Green's Function method for Tetraquarks

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13.05.2024

QCD basics

$$\mathcal{L} = -\frac{1}{4} G^a_{bc} G^{bc}_a + \sum_{ijf} \bar{q}_{if} (i \not{D} - m_f)_{ij} q_{jf}$$

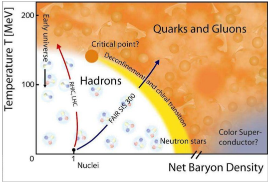
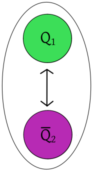
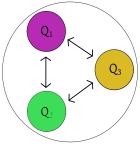


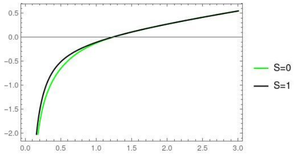
Fig.: QCD phase diagram: Aarts, Gert. (2015).

Meson:



Baryon:





One-gluon exchange:

$$V^G_{ij}(r_{ij}) = \kappa_s \frac{\alpha_s}{r_{12}}$$

Confinement:

$$V_{conf}(r_{ij}) = b r_{ij}$$

Exotic Hadrons

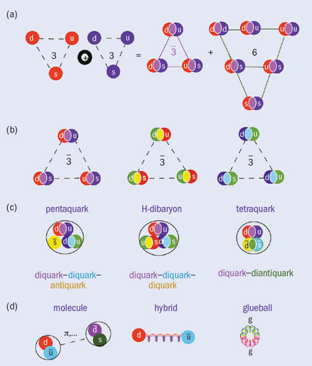


Fig.: Diquarks and exotic hadrons. Source: Front. Phys. 10 101401

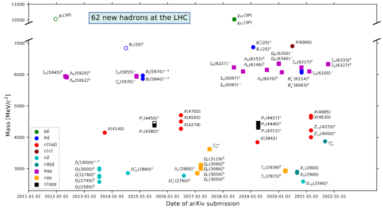


Fig.: New hadrons by the date of their discovery. Source:LHCb-PUB-2022-013

Green's Function

Definition

$AG(x, x') = \delta(x - x')$

In Heisenberg's picture we are dealing with time evolution of the operators:

$$A(\tau) = U^\dagger(\tau) A U(\tau),$$

Which leads to a special case that may be Fourier transformed.

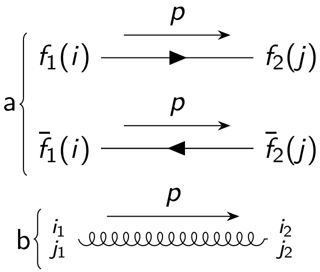
$$G(i_1, \tau_1; i_2, \tau_2) = -Tr[\rho T[a_i(\tau_1) a_i^\dagger(\tau_2)]]$$

$G(i_1, \tau_1; i_2, \tau_2) = G(i_1, i_2, \tau) = \frac{1}{\beta} \sum_{z_\nu} G(i_1 i_2, iz_\nu) e^{-iz_\nu \tau}$, where $G(i_1, i_2, \tau) = \frac{\delta^{i_1 i_2}}{iz_\nu - \epsilon_i}$.

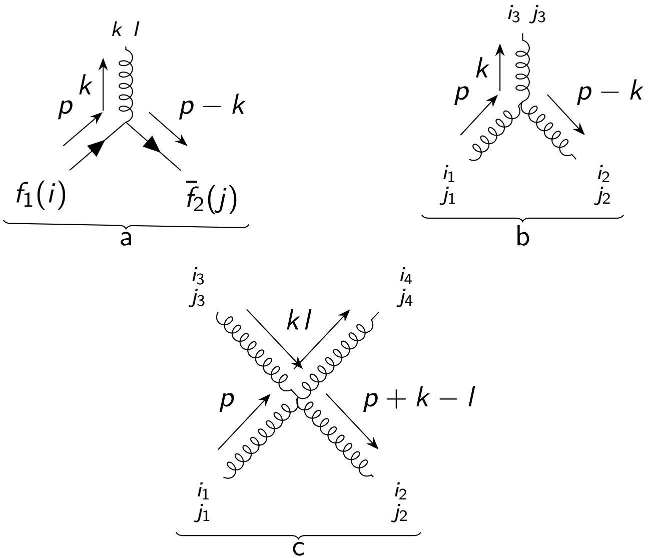
Green's Functions - propagators

We introduce the propagators:

