Internal Structure of Hadrons: Understanding Strong Interactions and the Standard Model

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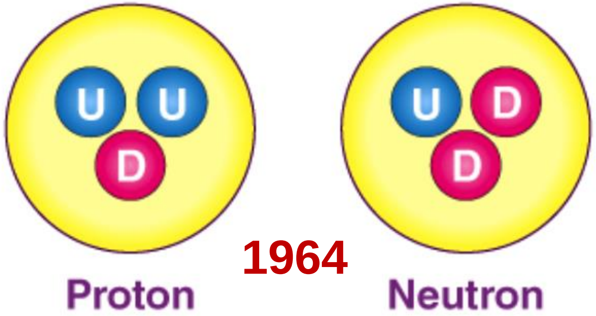
Strong interactions and QCD

When the **quarks** are really close to each other, the force is so weak that they behave almost as **free particles**. This discovery was expressed in **1973** in an elegant mathematical framework that led to a completely new theory **Quantum Chromodynamics**

PHYSICS NOBEL PRIZE 2004

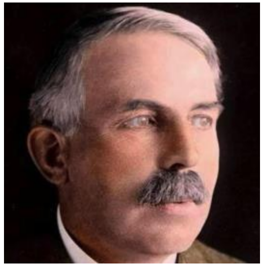


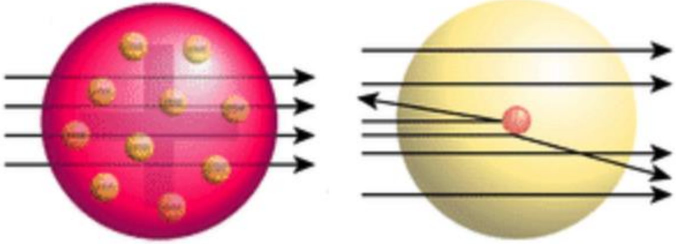
1964

The Quark Model

Murray Gell-Mann
The Nobel Prize 1969

Gell-Mann's **eightfold way** suggested all **hadrons**, including the **neutron** and **proton**, were composed of "**quarks**" with very unusual properties.





Pions

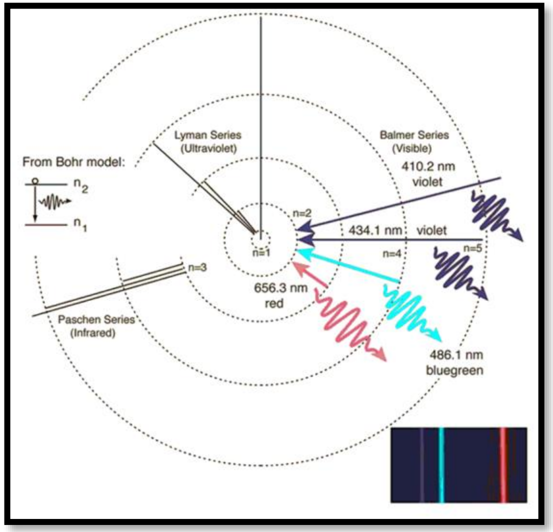
Gold
Nucleus
1911
1917
Proton

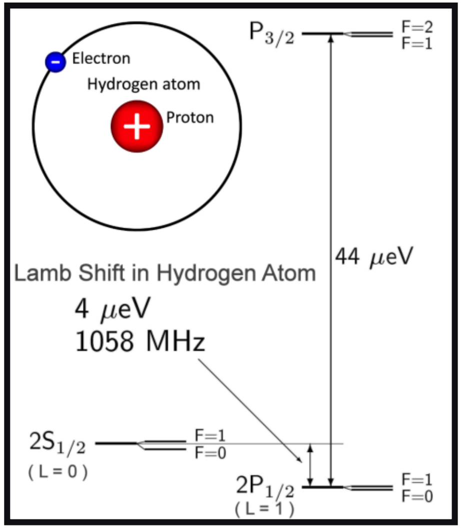
1934-1949

1947-1950

1960-2008

Two particle **bound states**

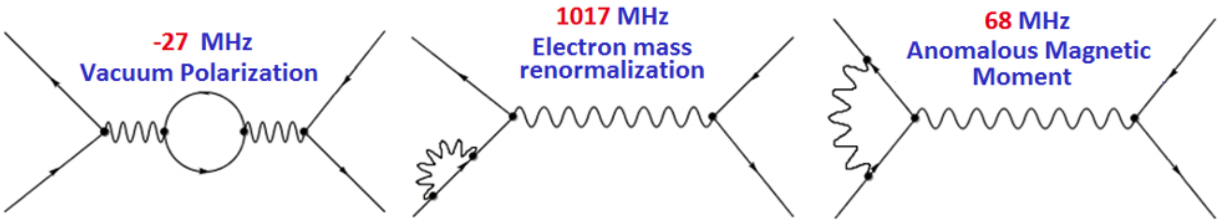




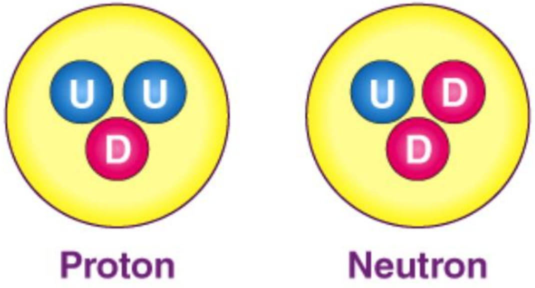
π and the **K** are the simplest two-body **bound states** in **QCD**. Unraveling their internal structure is a bigger challenge.

P and **N** are the simplest three-body **bound states** in **QCD**.

Dyson: If you don't understand the **Hydrogen atom** (in **QED**) you don't understand anything.

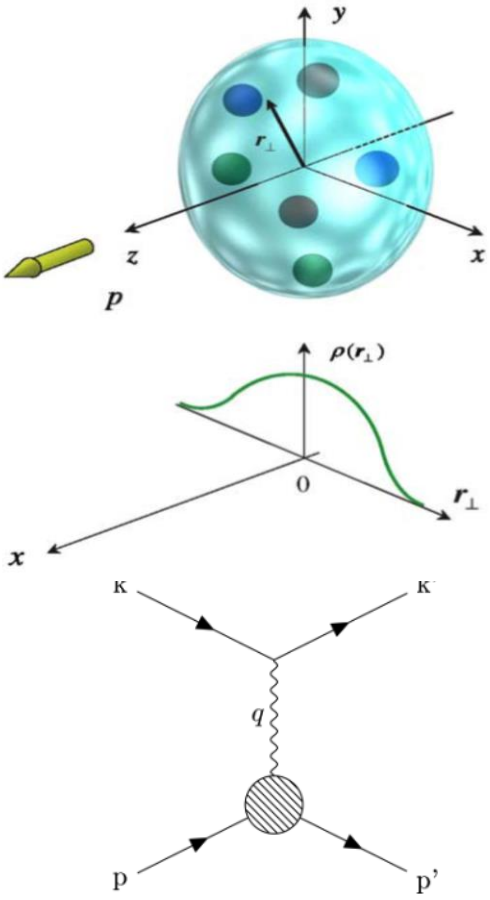


Renormalized QED

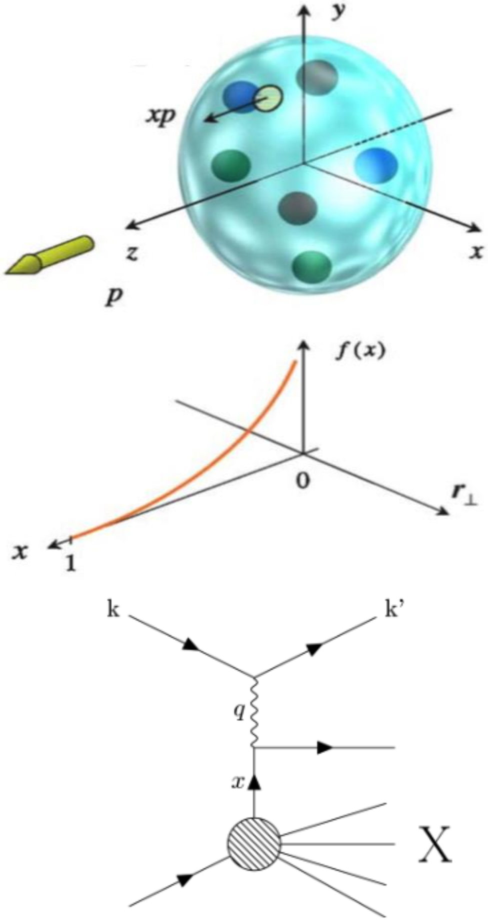


Confinement and **emergent hadron mass**

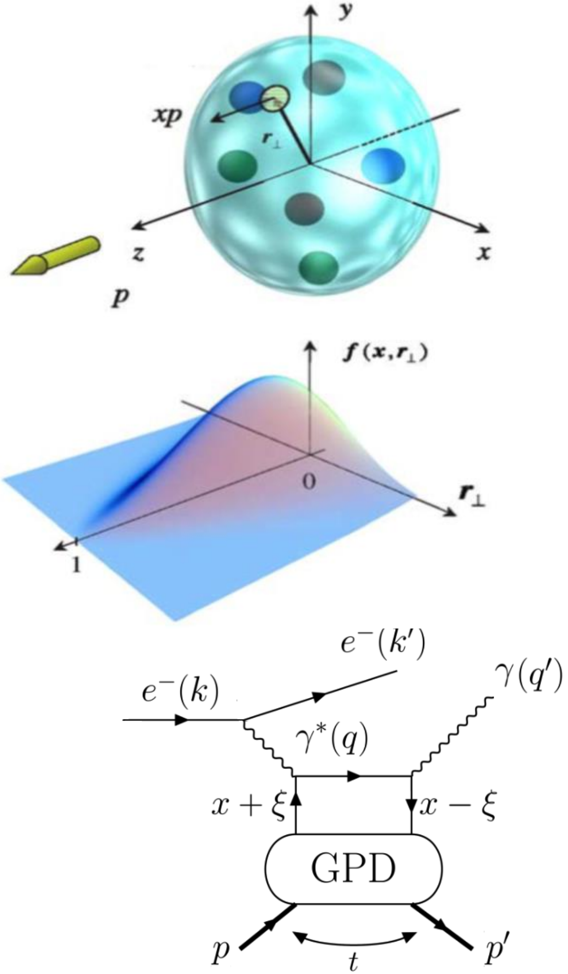
Hadrons Structure - QCD



Hofstadter, et. al., Phys. Rev. 91, 422 (1953)
Hofstadter, Rev. Mod. Phys. 28, 214 (1956)



Feynman, Phys. Rev. Lett. 23, 1415 (1969)



Hadron Structure: Footprints of QCD

3D Structure of Hadrons: **Global analysis** of **experimental data**, discrete **lattice studies**, **effective field theories** and continuum **Schwinger-Dyson equations**.

QCD is characterized by two **emergent** phenomena: **confinement** and **emergent hadron mass** (**EHM**).

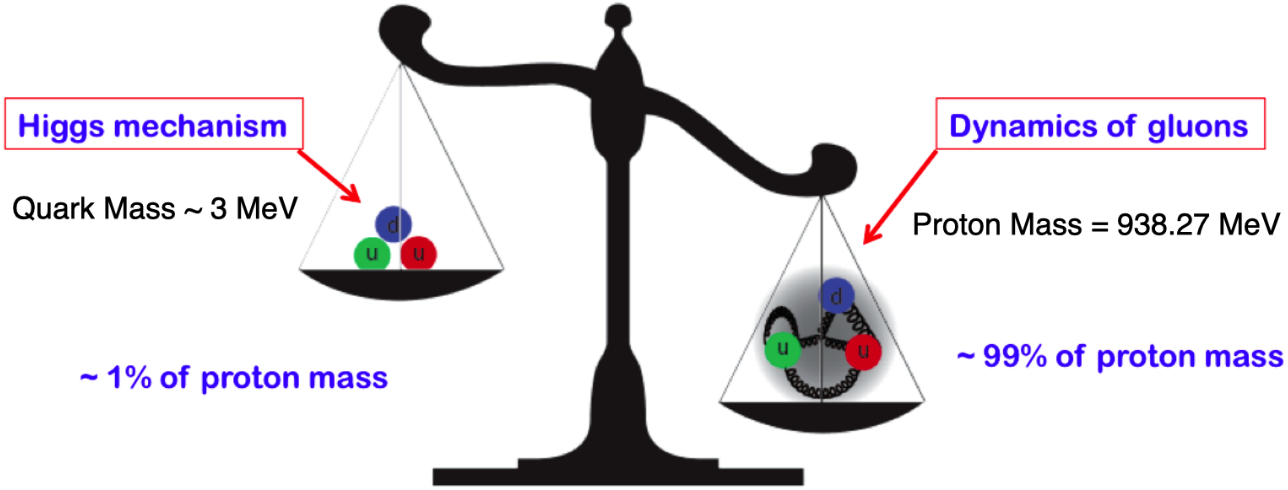
$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$

$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu\,,$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

Emergence of hadron masses **(EHM)** from QCD **dynamics**

- Formation of color-singlet bound states: “**Hadrons**”



