Transverse quark distributions in hadrons

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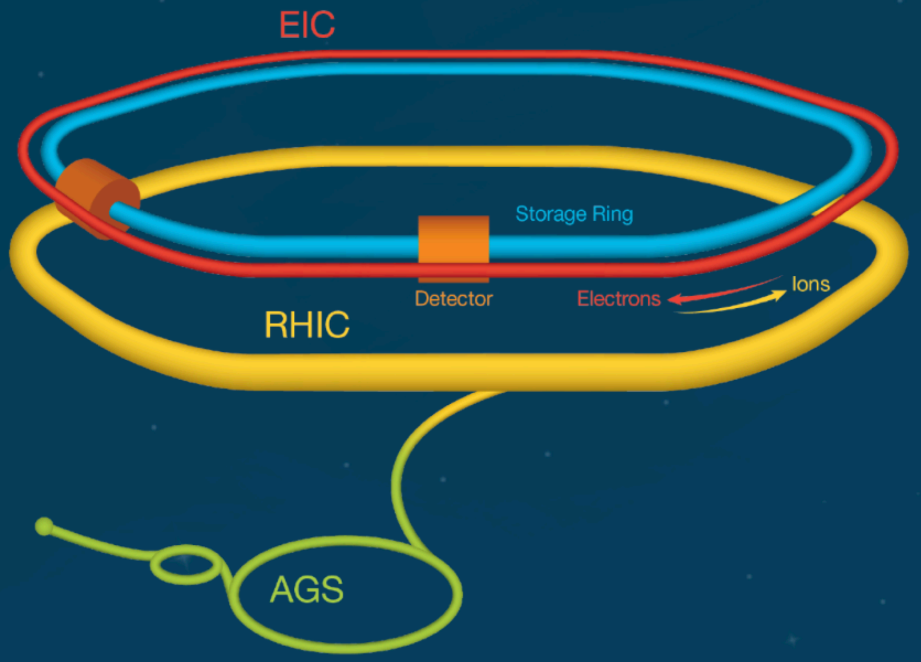
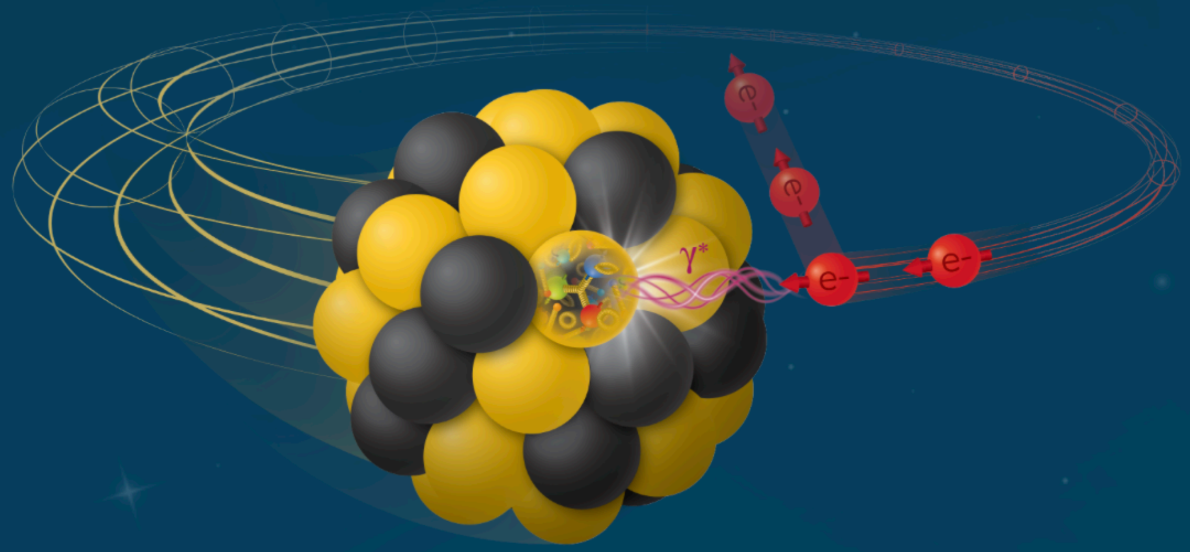
Before coming to the meat of the talk, let's find some motivation with the recently approved Electron-Ion Collider.

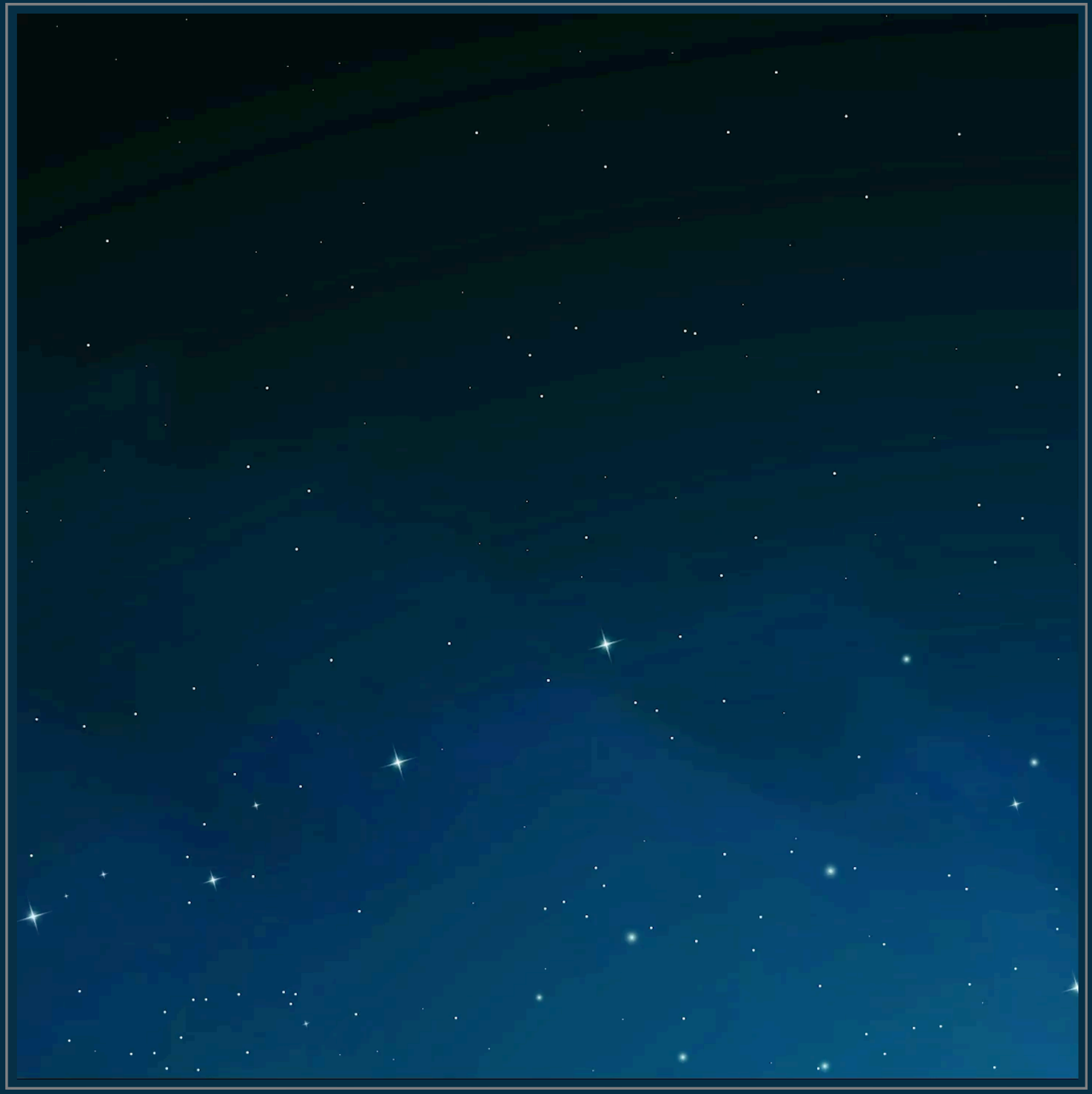
Why an Electron-Ion Collider ?

The Electron-Ion Collider (EIC) will be a 2.4-mile-circumference world-class particle collider, the first of its kind in the world. It will steer beams of high-energy polarized electrons into collisions with polarized protons and atomic nuclei to produce precision 3D snapshots of those particles' internal structure. Experiments at the EIC will help scientists unlock the secrets of the strongest force in nature and explore how tiny particles called quarks and gluons build up the mass, spin, and other properties of all visible matter.





The Science at the EIC

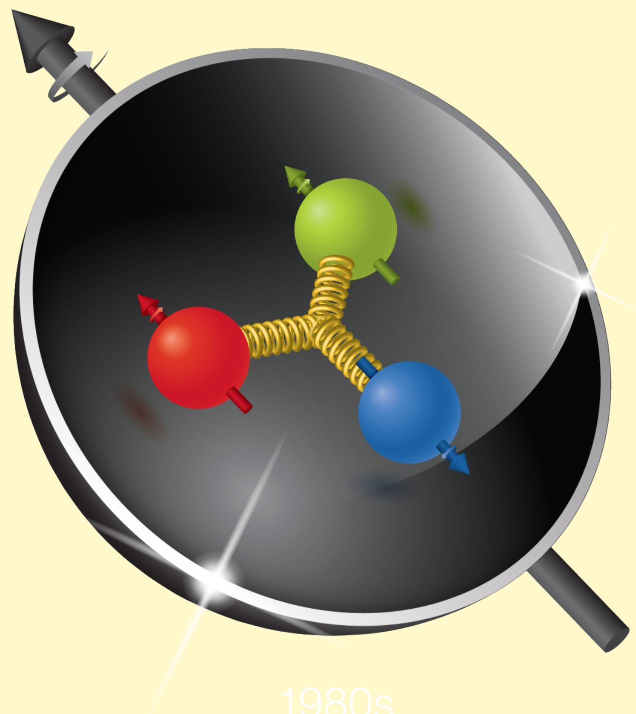


Courtesy of https://www.bnl.gov/eic

All of today's electronics and much of our economy depend on what scientists learned last century about atoms: the nucleus, its orbiting electrons, and the electromagnetic force. But there's a whole lot more going on inside the atomic nucleus and within its protons and neutrons.

The Electron-Ion Collider (EIC) will explore that inner microcosm. It will bring high-energy electrons into head-on collisions with high-energy protons or nuclei to reveal how the inner building blocks build up the properties and structure of all visible matter in the universe.

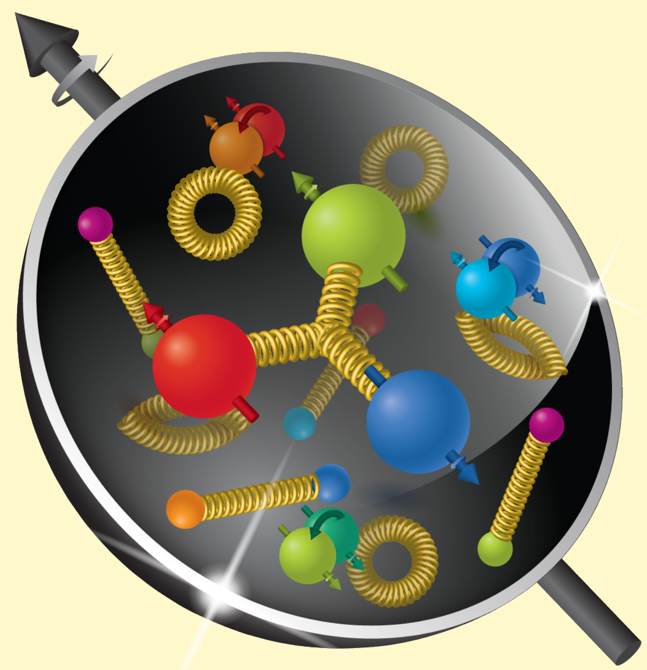
The origin of mass



1970s to 1980s

The simplest view of a proton shows only three quarks held together by gluons.

The origin of mass



1990s to 2000s

1990s to 2000s

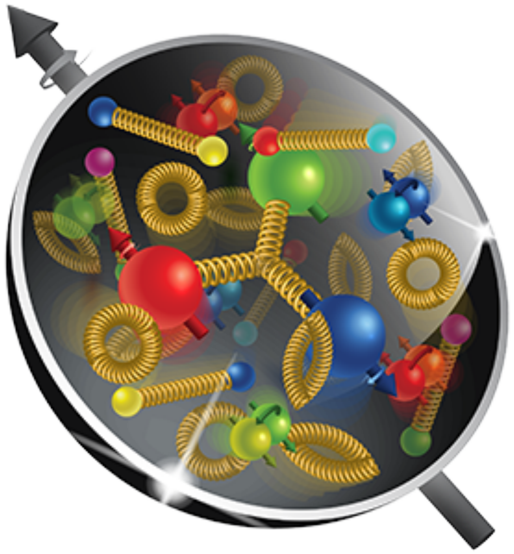
Experiments have revealed that the internal structure of a proton can be much more complicated.

The origin of mass

Quarks have very little mass and gluons have none.

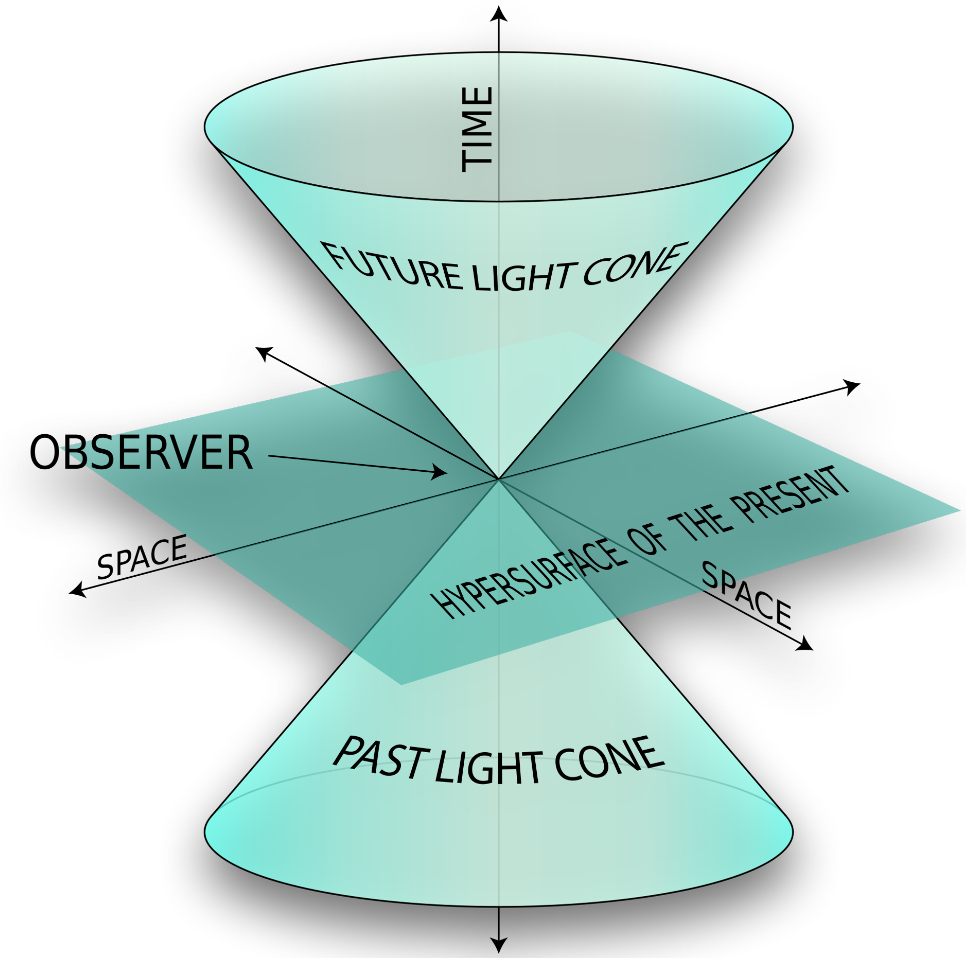
If you could weigh the mass of all the quarks and gluons outside of a proton, they'd account for only 1% of the total mass of the proton.

What creates the other 99%?



The 3d landscape of hadrons

- The Electron-Ion Collider will take three-dimensional precision snapshots of the internal structure of protons and atomic nuclei.
- By taking images at a range of energies, the EIC will reveal features of the “ocean” of gluons and the “sea” of quark-antiquark pairs that form when gluons interact.
- This will allow to map out the particles’ distribution and movement within protons and nuclei, similar to the way medical imaging technologies construct 3d dynamic images of the brain.
- May help reveal how the energy of the massless gluons is transformed through $E = mc^2$ to generate most of the mass of the visible universe.
- Simplest distributions are 1d: parton distribution amplitudes/functions.

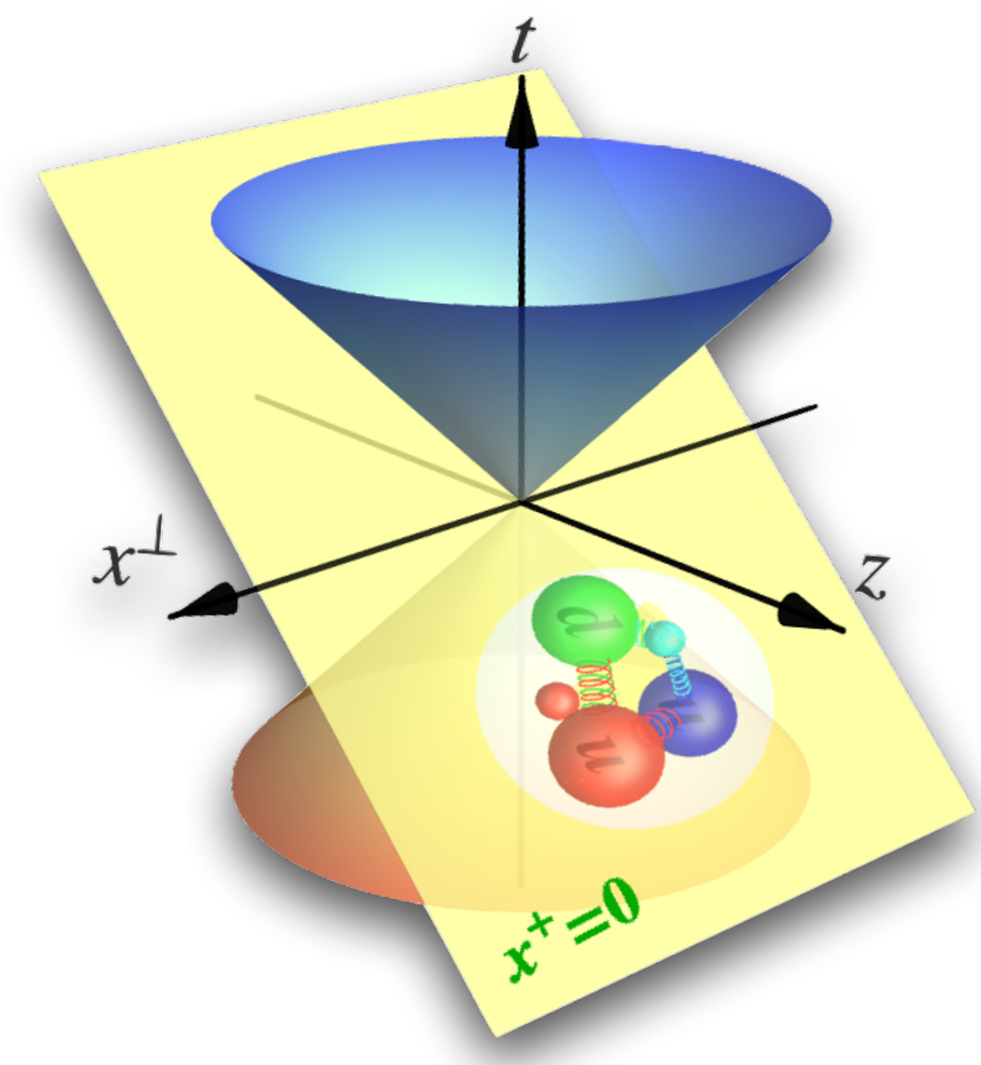


Light front mixes time and space

$$x = (x^+, \boldsymbol{x}_\perp, x^-)$$
$$x^\pm = \frac{1}{\sqrt{2}}(x_0 \pm x_3)$$
$$\boldsymbol{x}_\perp = (x_1, x_2)$$

