D}[\delta A^\mu_{\rm BG}(\mathbf{x})]$$

"White noise" distribution function

$$\mathcal{L}_{\rm FBG} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ - e A\!\!\!/_{\rm BG} - e A\!\!\!/ - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

"Free" contribution (Lagrangian of reference)

$$\mathcal{L} = \mathcal{L}_{\rm FBG} + \mathcal{L}_{\rm NBG}$$

$$\mathcal{L}_{\rm NBG} = \bar{\psi}(-e\delta A\!\!\!/_{\rm BG})\psi$$

"Noisy" contribution

$$\overline{Z^n[A]} = \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] \int \mathcal{D}[\delta A^\mu_{\rm BG}] e^{-\int d^3x \frac{[\delta A^\mu_{\rm BG}(\mathbf{x})]^2}{2\Delta_B}} e^{\mathrm{i} \int d^4x \sum_{a=1}^{n} \left(\mathcal{L}_{\rm FBG}[\bar{\psi}^a, \psi^a] + \mathcal{L}_{\rm NBG}[\bar{\psi}^a, \psi^a]\right)}$$

$$= \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] e^{\mathrm{i}\bar{S}[\bar{\psi}^a, \psi^a; A]}$$

Classical Magnetic Fluctuations

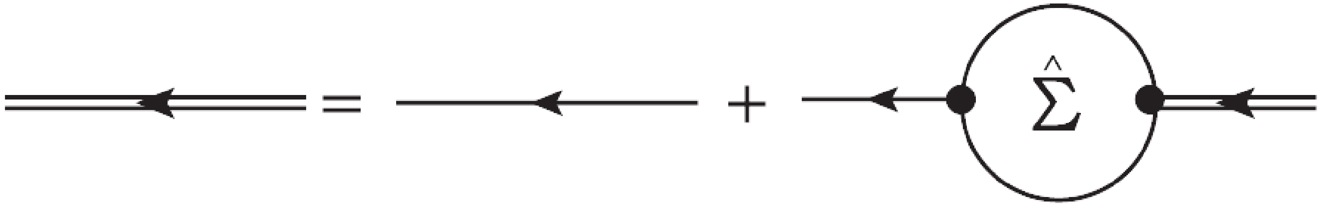
$$\overline{Z^n[A]} = \int \prod_{a=1}^{n} \mathcal{D}[\overline{\psi}^a, \psi^a] \int \mathcal{D}[\delta A^\mu_{\mathrm{BG}}] e^{-\int d^3x \frac{[\delta A^\mu_{\mathrm{BG}}(\mathbf{x})]^2}{2\Delta_B}} e^{\mathrm{i}\int d^4x \sum_{a=1}^{n} (\mathcal{L}_{\mathrm{FBG}}[\bar{\psi}^a,\psi^a]+\mathcal{L}_{\mathrm{NBG}}[\bar{\psi}^a,\psi^a])}$$

$$= \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] e^{\mathrm{i}\bar{S}[\bar{\psi}^a,\psi^a;A]}$$

