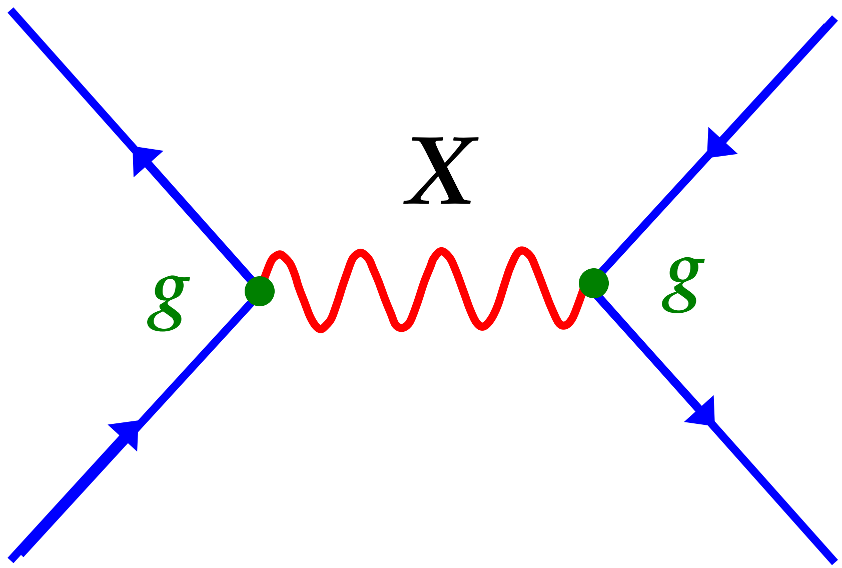
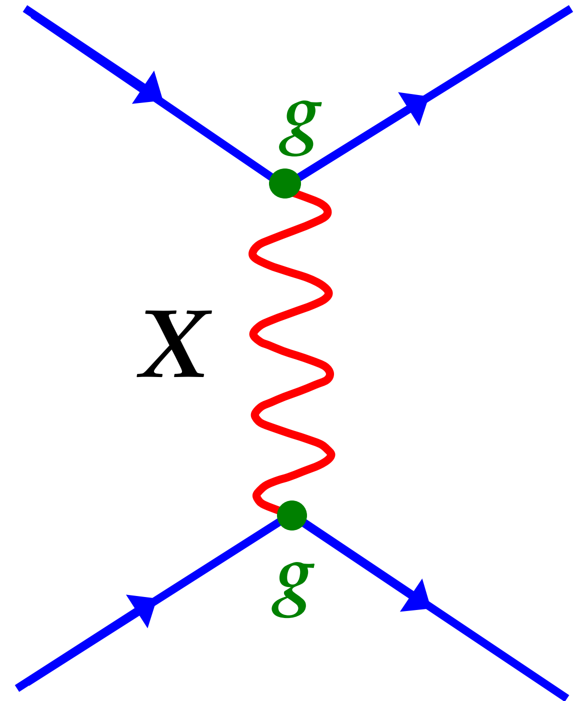
Foundations of the Standard Model

Prof Mark Thomson



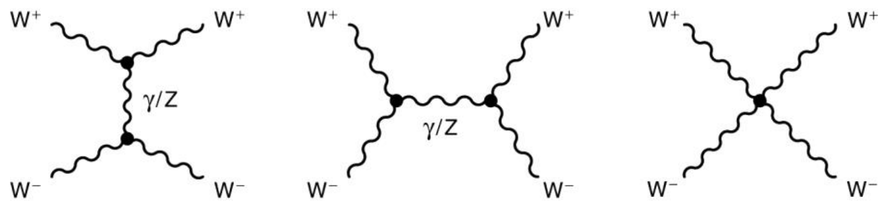


6. The Higgs Mechanism

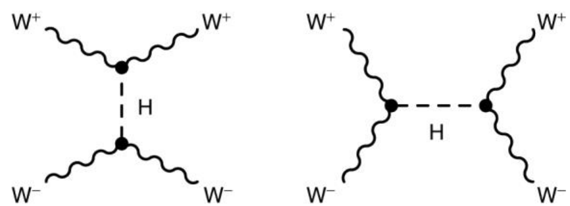
The Higgs Mechanism in 40 minutes…

★ **In the last lecture**
- Constructed a model of electroweak unification
- Its predictions have been well established at, for example, LEP and the LHC
- However, as it stands it does not work…

