Quark mixing model with S_3 modular symmetry and 3 Higgs doublets

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December 2024

Outline

Motivation

Some background

- The Standard Model has been successful in describing interactions.
- There are issues such as the mixing pattern, neutrino masses, the composition of dark matter...
- A possible solution: extending the model with one or more symmetry groups.
- Finite permutation groups: S_3 has provided a good approach to describe the mixing pattern.
- Finite groups based on modular symmetries have been proposed.

Quark mixing matrix

$$V_{ckm} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{sc} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

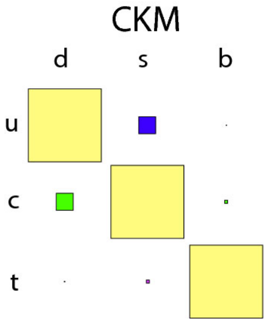


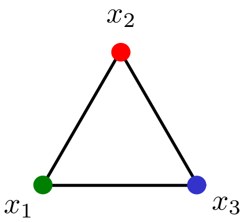
Figure 1: Quark mixing matrix pattern

Sheldon Stone, New physics from flavour. PoS ICHEP2012 (2013), 033 DOI: 10.22323/1.174.0033

Symmetry groups: S_3 Group

Permutation group of three elements

$$
\begin{aligned}
e &: \{x_1, x_2, x_3\} \to \{x_1, x_2, x_3\}, \\
a_1 &: \{x_1, x_2, x_3\} \to \{x_2, x_1, x_3\}, \\
a_2 &: \{x_1, x_2, x_3\} \to \{x_3, x_2, x_1\}, \\
a_3 &: \{x_1, x_2, x_3\} \to \{x_1, x_3, x_2\}, \\
a_4 &: \{x_1, x_2, x_3\} \to \{x_3, x_1, x_2\}, \\
a_5 &: \{x_1, x_2, x_3\} \to \{x_2, x_3, x_1\},
\end{aligned}
$$



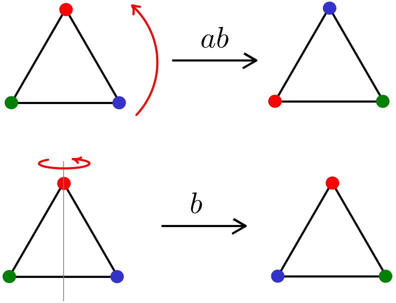
$$a_1 a_2 = a_5, \qquad a_2 a_1 = a_4, \qquad a_4 a_2 = a_2 a_1 a_2 = a_3.$$

If we redefine $a_1 = a$ and $a_2 = b$, then all the elements of the group are defined as follows.

$$\{e, a, b, ab, ba, bab\}, \tag{1}$$

Hajime Ishimori, Tatsuo Kobayashi, Hiroshi Ohki, Yusuke Shimizu, Hiroshi Okada, Morimitsu Tanimoto, Non-Abelian Discrete Symmetries in Particle Physics, Progress of Theoretical Physics Supplement, Volume 183, January 2010, Pages 1–163

Symmetry groups: S_3 Group



For the elements of S_3, it is also satisfied that

$$a^2 = b^2 = (bab)^2 = \mathbf{1}$$
$$(ab)^3 = (ba)^3 = \mathbf{1}, \quad (2)$$

This allows us to group the elements into three conjugacy classes (identity, rotations, and reflections).

$$C_1 : \{e\} \qquad C_2 : \{ab, ba\} \qquad C_3 : \{a, b, bab\} \quad (3)$$

Symmetry groups: S_3 Group

Matrix representation of S_3

$$a = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad bab = \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \tag{4}$$

$$ab = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad ba = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

Modular Groups

The modular group is defined as

$$\Gamma = SL_2(\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} | a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}. \tag{5}$$

From this group, the fractional transformation is defined as

$$\Gamma(\tau) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} (\tau) \rightarrow \frac{a\tau + b}{c\tau + d}. \tag{6}$$

with $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$.

