Synchrotron Light, Beam Dynamics and Light Sources

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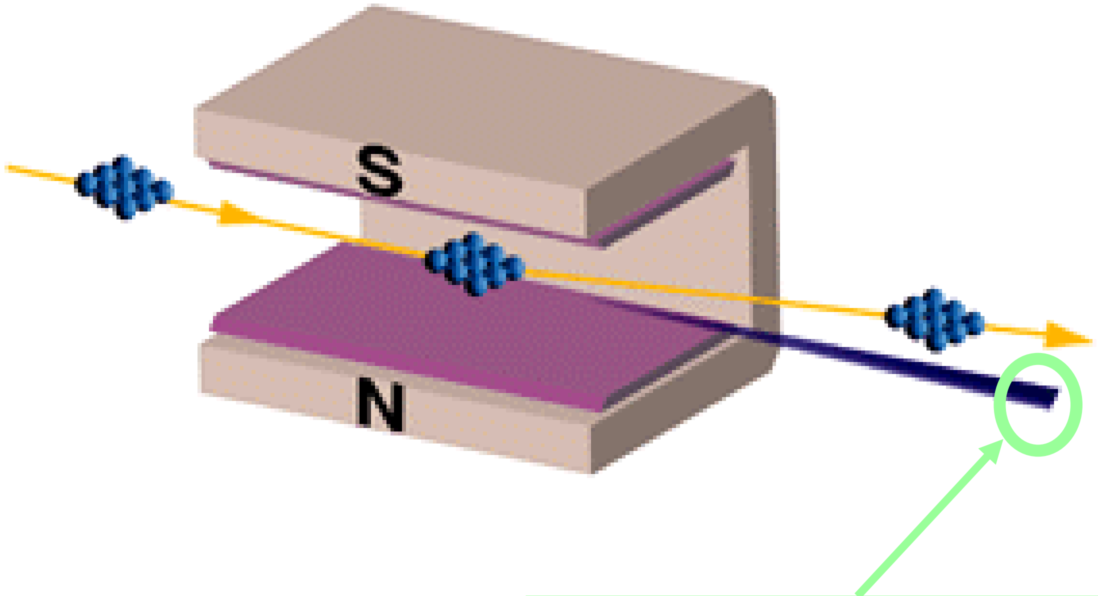
Curved orbit of electrons in magnetic field



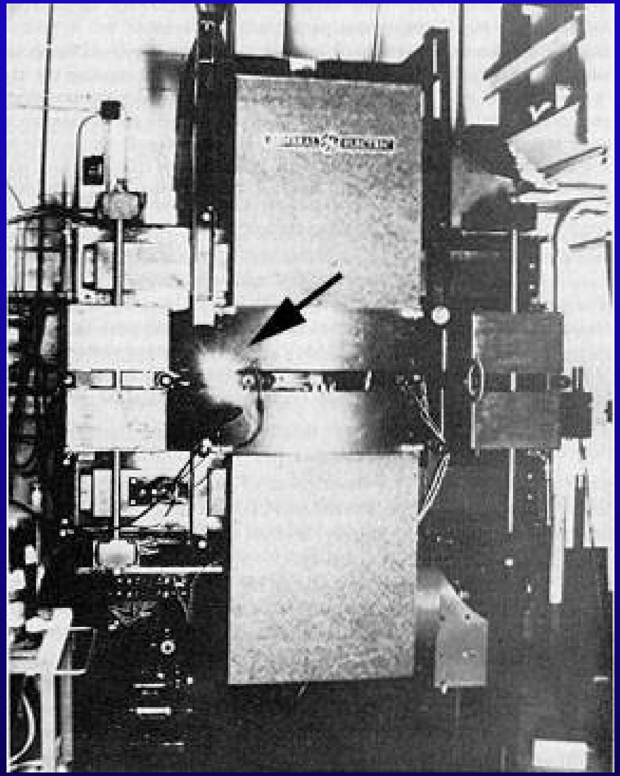
Accelerated charge → Electromagnetic radiation

Electromagnetic waves or photons

Crab Nebula
6000 light years away

First light observed
1054 AD

GE Synchrotron
New York State



First light observed
24 April, 1947

Synchrotron radiation: some dates

1873	Maxwell’s equations
1887	Hertz: electromagnetic waves
1898	Liénard: retarded potentials
1900	Wiechert: retarded potentials
1908	Schott: Adams Prize Essay

... waiting for accelerators …
1940: 2.3 MeV betatron,Kerst, Serber

Maxwell equations (poetry)

War es ein Gott, der diese Zeichen schrieb
Die mit geheimnisvoll verborg'nem Trieb
Die Kräfte der Natur um mich enthüllen
Und mir das Herz mit stiller Freude füllen.
Ludwig Boltzman

Was it a God whose inspiration
Led him to write these fine equations
Nature's fields to me he shows
And so my heart with pleasure glows.
translated by John P. Blewett

Synchrotron radiation: some dates

- 1873 Maxwell's equations
- 1887 Hertz: electromagnetic waves
- 1898 Liénard: retarded potentials
- 1900 Wiechert: retarded potentials
- 1908 Schott: Adams Prize Essay

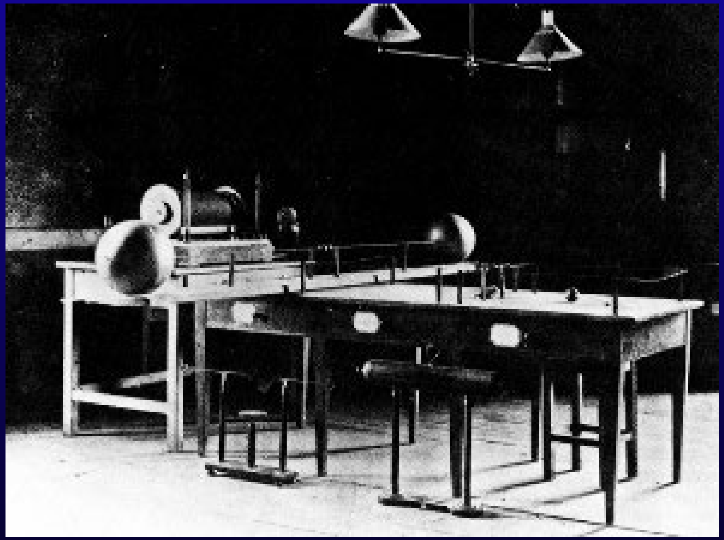
... waiting for accelerators …
1940: 2.3 MeV betatron,Kerst, Serber

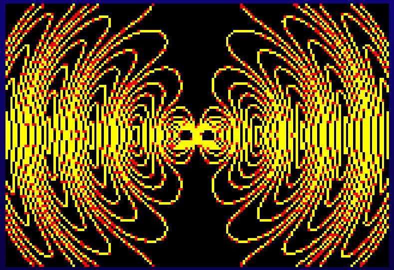
THEORETICAL UNDERSTANDING →

1873 **Maxwell's equations**

→ made evident that changing charge densities would result in electric fields that would radiate outward

1887 **Heinrich Hertz demonstrated such waves:**





It's of no use whatsoever[...] this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.

Synchrotron radiation: some dates

- 1873 Maxwell’s equations
- 1887 Hertz: electromagnetic waves
- 1898 Liénard: retarded potentials
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... waiting for accelerators …

1940: 2.3 MeV betatron,Kerst, Serber

Synchrotron radiation: some dates

- 1946 Blewett observes energy loss due to synchrotron radiation
  100 MeV betatron
- 1947 First visual observation of SR
  70 MeV **synchrotron**, GE Lab

NAME!

- 1949 Schwinger PhysRev paper

…

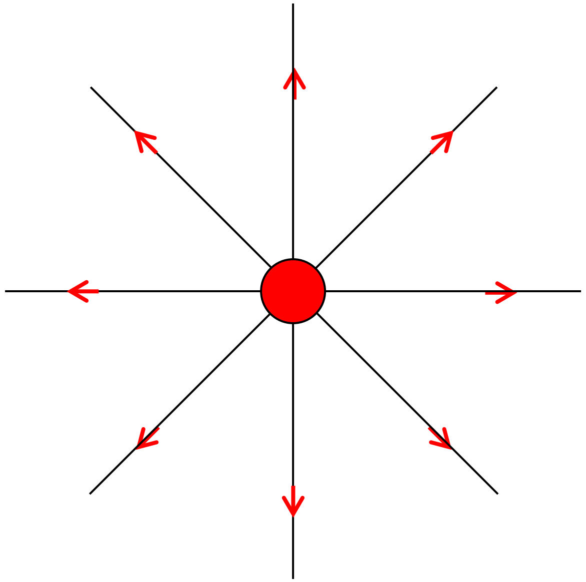
- 1976 Madey: first demonstration of Free Electron laser

Why do they radiate?

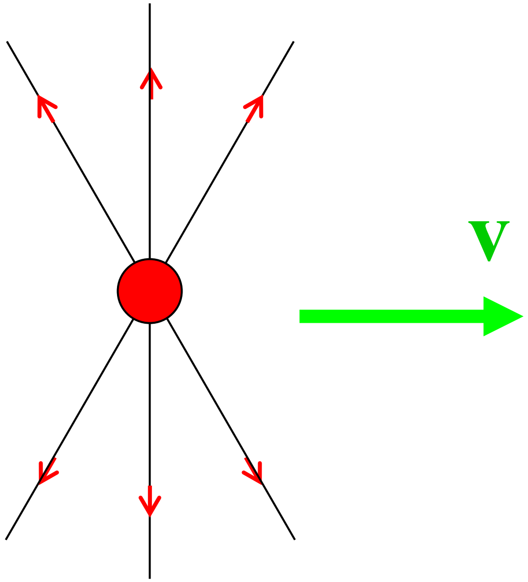
Synchrotron Radiation is
not as simple as it seems

… I will try to show
that it is much simpler

Charge at rest
Coulomb field, no radiation



Uniformly moving charge does not radiate



v = constant

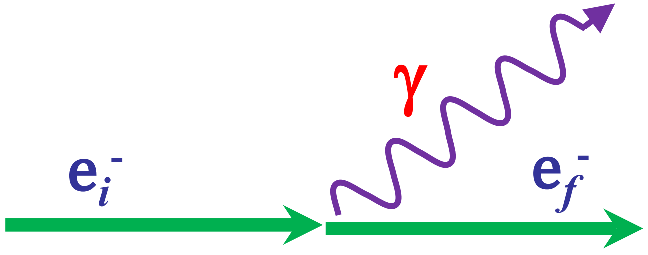
But! Cerenkov!

Free isolated electron cannot emit a photon

Easy proof using 4-vectors and relativity

- momentum conservation if a photon is emitted

$$\boldsymbol{P}_i = \boldsymbol{P}_f + \boldsymbol{P}_\gamma$$



- square both sides

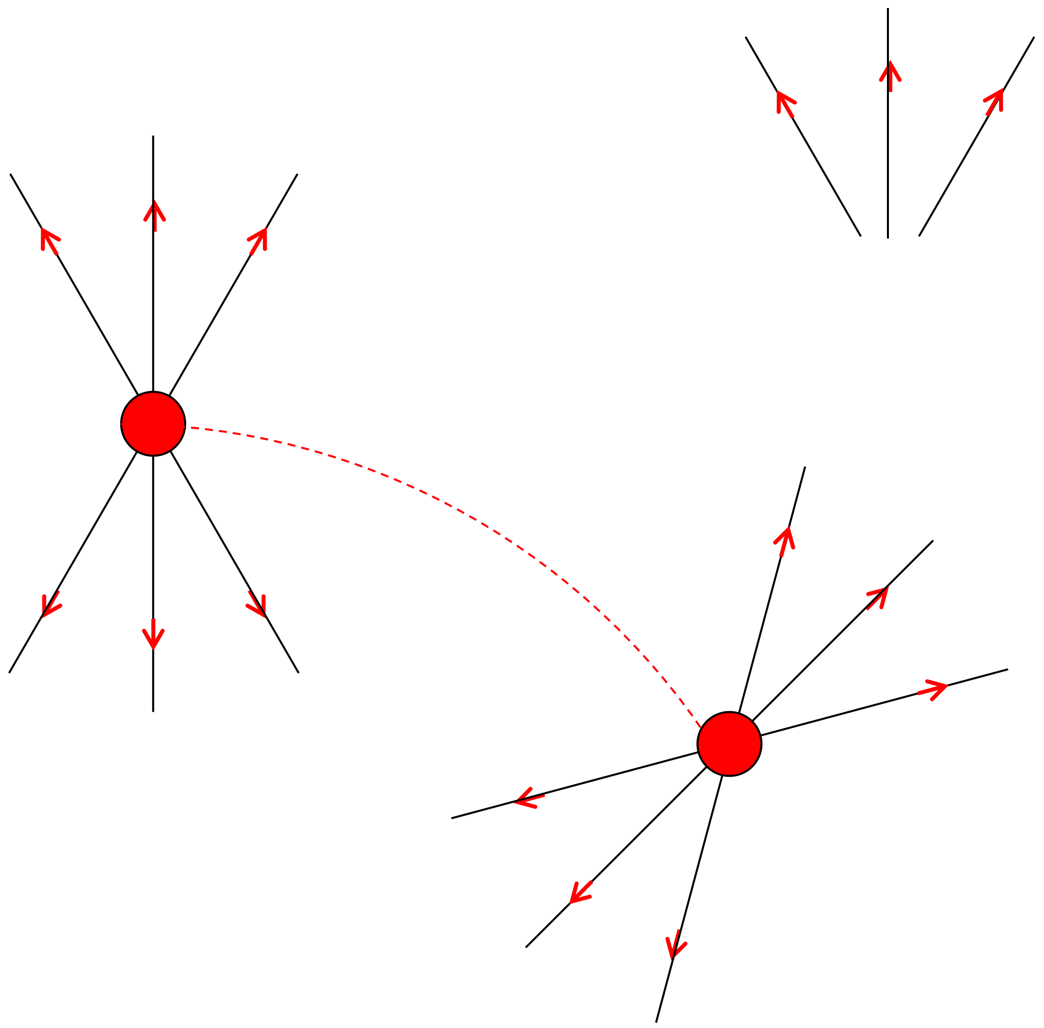
$$m^2 = m^2 + 2\boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma + 0 \;\Rightarrow\; \boldsymbol{P}_f \cdot \boldsymbol{P}_\gamma = 0$$

- in the rest frame of the electron

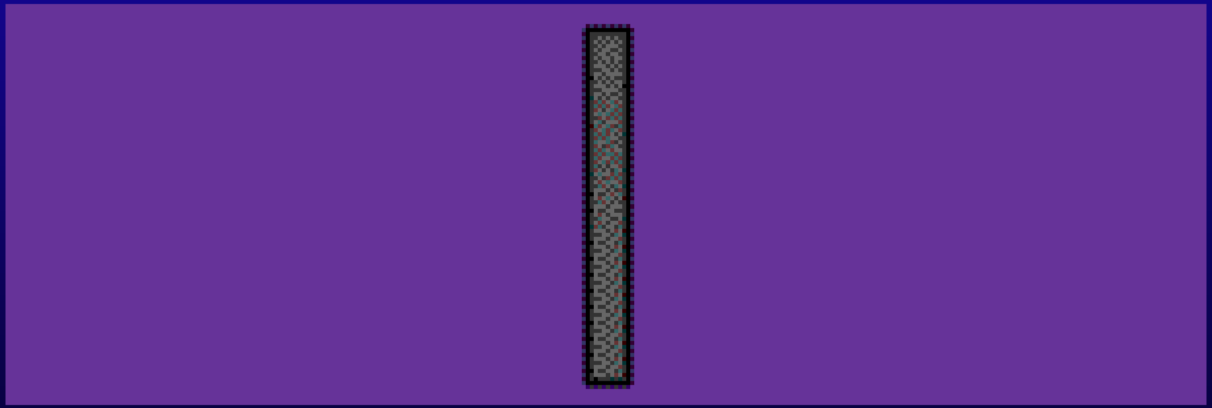
$$\boldsymbol{P}_f = (m, 0) \qquad \boldsymbol{P}_\gamma = (E_\gamma, p_\gamma)$$

this means that the photon energy must be zero.

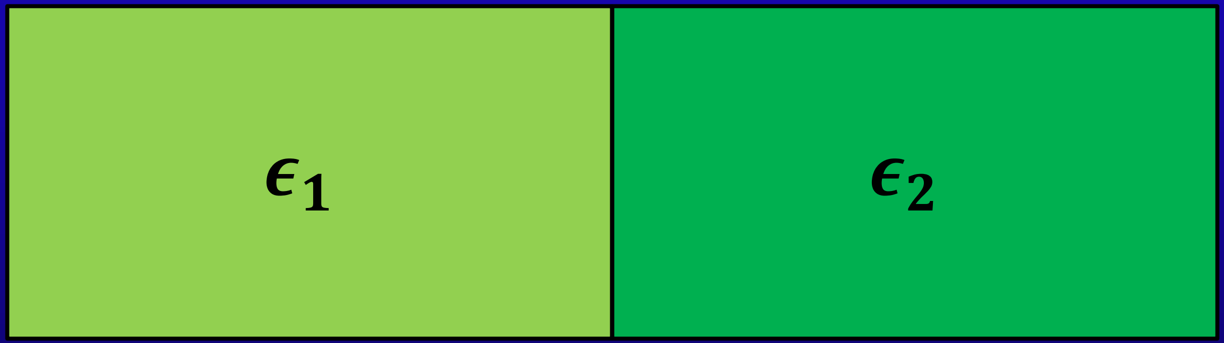
We need to separate the field from charge



Bremsstrahlung or “braking” radiation



Transition Radiation



$$c_1 = \frac{1}{\sqrt{\epsilon_1 \mu_1}} \qquad c_2 = \frac{1}{\sqrt{\epsilon_2 \mu_2}}$$

Liénard–Wiechert potentials

$$\varphi(t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[r\left(1-\vec{\mathbf{n}}\cdot\vec{\boldsymbol{\beta}}\right)\right]_{ret}} \qquad \vec{\mathbf{A}}(t) = \frac{q}{4\pi\varepsilon_0 c^2}\left[\frac{\vec{\mathbf{v}}}{r\left(1-\vec{\mathbf{n}}\cdot\vec{\boldsymbol{\beta}}\right)}\right]_{ret}$$

and the electromagnetic fields:

$$\nabla\cdot\vec{\mathbf{A}} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$$

(Lorentz gauge)

$$\vec{\mathbf{B}} = \nabla\times\vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\nabla\varphi - \frac{\partial\vec{\mathbf{A}}}{\partial t}$$

Fields of a moving charge

$$\vec{\mathbf{E}}(t) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{\mathbf{n}}-\vec{\boldsymbol{\beta}}}{\left(1-\vec{\mathbf{n}}\cdot\vec{\boldsymbol{\beta}}\right)^3\gamma^2}\cdot\boxed{\frac{\mathbf{1}}{\mathbf{r}^2}}\right]_{ret} +$$

"near field"

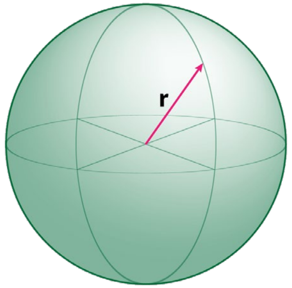
$$\frac{q}{4\pi\varepsilon_0 c}\left[\frac{\vec{\mathbf{n}}\times\left[\left(\vec{\mathbf{n}}-\vec{\boldsymbol{\beta}}\right)\times\boxed{\dot{\vec{\boldsymbol{\beta}}}}\right]}{\left(1-\vec{\mathbf{n}}\cdot\vec{\boldsymbol{\beta}}\right)^3\gamma^2}\cdot\boxed{\frac{\mathbf{1}}{\mathbf{r}}}\right]_{ret}$$

"far field"

$$\vec{\mathbf{B}}(\mathrm{t}) = \frac{1}{\mathrm{c}}\left[\vec{\mathbf{n}}\times\vec{\mathbf{E}}\right]$$