P. A. M. Dirac. ``Forms of Relativistic Dynamics'' (1949)

Light-Cone Distribution Amplitudes



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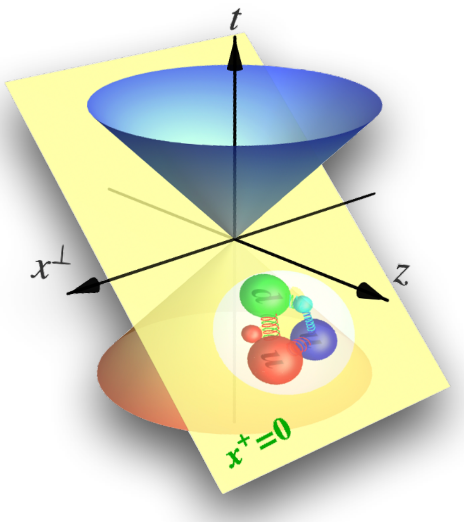
Light-Front Distribution Amplitudes



Light-Cone Distribution Amplitudes

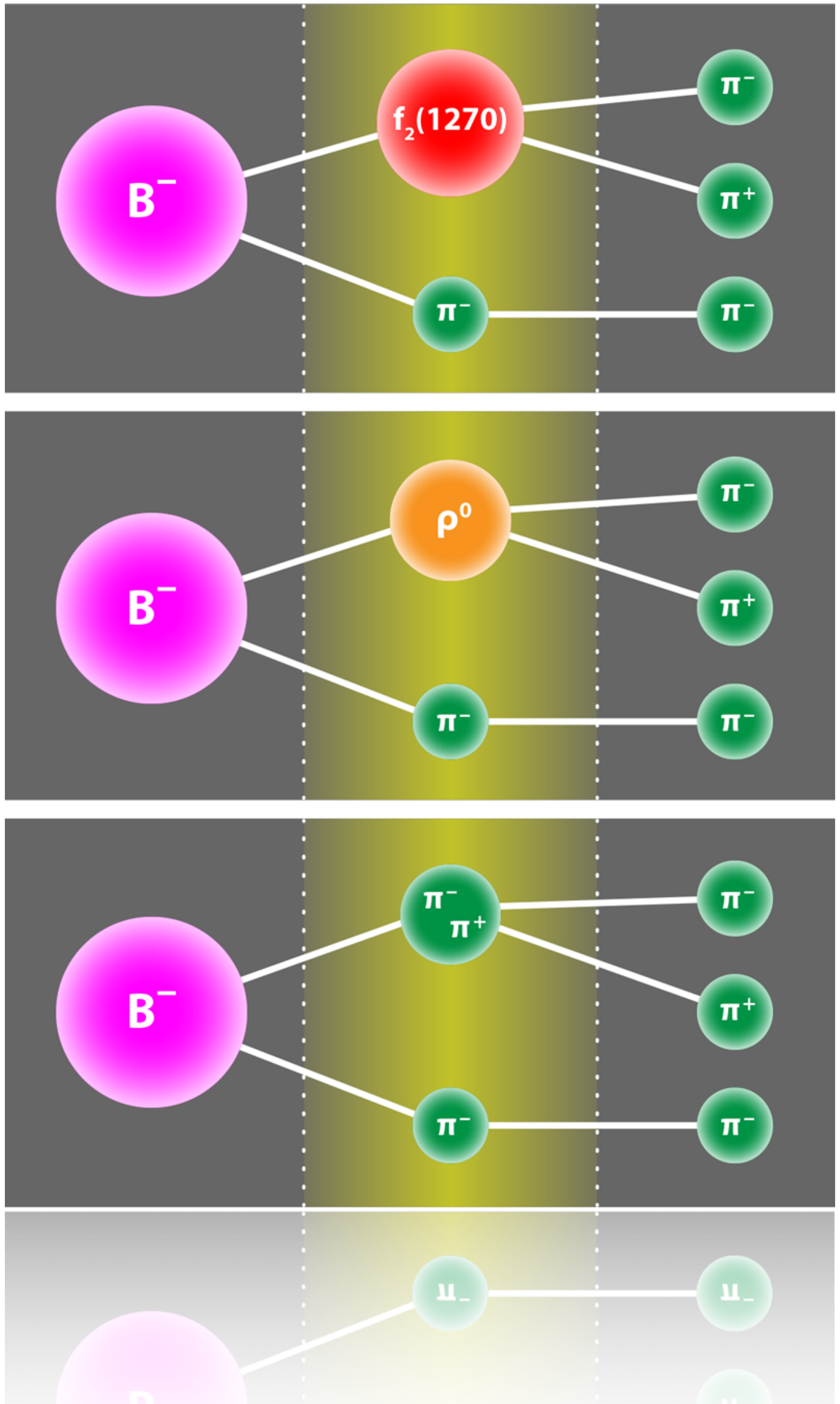
- Hadronic Light-Cone Distribution Amplitudes (LCDA) were introduced in the context of pQCD calculations of hard exclusive reactions [S. Brodsky and G. Lepage (1980)].

- The LCDAs are scale-dependent nonperturbative functions that can be interpreted as quantum-mechanical amplitudes.

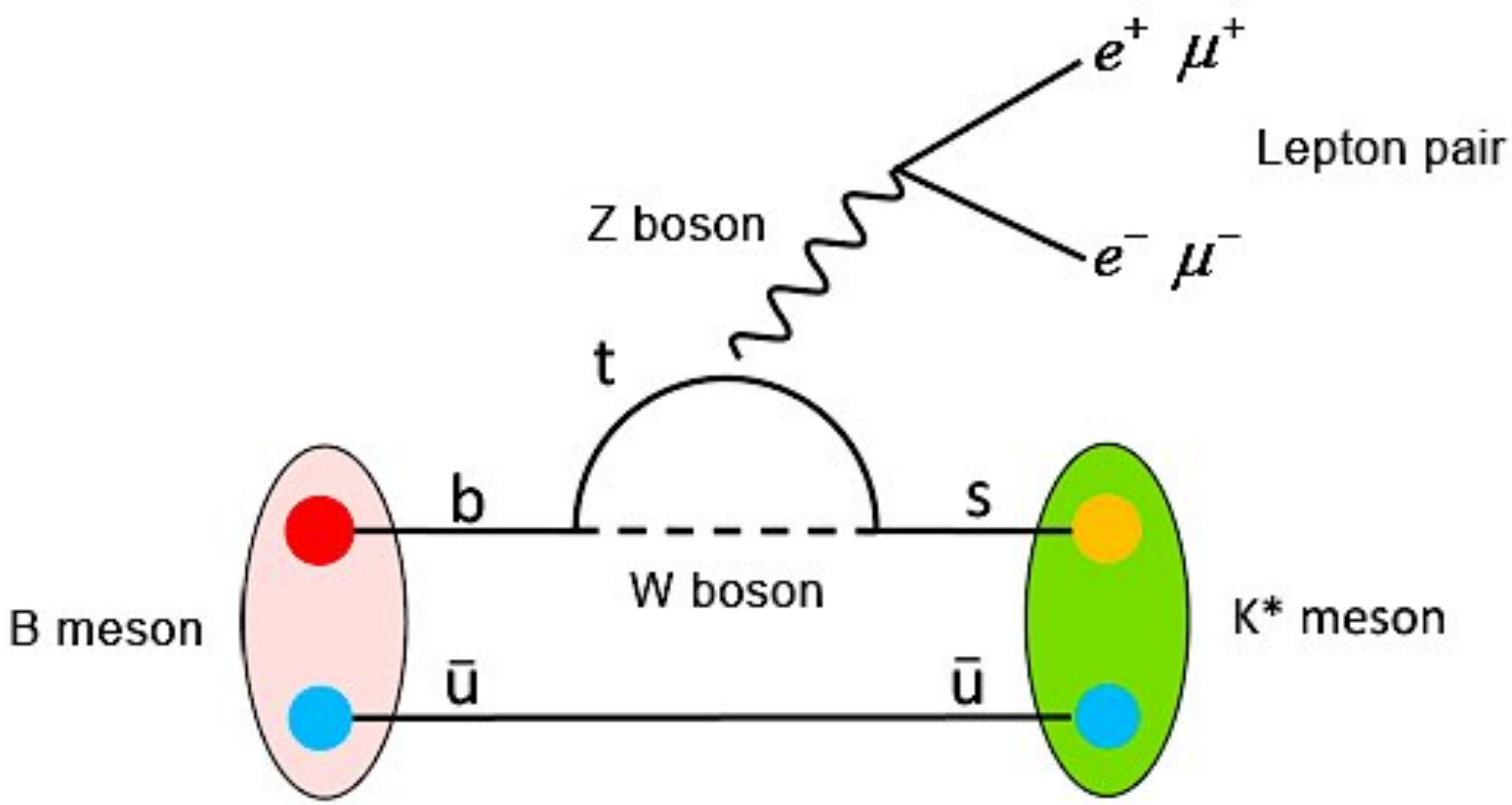
- For instance, the LCDA of a *B*-meson contains information about the probability of finding the light quark with a certain momentum fraction inside the heavy meson.

- In the light-front frame $x = k^{+}/P^{+}$ is the light-front momentum fraction of a fast-moving quark (Bjorken *x*),

Weak decays of heavy mesons



Weak decays of heavy mesons



Light-Front Distribution Amplitudes

Factorization of matrix elements in B-meson decays leads to convolution integrals:

$$\langle \pi^+\pi^- | (\bar{u}b)_{\rm V-A}(\bar{d}u)_{\rm V-A} | \bar{B}_d \rangle \implies \int_0^1 d\xi du dv \, \Phi_B(\xi)\Phi_\pi(u)\Phi_\pi(v) \, T(\xi, u, v; m_b)$$

LCDA of B and π

Hard-scattering kernel

$$\varphi_\pi(x;\mu) = \varphi_\pi^{\rm asy}(x)\left[1 + \sum_{j=2,4,\dots}^{\infty} a_j^{3/2}(\mu) C_j^{(3/2)}(2x-1)\right] \; ; \quad \varphi_\pi^{\rm asy}(x) = 6x(1-x)$$

S. Brodsky and G. Lepage (1980)

However, at accelerator scales: $\varphi_\pi(x) \neq \varphi^{\rm asy}(x) = 6x(1-x)$

Light-Front Distribution Amplitudes

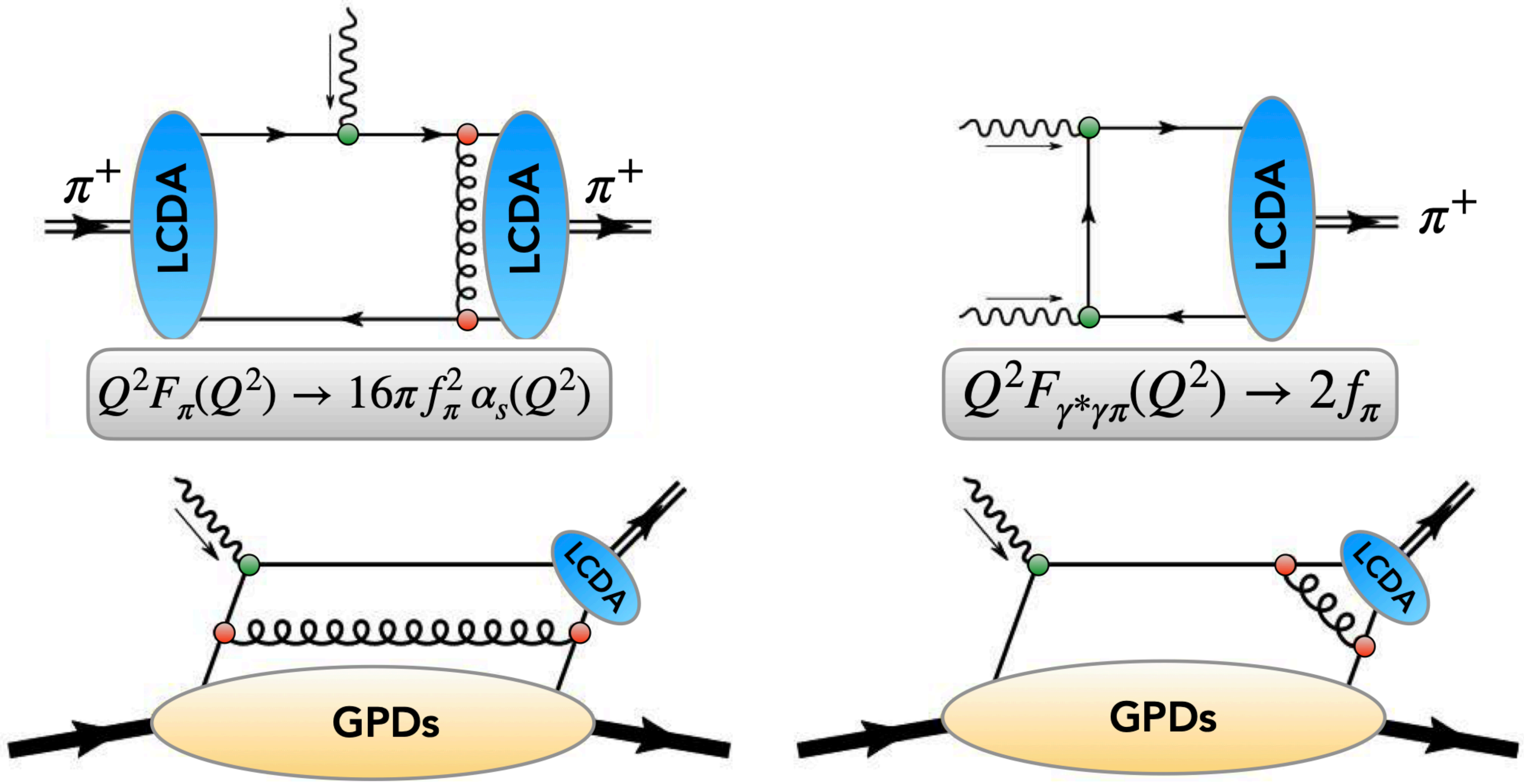
Factorization of matrix elements in B-meson decays leads to convolution integrals:

$$\langle \pi^+\pi^- \left| (\bar{u}b)_{\mathrm{V-A}}(\bar{d}u)_{\mathrm{V-A}} \right| \bar{B}_d \rangle \implies \int_0^1 d\xi du dv\, \Phi_B(\xi)\Phi_\pi(u)\Phi_\pi(v)\, T\,(\xi, u, v; m_b)$$

LCDA of B and π — Hard-scattering kernel

- LCDA were poorly known for light mesons. In recent years improved determinations of the first two moments of the pion and kaon by the RQCD Collaboration, Bali et al. (2019).
- B-meson LCDA is still a big challenge for lattice QCD due to disparate energy scales: $\Lambda_{\mathrm{QCD}} \ll m_H \ll P_Z$
- Current progress made with Large Momentum Effective Theory, X. Ji, PRL (2013).

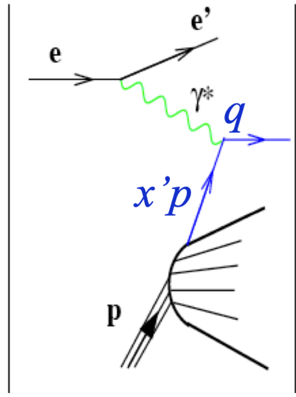
Hard exclusive scattering processes



DIS & Parton Distribution Functions

- Assumes fast moving hadron appears as a jet of partons moving in same direction and sharing its total momentum.
- DIS cross section is an incoherent sum of elastic scattering cross sections off individual partons.
- Parton model should work perfectly for $Q^2 \to \infty$ where coupling constant vanishes.

$$d\sigma \equiv \sum_i \int dx_i q_2^2 \left| \quad \right|^2$$



$$\mathrm{d}\sigma = \sum_i \mathrm{d}\hat{\sigma}(\hat{s}, \hat{t}, \hat{u})\ f_i(x)\mathrm{d}x$$

Define parton momentum distribution: $f_i(x) \equiv \dfrac{dP_i}{dx}\,, \quad \text{where} \quad \sum_i \int_0^1 dx\, f_i(x) = 1$

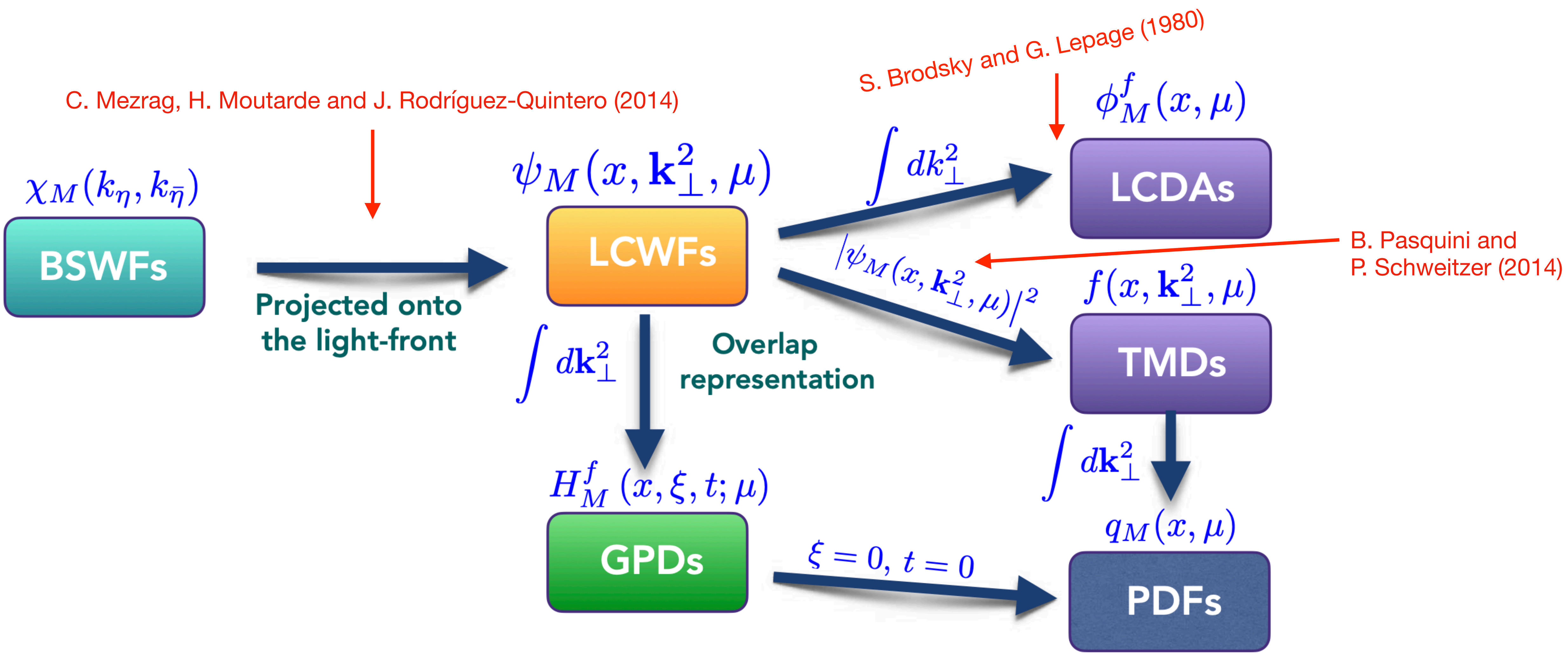
This defines the probability to find a parton with light-front momentum fraction $x = \dfrac{k^+}{P^+}$ of the hadron. PDF is related to structure functions:

$$F_2(x) = \sum_i q_i^2 x f_i(x) \qquad F_1(x) = \frac{1}{2x} F_2(x) \qquad \text{(Callan-Gross relation)}$$

What if we could obtain all kind of distribution functions from one unified calculation ?

Light-Front Wave Functions

With a particular projection of the Bethe-Salpeter wave functions we arrive the light-front wave functions, a more general object to describe probability amplitudes.