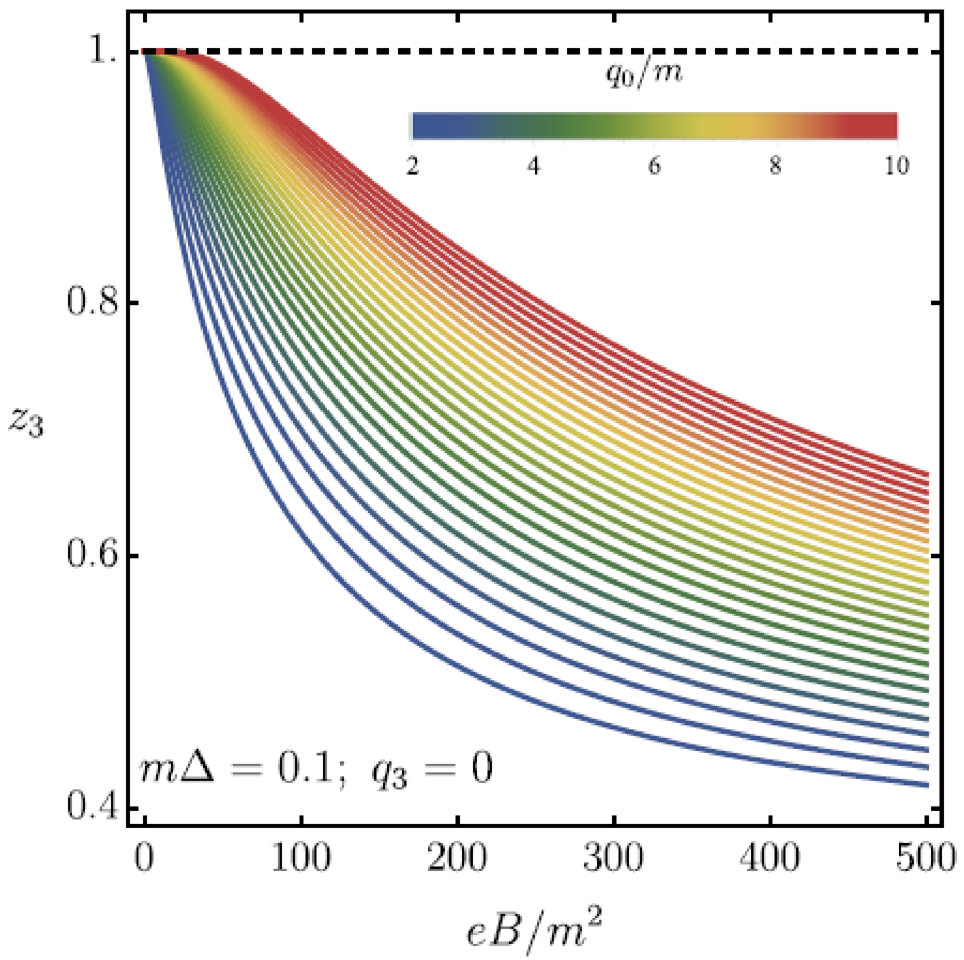
$$\bar{S}[\bar{\psi}^a, \psi^a; A] = \int d^4x \left(\sum_a \bar{\psi}^a (\mathrm{i}\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - eA\!\!\!/ - m)\psi^a - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \right)$$

$$+ \mathrm{i}\frac{e^2\Delta_B}{2} \int d^4x \int d^4y \sum_{a,b} \sum_{j=1}^{3} \bar{\psi}^a(x)\gamma^j\psi^a(x)\bar{\psi}^b(y)\gamma_j\psi^b(y)\delta^3(\mathbf{x}-\mathbf{y})$$