★ **The diagrams contributing to WW scattering lead to unitarity violation at ~1 TeV**
- Something is missing…



- Naturally fixed by the Higgs Boson contributions



★ **A bigger problem?**
- The $SU(2)_L$ Local Gauge Symmetry doesn't "allow" massive W/Z bosons
- Can't have both…
- Need to generate masses dynamically – the Higgs Mechanism

Dynamical generation of "mass"

★ **A simple analogy from classical electromagnetism**

- Consider EM radiation propagating through a plasma
- Because the plasma acts as a polarisable medium obtain the "dispersion relation"

$$n^2 = 1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

n = refractive index
ω = angular frequency
ω_p = plasma frequency

★ **Quanta only propagate if have frequency/energy greater than some minimum**

$$E > E_0 = \hbar\omega_p$$

★ **Above this energy, waves propagate with group velocity** $v_g = \frac{c^2}{v_p} = nc$

- **Dropping the subscript and using the previous expression for** n

$$v^2 = c^2 n^2 = c^2\left(1 - \frac{\hbar^2\omega_p^2}{\hbar^2\omega^2}\right) = c^2\left(1 - \frac{E_0^2}{E^2}\right)$$

- **Rearranging gives**

$$\frac{E_0^2}{E^2} = 1 - \frac{v^2}{c^2} \Rightarrow \boxed{E = E_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \gamma m c^2}$$

with $m = E_0/c^2$

- **Massless photons propagating through a plasma behave as massive particles propagating in a vacuum**

Gauge Invariance Revisited

★ **Recall the Lagrangian corresponding to the free particle Dirac equation**

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi$$

★ **Now require invariance under a U(1) local gauge transformation**

$$\psi(x) \to \psi'(x) = e^{iq\chi(x)}\psi(x)$$

under this transformation

$$\mathcal{L} \to \mathcal{L}' = ie^{-iq\chi}\overline{\psi}\gamma^\mu\left[e^{iq\chi}\partial_\mu\psi + iq\left(\partial_\mu\chi\right)e^{iq\chi}\psi\right] - me^{-iq\chi}\overline{\psi}e^{iq\chi}\psi$$

$$= \mathcal{L} - \boxed{q\overline{\psi}\gamma^\mu\left(\partial_\mu\chi\right)\psi}$$

★ **Can achieve the desired gauge invariance by replacing ∂_μ with the covariant derivative D_μ**

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$$

the desired cancellation of the term $q\overline{\psi}\gamma^\mu(\partial_\mu\chi)\psi$ can be achieved provided

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu\chi$$

★ **Nothing new here, just reformulating in terms of the Lagrangian density and the full gauge-invariant Lagrangian for QED is**

$$\mathcal{L}_{QED} = \overline{\psi}(i\gamma^\mu\partial_\mu\psi - m)\psi - q\overline{\psi}\gamma^\mu A_\mu\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where the normal field-strength tensor term is now included

The Problem with Mass

★ The corresponding "QED" Lagrangian with a massive photon would be

$$\mathcal{L}_{\mathrm{QED}'} \rightarrow \overline{\psi}(i\gamma^{\mu}\partial_{\mu}\psi - m_{\mathrm{e}})\psi + e\overline{\psi}\gamma^{\mu}A_{\mu}\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\tfrac{1}{2}m_{\gamma}A_{\mu}A^{\mu}}$$