The generators of $\Gamma(\tau)$ are

$$S : \tau \rightarrow -\frac{1}{\tau} \qquad \text{and} \qquad T : \tau \rightarrow \tau + 1, \tag{7}$$

which in Γ correspond to

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{8}$$

and satisfy in Γ

$$S^2 = \mathbf{1} \qquad \text{and} \qquad (ST)^3 = \mathbf{1}. \tag{9}$$

Modular Groups

The congruence subgroup is defined as

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma : \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\text{mod } N) \right\}. \tag{10}$$

It is worth noting that

$$\overline{\Gamma} \simeq PSL_2(\mathbb{Z}) = SL_2(\mathbb{Z})/\{I, -I\}. \quad \text{and} \quad \overline{\Gamma}(N) \simeq \Gamma(N)/\{I, -I\} \tag{11}$$

The finite modular group is defined as

$$\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N). \tag{12}$$

Some isomorphisms of the finite modular groups are

$$\begin{aligned} \Gamma_2 \simeq S_3 \qquad & \Gamma_3 \simeq A_4 \\ \Gamma_4 \simeq S_4 \qquad & \Gamma_5 \simeq A_5. \end{aligned} \tag{13}$$

It can be proof that

$$\begin{aligned} S^2 &= \mathbf{1} \\ (ST)^3 &= \mathbf{1} \\ T^N &= \mathbf{1}. \end{aligned} \tag{14}$$

Modular Groups

$$\Gamma_2 \simeq S_3$$

$$S^2 = \mathbf{1} \qquad\qquad a^2 = \mathbf{1}$$
$$(ST)^3 = \mathbf{1} \qquad\qquad (ab)^3 = \mathbf{1},$$
$$T^2 = \mathbf{1}. \qquad\qquad b^2 = \mathbf{1}$$

Modular Groups

Under $\overline{\Gamma}$, modular forms of weight k are defined as holomorphic functions $f(\tau)$ that satisfy

$$f(\tau) \to (c\tau + d)^k f(\tau), \tag{15}$$

forms of weight zero are invariant under Γ.

It can be shown that modular forms can be organized into multiplets that, under a finite group $\bar{\Gamma}$, transform as

$$\vec{f}(\tau) \to (c\tau + d)^k \rho(\gamma) f(\vec{\tau}), \tag{16}$$

where $\rho(\gamma)$ is a unitary representation of $\bar{\Gamma}$.

Elements for Model Construction

Extended the group with Γ_2

$$SU(3)_C \times SU_L(2) \times U_y(1) \times \Gamma_2 \tag{17}$$

$$\phi \to (c\tau + d)^{k_\phi} \phi \tag{18}$$

It is assumed as a hypothesis that modular symmetry is a residual symmetry of a more fundamental group at low energies.

Fields are not modular forms.

Elements for Model Construction

- Define the basis of S_3.
- Establish the assignments under S_3 and the modular weights of the fields.
- Construct the modular forms of S_3 with weight 2 and 4.
- Calculate the Lagrangian of the Yukawa sector.

Base definition

Base of S_3 for $\theta = 4\pi/3$

$$
\begin{aligned}
e &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad a = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \\
ab &= \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \quad ba = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \quad bab = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}.
\end{aligned} \tag{19}
$$

Tensor products of S_3

$$
\begin{aligned}
&\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} = (x_1y_1 + x_2y_2)_{\mathbf{1}} \oplus (x_1y_2 - x_2y_1)_{\mathbf{1'}} \oplus \begin{pmatrix} x_1y_2 + x_2y_1 \\ x_1y_1 - x_2y_2 \end{pmatrix}_{\mathbf{2}} \\
&\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes (y')_{\mathbf{1'}} = \begin{pmatrix} -x_2y' \\ x_1y' \end{pmatrix}_{\mathbf{2}} \\
&(x')_{\mathbf{1'}} \otimes (y')_{\mathbf{1'}} = (x'y')_{\mathbf{1}}.
\end{aligned} \tag{20}
$$

Higgs potential

Three Higgs doublets potential invariant under S_3

$$V = \mu_1^2 \left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2 \left(H_s^\dagger H_s\right) + \frac{a}{2}\left(H_s^\dagger H_s\right)^2 + b\left(H_s^\dagger H_s\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)$$
$$+ \frac{c}{2}\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 + \frac{d}{2}\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + e f_{ijk}\left(\left(H_s^\dagger H_i\right)\left(H_j^\dagger H_k\right) + h.c.\right)$$
$$+ f\left\{\left(H_s^\dagger H_1\right)\left(H_1^\dagger H_s\right) + \left(H_s^\dagger H_2\right)\left(H_2^\dagger H_s\right)\right\} + \frac{g}{2}\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)^2\right\}$$
$$+ \frac{h}{2}\left\{\left(H_s^\dagger H_1\right)\left(H_s^\dagger H_1\right) + \left(H_s^\dagger H_2\right)\left(H_s^\dagger H_2\right) + \left(H_1^\dagger H_s\right)\left(H_1^\dagger H_s\right) + \left(H_2^\dagger H_s\right)\left(H_2^\dagger H_s\right)\right\}; \tag{21}$$