Higgs mechanism

Quark Mass ~ 3 MeV

~ 1% of proton mass

Dynamics of gluons

Proton Mass = 938.27 MeV

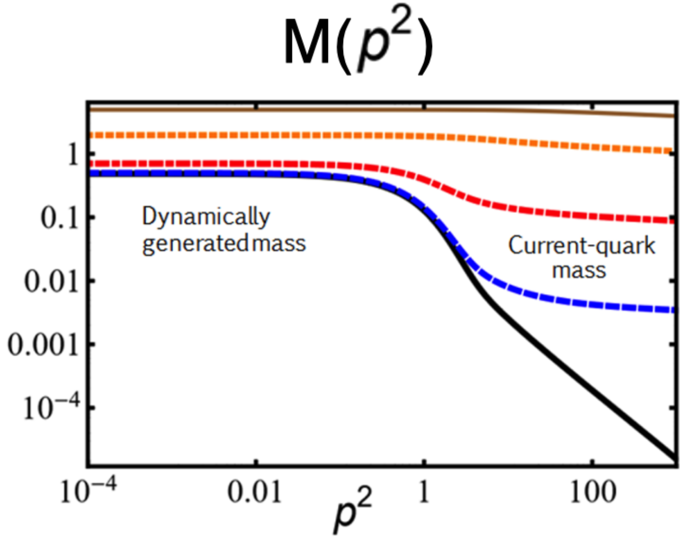
~ 99% of proton mass

QCD: Emergent Phenomena and Challenges

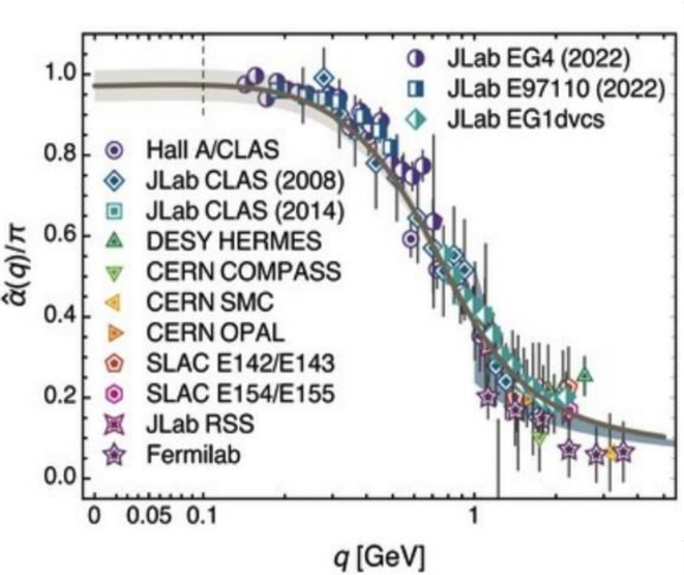
Origins of **confinement** and **dynamical mass generation** can perhaps be traced back to the Green functions of **quarks** and **gluons**.

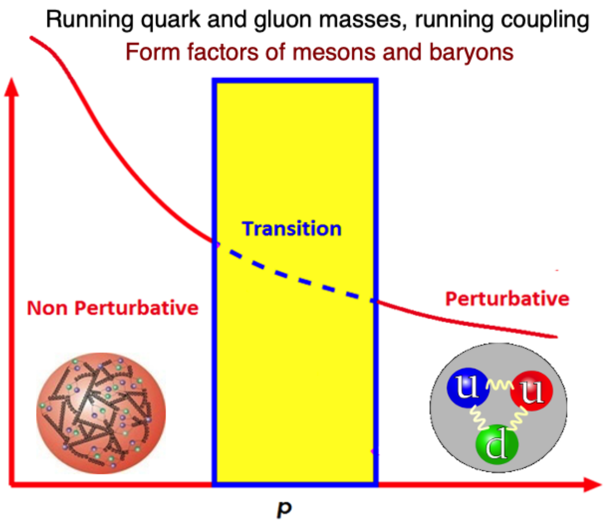
These emergent phenomena of **QCD**, non-existent in perturbation theory are naturally linked to the infrared enhancement of the **strong running coupling**.

The effects of the pattern of **dynamical mass generation** are traceable in the **Q^2 evolution** of the π and **K form factors** explored and planned in the **JLab** and the **EIC**.



$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$

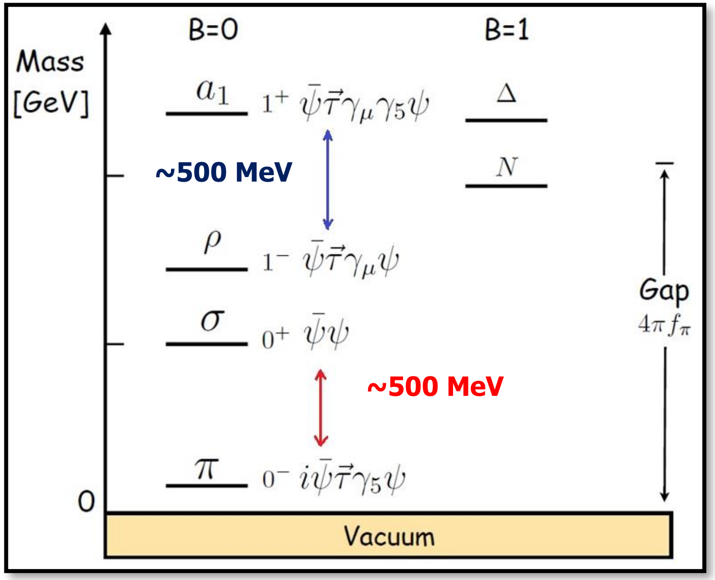


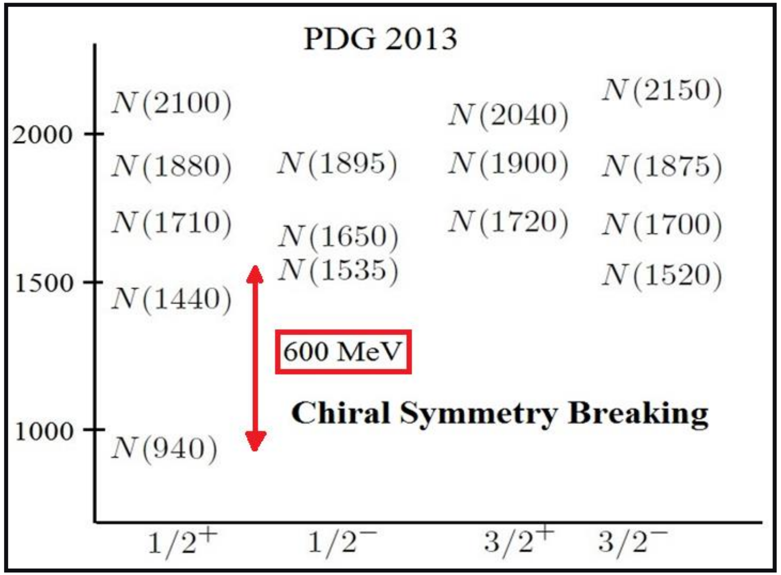


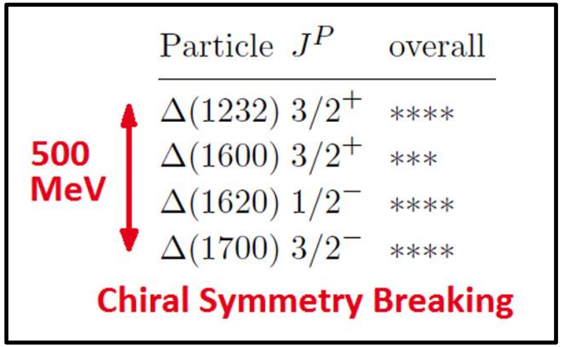
DCSB: Mass Spectrum of Mesons and Baryons

Without understanding the **spectrum** and **transitions** of **hadrons**, we expect neither **confinement** nor **DCSB** to be unraveled just like without studying the spectrum of hydrogen atom and its transitions, QED could not be fully understood.

Experimental signature and pattern of **DCSB** is reflected in **masses** and **form factors** of hadrons.







Particle	J^P	overall
$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****

Chiral Symmetry Breaking

QCD – Schwinger-Dyson Equations

Gauge Technique – Non Perturbative Solutios

- A. Salam, R. Delbourgo, Phys. Rev. 135 (1964) 6, B1398-B1427.

DCSB - Non-perturbative QED

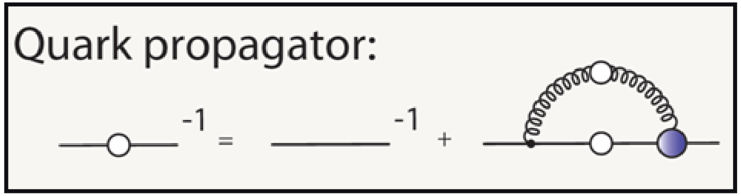
- P.I. Fomin, V.A. Miransky, Phys. Lett. B64 (1976) 166-168.

DCSB – Non-abelian Gauge Theories

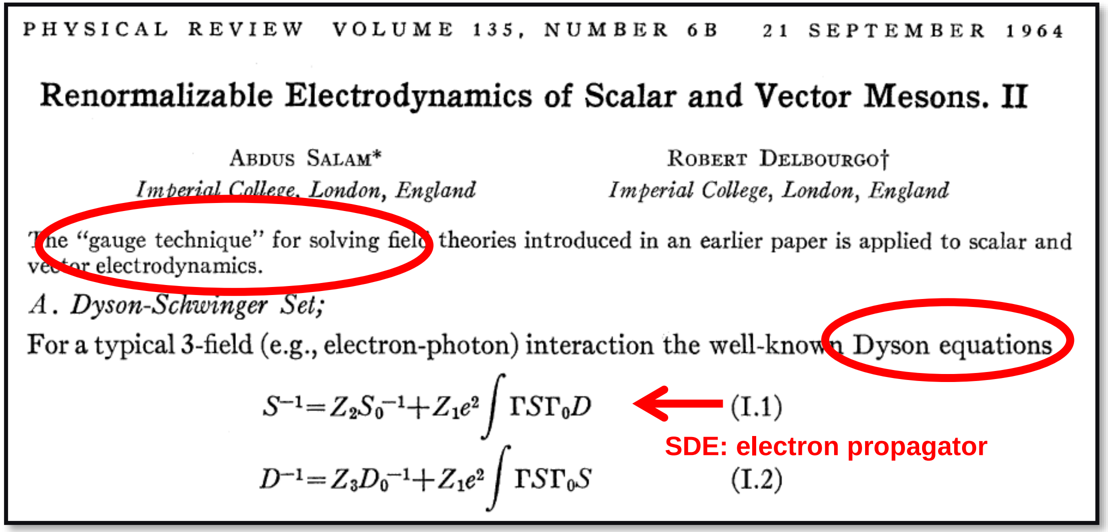
- V. Miransky, V. Gusynin, Y. Sitenko, Phys. Lett. B100 (1981) 157-162

DCSB – MT Model - Vector Mesons

- P. Maris, P. Tandy, Phys. Rev. C60 (1999)


Quark propagator:

$$S(p^2,\mu^2) = \frac{Z(p^2,\mu^2)}{i\,\gamma\cdot p + M(p^2)}$$


PHYSICAL REVIEW VOLUME 135, NUMBER 6B 21 SEPTEMBER 1964

Renormalizable Electrodynamics of Scalar and Vector Mesons. II

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Imperial College, London, England

Robert Delbourgo†
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The "gauge technique" for solving field theories introduced in an earlier paper is applied to scalar and vector electrodynamics.

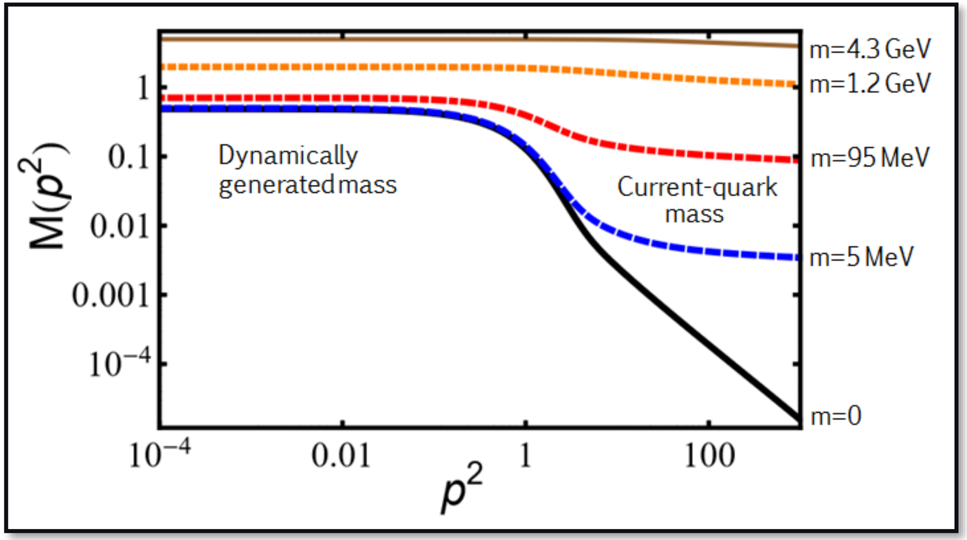
A. Dyson-Schwinger Set;

For a typical 3-field (e.g., electron-photon) interaction the well-known Dyson equations

$$S^{-1}=Z_2S_0^{-1}+Z_1e^2\int\Gamma S\Gamma_0 D \quad \leftarrow \quad (\mathrm{I.1})$$

SDE: electron propagator

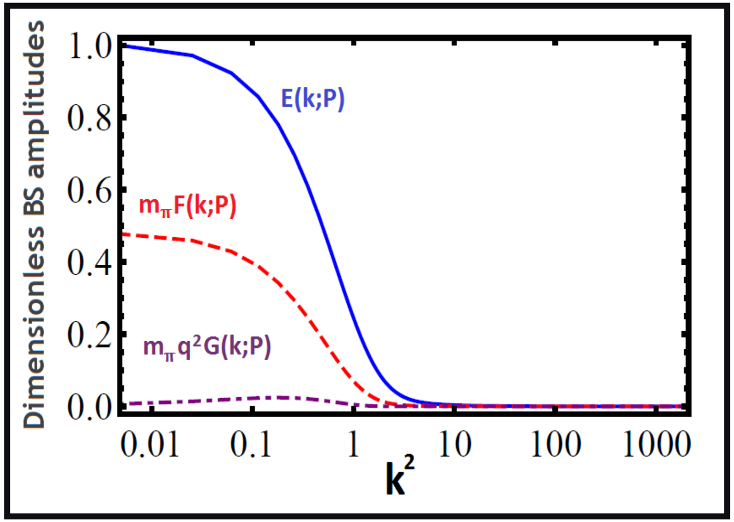
$$D^{-1}=Z_3D_0^{-1}+Z_1e^2\int\Gamma S\Gamma_0 S \quad (\mathrm{I.2})$$