Light-Front Wave Functions

- M. Burkardt, X.-D. Ji, and F. Yuan (2002) showed that for pseudoscalar mesons there are two independent light front wave functions for the leading Fock state, with $l_z = 0$ and $l_z = 1$.
- Two-particle Fock-state configuration is given by: $|M\rangle = |M\rangle_{l_z=0} + |M\rangle_{|l_z|=1}$

$$|M\rangle_{l_z=0} = i \int \frac{d^2\boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_0\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right) - b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right)\right] |0\rangle$$

$$|M\rangle_{|l_z|=1} = i \int \frac{d^2\boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_1\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[k_T^- b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right) + k_T^+ b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right)\right] |0\rangle$$

Light-Front Wave Functions

- M. Burkardt, X.-D. Ji, and F. Yuan (2002) showed that for pseudoscalar mesons there are two independent light front wave functions for the leading Fock state, with $l_z = 0$ and $l_z = 1$.
- Two-particle Fock-state configuration is given by: $|M\rangle = |M\rangle_{l_z=0} + |M\rangle_{|l_z|=1}$

LFWF can be obtained from the Bethe-Salpeter wave function via light-front projections:

$$\psi_0\left(x, \mathbf{k}_\perp^2\right) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[\gamma^+ \gamma_5 \chi(k, P)\right]$$

$$\psi_1\left(x, \mathbf{k}_\perp^2\right) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[i\sigma_{+i} k_T^i \gamma_5 \chi(k, P)\right]$$

C. Mezrag, H. Moutarde, and J. Rodriguez-Quintero, Few Body Syst. 57 (2016)
C. Shi and I. C. Cloët, Phys. Rev. Lett. 122, 082301 (2019)

$$\left(\Box_x + m^2\right) G(x,y) = -\delta(x-y)$$

Green functions and functional approaches to QCD

The QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

The key to complexity in QCD lies the gluon field strength tensor.

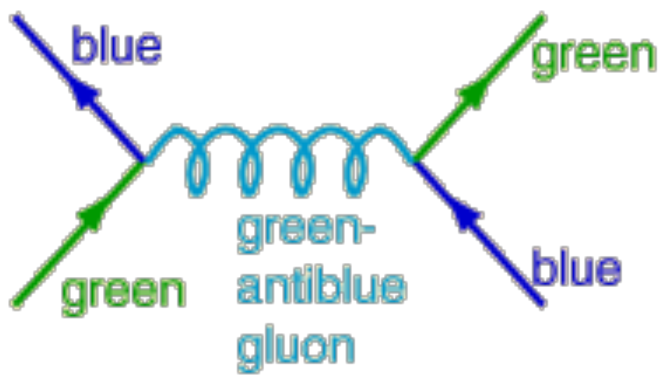
$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf^{abc}G^b_\mu G^c_\nu$$

It generates self-interactions with far-reaching consequences for hadron phenomenology.

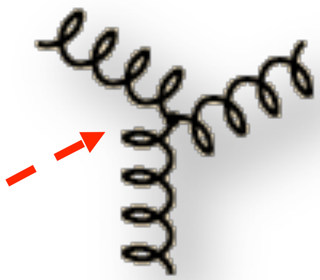
This complexity also affects the bare quark-gluon vertex in a nonperturbative manner !

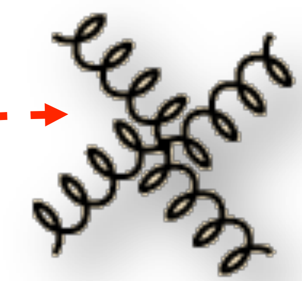
Relativistic Quantum Gauge Field Theory:

- Interactions mediated by vector boson exchange
- Vector bosons are perturbatively-massless

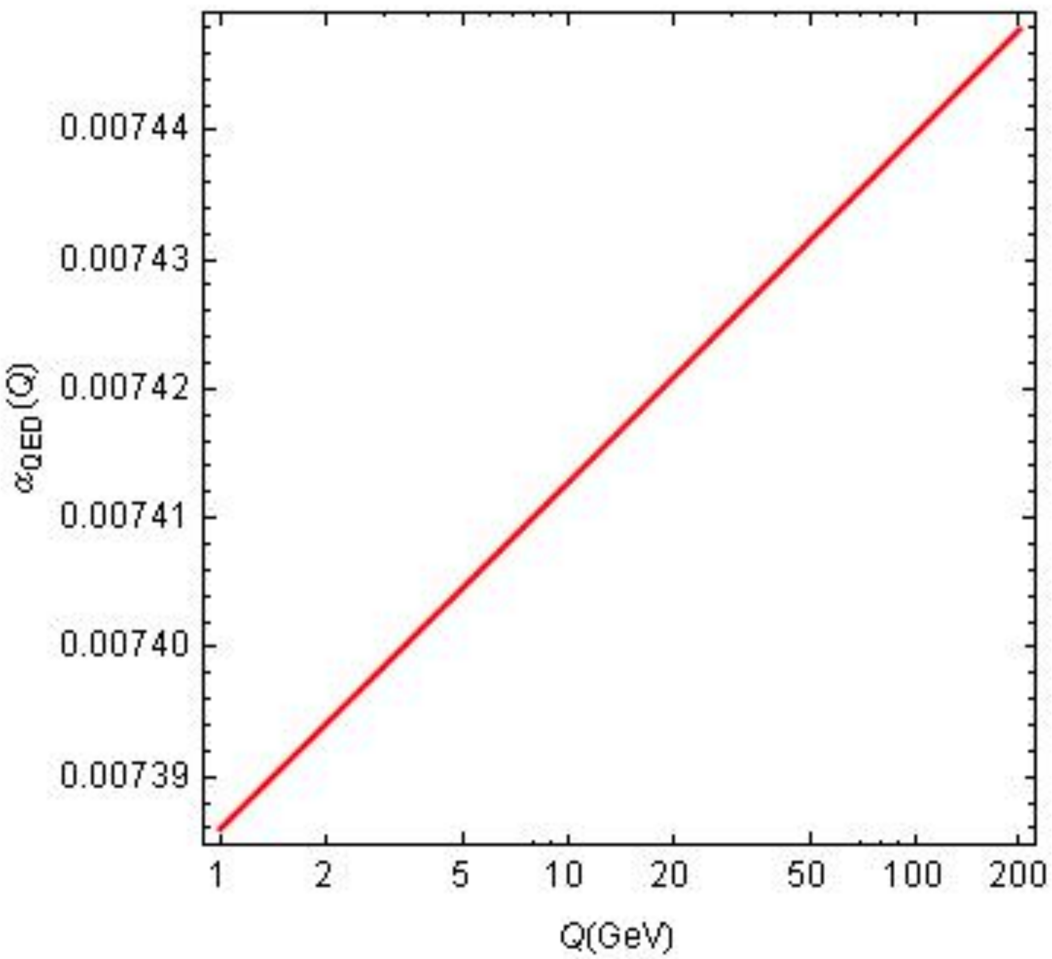


Feynman diagram for an interaction between quarks generated by a gluon.

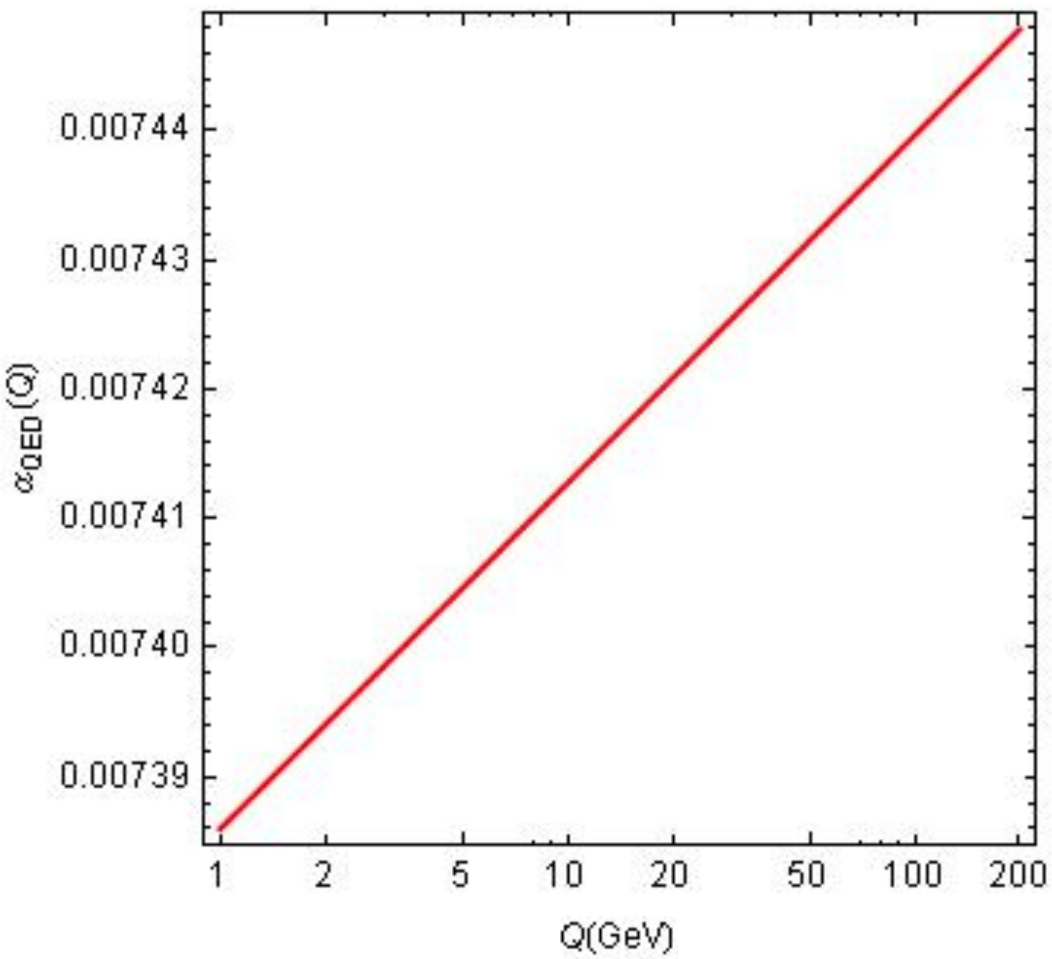




- Similar interaction in QED
- Special feature of QCD – gluon self-interactions

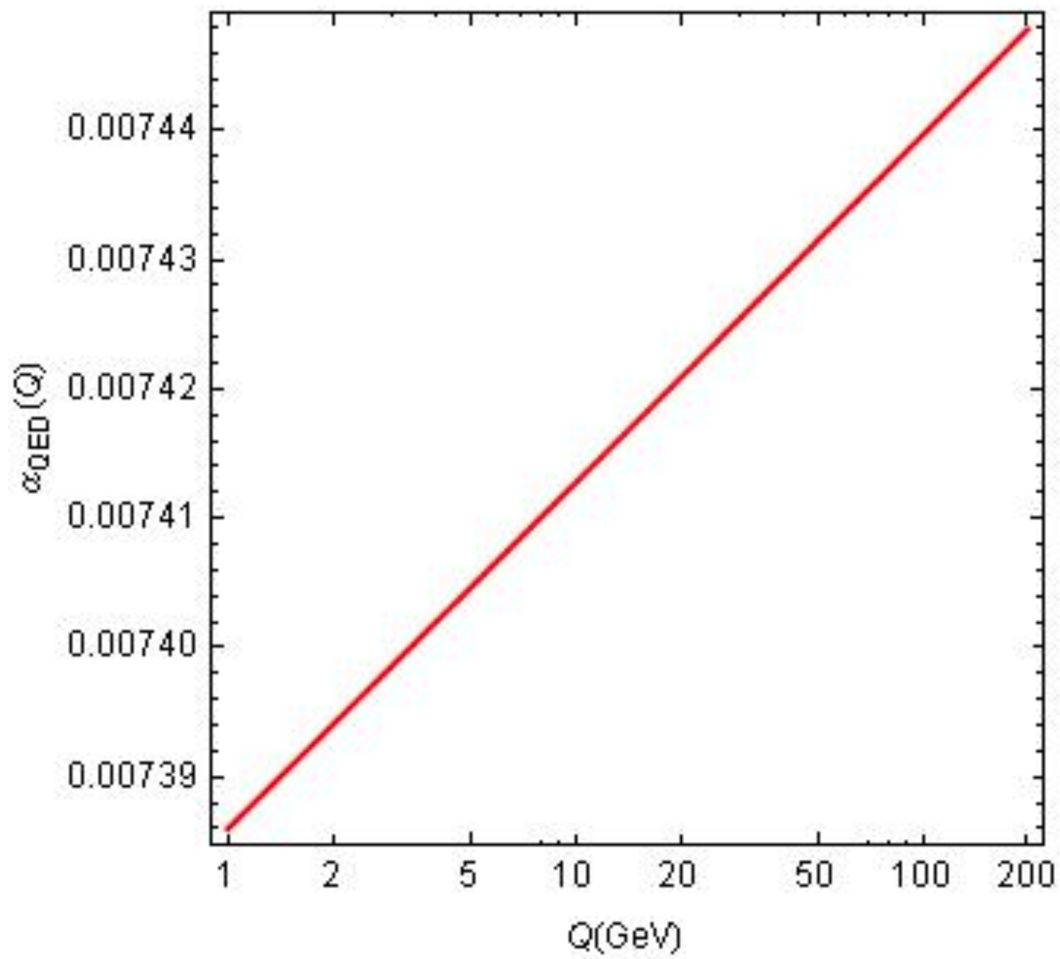


$$\alpha_{QED}(Q) = \frac{\alpha}{1 - \frac{2\alpha}{3\pi}\ln\frac{Q}{m_e}}$$



$$\alpha_{QED}(Q) = \frac{\alpha}{1 - \frac{2\alpha}{3\pi}\ln\frac{Q}{m_e}}$$

Add 3-gluon self-interaction



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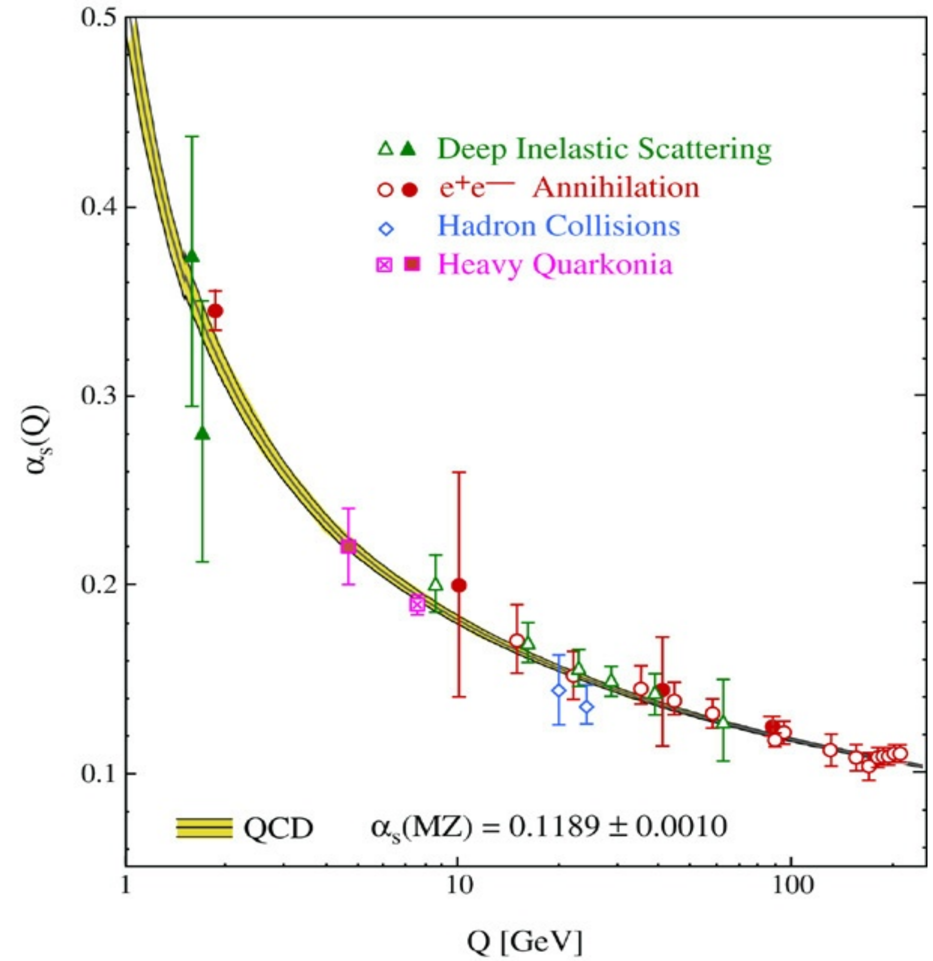


gluon antiscreening

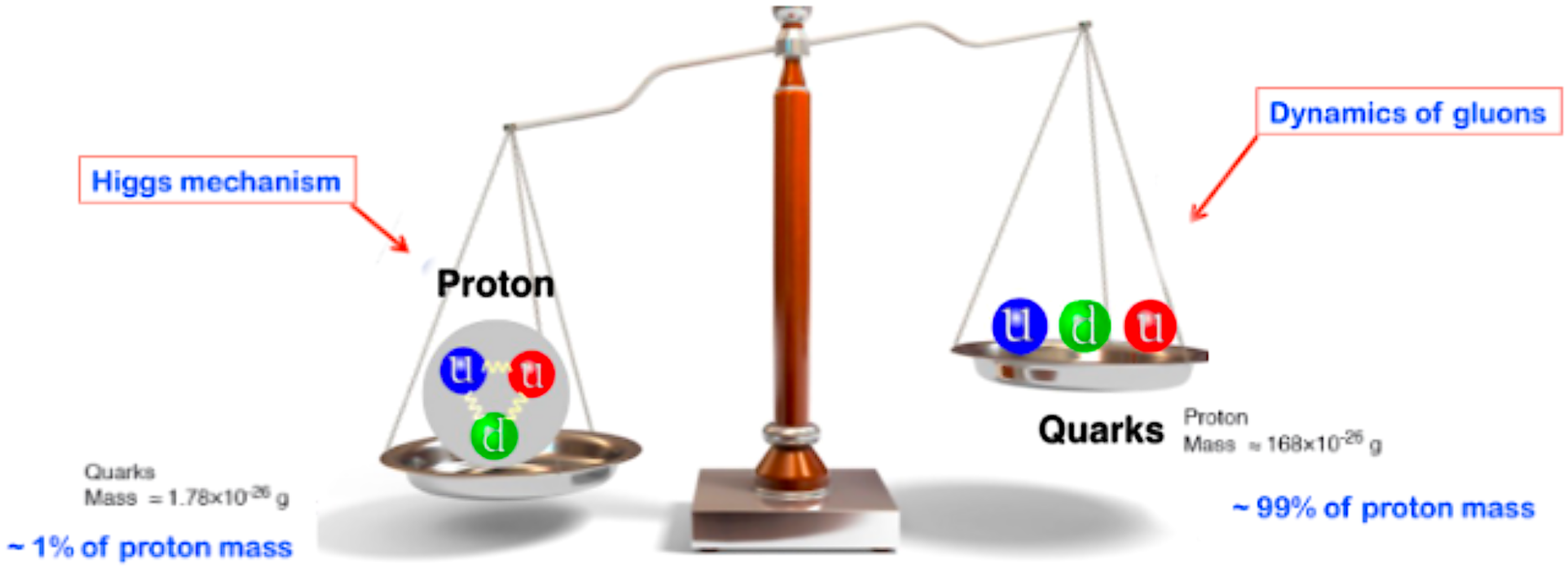
fermion screening

$$\alpha_{QCD}(Q) = \frac{6\pi}{(33 - 2N_f)\ln\frac{Q}{\Lambda}}$$

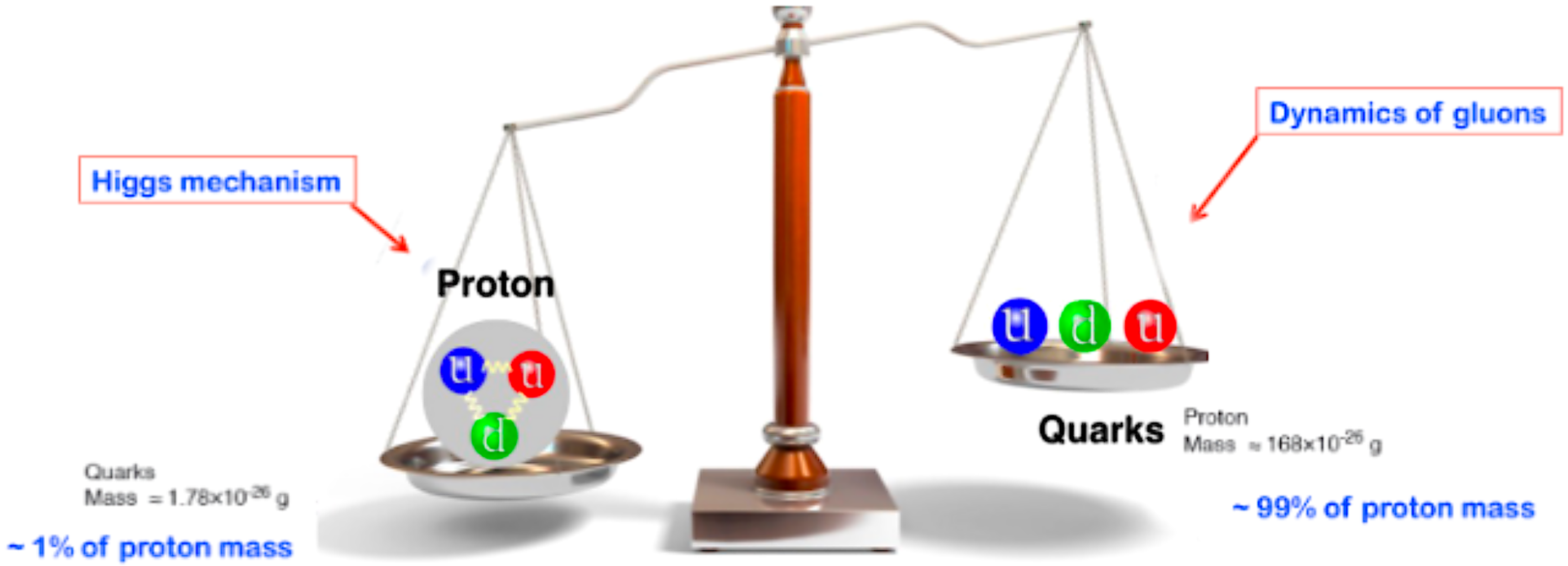
2004 Nobel Prize in Physics : Gross, Politzer and Wilczek