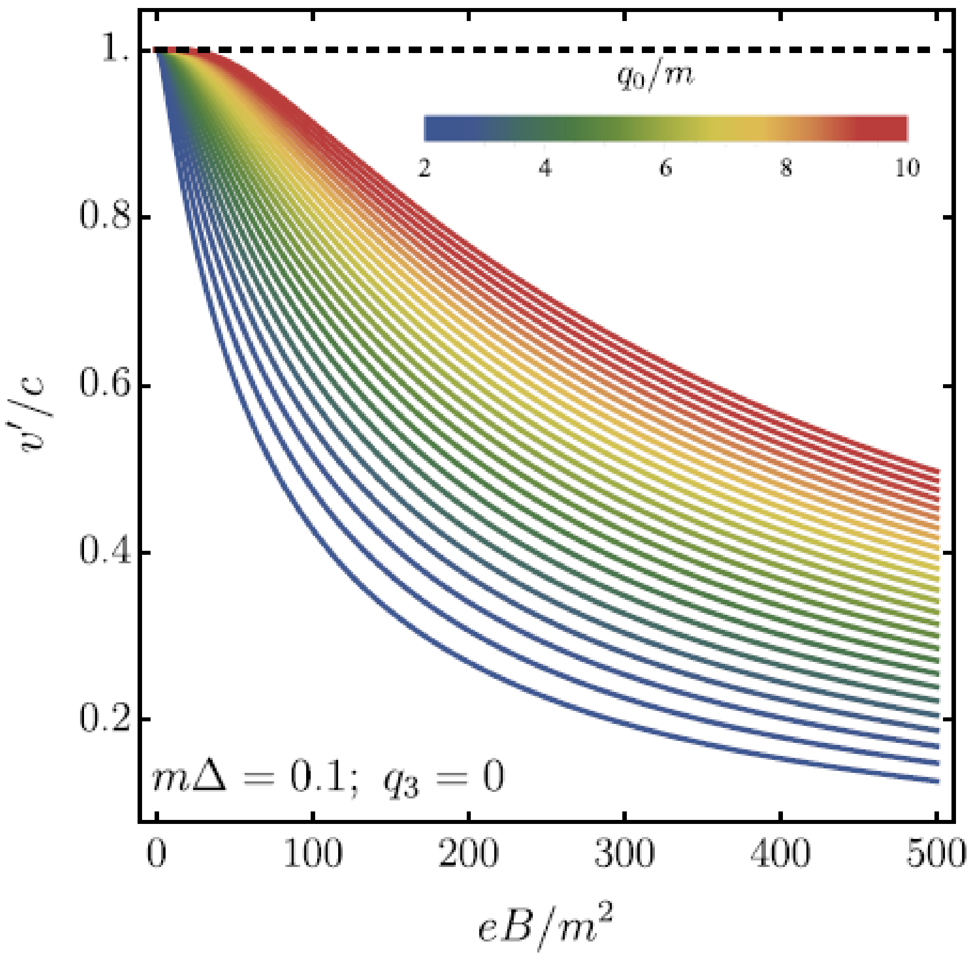
In a perturbative framework, the resulting interacting theory enables the renormalization of the fermion 'free' propagator (the Schwinger propagator)



Classical Magnetic Fluctuations



Electric charge renormalization factor



Inverse wave function renormalization factor
(birrefrigence effects)

Classical Magnetic Fluctuations

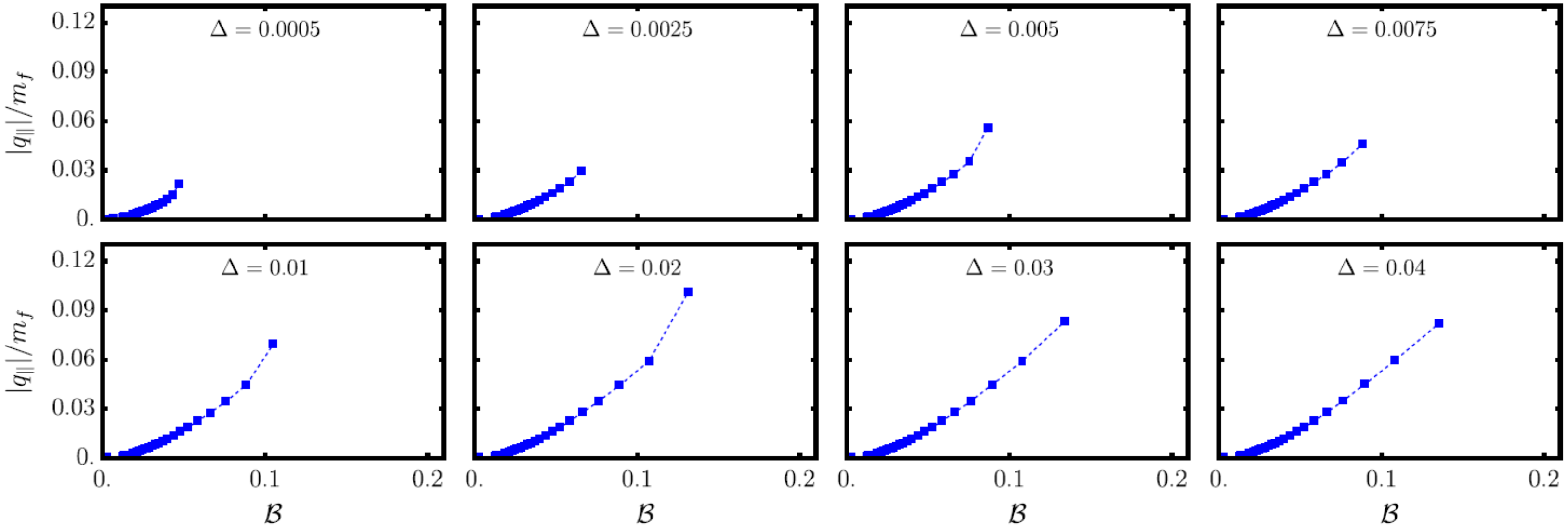
In an effective-action framework (bosonization), the resulting interacting theory enables the introduction of auxiliary fields

