# Bounce of spinning sports balls.

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# References


Rod Cross (2005)
Bounce of a spinning ball near normal incidence
*American Association of Physics Teachers* , doi: 10.1119/1.2008299.

Jacob Emil Mencke, Mirko Salewski, Ole L. Trinhammer, Andreas T. Adler (2020)
Flight and bounce of spinning sports balls
*American Association of Physics Teachers*, doi: 10.1119/10.0001659

# Angular and linear Newton's second law

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\vec{F}\,dt = m\,d\vec{v}$$

$$\int \sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1 \tag{1}$$

$$\vec{M} = \vec{R} \times \vec{F}$$

$$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt}$$

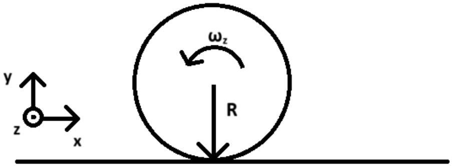
$$\vec{M}\,dt = I\,d\vec{\omega}$$

$$\int \sum (\vec{R} \times \vec{F})\,dt = I\vec{\omega}_1 - I\vec{\omega}_2 \tag{2}$$

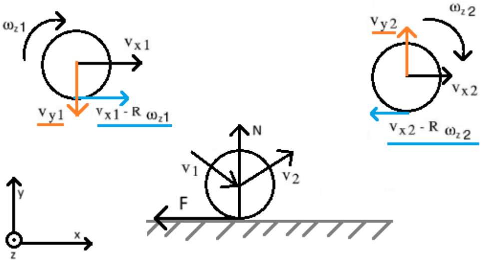
$$I = \beta m R^2$$

For hollow ball $\beta = \frac{2}{3}$, For solid balls, $\beta = \frac{2}{5}$

$$\vec{R} \times (m\vec{v}_2 - m\vec{v}_1) = I\vec{\omega}_2 - I\vec{\omega}_1 \tag{3}$$



# Coefficients of restitution




$$e_y = -\frac{v_{y,2}}{v_{y,1}}$$

$$e_x = -\frac{v_{x,2} + R\omega_{z,2}}{v_{x,1} + R\omega_{z,1}} \qquad e_z = -\frac{v_{z,2} - R\omega_{x,2}}{v_{z,1} - R\omega_{x,1}}$$

# After bounce values

$$v_{x,2} = \frac{1 - \beta e_x}{\beta + 1} v_{x,1} + \frac{\beta(1 + e_x)}{\beta + 1} R\omega_{z,1}$$

$$v_{y,2} = -e_y v_{y,1}$$

$$v_{z,2} = \frac{1 - \beta e_z}{\beta + 1} v_{z,1} + \frac{\beta(1 + e_z)}{\beta + 1} R\omega_{x,1}$$

# After bounce values

$$\omega_{x,2} = \frac{\beta - e_z}{\beta + 1} v_{x,1} + \frac{e_z + 1}{\beta + 1} \frac{v_{z,1}}{R}$$

$$\omega_{y,2} = \omega_{y,1}$$

$$\omega_{z,2} = \frac{\beta - e_x}{\beta + 1} \omega_{z,1} - \frac{e_x + 1}{\beta + 1} \frac{v_{x,1}}{R}$$

$$\begin{pmatrix} v_{x,2} \\ v_{y,2} \\ v_{z,2} \\ \omega_{x,2} \\ \omega_{y,2} \\ \omega_{z,2} \end{pmatrix} = \mathbf{B} \begin{pmatrix} v_{x,1} \\ v_{y,1} \\ v_{z,1} \\ \omega_{x,1} \\ \omega_{y,1} \\ \omega_{z,1} \end{pmatrix}$$

# Bounce matrix

$$\mathbf{B} = \begin{pmatrix} \frac{1-\beta e_x}{\beta+1} & 0 & 0 & 0 & 0 & -\frac{\beta(1+e_x)}{\beta+1}R \\ 0 & -e_y & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\beta e_z}{\beta+1} & \frac{\beta(1+e_z)}{\beta+1}R & 0 & 0 \\ 0 & 0 & \frac{e_z+1}{\beta+1}\frac{1}{R} & \frac{\beta-e_z}{\beta+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{e_x+1}{\beta+1}\frac{1}{R} & 0 & 0 & 0 & 0 & \frac{\beta-e_x}{a+1} \end{pmatrix}$$

$$\mathbf{B} = \mathbf{Q}\mathbf{A}\mathbf{Q}^{-1} \qquad \mathbf{B}^N = \mathbf{Q}\mathbf{A}^N\mathbf{Q}^{-1}$$

$$\mathbf{A} = \begin{pmatrix} -e_x & 0 & 0 & 0 & 0 & 0 \\ 0 & -e_y & 0 & 0 & 0 & 0 \\ 0 & 0 & -e_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} \beta R & 0 & 0 & 0 & 0 & -R \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta R & R & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{B}^N = \mathbf{Q} \begin{pmatrix} (-e_x)^N & 0 & 0 & 0 & 0 & 0 \\ 0 & (-e_y)^N & 0 & 0 & 0 & 0 \\ 0 & 0 & (-e_z)^N & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \mathbf{Q}^{-1}$$

$$\lim_{N\to\infty} B^N = \begin{pmatrix} \frac{1}{1+\beta} & 0 & 0 & 0 & 0 & -\frac{\beta R}{1+\beta} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1+\beta} & \frac{\beta R}{1+\beta} & 0 & 0 \\ 0 & 0 & \frac{1}{1+\beta}\frac{1}{R} & \frac{a}{1+\beta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{1}{1+\beta}\frac{1}{R} & 0 & 0 & 0 & 0 & \frac{a}{1+\beta} \end{pmatrix}$$

