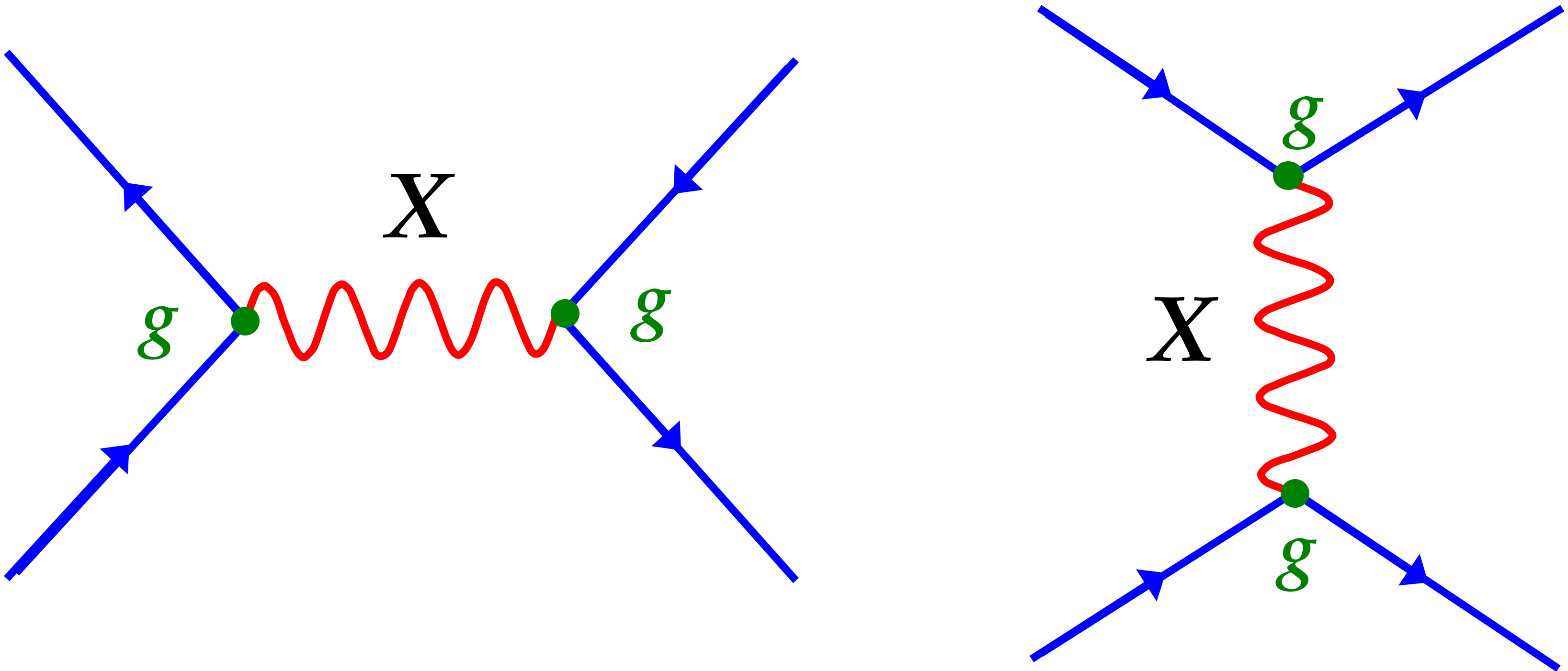
Foundations of the Standard Model

Prof Mark Thomson



5. The Weak Interaction

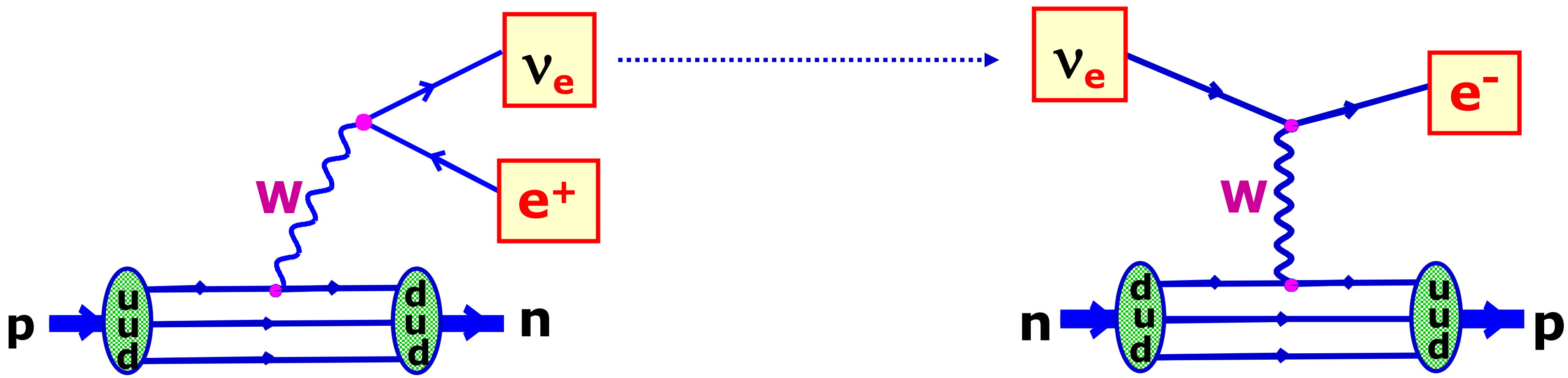
The Weak Interaction

★ **QED and QCD share many common features**

- Mediated by massless spin-1 bosons
- A vector interaction with a vertex factor of the form $\overline{u}(p')\gamma^{\mu}u(p)$
- No change of flavour in the interaction
- Parity conserved