Energy flow integrated over a sphere

$$Power \sim E^2 \cdot \text{Area}$$



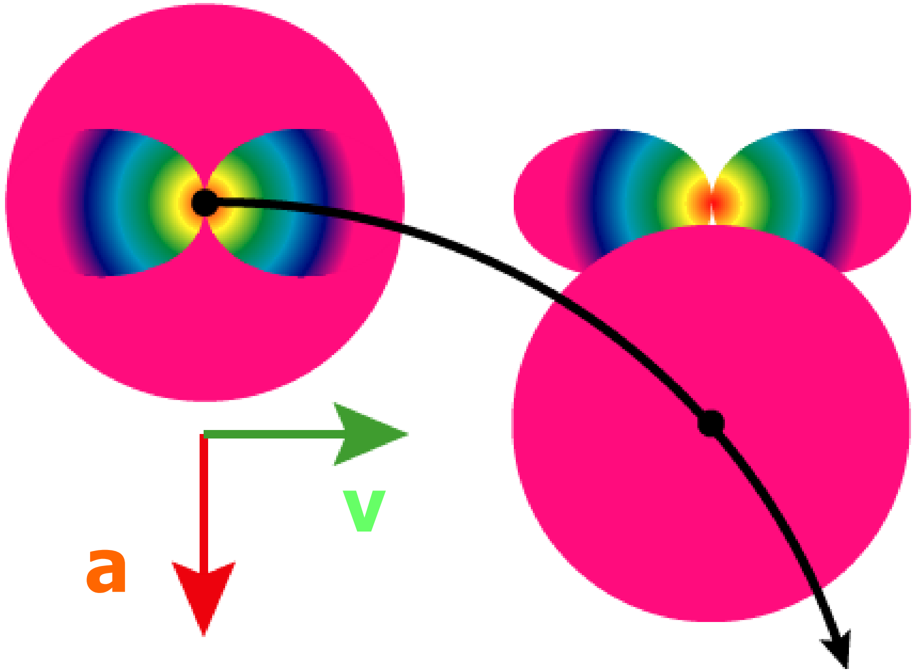
$$A = 4\pi r^2$$

Near field $$P \propto \frac{1}{r^4} r^2 \propto \frac{1}{r^2}$$

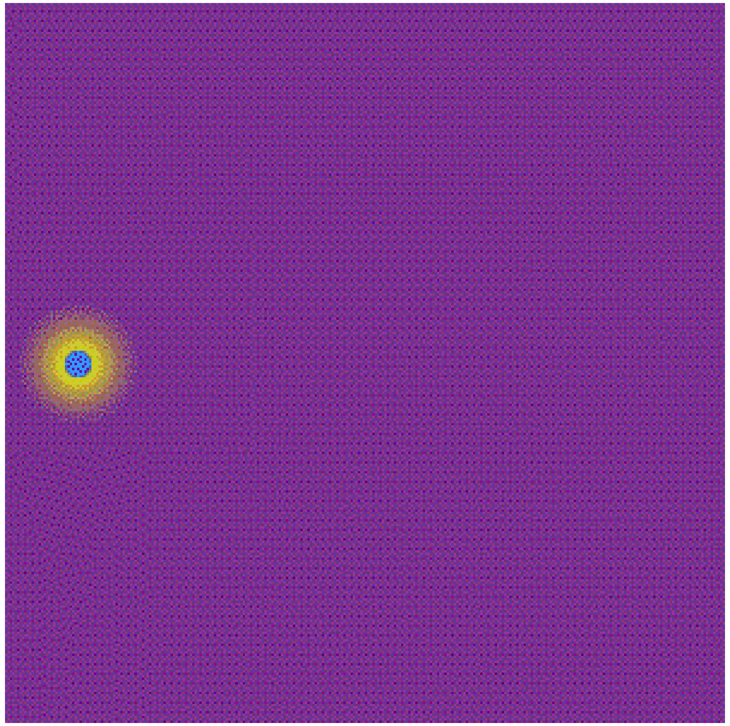
Far field $$P \propto \frac{1}{r^2} r^2 \propto const$$

Radiation = constant flow of energy to infinity

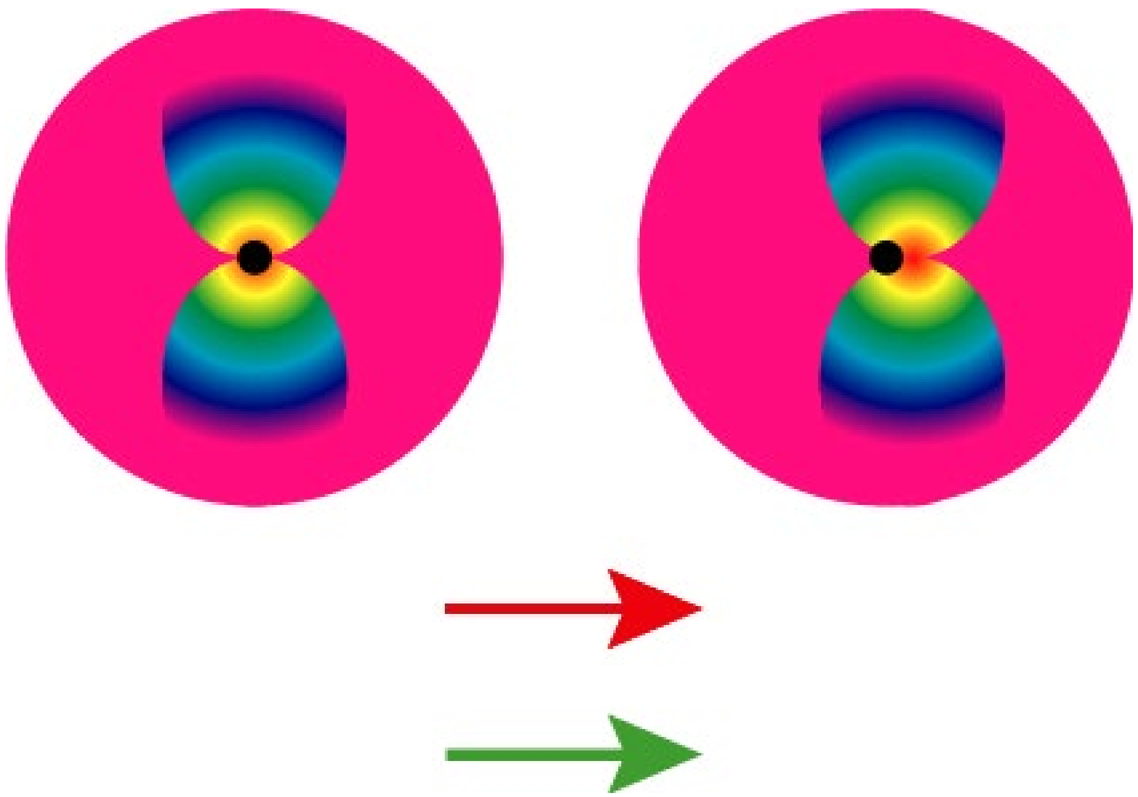
Transverse acceleration



Radiation field quickly separates itself from the Coulomb field



Longitudinal acceleration

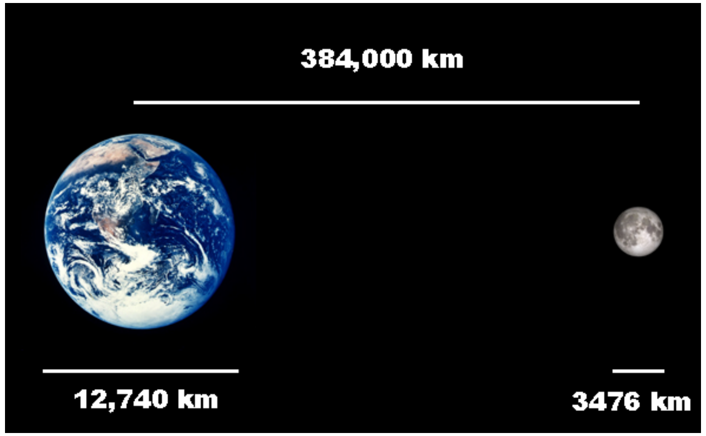


Radiation field cannot separate itself from the Coulomb field

Synchrotron Radiation
Basic Properties

Beams of ultra-relativistic particles: e.g. a race to the Moon

An electron with energy of a few GeV emits a photon… a race to the Moon!



$$\Delta t = \frac{L}{\beta c} - \frac{L}{c} = \frac{L}{\beta c}(1-\beta) \sim \frac{L}{\beta c} \cdot \frac{1}{2\gamma^2}$$

$$\beta \equiv \frac{v}{c}$$

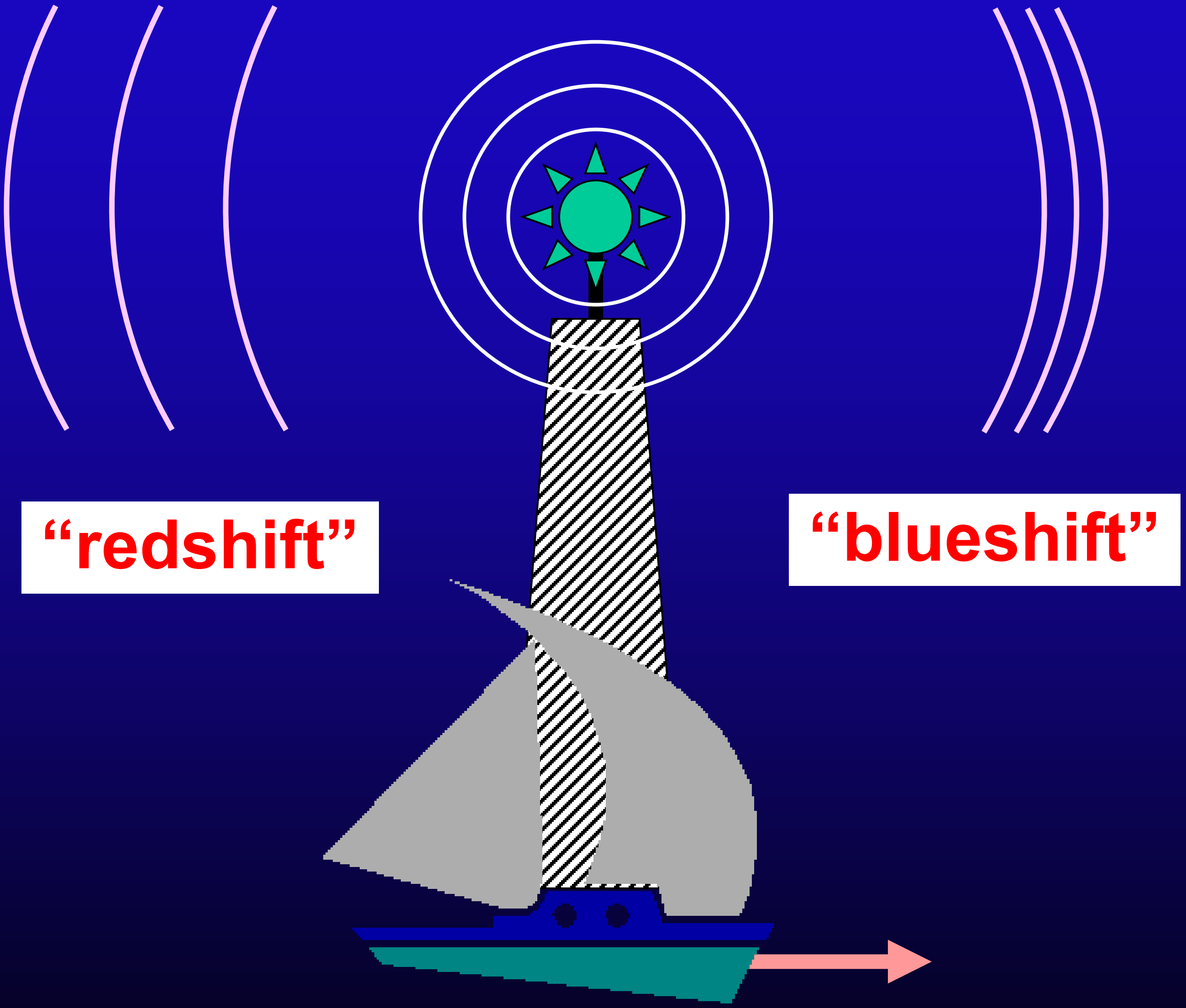
Electron will lose

- by only 8 meters
- the race will last only 1.3 seconds

$$\gamma \equiv \frac{E}{mc^2} = \frac{1}{\sqrt{1-\beta^2}}$$

$$\Delta L = L(1-\beta) \cong \frac{L}{2\gamma^2}$$

Moving Source of Waves: Doppler effect

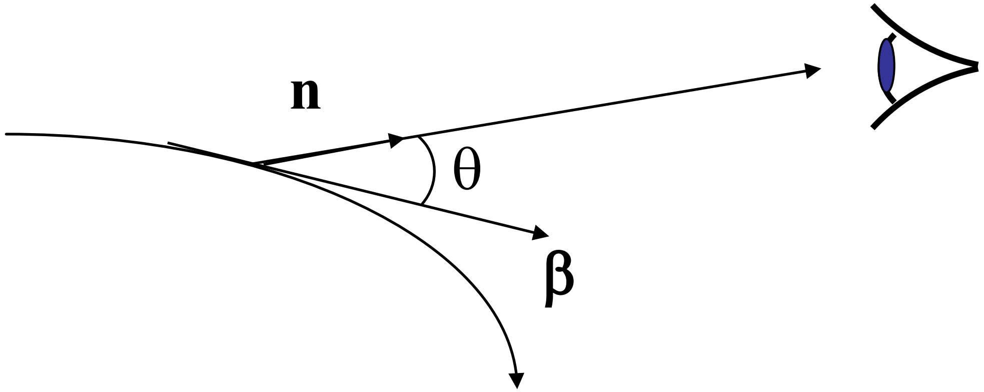


“redshift”

“blueshift”

Time compression

Electron with velocity β emits a wave with period T_{emit} while the observer sees a different period T_{obs} because the electron was moving towards the observer



$$T_{obs} = (1 - \mathbf{n} \cdot \boldsymbol{\beta})\, T_{emit}$$

The wavelength is shortened by the same factor

$$\lambda_{obs} = (1 - \beta \cos\theta)\, \lambda_{emit}$$

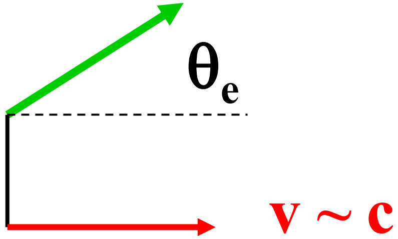
in ultra-relativistic case, looking along a tangent to the trajectory

$$\boxed{\lambda_{obs} = \frac{1}{2\gamma^2} \lambda_{emit}}$$

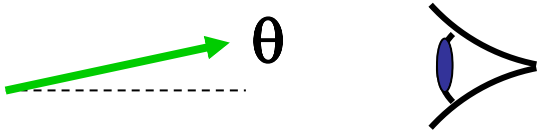
since

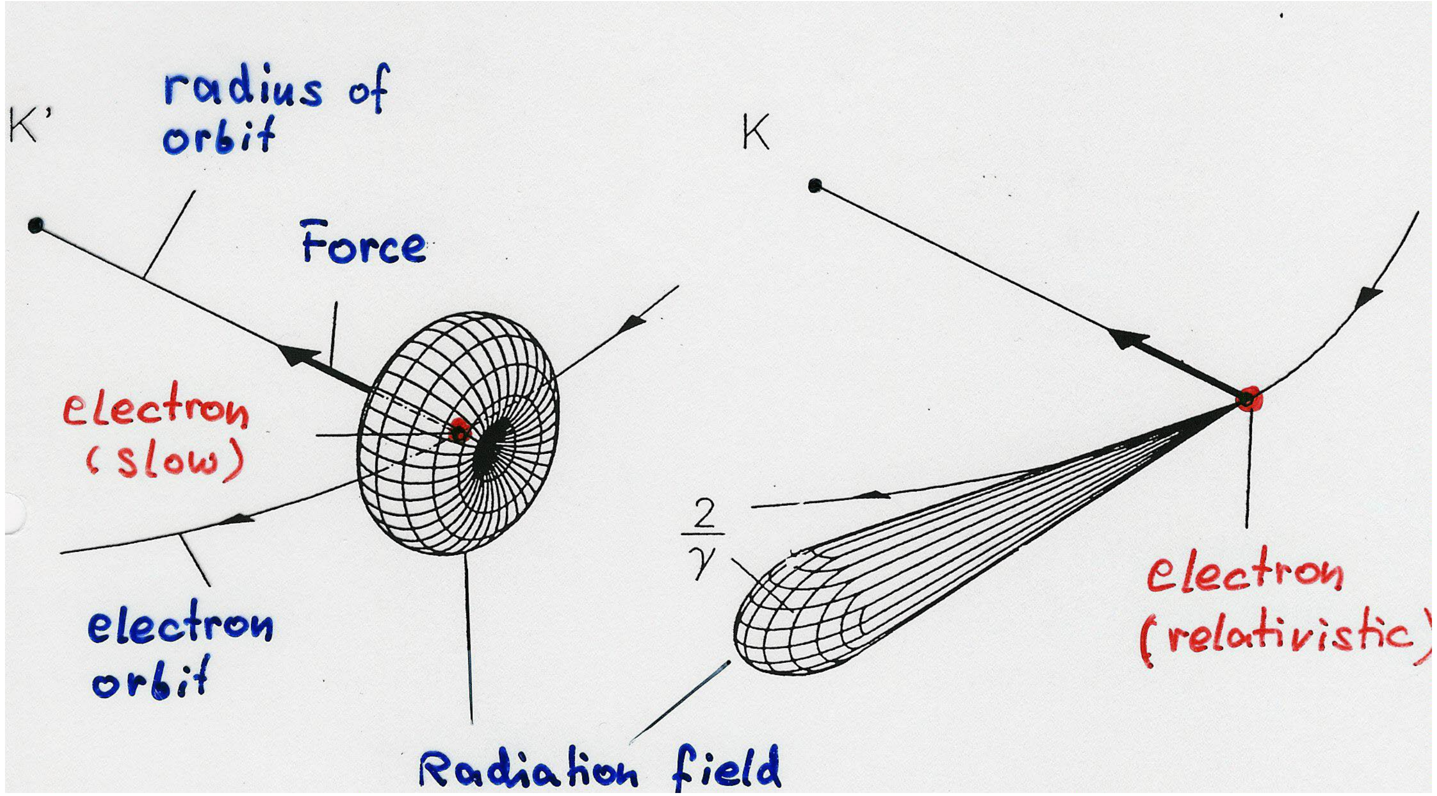
$$\boxed{1 - \beta = \frac{1 - \beta^2}{1 + \beta} \cong \frac{1}{2\gamma^2}}$$

Radiation is emitted into a narrow cone



$$\boxed{\theta = \frac{1}{\gamma} \cdot \theta_e}$$



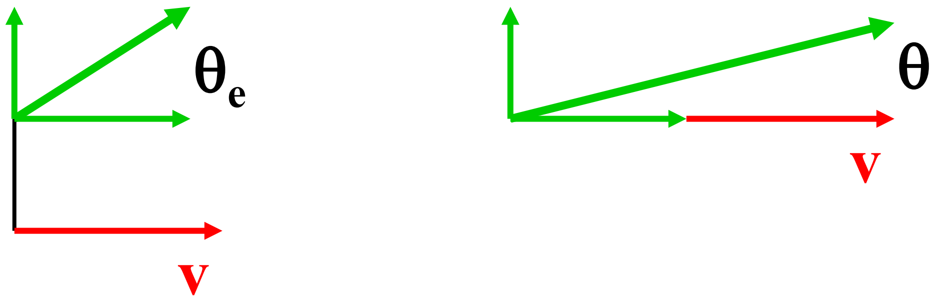


v << c

v ≈ c

Sound waves (non-relativistic)

Angular collimation



$$\theta = \frac{v_{s\perp}}{v_{s\parallel} + \mathbf{v}} = \frac{v_{s\perp}}{v_{s\parallel}} \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}}$$

Doppler effect (moving source of sound)

$$\lambda_{heard} = \lambda_{emitted}\left(1 - \frac{\mathbf{v}}{v_s}\right)$$

Synchrotron radiation power

Power emitted is proportional to:

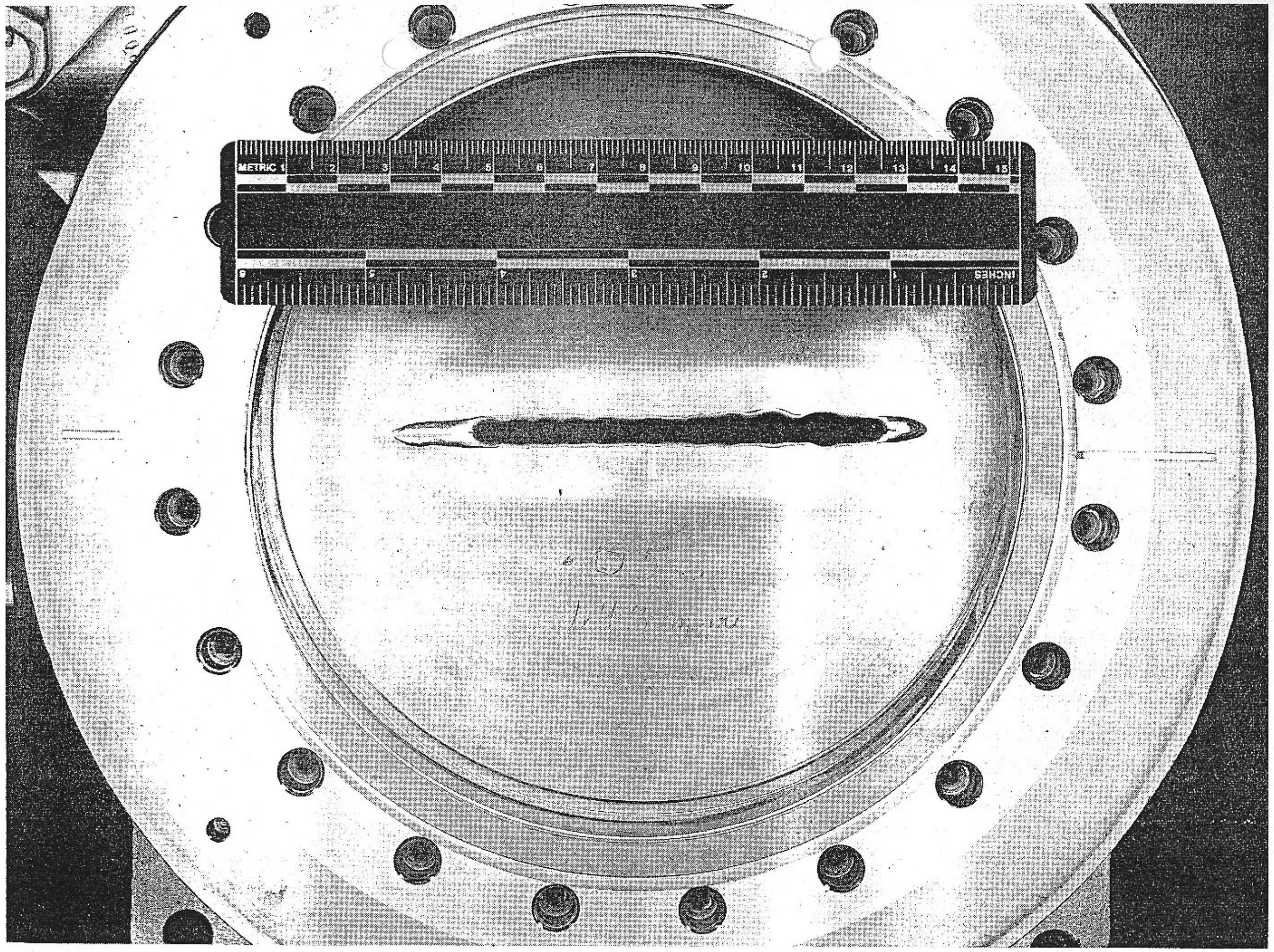
$$\boxed{P \propto E^2 B^2}$$

$$\boxed{P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}}$$

E = Energy!

