Modern machine learning methods in HEP Lecture I — Introduction to Machine Learning



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Why care about ML in physics?

Why ML in HEP?

Data volume

Large amounts of data

- 1. labeled (Simulation)
- 2. unlabeled (Detector)

ML wants lots of data





Signal detection

Rare and elusive signals among large backgrounds

Complexity

High-dimensional & highly correlated data structure

ML is expressive and flexible



1987

ML has high accuracy and sensitivity





Increasing interest

- > 150 paper/year
 - Future of HEP?

ML is fun!



Computing Budget

Simulation & analysis is computationally expensive

ML is fast



2014





























































































LHC analysis + ML









HEPML Living Review



GitHub ☆ 246 ¥ 78

>

Table of contents Reviews Modern reviews Specialized reviews Classical papers Datasets Classification Parameterized classifiers Representations Targets Learning strategies Fast inference / deployment Regression Pileup Calibration Recasting Matrix elements Parameter estimation **Parton Distribution Functions** (and related) Lattice Gauge Theory **Function Approximation** Symbolic Regression Equivariant networks.

Check LivingReview for many **ML4HEP** applications

HEPML

Equivariant networks.





1. Introduction to Machine Learning

- Basic concepts of machine learning
- Classification and Regression
- Example: Top Tagging, MadMiner \bullet

2. Generative Models for the LHC

Normalizing flows \bullet

Today

Friday

- **CWoLA and Anomaly detection** \bullet
- Examples: MadNIS, CWoLA-Hunting,...



Lecture I (90min)

Lecture II (90min)



1. Introduction to Machine Learning

- Basic concepts of machine learning
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2. Generative Models for the LHC

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Lecture I (90min)

Lecture II (90min)



Introduction to Machine Learning





Tom Mitchell

ML pioneer



"Machine learning (ML) is the study of computer algorithms that can improve automatically through experience and by the use of data"





Tom Mitchell

ML pioneer

"Machine learning (ML) is the study of computer algorithms that can improve automatically through experience and by the use of data"











Tom Mitchell

ML pioneer

"Machine learning (ML) is the study of computer **algorithms** that can improve automatically through **experience** and by the use of data"

- 1. algorithm: a method to perform a task of interest
- 2. experience: training data, which the algorithm can use to learn how to perform a task
- 3. **improve**: a way to measure the performance on the training data
- 4. **automatically**: a strategy to exploit the training data, without external input



"Machine learning is just statistics on steroids. Lots and lots of steroids"

Ilya Sutskever

Co-founder of OpenAl





What if feels like... (sometimes)

Aim of the lectures:

Giving you the tools and ideas to use it right!





What if feels like... (sometimes)

Aim of the lectures:

Giving you the tools and ideas to use it right!

Be aware!

The core of machine learning is to find structure in data - **no more no less!**



Thanks to machine-learning algorithms, the robot apocalypse was short-lived.



Artificial Intelligence

ARTIFICIAL INTELLIGENCE

A technique which enables machines to mimic human behaviour





Artificial Intelligence

Machine Learning

ARTIFICIAL INTELLIGENCE

A technique which enables machines to mimic human behaviour

MACHINE LEARNING

Subset of AI technique which use statistical methods to enable machines to improve with experience





Artificial Intelligence

Machine Learning

Deep Learning

ARTIFICIAL INTELLIGENCE

A technique which enables machines to mimic human behaviour

MACHINE LEARNING

Subset of AI technique which use statistical methods to enable machines to improve with experience

DEEP LEARNING

Subset of ML which make the computation of neural networks feasible





What are neural networks?

Components of artificial neurons







Deep neural network





Training objective and optimization

Performance measure

In order to train the neural network, we need a training objective or loss function:



 $\mathscr{L}_{\text{tot}} = \sum_{i}^{N} \mathscr{L}(f_{\omega}(x_{i}), y_{i})$



Training objective and optimization

Performance measure

In order to train the neural network, we need a training objective or loss function:



Backpropagation



$$\mathscr{L}(f_{\omega}(x_i), y_i)$$

$$\frac{\partial \mathscr{L}}{\partial \omega} = \sum_{ij} \frac{\partial \mathscr{L}}{\partial f_{j,\omega}} \cdot \frac{\partial f_{j,\omega}}{\partial \omega_i} \stackrel{!}{=} 0$$


Training objective and optimization

Performance measure

In order to train the neural network, we need a training objective or loss function:



Backpropagation



Stochastic gradient descent

$$\omega_{t+1} = \omega_t - \eta \nabla_{\omega_t} \mathscr{L}_b(\omega),$$

$$\mathscr{L}(f_{\omega}(x_i), y_i)$$

$$\frac{\partial \mathscr{L}}{\partial \omega} = \sum_{ij} \frac{\partial \mathscr{L}}{\partial f_{j,\omega}} \cdot \frac{\partial f_{j,\omega}}{\partial \omega_i} \stackrel{!}{=} 0$$

with
$$\mathscr{L}_b(\omega) = \sum_{i}^{b} \mathscr{L}(f_{\omega}(x_i), y_i)$$



MODE CONNECTIVITY

OPTIMA OF COMPLEX LOSS FUNCTIONS CONNECTED BY SIMPLE CURVES OVER WHICH TRAINING AND TEST ACCURACY ARE NEARLY CONSTANT

BASED ON THE PAPER BY TIMUR GARIPOV, PAVEL IZMAILOV, DMITRII PODOPRIKHIN, DMITRY VETROV, ANDREW GORDON WILSON VISUALIZATION & ANALYSIS IS A COLLABORATION BETWEEN TIMUR GARIPOV, PAVEL IZMAILOV AND JAVIER IDEAMI@LOSSLANDSCAPE.COM

3.6

NeurIPS 2018, ARXIV:1802.10026 | LOSSLANDSCAPE.COM



Optimizer and loss landscape

Finding the global minimum in a sea of local minima



- Loss function
- Lack of exploration
- Lack of exploitation





Overfitting und regularization





Overfitting und regularization



Identify

 Test/validation set: independent samples for testing/ validation



Overfitting und regularization



Identify

 Test/validation set: independent samples for testing/ validation

Regularization

- L1 & L2 Regularization: penalty term in the loss function
- Dropout: randomly sets the inputs to some nodes to zero



How to choose the loss function?

Fits and interpolations

Approximate function

$$f_{\omega}(x) \approx f(x)$$

Maximize probability for fit output

$$p(x \mid \omega) = \prod_{j} \frac{1}{\sqrt{2\pi\sigma_j}} \exp\left(\frac{-\frac{|f_j - f_{\omega}(x_j)|^2}{2\sigma_j}}{2\sigma_j}\right)$$

$$\Rightarrow \quad \log p(x \mid \omega) = -\sum_{j} \frac{|f_j - f_{\omega}(x_j)|^2}{2\sigma_j} + \text{const}$$





Fits and interpolations

Approximate function

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Assumes Gaussian probablity distribution





Fits and interpolations

Approximate function

$$f_{\omega}(x) \approx f(x)$$

Maximize probability for fit output

$$p(x \mid \omega) = \prod_{j} \frac{1}{\sqrt{2\pi\sigma_{j}}} \exp\left(-\frac{|f_{j} - f_{\omega}(x_{j})|^{2}}{2\sigma_{j}}\right)$$

$$\Rightarrow \quad \log p(x \mid \omega) = -\sum_{j} \frac{|f_{j} - f_{\omega}(x_{j})|^{2}}{2\sigma_{j}} + \text{const}$$

$$Minimize \text{ negative log-likelihood}$$

Assumes Gaussian probablity distribution





Loss function in fits

$$\mathscr{L}_{\text{fit}} = \frac{1}{N} \sum_{j} \mathscr{L}_{j} = \frac{1}{N} \sum_{j} \frac{|f_{j} - f_{\omega}(x_{j})|^{2}}{2\sigma_{j}}$$



Loss function in fits

$$\mathscr{L}_{\text{fit}} = \frac{1}{N} \sum_{j} \mathscr{L}_{j} = \frac{1}{N} \sum_{j} \frac{|f_{j} - f_{\omega}(x_{j})|^{2}}{2\sigma_{j}}$$

Typical ML Regression loss



$$\mathscr{L} = \frac{1}{2\sigma N} \sum_{j} |f_j - f_\omega(x_j)|^2 \equiv \frac{1}{2\sigma} \text{MSE}$$























Loss function in fits

$$\mathscr{L}_{\text{fit}} = \frac{1}{N} \sum_{j} \mathscr{L}_{j} = \frac{1}{N} \sum_{j} \frac{|f_{j} - f_{\omega}(x_{j})|^{2}}{2\sigma_{j}}$$



Puts more weight solely on deviations of the function values!