★ **The weak interaction is different in many ways**

- Mediated by massive spin-1 bosons
- The (charged-current) weak interaction always changes flavour
- The charged-current W bosons carry the charge of the U(1) interaction
- Parity is violated, which implies that the vertex factor different from QED/QCD



Parity Violation in β-Decay

★The parity operator $\hat{P}$ corresponds to a discrete transformation $x \rightarrow -x,\ etc.$

★Under the parity transformation:

Vectors change sign

$$\vec{r} \xrightarrow{\hat{P}} -\vec{r}$$

$$\vec{p} \xrightarrow{\hat{P}} -\vec{p} \qquad \left(p_x = \frac{\partial}{\partial x},\ etc.\right)$$

Axial-Vectors unchanged

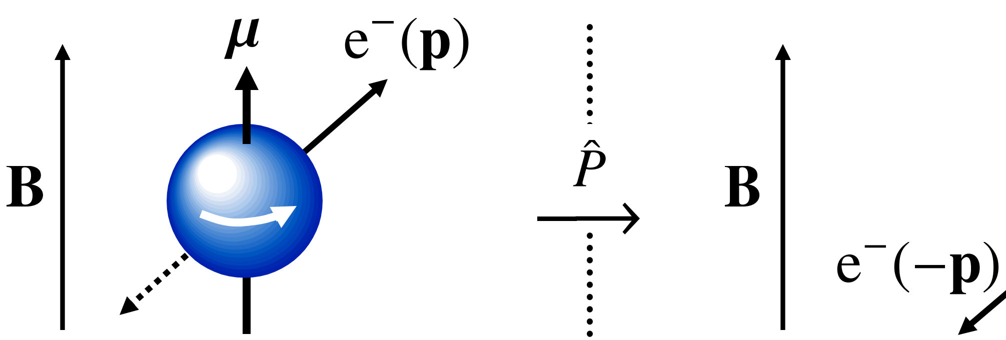
$$\vec{L} \xrightarrow{\hat{P}} \vec{L} \qquad (\vec{L} = \vec{r} \wedge \vec{p})$$

$$\vec{\mu} \xrightarrow{\hat{P}} \vec{\mu} \qquad (\vec{\mu} \propto \vec{L})$$

Note B is an axial vector $\mathrm{d}\vec{B} \propto \vec{J} \wedge \vec{r}\, \mathrm{d}^3\vec{r}$

★**1957:** C.S.Wu et al. studied β-decay of polarized nuclei: $^{60}\mathrm{Co} \rightarrow {}^{60}Ni^{*} + e^{-} + \overline{\nu}_e$

★Observed **electrons emitted preferentially** in direction opposite to applied field



more e- in ↙ c.f. ↗

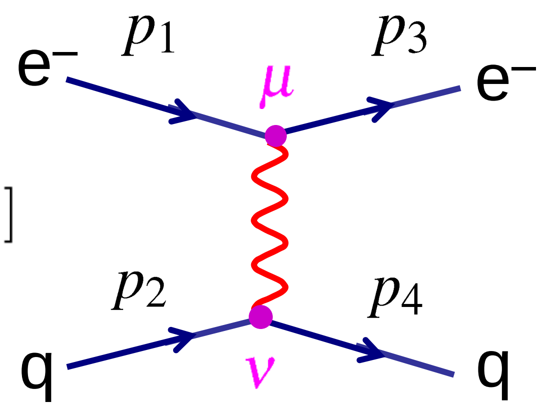
If parity were conserved: expect equal rate for producing e⁻ in directions along and opposite to the nuclear spin.

★Conclude **parity is violated** in WEAK INTERACTION

→ the WEAK interaction vertex is **NOT** of the form $\overline{u}_e \gamma^{\mu} u_{\nu}$

Parity Conservation in QED and QCD

- Consider the QED process $e^-q \to e^-q$
- The Feynman rules for QED give:



$$-iM = [\overline{u}_e(p_3)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}_q(p_4)ie\gamma^\nu u_q(p_2)]$$

- Which can be expressed in terms of the electron and quark 4-vector currents:

$$M = -\frac{e^2}{q^2}g_{\mu\nu}j_e^\mu j_q^\nu = -\frac{e^2}{q^2}j_e.j_q$$

with $\quad j_e = \overline{u}_e(p_3)\gamma^\mu u_e(p_1)$ and $j_q = \overline{u}_q(p_4)\gamma^\mu u_q(p_2)$

★ Consider the what happen to the matrix element under the parity transformation

- Spinors transform as $\quad u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$
- Adjoint spinors transform as

$$\overline{u} = u^\dagger\gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger\gamma^0 = u^\dagger\gamma^{0\dagger}\gamma^0 = u^\dagger\gamma^0\gamma^0 = \overline{u}\gamma^0$$

$$\overline{u} \xrightarrow{\hat{P}} \overline{u}\gamma^0$$

- Hence $\quad j_e = \overline{u}_e(p_3)\gamma^\mu u_e(p_1) \xrightarrow{\hat{P}} \overline{u}_e(p_3)\gamma^0\gamma^\mu\gamma^0 u_e(p_1)$

★ Consider the components of the four-vector current

0: $$j_e^0 \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^0\gamma^0 u = \overline{u}\gamma^0 u = j_e^0$$ since $\gamma^0\gamma^0 = 1$

k=1,2,3: $$j_e^k \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^k\gamma^0 u = -\overline{u}\gamma^k\gamma^0\gamma^0 u = -\overline{u}\gamma^k u = -j_e^k$$ since $\gamma^0\gamma^k = -\gamma^k\gamma^0$

- The time-like component remains unchanged and the space-like components change sign
- Similarly $j_q^0 \xrightarrow{\hat{P}} j_q^0 \qquad j_q^k \xrightarrow{\hat{P}} -j_q^k \quad k = 1,2,3$