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

The power is all too real!



ig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

Synchrotron radiation power

Power emitted is proportional to:

Energy

$$P \propto E^2 B^2$$

Magnetic field

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3}\alpha\hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5}\left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197\ \mathrm{Mev} \cdot \mathrm{fm}$$

Energy loss per turn:

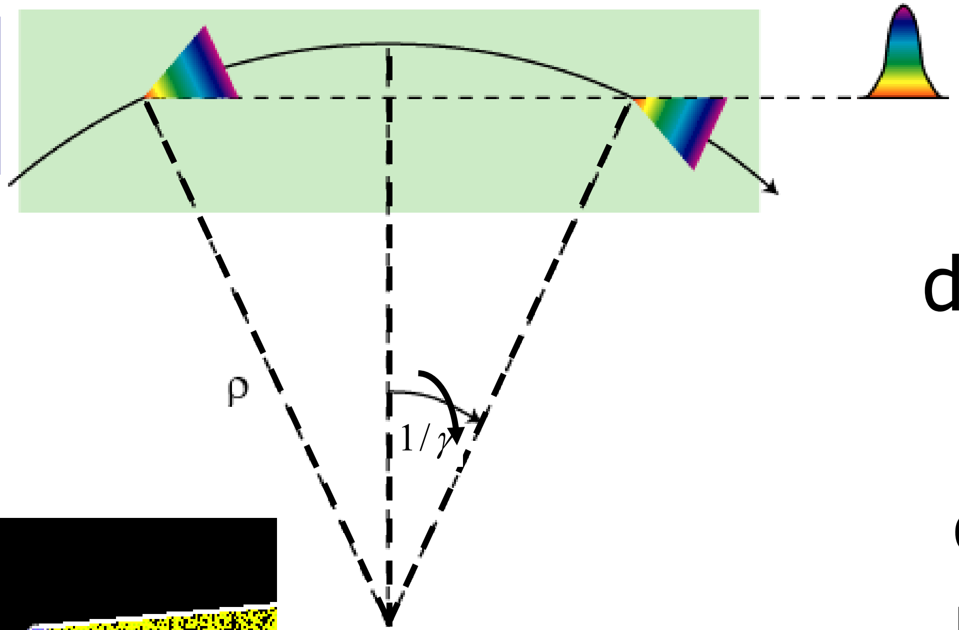
$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3}\alpha\hbar c\frac{\gamma^4}{\rho}$$

Typical frequency of synchrotron light

Due to extreme collimation of light observer sees only a small portion of electron trajectory **(a few mm)**

$$l \sim \frac{2\rho}{\gamma}$$

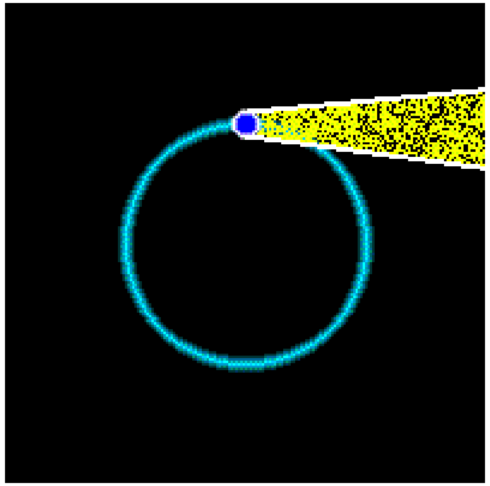


Pulse length:
difference in times it takes an electron and a photon to cover this distance

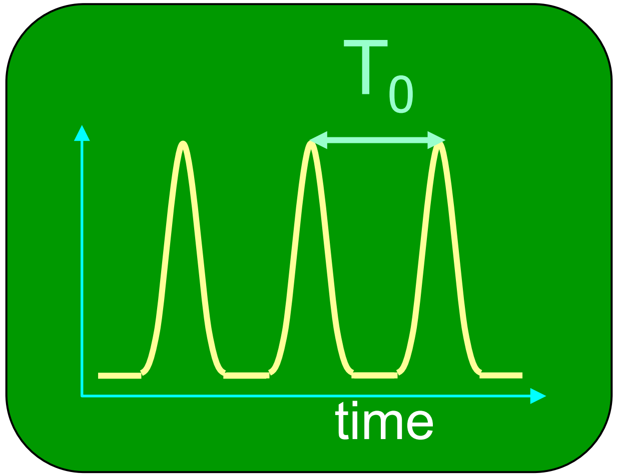
$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1-\beta)$$

$$\Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$

$$\omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0$$



Spectrum of synchrotron radiation



- Synchrotron light comes in a series of flashes every T_0 (revolution period)

- the spectrum consists of harmonics of

$$\omega_0 = \frac{1}{T_0}$$

- flashes are extremely short: harmonics reach up to very high frequencies

$$\omega_{typ} \cong \gamma^3 \omega_0$$

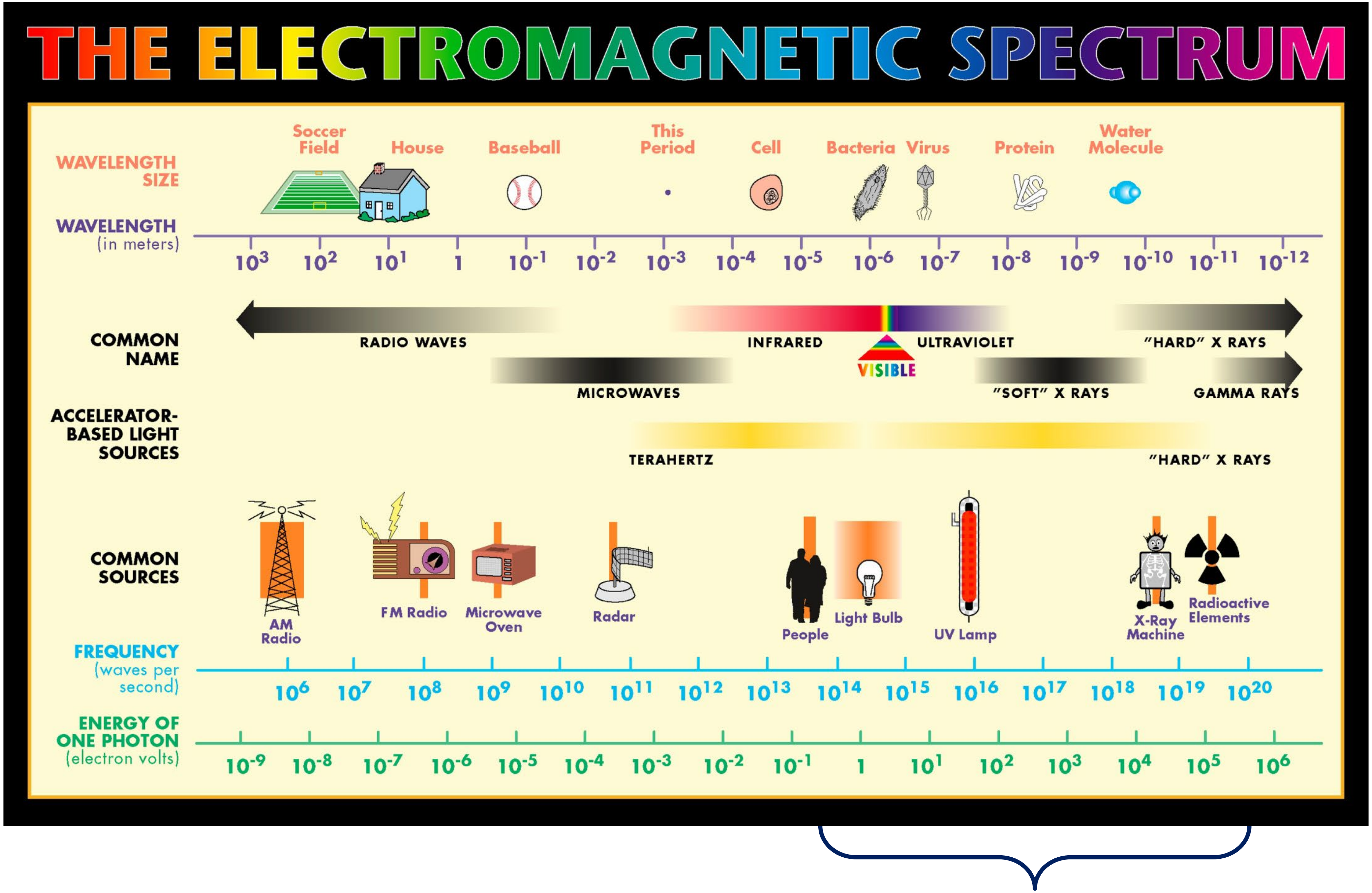
$$\omega_0 \sim 1\ \mathrm{MHz}$$

$$\gamma \sim 4000$$

$$\omega_{\mathrm{typ}} \sim 10^{16}\ \mathrm{Hz}\,!$$

- At high frequencies the individual harmonics overlap

continuous spectrum !



Wavelength continuously tunable !

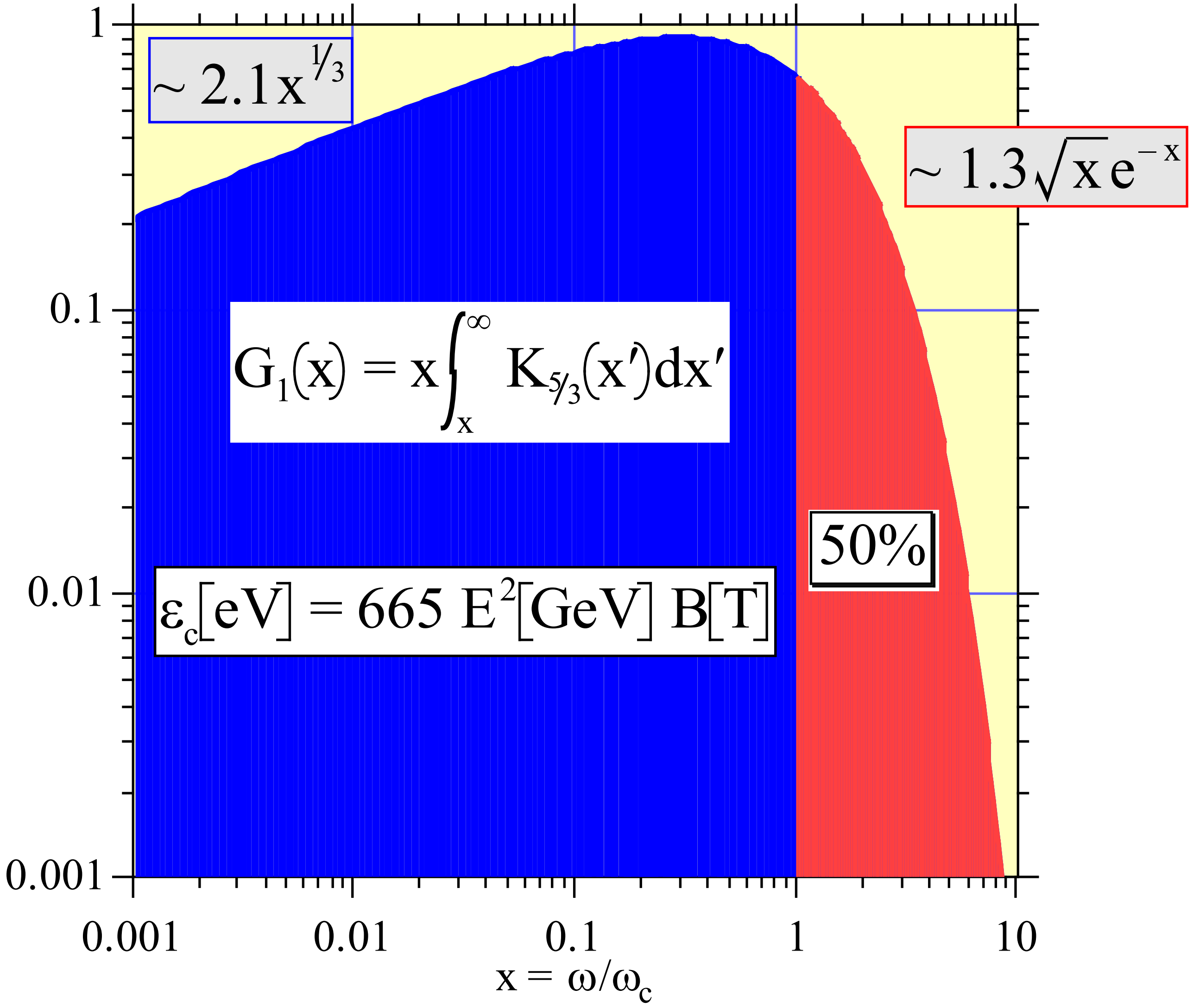
$$\boxed{\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)}$$

$$S(x) = \frac{9\sqrt{3}}{8\pi}\, x \int_x^\infty K_{5/3}(x')\,dx'$$

$$\int_0^\infty S(x')\,dx' = 1$$

$$P_{tot} = \frac{2}{3}\hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

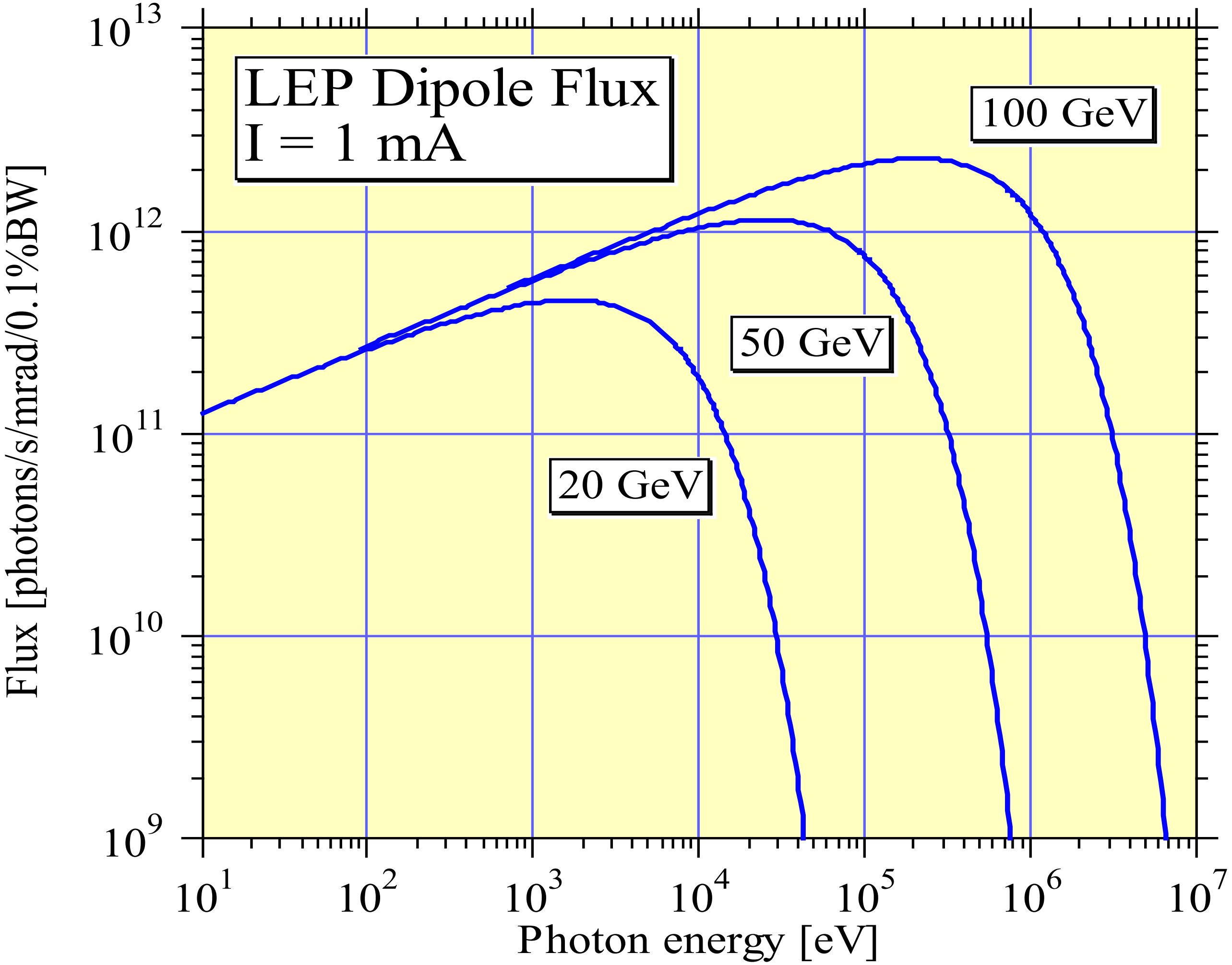
$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$



$$G_1(x) = x\int_x^\infty K_{5/3}(x')\,dx'$$

$$\varepsilon_c[\text{eV}] = 665\ E^2[\text{GeV}]\ B[\text{T}]$$

Synchrotron radiation flux for different electron energies



Useful books and references

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Cambridge University Press 2004