$$\boxed{v_1^2 = 3v_2^2},$$

where the VEVs are denoted as

$$\langle 0| H_1 |0\rangle = \frac{1}{\sqrt{2}} v_1; \quad \langle 0| H_2 |0\rangle = \frac{1}{\sqrt{2}} v_2; \quad \langle 0| H_s |0\rangle = \frac{1}{\sqrt{2}} v_3, \tag{22}$$

M. Gómez-Bock, M. Mondragón, A. Pérez-Martínez, Scalar and gauge sectors in the 3-Higgs Doublet Model under the S 3 symmetry, Eur. Phys. J. C 81 (10) (2021) 942.

Assignments in S_3 and modular weights

	(Q_1, Q_2)	(q_1, q_2)	Q_3	q_3
$SU(2)$	2	1	2	1
S_3	2	2	1	1
k	-2	-2	0	0
	(H_1, H_2)	H_s	$(Y_1^{2(4)}(\tau), Y_2^{2(4)}(\tau))$	$Y_s^{(4)}(\tau)$
$SU(2)$	2	2	1	1
S_3	2	1	2	1
k	0	0	2(4)	4

Table 1: Charges, assignments, and modular weights of $SU(2)$ and S_3. The superindex (4) on the modular forms indicates that they are of modular weight 4. The subindex s indicates that it is the symmetric singlet of the modular form of weight 4.

Modular forms in S_3

The modular forms will be constructed from the following expression.

$$\sum_i \frac{d}{d\tau} \log f_i(\tau) \to \sum_i (c\tau + d) k_i c + \sum_i (c\tau + d)^2 \frac{d}{d\tau} \log f_i(\tau). \qquad (23)$$

A useful type of modular form is

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \qquad q = e^{2\pi i \tau} \qquad (24)$$

Ferruccio Feruglio.Are neutrino masses modular forms? arXiv preprint arXiv:1706.08749-(2019). DOI: 10.1142/97898132380530012

Modular forms in S_3

Under the generators of the modular group, $\eta(\tau)$ transforms as

Under T

$$\begin{aligned} \eta(2\tau) &\to e^{i\pi/6}\eta(2\tau), \\ \eta(\tau/2) &\to \eta((\tau+1)/2), \\ \eta((\tau+1)/2) &\to e^{i\pi/12}\eta(\tau/2). \end{aligned} \tag{25}$$

Under S

$$\begin{aligned} \eta(2\tau) &\to \sqrt{-i\tau/2}\,\eta(\tau/2), \\ \eta(\tau/2) &\to \sqrt{-i3\tau}\,\eta(2\tau), \\ \eta((\tau+1)/2) &\to e^{-i\pi/12}\sqrt{-i\tau}\,\eta((\tau+1)/2). \end{aligned} \tag{26}$$

Modular Forms in S_3

The general modular form in terms of the functions $\eta(\tau)$ can be written as

$$Y(\alpha,\beta,\gamma|\tau) = \frac{d}{d\tau}\left(\alpha \log\eta\left(\frac{\tau}{2}\right) + \beta\log\eta\left(\frac{\tau+1}{2}\right) + \gamma\log\eta\left(2\tau\right)\right), \tag{27}$$

which satisfies

$$\begin{array}{rcl} \alpha+\beta+\gamma & = & 0 \\ Y(\alpha,\beta,\gamma|\tau) & \xrightarrow{S} & \tau^2 Y(\gamma,\beta,\alpha|\tau) \\ Y(\alpha,\beta,\gamma|\tau) & \xrightarrow{T} & Y(\beta,\alpha,\gamma|\tau) \end{array} \tag{28}$$

with the generators of S_3 for the representation $\mathbf{2}$

$$\rho(S) = \frac{1}{2}\left(\begin{array}{cc} 1 & -\sqrt{3} \\ -\sqrt{3} & -1 \end{array}\right), \qquad \rho(T) = \left(\begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array}\right), \tag{29}$$