$$e^{-\frac{e^2\Delta_B}{2}\int d^4x \int d^4y \sum_{a,b}^{n}\sum_{j=1}^{3}\bar{\psi}^a(x)\gamma^j\psi^a(x)\bar{\psi}^b(y)\gamma_j\psi^b(y)}$$

$$= \mathcal{N}\left[\prod_{j=1}^{3}\int \mathcal{D}Q_j(x)\mathcal{D}Q_j^*(x)\right] e^{-\frac{2}{\Delta_B}\int d^4x|Q_j(x)|^2 + \mathrm{i}e\int d^4x Q_j(x)\sum_{a=1}^{n}\bar{\psi}^a(x)\gamma^j\psi^a(x) - \mathrm{i}e\int d^4x Q_j^*(x)\sum_{a=1}^{n}\bar{\psi}^a(x)\gamma^j\psi^a(x)}$$

$$\mathrm{i}S_{\mathrm{eff}}[A_{\mathrm{BG}}] - \mathrm{i}S_0[A_{\mathrm{BG}}] = \mathcal{N}\left[\prod_{j=1}^{3}\int \mathcal{D}Q_j(x)\mathcal{D}Q_j^*(x)\right] e^{-\frac{2}{\Delta_B}\int d^4x|Q_j(x)|^2 + \ln[\mathrm{Tr}\,\ln(\mathbf{1} - eS_{\mathrm{F}}\gamma^j(Q_j - Q_j^*))]}$$

By looking the saddle-point, we promote Q as an order parameter



$$Q_j = \mathrm{i}e\Delta_B\langle\langle\bar{\psi}\gamma_j\psi\rangle\rangle_\Delta$$

There are values of the noise and magnetic field that break the $U(1)$ symmetry by generating vectorial currents

Classical Magnetic Fluctuations

$$Q_j = \mathrm{i} e \Delta_B \langle\langle \bar{\psi} \gamma_j \psi \rangle\rangle_\Delta$$

There are values of the noise and magnetic field that break the $U(1)$ symmetry by generating vectorial currents

(Effective action approach)

Is this symmetry broken from the perturbative point of view?

PHYSICAL REVIEW D **109,** 056007 (2024)

Exploring magnetic fluctuation effects in QED gauge fields: Implications for mass generation

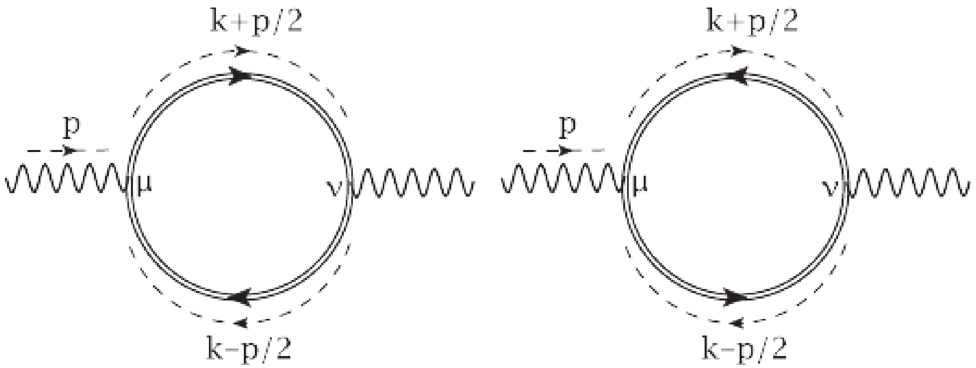
Jorge David Castaño-Yepes[1,*] and Enrique Muñoz[1,2,†]