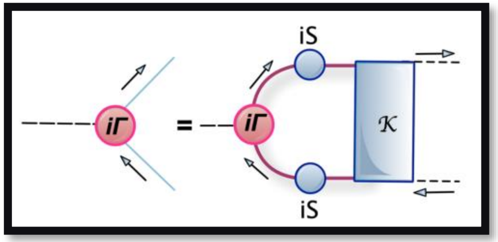


Bethe-Salpeter Amplitudes – Quark Photon Vertex

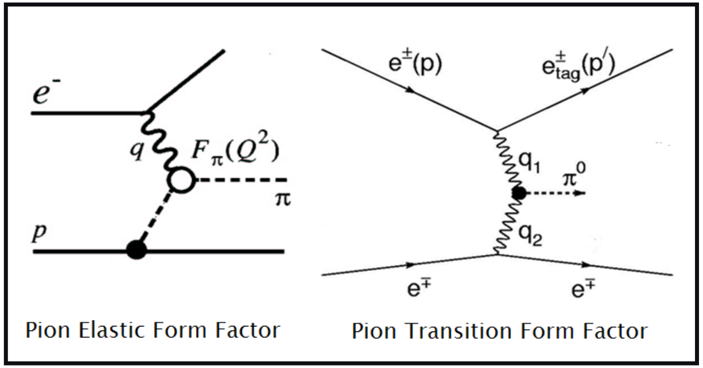
The pattern of **DCSB** is traceable in the **momentum evolution** of the π and **K Bethe-Salpeter Amplitudes (BSAs)** and **form factors** explored and planned in the **JLab** and the **EIC**.

$$\Gamma_\pi(k,P) = \gamma_5 \left[iE_\pi(k;P) + \gamma \cdot P F_\pi(k;P) + \gamma \cdot k k \cdot P G_\pi(k;P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k;P) \right]$$

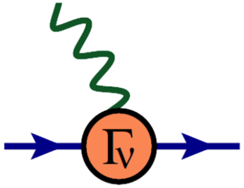




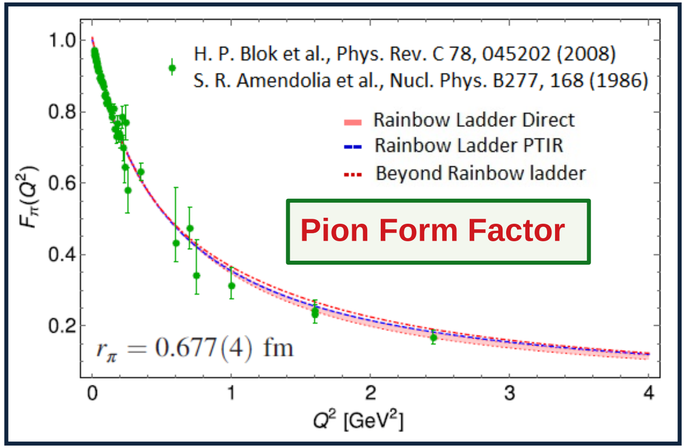
Qark-Photon Vertex

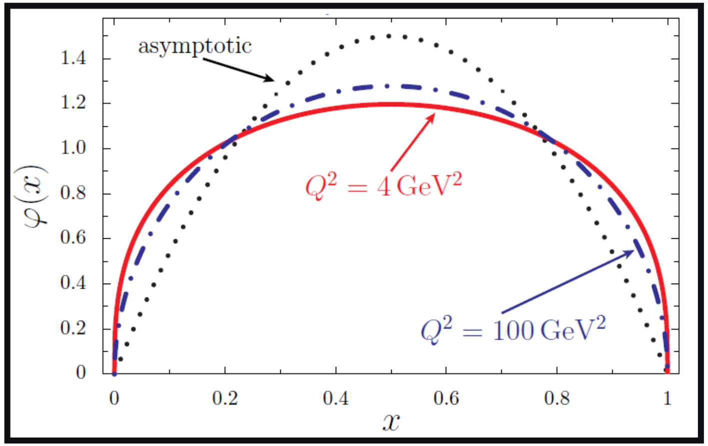


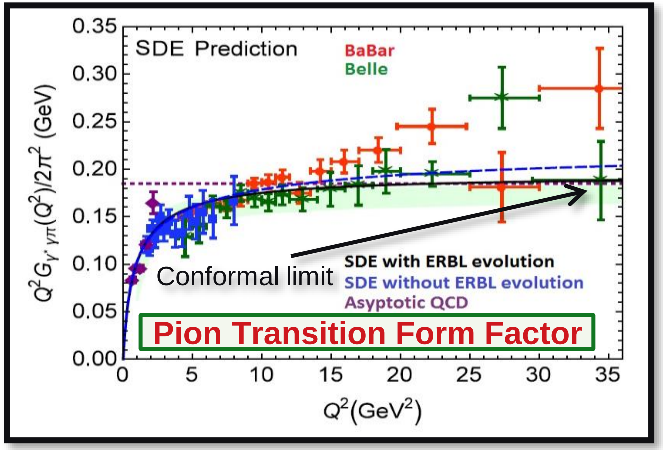
AB, M.R. Pennington, Phys. Rev. D50 7679 (1994)
R. Bermudez et. al., Phys. Rev. C85, 045205 (2012)
L. Albino et. al., Phys. Rev. D100 (2019) 5, 054028
V. Banda, AB, Phys. Rev. D107 073008 (2023)



Electromagnetic and Transition Form Factors

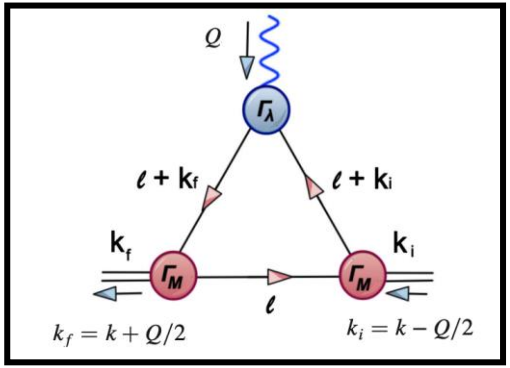


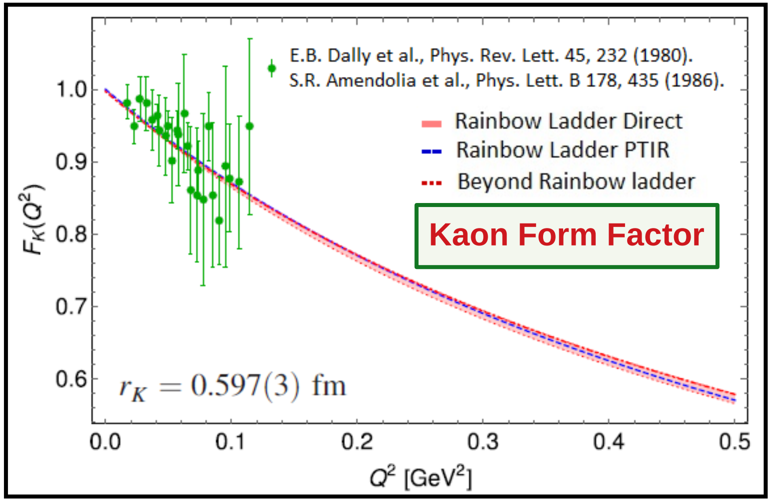


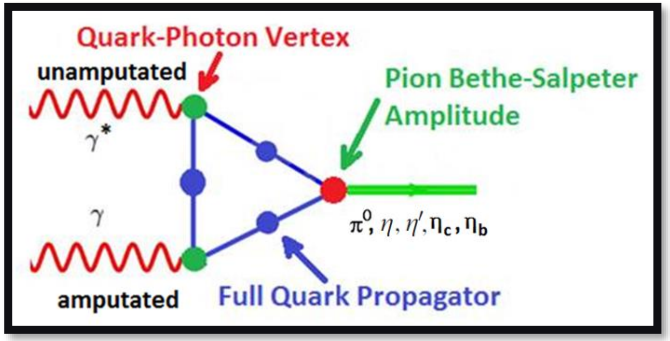


A. Miramontes et. al., Phys. Rev. D 105 (2022) 7, 074013

K. Raya, et. al. Phys. Rev. D 93 (2016) 7, 074017







Mon Anomalous Magnetic Moment

A muon with spin s has a **magnetic moment**: $$\boxed{\mu = g\,\frac{e}{2m}\,s}$$

The factor **g** is called the gyro-magnetic factor. The **Dirac equation** for a charged elementary fermion with spin 1/2 implies **g = 2**.

The **anomalous magnetic moment** is the deviation from g = 2, parameterized by **a_μ = (g−2)/2**. It appears due to radiative corrections. **Renormalization** of **QED** was established in 1943 and 1947-1948 by Tomonaga, Schwinger and Feynman.

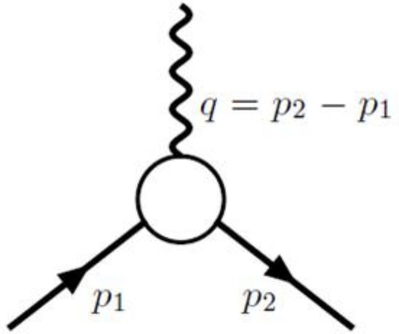
The leading contribution to **a_μ**, calculated by Schwinger in 1949, is: $$\boxed{a_\mu = \frac{\alpha}{2\pi}}$$

The **amplitude** of a muon scattering off an external electromagnetic field A is: ($q=p_2-p_1$):

$$\boxed{\mathcal{M} = -ie\langle \mu_{p_2}|J^\mu(0)|\mu_{p_1}\rangle A_\mu(q)}$$

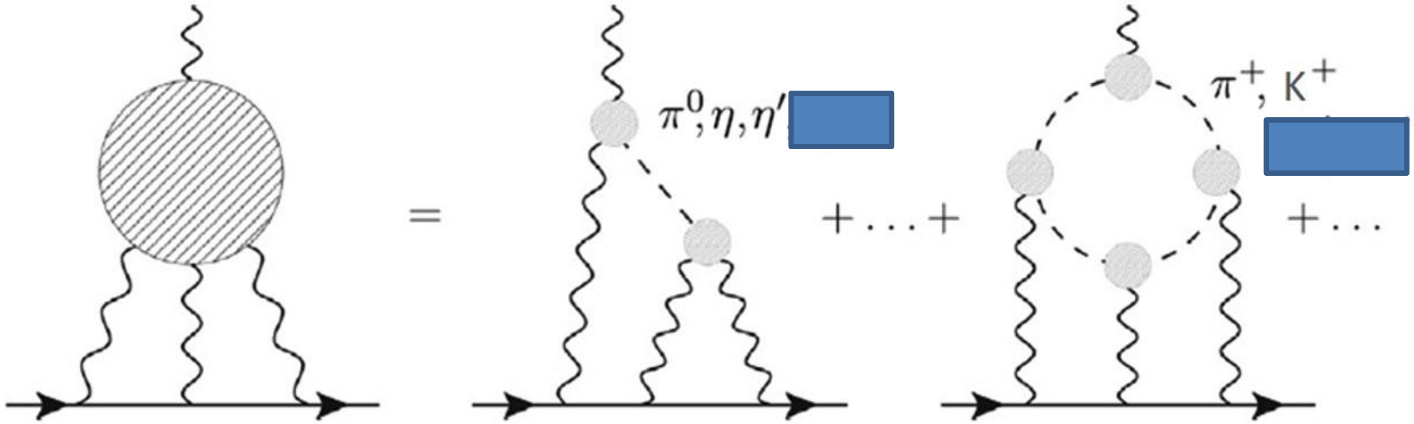
$$\langle \mu_{p_2}|J^\mu(0)|\mu_{p_1}\rangle = \bar{u}_{p_2}\Gamma^\mu(p_2,p_1)u_{p_1}$$

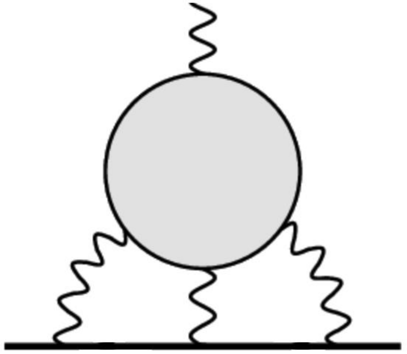
$$\Gamma^\mu(p_2,p_1) = \left[F_D(q^2)\gamma^\mu + F_P(q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]$$

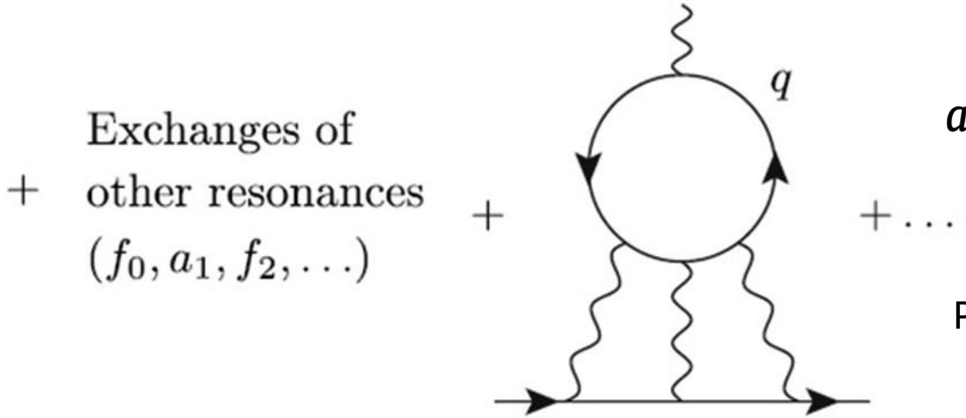


QCD - Hadronic Light by Light Scattering

The **hadronic light-by-light** scattering contribution appears at $\mathcal{O}(\alpha^3)$.