★ Consequently the four-vector scalar product

$$j_e.j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e.j_q \qquad k = 1,3$$

or $j^\mu \xrightarrow{\hat{P}} j_\mu$

$$j^\mu.j^\nu \xrightarrow{\hat{P}} j_\mu.j_\nu \xrightarrow{\hat{P}} j^\mu.j^\nu$$

QED Matrix Elements are Parity Invariant

➡ **Parity Conserved in QED**

★ The QCD vertex has the same form and, thus,

Parity Conserved in QCD

Recall Bilinear Covariants

★ The requirement of Lorentz invariance of the matrix element severely restricts the form of the interaction vertex. QED and QCD are "VECTOR" interactions:

$$j^{\mu} = \overline{\psi}\gamma^{\mu}\phi$$

★ This is a parity conserving interaction

★ In general, there are only 5 possible combinations of two spinors and the gamma matrices that form Lorentz invariant currents, called "bilinear covariants":

Type	Form	Components	"Boson Spin"
♦ SCALAR	$\overline{\psi}\phi$	1	0
♦ PSEUDOSCALAR	$\overline{\psi}\gamma^5\phi$	1	0
♦ VECTOR	$\overline{\psi}\gamma^{\mu}\phi$	4	1
♦ AXIAL VECTOR	$\overline{\psi}\gamma^{\mu}\gamma^5\phi$	4	1
♦ TENSOR	$\overline{\psi}(\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu})\phi$	6	2

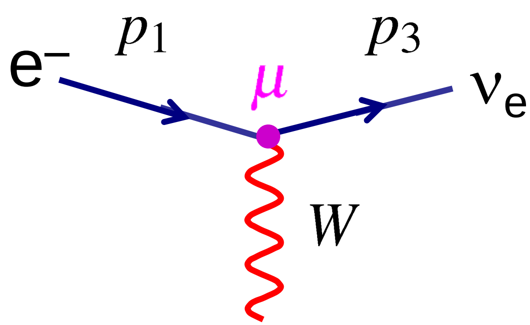
★ Since parity is observed to be violated in the weak interaction, e.g. in beta decay, the interaction cannot simply be VECTOR in nature

V-A Structure of the Weak Interaction

★ The most general form for the interaction between a fermion and a boson is a linear combination of bilinear covariants

★ For an interaction corresponding to the **exchange of a spin-1** particle the most general form is a linear combination of **VECTOR** and **AXIAL-VECTOR**

★ The form for WEAK interaction is <u>determined from experiment</u> to be **VECTOR – AXIAL-VECTOR** **(V – A)**



$$j^\mu \propto \bar{u}_{\nu_e}(\gamma^\mu - \gamma^\mu\gamma^5)u_e$$

V – A

★ Can this account for parity violation?

★ First consider parity transformation of a **pure AXIAL-VECTOR** current

- The space-like components remain unchanged and the time-like components change sign (the opposite to the parity properties of a vector-current)

$$j_A^0 \xrightarrow{\hat{P}} -j_A^0; \quad j_A^k \xrightarrow{\hat{P}} +j_A^k; \qquad j_V^0 \xrightarrow{\hat{P}} +j_V^0; \quad j_V^k \xrightarrow{\hat{P}} -j_V^k$$

- Consequently, parity is conserved a **pure** vector and **pure** axial-vector interactions

V-A Structure of the Weak Interaction

★ The most general form for the interaction between a fermion and a boson is a linear combination of bilinear covariants

★ For an interaction corresponding to the **exchange of a spin-1** particle the most general form is a linear combination of **VECTOR** and **AXIAL-VECTOR**

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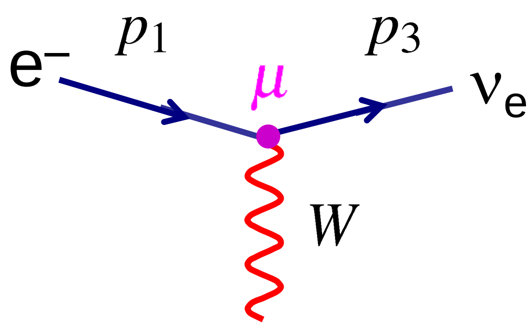


$$j^{\mu} \propto \overline{u}_{\nu_e}(\gamma^{\mu} - \gamma^{\mu}\gamma^{5})u_e$$

V – A

★ Can this account for parity violation?

- However, the combination of a vector current and an axial vector current

$$j_{V1}\cdot j_{A2} \xrightarrow{\hat{P}} (j_1^0)(-j_2^0) - \sum_{k=1,3}(-j_1^k)(j_2^k) = -j_{V1}\cdot j_{A2}$$

changes sign under parity – can give parity violation !

★ Maximal Parity violation for V-A and V+A

Chiral Structure of QED (Reminder)

★ **Recall previously introduced CHIRAL projections operators**

$$P_R = \tfrac{1}{2}(1+\gamma^5); \qquad P_L = \tfrac{1}{2}(1-\gamma^5)$$

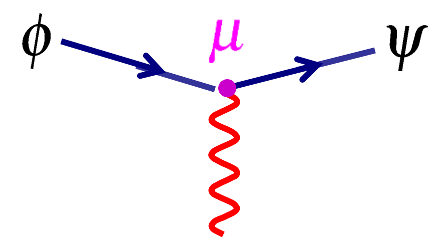
project out chiral right- and left- handed states

★ **Only in the ultra-relativistic limit, chiral states correspond to helicity states**

★ **Any spinor can be expressed as:**

$$\psi = \tfrac{1}{2}(1+\gamma^5)\psi + \tfrac{1}{2}(1-\gamma^5)\psi = P_R\psi + P_L\psi = \psi_R + \psi_L$$

•**The QED vertex** $\overline{\psi}\gamma^\mu\phi$ **in terms of chiral states:**