# After many bounce

$$v_{x,N\to\infty} = \frac{1}{1+\beta} v_{x,1} - \frac{\beta R}{1+\beta} \omega_{z,1}$$

$$v_{z,N\to\infty} = \frac{1}{1+\beta} v_{z,1} + \frac{\beta R}{1+\beta} \omega_{x,1}$$

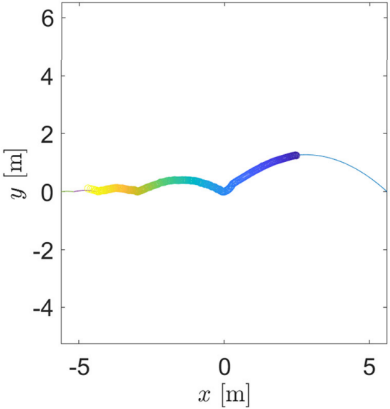
$$\omega_{x,N\to\infty} = \frac{1}{1+\beta} \frac{1}{R} v_{z,1} + \frac{\beta}{1+\beta} \omega_{x,1}$$

$$\omega_{z,N\to\infty} = -\frac{1}{1+\beta} \frac{1}{R} v_{x,1} + \frac{\beta}{1+\beta} \omega_{z,1}$$

$$v_{x,N\to\infty} = -\omega_{z,N\to\infty} R$$
$$v_{z,N\to\infty} = \omega_{x,N\to\infty} R$$

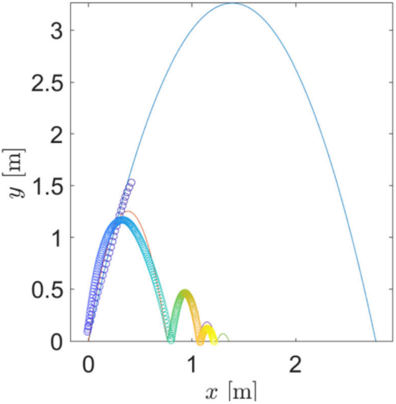
# Soccer ball on wet grass, no spin

| $v_{x0}$ (m/s) | $v_{y0}$ (m/s) | $\omega_{z0}$ (1/s) | $e_{x,z}$ | $e_y$ |
|---|---|---|---|---|
| -5.5 | -5 | 0 | -0.75 | 0.57 |



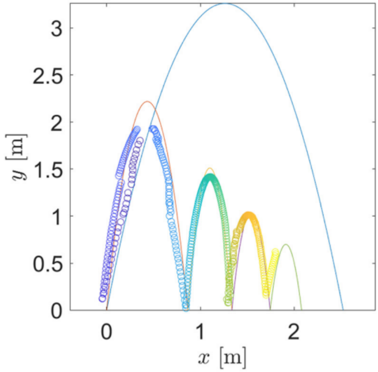
# Soccer ball on dry grass, backspin

| $v_{x0}$ (m/s) | $v_{y0}$ (m/s) | $\omega_{z0}$ (1/s) | $e_{x,z}$ | $e_y$ |
|---|---|---|---|---|
| -1.7 | -8 | -32.8 | 0.12 | 0.62 |



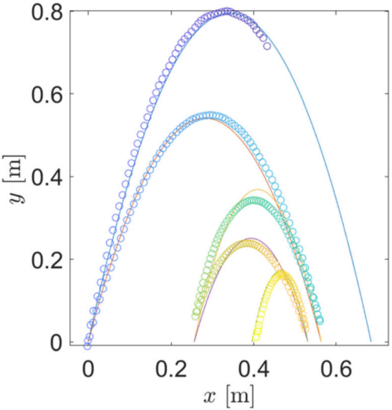
# Basketball on a basketball court, backspin

| $v_{x0}$ (m/s) | $v_{y0}$ (m/s) | $\omega_{z0}$ (1/s) | $e_{x,z}$ | $e_y$ |
|---|---|---|---|---|
| -1.55 | -8 | -30 | 0.1 | 0.825 |



# Superball (rubber ball) on wood, backspin

| $v_{x0}$ (m/s) | $v_{y0}$ (m/s) | $\omega_{z0}$ (1/s) | $e_{x,z}$ | $e_y$ |
|---|---|---|---|---|
| -0.85 | -3.95 | -49 | 0.83 | 0.825 |



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