★ But the mass term, breaks the U(1) gauge invariance from $A_{\mu} \rightarrow A'_{\mu} = A_{\mu} - \partial_{\mu}\chi$

$$\tfrac{1}{2}m_{\gamma}^{2}A_{\mu}A^{\mu} \rightarrow \tfrac{1}{2}m_{\gamma}^{2}\left(A_{\mu} - \partial_{\mu}\chi\right)(A^{\mu} - \partial^{\mu}\chi) \neq \tfrac{1}{2}m_{\gamma}^{2}A_{\mu}A^{\mu}$$

★ This is not a problem for QED and QCD where the gauge bosons are massless, but breaks the $SU(2)_L$ gauge symmetry of the weak interaction with massive gauge bosons

★ There is also a problem with fermion masses…
e.g. consider the electron mass term in the QED Lagrangian

$$\begin{aligned} -m_{\mathrm{e}}\bar{\mathrm{e}}\mathrm{e} &= -m_{\mathrm{e}}\bar{\mathrm{e}}\left[\tfrac{1}{2}(1-\gamma^{5}) + \tfrac{1}{2}(1+\gamma^{5})\right]\mathrm{e} \\ &= -m_{\mathrm{e}}\bar{\mathrm{e}}\left[\tfrac{1}{2}(1-\gamma^{5})\mathrm{e}_{L} + \tfrac{1}{2}(1+\gamma^{5})\mathrm{e}_{R}\right] \\ &= -m_{\mathrm{e}}(\bar{\mathrm{e}}_{R}\mathrm{e}_{L} + \bar{\mathrm{e}}_{L}\mathrm{e}_{R})\,. \end{aligned}$$

this also does not respect the SU(L) gauge symmetry, because LH particles transform as weak isospin doublets and RH particles transform as singlets

Towards the Higgs

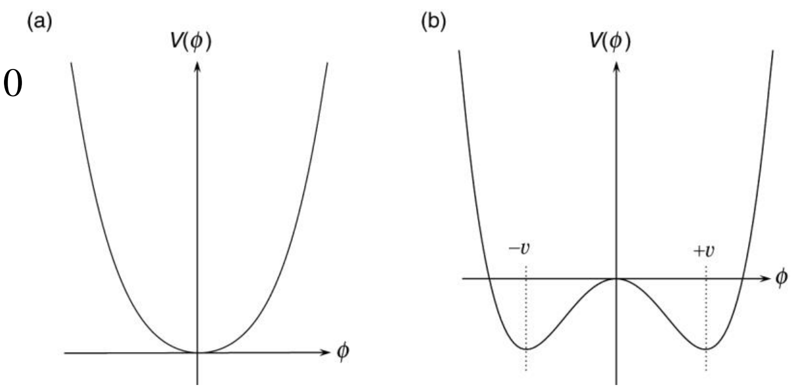
★ The algebra of Higgs mechanism is quite fiddly to derive, mostly

★ Starting gently… a scalar field with a potential $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) \\ &= \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4\end{aligned}$$

★ For lambda positive, the potential can take two forms

$\mu^2 > 0$



$\mu^2 < 0$

Although the potential is symmetric, for $\mu^2 < 0$ the lowest energy "vacuum" states occur at either

$$\phi = \pm v = \pm\left|\sqrt{\frac{-\mu^2}{\lambda}}\right|$$

★ The choice of vacuum spontaneously breaks the symmetry of the Lagrangian

Towards the Higgs

★ **If the vacuum is chosen to be at $\phi = +v$, expanding about this point**

$$\phi(x) = v + \eta(x)$$

★ **The Lagrangian now can be written**

$$\mathcal{L}(\eta) = \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - V(\eta)$$
$$= \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \tfrac{1}{2}\mu^2(v+\eta)^2 - \tfrac{1}{4}\lambda(v+\eta)^4$$

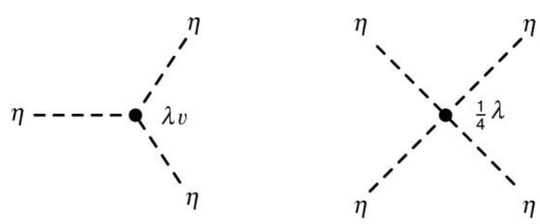
Since the minimum of the potential is given by $\mu^2 = \lambda v^2$ this is equivalent to

$$\mathcal{L}(\eta) = \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \tfrac{1}{4}\lambda\eta^4 + \tfrac{1}{4}\lambda v^4$$

