Spontaneous breaking of global symmetries

Main scope: learn the concept and selected examples in high-energy particle physics

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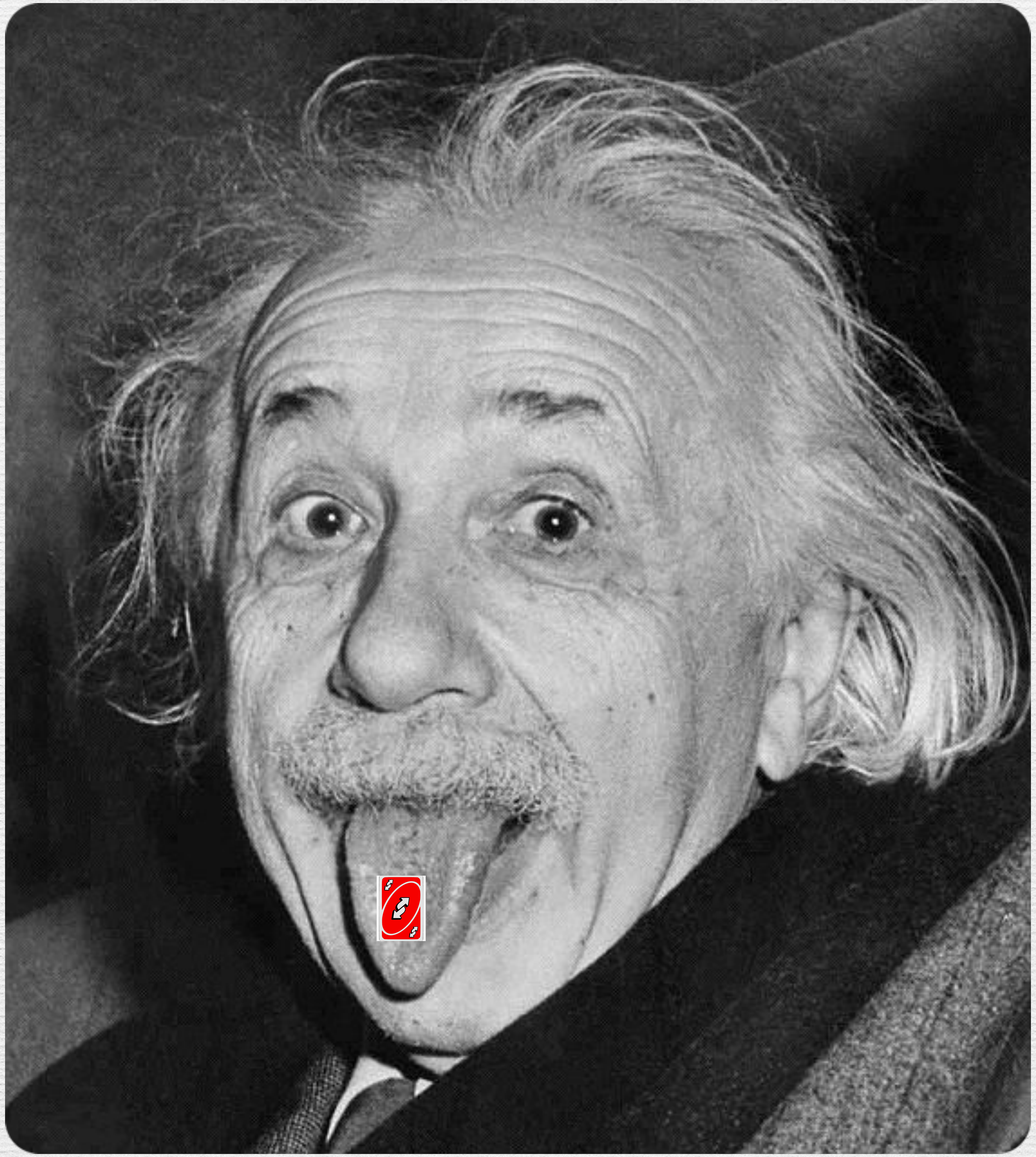
Symmetric timeline

- Ancient Greeks - geometrical symmetry: proportions based on integer numbers, with «harmonizing» all to unity.
- XVII century - based on the equality of opposed elements.
- Mathematical operations appear: reflection, rotations, translations -> invariance.

Symmetric timeline: XIX century

- Hamilton dynamic equations
- Jacobi: transformational properties of theories (e.g. canonical Poisson)
- Algebraic concept of group is created, symmetry satisfying group conditions

Symmetric timeline: Einstein happens



Symmetric timeline: Einstein happens

E. P. Wigner (1967) writes:

«..the reversal of a trend: until then, the principles of invariance were derived from the laws of motion … It is now natural for us to derive the laws of nature and to test their validity by means of the laws of invariance, rather than to derive the laws of invariance from what we believe to be the laws of nature…».