Loss function in fits

$$\mathscr{L}_{\text{fit}} = \frac{1}{N} \sum_{j} \mathscr{L}_{j} = \frac{1}{N} \sum_{j} \frac{|f_{j} - f_{\omega}(x_{j})|^{2}}{2\sigma_{j}} \quad \text{if errors}$$

Preprocessing of the training data

$$f_j \to \log f_j$$
 $f_j \to f_j - \langle f_j \rangle$ $f_j \to \frac{f_j}{\langle f_j \rangle} \cdots$

























Loss function in fits

$$\mathscr{L}_{\text{fit}} = \frac{1}{N} \sum_{j} \mathscr{L}_{j} = \frac{1}{N} \sum_{j} \frac{|f_{j} - f_{\omega}(x_{j})|^{2}}{2\sigma_{j}} \quad \text{if errors}$$

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What about Classification?

Approximate data probability

 $p_{\omega}(x) \approx p_{\text{data}}(x)$



What about Classification?

Approximate data probability

 $p_{\omega}(x) \approx p_{\text{data}}(x)$

IX.

On the Problem of the most Efficient Tests of Statistical Hypotheses.

By J. NEYMAN, Nencki Institute, Soc. Sci. Lit. Varsoviensis, and Lecturer at the Central College of Agriculture, Warsaw, and E. S. PEARSON, Department of Applied Statistics, University College, London.

(Communicated by K. PEARSON, F.R.S.)

(Received August 31, 1932.-Read November 10, 1932.)



ratio of the two likelihoods is the **most powerful** test statistic





What about Classification?



Kullback-Leibler divergence

$$D_{\rm KL}(p_{\rm data} | p_{\omega}) = \left\langle \log \frac{p_{\rm data}(x)}{p_{\omega}(x)} \right\rangle_{p_{\rm data}} = \int dx \, p_{\rm data}(x)$$

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ratio of the two likelihoods is the most powerful test statistic

$$\log \frac{p_{\text{data}}(x)}{p_{\omega}(x)}$$





Classification loss function

Classification loss

$$\begin{aligned} \mathscr{L}_{\text{class}} &= \sum_{j=S,B} D_{\text{KL}}(p_{\text{data},j} | p_{\omega,j}) \\ &= -\sum_{\{x\}} \left[p_{\text{data},S} \log p_{\omega,S} + p_{\text{data},B} \log p_{\omega,B} \right] - \end{aligned}$$





Classification loss function

Classification loss

$$\mathscr{L}_{class} = \sum_{j=S,B} D_{KL}(p_{data,j} | p_{\omega,j})$$
$$= -\sum_{\{x\}} \left[p_{data,S} \log p_{\omega,S} + p_{data,B} \log p_{\omega,B} \right] \cdot$$

Using $p_B = 1 - p_S$

Binary cross-entropy loss

$$\mathcal{L}_{\text{BCE}} = -\sum_{\{x\}} \left[p_{\text{data},S} \log p_{\omega,S} + (1 - p_{\text{data},S}) \log(1 + 1) \right]$$





Classification loss function

Classification loss

$$\mathscr{L}_{class} = \sum_{j=S,B} D_{KL}(p_{data,j} | p_{\omega,j})$$
$$= -\sum_{\{x\}} \left[p_{data,S} \log p_{\omega,S} + p_{data,B} \log p_{\omega,B} \right] \cdot$$

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Binary cross-entropy loss

$$\mathcal{L}_{\text{BCE}} = -\sum_{\{x\}} \left[p_{\text{data},S} \log p_{\omega,S} + (1 - p_{\text{data},S}) \log(1 + 1) \right]$$





Useful libraries + algorithms

 scikit-learn: For most of the "basic" algorithms like Linear Regression and Boosted Decision Trees. Also useful for preprocessing, model combination, etc.



• XGBoost, LightGBM, CatBoost: For optimized tree-based models



• TensorFlow, PyTorch, Jax: For deep learning models











The Landscape of Machine Learning





Break & Questions

3 Minutes



Classification with Top Tagging

Example

LHC analysis + ML











Run: 282712

A jet is collimated shower of particles in the collider



Landscape Dataset

- Open dataset for the devolopment of better tagging algorithms for particle physics
- 2 million simulated examples
- Perfect class labels:
 S=top jet or B=light quark/gluon jet
- Input: momentum sorted list of 200 particles/jets with 3 features/particle

 (p_x, p_y, p_z)

SciPost Physics

Submission

The Machine Learning Landscape of Top Taggers

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> > July 24, 2019

Abstract

Based on the established task of identifying boosted, hadronically decaying top quarks, we compare a wide range of modern machine learning approaches. Unlike most established methods they rely on low-level input, for instance calorimeter output. While their network architectures are vastly different, their performance is comparatively similar. In general, we find that these new approaches are extremely powerful and great fun.