★ **Dropping the constant**

$$\mathcal{L}(\eta) = \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \tfrac{1}{2}m_\eta^2\eta^2 - V(\eta)\,, \quad \text{with} \quad V(\eta) = \lambda v\eta^3 + \tfrac{1}{4}\lambda\eta^4$$

which is the Lagrangian for a massive scalar field $m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$ with triple and quartic self-interactions



One step closer

★ **Now consider a complex scalar field** $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

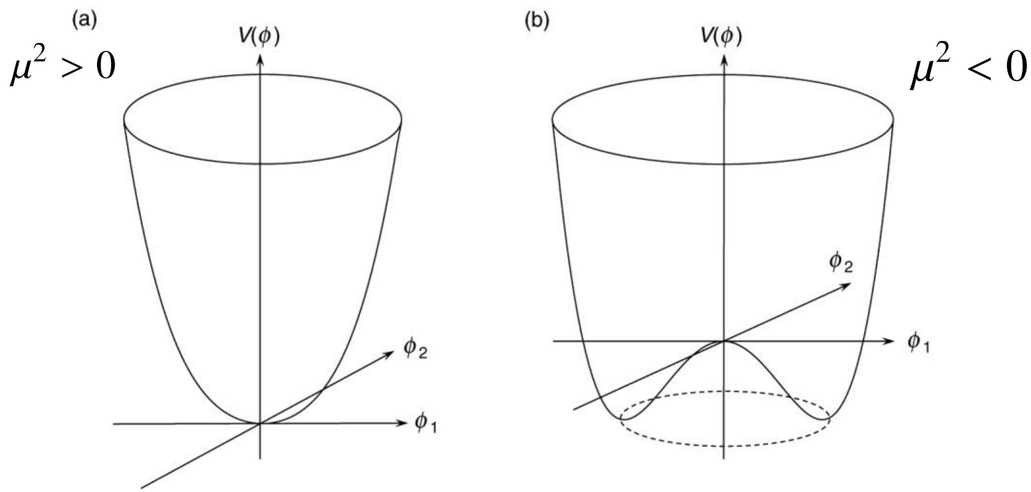
for which the Lagrangian is

$$\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - V(\phi) \quad \text{with} \quad V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$$

In terms of the two field components

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \tfrac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) - \tfrac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \tfrac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

★ **The shape of the potential takes two forms**



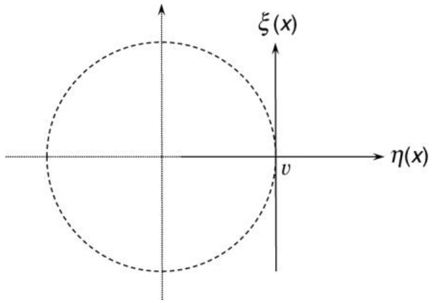
for $\mu^2 < 0$ **there are a continuous (infinite) set of minima**

$$\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} = v^2$$

One step closer

★ **Break the symmetry: expand around the minimum** $(\phi_1, \phi_2) = (v, 0)$

$$\phi(x) = \frac{1}{\sqrt{2}}(\eta(x) + v + i\xi(x))$$



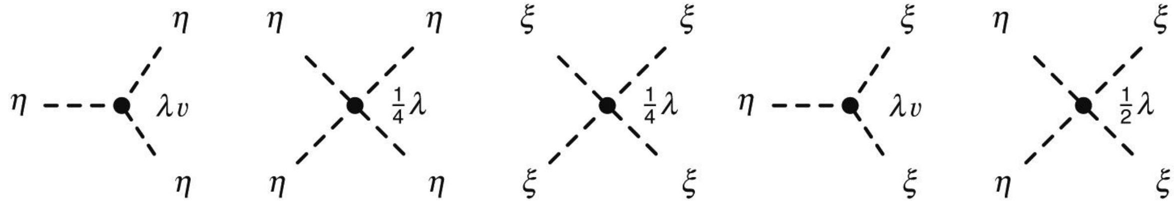
★ **Then express the original Lagrangian in terms of these new fields. After some simple algebra**

