Noncommutative geometries-based grand unification theories in the low-energy regime

Inversigating a transition between the extended Standard Model and effective theories of nuclear structure

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1. Noncommutative geometries

2. Nuclear astrophysics

Spacetime and noncommutativity

- S. Doplicher, K. Fredenhagen, J.E. Roberts The quantum structure at the Planck scale and quantum fields Comm.Math.Phys. 172 (1995) 187-220
- QM: measuring precise location needs higher energy
- GR: localising big energy in a small space leads to a collapse into a black hole
- Using these arguments one can derive approximate *uncertainty relations* between the coordinates:

$$[x^\mu, x^\nu] = q^{\mu\nu}$$

- Description of the Standard Model as originating from finite spectral triple

Gelfand Naimark Theorem

Theorem

Every commutative C^*-algebra A is *-isomorphic to the algebra $C_0(K)$ of continuous functions vanishing at infinity on a locally compact Hausdorff space

Que?

C^*-algebra

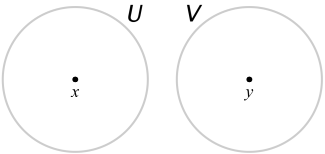
a complex algebra A of continuous linear operators on a complex Hilbert space with two additional properties:

- A is a topologically closed set in the norm topology of operators
- A is closed under the operation of taking adjoints of operators

Que?

Haussdorf space

A topological space where, for any two distinct points, there exist neighbourhoods of each that are disjoint from each other



Spectral triples (H, A, D)

- H - a complex Hilbert space
- A - a (real or comlex) unital $*$-algebra of bounded operators on H
- D - Dirac operator: a self adjoint operator on H with compact resolvent, such that:

$$\forall_{a \in A} \quad a \cdot Dom(D) \subseteq Dom(D),$$

and $[D, a]$ extends to a bounded operator on H

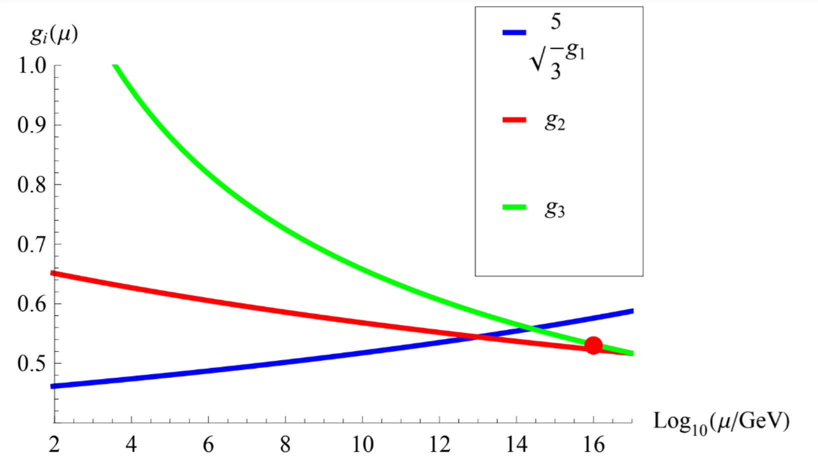
The Standard Model spectral triple

- Let us arrange particles in a 4×4 matrix in the following way:

$$\Psi = \begin{bmatrix} \nu_R & u_R^1 & u_R^2 & u_R^3 \\ {}_R & d_R^1 & d_R^2 & d_R^3 \\ \nu_L & u_L^1 & u_L^2 & u_L^3 \\ {}_L & d_L^1 & d_L^2 & d_L^3 \end{bmatrix}$$

  (the Hilbert (sub)space representing particles is $F = M_4(\mathbb{C})$)
- our H is $H \simeq F \bigoplus F^*$: $H = \left\{ \begin{bmatrix} v \\ w \end{bmatrix} \middle| v, w \in M_4(\mathbb{C}) \right\}$
- $A \simeq \mathbb{C} \bigoplus \mathbb{H} \bigoplus M_3(\mathbb{C})$
- I am to lazy to show you how D looks like

Ok, but does it require another collider?



What should we do?

Cosmological principle

The universe is isotropic, homogenous and the laws of physics are universal*

*apart from the Strazacka street in Karpacz

Matter inside a star

- metric:

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\Omega^2$$

- perfect fluid:

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}$$

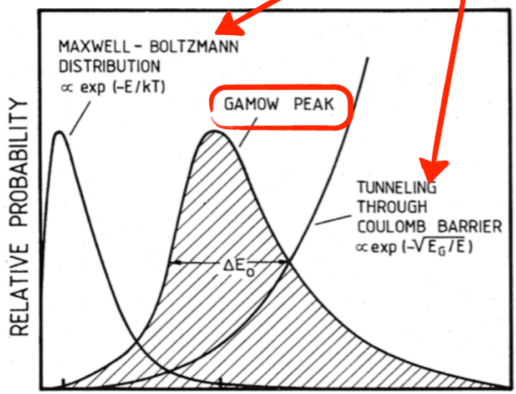
- $u_r = u_\theta = u_\phi = 0$ and $g_{\mu\nu}u^\mu u^\nu = -1$
- $T^{00} = \rho e^{-2\Phi}, T^{rr} = pe^{-2\Lambda} T^{\theta\theta} = pr^{-2}, T^{\phi\phi} = pr^{-2} sin^{-2}\theta$
- $G^{\mu\nu} = 8\pi T^{\mu\nu},\ T^{\mu\nu}_{;\nu} = 0$

Cross section

- $\sigma = \pi \lambda^2 \implies \sigma = \sigma(E) = \sigma(v)$
- for a reaction $X(x, y)Y$ and N_i - no. particles/ cm^3 of type i we obtain a rate of nuclear reactions: $r = N_x v N_X \sigma(v)$
- velocity distribution $\int_0^\infty \Phi(v) dv = 1$
- $\langle \sigma v \rangle = \int_0^\infty \Phi(v) v \sigma(v) dv$
- $\Phi(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T} \right)^{3/2} exp\left(-\frac{mv^2}{2k_B T} \right) \propto E \cdot exp\left(-\frac{E}{k_B T} \right)$

Cross section

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{1/2}\left(\frac{1}{K_B T}\right)^{3/2}\int_0^\infty S(E)\exp\left(-\frac{E}{K_B T}-\sqrt{\frac{E_G}{E}}\right)dE$$



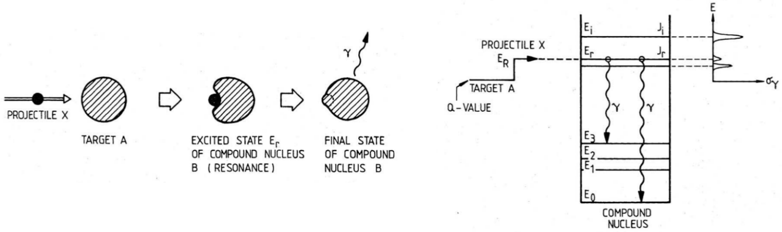
Gamow energy

$$E_G = 2\mu\frac{\pi^2 e^4}{\hbar^2}(Z_1 Z_2)^2\ MeV$$

Non-resonant reaction

- $\sigma(E) \propto \frac{1}{v} \implies \langle \sigma v \rangle = const$
- $\sigma v = S(E=0) + S'(E=0)v + S''(E=0)v^2 + \ldots$

Resonant-reaction



- $\sigma \propto | < E_f | H_\gamma | E_r > |^2 | < E_r | H_f | A + x > |^2$

Thanks!
rosmarinus debet crescere, et fistula debet strepere