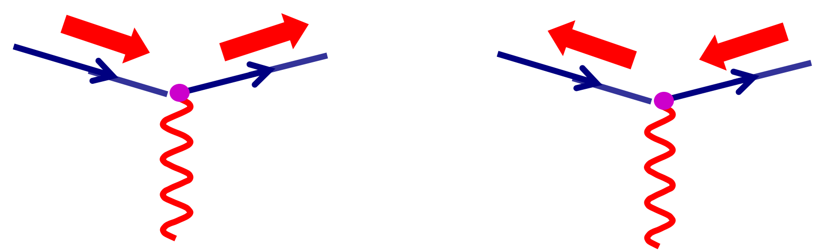


$$\overline{\psi}\gamma^\mu\phi = \overline{\psi}_R\gamma^\mu\phi_R + \overline{\psi}_R\gamma^\mu\phi_L + \overline{\psi}_L\gamma^\mu\phi_R + \overline{\psi}_L\gamma^\mu\phi_L$$

"conserves chirality", e.g.

$$\begin{aligned}
\overline{\psi}_R\gamma^\mu\phi_L &= \tfrac{1}{2}\psi^\dagger(1+\gamma^5)\gamma^0\gamma^\mu\tfrac{1}{2}(1-\gamma^5)\phi \\
&= \tfrac{1}{4}\psi^\dagger\gamma^0(1-\gamma^5)\gamma^\mu(1-\gamma^5)\phi \\
&= \tfrac{1}{4}\overline{\psi}\gamma^\mu(1+\gamma^5)(1-\gamma^5)\phi = 0
\end{aligned}$$

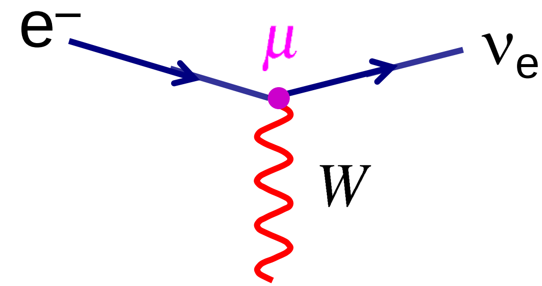
★**In the ultra-relativistic limit only two helicity combinations are non-zero**



Chrial Structure of the WEAK Interaction

★**The charged current (W$^{\pm}$) weak vertex is:**

$$\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)$$



★**Since** $\frac{1}{2}(1-\gamma^5)$ **projects out left-handed chiral particle states:**

$$\overline{\psi}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)\phi = \overline{\psi}\gamma^{\mu}\phi_L$$

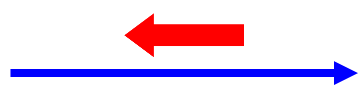
★**Writing** $\overline{\psi} = \overline{\psi}_R + \overline{\psi}_L$ **and from discussion of QED,** $\overline{\psi}_R\gamma^{\mu}\phi_L = 0$ **gives**

$$\overline{\psi}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)\phi = \overline{\psi}_L\gamma^{\mu}\phi_L$$

Only the left-handed chiral components of particle spinors and right-handed chiral components of anti-particle spinors participate in charged current weak interactions

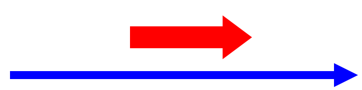
★**At very high energy** $(E \gg m)$ **, the left-handed chiral components are helicity eigenstates :**

$\frac{1}{2}(1-\gamma^5)u \Rightarrow$



LEFT-HANDED PARTICLES
Helicity = -1

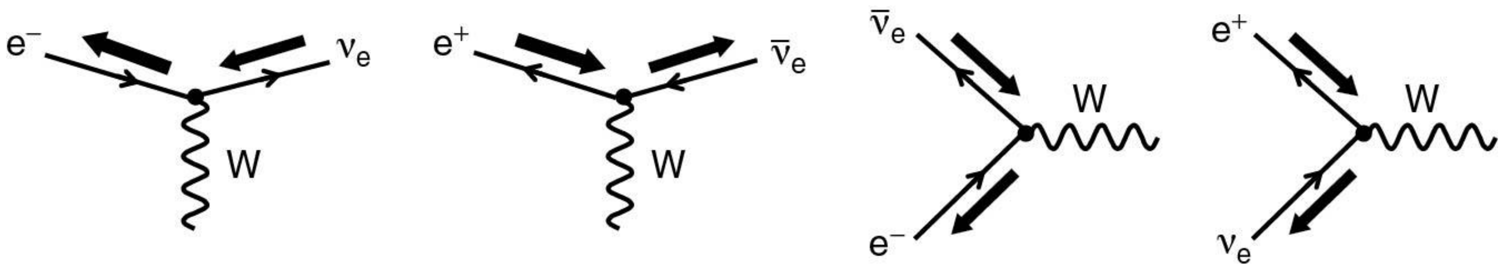
$\frac{1}{2}(1-\gamma^5)v \Rightarrow$



RIGHT-HANDED ANTI-PARTICLES
Helicity = +1

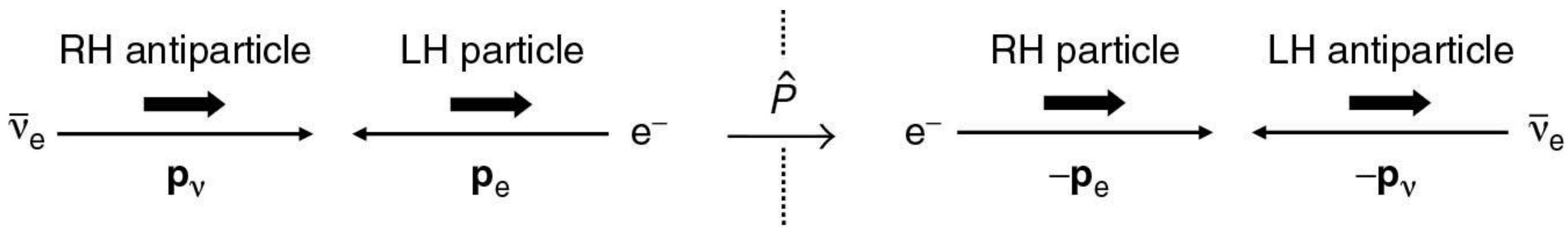
In the ultra-relativistic limit only left-handed particles and right-handed antiparticles participate in charged current weak interactions

e.g. **In the relativistic limit, the only possible electron – neutrino interactions are:**