Modular Forms in S_3

The system of equations generated by is solved

$$\begin{aligned} \alpha + \beta + \gamma &= 0 \\ Y(S\tau) &= Y(-1/\tau) = \tau^2 \rho(S) Y(\tau) \\ Y(T\tau) &= Y(\tau+1) = \rho(T) Y(\tau) \end{aligned} \tag{30}$$

The modular forms of weight 2 for S_3, with $C_1 = i/2\pi$, are

$$\begin{aligned} Y_1(\tau) &= \frac{\sqrt{3}i}{4\pi}\left(\frac{\eta'(\tau/2)}{\eta(\tau/2)} - \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)}\right), \\ Y_2(\tau) &= \frac{i}{4\pi}\left(\frac{\eta'(\tau/2)}{\eta(\tau/2)} + \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)} - \frac{8\eta'(2\tau)}{\eta(2\tau)}\right), \end{aligned} \tag{31}$$

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \otimes \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = Y_s^{(4)} + \begin{pmatrix} Y_1^{(4)} \\ Y_2^{(4)} \end{pmatrix},$$

$$\begin{aligned} Y_s^{(4)} &= Y_1^2 + Y_2^2 \\ Y_1^{(4)} &= 2Y_1Y_2 \\ Y_2^{(4)} &= Y_1^2 - Y_2^2. \end{aligned}$$

Yukawa Sector

To compress the notation, the doublets of S_3 can be redefined as

$$Q = \begin{pmatrix} \overline{Q}_1 \\ \overline{Q}_2 \end{pmatrix}; \; u = \begin{pmatrix} u_{1R} \\ u_{2R} \end{pmatrix}; \; H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}; \; Y^{(4)} = \begin{pmatrix} Y_1^{(4)} \\ Y_2^{(4)} \end{pmatrix}; \; Y^{(2)} = \begin{pmatrix} Y_1^{(2)} \\ Y_2^{(2)} \end{pmatrix};$$

Thus, the Lagrangian in the Yukawa sector is

$$\begin{aligned} \mathcal{L}_y^{(u)} &= C_1 \overline{Q} \otimes u \otimes \tilde{H} \otimes Y^{(4)} + C_2 \overline{Q} \otimes u \otimes \tilde{H} \otimes Y_s^{(4)} \\ &+ C_3 \overline{Q} \otimes u \otimes \tilde{H}_s \otimes Y^{(4)} + C_4 \overline{Q} \otimes u \otimes \tilde{H}_s \otimes Y_s^{(4)} \\ &+ C_5 \overline{Q} \otimes u_{3R} \otimes \tilde{H} \otimes Y^{(2)} + C_6 \overline{Q} \otimes u_{3R} \otimes \tilde{H}_s \otimes Y^{(2)} \\ &+ C_7 \overline{Q}_3 \otimes u \otimes \tilde{H} \otimes Y^{(2)} + C_8 \overline{Q}_3 \otimes u \otimes \tilde{H}_s \otimes Y^{(2)} \\ &+ C_9 \overline{Q}_3 \otimes u_{3R} \otimes \tilde{H}_s + \text{h.c.} \end{aligned} \tag{32}$$

Only the S_3 invariant terms are allowed, and their modular weight must be zero.