Asymptotic freedom — perturbation theory is valid at large Q^2

QCD becomes *nonperturbative for* $Q^2 < 2$ GeV^2

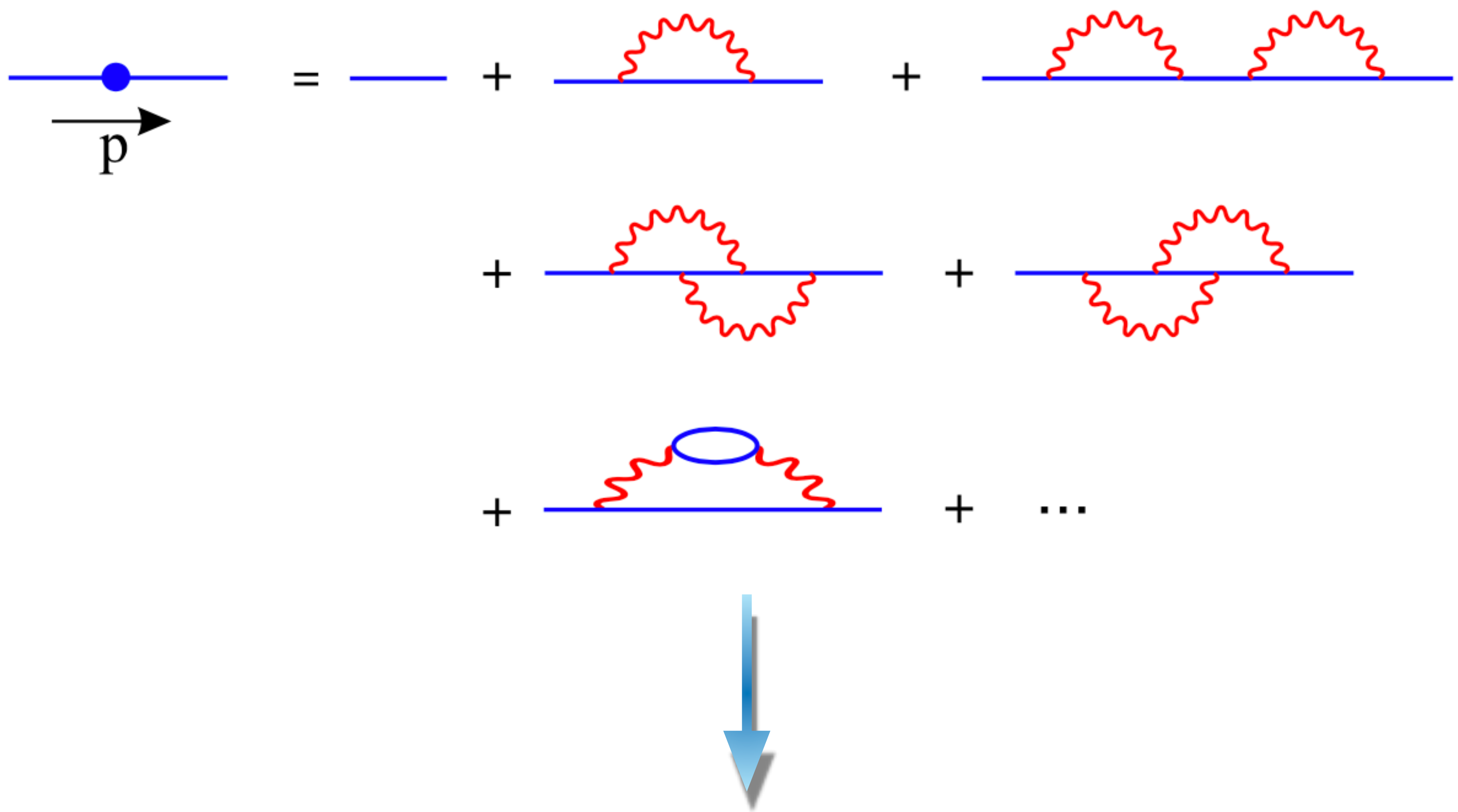


So where do the Hadron's masses come from after all?! The Higgs boson isn't doing the job !



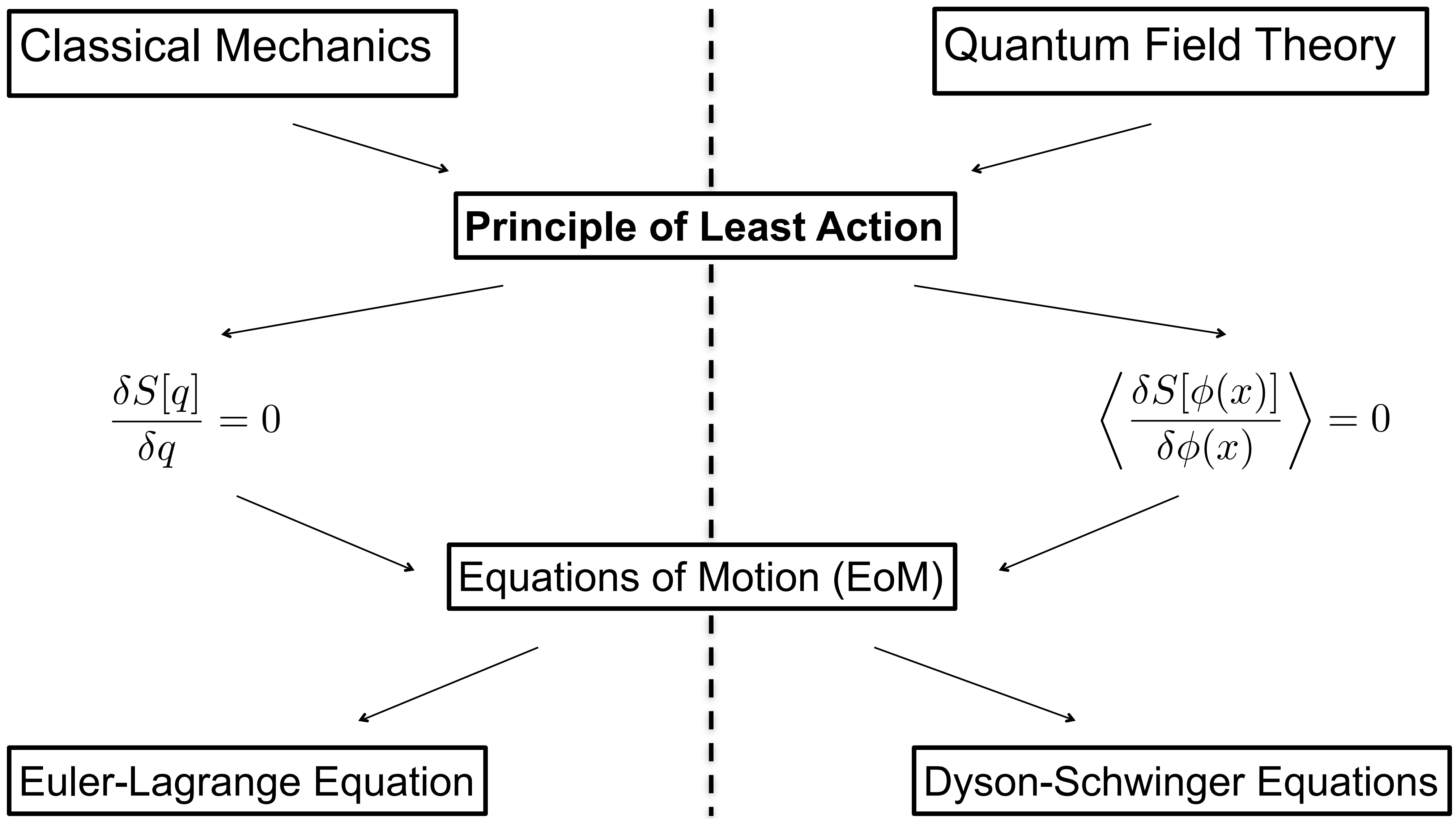
Hint: the gluons interact with each other and have infinite ways to interact with the quark and "dress it".

Answer: not with a perturbative re-summation!



$$\mathcal{M}(p) \;=\; m_0 \left(1 \;+\; c_1\,\alpha\,\ln\left(\frac{p^2}{\mu^2}\right) \;+\; c_2\,\alpha^2\,\ln^2\left(\frac{p^2}{\mu^2}\right) \;+\; \cdots \right)$$

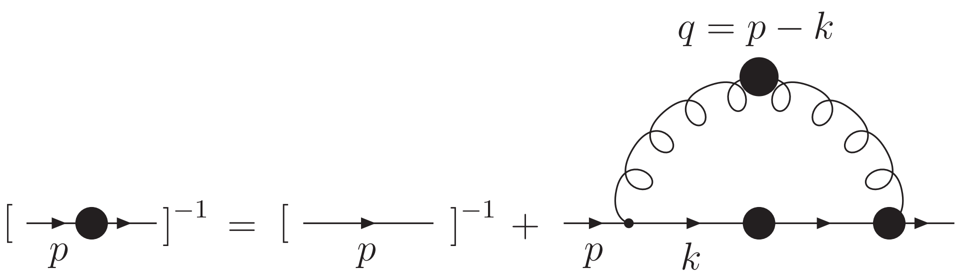
NONPERTURBATIVE CONTINUUM TOOLS FOR QCD



Quark-Gap Equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\rm bm}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)
\end{aligned}
$$



with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

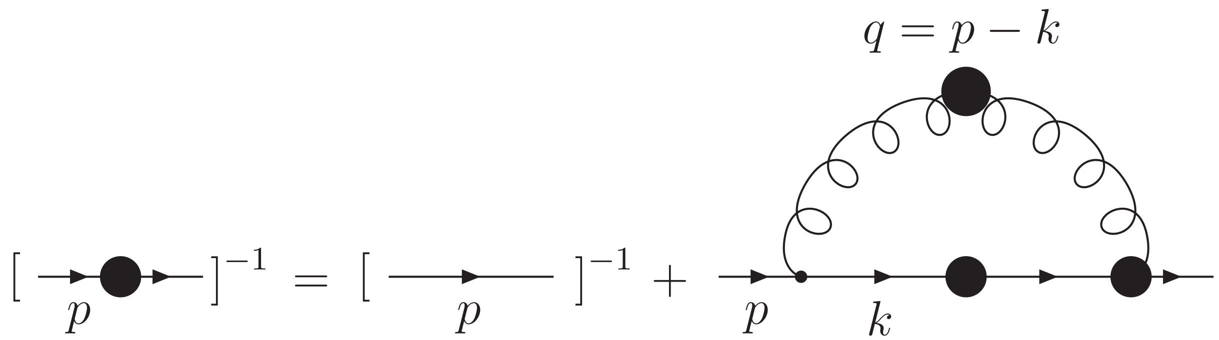
$D_{\mu\nu}$	:	dressed-gluon propagator
$\Gamma^a_\nu(q,p)$	:	dressed quark-gluon vertex
Z_2	:	quark wave function renormalization constant
Z_1	:	quark-gluon vertex renormalization constant

Each satisfies it's own DSE !

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

where ζ is the renormalization point.

Quark-Gap Equation in QCD



$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\rm bm}) + \Sigma(p) := i\gamma \cdot p\, A(p^2) + B(p^2)$$

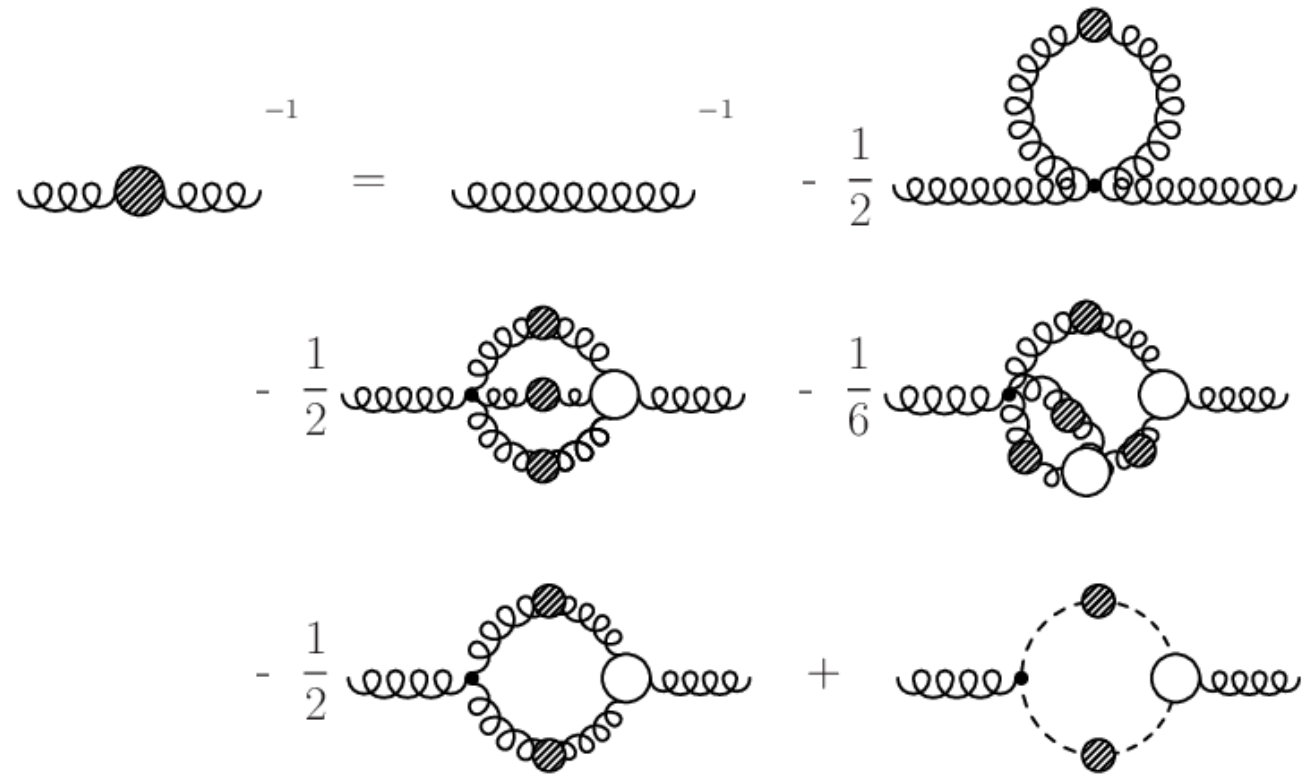
$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p)$$

$$S(p) = \frac{Z\left(p^2\right)}{i\gamma \cdot p + M\left(p^2\right)}$$

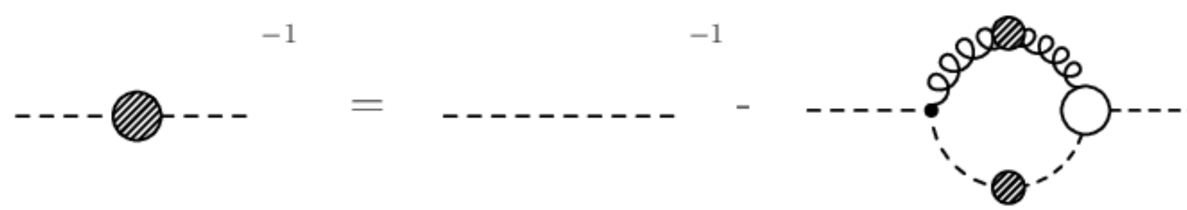
Running quark mass

QCD's Dyson-Schwinger Equations

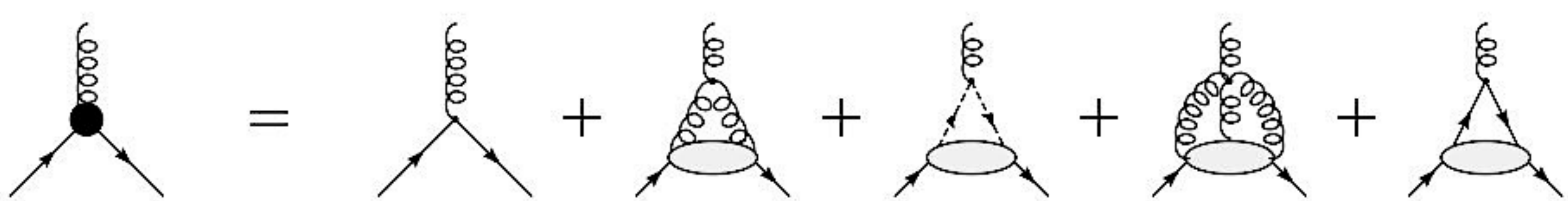
Gluon



Ghost

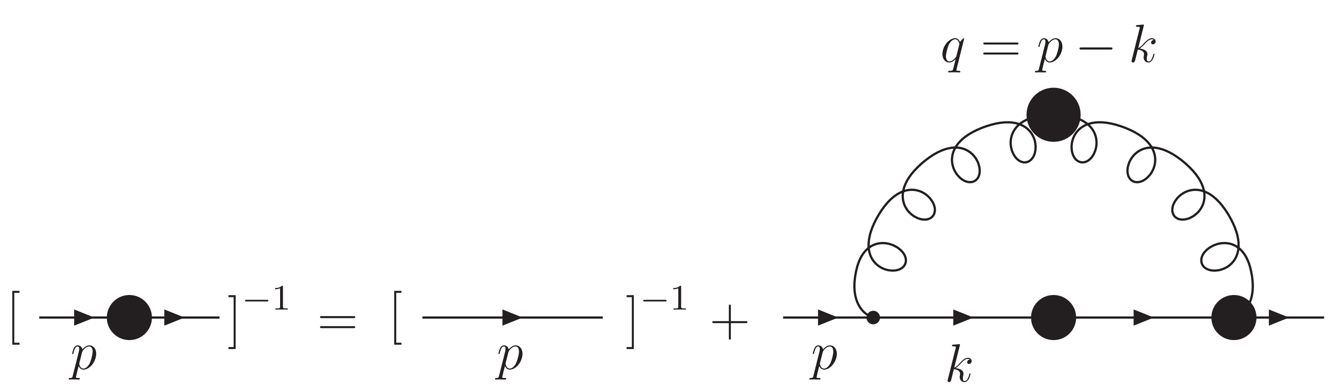


Quark-Gluon Vertex

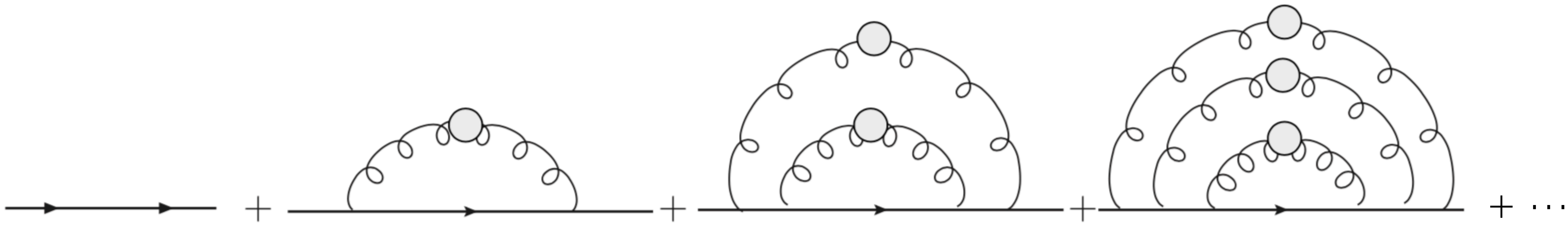


Truncation schemes and symmetries

- Any DSE (quark, gluon …) describes an infinite tower coupled integral equations — *n*-point function involves *n*+1 and higher Green function!
- Kernel of the equation for the quark self-energy involves the dressed-gluon propagator and the dressed-quark-gluon vertex.
- Coupling between equations ***requires*** a truncation!
- Truncation must preserve essential symmetries of QCD, in particular chiral symmetry.
- Ward-Takahashi identity must be satisfied, that is in the chiral limit the pion must be massless!