Thus the universality of global continuous spacetime symmetries is postulated, setting the trend for the rest of the physics.

Timeline: finally, particles

- Finally, particles: continuous internal symmetries: approximately same mass from the one system, connected by underlying symmetry group.

- Started from the Heisenberg's permutation symmetry:
  -1932 paper on SU(2) p-n symmetry
  -1937 isotropic spin (isospin) symmetry by Wigner.

Let's enGAUGE in discussion

The struggle is real:

- How to properly define gauge symmetry?
- Until what point does local symmetry allow us to deal with new physics?
- At what point do we go global?
- What extent are the gauge symmetry parameters «physically real»?

Breaking stuff is natural

- First physical symmetry breaking study - Pierre Curie:
  In order for a phenomenon to occur in a medium, the original symmetry of this specific medium has to be «lowered» to the symmetry level of this phenomenon. The «lowering» of the symmetry is what allows for phenomenon to happen (sounds familiar, Mr. Higgs?).

Explicit vs Spontaneous breaking

- Explicit breaking = BROKEN symmetry
  - ➡ symmetry is not realised at all
- Spontaneous breaking = HIDDEN symmetry
  - ➡ symmetry is **present** in Hamiltonian or Lagrangian, but is **not respected** by ground state

Symmetry can be

- Realised, i.e. presence of symmetry is obvious.
- Broken
  - Explicitly broken
  - Spontaneously broken

Breaking stuff is explicit fun

(a) Introduce the breaking by hand on the basis of theoretical/experimental results (e.g. QFT of weak interactions, constructed to manifestly violate mirror symmetry or parity -> parity violation of weak interactions (T. D. Lee, C.N. Yang, 1956).

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(c) Symmetry-breaking terms may appear because of non-renormalizable effects: these effects are small and can therefore be ignored at the low-energy regime (due to the heavy particles not included in the theory). It may then happen that the coarse-grained description thus obtained possesses more symmetries than the deeper theory.

What am I doing?

- ➡ Why QCD?
- ➡ QCD effective models
- ➡ What's next?
- ➡ Take away message

Why QCD?

- ✓describe all particles as quark-gluon DOFs
- ✓high-energy regime is perturbative (asymptotic freedom of QCD)
- ✓low-energy regime is less nice: because non-perturbative behaviour (confinement of quarks and gluons, breaking of chiral symmetry); analytical up to some point
- ✓bunch of particles, can «experiment» with exotics
- ✓very fundamental in nature (or at least we believe it to be)
- ✓bunch of fun and fundamental symmetries (breaking and restoring them)

QCD symmetries

- Isospin (up and down quarks are the two states of a singlet)
- U(1) symmetry (idk, ask ‘tHooft)
- Chiral symmetry
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  «..QCD enjoys Poincaré symmetry...» © Wikipedia

Why chiral symmetry(χ)?

- Chiral symmetry ~ symmetry of strong interactions at low energy (lightest quarks)
- QCD has hidden chiral symmetry
- Is well researched, which allows to explore in great detail and understand the general mechanism of symmetries better

Consequences of symmetry breaking

- quark mass generation (current quark mass vs composite quark mass): proton is uud
- $m_p \approx 900\ MeV$, but
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- quark mass generation (current quark mass vs composite quark mass): proton is uud $m_p \approx 900\ MeV$, but $m_u \approx 4\ MeV$ $m_d \approx 8\ MeV$
- energy spectrum (will give you a hint towards SSB)
- pions are Goldstone bosons

How to go about exploring?

- Make use of Noether's theorem to detect if there is a symmetry in the system and if it is broken:

$$\partial_\mu j^\mu = 0 \text{ if symmetry is conserved}$$
$$\partial_\mu j^\mu \neq 0 \text{ if symmetry is broken}$$

- Consider different existent models which have the symmetry:

  - ➡ σ-model (linear, non-linear) - χ-symmetry in terms of mesons

  - ➡ NJL (Nambu - Jona-Lasinio) model for QCD - χ-symmetry in terms of quarks

Why consider NJL? [1]

- Has all the important symmetries of QCD (which are also observed in nature), including chiral
- QCD symmetries' breaking is well observed in NJL (fermion mass generation, special role of Goldstone modes)
- Is formulated in terms of quarks, allows to consider two-flavour and three-flavour approaches
- Easy to understand, recreate and restore chiral symmetry
- Shortcomings: fermion interaction is assumed to be point-like (non-renormalizable FT), only effective model, no confinement