A. W. Chao, *Lectures on Accelerator Physics*, World Scientific 2020

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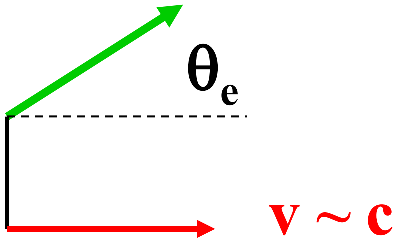
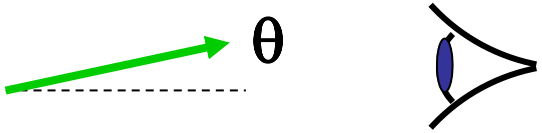
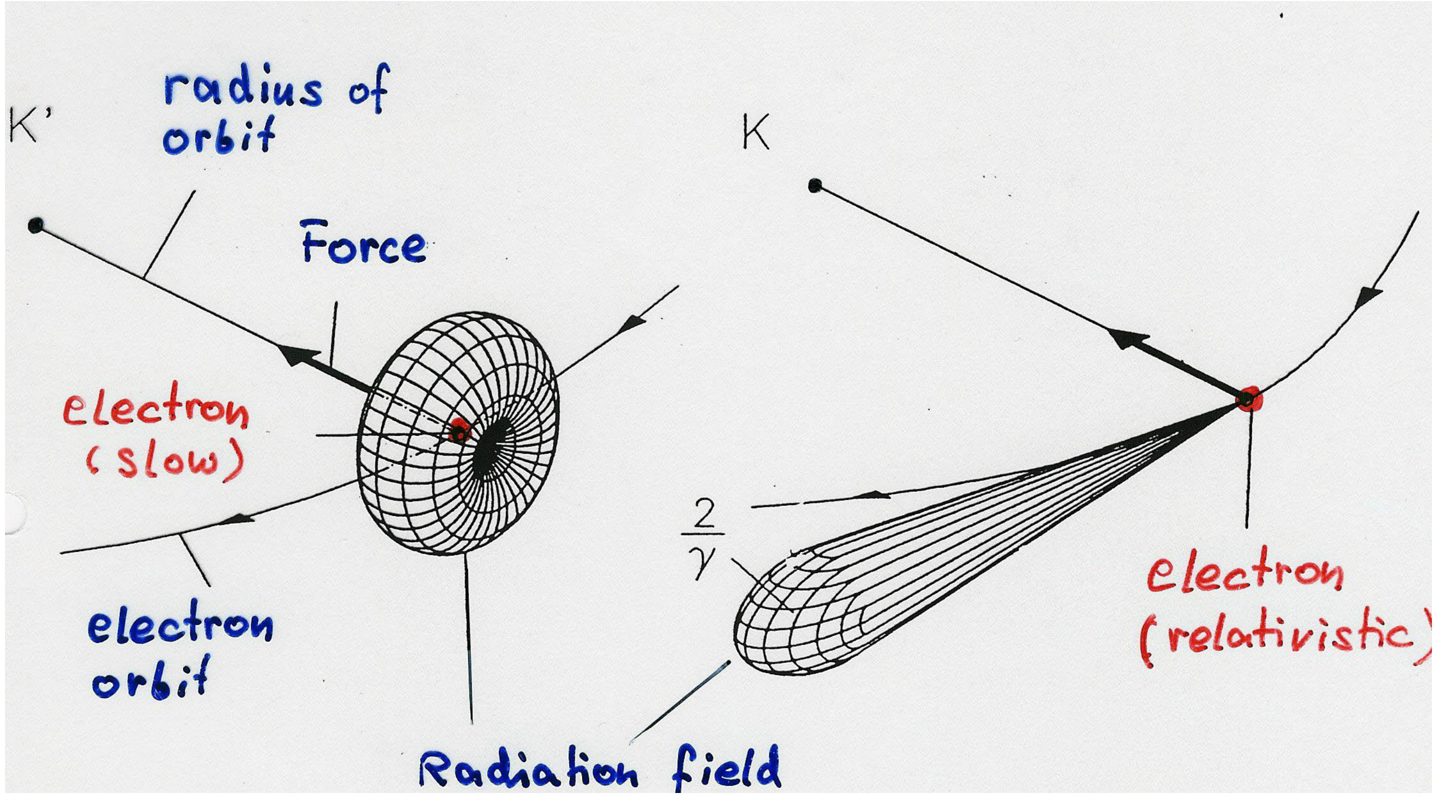
World Scientific 2013

Radiation is emitted into a narrow cone



$$\boxed{\theta = \frac{1}{\gamma} \cdot \theta_e}$$





v << c

v ≈ c

Radiation effects in electron storage rings

Average radiated power restored by RF

- Electron loses energy each turn to synchrotron radiation $\boxed{U_0 \cong 10^{-3} \text{ of } E_0}$
- RF cavities accelerate electrons back to the nominal energy $\boxed{V_{RF} > U_0}$

Radiation damping

- Average rate of energy loss produces **DAMPING** of electron oscillations in all three degrees of freedom (if properly arranged!)

Quantum fluctuations

- Statistical fluctuations in energy loss (from quantized emission of radiation) produce **RANDOM EXCITATION** of these oscillations

Equilibrium distributions

- The balance between the damping and the excitation of the electron oscillations determines the equilibrium distribution of particles in the beam

Radiation damping

Transverse oscillations

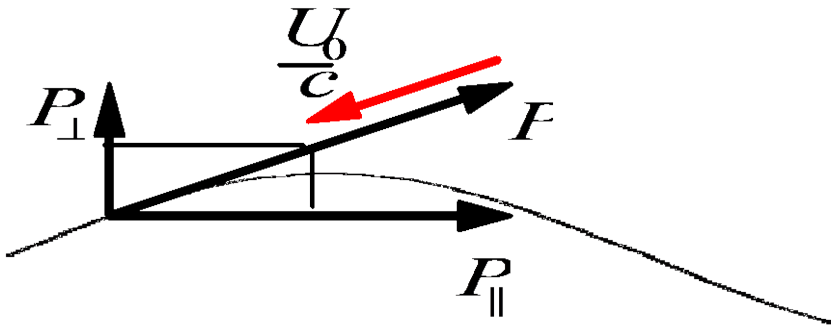
Average energy loss and gain per turn

- Every turn electron radiates small amount of energy

$$E_1 = E_0 - U_0 = E_0\left(1 - \frac{U_0}{E_0}\right)$$

- only the amplitude of the momentum changes

$$P_1 = P_0 - \frac{U_0}{c} = P_0\left(1 - \frac{U_0}{E_0}\right)$$

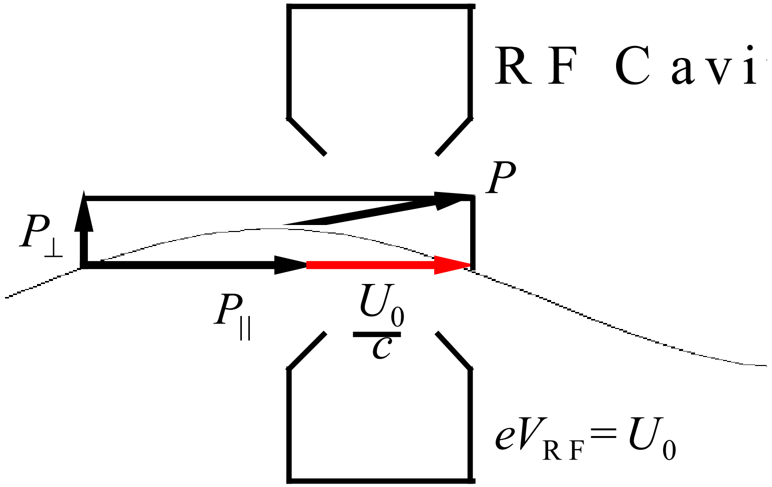


- Only the longitudinal component of the momentum is increased in the RF cavity

- Energy of betatron oscillation

$$E_\beta \propto A^2$$

$$A_1^2 = A_0^2\left(1 - \frac{U_0}{E_0}\right) \quad \text{or} \quad A_1 \cong A_0\left(1 - \frac{U_0}{2E_0}\right)$$



Damping of vertical oscillations

- But this is just the exponential decay law!

$$\frac{\Delta A}{A} = -\frac{U_0}{2E}$$

$$A = A_0 \cdot e^{-t/\tau}$$

- The oscillations are exponentially **damped** with the **damping time (milliseconds!)**

$$\tau = \frac{2ET_0}{U_0}$$

the time it would take particle to
'lose all of its energy'

- In terms of radiation power

$$\tau = \frac{2E}{P_\gamma}$$

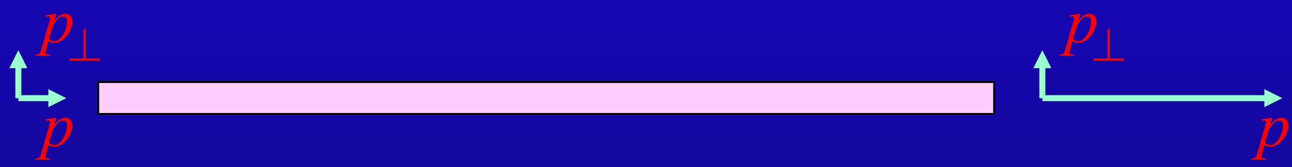
and since $P_\gamma \propto E^4$

$$\tau \propto \frac{1}{E^3}$$

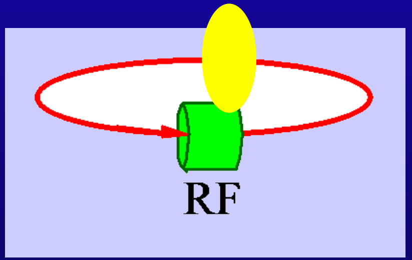
Adiabatic damping in linear accelerators

In a **linear accelerator**:

$$x' = \frac{p_\perp}{p} \quad \text{decreases} \quad \propto \frac{1}{E}$$

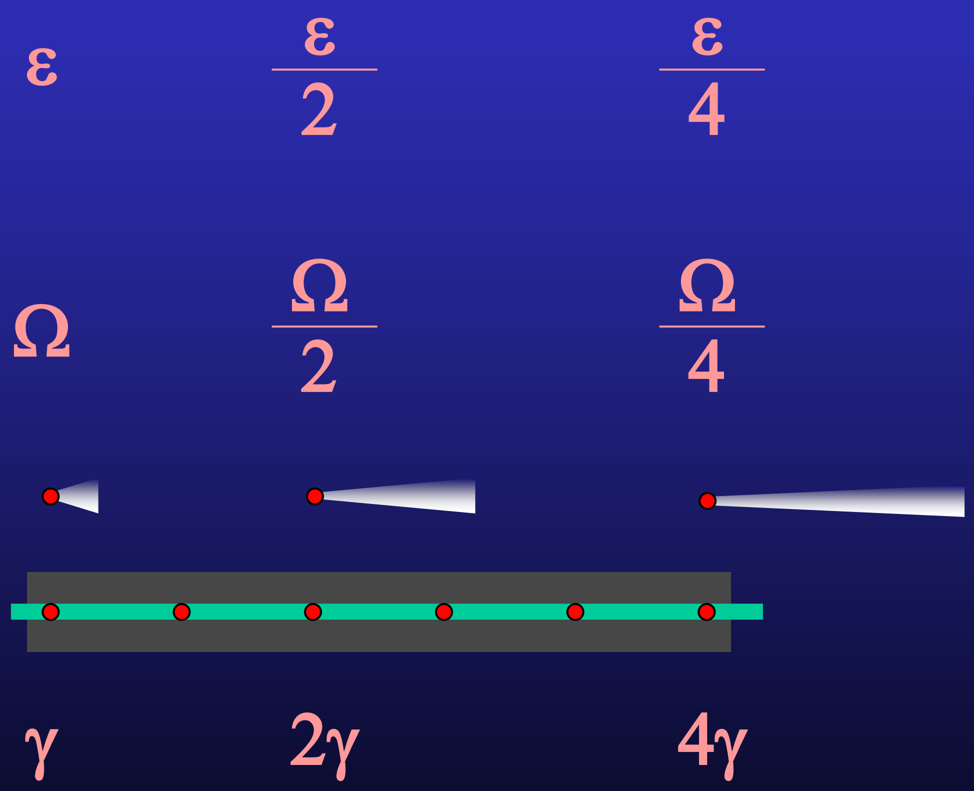


In a **storage ring** beam passes many times through same RF cavity



- **Clean loss of energy every turn (no change in x')**
- **Every turn is re-accelerated by RF (x' is reduced)**
- **Particle energy on average remains constant**

Emittance damping in linacs:



$\varepsilon \qquad \frac{\varepsilon}{2} \qquad \frac{\varepsilon}{4}$

$\Omega \qquad \frac{\Omega}{2} \qquad \frac{\Omega}{4}$

$\gamma \qquad 2\gamma \qquad 4\gamma$

$$\varepsilon \propto \frac{1}{\gamma}$$

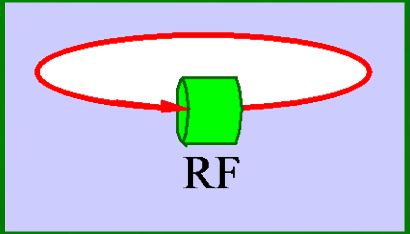
or

$$\gamma\varepsilon = \text{const.}$$

Radiation damping

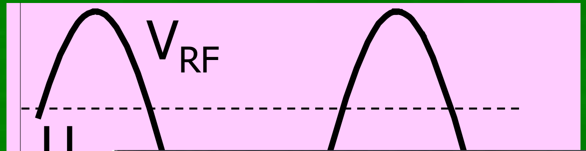
Longitudinal oscillations

Longitudinal motion: compensating radiation loss U_0



- RF cavity provides accelerating field with frequency
  - h – harmonic number

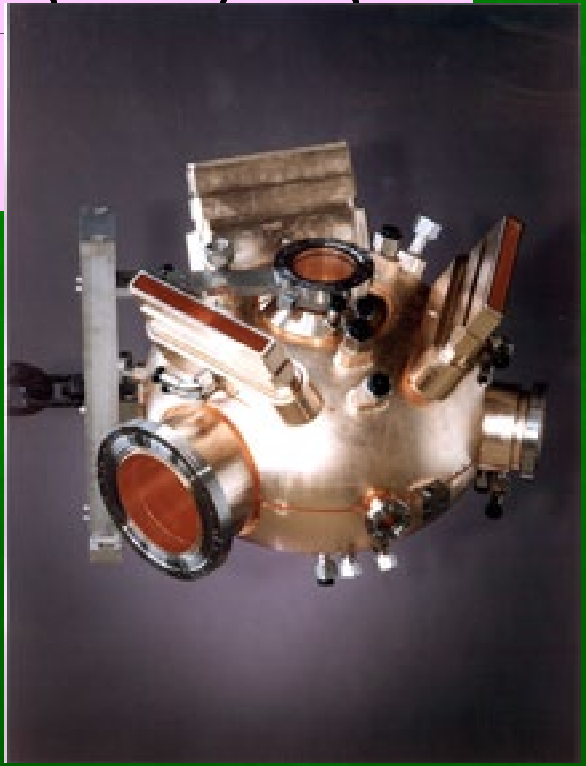
$$f_{RF} = h \cdot f_0$$



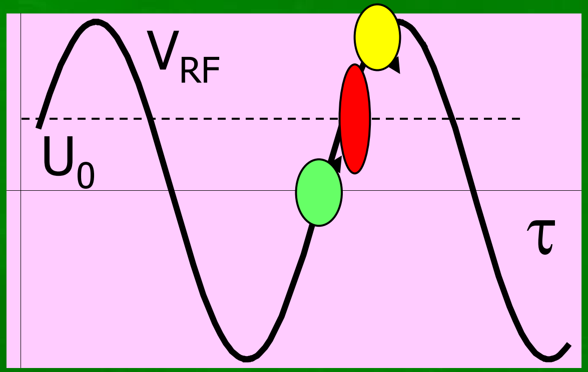
- The energy gain:

$$U_{RF} = eV_{RF}(\tau)$$

- Synchronous particle:
  - has design energy
  - gains from the RF on the average as much as it loses per turn U_0



Longitudinal motion: phase stability

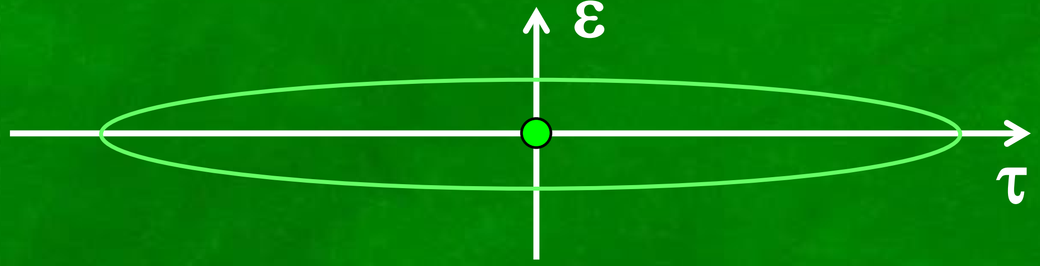


- Particle ahead of synchronous one
  - gets too much energy from the RF
  - goes on a longer orbit (not enough B)
    >> takes longer to go around
  - comes back to the RF cavity closer to synchronous part.

- Particle behind the synchronous one
  - gets too little energy from the RF
  - goes on a shorter orbit (too much B)
  - catches-up with the synchronous particle

Longitudinal motion: energy-time oscillations

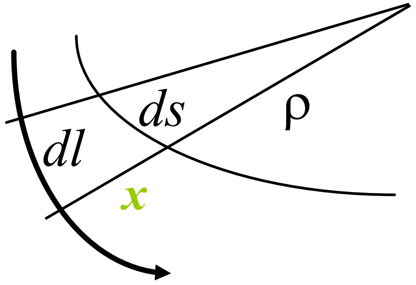
energy deviation from the design energy,
or the energy of the synchronous particle



longitudinal coordinate measured from the
position of the synchronous electron

Orbit Length

Length element depends on x



$$dl = \left(1 + \frac{x}{\rho}\right) ds$$

Horizontal displacement has two parts:

$$x = x_\beta + x_\varepsilon$$

- To first order x_β does not change L
- x_ε – has the same sign around the ring

Length of the off-energy orbi

$$L_\varepsilon = \oint dl = \oint \left(1 + \frac{x_\varepsilon}{\rho}\right) ds = L_0 + \Delta L$$

$$\Delta L = \delta \cdot \oint \frac{D(s)}{\rho(s)} ds \quad \text{where} \quad \delta = \frac{\Delta p}{p} = \frac{\Delta E}{E}$$

$$\frac{\Delta L}{L} = \alpha \cdot \delta$$

Something funny happens on the way around the ring...

Revolution time changes with energy

$$T_0 = \frac{L_0}{c\beta}$$

$$\frac{\Delta T}{T} = \frac{\Delta L}{L} - \frac{\Delta\beta}{\beta}$$

- Particle goes faster (not much!)

$$\frac{d\beta}{\beta} = \frac{1}{\gamma^2} \cdot \frac{dp}{p} \quad \text{(relativity)}$$

- while the orbit length increases (more!)

$$\frac{\Delta L}{L} = \alpha \cdot \frac{dp}{p}$$

- The "slip factor" $\eta \cong \alpha$ since $\alpha >> \frac{1}{\gamma^2}$

$$\frac{\Delta T}{T} = \left(\alpha - \frac{1}{\gamma^2}\right) \cdot \frac{dp}{p} = \eta \cdot \frac{dp}{p}$$

- Ring is above "transition energy"

$$\alpha \equiv \frac{1}{\gamma_{tr}^2}$$

isochronous ring: $\eta = 0 \ \text{ or } \ \gamma = \gamma_{tr}$

Not only accelerators work above transition

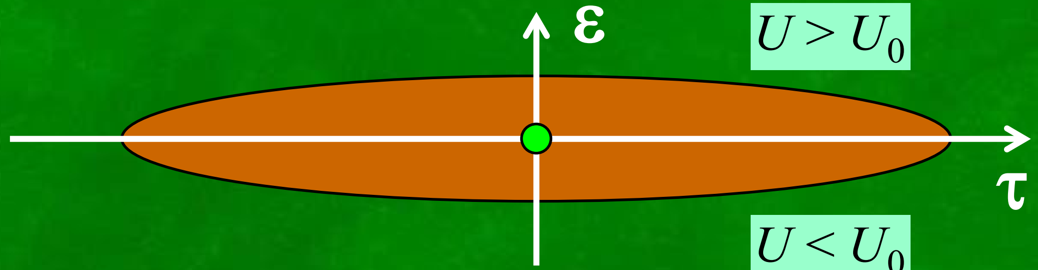
Dante Aligieri
Divine Comedy

Longitudinal motion: damping of synchrotron oscillations

$P_\gamma \propto \boldsymbol{E}^2 B^2$

During one period of synchrotron oscillation:

- when the particle is in the upper half-plane, it loses more energy per turn, its energy gradually reduces



- when the particle is in the lower half-plane, it loses less energy per turn, but receives U_0 on the average, so its energy deviation gradually reduces

The synchrotron motion is damped

- the phase space trajectory is spiraling towards the origin

Robinson theorem: Damping partition numbers

- Transverse betatron oscillations are damped with

$$\tau_x = \tau_z = \frac{2ET_0}{U_0}$$

- Synchrotron oscillations are damped twice as fast

$$\tau_\varepsilon = \frac{ET_0}{U_0}$$

- The total amount of damping (Robinson theorem) depends only on energy and loss per turn

$$\boxed{\frac{1}{\tau_x} + \frac{1}{\tau_y} + \frac{1}{\tau_\varepsilon} = \frac{2U_0}{ET_0} = \frac{U_0}{2ET_0}\left(J_x + J_y + J_\varepsilon\right)}$$

the sum of the partition numbers $\boxed{J_x + J_z + J_\varepsilon = 4}$

Equilibrium beam sizes

Radiation effects in electron storage rings

Average radiated power restored by RF

- Electron loses energy each turn to synchrotron radiation $\boxed{U_0 \cong 10^{-3} \text{ of } E_0}$
- RF cavities accelerate electrons back to the nominal energy $\boxed{V_{RF} > U_0}$

Radiation damping

- Average rate of energy loss produces **DAMPING** of electron oscillations in all three degrees of freedom (if properly arranged!)