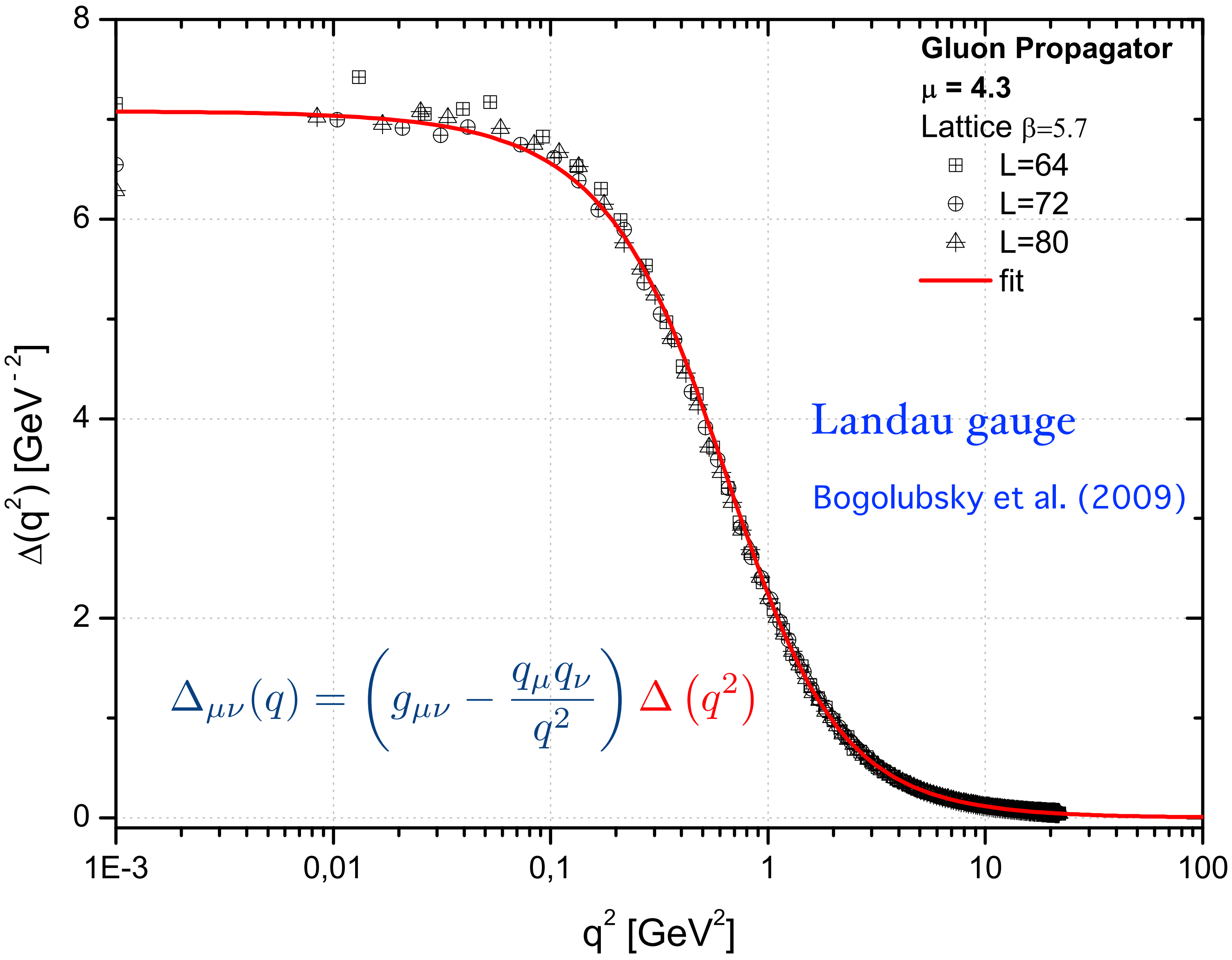
Leading truncation: *rainbow*



- Only bare structure of the quark-gluon vertex is kept.
- Leads to planar diagrams (large N_c limit).
- Effective model dressing accounts for both, gluon and vertex.
- Nonperturbative dressing controls the integral kernel's strength.
- Sufficient strength $\Longrightarrow$ *Dynamical Chiral Symmetry Breaking*

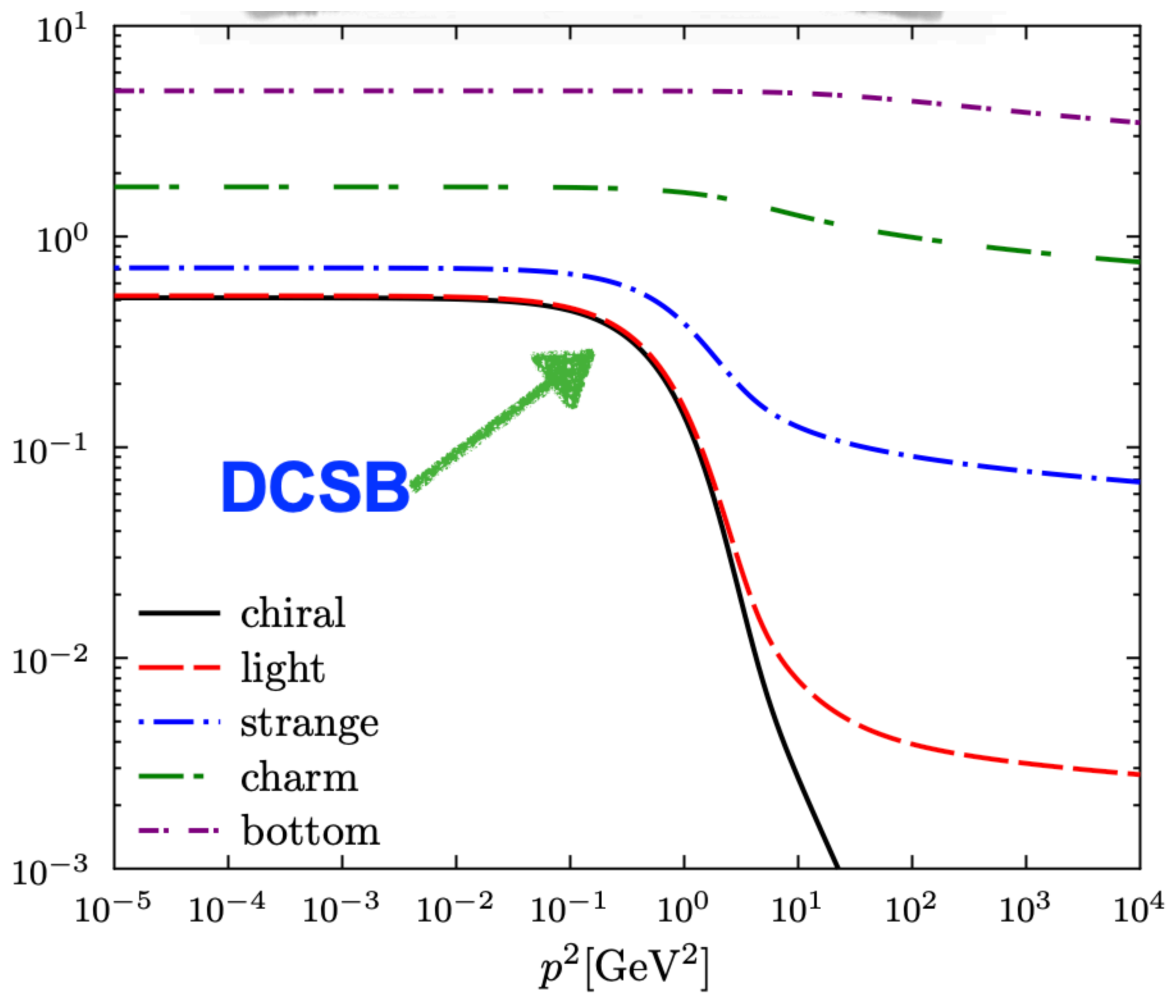
  $\Longrightarrow$ **Constituent quark mass**

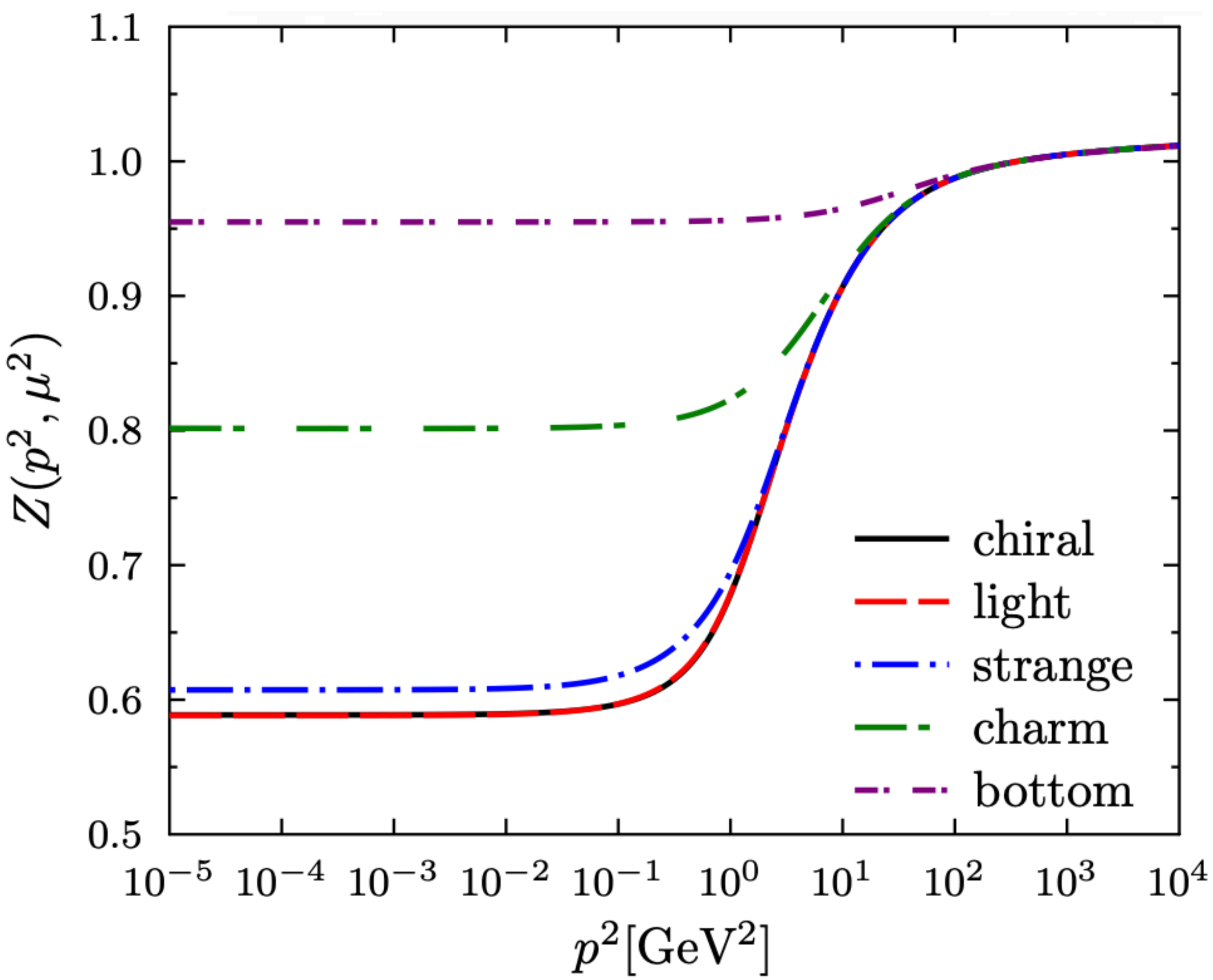
Lattice QCD gluon dressing function



Phenomenological applications: use of effective interaction that reproduces Lattice QCD and DSE gluon propagators: *infrared massive fixed point; ultraviolet massless propagator*.

DSE solutions — flavor dependence

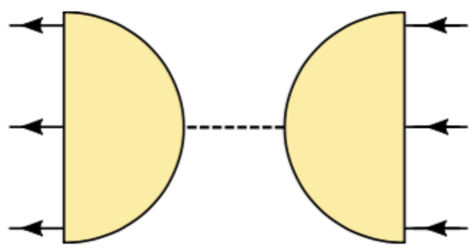




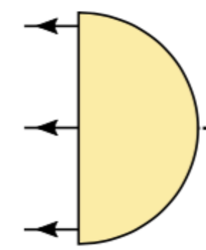
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

L. Albino, A. Bashir, B.E., L.X. Gutiérrez Guerrero, E. Rojas PRD 100 (2019)
L. Albino, A. Bashir, B.E., E. Rojas, F. E. Serna, R.C. Silveira, JHEP11 (2021)
J. R. Lessa, F. E. Serna, B.E., A. Bashir, O. Oliveira, PRD 107 (2023)

Everything we need to know is encoded in *n*-point Green functions



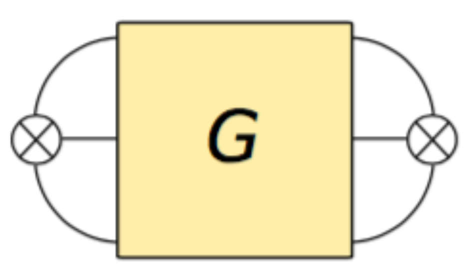
$P^2 \longrightarrow -m_\lambda{}^2$



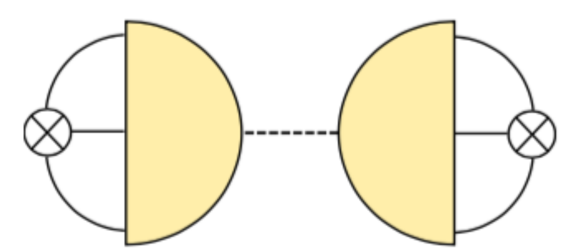
$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} \simeq \sum_\lambda \frac{\mathbf{\Psi}^\lambda_{\alpha\beta\gamma}\,\bar{\mathbf{\Psi}}^\lambda_{\alpha'\beta'\gamma'}}{P^2+m_\lambda^2}$$

$$\mathbf{\Psi}^\lambda_{\alpha\beta\gamma} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\,|\lambda\rangle$$

Spectral decomposition: extract baryon poles from quark 6-point functions $\Longrightarrow$ Residue at pole: Bethe-Salpeter wave function, contains all information about baryon



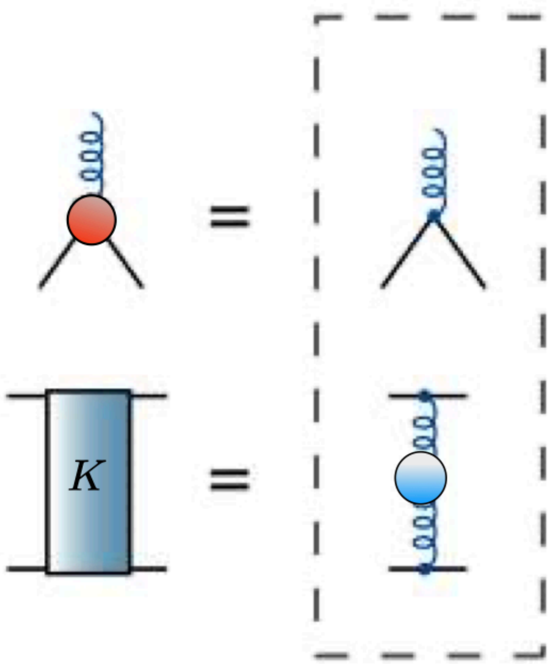
$P^2 \longrightarrow -m_\lambda{}^2$

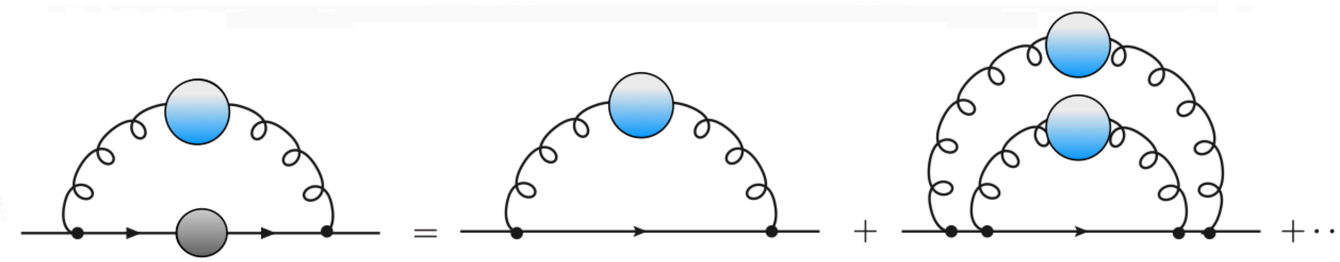


$\sim \exp(-m_N t)$

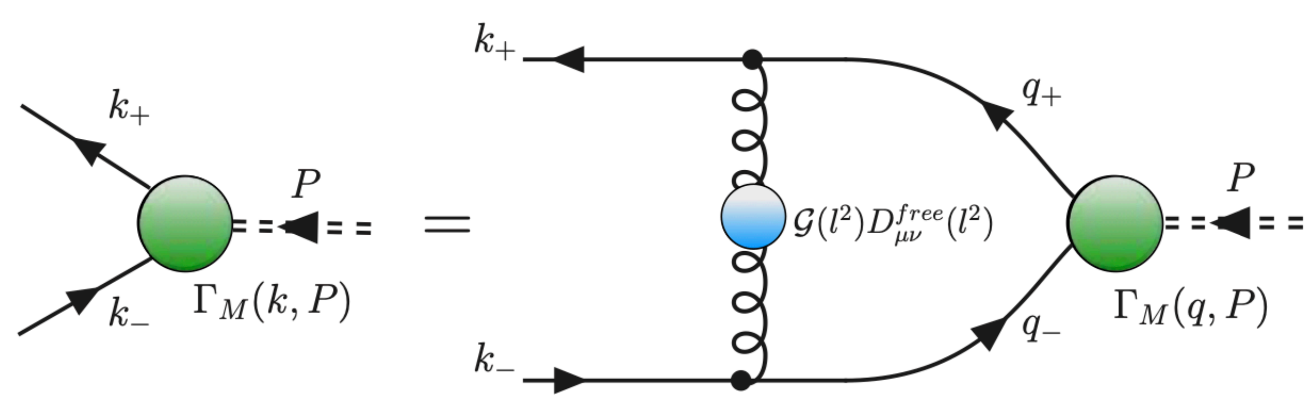
Lattice: extract baryon poles from correlators $\Longrightarrow$ exponential Euclidean time decay

Bethe-Salpeter equation for QCD bound states





$$K(q,k;P) = -Z_2^2\,\mathcal{G}\left(l^2\right) D_{\mu\nu}^{\text{free}}(l)\,\gamma_\mu \frac{\lambda^a}{2} \gamma_\nu \frac{\lambda^a}{2}$$

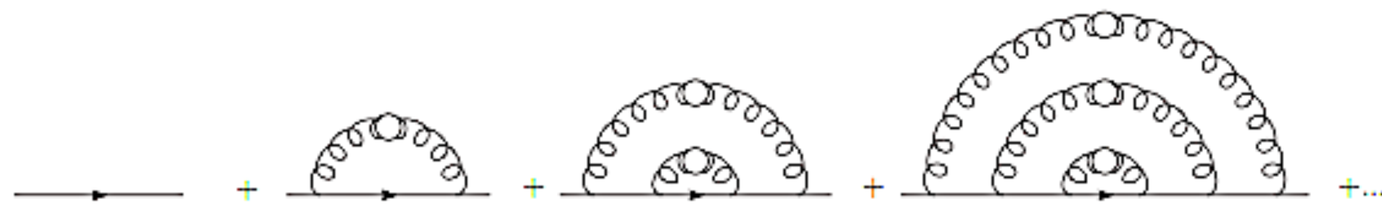


- Quark propagators from solving the gap equation (DSE) for space-like momenta.
- In Euclidean space, the propagators are functions of the meson momentum $P = (\vec{0}, iM)$
- Extension to complex plane via Cauchy's integral theorem.