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \frac{1}{2}m_\eta^2\eta^2 + \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) - V_{int}(\eta, \xi)$$

with $m_\eta = \sqrt{2\lambda v^2}$ **and interactions**

$$V_{int}(\eta, \xi) = \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \frac{1}{2}\lambda\eta^2\xi^2$$

★ **This the Lagrangian for a massive scalar field** $\eta(x)$ **+ massless scalar** $\xi(x)$**, known as a Goldstone Boson, and interactions**



The Higgs Mechanism

★ **Repeat the previous exercise for a complex scalar field, but now embedded in a theory with a U(1) local gauge symmetry and the same potential**

$$V(\phi) = \mu^2\phi^2 + \lambda\phi^4$$

★ **Because of the derivatives in**

$$\mathcal{L} = (\partial_\mu\phi)^*(\partial^\mu\phi) - V(\phi)$$

the Lagrangian is not invariant under

$$\phi(x) \rightarrow \phi'(x) = e^{ig\chi(x)}\phi(x)$$

★ **As before, the gauge invariance can be achieved using the covariant derivative**

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + igB(x)$$

★ **The resulting Lagrangian**

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - V(\phi^2)$$

has the required local gauge invariance provided

$$B_\mu \rightarrow B'_\mu = B_\mu - \partial_\mu\chi(x)$$

★ **The U(1) local gauge invariance requires a new field with well-defined gauge transformation properties – nothing new here**

The Higgs Mechanism

★ **The combined Lagrangian for the complex scalar field and the new *massless vector* field *B is***

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4$$

where $F^{\mu\nu}F_{\mu\nu}$ **is the kinetic term associated with the field-strength tensor**

$$F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

★ **Again, adding an explicit mass term** $\frac{1}{2}m_B B_\mu B^\mu$ **would break the gauge invariance**

★ **The full expression for the Lagrangian (expanding the covariant derivative) is**

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4$$
$$- igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2B_\mu B^\mu\phi^*\phi\,.$$

★ **Breaking the symmetry and repeating the previous expansion around the minimum**

$$\phi(x) = \tfrac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$

★ **After some straightforward algebra**

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2}_{\text{massive }\eta} + \underbrace{\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)}_{\text{massless }\xi} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B_\mu B^\mu}_{\text{massive gauge field}} - V_{int} + gvB_\mu(\partial^\mu\xi)$$

we now have a local gauge invariant mass term for the gauge boson!

The Higgs Mechanism

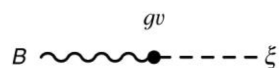
★ **It all looks so good…**

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2}_{\text{massive } \eta} + \underbrace{\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)}_{\text{massless } \xi} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B_\mu B^\mu}_{\text{massive gauge field}} - V_{int} + \boxed{gvB_\mu(\partial^\mu\xi)}$$

but we seem to have two problems

- **We started with 4 degrees of freedom (two for the complex scalar field + two for the transverse polarisations of the massless vector gauge boson). We now appear to have 5. Two scalar fields and 3 polarisations – an additional degree of freedom associated with the longitudinal polarisation of the massive gauge boson**

- **There appears to be a coupling between a massless Goldstone scalar and massive spin-1 vector field**



★ **The solution is subtle. Both the above problems are indicative of the fact we are not working with fields that represent the physical particles. The $gvB_\mu(\partial^\mu\xi)$ term is reminiscent of an off-diagonal term in a mass matrix for a couple quantum system. We need to work in the diagonal basis if the fields are to represent the physical particles**