Quantum fluctuations

- Statistical fluctuations in energy loss (from quantized emission of radiation) produce **RANDOM EXCITATION** of these oscillations

Equilibrium distributions

- The balance between the damping and the excitation of the electron oscillations determines the equilibrium distribution of particles in the beam

Quantum nature of synchrotron radiation

Damping only

- If damping was the whole story, the beam emittance (size) would shrink to microscopic dimensions!*
- Lots of problems! (e.g. **coherent radiation**)

---

* How small? On the order of electron wavelength

$$E = \gamma mc^2 = h\nu = \frac{hc}{\lambda_e} \quad \Rightarrow \quad \lambda_e = \frac{1}{\gamma}\frac{h}{mc} = \frac{\lambda_C}{\gamma}$$

$\lambda_C = 2.4 \cdot 10^{-12} m$ – Compton wavelength

Diffraction limited electron emittance

$$\varepsilon \geq \frac{\lambda_C}{4\pi\gamma}\left(\times N^{1/3} \text{ – fermions}\right)$$

Quantum nature of synchrotron radiation

Quantum fluctuations

- Because the radiation is emitted in quanta, radiation itself takes care of the problem!

- It is sufficient to use quasi-classical picture:
  - » *Emission time is very short*
  - » *Emission times are statistically independent (each emission - only a small change in electron energy)*

Purely stochastic (Poisson) process

Visible quantum effects

I have always been somewhat amazed that a purely quantum effect can have gross macroscopic effects in large machines;

and, even more,

that Planck's constant has just the right magnitude needed to make practical the construction of large electron storage rings.

A significantly larger or smaller value of $\hbar$

would have posed serious -- perhaps insurmountable -- problems for the realization of large rings.

Mathew Sands

Quantum excitation of energy oscillations

Photons are emitted with typical energy $$u_{ph} \approx \hbar\omega_{typ} = \hbar c \frac{\gamma^3}{\rho}$$

at the rate (photons/second) $$\mathcal{N} = \frac{P_\gamma}{u_{ph}}$$

Fluctuations in this rate excite oscillations

During a small interval Δt electron emits photons $N = \mathcal{N} \cdot \Delta t$

losing energy of $N \cdot u_{ph}$

Actually, because of fluctuations, the number is $N \pm \sqrt{N}$

resulting in **spread in energy loss** $\pm\sqrt{N} \cdot u_{ph}$

For large time intervals RF compensates the energy loss, providing damping towards the design energy E_0

Steady state: typical deviations from E_0

≈ typical fluctuations in energy during a damping time τ_ε

Equilibrium energy spread: rough estimate

We then expect the rms energy spread to be $\sigma_\varepsilon \approx \sqrt{N \cdot \tau_\varepsilon} \cdot u_{ph}$

and since $\tau_\varepsilon \approx \frac{E_0}{P_\gamma}$ and $P_\gamma = N \cdot u_{ph}$

$\sigma_\varepsilon \approx \sqrt{E_0 \cdot u_{ph}}$ geometric mean of the electron and photon energies!

Relative energy spread can be written then as:

$$\frac{\sigma_\varepsilon}{E_0} \approx \gamma \sqrt{\frac{\lambda\!\!\!^{-}e}{\rho}}$$

$$\lambda\!\!\!^{-}e = \frac{\hbar}{m_e c} \simeq 4 \cdot 10^{-13} m$$

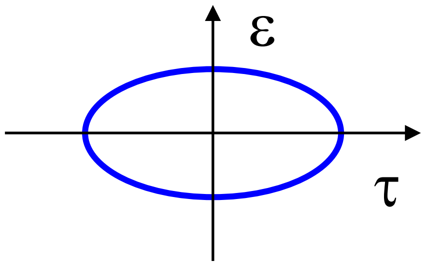
it is roughly constant for all rings

$$\frac{\sigma_\varepsilon}{E_0} \sim const \sim 10^{-3}$$

- typically $\rho \propto E^2$

Equilibrium bunch length

Bunch length is related to the energy spread



- Energy deviation and time of arrival
(or position along the bunch)
are conjugate variables (synchrotron oscillations)

$$\hat{\tau} = \frac{\alpha}{\Omega_s}\left(\frac{\hat{\varepsilon}}{E}\right)$$

$$\boxed{\sigma_\tau = \frac{\alpha}{\Omega_s}\left(\frac{\sigma_\varepsilon}{E}\right)}$$

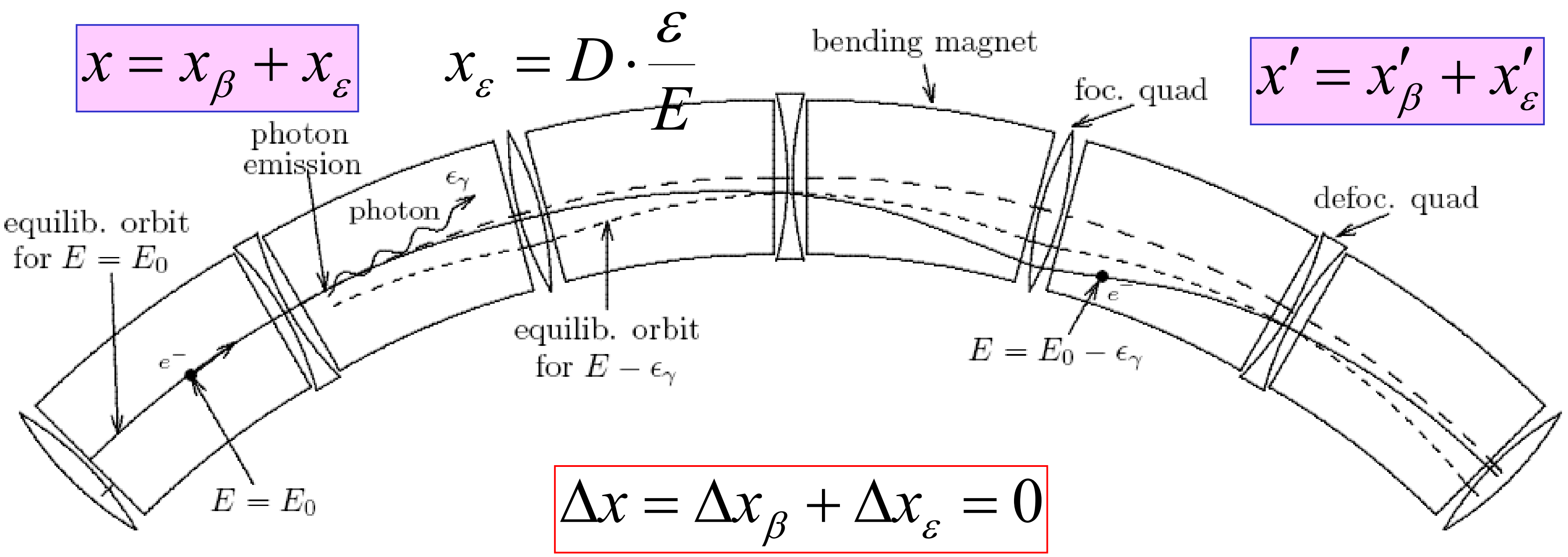
- recall that $\boxed{\Omega_s \propto \sqrt{V_{RF}}}$

Two ways to obtain short bunches:

- RF voltage (power!) $\boxed{\sigma_\tau \propto 1/\sqrt{V_{RF}}}$
- Momentum compaction factor in the limit of $\alpha = 0$
isochronous ring: particle position along the bunch is frozen

$$\boxed{\sigma_\tau \propto \alpha}$$

Excitation of betatron oscillations



$$x = x_\beta + x_\varepsilon$$

$$x_\varepsilon = D \cdot \frac{\varepsilon}{E}$$

$$x' = x'_\beta + x'_\varepsilon$$

$$\Delta x = \Delta x_\beta + \Delta x_\varepsilon = 0$$

$$\Delta x_\beta = -D \cdot \frac{\varepsilon_\gamma}{E}$$

Courant Snyder invariant

$$\Delta x'_\beta = -D' \cdot \frac{\varepsilon_\gamma}{E}$$

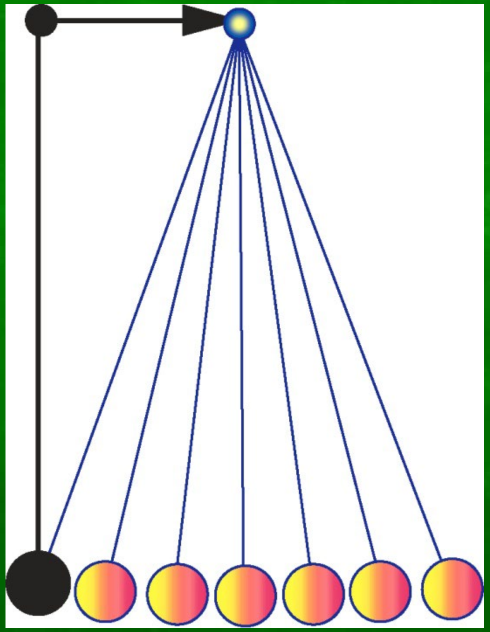
$$\Delta \varepsilon = \gamma \Delta x_\beta^2 + 2\alpha \Delta x_\beta \Delta x'_\beta + \beta \Delta x'^2_\beta = \left[\gamma D^2 + 2\alpha D D' + \beta D'^2\right] \cdot \left(\frac{\varepsilon_\gamma}{E}\right)^2$$

Excitation of betatron oscillations

Electron emitting a photon

- at a place with non-zero dispersion
- starts a betatron oscillation around a new reference orbit

$$\boxed{x_\beta \approx D \cdot \frac{\varepsilon_\gamma}{E}}$$



Horizontal oscillations: equilibrium

Emission of photons is a random process

- Again we have random walk, now in **x**. How far particle will wander away is limited by the radiation damping
- The balance is achieved on the time scale of the damping time $\tau_x = 2\ \tau_\varepsilon$

$$\sigma_{x\beta} \approx \sqrt{\mathcal{N} \cdot \tau_x} \cdot D \cdot \frac{\varepsilon_\gamma}{E} = \sqrt{2} \cdot D \cdot \frac{\sigma_\varepsilon}{E}$$

- Typical horizontal beam size ~ 1 mm

Quantum effect visible to the naked eye!

- Vertical size - determined by coupling

Equilibrium horizontal emittance

Detailed calculations for isomagnetic lattice

$$\varepsilon_{x0} \equiv \frac{\sigma_{x\beta}^2}{\beta} = \frac{C_q E^2}{J_x} \cdot \frac{\langle \mathcal{H} \rangle_{mag}}{\rho}$$

where

$$\begin{aligned}\mathcal{H} &= \gamma D^2 + 2\alpha D D' + \beta D'^2 \\ &= \frac{1}{\beta}\left[D^2 + (\beta D' + \alpha D)^2\right]\end{aligned}$$

and $\langle \mathcal{H} \rangle_{mag}$ is average value in the bending magnets

Beam emittance

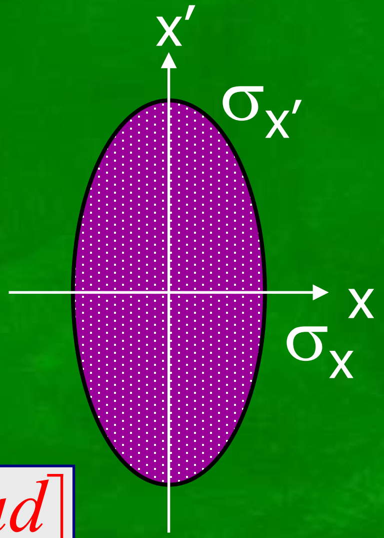
Betatron oscillations

- Particles in the beam execute betatron oscillations with different amplitudes.

Transverse beam distribution

- Gaussian (electrons)
- "Typical" particle: 1 - σ ellipse
  (in a place where $\alpha = \beta' = 0$)

$$\boxed{\text{Area} = \pi \cdot \varepsilon}$$



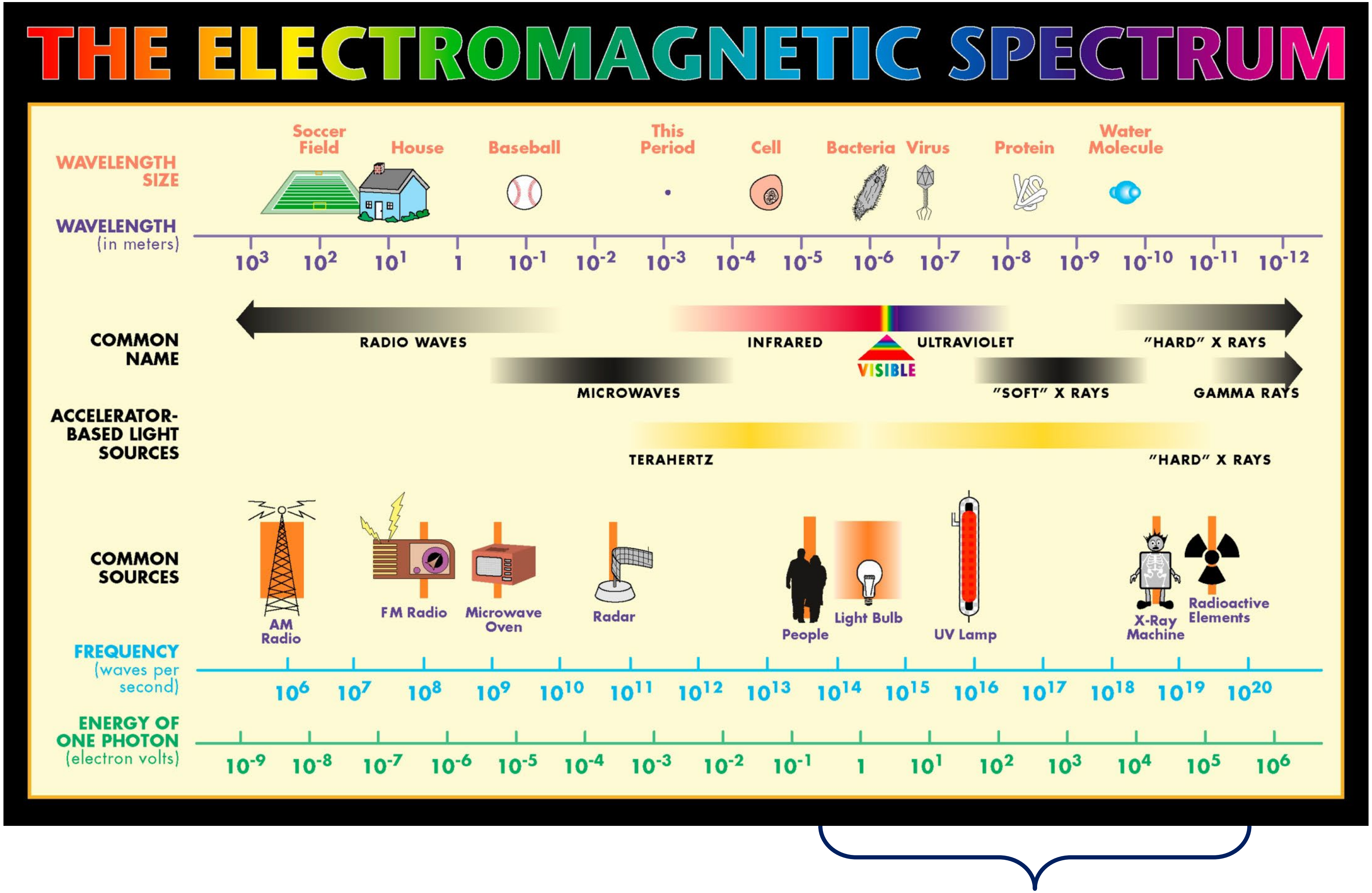
$$\boxed{\text{Units of } \varepsilon \quad [m \cdot rad]}$$

$$\boxed{\text{Emittance} \equiv \frac{\sigma_x^2}{\beta}}$$

$$\boxed{\varepsilon = \sigma_x \cdot \sigma_{x'}}$$

$$\boxed{\begin{aligned} \sigma_x &= \sqrt{\varepsilon\,\beta} \\ \sigma_{x'} &= \sqrt{\varepsilon / \beta} \end{aligned}}$$

$$\boxed{\beta = \frac{\sigma_x}{\sigma_{x'}}}$$

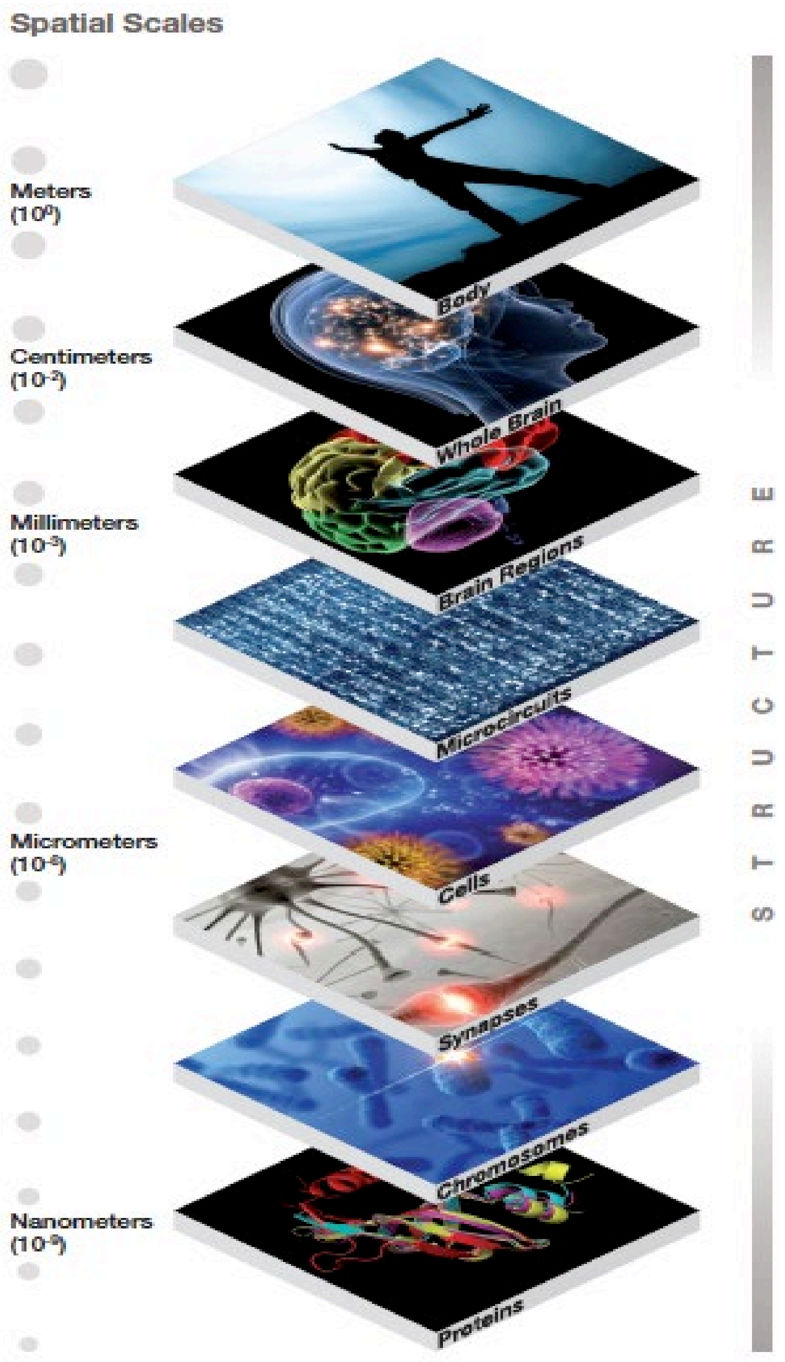


Wavelength continuously tunable !