★ **The helicity dependence of the weak interaction ⟷ parity violation**

e.g. $\overline{\nu}_e + e^- \rightarrow W^-$



Valid weak interaction

Does not occur

Weak Charged Current Propagator

★The charged-current Weak interaction is different from QED and QCD in that it is mediated by massive W-bosons (80.3 GeV)

★This results in a more complicated form for the propagator:

- in handout 4 showed that for the exchange of a massive particle:

massless massive

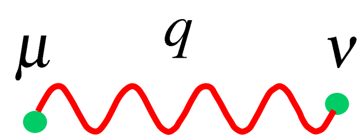
$$\frac{1}{q^2} \longrightarrow \frac{1}{q^2 - m^2}$$

- In addition, the sum over W boson polarization states modifies the numerator

W-boson propagator

spin 1 $W^{\pm}$

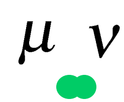
$$\frac{-i\left[g_{\mu\nu} - q_\mu q_\nu / m_W^2\right]}{q^2 - m_W^2}$$



★ However, in the limit where q^2 is small compared with $m_W = 80.3\,\mathrm{GeV}$ the interaction takes a simpler form.

W-boson propagator ($q^2 \ll m_W^2$)

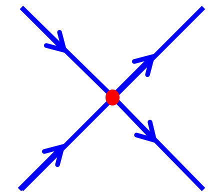
$$\frac{ig_{\mu\nu}}{m_W^2}$$



- The interaction appears point-like (i.e no q^2 dependence)

Connection to Fermi Theory

★ In 1934, before the discovery of parity violation, Fermi proposed, in analogy with QED, that the invariant matrix element for β-decay was of the form:



$$M_{fi} = G_F\, g_{\mu\nu} [\overline{\psi}\gamma^\mu \psi][\overline{\psi}\gamma^\nu \psi]$$

where $G_F = 1.166\times 10^{-5}\,\mathrm{GeV}^{-2}$

• Note the absence of a propagator : i.e. this represents an interaction at a point

★ After the discovery of parity violation in 1957 this was modified to

$$M_{fi} = \frac{G_F}{\sqrt{2}} g_{\mu\nu} [\overline{\psi}\gamma^\mu (1-\gamma^5)\psi][\overline{\psi}\gamma^\nu (1-\gamma^5)\psi]$$

(the factor of √2 was included so the numerical value of G_F did not need to be changed)

★ Compare to the prediction for W-boson exchange

$$M_{fi} = [\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^\mu(1-\gamma^5)\psi] \frac{g_{\mu\nu} - q_\mu q_\nu/m_W^2}{q^2 - m_W^2} [\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^\nu(1-\gamma^5)\psi]$$

which for $q^2 \ll m_W^2$ becomes:

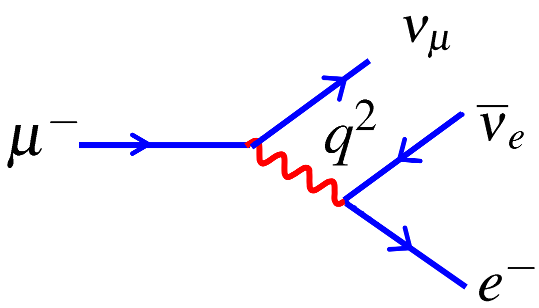
$$M_{fi} = \frac{g_W^2}{8m_W^2} g_{\mu\nu} [\overline{\psi}\gamma^\mu (1-\gamma^5)\psi][\overline{\psi}\gamma^\nu (1-\gamma^5)\psi]$$

➡ $$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

Still usually use G_F to express strength of weak interaction as the is the quantity that is precisely determined in muon decay

Strength of Weak Interaction

★ **Strength of weak interaction most precisely measured in muon decay**



- **Here** $q^2 < m_\mu \ (0.106\,\text{GeV})$
- **To a very good approximation the W-boson propagator can be written**

$$\frac{-i\left[g_{\mu\nu} - q_\mu q_\nu / m_W^2\right]}{q^2 - m_W^2} \approx \frac{i g_{\mu\nu}}{m_W^2}$$

- **In muon decay measure** g_W^2 / m_W^2
- **Muon decay** ➡ $G_F = 1.16639(1) \times 10^{-5}\,\text{GeV}^{-2}$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2}$$

★ **To obtain the intrinsic strength of weak interaction need to know mass of W-boson:** $m_W = 80.403 \pm 0.029\,\text{GeV}$

➡ $$\alpha_W = \frac{g_W^2}{4\pi} = \frac{8 m_W^2 G_F}{4\sqrt{2}\pi} = \frac{1}{30}$$

★ **The intrinsic strength of the weak interaction is similar to, but greater than, the EM interaction ! It is the massive W-boson in the propagator which makes it appear weak. For** $q^2 \gg m_W^2$ **weak interactions are more likely than EM.**

$SU(2)_L$: The Weak Interaction

★ The Weak Interaction arises from **SU(2)** local gauge symmetry

$$\psi \to \psi' = \psi e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}}$$

where the $\vec{\sigma}$ are the generators of the SU(2) symmetry, i.e the **three** Pauli spin matrices