Yukawa Sector

Matrix elements

$$
\begin{aligned}
M_{11}^{(u)} &= (\alpha+\gamma)v_1 Y_1^{(4)} + (\alpha-\gamma)v_2 Y_2^{(4)} + C_2 v_2 Y_s^{(4)} + C_3 v_s Y_2^{(4)} + C_4 v_s Y_s^{(4)} \\
M_{12}^{(u)} &= (\beta+\gamma)v_2 Y_1^{(4)} + (\gamma-\beta)v_1 Y_2^{(4)} + C_2 v_1 Y_s^{(4)} + C_3 v_s Y_1^{(4)} \\
M_{13}^{(u)} &= C_5(v_2 Y_1^{(2)} + v_1 Y_2^{(2)}) + C_6 v_s Y_1^{(2)} \\
M_{21}^{(u)} &= (\beta+\gamma)v_1 Y_2^{(4)} + (\gamma-\beta)v_2 Y_1^{(4)} + C_2 v_1 Y_s^{(4)} + C_3 v_s Y_1^{(4)} \\
M_{22}^{(u)} &= (\alpha+\gamma)v_2 Y_2^{(4)} + (\alpha-\gamma)v_1 Y_1^{(4)} - C_2 v_2 Y_s^{(4)} - C_3 v_s Y_2^{(4)} + C_4 v_s Y_s^{(4)} \\
M_{23}^{(u)} &= C_5(v_1 Y_1^{(2)} - v_2 Y_2^{(2)}) + C_6 v_s Y_2^{(2)} \\
M_{31}^{(u)} &= C_7(v_2 Y_1^{(2)} + v_1 Y_2^{(2)}) + C_8 v_s Y_1^{(2)} \\
M_{32}^{(u)} &= C_7(v_1 Y_1^{(2)} - v_2 Y_2^{(2)}) + C_8 v_s Y_2^{(2)} \\
M_{33}^{(u)} &= C_9 v_s .
\end{aligned}
\tag{33}
$$

In this model, the free parameters are v_1, v_2, v_s, α, β, γ, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9 y τ,

V_{CKM} Matrix

The goal is to construct a matrix of the form

$$\begin{pmatrix} 0 & a & 0 \\ a^* & b & c \\ 0 & c^* & d \end{pmatrix}, \tag{34}$$

Known as texture zeros.

To satisfy this form in the mass matrix, the following conditions must be imposed

$$M_{11} = 0 \quad M_{12} = M_{21}^* \quad M_{32} = M_{23}^* \quad M_{13} = M_{31} = 0. \tag{35}$$

$$\begin{aligned} M_{13}^{(u)} &= C_5(v_2 Y_1^{(2)} + v_1 Y_2^{(2)}) + C_6 v_s Y_1^{(2)} = 0, \\ M_{31}^{(u)} &= C_7(v_2 Y_1^{(2)} + v_1 Y_2^{(2)}) + C_8 v_s Y_1^{(2)} = 0, \end{aligned} \tag{36}$$

Using the relation from the minimization of the Higgs potential

$$Y_2^{(2)}(\tau) - \sqrt{3} Y_1^{(2)}(\tau) = 0. \tag{37}$$

H. Fritzsch, Z.-Z. Xing, Mass and flavor mixing schemes of quarks and leptons, Progress in Particle and Nuclear Physics 45 (1) (2000) 1–81. doi:https://doi.org/10.1016/S0146-6410(00)00102-2.

V_{CKM} Matrix

We make the texture zeros matrix if

$$
\begin{array}{lll}
Re(\beta) = 0 & C_3 = 0 & \gamma = -(1/2)(v_s/v_2)C_4 \\
\alpha = -C_2 \in \mathbb{R} & C_6 = -4(v_2/v_s)C_5 & C_8 = -4(v_2/v_s)C_7 \\
C_5 = C_7^* & \tau = i & C_9, v_{1,2}, v_s \in \mathbb{R}
\end{array}
$$

For $\tau = i$ we have

$$y_2 = \sqrt{3}y_1 \qquad y_1^{(4)} = 2\sqrt{3}y_1^2 \qquad y_2^{(4)} = -2y_1^2 \qquad y_s^{(4)} = 4y_1^2, \tag{38}$$

with $y_k = Y_k(i)$. The mass matrix takes the form.
For simplify the notation, we have defined the parameters as

$$
\begin{aligned}
C' &= 4\sqrt{3}v_2 y_1^2 (C_2 + \beta), \\
C'_4 &= 8y_1^2 (C_4 v_s - C_2 v_2), \\
C'_5 &= -4\sqrt{3}v_2 y_1 C_5, \\
C'_9 &= C_9 vs
\end{aligned}
\qquad
M^{(u)} = \begin{pmatrix} 0 & C' & 0 \\ C'^* & C'_4 & C'_5 \\ 0 & C_5'^* & C'_9 \end{pmatrix}, \tag{39}
$$

V_{CKM} Matrix

The information about the phase is extracted through the matrix

$$P_f = \textit{diag}(1, e^{i\phi_1}, e^{i(\phi_1-\phi_2)}) \qquad (40)$$

where ϕ_1 is C' phase and ϕ_2 is C_5' phase.