PHYSICAL REVIEW D **101,** 036016 (2020)

Gluon polarization tensor in a magnetized medium: Analytic approach starting from the sum over Landau levels

Alejandro Ayala,[1,2] Jorge David Castaño-Yepes,[1,*] M. Loewe,[3,2,4] and Enrique Muñoz[3,5]

No photon magnetic mass is generated in the equilibrium case



$$M_\parallel^2 \equiv \frac{43\alpha_{\mathrm{em}} \mathcal{B}^2 \tilde{\Delta}}{96\pi} m^2,$$

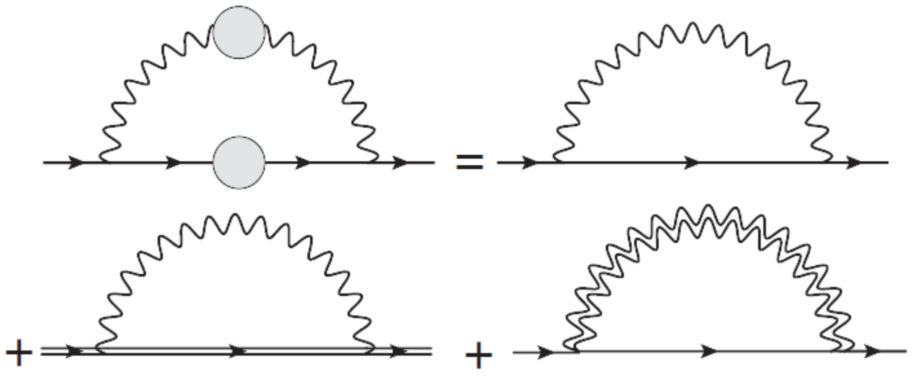
$$M_\perp^2 \equiv \frac{\alpha_{\mathrm{em}} \mathcal{B}^2 \tilde{\Delta}}{3\pi} m^2.$$

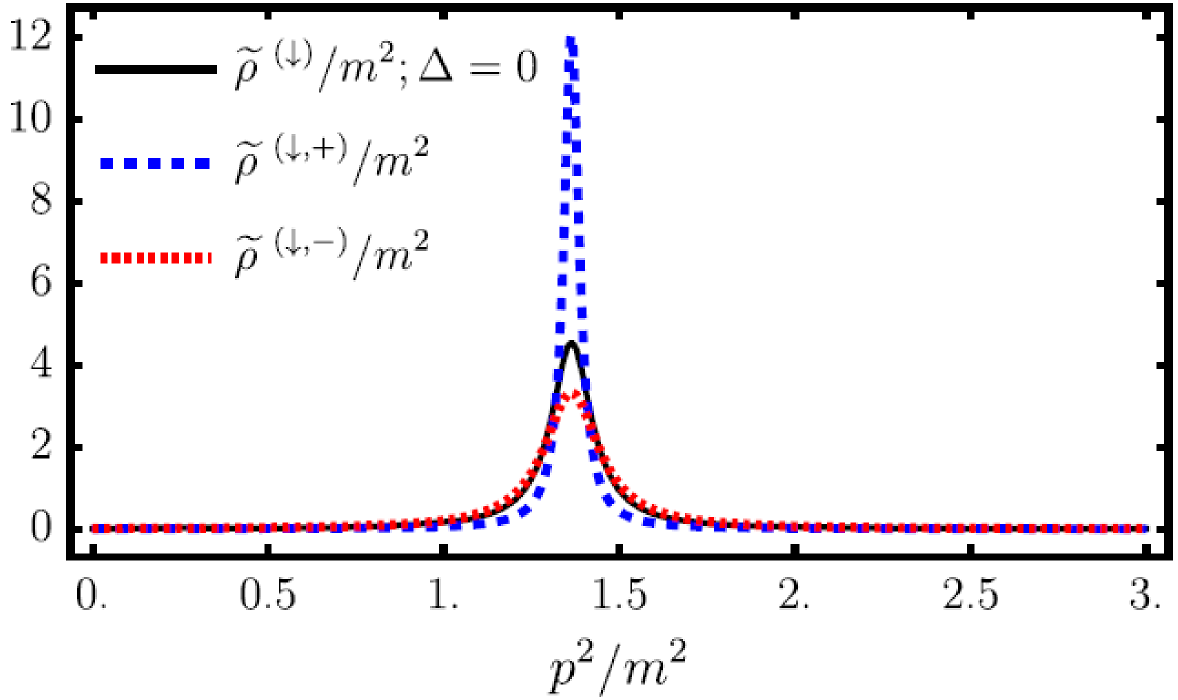
A finite photon mass implies the breaking of the $U(1)$ symmetry by the noisy magnetic field