⇨ **3 Gauge Fields** $W_1^\mu,\ W_2^\mu,\ W_3^\mu$

★ The wave-functions have two components which, in analogy with isospin, are represented by **"weak isospin", constructed to account for flavour change**

★ The fermions are placed in isospin doublets and the local phase transformation corresponds to

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \to \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}' = e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

★ Weak Interaction only couples to **LH particles**/**RH anti-particles**, hence only place **LH particles**/**RH anti-particles** in weak isospin doublets: $I_W = \frac{1}{2}$
RH particles/**LH anti-particles** placed in weak isospin singlets: $I_W = 0$

Weak Isospin

$I_W = \frac{1}{2}$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

← $I_W^3 = +\frac{1}{2}$ (upper components)
← $I_W^3 = -\frac{1}{2}$ (lower components)

$I_W = 0$

$$(\nu_e)_R,\ (e^-)_R, \ldots (u)_R,\ (d)_R, \ldots$$

Note: RH/LH refer to chiral states

SU(2)$_L$: The Weak Interaction

★ The Weak Interaction arises from SU(2) local gauge symmetry

$$\varphi(x) \rightarrow \varphi'(x) = \exp\left[ig_{\mathrm{W}}\,\boldsymbol{\alpha}(x)\cdot\mathbf{T}\right]\varphi(x)$$

where T are the three generators of SU(2) symmetry, namely the familiar three Pauli spin matrices

$$\mathbf{T} = \tfrac{1}{2}\boldsymbol{\sigma}$$

★ Hence, the fields must appear as two component “weak isospin” doublets

★ To reproduce the observed properties of the charged-current weak interaction, the weak isospin doublets are, for example,

$$\varphi(x) = \begin{pmatrix} \nu_{\mathrm{e}}(x) \\ \mathrm{e}^{-}(x) \end{pmatrix}_L$$

★ Following the approach for QED and QCD, the SU(2)$_L$ interaction is

$$ig_{\mathrm{W}} T_k \gamma^{\mu} \mathrm{W}^k_{\mu} \varphi_L = ig_{\mathrm{W}} \tfrac{1}{2}\sigma_k \gamma^{\mu} \mathrm{W}^k_{\mu} \varphi_L$$

The sum over k implies three interaction terms

★ **For simplicity only consider** $\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$

- **The gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2)**

$$j^1_\mu = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_1 \chi_L \qquad j^2_\mu = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_2 \chi_L \qquad j^3_\mu = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_3 \chi_L$$

★ **The charged current interaction enters as a linear combinations of W_1, W_2**

$$W^{\pm\mu} = \frac{1}{\sqrt{2}}(W_1^\mu \pm W_2^\mu)$$

★ **The $W^\pm$ interaction terms**

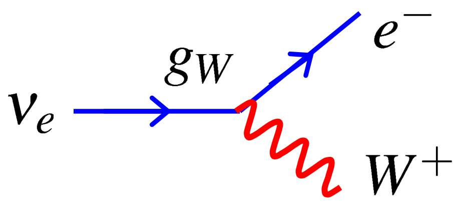
$$j^\mu_\pm = \frac{g_W}{\sqrt{2}}(j_1^\mu \pm i j_2^\mu) = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \tfrac{1}{2}(\sigma_1 \pm i\sigma_2)\chi_L$$

★ **Express in terms of the weak isospin ladder operators** $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$

$$\boxed{j^\mu_\pm = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_\pm \chi_L}$$

Origin of $\frac{1}{\sqrt{2}}$ in Weak CC

W⁺



corresponds to $\left[j^\mu_+ = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_+ \chi_L \right]$

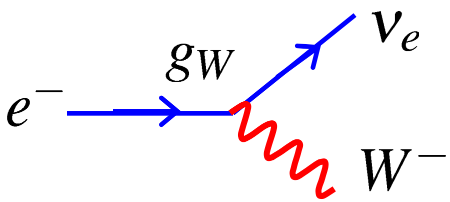
Bars indicates adjoint spinors

which can be understood in terms of the weak isospin doublet

$$j^\mu_+ = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_+ \chi_L = \frac{g_W}{\sqrt{2}} (\overline{\nu}_L, \overline{e}_L) \gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{g_W}{\sqrt{2}} \overline{\nu}_L \gamma^\mu e_L = \boxed{\frac{g_W}{\sqrt{2}} \overline{\nu} \gamma^\mu \tfrac{1}{2}(1-\gamma^5) e}$$

★ **Similarly**

W⁻



corresponds to $\left[j^{\mu}_{-} = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^{\mu} \sigma_{-} \chi_L \right]$

$$j^{\mu}_{-} = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^{\mu} \sigma_{-} \chi_L = \frac{g_W}{\sqrt{2}} (\overline{\nu}_L, \bar{e}_L) \gamma^{\mu} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{g_W}{\sqrt{2}} \bar{e}_L \gamma^{\mu} \nu_L = \frac{g_W}{\sqrt{2}} \bar{e} \gamma^{\mu} \tfrac{1}{2}(1-\gamma^5)\nu$$

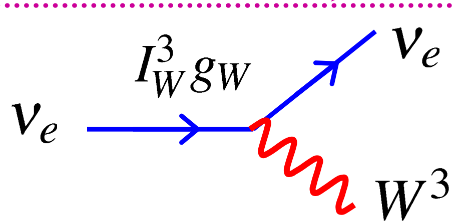
★ **However, left with have an additional interaction due to W³**

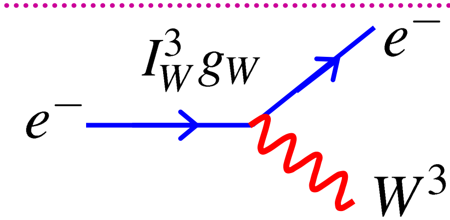
$$\boxed{j^{\mu}_{3} = g_W \overline{\chi}_L \gamma^{\mu} \tfrac{1}{2} \sigma_3 \chi_L}$$

expanding this:

$$\boxed{I^3_W = \pm\tfrac{1}{2}}$$

$$j^{\mu}_{3} = g_W \frac{1}{2} (\overline{\nu}_L, \bar{e}_L) \gamma^{\mu} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = g_W \tfrac{1}{2} \overline{\nu}_L \gamma^{\mu} \nu_L - g_W \tfrac{1}{2} \bar{e}_L \gamma^{\mu} e_L$$





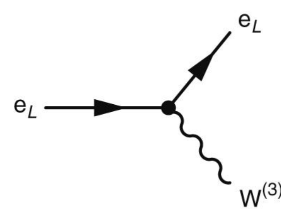
NEUTRAL CURRENT INTERACTIONS !