NJL. Symmetries

- original pre-QCD symmetry of NJL model:

$$SU_V(2) \otimes SU_A(2) \otimes U_V(1) \otimes U_A(1)$$

$SU_V(2)$ - isospin symmetry, approximately conserved
$U_V(1)$ - baryonic, always conserved
$SU_A(2)$ - chiral, CSB and Goldstone mode
$U_A(1)$ - axial, $U_A(1)$ «puzzle» ('t Hooft 1976)

- in further considerations of quark NJL use:

$$SU_V(2) \otimes SU_A(2) \otimes U_V(1)$$

NJL. Chiral rotations

$$\psi \to e^{-\frac{1}{2}i\vec{\sigma}\cdot\vec{\alpha}\gamma_5}\,\psi \qquad\qquad \bar{\psi} \to \bar{\psi}\,e^{-\frac{1}{2}i\vec{\sigma}\cdot\vec{\alpha}\gamma_5}$$

where $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ each is a four spin

Important: mixing two different structures

$\vec{\sigma}$ - Pauli matrices (flavour DOF)

γ_5 - Dirac matrix (Dirac DOF)

These are transformations of $SU_A(2)$ chiral symmetry (global)

NJL. Lagrangian invariance

$$\mathscr{L}_{NJL} = \bar{\psi} i(\gamma^\mu \partial_\mu - m)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\sigma} \psi)^2]$$

Under chiral rotation:

- free term invariant iff $m_u = m_d = 0$ since after transformation:

$$\bar{\psi} i\gamma^\mu \partial_\mu \psi - m\bar{\psi} e^{-\vec{\sigma}\cdot\vec{\alpha}\gamma_5}$$

- each interacting term is not invariant individually, but the combination is:

$$\bar{\psi}\psi \longrightarrow \bar{\psi}\psi - i\bar{\psi}\gamma_5 \vec{\sigma} \cdot \vec{\alpha} \ \psi$$

$$\bar{\psi}(i\gamma_5 \vec{\sigma})\psi \longrightarrow \bar{\psi}(i\gamma_5 \vec{\sigma})\psi + \bar{\psi}\psi\vec{\alpha}$$

NJL. Noether current

$$\partial_m \vec{j}^{\mu} = im\bar{\psi}\gamma_5 \overrightarrow{\alpha} \psi$$

- non-zero divergence $\longrightarrow$ symmetry is broken
- chiral current

$$J_5^k \mu = \bar{\psi} m \gamma_\mu \gamma_5 \sigma^k \psi$$

Gell-Mann - Oakes - Renner eq.

$$f_\pi^2 m_\pi^2 = -\frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle$$

- appears for any theory with chiral symmetry

Further goals:

- NJL considerations to three flavours
- Be able to calculate the observables:
  - NJL quark propagator $iS_q = \langle 0 | T\{q, \bar{q}\} | 0 \rangle$
  - finite thermal parameters
  - excitations in the system

σ-model. Briefly

- Why? $\longrightarrow$ Chiral symmetry in meson «picture»
- Define: pion-like state $\vec{\pi} \equiv i\bar{\psi}\vec{\tau}\gamma_5\psi$ and sigma-like state $\sigma \equiv \bar{\psi}\psi$
- Apply chiral rotations to obtain:

$$\vec{\pi} \longrightarrow \vec{\pi} + \vec{\alpha}\sigma \qquad \text{and} \qquad \sigma \longrightarrow \sigma + \vec{\alpha} \cdot \vec{\pi}$$

σ-nonlinear model. Briefly

- Why? $\longrightarrow$ Chiral symmetry in meson «picture»
- But….

σ-nonlinear model. Briefly

- Why? ⟶ Chiral symmetry in meson «picture»
- But….

Theory is broken from the start, we want to «unbreak» it.

Literature

1. S.P. Klevansky, *The Nambu - Jona-Lasinio model of quantum chromodynamics,* Reviews of Modern Physics, Vol. 64, No. 3, July 1992

2. V. Koch, *Aspects of Chiral Symmetry*, Berkley 2008

3. T. Hatsuda, T. Kunihiro, *QCD phenomenology based on a chiral effective Lagrangian,* Phys. Rep. 247 (1994) 221-367

Bonus: ASS SYMMETRY

A paradox whereby a hungry and thirsty donkey, placed between a bundle of hay and a pail of water, would die of hunger and thirst because there was no reason for him to choose one resource over the other. By Jean Buridan