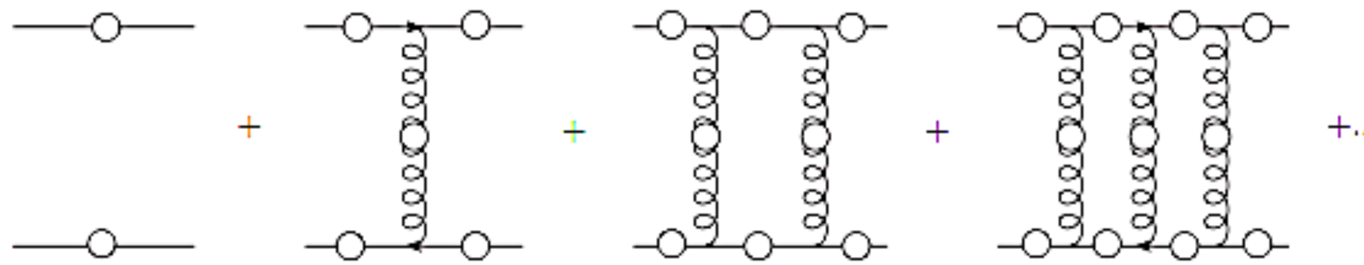
Bethe-Salpeter equation for QCD bound states

Rainbow-ladder truncation (leading symmetry-preserving approximation)

$S(p) =$



$G^4(k, q, P) =$



Model gluon propagator, solve quark propagator and 4-point Green function.

- Quark propagators from solving the gap equation (DSE) for space-like momenta.
- In Euclidean space, the propagators are functions of the meson momentum $P = (\vec{0}, iM)$
- Extension to complex plane via Cauchy's integral theorem.

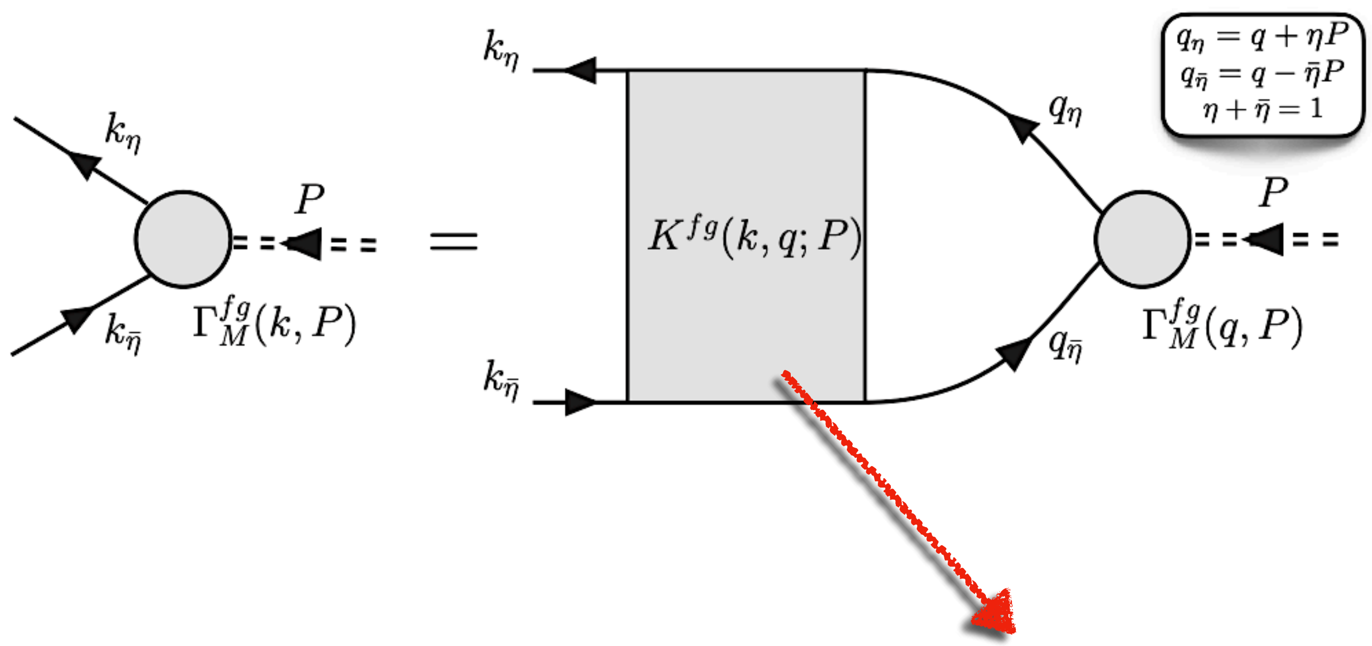
Bethe-Salpeter Equation for QCD Bound States

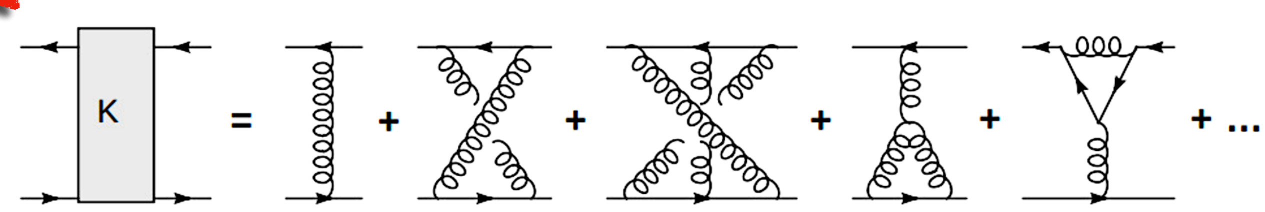
$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

$K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel

$S_f(q_\eta)$ = Dressed quark propagator

$\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)





Bethe-Salpeter Equation for QCD Bound States

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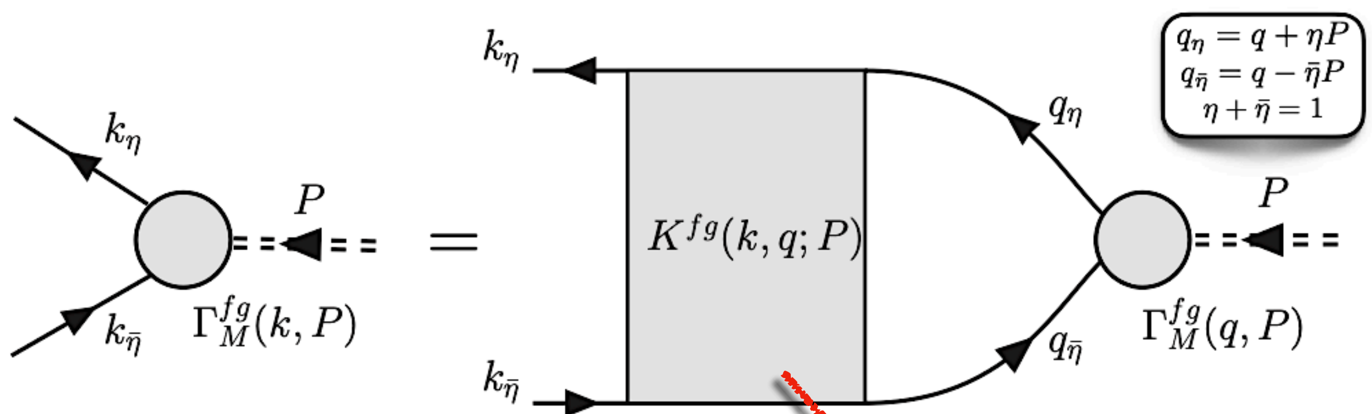
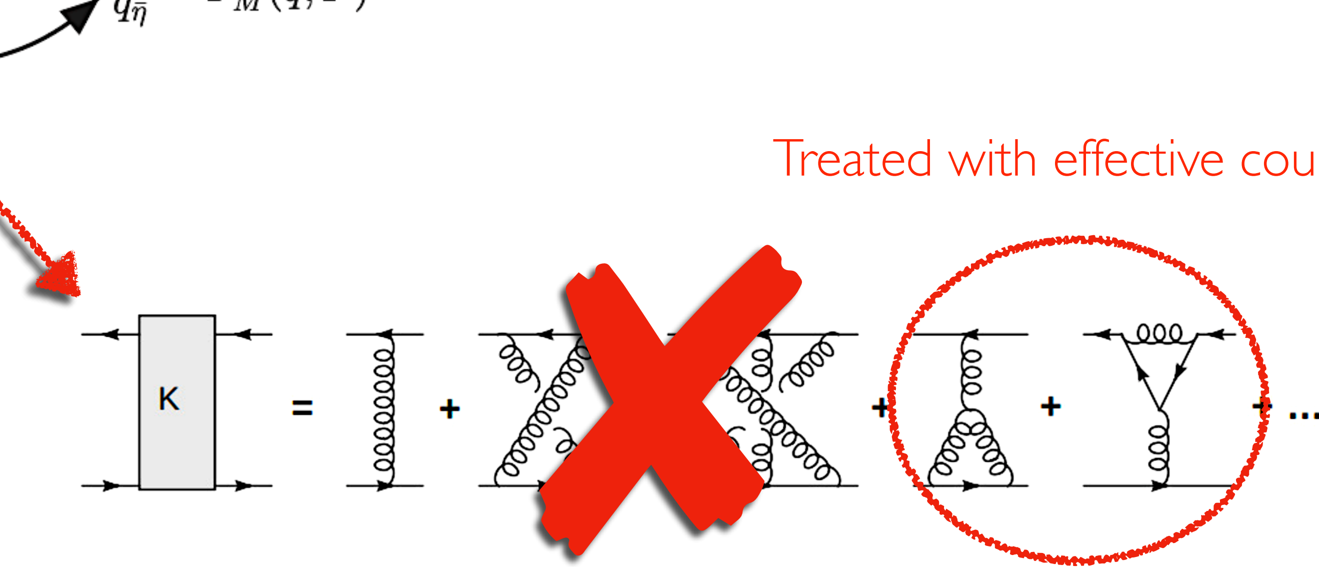


Treated with effective couplings



Bethe-Salpeter Amplitudes

- The general form of $\Gamma_M(k;P)$ is given by

$$\Gamma(k;P) = \sum_{i=1}^{N} \mathcal{T}^i(k,P)\mathcal{F}_i(k^2, z_k, P^2), \quad z_k = k.P/|k||P|,$$

- where $\mathcal{T}^i$(k,P) are Dirac's covariants;
- $\mathcal{F}_i(k^2, z_k, P^2)$ are Lorentz invariant amplitudes;
- N denotes the number of covariants which are different for different meson's channel.
- For the case of pseudoscalar mesons we have $N = 4$ and for vector mesons one has $N = 8$

Beyond S-wave amplitudes

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

relativistic $q\bar{q}$

$$\Gamma_\pi(P,p) = \gamma_5 \,[F_1(P,p) \quad \text{s-wave}$$
$$+ F_2(P,p) i\not{P}$$
$$+ F_3(P,p) pP i\not{p} \quad \text{p-wave}$$
$$+ F_4(P,p)[\not{p}, \not{P}]]$$

~~$P : (-1)^{L+1}$~~

Llewellyn-Smith, Annals Phys. 53 (1969) 521–558

Pseudoscalar meson spectrum

	M_P	M_P^{exp}	ϵ_{M_P} [%]	f_P	$f_P^{\text{exp/lQCD}}$	ϵ_{f_P} [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17

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Vector meson spectrum

	M_V	M_V^{exp}	ϵ_{M_V} [%]	f_V	$f_V^{\text{exp/lQCD}}$	ϵ_{f_V} [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62

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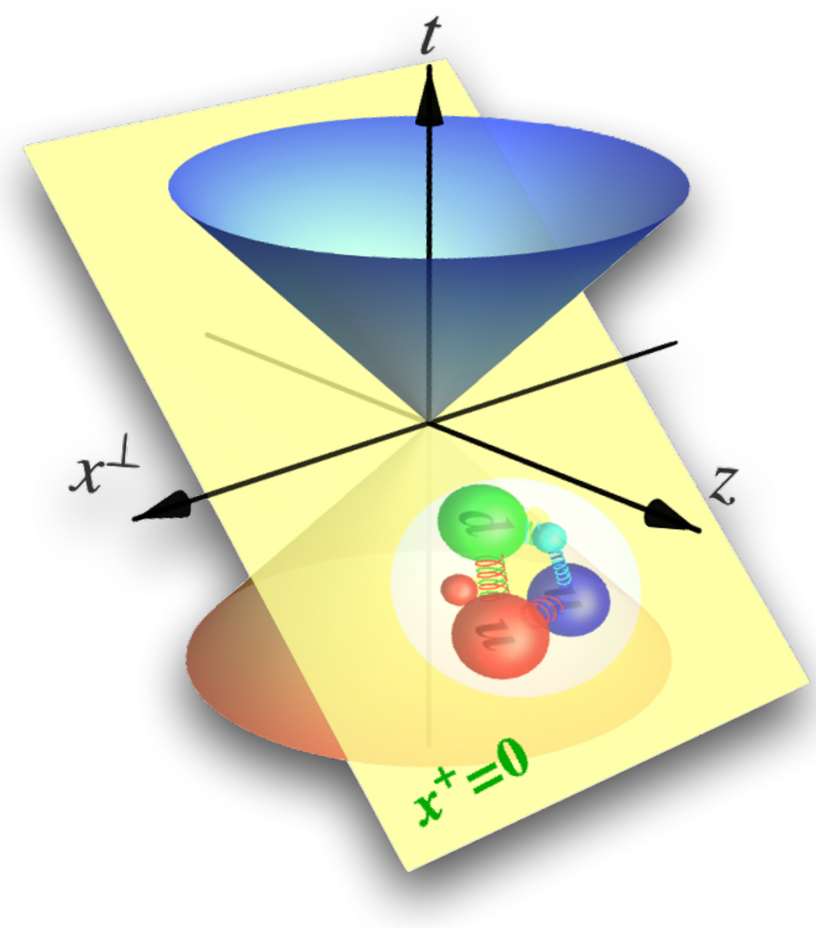


Light-Front Wave Functions

Light-Front Wave Functions

- The **LCWFs** are obtained from the **BSWF** via the light front projections:

$$\psi_M^{\uparrow\downarrow}(x, \mathbf{k}_\perp^2) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \mathrm{Tr}_D[\gamma^+ \gamma_5 \chi(k, P)],$$

$$\psi_M^{\uparrow\uparrow}(x, \mathbf{k}_\perp^2) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta(xP^+ - k^+) \mathrm{Tr}_D[i\sigma_{+i} k_T^i \gamma_5 \chi(k, P)]$$

- With the **LCWF** one can readily derive two distributions:

  - The leading-twist **TMD** [B. Pasquini and P. Schweitzer (2014)]

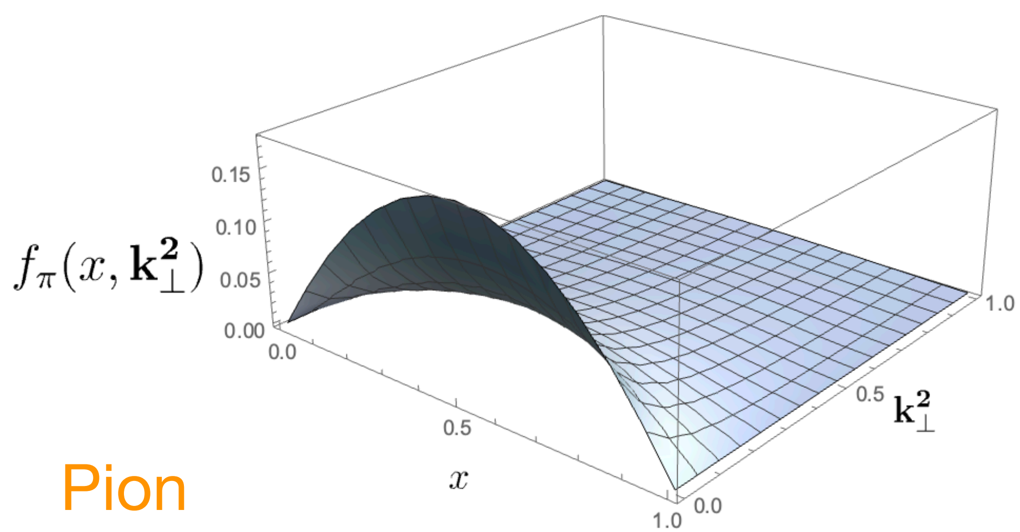
$$f_M\left(x, \mathbf{k}_\perp^2, \mu\right) = \frac{1}{(2\pi)^3} \left| \psi_M^{\uparrow\downarrow}\left(x, \mathbf{k}_\perp^2, \mu\right) + \mathbf{k}_\perp^2 \psi_M^{\uparrow\uparrow}\left(x, \mathbf{k}_\perp^2, \mu\right) \right|^2$$

  - The **PDF**

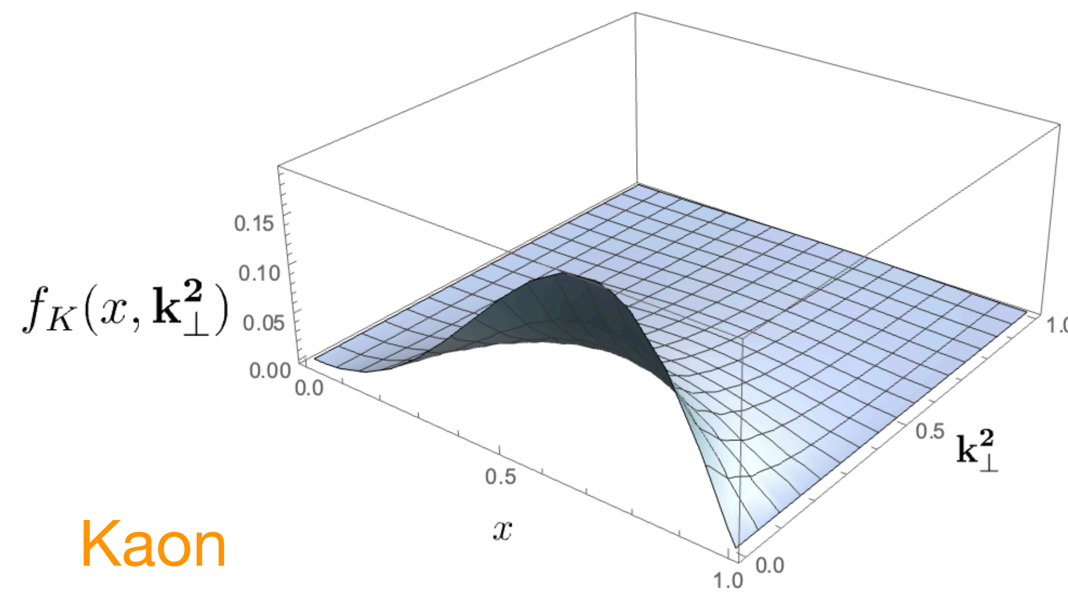
$$q_M(x, \mu) = \int d\mathbf{k}_\perp^2 f_M(x, \mathbf{k}_\perp^2, \mu)$$