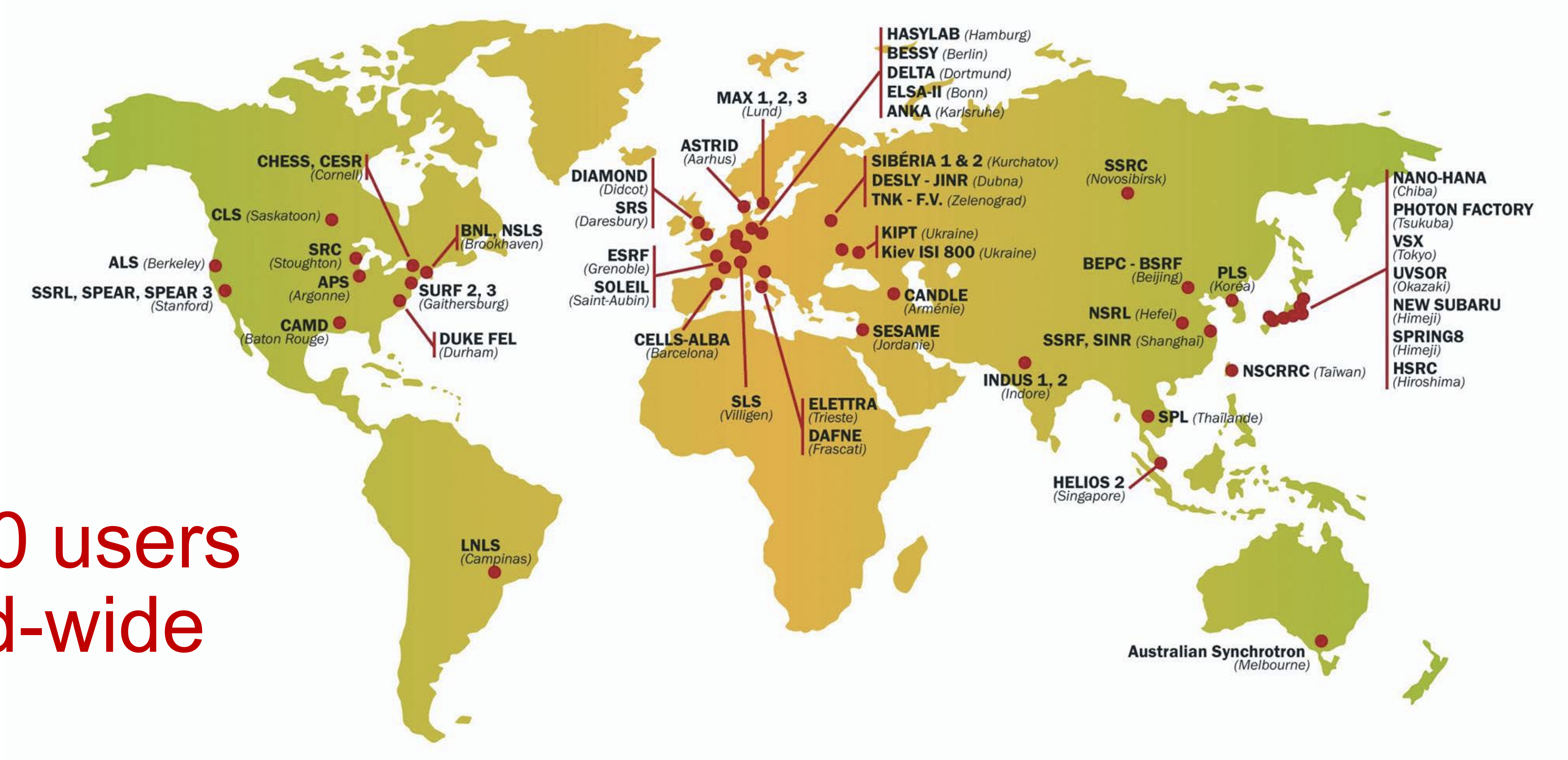
Imaging things

on all length and time scales using accelerators,

e.g. latest X-Ray and computational technologies (developed at accelerators)



Synchrotron Light Sources: about 50 storage ring based



60‘000 users world-wide

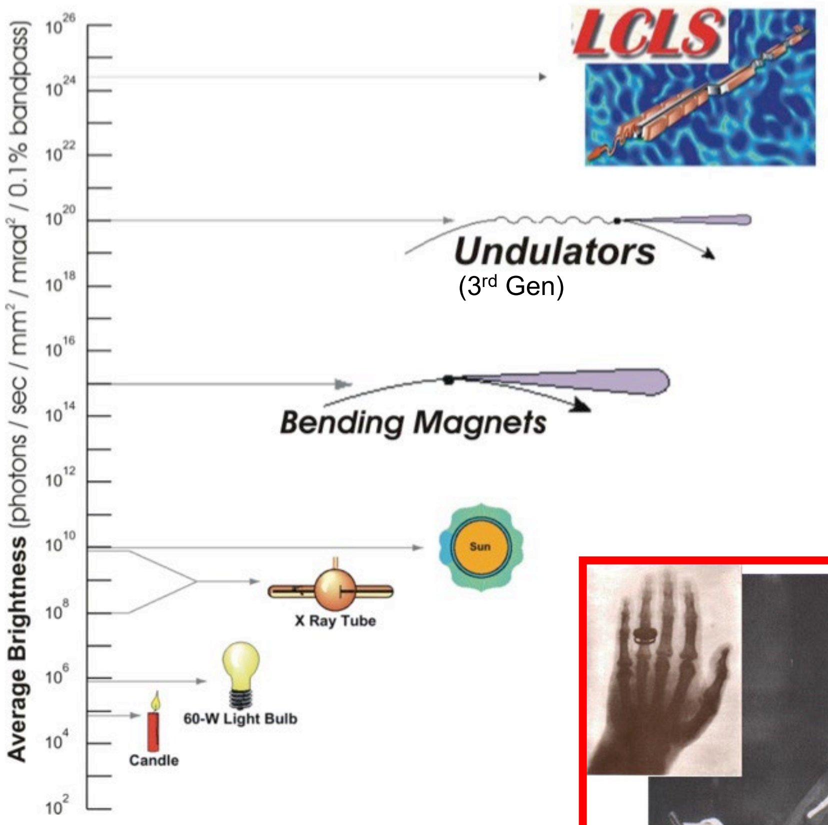
Established, mature technology

The «brightness» of a light source

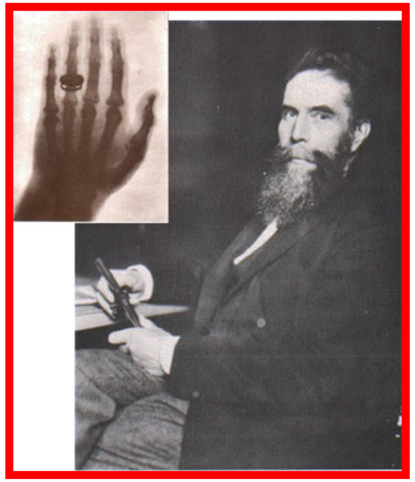
$$\text{Brightness} = \text{constant} \times \frac{F}{S \times \Omega}$$

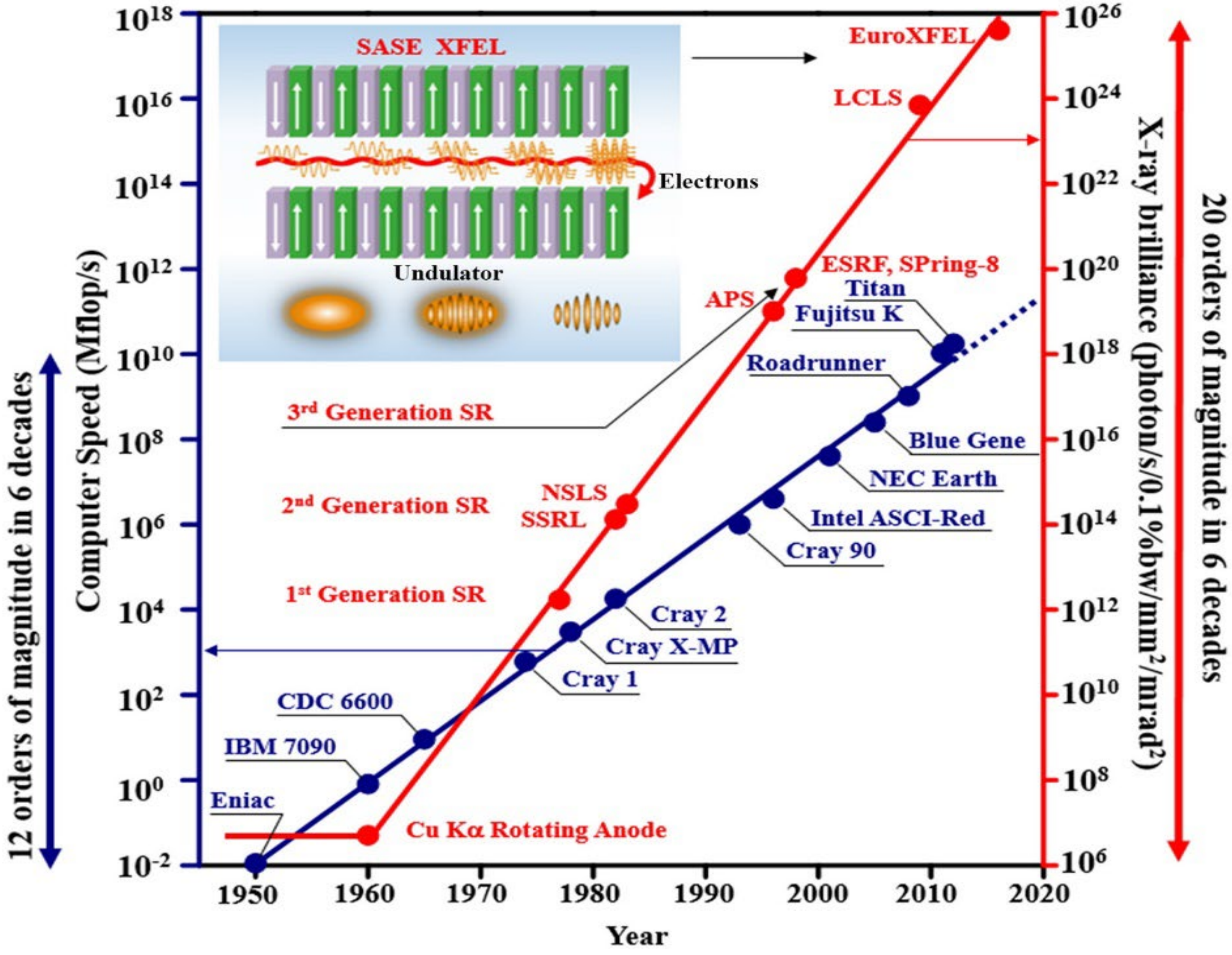
X – Rays Brightness

Average Brightness



Bertha Roentgen's hand
(exposure: 20 min)



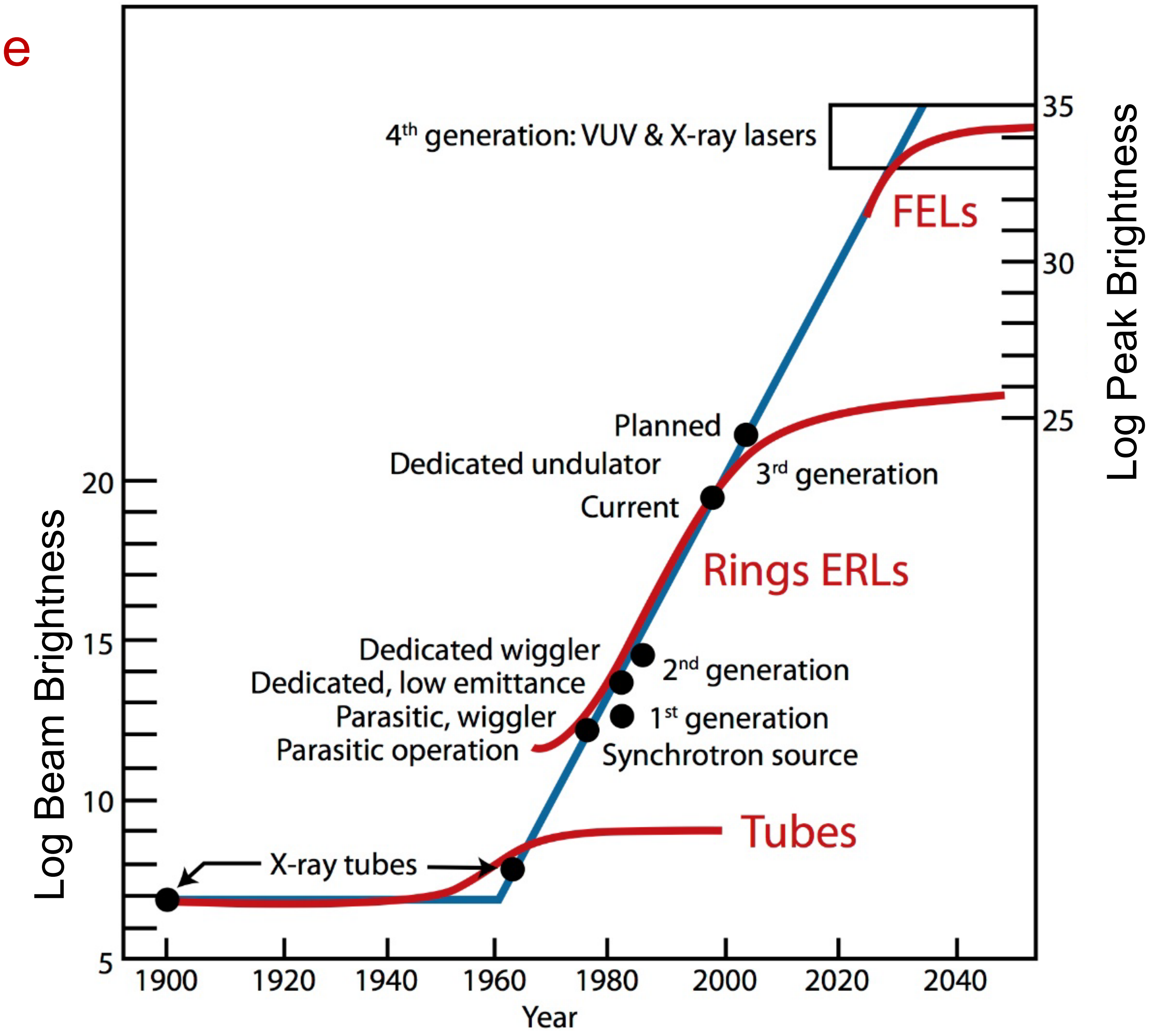


Easter morning 1900: 5th
the automobile.

Easter morning 1913: 5th Ave, New York City.
Spot the horse.

Brightness: disruptive change

- X-ray Tubes
- Storage Rings
- FELs
- ? Compact sources ?



Particle beam emittance

Emittance = S x Ω

LEAPS is the largest consortium of analytical facilities world-wide and further expanding its service to an interdisciplinary European user community

at a glance

19 facilities - **16** institutions - **10** countries

> 300 operating End Stations

> 1.000.000 h beamtime /year

> 7'000 publications/year

> 15 spin off companies

> 35.000 users from all EU & beyond
researchers from all research areas

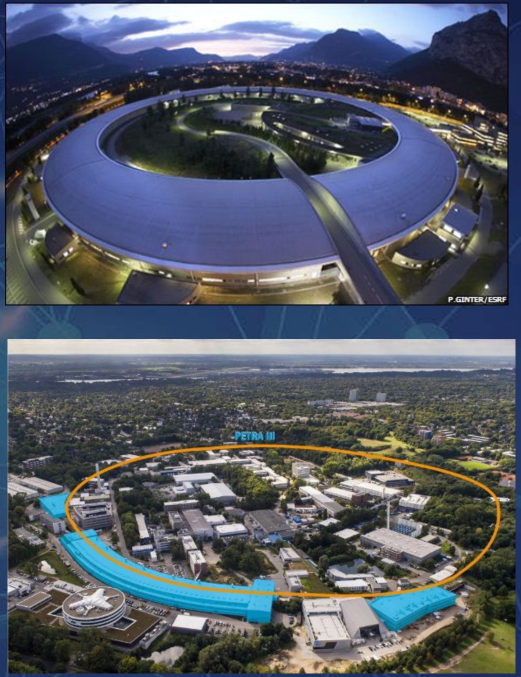
Construction and Operation (~ 800 M€/year) through national funding
Investment: 1.6 B€; 1.5 B€ upgrade programs

LEAPS League of European Accelerator-based Photon Sources

LEAPS League of European Accelerator-based Photon Sources

Synchrotrons

ESRF & PETRA III
6 GeV



Alba, Diamond, Elettra, Max IV, SLS, Soleil
2-3 GeV

ASTRID, BESSY II, DAFNE, Max IV,
PTB, Solaris < 2 GeV

LEAPS League of European Accelerator-based Photon Sources

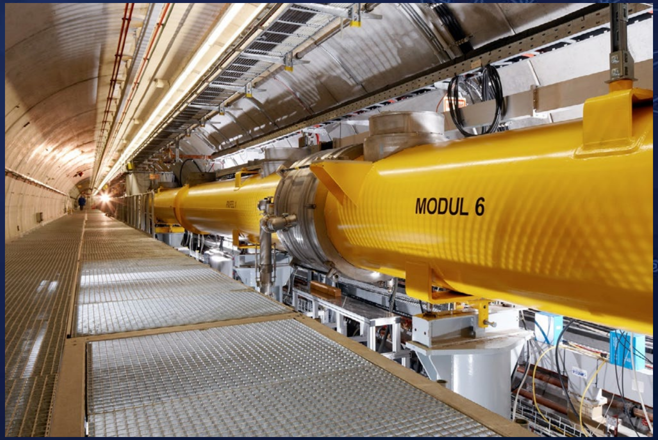
FELs from Hard X rays to IR

European XFEL

SwissFEL

FERMI

FELBE



FLASH

FELIX

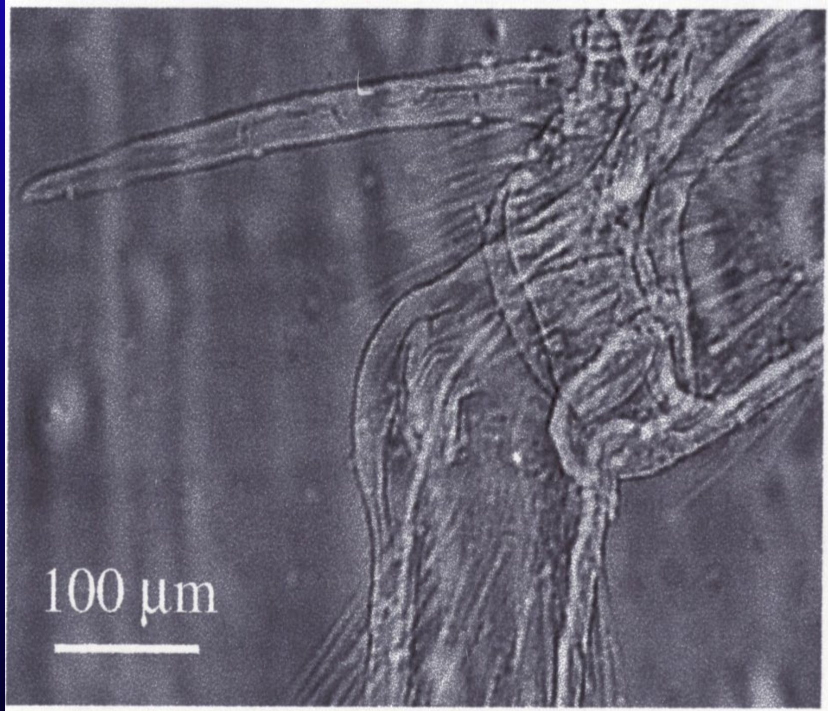
Transverse coherence

- High brightness gives coherence

- Wave optics methods for X-rays (all chapters in Born & Wolf)