5

Landscape Dataset

 Open d of bette for parti

- 2 millio
- Perfect
 S=top jet
- Input: n I 200 par s 3 featur $\begin{pmatrix} p_x, p_y, p_z \end{pmatrix}$

Signal 40 40 35 35 : 10⁻¹ 30 30 · 25 · 25 • 20 • 20 -10⁻² 15 15 10 10 5 5 - 10⁻³ 0 + 0 0 -10 10 40 20 30 0

Figure 1: Left: typical single jet image in the rapidity vs azimuthal angle plane for the top signal after pre-processing. Center and right: signal and background images averaged over 10,000 individual images.



[1902.09914]

Based on the established task of identifying boosted, hadronically decaying top quarks, we compare a wide range of modern machine learning approaches. Unlike most established methods they rely on low-level input, for instance calorimeter output. While their network architectures are vastly different, their performance is comparatively similar. In general, we find that these new approaches are extremely powerful and great fun.





Cut so that **30%** of top quarks pass selection:

 R_{30} is the inverse of the number of background jets that also pass

















Jets as images [1701.08784, 1803.00107]









Jets as images [1701.08784, 1803.00107]



Better

Point cloud [PFN 1810.05165]







• Jets as images [1701.08784, 1803.00107]

- Point cloud [PFN 1810.05165]
- Point cloud with Graphs [1902.08570, 2007.13681]







Jets as images [1701.08784, 1803.00107]

- **Point cloud** [PFN 1810.05165]
- **Point cloud with Graphs** [1902.08570, 2007.13681]
- **Point cloud with attention** [2202.03772]







Jets as images [1701.08784, 1803.00107]

Better

- **Point cloud** [PFN 1810.05165]
- **Point cloud with Graphs** [1902.08570, 2007.13681]
- **Point cloud with attention** [2202.03772]
- **Lorentz symmetries** [1707.08966, 2201.08187, 2211.00454]

[2212.00046] + G. Kasieczka







Example II **Regression with MadMiner**

Brehmer, Cranmer, Louppe, Pavez [1805.00013, 1805.00020, 1805.12244]



LHC analysis + ML













Theory parameters

 θ









































Prediction (Simulation)

Recap – **HEP** analysis

































Why has that not stopped us so far?

Recap – **HEP** analysis



Solve it by histogramming summary statistics







Solve it by histogramming summary statistics



$\hat{p}(x \mid \theta) = p(x' \mid \theta) =$





Solve it by histogramming summary statistics



$\hat{p}(x \mid \theta) = p(x' \mid \theta) =$

- How to choose x'? Standard variables often lose information [1612.05261,1712.02350]
- Curse of dimensionality: Histograms don't scale to high-dimensional x











into transfer function

approximate **shower + detector effects**











Detector **Observables** Interactions

Matrix Element Method [K. Kondo 1988]











Solving it with likelihood-free inference



1. Simulation

"Mining gold": Extract additional information from simulator

Brehmer, Cranmer, Louppe, Pavez [1805.00013, 1805.00020, 1805.12244]





"Mining gold": Extract additional information from simulator

Use this information to train estimator for likelihood ratio

Brehmer, Cranmer, Louppe, Pavez [1805.00013, 1805.00020, 1805.12244]





"Mining gold": Extract additional information from simulator

Use this information to train estimator for likelihood ratio

Brehmer, Cranmer, Louppe, Pavez [1805.00013, 1805.00020, 1805.12244]

Set new constrains with standard hypothesis tests





1. Simulation

2. Machine Learning

3. Inference



Mining gold from the simulator



Intractable integrals


Mining gold from the simulator



 \Rightarrow For each generated event, we can calculate the joint likelihood ratio conditional on its evolution:

$$r(x, z \mid \theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p \mid \theta_0)}{p(x, z_d, z_s, z_p \mid \theta_1)} = \frac{p(x \mid z_d)}{p(x \mid z_d)} \frac{p(z_d \mid z_s)}{p(z_d \mid z_s)} \frac{p(z_s \mid z_p)}{p(z_s \mid z_p)} \frac{p(z_p \mid \theta_0)}{p(z_s \mid z_p)}$$



The MadMiner approach



1. Simulation

2. Machine Learning

3. Inference





We can calculate the joint likelihood ratio

$$r(x, z \mid \theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p \mid \theta_0)}{p(x, z_d, z_s, z_p \mid \theta_1)}$$

We want the **likelihood ratio function**

$$r(x \mid \theta_0, \theta_1) \equiv \frac{p(x \mid \theta_0)}{p(x \mid \theta_1)}$$