$$a_\mu^{\mathrm{HLbL}} = 92(19) \times 10^{-11}$$

$$a_\mu^{\mathrm{HLbL}} = 90(17) \times 10^{-11}$$

(phenomenology + lattice QCD)

Phys. Rep. 126 887 1 (2020)

QCD - Hadronic Light by Light Scattering

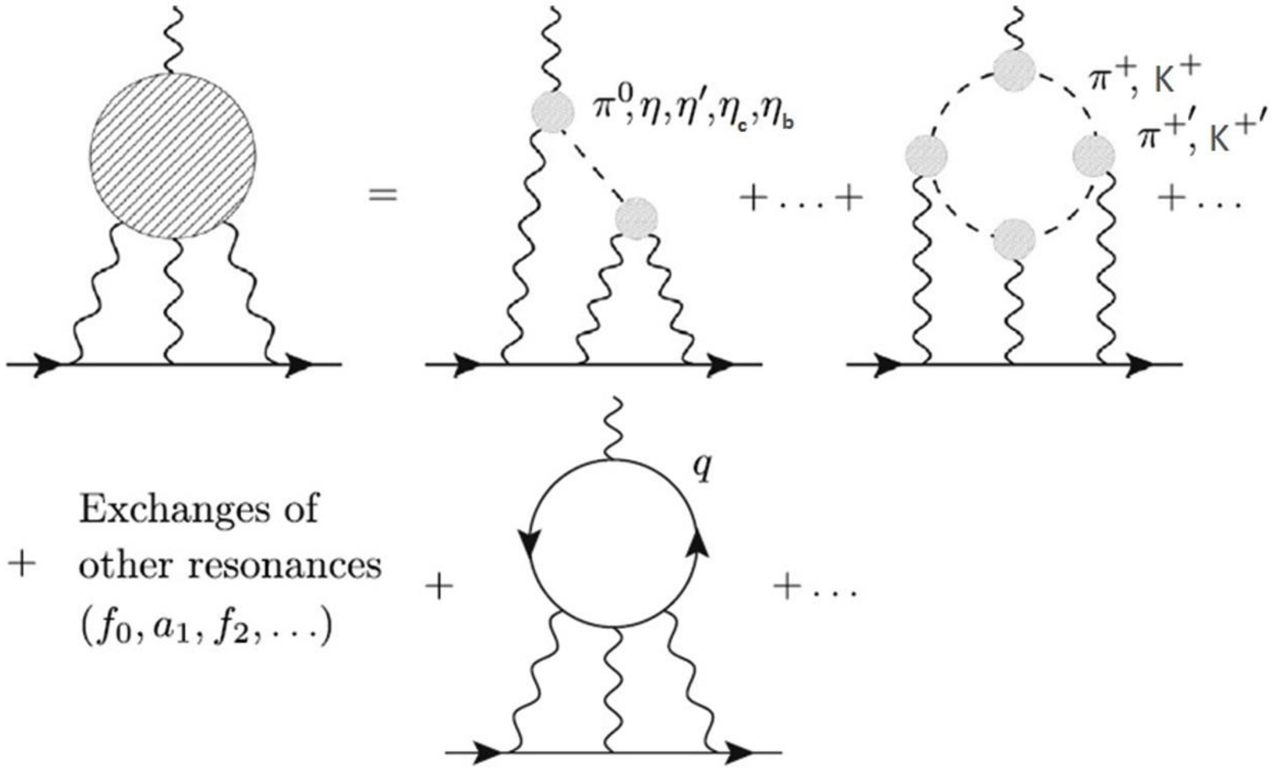
"Contribution of neutral pseudoscalar mesons to a_μ^{HLbL} within a Schwinger-Dyson equations approach to QCD",
K. Raya, AB, P. Roig,
Phys. Rev. D101 (2020) 7, 074021.

"Pion and Kaon box contribution to a_μ^{HLbL}"
A. Miramontes, AB, K. Raya, P. Roig,
Phys. Rev. D105 (2022) 7, 074013.

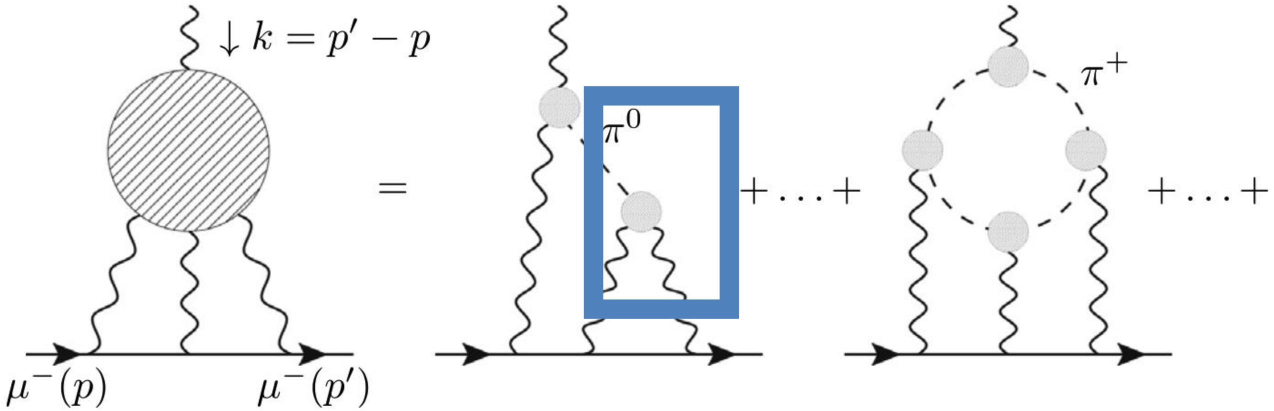
"The anomalous magnetic moment of the muon in the Standard Model"
T. Aoyama et. al.,
Phys. Rept. 887 (2020) 1-166.

"Prospects for precise predictions of a_μ",
G. Colangelo et. al. Contribution to: 2022 Snowmass Summer Study,
e-Print: 2203.15810 [hep-ph].

"The box diagram contribution of the radially excited pion and kaon to a_μ^{HLbL}"
e-Print: 2411.02218 [hep-ph].



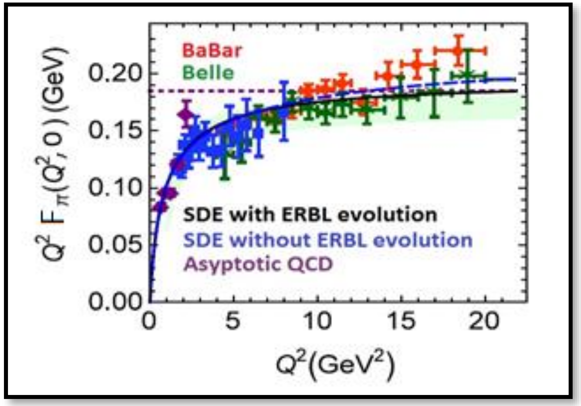
Neutral Pseudoscalar Pole Contributions

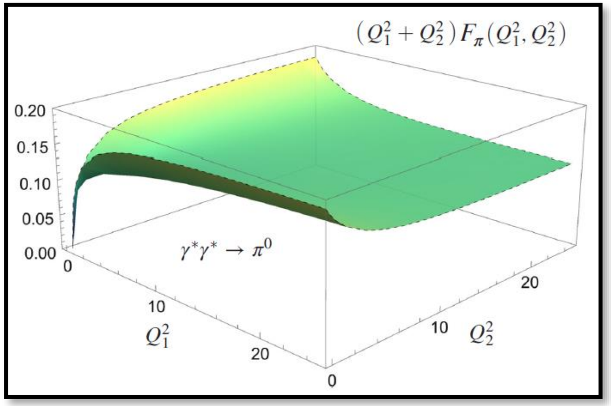


$$
\begin{aligned}
a_\mu^{\pi^0-\text{pole}} &= (61.4 \pm 2.1) \times 10^{-11} \\
a_\mu^{\eta-\text{pole}} &= (14.7 \pm 1.9) \times 10^{-11} \\
a_\mu^{\eta'-\text{pole}} &= (13.6 \pm 0.8) \times 10^{-11} \\
a_\mu^{\eta_c-\text{pole}} &= (0.9 \pm 0.1) \times 10^{-11} \\
a_\mu^{\eta_b-\text{pole}} &= (0.0026 \pm 0.0001) \times 10^{-11} \\
\hline
a_\mu^{\text{PS}-\text{pole}} &= (90.6 \pm 4.9) \times 10^{-11}
\end{aligned}
$$

The form factor $\pi^0 \to \gamma^*\gamma$ extended to $\pi^0 \to \gamma^*\gamma^*$.

K. Raya, AB, P. Roig, Phys. Rev. D 101 (2020) 7, 074021





Dispersive methods:

$$a_\mu^{\pi^0\text{-pole}} = 63.6(2.7) \times 10^{-11}$$

$$a_\mu^{\eta\text{-pole}} = 16.3(1.4) \times 10^{-11}$$

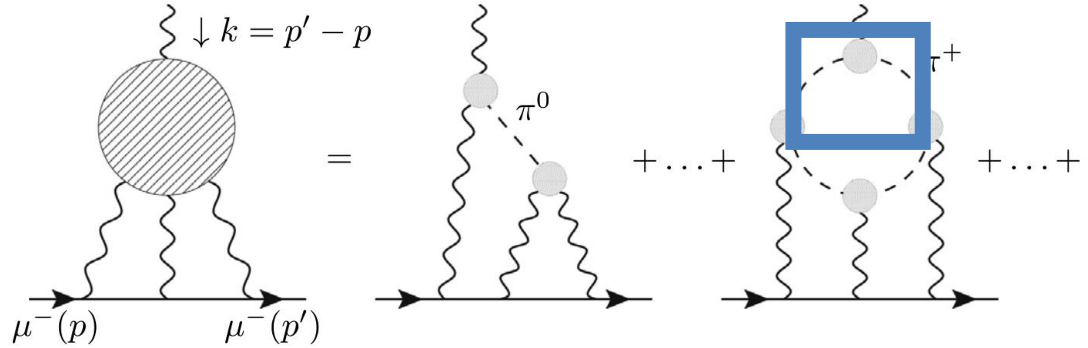
$$a_\mu^{\eta'\text{-pole}} = 14.5(1.9) \times 10^{-11}$$

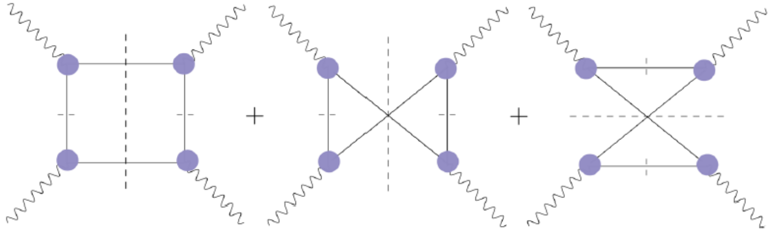
Lattice: $a_\mu^{\pi^0\text{-pole}} = 59.7(3.6) \times 10^{-11}$

SDE: $a_\mu^{\text{PS-pole}} = 91.6\,(1.9) \times 10^{-11}$

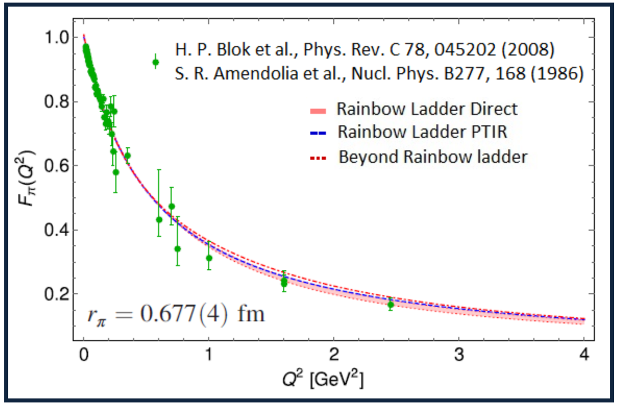
G. Eichmann et. al. Phys. Lett. B 797 (2019)

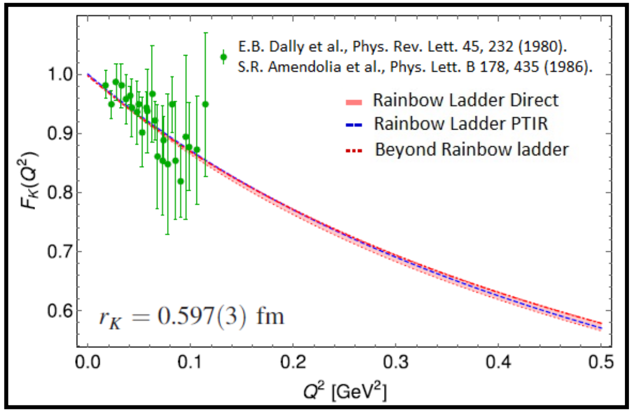
π and K **Form Factors** Contributions





Leading order **P**-box contributions to $a_\mu^{\rm HLbL}$, where the corresponding **P** meson EFFs are highlighted by the purple filled circles.