PHYSICAL REVIEW D **110,** 056003 (2024)

Fermion self-energy and effective mass in a noisy magnetic background

Jorge David Castaño-Yepes[1,*] and Enrique Muñoz[1,2,†]





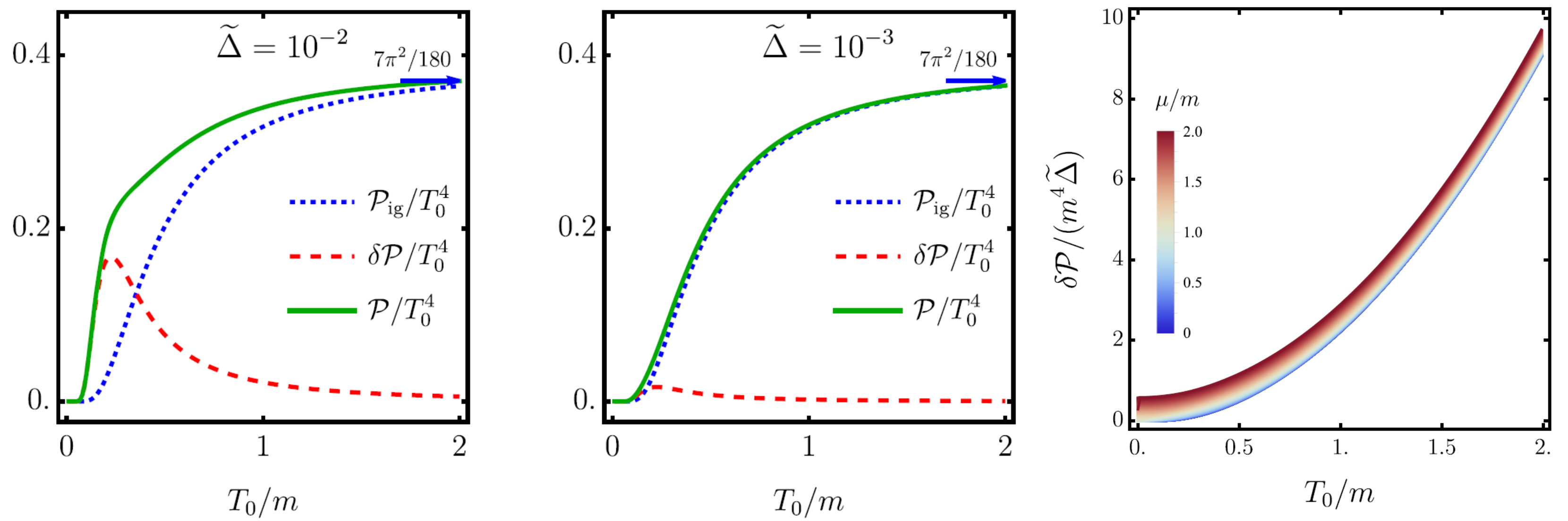
The presence of magnetic fluctuations breaks totally the Lorentz symmetry impliying the existence of 4 fermión quasi-particle modes