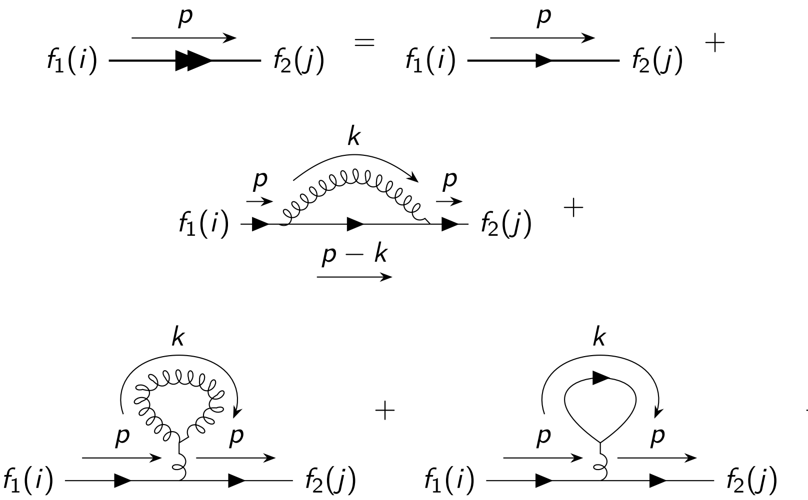
- a) for quarks and antiquarks
- b) gluons



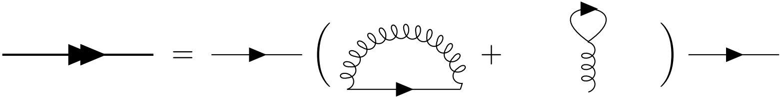
Lagrangian QCD - vertices



Dressed Propagator - quark



Dressed propagaror - simplified notaion



$$G_q = G_q^0 \times (\Gamma \Sigma_F \Gamma + \Sigma_H) \times G_q^0$$

- Σ_F - Fock diagram
- Σ_H - Hartree diagram

HF- approximation

$$\Sigma_F = \int d^3\vec{q}(-\Omega(\tfrac{1}{2\pi})^3)f(\epsilon_q)V(\vec{k} - \vec{q}, z_\nu - \omega_\lambda)$$

$$V_{qq}^{OGE}(r) = \frac{-a}{r}$$

$$G_1^{HF}(\vec{k}, z_\nu) = \frac{1}{iz_\nu - \epsilon_k - \Sigma_1^{HF}(\vec{k})}$$

$$\partial\Phi = Tr\Sigma\partial G$$

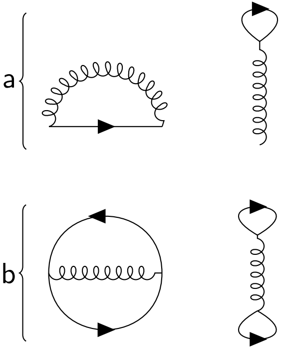
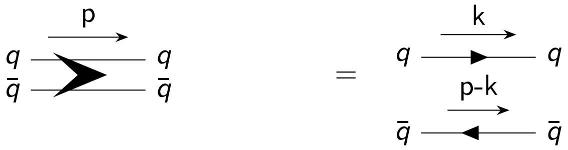
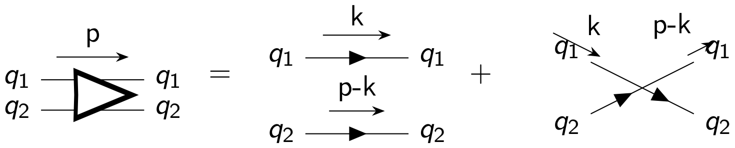


Fig.: a) Contribution to the quark self energy and b) their corresponding Φ functionals.