- Holography

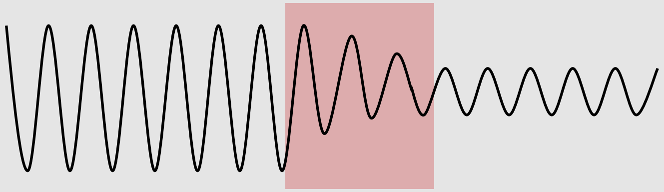
The knee of a spider



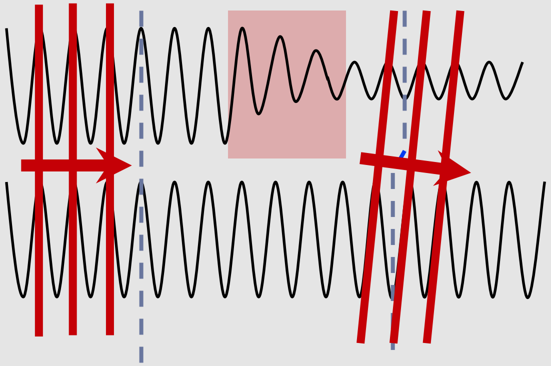
phase contrast imaging

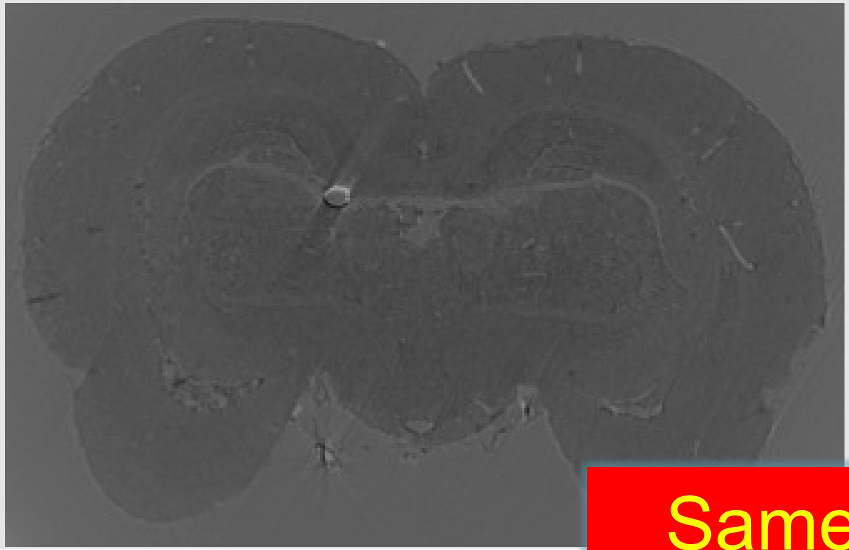
Phase contrast X-Ray imaging: improved soft tissue contrast

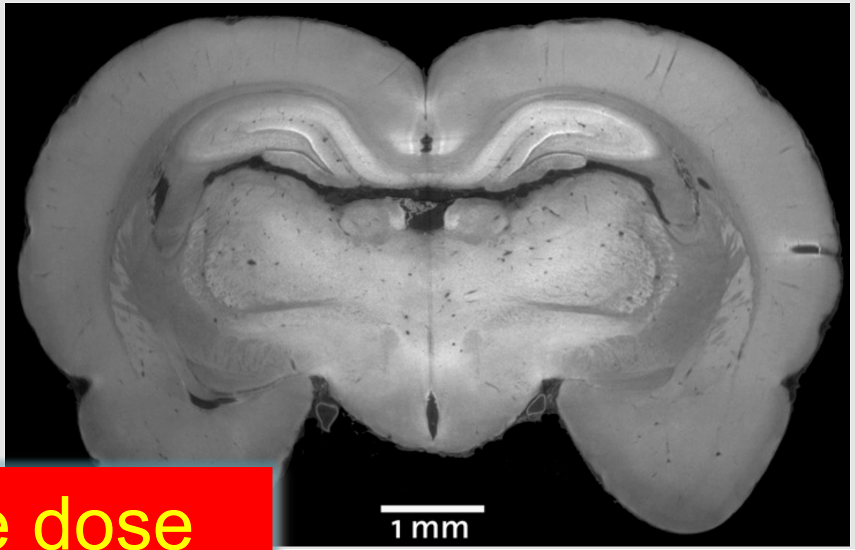
Absorption



Phase contrast







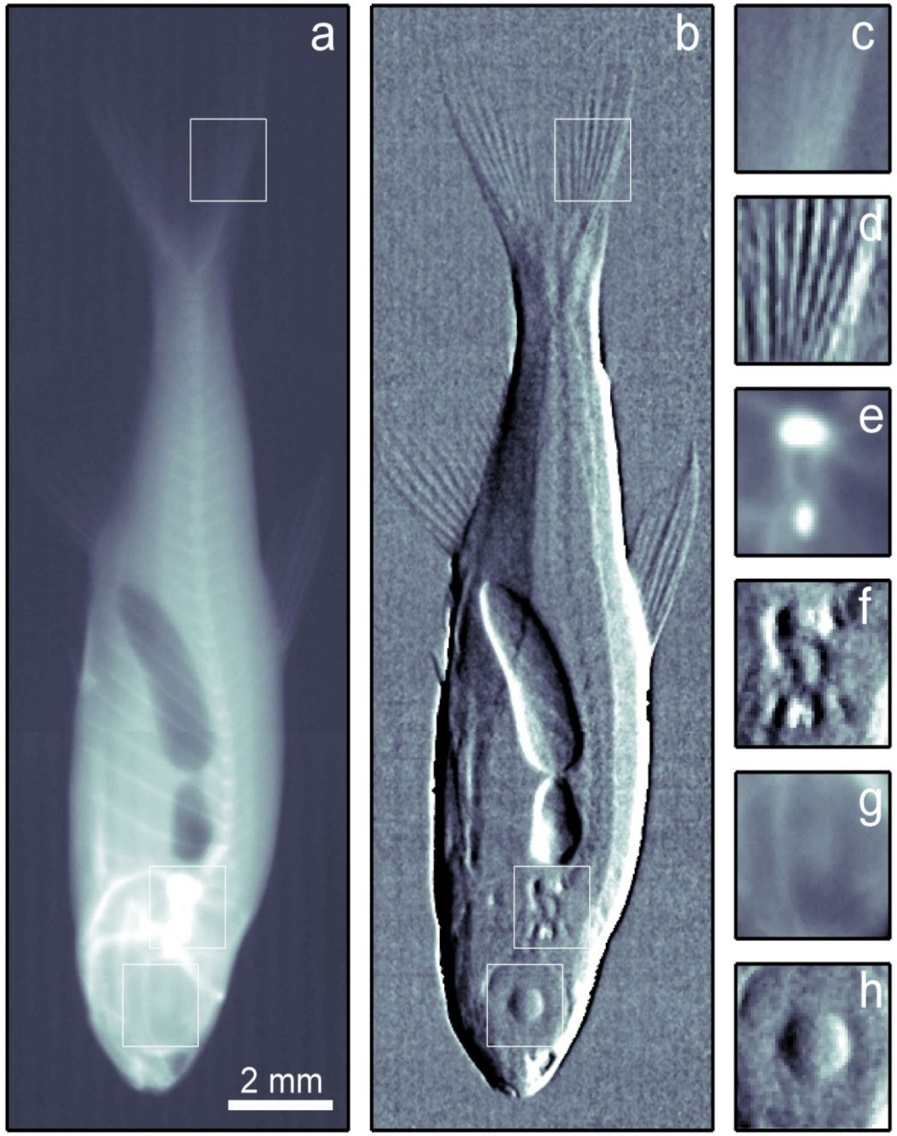
Same dose

X-ray Radiography of a fish

conventional
Absorption

a

(+ details c , e, g)



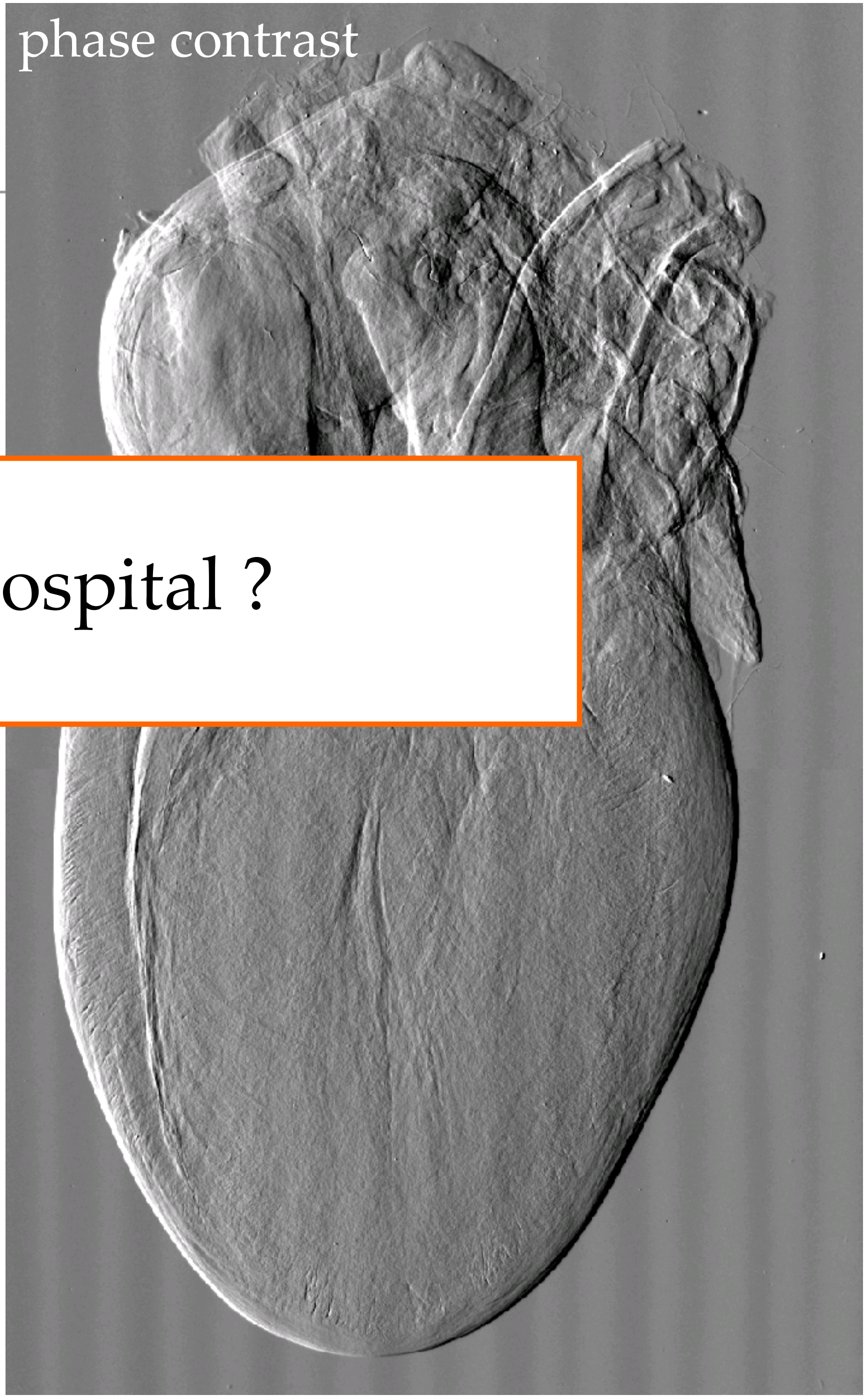
Phase contrast
Microscopy

b

(+ details d, f, h)

Imaging for Life Science Applications

17.5 keV,
synchrotron results



Into the hospital ?

Muscles and tracheal network *during* flight

Surreal X-ray movie reveals how a fly beats its wings

21:00 25 March 2014 by Jacob Aron
For similar stories, visit the Picture of the day Topic Guide

Scientists film inside a flying insect

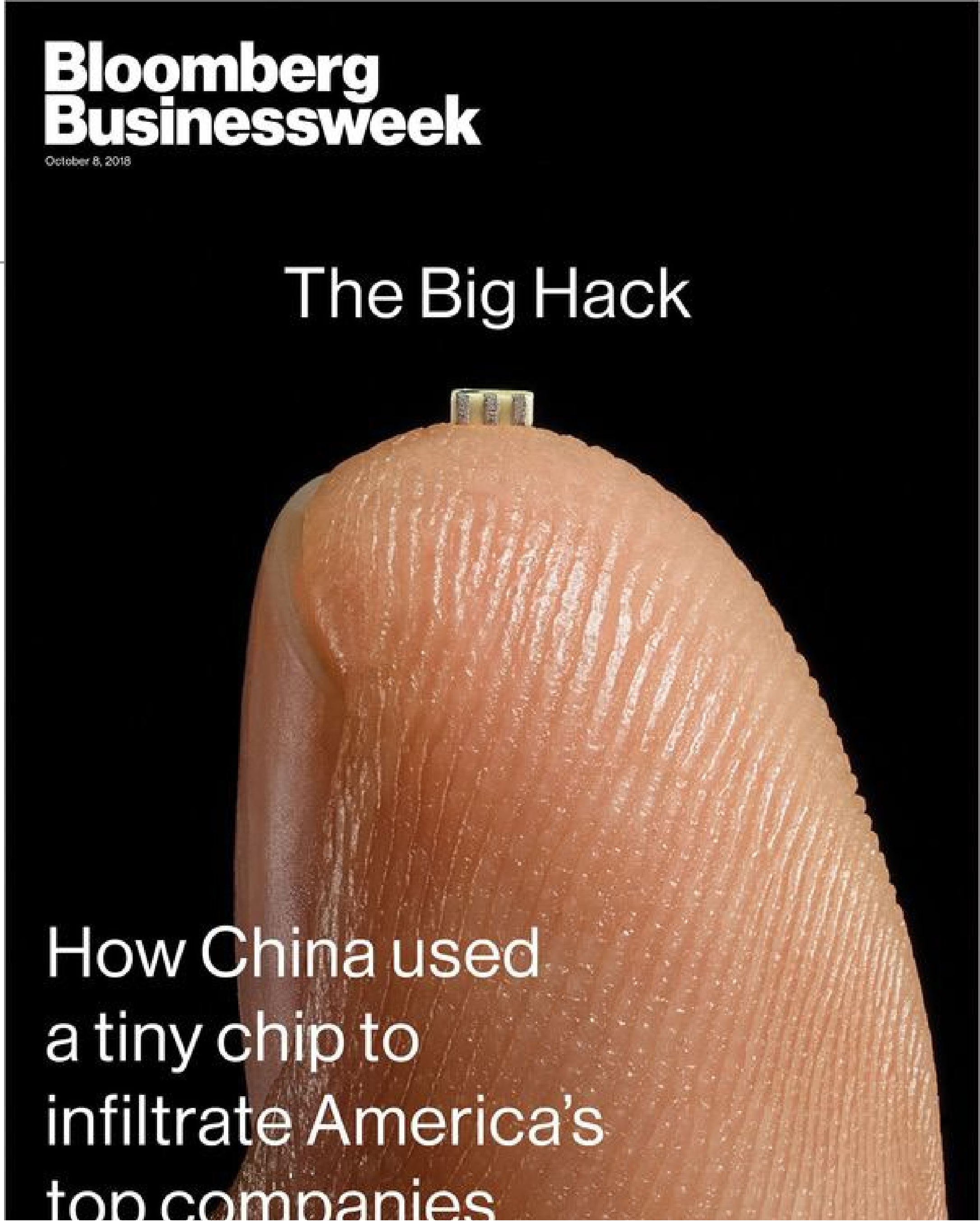
By Victoria Gill
Science reporter, BBC News

26 March 2014 Science & Environment

R. Mokso *et al.*, Scientific Reports **5** 8727 (2015)

Fake news?

Intel Core Pentium G3260 (3300)
Dual Core, 22 nm

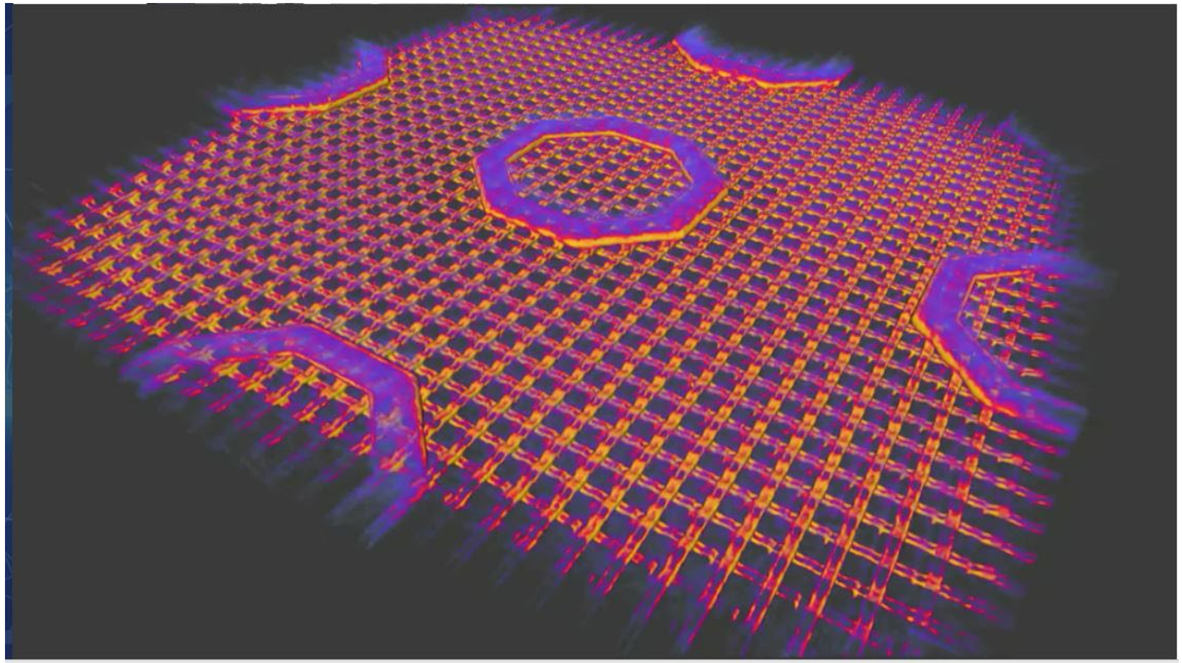


Holler et al., Nature 543, 402–406 (16 March 2017)

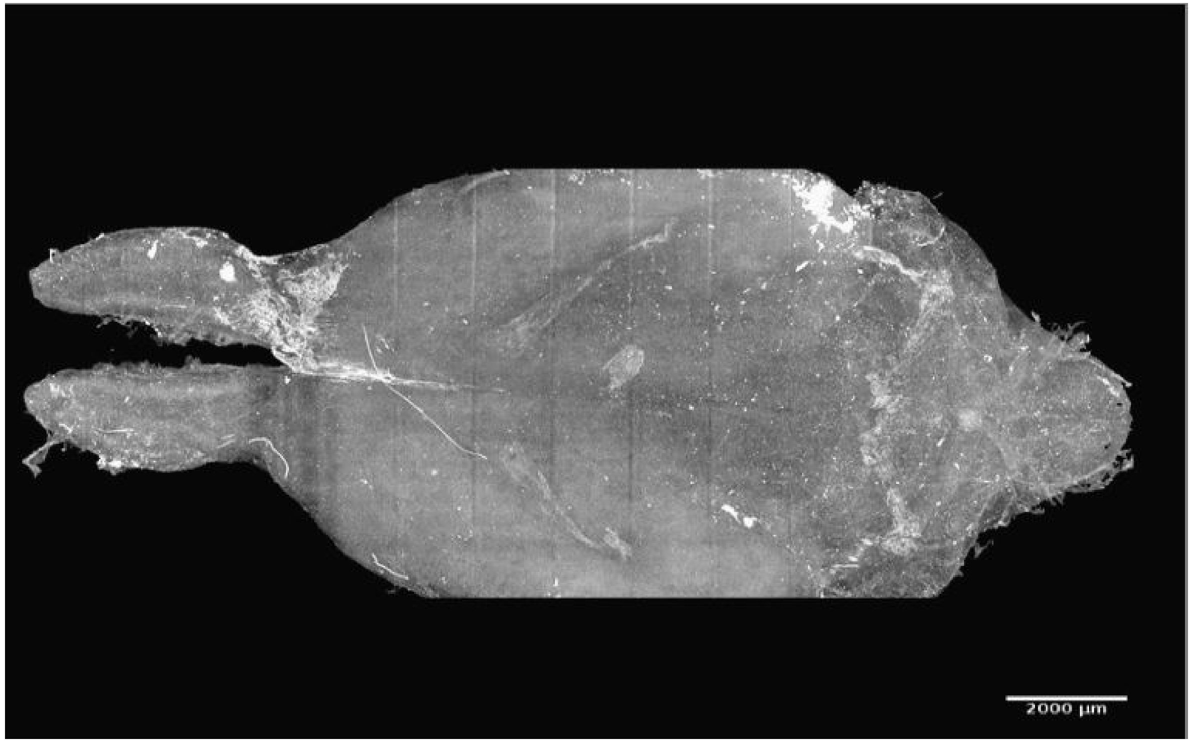
X-Ray tomography

Architecture of artificial and natural intelligence on all scales

Nature Electronics 2, 464-470 (2019)
New X-ray world record: Looking inside a microchip with 4 nanometre precision | News & Events | PSI



Brain of a mouse in 3-D
Miettinen et al.