The Higgs Mechanism

★ **The Lagrangian**

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2}_{\text{massive } \eta} + \underbrace{\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)}_{\text{massless } \xi} - \underbrace{\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} g^2 v^2 B_\mu B^\mu}_{\text{massive gauge field}} - V_{int} + \boxed{g v B_\mu (\partial^\mu \xi)}$$

is gauge invariant. Can we find a gauge in which the non-physical fields do not appear? Yes… the unitary gauge

★ **Noting that**

$$\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + g v B_\mu (\partial^\mu \xi) + \tfrac{1}{2} g^2 v^2 B_\mu B^\mu = \tfrac{1}{2} g^2 v^2 \left[B_\mu + \frac{1}{gv}(\partial_\mu \xi) \right]^2$$

it is clear that the massless Goldstone field can be eliminated by making the gauge transformation

$$B_\mu(x) \rightarrow B'_\mu(x) = B_\mu(x) + \frac{1}{gv} \partial_\mu \xi(x)$$

★ **Because of the built in U(1) gauge invariance of the Lagrangian, nothing has physically changed. This is just equivalent of a U(1) transformation**

$$\phi(x) \rightarrow \phi'(x) = e^{-ig\frac{\xi(x)}{gv}} \phi(x) = e^{-i\xi(x)/v} \phi(x)$$

★ **To first order our original expansion about the SSB minimum can be written**

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)) \approx \frac{1}{\sqrt{2}} [v + \eta(x)]\, e^{i\xi(x)/v}$$

and it is clear that in the Unitary gauge the field $\xi(x)$ has been "gauged away"

The Higgs Mechanism

★ **The last step is equivalent to writing**

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) \equiv \frac{1}{\sqrt{2}}(v + h(x))$$

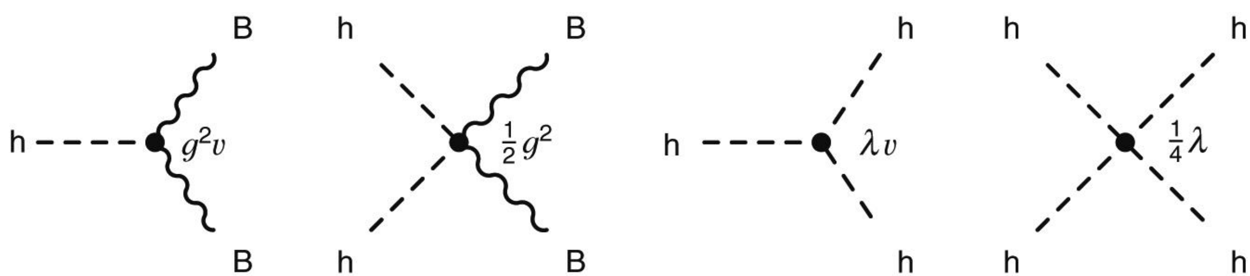
where the field η(x) has been rewritten as the Higgs field h(x) to emphasise the fact that this is the physical field

★ **In the Unitary gauge**

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2}_{\text{massive } h \ scalar} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2 v^2 B_\mu B^\mu}_{\text{massive gauge boson}}$$

$$+ \underbrace{g^2 v B_\mu B^\mu h + \tfrac{1}{2}g^2 B_\mu B^\mu h^2}_{h,B \text{ interactions}} - \underbrace{\lambda v h^3 - \tfrac{1}{4}\lambda h^4}_{h \text{ self-interactions}}$$

with $\boxed{m_B = g\,v}$ **and** $\boxed{m_{\mathrm{H}} = \sqrt{2\lambda}\,v}$ **and well-defined boson interactions**



The Higgs and Electroweak Unification

★ **Electroweak unification is achieved by embedding the Higgs Mechanism in the local gauge symmetry** $\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L$

★ **The scalar fields now need to be placed in a weak isospin doublet and the simplest model uses two complex scalar fields in a weak isospin doublet**

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \tfrac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

★ **The Lagrangian for the scalar fields can be written**

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi)$$

with $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$

and a continuum of minima at $\phi^\dagger \phi = \tfrac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = v^2 = -\dfrac{\mu^2}{2\lambda}$