A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013

Radial excitations of π and **K**:

A.S. Miramontes, K. Raya, AB, P. Roig, G. Paredes-Torres,e-Print: 2411.02218 [hep-ph].

$$a_\mu^{\pi_1-\text{box}} = (-3.2 \pm 0.6) \times 10^{-13}$$

$$a_\mu^{K_1-\text{box}} = -6.8 \times 10^{-14}$$

$$a_\mu^{\pi^\pm-\text{box}} = -(15.6 \pm 0.2) \times 10^{-11}$$

$$a_\mu^{K^\pm-\text{box}} = -(0.48 \pm 0.02) \times 10^{-11}$$

Dispersive methods:

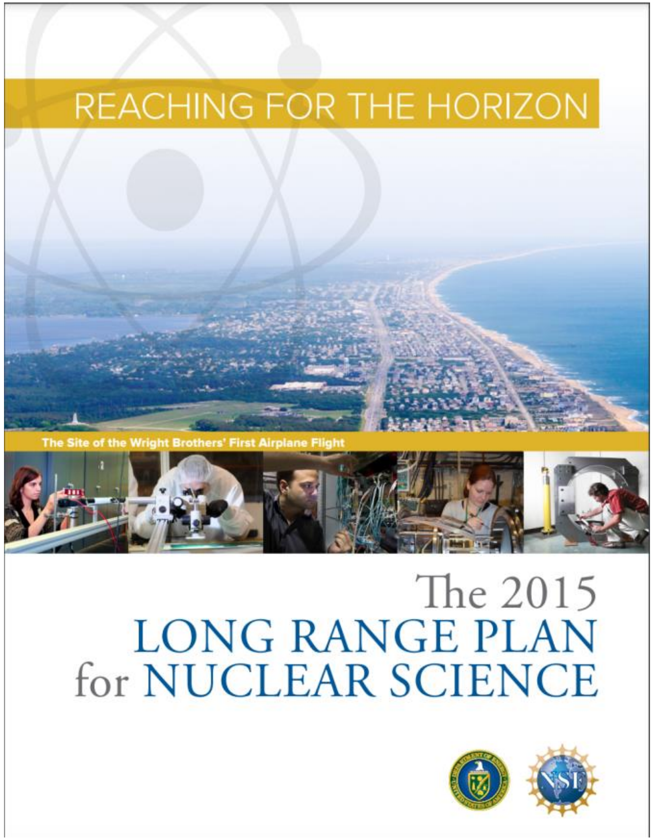
$$a_\mu^{\pi\text{-box}} = -15.9(2) \times 10^{-11}$$

$$a_\mu^{K^+\text{-box,VMD}} = -0.50 \times 10^{-11}$$

$$a_\mu^{K^+\text{-box,DSE}} = -0.48(2) \times 10^{-11}$$

Eichmann, et. al. Phys.Rev.D 101 (2020) 5, 054015

π and K **Form Factor - JLab 12 GeV Upgrade**



REACHING FOR THE HORIZON

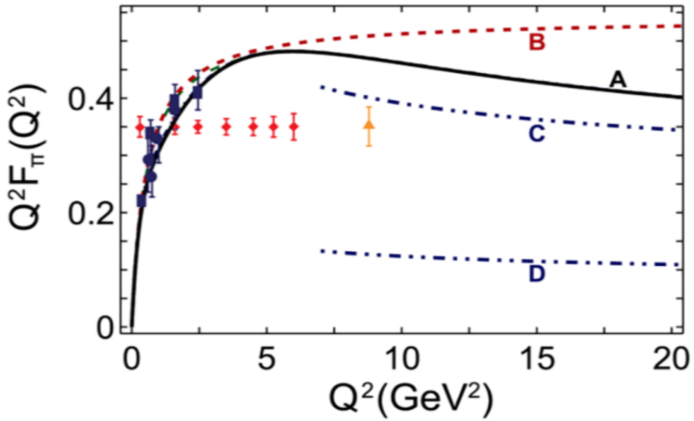
The Site of the Wright Brothers' First Airplane Flight

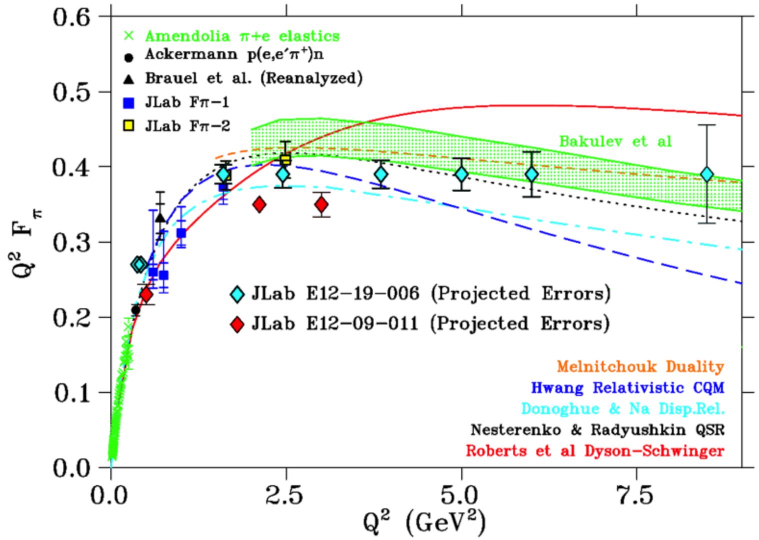
The 2015
LONG RANGE PLAN
for NUCLEAR SCIENCE

The study of the **pion form factor** is one of the **flagship goals** of the **JLab 12-GeV upgrade**… regime in which the phenomenology of **QCD** begins a **transition** from **large-** to **short-distance** scale behavior.

The **pion form factor** can potentially be measured till **$Q^2 \sim 6\text{-}8$** in the **12 GeV** upgrade of the **JLab**.

Courtesy Garth Huber

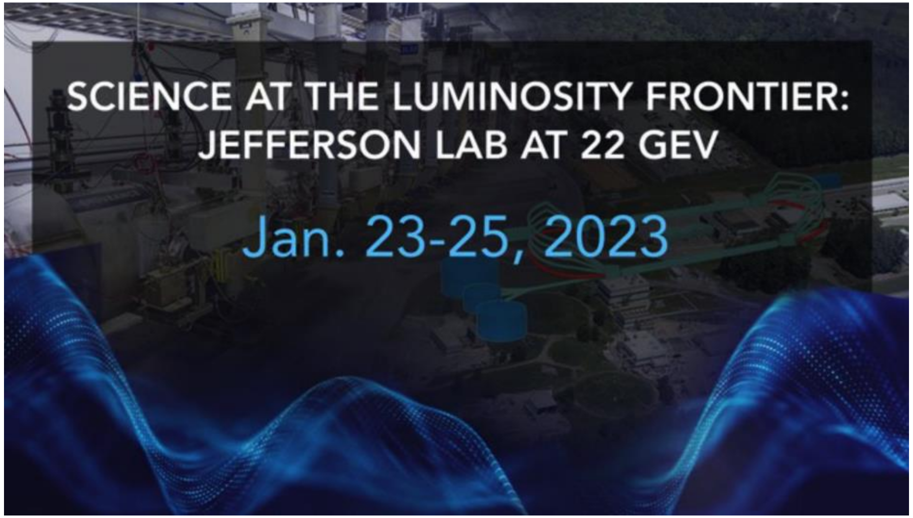


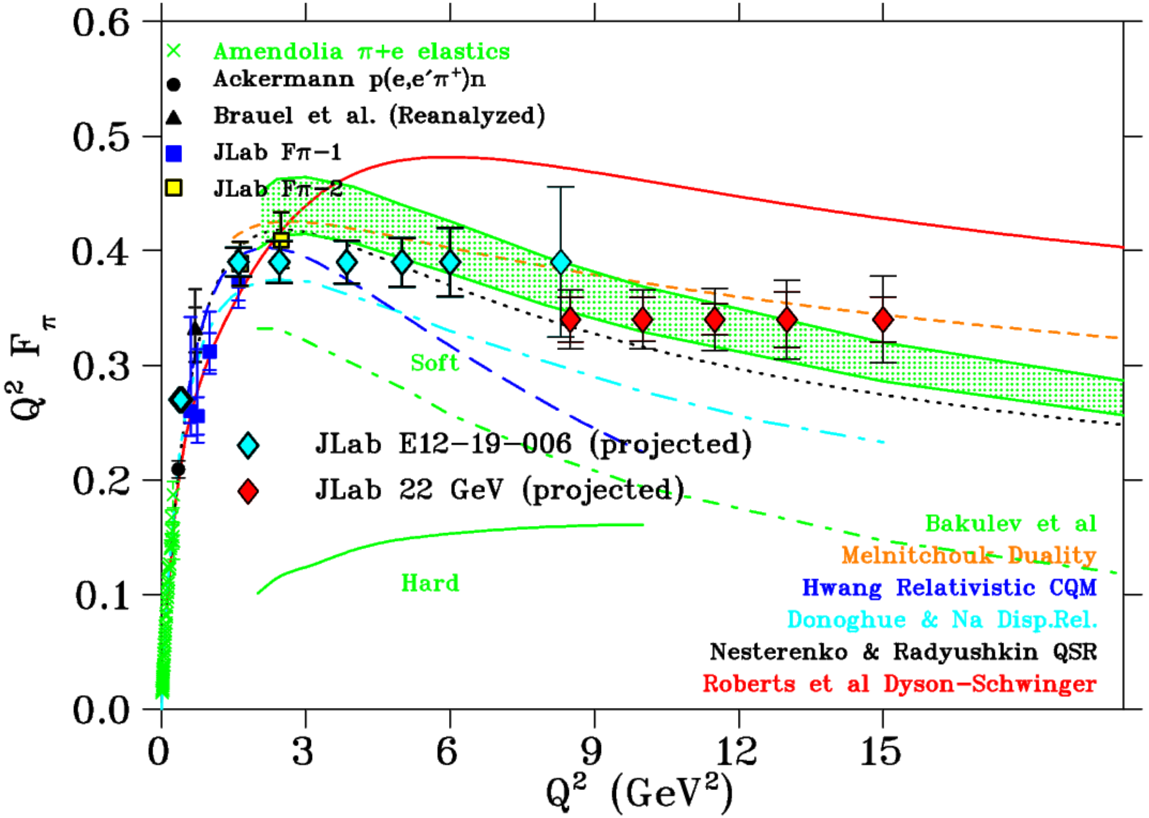


The **electromagnetic form factors** of **K** can be measured till **5 GeV^2** in the **12 GeV** upgrade of **JLab**

π and K Form Factor - JLab 22 GeV Upgrade

With a potential next **22-GeV** upgrade of the **JLab**, the **pion electromagnetic form factor** could be measured till **$Q^2 \sim 15$ GeV^2**.





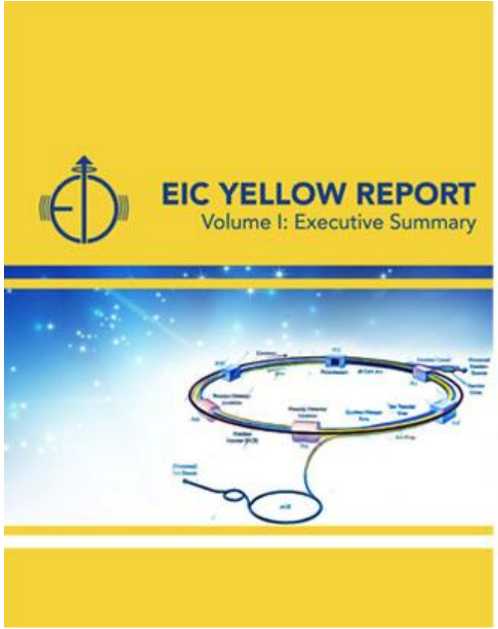
The **form factors** of **K** can be measured till **10 GeV^2** in the **22 GeV** upgrade of **JLab**

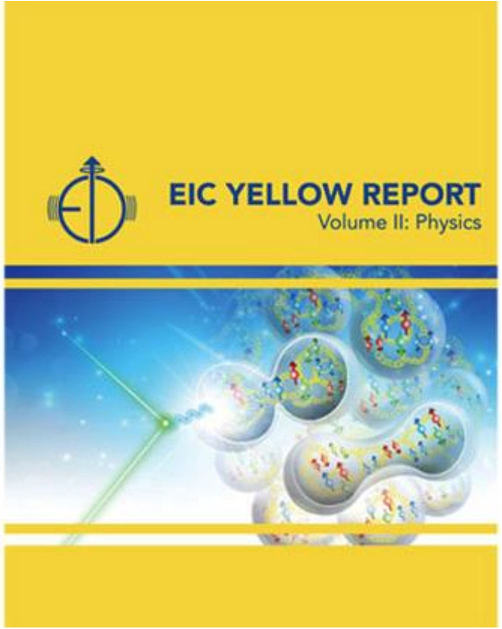
Courtesy Garth Huber

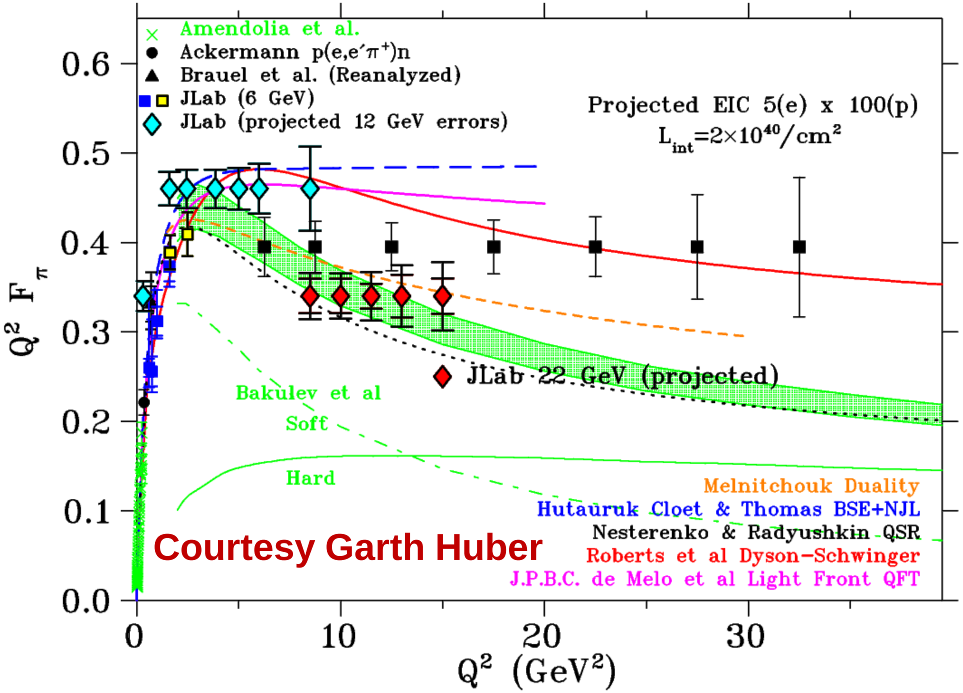
π and K Form Factor at Large Q^2 in EIC Era

Science Question: Can we get quantitative guidance on the **emergent pion mass** mechanism**?**
Key measurement: **Pion form factor** data for **Q^2: 10-40 GeV^2**.