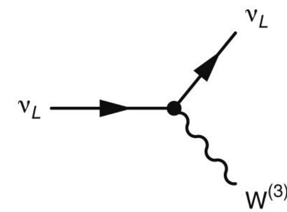
Electroweak Unification

★ Tempting to identify the W^3 as the Z boson

★ This would imply that the weak neutral current had the form

$$j_3^{\mu} = I_W^{(3)} g_W \bar{f} \gamma^{\mu} \tfrac{1}{2}(1-\gamma^5) f$$





and the weak neutral current would violate parity maximally, contrary to observation at, for example, the Z pole

★ At this point we have two neutral electroweak bosons,

$W_{\mu}^{(3)}$ of the $SU(2)_L$ symmetry

A_{μ} of the U(1) symmetry of electromagnetism

★ We have also seen that

- the intrinsic strengths of the weak interaction and EM are similar
- and the charged W bosons curry the charge of the U(1) symmetry

★ There is something else going on here

- **Electroweak Unification** of Glashow, Salam and Weinberg

Electroweak Unification

★ **Suppose the observed physical neutral electroweak bosons, the photon and the Z boson arise from the mass matrix of the W^3 and a new neutral boson B associated with a different U(1) symmetry**

★ **The physical bosons (the Z and photon field, A) are:**

$$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

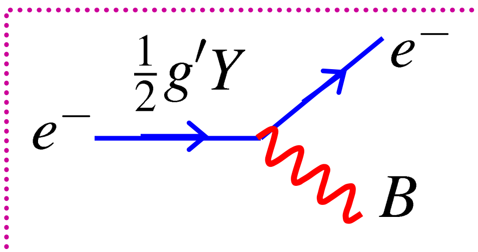
θ_W **is the weak mixing angle**

★ **The new boson is associated with a new gauge symmetry similar to that of electromagnetism : $U(1)_Y$**

★ **The charge of this symmetry is called WEAK HYPERCHARGE Y**

$$Y = 2Q - 2I^3_W$$

Q is the EM charge of a particle
I^3_W is the third comp. of weak isospin



- **By convention the coupling to the B_μ $\frac{1}{2}g'Y$**

$e_L: \; Y = 2(-1) - 2(-\tfrac{1}{2}) = -1$ $\qquad$ $\nu_L: \; Y = +1$

$e_R: \; Y = 2(-1) - 2(0) = -2$ $\qquad$ $\nu_R: \; Y = 0$

(this identification of hypercharge in terms of Q and I_3 makes all of the following work out)

★ To work the coupling constants of the **W³**, **B**, & **photon** must be related

e.g. consider contributions involving the neutral interactions of electrons:

γ $$j_\mu^{em} = e\overline{\psi}Q_e\gamma_\mu\psi = e\bar{e}_L Q_e \gamma_\mu e_L + e\bar{e}_R Q_e \gamma_\mu e_R$$

W^3 $$j_\mu^{W^3} = -\frac{g_W}{2}\bar{e}_L\gamma_\mu e_L$$

B $$j_\mu^Y = \frac{g'}{2}\overline{\psi}Y_e\gamma_\mu\psi = \frac{g'}{2}\bar{e}_L Y_{e_L}\gamma_\mu e_L + \frac{g'}{2}\bar{e}_R Y_{e_R}\gamma_\mu e_R$$

★ The relation $A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$ is equivalent to requiring

$$j_\mu^{em} = j_\mu^Y \cos\theta_W + j_\mu^{W^3}\sin\theta_W$$

•Writing this in full:

$$e\bar{e}_L Q_e\gamma_\mu e_L + e\bar{e}_R Q_e\gamma_\mu e_R = \frac{1}{2}g'\cos\theta_W[\bar{e}_L Y_{e_L}\gamma_\mu e_L + \bar{e}_R Y_{e_R}\gamma_\mu e_R] - \frac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$$

$$-e\bar{e}_L\gamma_\mu e_L - e\bar{e}_R\gamma_\mu e_R = \frac{1}{2}g'\cos\theta_W[-\bar{e}_L\gamma_\mu e_L - 2\bar{e}_R\gamma_\mu e_R] - \frac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$$

which works if: $e = g_W\sin\theta_W = g'\cos\theta_W$ (i.e. equate coefficients of L and R terms)

★ Couplings of electromagnetism, the weak interaction and the interaction of the $U(1)_Y$ symmetry are therefore related.