Thermal Fluctuations

$$\beta = (T_0 + \delta T)^{-1} = T_0^{-1} - \frac{\delta T}{T_0^2} = \beta_0 + \delta\beta \longrightarrow \begin{aligned} \delta\beta &= -\frac{\delta T}{T_0^2}, \\ \overline{\delta\beta} &= -T_0^{-2}\overline{\delta T} = 0, \\ \overline{\delta\beta^2} &= T_0^{-4}\overline{\delta T^2} = \beta_0^4\Delta = \Delta_\beta \end{aligned} \longrightarrow dP[\delta\beta] = \frac{d(\delta\beta)}{\sqrt{2\pi\Delta_\beta}} e^{-\frac{\delta\beta^2}{2\Delta_\beta}}$$

$$\overline{Z^n} = \left(1 + \sum_{j=1}^{\infty} \frac{(\Delta_\beta/2)^j}{j!} \frac{\partial^{2j}}{\partial\beta_0^{2j}}\right) Z_0^n \longrightarrow -\beta_0\bar{\Omega} = \overline{\ln Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n} = \exp\left[\frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\right] \ln Z_0$$

$$\ln Z_{F0} = 2\mathcal{V} \int \frac{d^3p}{(2\pi)^3} \left\{ \ln\left(1 + e^{\beta_0(\mu - E_{\mathbf{p}})}\right) + \ln\left(1 + e^{\beta_0(\mu + E_{\mathbf{p}})}\right) \right\}.$$



Thermal Fluctuations

Finally, we computed the pressure of bosons (gluons) from the equilibrium relation $\Omega_0^B = -T_0 \ln Z_{B0} = -\nu_B \mathcal{V} \frac{\pi^2 T_0^4}{90}$

$$\delta\mathcal{P}^B = \mathcal{P} - \mathcal{P}_{\text{ig}}^B = \frac{\Delta_\beta}{2\beta_0 \mathcal{V}} \frac{\partial^2}{\partial \beta_0^2} \ln Z_{B0}$$

$$= \nu_B \frac{\pi^2}{15} \Delta_\beta \beta_0^{-6} = \nu_B \frac{\pi^2}{15} \Delta T_0^2 > 0$$

Implications for the deconfinement temperature

Assuming that the hadronic phase is mainly constituted by pions (bosons) $\mathcal{P}_{\text{Had}} = 3\frac{\pi^2 T_0^4}{90} + \delta\mathcal{P}^{\text{Had}}$

And the plasma pase has quarks (fermions) and gluons (bosons) $\mathcal{P}_{\text{Plasma}} = \left(\nu_B + \frac{7}{4}\nu_F\right)\frac{\pi^2 T_0^4}{90} + \delta\mathcal{P}^{\text{Plasma}} - B$

The critical temperature is obtained by imposing the condition of equal pressures at both phases at the phase transition

$$T_c = T_c^0 \left(1 - \frac{\delta\mathcal{P}^{\text{Net}}}{(T_c^0)^4}\right)^{1/4} \leq T_c^0$$

Nonequilibrium temperature fluctuations will in principle decrease the critical temperature for the deconfinement transition

Conclusions

- We developed a theoretical framework to incorporate fluctuations of any kind into quantum field theories, enabling a wide range of applications in high-energy physics.

- The explored phenomenology has the potential to influence the interpretation of current and future observables in high-precision experiments.

- Our findings are applicable to heavy-ion collision experiments, as well as to the conditions of the early universe and compact astronomical objects, where fluctuations play a crucial role in describing the underlying phenomenology.

Thank you!