The value of gold



The value of gold

We can calculate the joint likelihood ratio



We want the likelihood ratio function

$$r(x \mid \theta_0, \theta_1) \equiv \frac{p(x \mid \theta_0)}{p(x \mid \theta_1)}$$



 $r(x, z \mid \theta_0, \theta_1) \text{ are scattered around } r(x \mid \theta_0, \theta_1)$







We can calculate the joint likelihood ratio

$$r(x, z \mid \theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p \mid \theta_0)}{p(x, z_d, z_s, z_p \mid \theta_1)}$$

 $L_r[\hat{r}(x \mid \theta_0, \theta_0)]$

We want the **likelihood ratio function**

$$r(x \mid \theta_0, \theta_1) \equiv \frac{p(x \mid \theta_0)}{p(x \mid \theta_1)}$$

With $r(x, z | \theta_0, \theta_1)$, we define the functional

$$\theta_1)] = \int dx \int dz \, p(x, z \,|\, \theta_1) \, \left[\hat{r}(x \,|\, \theta_0, \theta_1) - r(x, z \,|\, \theta_0, \theta_1) \right]$$

One can show it is minimized by

 $\hat{r}(x \mid \theta_0, \theta_1) = r(x \mid \theta_0, \theta_1)$



Machine learning = applied calculus

We can get a precise estimator by numerically minimizing a functional:

$$\hat{r}(x \mid \theta_0, \theta_1) = \arg \min_{\hat{r}(x \mid \theta_0, \theta_1)} \int dx \int dz$$

 $\left[r(x, z \mid \alpha_1) \left[\hat{r}(x \mid \theta_0, \theta_1) - r(x, z \mid \theta_0, \theta_1)\right]^2\right]$ $L_r[\hat{r}(x \mid \theta_0, \theta_1)]$



Machine learning = applied calculus

We can get a precise estimator by numerically minimizing a functional:

$$\hat{r}(x \mid \theta_0, \theta_1) = \underset{\hat{r}(x \mid \theta_0, \theta_1)}{\operatorname{arg\,min}} \underbrace{\int dx \int dz \, p(x, z \mid \alpha_1) \left[\hat{r}(x \mid \theta_0, \theta_1) - r(x, z \mid \theta_0, \theta_1) \right]^2}_{L_r[\hat{r}(x \mid \theta_0, \theta_1)]}$$

We do this via **Machine Learning**:



Loss function

$$= \frac{1}{N} \sum_{i} \left[\hat{r}_{\omega}(x_i | \theta_0, \theta_1) - r(x_i, z_i | \theta_0, \theta_1) \right]^2$$

Flexible parametric function (e.g. neural network)

Numerical optimization algorithm (eg. Stochastic gradient descent)





The MadMiner approach



1. Simulation

2. Machine Learning

3. Inference



Constraining EFT parameters with ML

Higgs production in weak boson fusion:



at least 16-dimensional observable space

$$\sigma^a D^{\nu} \phi W^a_{\mu\nu} - rac{f_{WW}}{\Lambda^2} rac{g^2}{4} (\phi^{\dagger} \phi) W^a_{\mu\nu} W^{\mu\nu a}$$



Better sensitivity to new physics



Results are based on 36 observed events, assuming SM



Better sensitivity to new physics

Expected exclusion limits at 68%, 95%, 99.7% CL



Results are based on 36 observed events, assuming SM



Better sensitivity to new physics

Expected exclusion limits at 68%, 95%, 99.7% CL



Results are based on 36 observed events, assuming SM







Appendix

Bonus material

$$L[\hat{g}(x)] = \int dx \int dz \, p(x, z \mid \theta) \left[g(x, z) - \hat{g}(x) \right]$$

=
$$\int dx \left[\hat{g}^2(x) \int dz \, p(x, z \mid \theta) - 2\hat{g}(x) \int dz \, p(x, z \mid \theta) \, g(x, z) + \int dz \, p(x, z \mid \theta) \, g^2(x, z) \right]$$

$$0 = \frac{\delta F}{\delta \hat{g}} \bigg|_{g^*} = 2\hat{g} \underbrace{\int dz \, p(x, z \,|\, \theta) - 2 \int dz}_{=p(x|\theta)}$$

$$g^*(x) = \frac{1}{p(x \mid \theta)} \int dz \, p(x, z, \mid \theta) \, g(x, z)$$

=F(x)

 $z p(x, z \mid \theta) g(x, z)$

Choose

$$g(x, z) = \frac{p(x, z, | \theta_0)}{p(x, z, | \theta_1)}$$