★ **After symmetry breaking, we expect a neutral massless vector boson (the photon), so anticipating this expand the fields around the vacuum minimum**

$$\langle 0|\phi|0\rangle = \tfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{i.e.} \quad \phi(x) = \tfrac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{pmatrix}$$

The Higgs and Electroweak Unification

★ **The local gauge symmetry can be imposed by replacing**

$$\partial_\mu \to D_\mu = \partial_\mu + ig_\mathrm{W}\mathbf{T}\cdot\mathbf{W}_\mu + ig'\frac{Y}{2}B_\mu$$

where $\mathbf{T} = \frac{1}{2}\boldsymbol{\sigma}$ **are the three SU(2) generators, represented by 2x2 matrices**

★ **The rest is a slog. However, anticipating that three Goldstone bosons will form the longitudinal polarisation states of three massive gauge bosons and the physical states will emerging after moving to the Unitary gauge we can cheat a bit and immediately write the Higgs doublet in the Unitary Gauge**

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

★ **Putting this all together, get**

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) + \tfrac{1}{8}g_\mathrm{W}^2(W_\mu^{(1)} + iW_\mu^{(2)})(W^{(1)\mu} - iW^{(2)\mu})(v+h)^2$$
$$+ \tfrac{1}{8}(g_\mathrm{W}W_\mu^{(3)} - g'B_\mu)(g_\mathrm{W}W^{(3)\mu} - g'B^\mu)(v+h)^2$$

★ **Gauge boson masses are quadratic in the fields, i.e. are contained in**

$$\tfrac{1}{8}v^2g_\mathrm{W}^2\left(W_\mu^{(1)}W^{(1)\mu} + W_\mu^{(2)}W^{(2)\mu}\right) + \tfrac{v^2}{8}\left(g_\mathrm{W}W_\mu^{(3)} - g'B_\mu\right)\left(g_\mathrm{W}W^{(3)\mu} - g'B^\mu\right)$$

therefore

$$\boxed{m_\mathrm{W} = \tfrac{1}{2}g_\mathrm{W}v}$$

The Neutral Boson Masses

★ the masses of the neutral bosons are obtained from

$$\frac{v^2}{8}\left(g_W W^{(3)}_\mu - g' B_\mu\right)\left(g_W W^{(3)\mu} - g' B^\mu\right) = \frac{v^2}{8}\left(W^{(3)}_\mu \; B_\mu\right)\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} W^{(3)\mu} \\ B^\mu \end{pmatrix}$$

$$= \frac{v^2}{8}\left(W^{(3)}_\mu \; B_\mu\right)\mathbf{M}\begin{pmatrix} W^{(3)\mu} \\ B^\mu \end{pmatrix}$$

★ in the mass matrix there are terms that couple together the $W^{(3)}$ and B fields, which means that these are not the physical fields that are eigenstates of the free particle Hamiltonian. To obtain the physical fields, need to diagonalise the mass matrix using the characteristic equation $\det(\mathbf{M} - \lambda I) = 0$ giving eigenvalues from

$$(g_W^2 - \lambda)(g'^2 - \lambda) - g_W^2 g'^2 = 0$$

giving $\lambda = 0$ or $\lambda = g_W^2 + g'^2$

★ In the diagonal basis the mass matrix is

$$\frac{1}{8}v^2\left(A_\mu \;\; Z_\mu\right)\begin{pmatrix} 0 & 0 \\ 0 & g_W^2 + g'^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}$$

a massless field (the photon) and a massive neutral gauge boson the Z with

$$\boxed{m_Z = \tfrac{1}{2}\sqrt{(g_W^2 + g'^2)}\,v}$$

The Neutral Boson Masses

★ **The physical fields, that correspond to the normalised eigenvalues of the mass matrix are:**

$$A_\mu = \frac{g' W^{(3)}_\mu + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \quad \text{with} \quad m_A = 0\,,$$

$$Z_\mu = \frac{g_W W^{(3)}_\mu - g' B_\mu}{\sqrt{g_W^2 + g'^2}} \quad \text{with} \quad m_Z = \tfrac{1}{2} v \sqrt{g_W^2 + g'^2}$$