Science Question: How much interference is between emergent and Higgs mass mechanism?
Key measurement: **Kaon form factor** data for **Q^2: 10-20 GeV^2**.

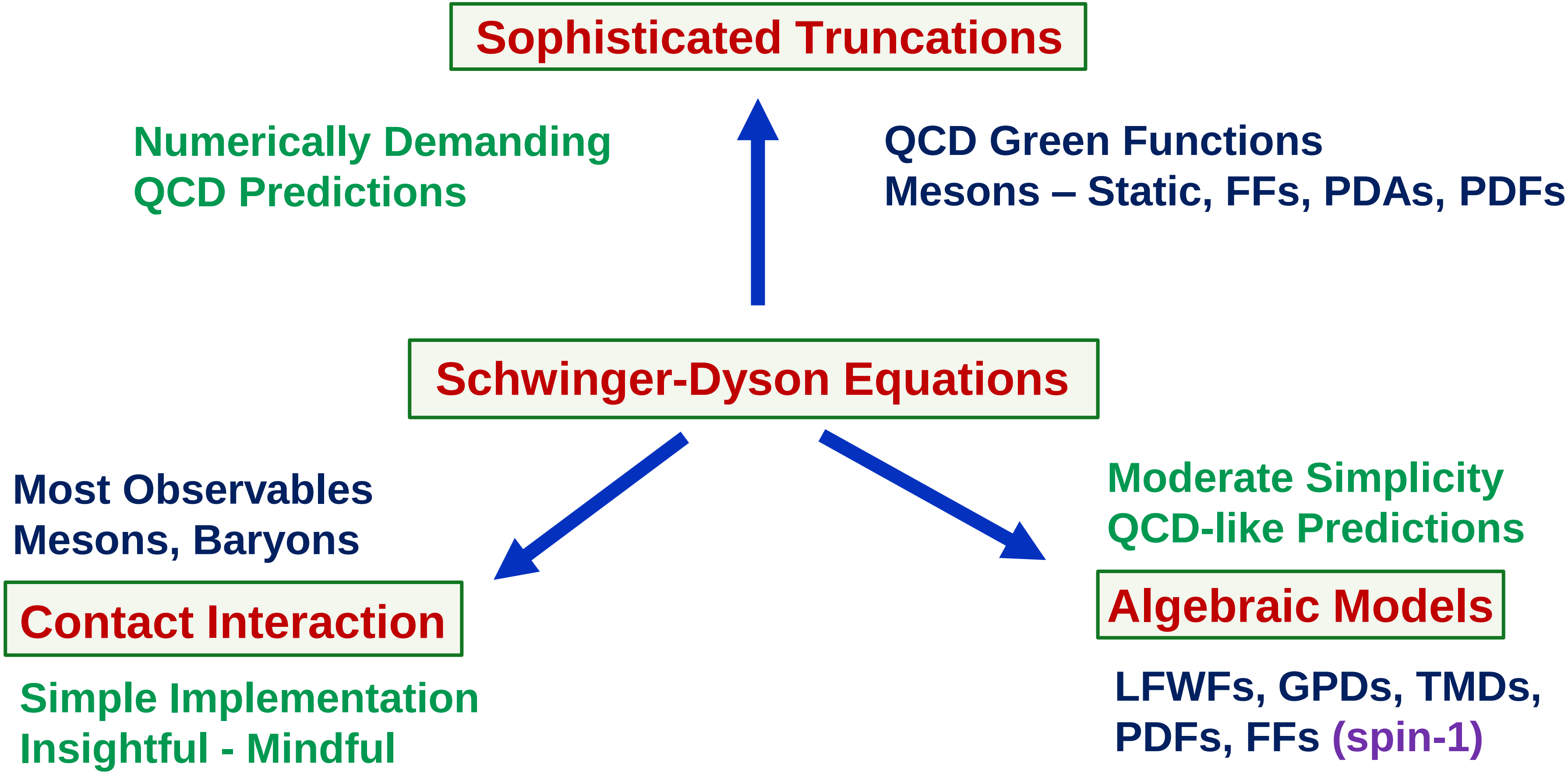

EIC YELLOW REPORT
Volume I: Executive Summary


EIC YELLOW REPORT
Volume II: Physics



Courtesy Garth Huber

Towards Algebraic Models



Sophisticated Truncations

Numerically Demanding
QCD Predictions

QCD Green Functions
Mesons – Static, FFs, PDAs, PDFs

Schwinger-Dyson Equations

Most Observables
Mesons, Baryons

Contact Interaction

Simple Implementation
Insightful - Mindful

Moderate Simplicity
QCD-like Predictions

Algebraic Models

LFWFs, GPDs, TMDs,
PDFs, FFs (spin-1)

The Algebraic Model (AM)

- It retains the **constant term** from original models, setting it to **M_q**.

- There is a **term linear** in **w** with the coefficient **$(M_h^2 - M_q^2)$**. For same **flavored quarks**, it ceases to contribute by construction.

- There is a **quadratic term w^2** with coefficient **m_M^2**. The condition

$$|M_{\bar{h}} - M_q| \leq m_{\mathrm{M}} \leq M_{\bar{h}} + M_q$$

- It guarantees the **positivity** of

$$\Lambda^2(w)$$

The quark propagator:

$$S_{q(\bar{h})}(k) = [-i\gamma \cdot k + M_{q(\bar{h})}]\Delta(k^2, M^2_{q(\bar{h})})$$

$$\Delta(s,t) = (s+t)^{-1}$$

Bethe-Salpeter Amplitude:

$$n_{\mathrm{M}}\Gamma_{\mathrm{M}}(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho_{\mathrm{M}}(w)[\hat{\Delta}(k_w^2, \Lambda_w^2)]^\nu$$

$$\hat{\Delta}(s,t) = t\Delta(s,t), \quad k_w = k + (w/2)P$$

$M_{q(\bar{h})}$ is constituent quark mass for a given flavor

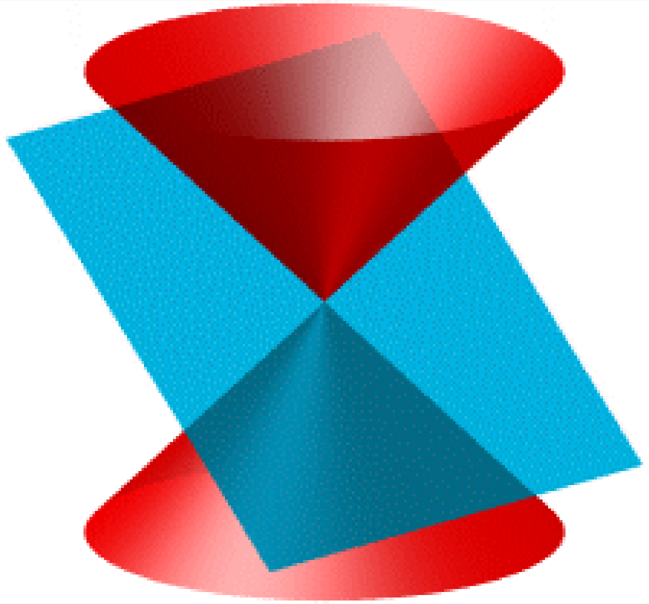
n_{M} is a normalization constant

$\rho_{\mathrm{M}}(w)$ is a spectral density

$$\Lambda^2(w) = M_q^2 - \frac{1}{4}(1-w^2)m_{\mathrm{M}}^2 + \frac{1}{2}(1-w)(M_{\bar{h}}^2 - M_q^2)$$

BSA - Light Front Wavefunction - GPDs

For a quark in pion/kaon **M**, the **leading twist** (2-particle) **light front wave function**, ψ_{M}, can be obtained via light front projection of the meson’s **BSWF**.

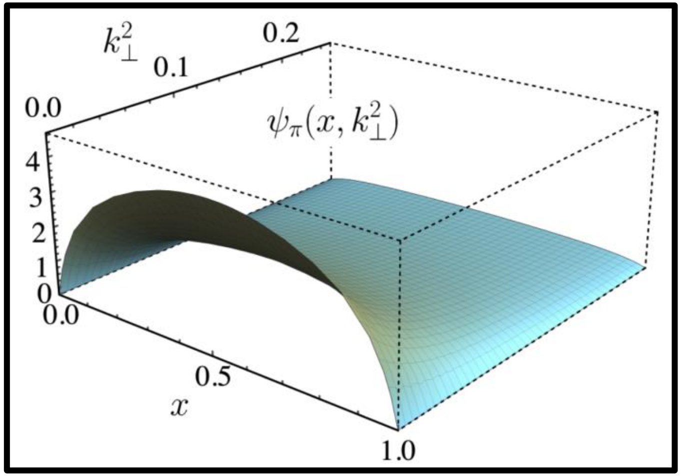


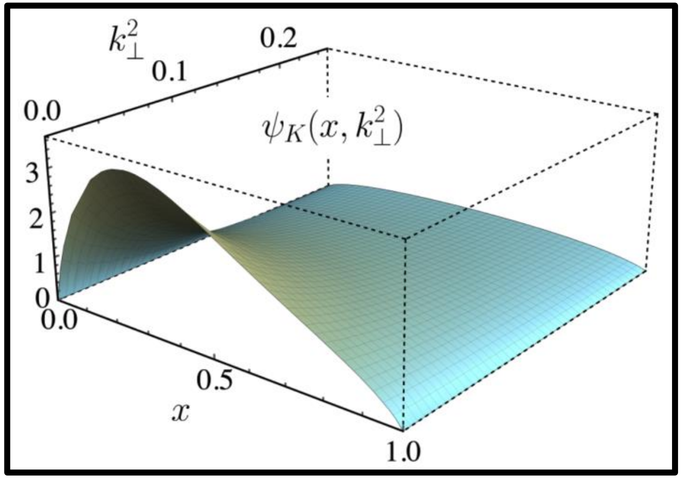
Bethe-Salpeter Wavefunction:

$$\chi_{\mathrm{M}}(k,P) = S_q(k+P/2)\Gamma_{\mathrm{M}}(k,P)S_{\bar{h}}(k-P/2)$$

Light Front Wavefunction:

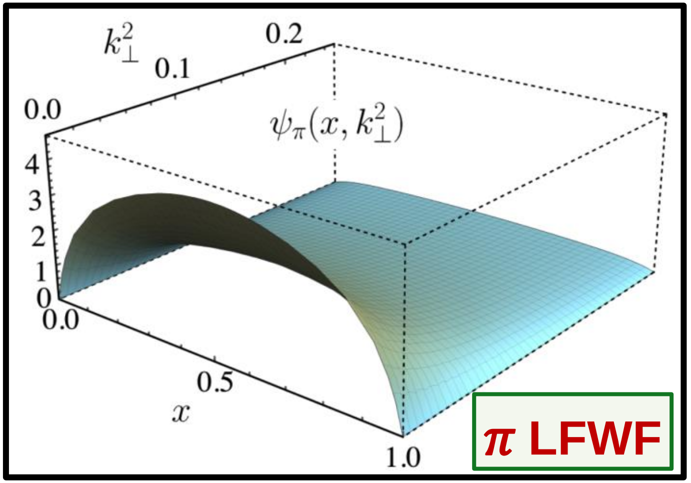
$$\psi^q_{\mathrm{M}}(x,k_\perp^2) = \mathrm{tr}\int_{dk_\parallel}\delta(n\cdot k - x\, n\cdot P)\gamma_5\gamma\cdot n\,\chi_{\mathrm{M}}(k-P/2,P)$$