Normalization condition: $\int_0^1 dx\, q_M\left(x, \mu\right) = 1.$

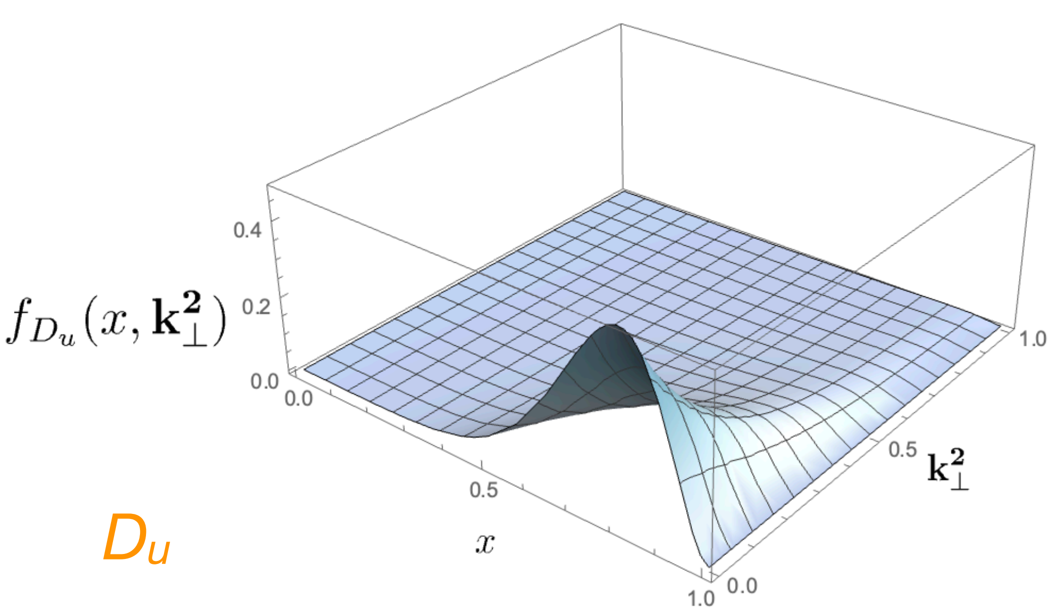
Transverse Distribution Functions



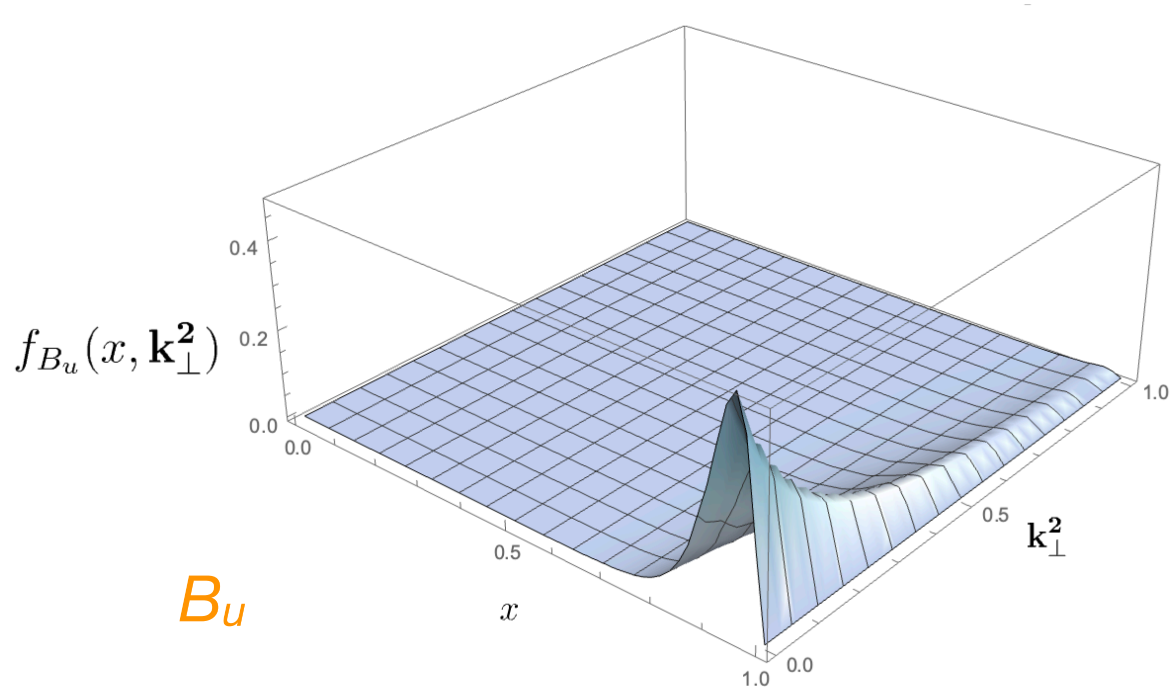
Pion



Kaon

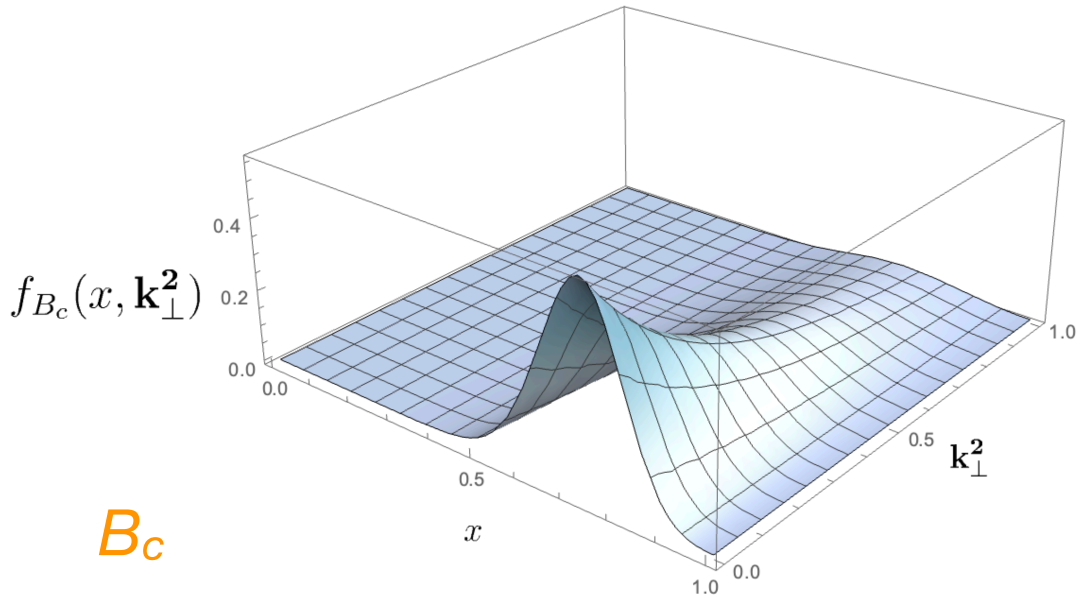


D_u

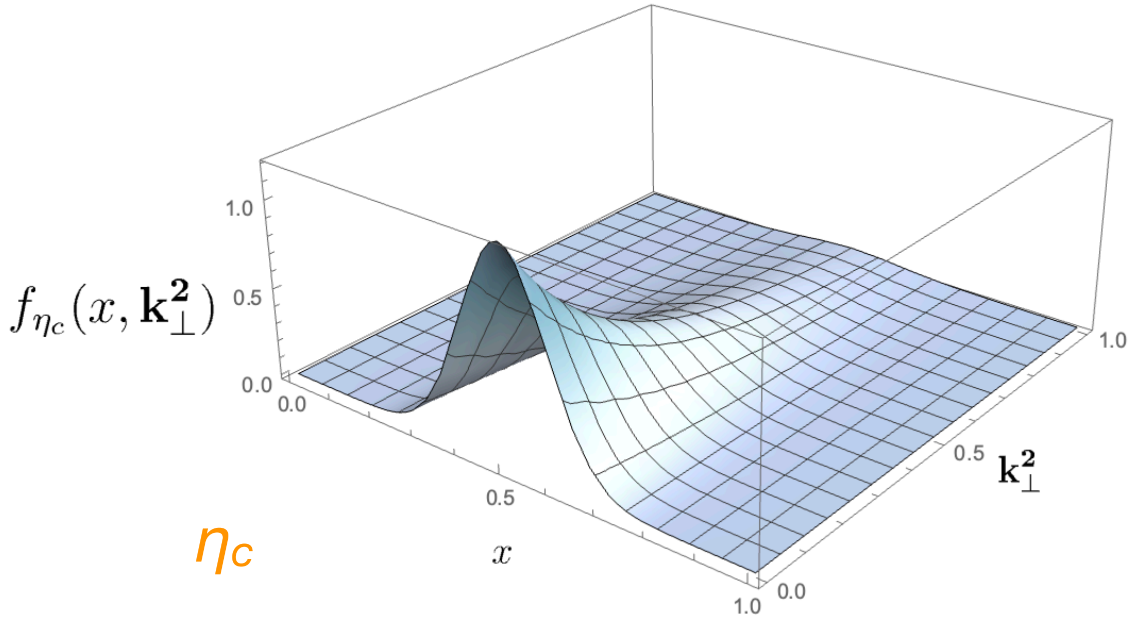


B_u

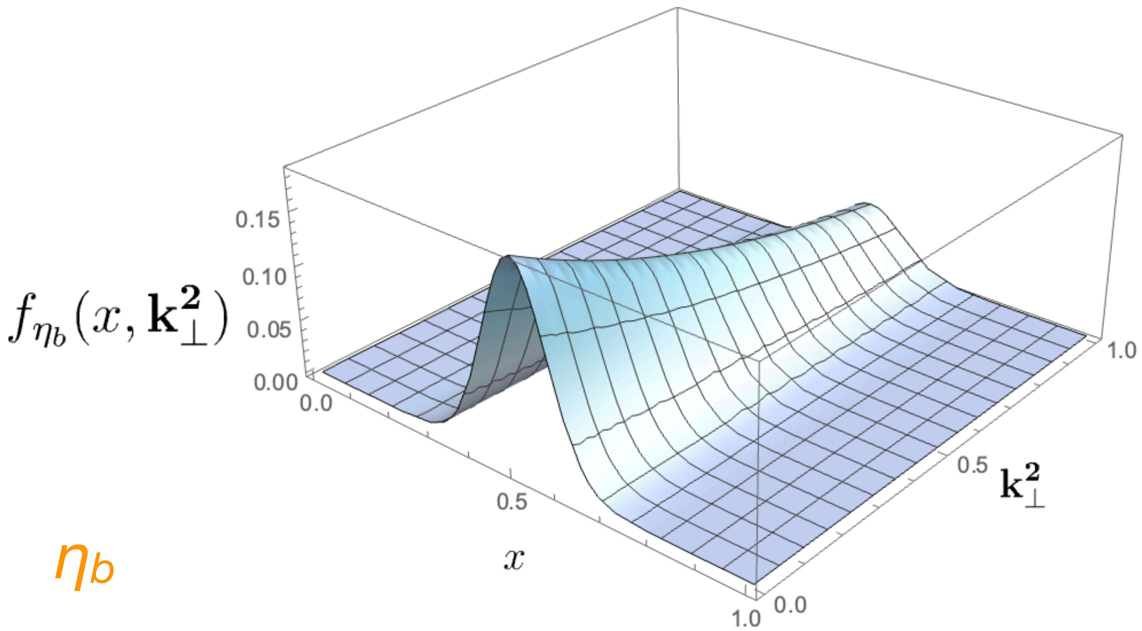
Transverse Distribution Functions



B_c

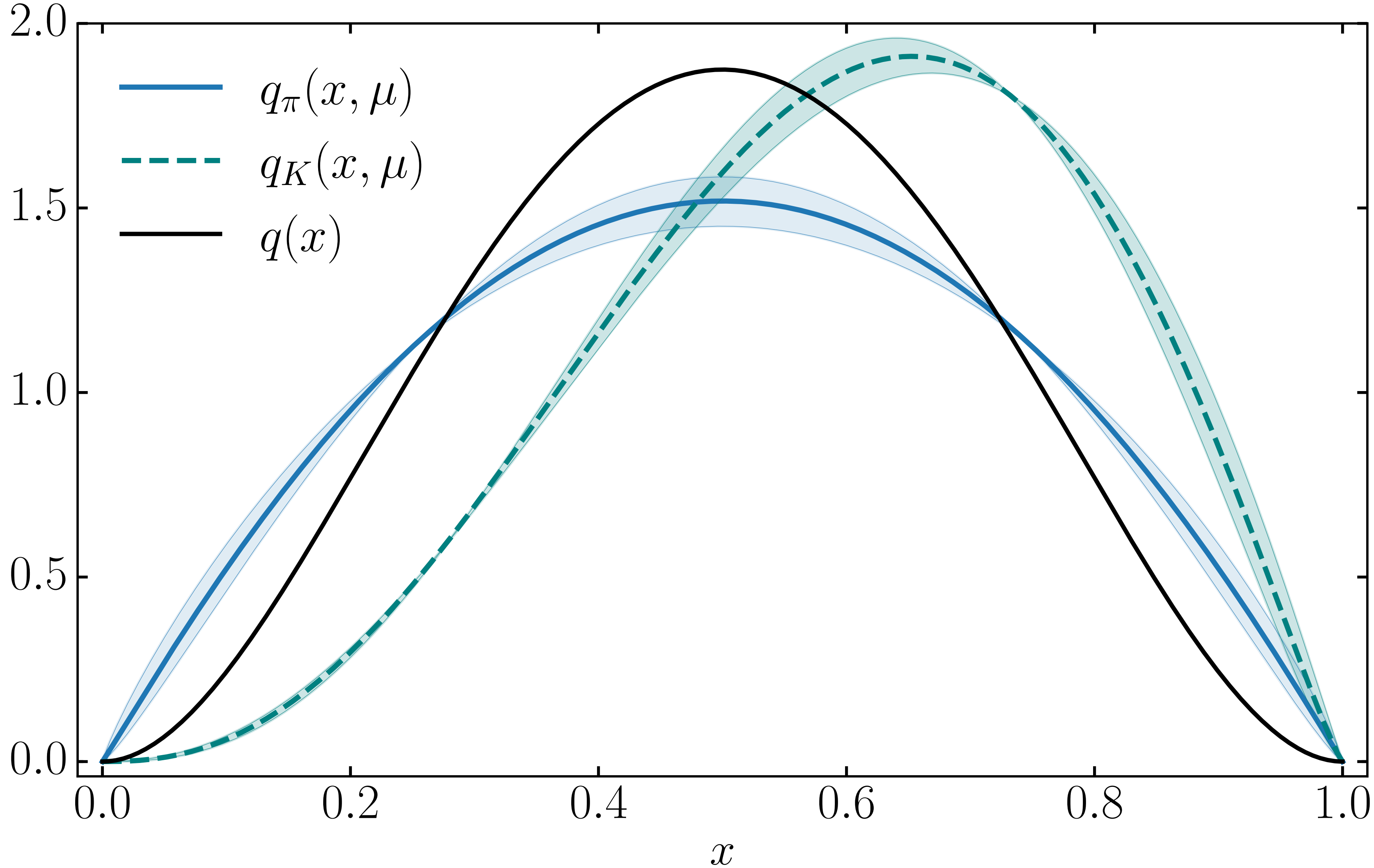


η_c



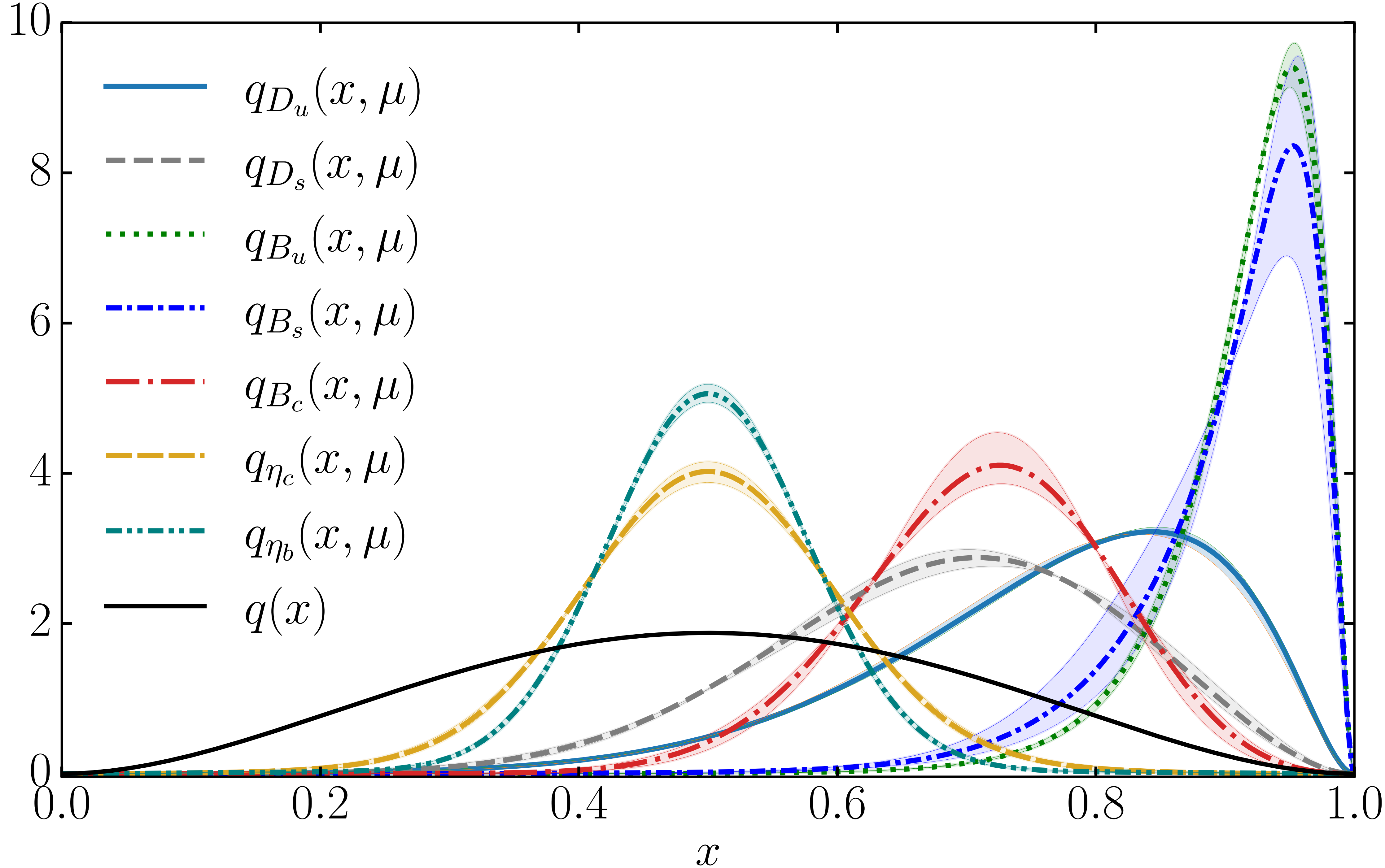
η_b

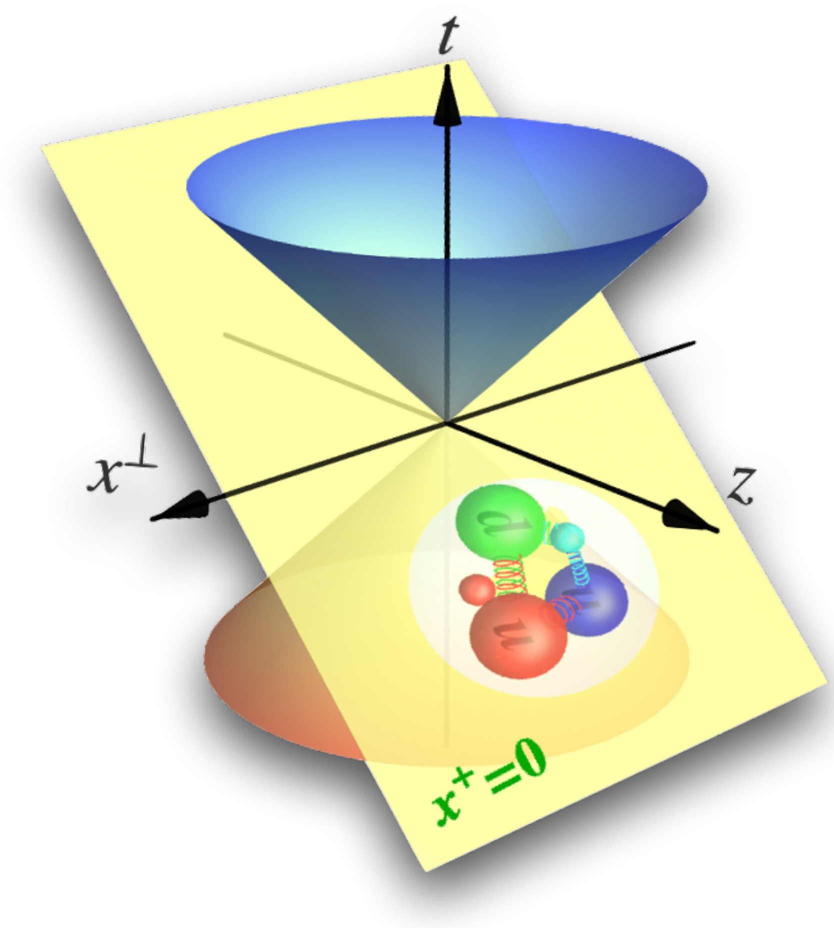
Parton Distribution Functions



The PDFs can be parametrized by: $q(x;\mu_0) = 30[x(1-x)]^2 \left[1 + \sum_{j=1}^{j_m} a_j C_j^{5/2}(2x-1)\right]$