$$M^{(u)} = P_f^{\dagger} \bar{M}^{(u)} P_f, \qquad (41)$$

Therefore,

$$\bar{M}^{(u)} = \begin{pmatrix} 0 & |C| & 0 \\ |C| & C_4' & |C_5'| \\ 0 & |C_5'| & C_9' \end{pmatrix}, \qquad (42)$$

F González Canales, A Mondragón, M Mondragón, UJ Saldaña Salazar, and L Velasco-Sevilla. Quark sector of S 3 models: classification and comparison with experimental data.Physical Review D88(9), 096004 (2013)

V_{CKM} Matrix

If we relate the mass matrix $M_D = \text{diag}(\widetilde{\sigma}_1, -\widetilde{\sigma}_2, 1)$ and using the invariants of a matrix, one obtains

$$|C| = \sqrt{\frac{\widetilde{\sigma}_1 \widetilde{\sigma}_2}{C_9'}}$$

$$C_4' = (\widetilde{\sigma}_1 - \widetilde{\sigma}_2 + 1 - C_9') \tag{43}$$

$$|C_5'| = \sqrt{\frac{(1 - C_9')(C_9' - \widetilde{\sigma}_1)(C_9' + \widetilde{\sigma}_2)}{C_9'}}.$$

where $\widetilde{\sigma}_i = \sigma_i/\sigma_3$, con $i = 1, 2$.

This calculation applies to both up and down quarks.

V_{CKM} Matrix

$$
\begin{aligned}
V_{ud}^{th} &= \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_s \xi_1^u \xi_1^d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u \xi_1^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{us}^{th} &= -\sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d \xi_1^u \xi_2^d}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} + \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_s}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u \xi_2^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_1^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{ub}^{th} &= \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d \tilde{\sigma}_s \delta_d \xi_1^u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} + \sqrt{\frac{\tilde{\sigma}_u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d \xi_1^u} - \sqrt{\delta_u \xi_2^u \xi_1^d \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{cd}^{th} &= -\sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_s \xi_2^u \xi_1^d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_2^u \xi_1^d} + \sqrt{\delta_u \delta_d \xi_1^u \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{cs}^{th} &= \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_d \xi_2^u \xi_2^d}{\mathcal{D}_{2u}\mathcal{D}_{2d}}} + \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_s}{\mathcal{D}_{2u}\mathcal{D}_{2d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_2^u \xi_2^d} + \sqrt{\delta_u \delta_d \xi_1^u \xi_1^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{cb}^{th} &= -\sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_d \tilde{\sigma}_s \delta_d \xi_2^u}{\mathcal{D}_{2u}\mathcal{D}_{3d}}} + \sqrt{\frac{\tilde{\sigma}_c}{\mathcal{D}_{2u}\mathcal{D}_{3d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d \xi_2^u} - \sqrt{\delta_u \xi_1^u \xi_1^d \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{td}^{th} &= \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_c \tilde{\sigma}_s \delta_u \xi_1^d}{\mathcal{D}_{3u}\mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_d}{\mathcal{D}_{3u}\mathcal{D}_{1d}}} \left(\sqrt{\delta_u (1-\delta_u)(1-\delta_d)\,\xi_1^d} - \sqrt{\delta_d \xi_1^u \xi_2^u \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{ts}^{th} &= -\sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_c \tilde{\sigma}_d \delta_u \xi_2^d}{\mathcal{D}_{3u}\mathcal{D}_{2d}}} + \sqrt{\frac{\tilde{\sigma}_s}{\mathcal{D}_{3u}\mathcal{D}_{2d}}} \left(\sqrt{\delta_u (1-\delta_u)(1-\delta_d)\,\xi_2^d} - \sqrt{\delta_d \xi_1^u \xi_2^u \xi_1^d}\, e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{tb}^{th} &= \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_c \tilde{\sigma}_d \tilde{\sigma}_s \delta_u \delta_d}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \left(\sqrt{\frac{\xi_1^u \xi_2^u \xi_1^d \xi_2^d}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \sqrt{\frac{\delta_u \delta_d (1-\delta_u)(1-\delta_d)}{\mathcal{D}_{3u}\mathcal{D}_{3d}}}\, e^{i\phi_2} \right) e^{i\phi_1}.
\end{aligned}
\tag{44}
$$

F. González Canales, A Mondragón, M Mondragón, UJ Saldaña Salazar, and L Velasco-Sevilla.Quark sector of S 3 models: classification and comparison with experimental data.Physical Review D88(9), 096004 (2013).