The Z Boson

★ **In this model we can now derive the couplings of the Z Boson**

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

I^3_W **for the electron** $I^3_W = \frac{1}{2}$

$$j^Z_\mu = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L Y_{e_L} \gamma_\mu e_L + \bar{e}_R Y_{e_R} \gamma_\mu e_R] - \tfrac{1}{2} g_W \cos\theta_W [e_L \gamma_\mu e_L]$$

- **Writing this in terms of weak isospin and charge:**

$$j^Z_\mu = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L (2Q - 2I^3_W) \gamma_\mu e_L + \bar{e}_R (2Q) \gamma_\mu e_R] + I^3_W g_W \cos\theta_W [e_L \gamma_\mu e_L]$$

For RH chiral states I_3=0

- **Gathering up the terms for LH and RH chiral states:**

$$j^Z_\mu = [g' I^3_W \sin\theta_W - g' Q \sin\theta_W + g_W I^3_W \cos\theta_W]\, \bar{e}_L \gamma_\mu e_L - [g' Q \sin\theta_W]\, e_R \gamma_\mu e_R$$

- **Using:** $e = g_W \sin\theta_W = g' \cos\theta_W$ **gives**

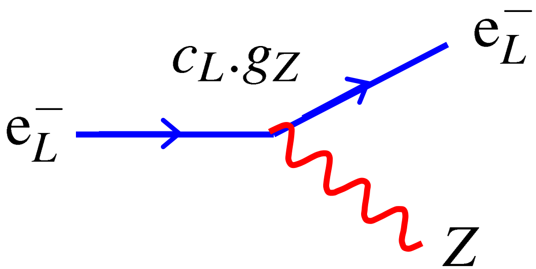
$$j^Z_\mu = \left[g' \frac{(I^3_W - Q\sin^2\theta_W)}{\sin\theta_W} \right] \bar{e}_L \gamma_\mu e_L - \left[g' \frac{Q\sin^2\theta_W}{\sin\theta_W} \right] e_R \gamma_\mu e_R$$

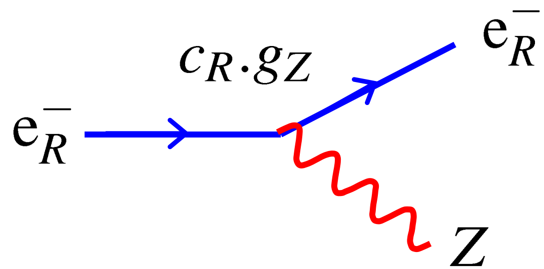
$$j^Z_\mu = g_Z (I^3_W - Q\sin^2\theta_W)[\bar{e}_L \gamma_\mu e_L] - g_Z Q \sin^2\theta_W [e_R \gamma_\mu e_R]$$

with $e = g_Z \cos\theta_W \sin\theta_W$ **i.e.** $g_Z = \dfrac{g_W}{\cos\theta_W}$

★ **Unlike for the Charged Current Weak interaction (W) the Z Boson couples to both LH and RH chiral components, but not equally…**

$$
\begin{aligned}
j^Z_\mu &= g_Z(I^3_W - Q\sin^2\theta_W)[\overline{\mathrm{e}}_L\gamma_\mu \mathrm{e}_L] - g_Z Q\sin^2\theta_W[\mathrm{e}_R\gamma_\mu e_R] \\
&= g_Z c_L[\overline{\mathrm{e}}_L\gamma_\mu \mathrm{e}_L] + g_Z c_R[\mathrm{e}_R\gamma_\mu e_R]
\end{aligned}
$$





$$c_L = I^3_W - Q\sin^2\theta_W \qquad c_R = -Q\sin^2\theta_W$$

W^3 part of Z couples only to LH components (like $W^\pm$)

B_μ part of Z couples equally to LH and RH components

★ **Use projection operators to obtain vector and axial vector couplings**

$$\overline{u}_L\gamma_\mu u_L = \overline{u}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)u \qquad \overline{u}_R\gamma_\mu u_R = \overline{u}\gamma_\mu \tfrac{1}{2}(1+\gamma_5)u$$

$$j^Z_\mu = g_Z\overline{u}\gamma_\mu\left[c_L\tfrac{1}{2}(1-\gamma_5) + c_R\tfrac{1}{2}(1+\gamma_5))\right]u$$

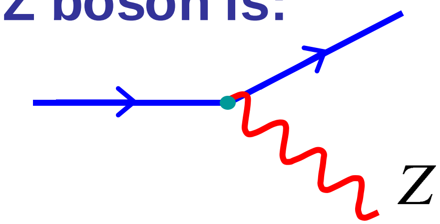
$$j_\mu^Z = \frac{g_Z}{2}\bar{u}\gamma_\mu\left[(c_L+c_R)+(c_R-c_L)\gamma_5)\right]u$$

★ Which in terms of **V** and **A** components gives: $j_\mu^Z = \frac{g_Z}{2}\bar{u}\gamma_\mu\left[c_V - c_A\gamma_5\right]u$

with $c_V = c_L + c_R = I_W^3 - 2Q\sin^2\theta_W$ $\quad c_A = c_L - c_R = I_W^3$

★ Hence the vertex factor for the Z boson is:

$$-ig_Z\tfrac{1}{2}\gamma_\mu\left[c_V - c_A\gamma_5\right]$$



★ Using the experimentally determined value of the weak mixing angle:

$\sin^2\theta_W \approx 0.23$

Fermion	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

Summary

★ The Standard Model interactions are mediated by spin-1 **gauge bosons**

★ The form of the interactions are completely specified by the assuming an underlying local phase transformation ➡ **GAUGE INVARIANCE**

$U(1)_{em}$	⇨	QED
$SU(2)_L$	⇨	Charged Current Weak Interaction + W^3
$SU(3)_{col}$	⇨	QCD

★ In order to "unify" the electromagnetic and weak interactions, introduced a new symmetry gauge symmetry : U(1) hypercharge

$U(1)_Y$	⇨	B_μ

★ The physical Z boson and the photon are mixtures of the neutral W boson and B determined by the **Weak Mixing angle**

$$\sin\theta_W \approx 0.23$$

★ **Have we really unified the EM and Weak interactions ? Well not really…**

- Started with two independent theories with coupling constants g_W, e
- Ended up with coupling constants which are related but at the cost of introducing a new parameter in the Standard Model θ_W
- Interactions not unified from any higher theoretical principle… **but the Higgs…**