Parton Distribution Functions



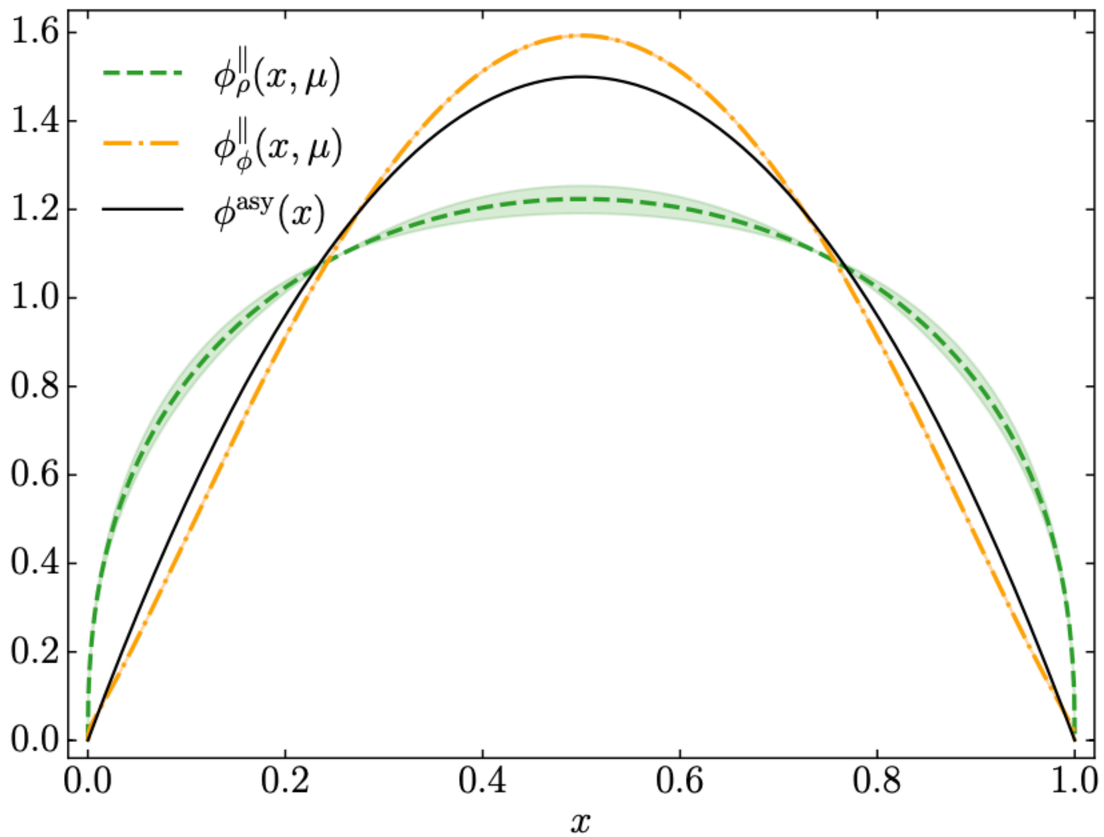


Meson Distribution Amplitudes on the Light Front

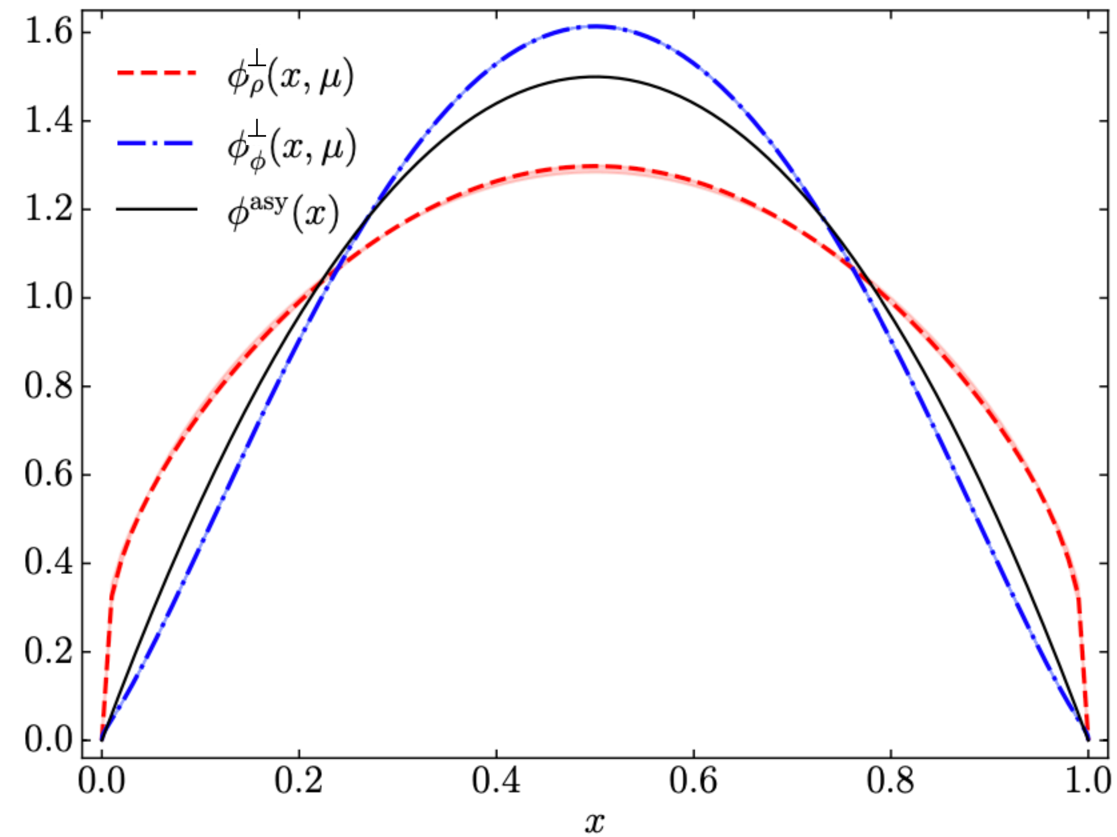
Vector Meson LCDA

Vector mesons are described by two distribution amplitudes: $\phi_V^{\parallel}(x;\mu)$ and $\phi_V^{\perp}(x;\mu)$

ρ and ϕ mesons: longitudinal LCDA



ρ and ϕ mesons: transverse LCDA



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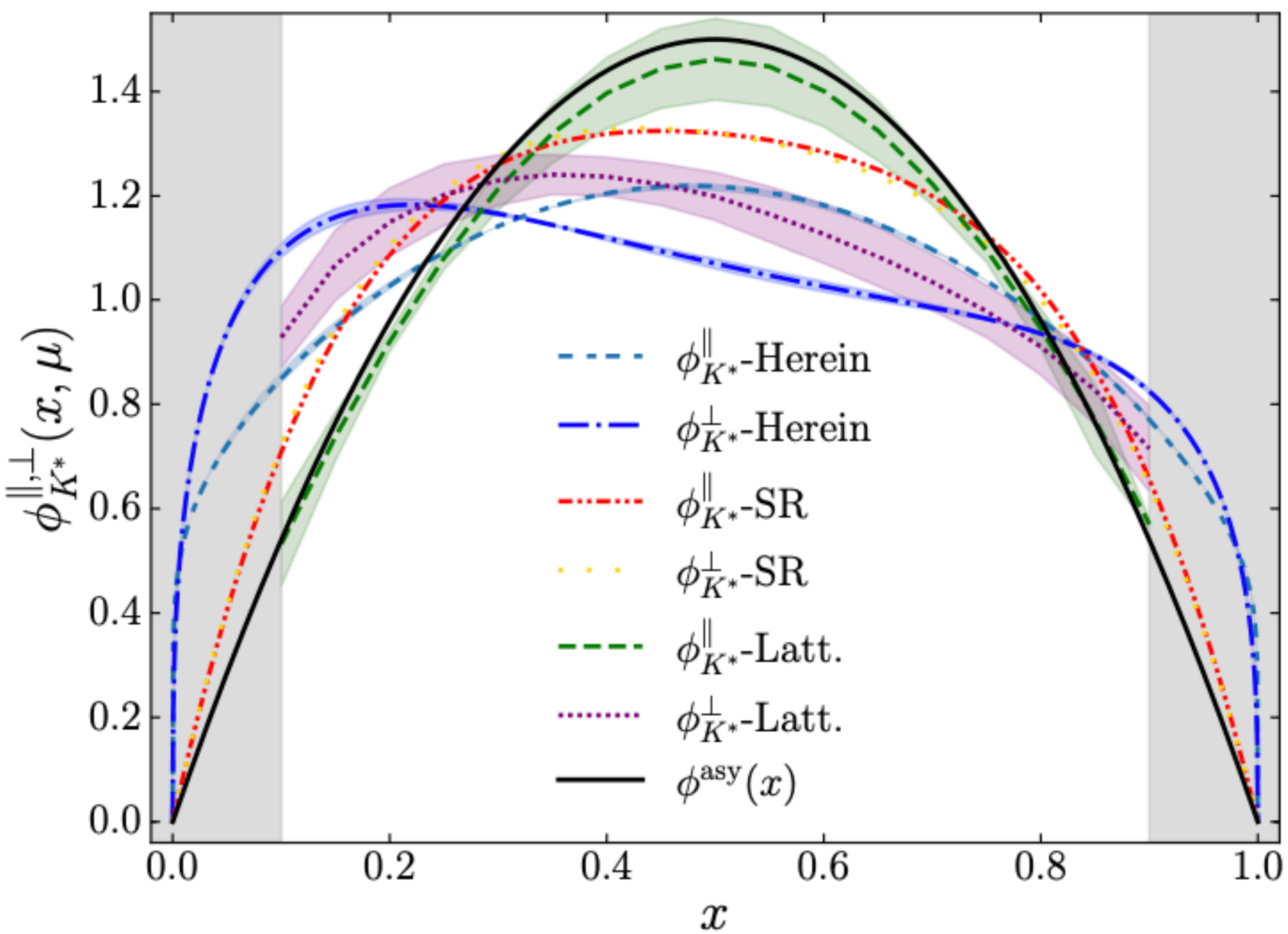
We find: $\phi_\phi^{\parallel}(x,\mu) \approx \phi_\phi^{\perp}(x,\mu)$

Vector Meson LCDA



K^* meson longitudinal and transverse LCDA

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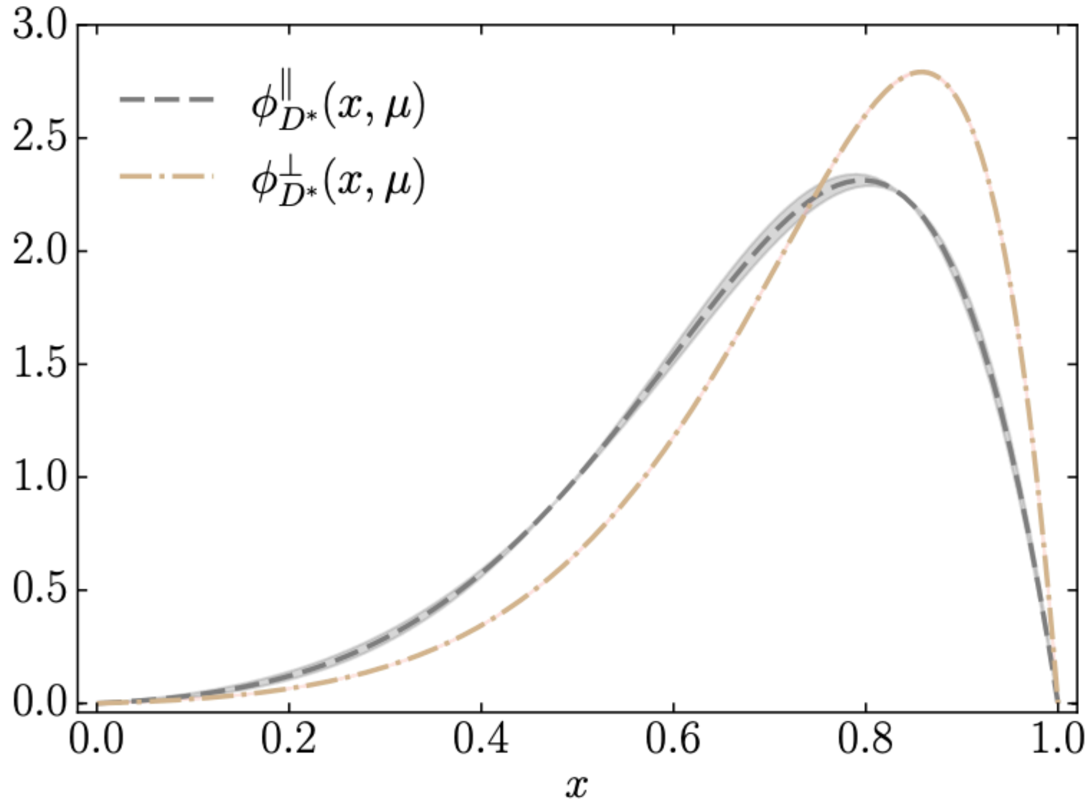
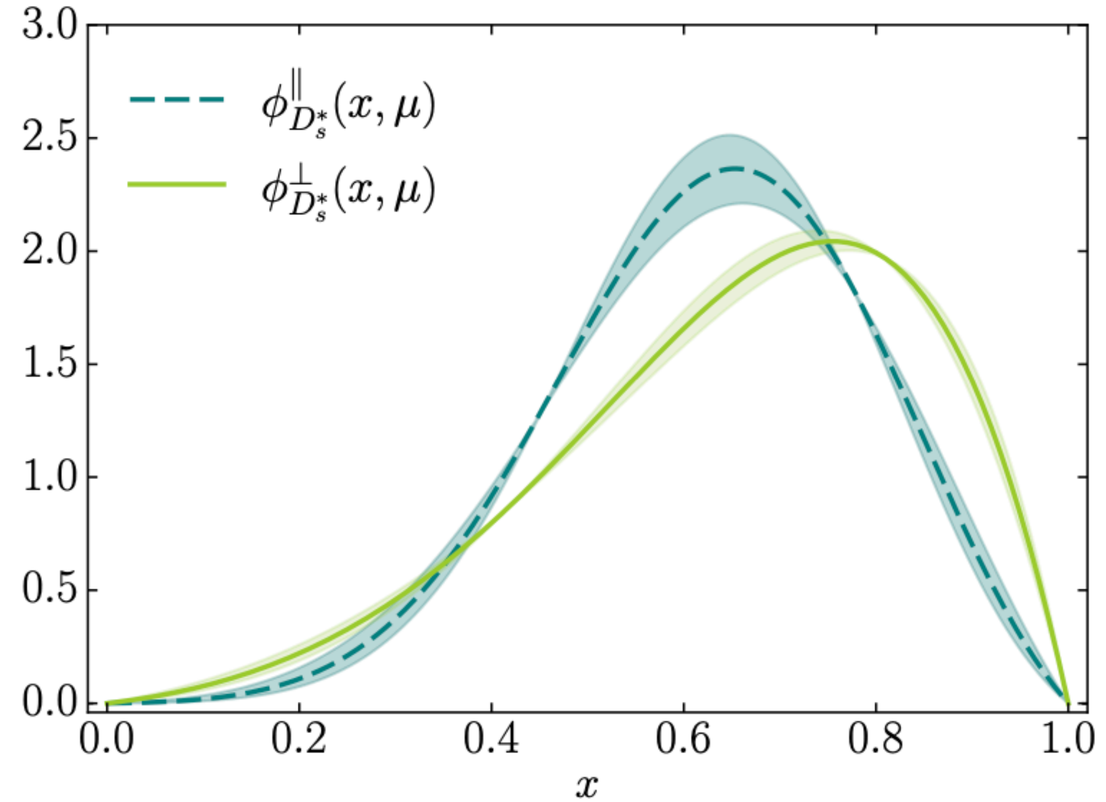
Vector Meson LCDA





D^* and D_s^* mesons longitudinal and transverse LCDA

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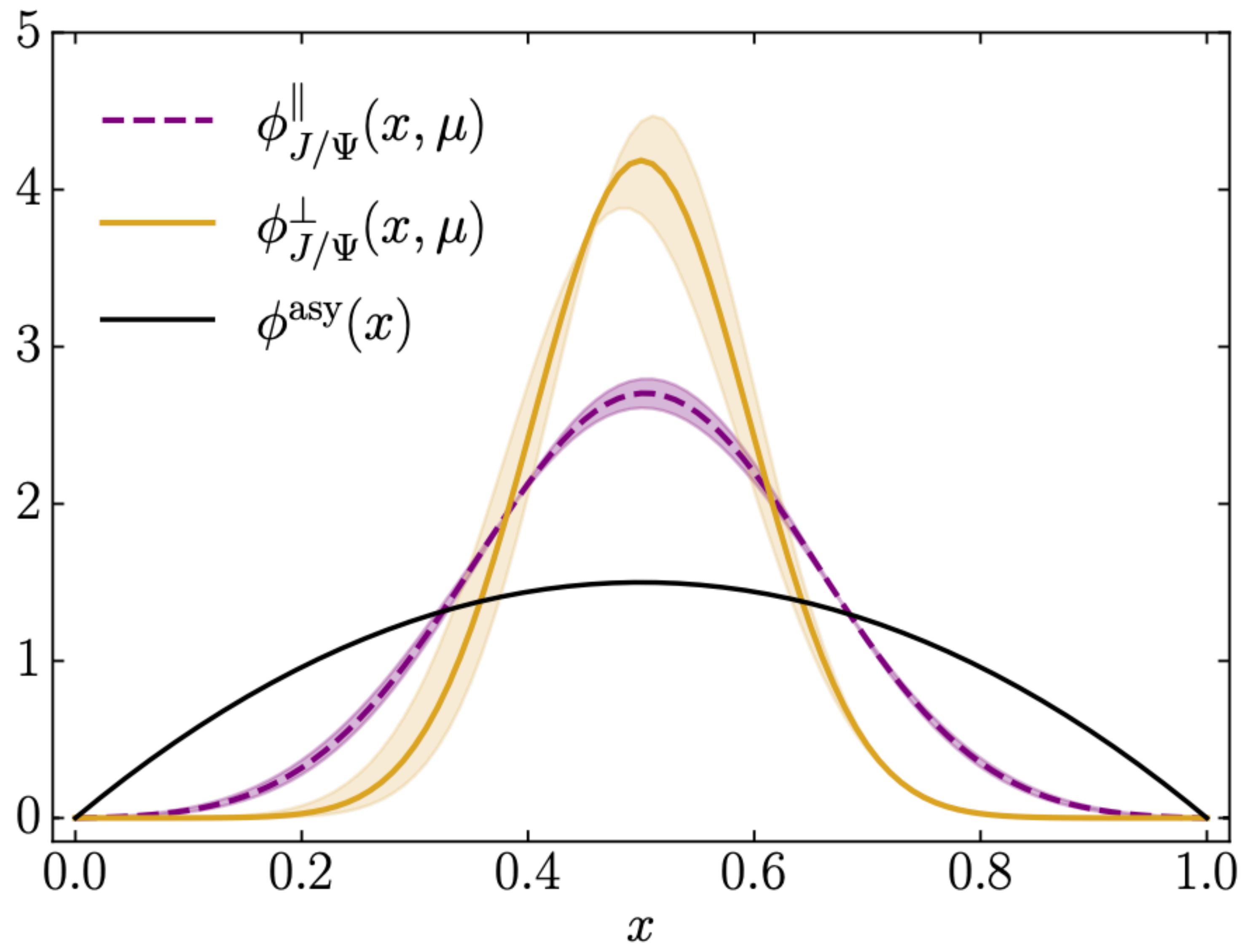
Vector Meson LCDA



J/ψ meson longitudinal and transverse LCDA

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Conclusions & Progress

- Much progress was made from QCD-based modeling toward more sophisticated truncation that preserve symmetries
- We obtain nonperturbative quark propagators and quark-antiquark bound states for flavored mesons satisfying chiral symmetry and Poincaré covariance.
- This approach reproduces very well the charmonium, bottonium, D and B meson mass spectrum and their weak decay constants (also excited states).
- The three-dimensional momentum landscape of light and heavy mesons is obtained from different light-front projection of their Bethe-Salpeter wave function and don't involve the calculation of diagrams.
- For all LCDA, PDF and TMD we can readily provide parametrized expressions.
- Current progress: *improve beyond-leading corrections in BSE kernel for heavy-light mesons; computing GPDs and gravitational form factors; extension to nucleons.*