★ **Writing the ratio of the couplings of the underlying U(1)$_Y$ and SU(2)$_L$ local gauge symmetries as**

$$\frac{g'}{g_W} = \tan\theta_W$$

we obtain the relations

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^{(3)}_\mu\,,$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W^{(3)}_\mu$$

★ **In GSW electroweak model everything is determined by the two couplings of the underlying gauge symmetries and the two free parameters of the Higgs potential**

$$\boxed{g_W,\ g',\ \mu,\ \lambda}$$

Wrapping Up I

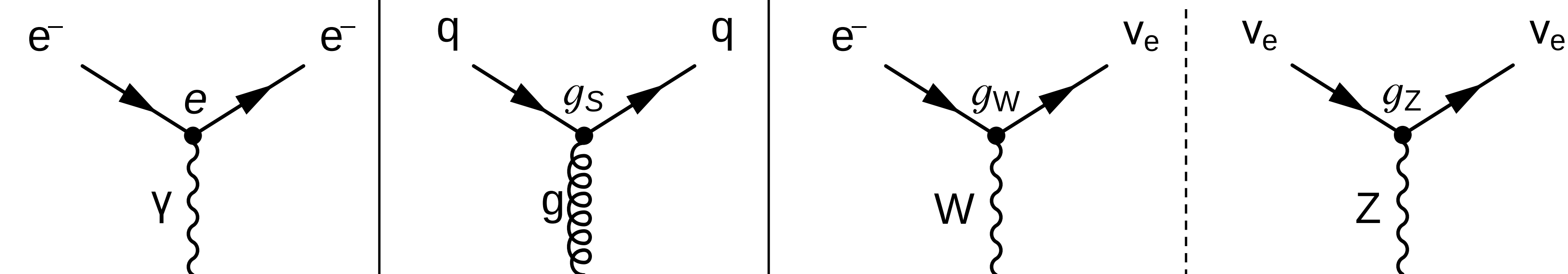
- The Higgs Mechanism is central to the electroweak unification in the standard model, fixing the properties of the W, Z and Higgs

- Can also be used to write down gauge invariant terms in the Lagrangian for fermion masses – but has no power in determining the masses

$$\overline{L}\phi_c R + \overline{R}\phi_c^\dagger L$$

- In the SM, in some sense all particles are massless, the mass terms in the Lagrangian are generated dynamically through symmetry breaking

Electromagnetism | Strong Interaction | Weak Interaction



Electromagnetism	Strong Interaction	Weak Interaction	
All charged particles	Only quarks	All fermions	All fermions
Never changes flavour	Never changes flavour	Always changes flavour	Never changes flavour
$\alpha \approx 1/137$	$\alpha_S \approx 1$	$\alpha_{W/Z} \approx 1/30$	

Wrapping Up 2

★ **What are the Foundations of the SM?**

- **Lorentz Invariance** and Relativistic Quantum Mechanics
- **Quantum Field Theory** – a framework for relativistic calculation
- **Local Gauge Invariance** – determines the nature of the SM forces
- **Higgs Mechanism** – the glue that makes electroweak unification work

  and much much more

★ **Really only scratched the surface of the SM - many topics not covered:**

- **CKM and PNMS matrices –** weak interaction mixing
- **CP Violation** – a framework for relativistic calculation
- **Neutrinos and neutrino oscillations** – masses and CP violation
- ….

★ **Also, many questions**

- **Why 3 generations of fermions –** why not one or two, are there more
- **Why are the fermion masses so different –** $m_e \sim 0.0005$ GeV, $m_t \sim 175$ GeV
- **Why $U(1)xSU(2)_L xSU(3)_c$** – chosen to reflect experiment, e.g.
- **Neutrinos masses** – very small, perhaps something else is going on
- ….

★ **More than enough to keep the next generation of experimentalists, theorists and accelerator physicists extremely busy…**