Towards understanding the healthy and diseased brain

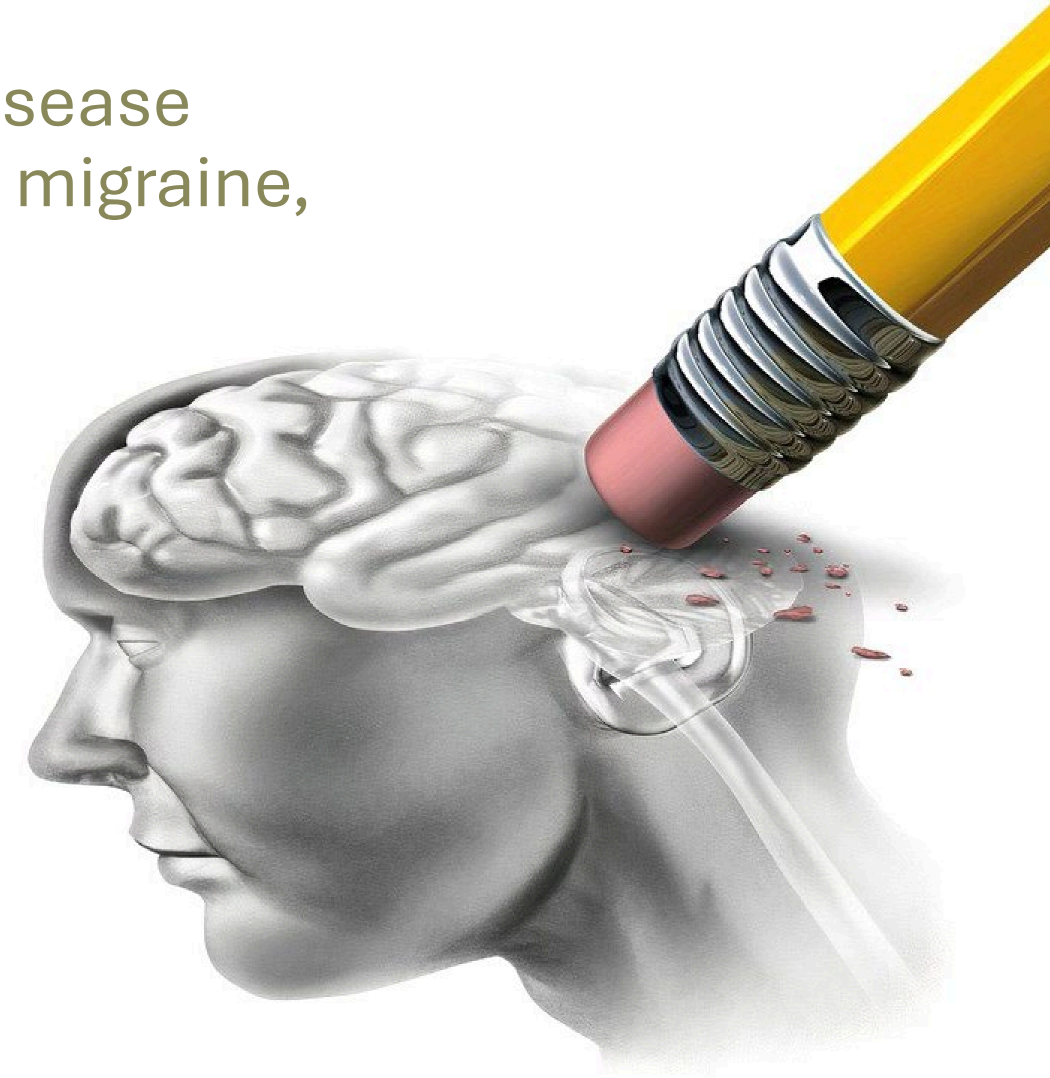
1 in 6 humans suffer from a neurological disease
e.g. Alzheimer's, MS, depression, epilepsy, migraine,
drug addiction, etc.

> 1 billion people affected

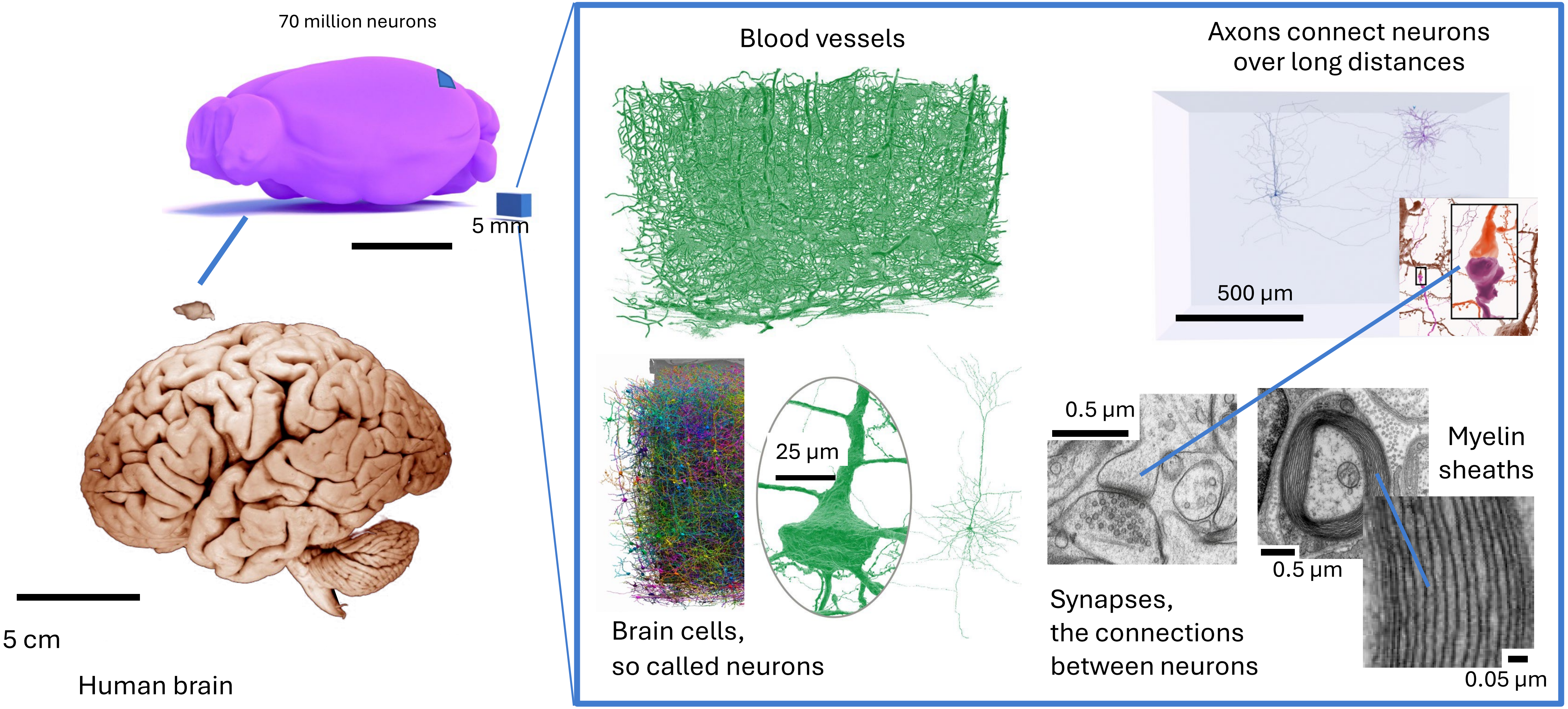
> 1,000 known brain diseases

Cures are lacking

Symptomatic treatments are scarce



The challenge: Visualizing the structure across scales



Mouse brain

70 million neurons

Human brain

171 billion brain cells
1,000 trillion synapses

1mm^3 of the mouse brain 100,000 neurons, 700 million synapses

Blood vessels

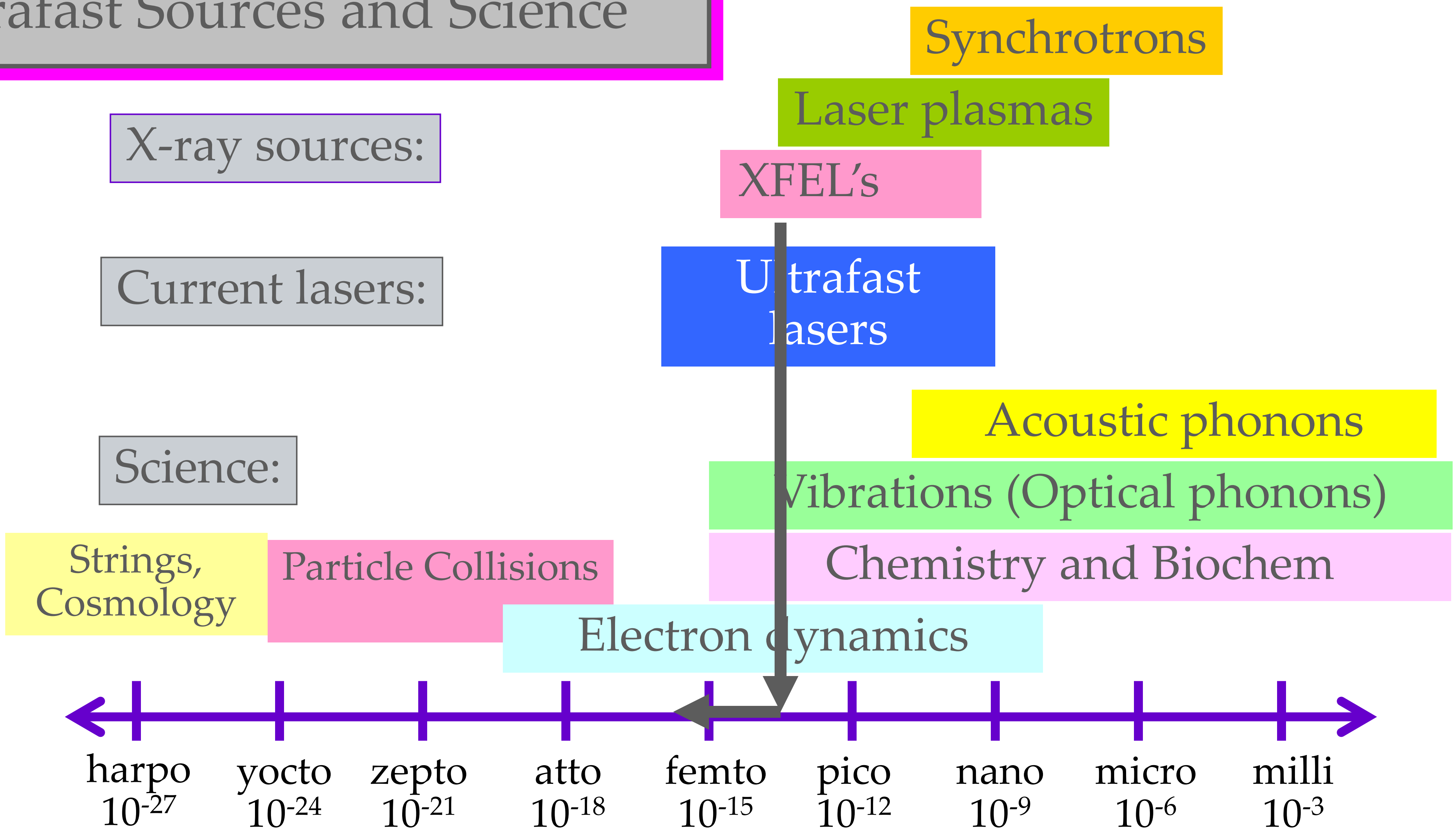
Axons connect neurons over long distances

Brain cells, so called neurons

Synapses, the connections between neurons

Myelin sheaths

Ultrafast Sources and Science



X-ray sources:

Synchrotrons

Laser plasmas

XFEL's

Current lasers:

Ultrafast lasers

Science:

Acoustic phonons

Vibrations (Optical phonons)

Chemistry and Biochem

Strings, Cosmology

Particle Collisions

Electron dynamics

harpo 10^{-27}

yocto 10^{-24}

zepto 10^{-21}

atto 10^{-18}

femto 10^{-15}

pico 10^{-12}

nano 10^{-9}

micro 10^{-6}

milli 10^{-3}

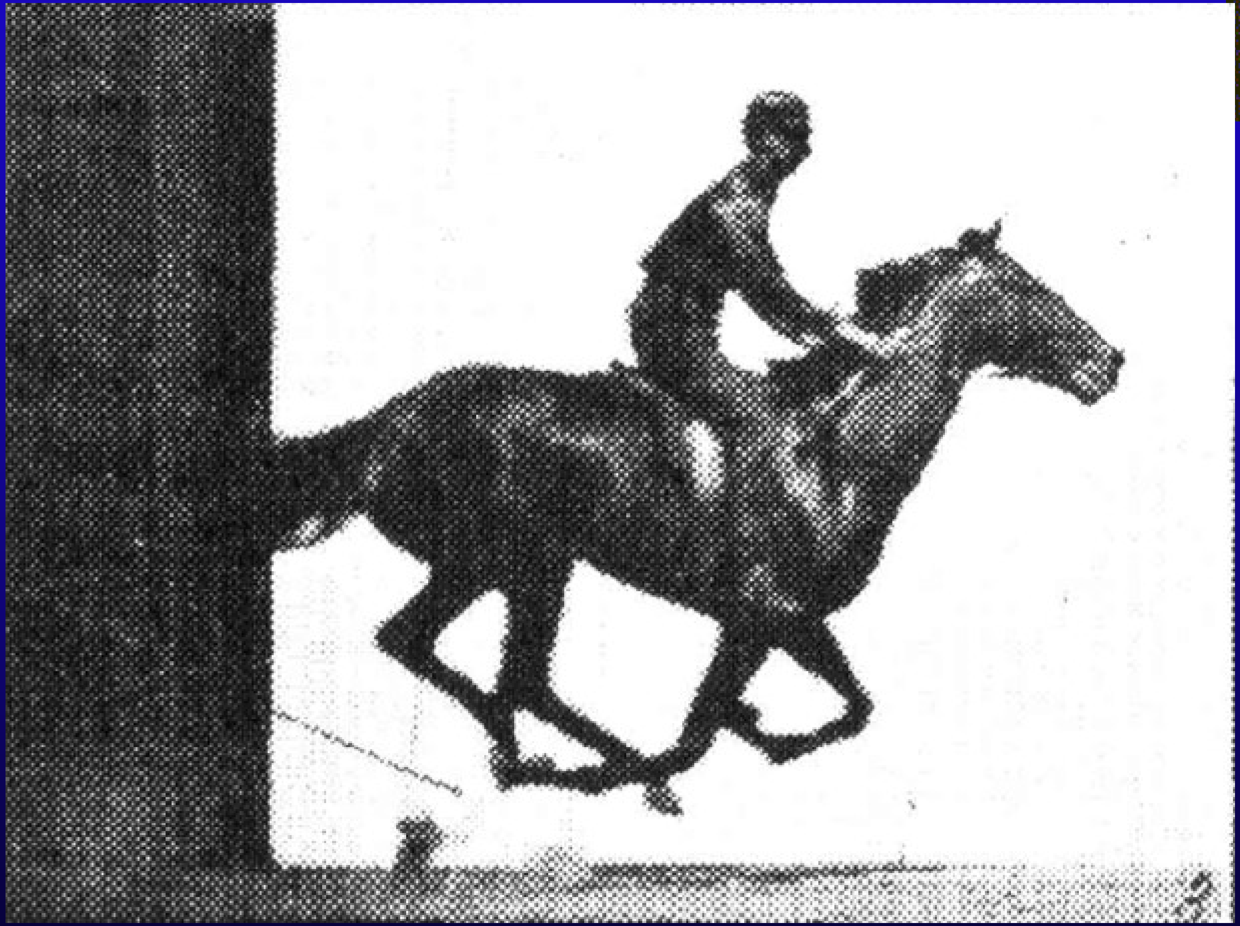
J. Hastings

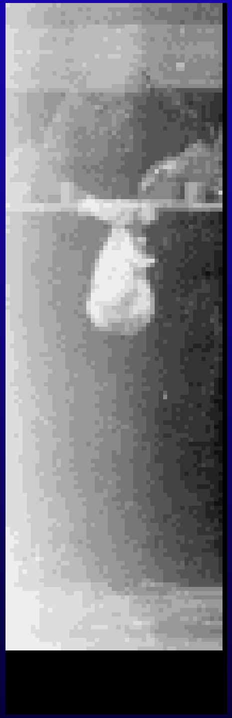
1878: E. Muybridge at Stanford

Tracing motion of animals by spark photography

E. Muybridge

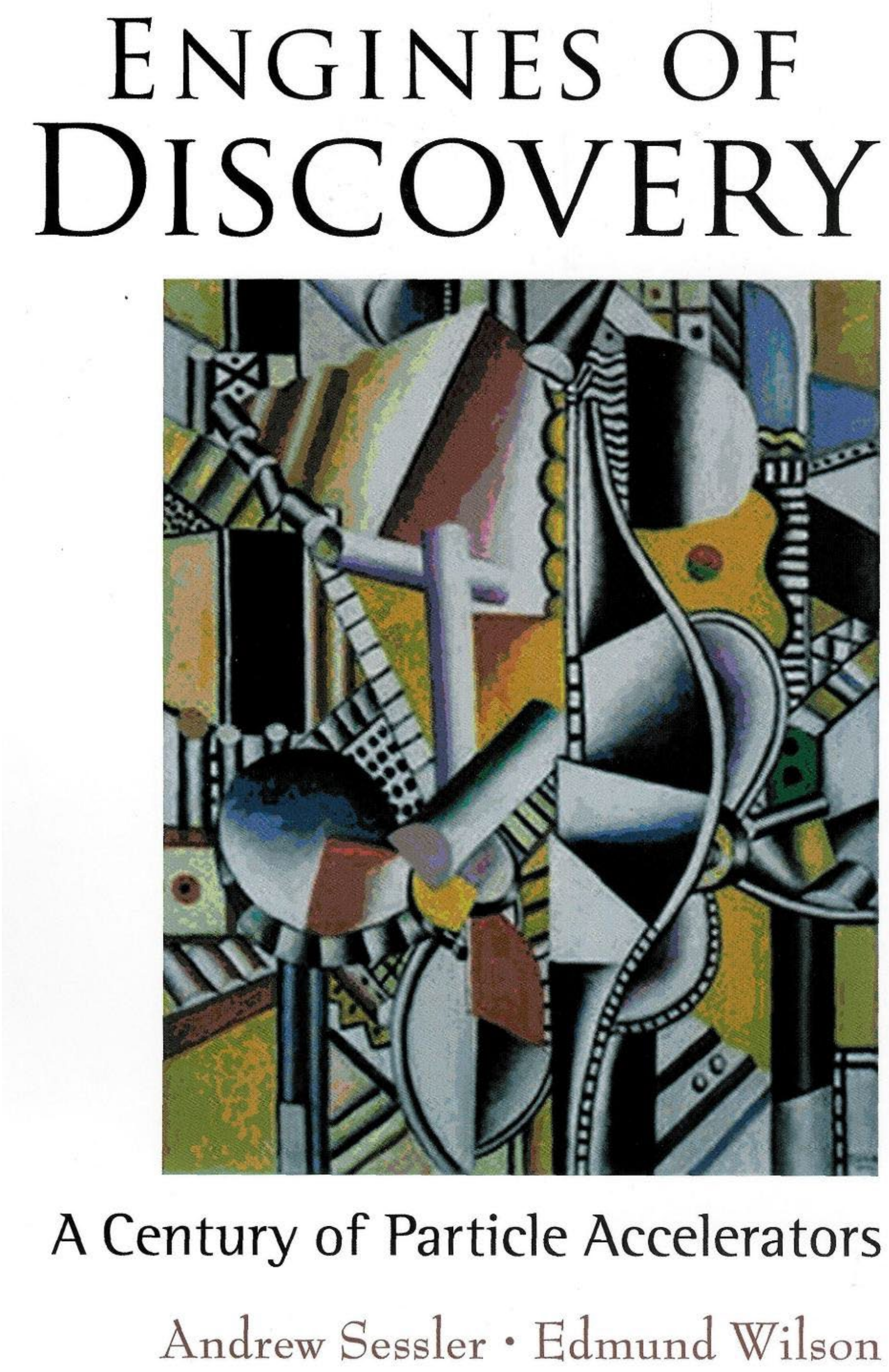
L. Stanford





Muybridge and Stanford disagree whether all feet leave the ground at one time during the gallop…

E. Muybridge, *Animals in Motion*, ed. by L. S. Brown (Dover Pub. Co., New York 1957).



ENGINES OF DISCOVERY

A Century of Particle Accelerators

Andrew Sessler · Edmund Wilson

« Le seul véritable voyage … ce ne serait pas d'aller vers de nouveaux paysages, mais d'avoir d'autres yeux, de voir l'univers avec les yeux d'un autre, de cent autres, de voir les cent univers que chacun d'eux voit, que chacun d'eux est. »

(Marcel Proust, La Prisonnière, 1923)

"The real voyage of discovery consists not in seeking new landscapes but in having new eyes"

Marcel Proust