V_{CKM} Matrix

where it has been defined

$$\begin{aligned}
\delta_{u,d} &= 1 - C'_{9u,d} \\
\xi_1^{u,d} &= 1 - \widetilde{\sigma}_{u,d} - \delta_{u,d}, \\
\xi_2^{u,d} &= 1 + \widetilde{\sigma}_{c,s} - \delta_{u,d}, \\
\mathcal{D}_{1(u,d)} &= (1 - \delta_{u,d})(\widetilde{\sigma}_{u,d} + \widetilde{\sigma}_{c,s})(1 - \widetilde{\sigma}_{u,d}), \\
\mathcal{D}_{2(u,d)} &= (1 - \delta_{u,d})(\widetilde{\sigma}_{u,d} + \widetilde{\sigma}_{c,s})(1 + \widetilde{\sigma}_{c,s}), \\
\mathcal{D}_{3(u,d)} &= (1 - \delta_{u,d})(1 - \widetilde{\sigma}_{u,d})(1 + \widetilde{\sigma}_{c,s}).
\end{aligned} \tag{45}$$

V_{CKM} Matrix

The comparison is made with the χ^2 function defined as

$$\chi^2 = \sum_{i=u,c,t} \sum_{j=d,s,b} \frac{\left(|V_{ij}^{\text{th}}| - |V_{ij}|\right)^2}{\sigma_{V_{ij}}^2} \tag{46}$$

Parameters	Values in the fit
C'_{9u}	0.816393
C'_{9d}	0.828604
ϕ_{1u}	1.63797
ϕ_{1d}	0
ϕ_{2u}	0.0981477
ϕ_{2d}	0
χ^2	0.00070

Table 2: Values of the free parameters for the adjustment with the values of the ratios of the masses fixed in their central value and the respective obtaining of χ^2.

Matriz V_{CKM}

VCKM matrix for this fit

$$V^{th}_{CKM} = \begin{pmatrix} 0.97435 & 0.2250 & 0.00369 \\ 0.22486 & 0.97349 & 0.04182 \\ 0.00857 & 0.04110 & 0.999118 \end{pmatrix}, \qquad (47)$$

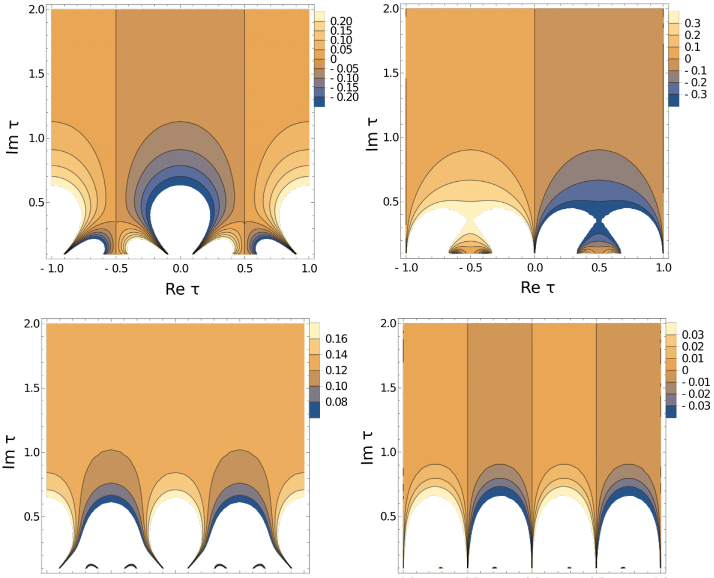
Conclusions and Perspectives

- When we use S_3 (alone) it's diffcult to obtain a proper V_{ckm} matrix. However, with S_3 derived from a modular symmetry the constrains vanish and we obtain a accurate V_{ckm} matrix.
- In this framework, the Yukawa's are modular functions that can be obtained from the modular symmetry.
- It is possible to test other assignments or symmetry groups based on modular symmetry.
- New models can be constructed by combining some symmetry groups and modular groups.

Thank you for your attention!!!

Modular forms in S_3

Real and imaginary part of the modular form



	χ_1	$\chi_{1'}$	χ_2
C_1	1	1	2
C_2	1	1	-1
C_3	1	-1	0