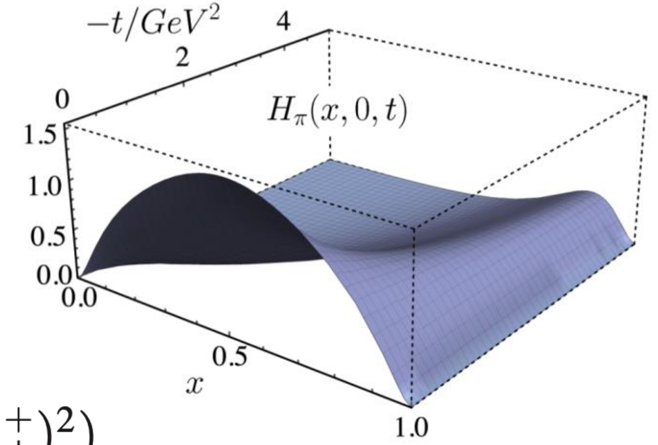




L. Albino, M. Higuera, K. Raya, AB, Phys. Rev. D 106 (2022) 3, 034003

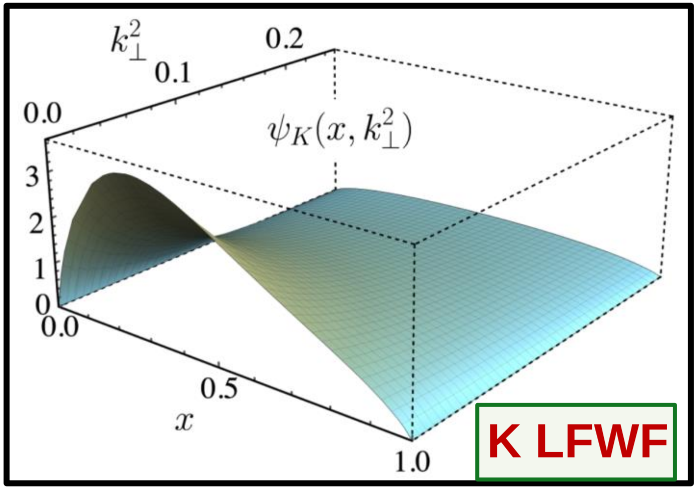
From the LFWFs to the GPDs





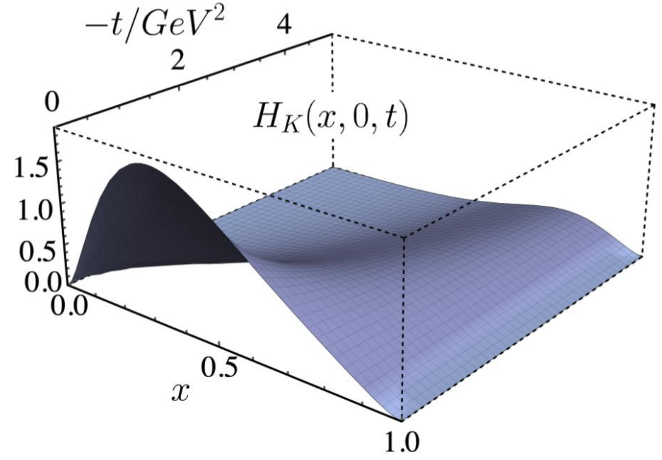
$$H^q_{\mathrm{M}}(x,\xi,t)$$

$$\int \frac{d^2k_\perp}{16\pi^3}\psi^{q*}_{\mathrm{M}}(x^-,(\mathbf{k}^-_\perp)^2)\psi^q_{\mathrm{M}}(x^+,(\mathbf{k}^+_\perp)^2)$$

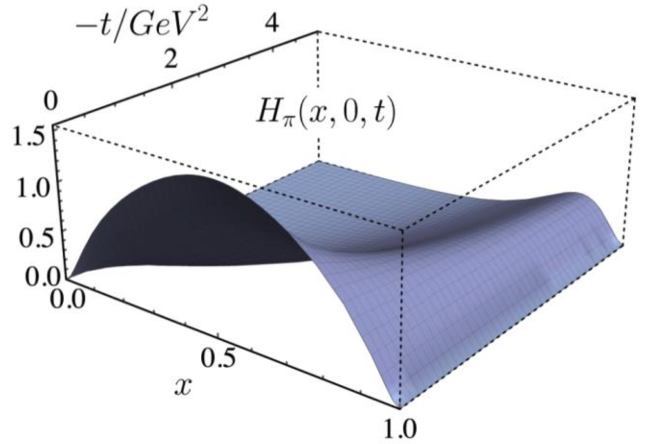


Overlap Representation of the GPDS

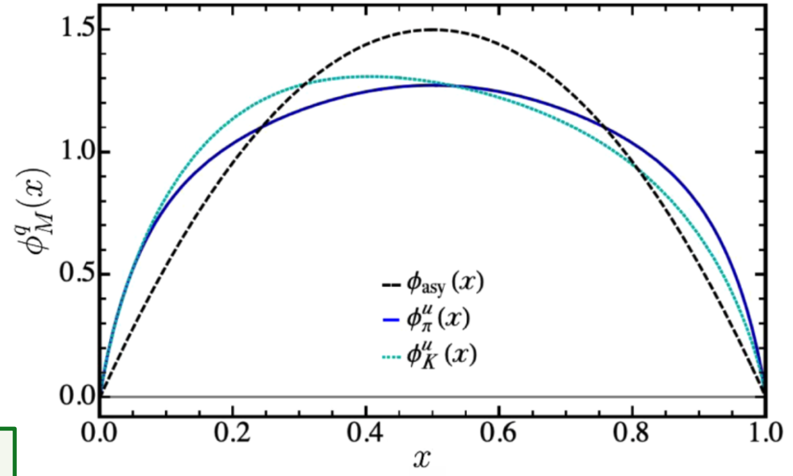
L. Albino, M. Higuera, K. Raya, AB
Phys. Rev. D 106 (2022) 3, 034003



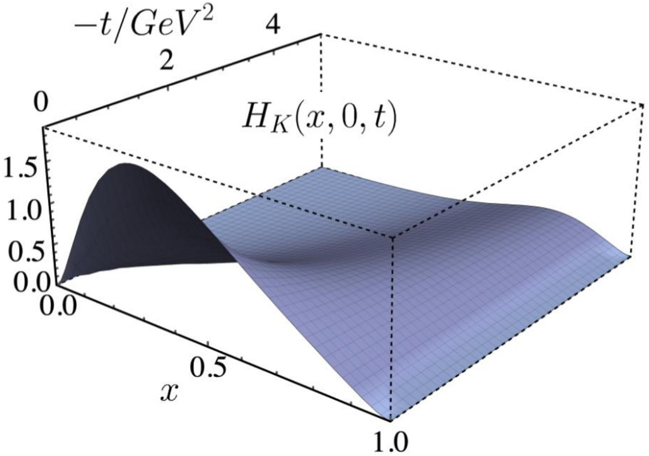
The GPDs and the PDFs



$$q_{\mathrm{M}}(x) \equiv H^q_{\mathrm{M}}(x,0,0)$$

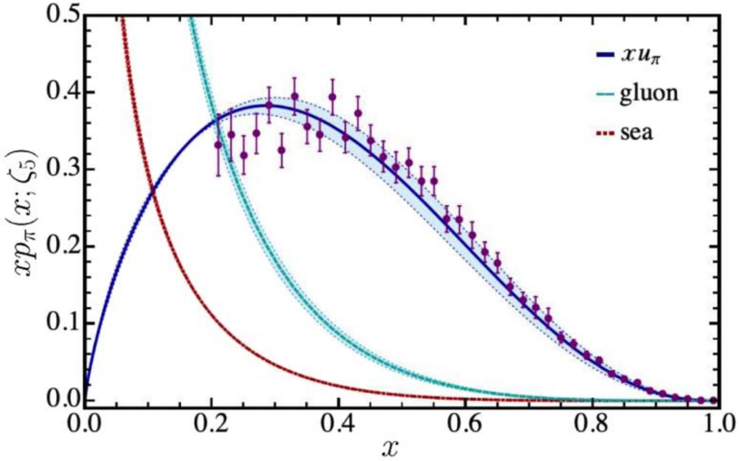


DGLAP Evolution Equations



L. Albino, M. Higuera, K. Raya, AB
Phys. Rev. D 106 (2022) 3, 034003

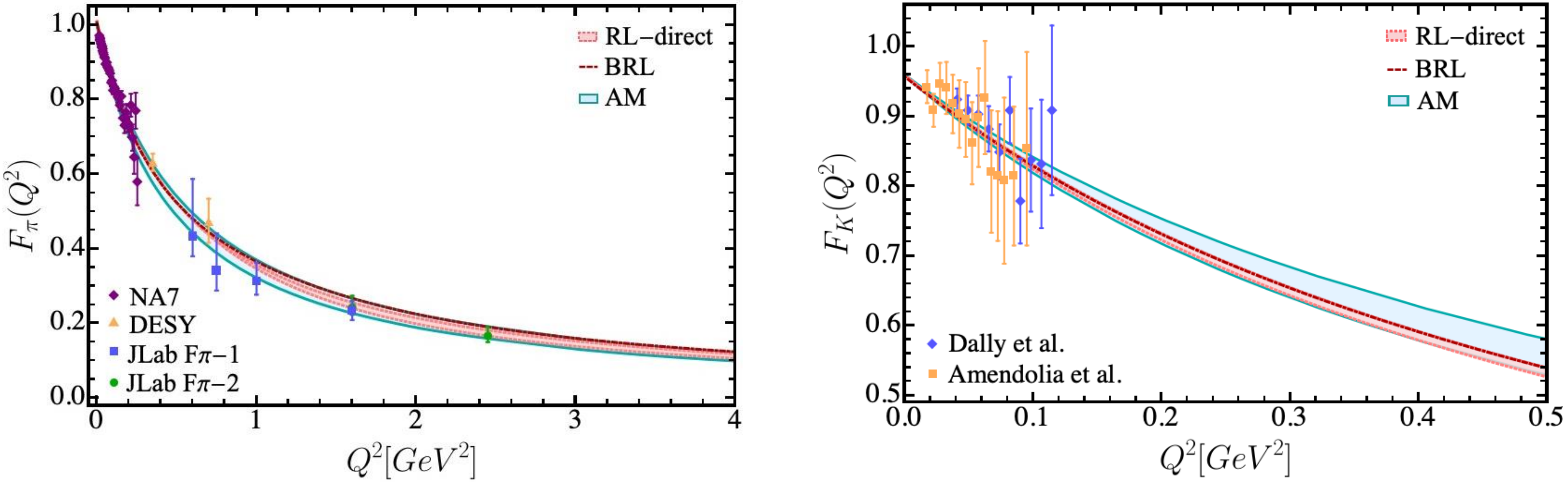
M. Aicher, A. Schäfer, W. Vogelsang,
Phys. Rev. Lett.105, 252003 (2010)



Completing the Cycle – Back to Form Factors

The **electromagnetic form factors** using our **algebraic model** can be obtained either through the knowledge of the **GPDs** or the direct evaluation of the **triangle diagram**.

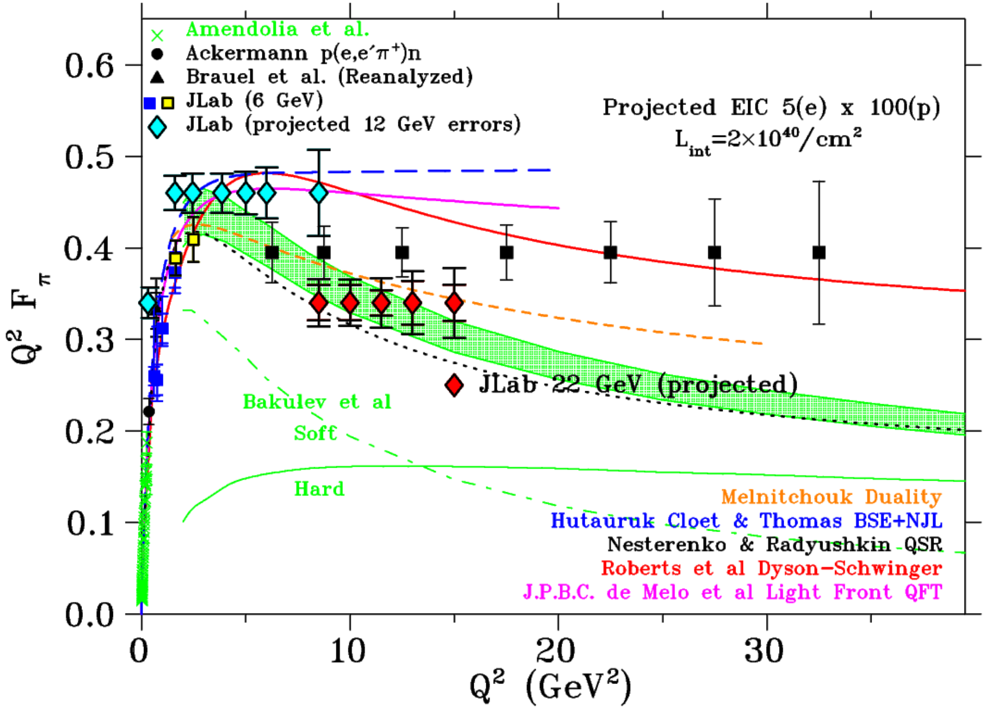
Such an exercise provides stringent constraints on the efficacy of the **algebraic model** we have constructed by direct comparison with the refined calculation of these **form factors.**

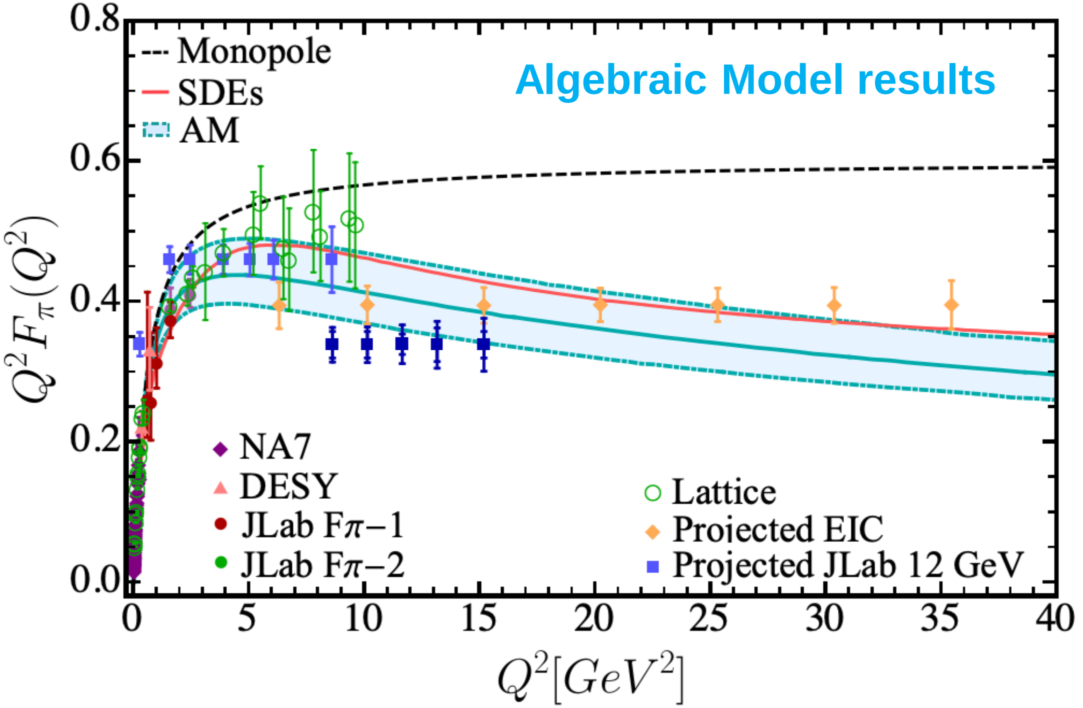


I. M. Higuera, R. J. Hernández, K. Raya, AB, Phys. Rev. D 110, 034013, (2024)

Completing the Cycle – Back to Form Factors

We can extend this analysis of the **Algebraic Model** to compute the **pion electromagnetic form factors** to larger **Q^2** range: **0-40 GeV^2** which would likely cover the photon virtualities accessible to the **JLab12**, **JLab22** and **EIC programs**:

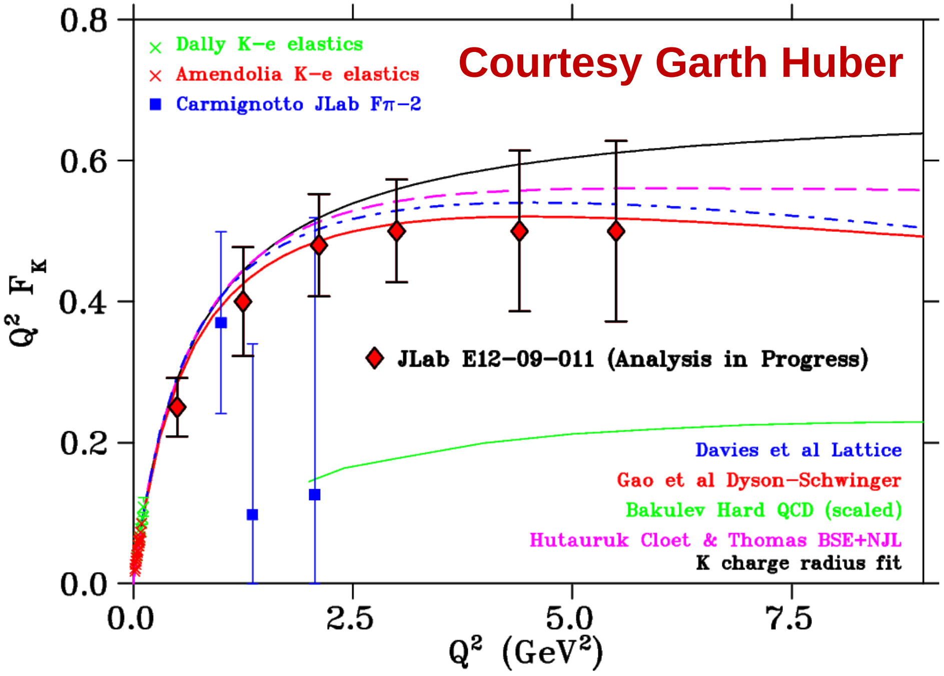




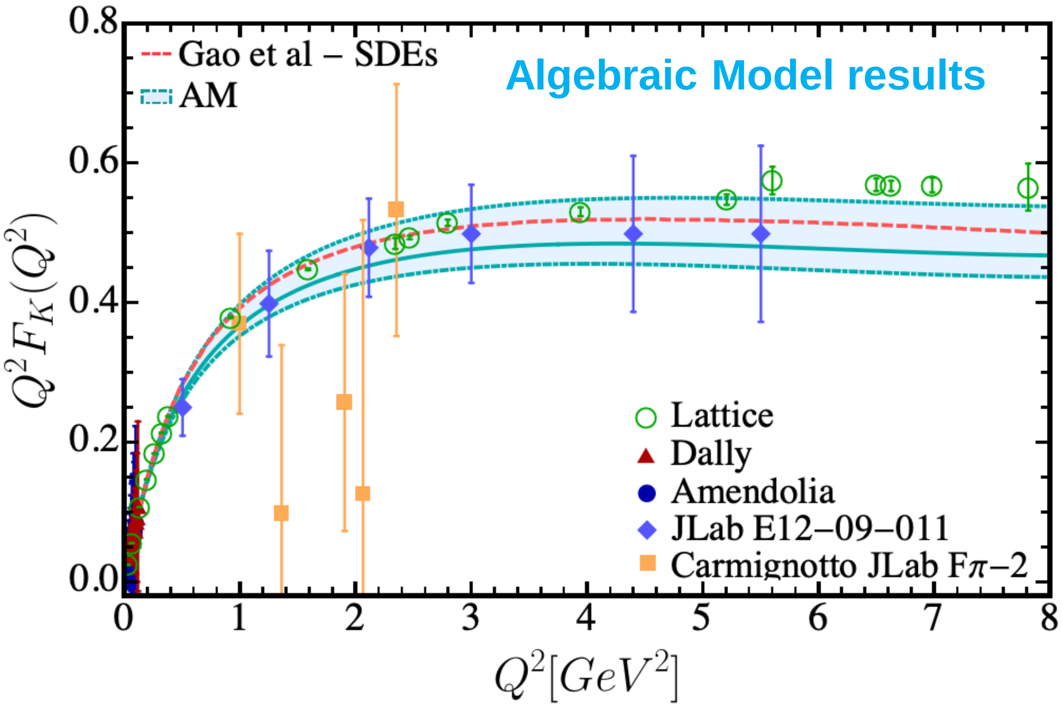
I. M. Higuera, R. J. Hernández, K. Raya, AB, Phys. Rev. D 110, 034013, (2024)

Completing the Cycle – Back to Form Factors

There is an analysis underway of the **kaon electromagnetic form factor** till **5.5 GeV^2** of the data obtained in **JLab E12-09-011** experiment in its **12 GeV upgrade**.



Courtesy Garth Huber



Algebraic Model results

I. M. Higuera, R. J. Hernández, K. Raya, AB, Phys. Rev. D 110, 034013, (2024)

Summary and Outlook

- The interplay of **QCD akin** truncations of **Schwinger-Dyson equations** and **algebraic model** based upon these studies shed important light on the **internal structure** of **pion** and **kaon**.

- **QCD akin** refined computation of **pion** and **kaon electromagnetic form factors** at low and intermediate virtualities of the probing photon in electroproduction processes:

A. Miramontes AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013
L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802

- Results for the **pion electromagnetic form factor** at large photon virtualities accessible to the potential **22GeV upgrade** of the **JLab** and **EIC** are also available:

L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802
J. Arrington, et al. (Feb 23, 2021, J.Phys. G 48 (2021) 7, 075106

- More recently, **pion** and **kaon form factors** have been computed in the the **time-like region**

A.S. Miramontes, H. Sanchis Alepuz, R. Alkofer, Phys. Rev. D 103 (2021) 11, 116006
A.S. Miramontes, AB, Phys. Rev. D 107 (2023) 1, 014016

Summary and Outlook

- Carefully constructed **Algebraic Models** can enable computation of the **GPDs**, **PDFs** and **EFF** with relative ease which is reminiscent of a **contact interaction** while mimicking the reliability of **QCD akin** refined truncations of **Schwinger-Dyson equations**.

L. Albino, M. Higuera, K. Raya, AB Phys. Rev. D 106 (2022) 3, 034003
I. M. Higuera, R. J. Hernández, K. Raya, AB, Phys. Rev. D 110, 034013, (2024)
M.A. Sultan, J. Kang, AB, L. Chang, e-Print: 2409.09595 [hep-ph]

- Despite these encouraging results and synergy with experimental endeavors at **JLab** and **EIC**, further improvements and extensions in the **continuum QCD approach** are desirable.

- Deeper research into the theoretical foundations of the truncations involved at the level of the **Green functions** of the fundamental degrees of freedom, i.e., **quarks**, **gluons**, as well as **quark-gluon** and **gluon-gluon** interactions continues vigorously.

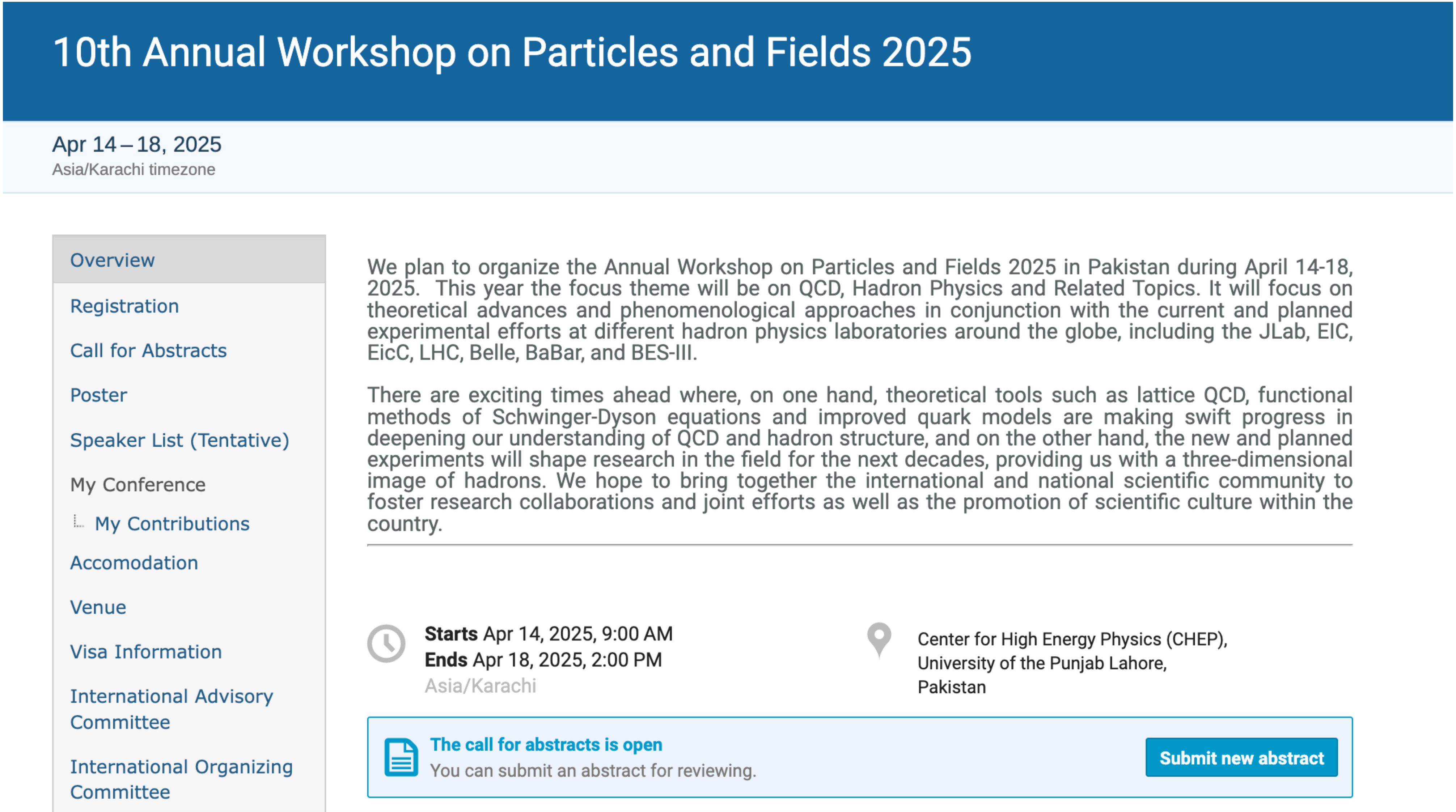
- **Schwinger-Dyson equations** have also been of substantial success in the studies of **baryons** such as the **transition form factors** of **nucleon** to its **excited states** which is a hallmark of **CLAS**, **CLAS12** and **CLAS22** programs at **JLab** and hold the promise to offer a reliable tool for the future **JLab** and **EIC era** research into the heart of **hadronic matter**.

Thank you for your attention

U.S. DEPARTMENT OF ENERGY

22 GEV (LNF-INFN 2024)

The five-day long workshop “Science at the luminosity frontier: Jefferson Lab at 22 GEV” will be held in Frascati, Italy, on December 9 - 13, 2024 (Monday 9:00am – Friday 1:30pm)



10th Annual Workshop on Particles and Fields 2025

Apr 14 – 18, 2025
Asia/Karachi timezone

We plan to organize the Annual Workshop on Particles and Fields 2025 in Pakistan during April 14-18, 2025. This year the focus theme will be on QCD, Hadron Physics and Related Topics. It will focus on theoretical advances and phenomenological approaches in conjunction with the current and planned experimental efforts at different hadron physics laboratories around the globe, including the JLab, EIC, EicC, LHC, Belle, BaBar, and BES-III.

There are exciting times ahead where, on one hand, theoretical tools such as lattice QCD, functional methods of Schwinger-Dyson equations and improved quark models are making swift progress in deepening our understanding of QCD and hadron structure, and on the other hand, the new and planned experiments will shape research in the field for the next decades, providing us with a three-dimensional image of hadrons. We hope to bring together the international and national scientific community to foster research collaborations and joint efforts as well as the promotion of scientific culture within the country.

Starts Apr 14, 2025, 9:00 AM
Ends Apr 18, 2025, 2:00 PM
Asia/Karachi

Center for High Energy Physics (CHEP),
University of the Punjab Lahore,
Pakistan

The call for abstracts is open
You can submit an abstract for reviewing.