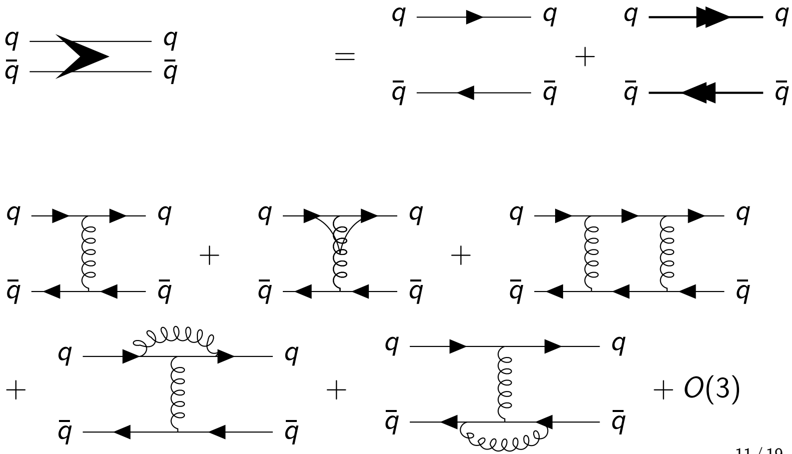
Non interacting two quark propagator.

We need to include additional quark exchange diagram if we have two identical particles - to account for the Pauli exclusion principle.





Meson



The discovery of X(6900)

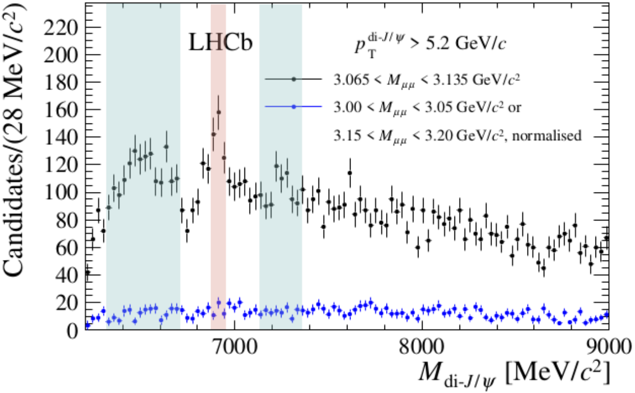
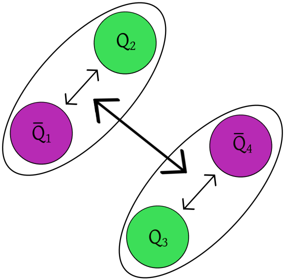


Fig.: Resonance X(6900) with itsparteners.
Źródło:LHCb-PAPER-2020-011

Tetraquark structure



- Meson-meson:
  $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$
- Diquark-antidiquark:
  - $6 \otimes \bar{6} = 1 \oplus 8 \oplus 27$
  - $3 \otimes \bar{3} = 1 \oplus 8$

  Reference:
  - R. Jaffe: Phys. Rev. D 15, 267 (1977)
  - M. Kuchta: https://arxiv.org/abs/2309.04794

Quark currents

$$[QQ]_{\bar{\mathbf{3}}}[\bar{Q}\bar{Q}]_{\mathbf{3}} = T_a T^b = \tfrac{1}{9}\delta_a^b \epsilon_{akl} Q^k Q^l \epsilon^{bxy} \bar{Q}_x \bar{Q}_y + (\epsilon_{akl} Q^k Q^l \epsilon^{bxy} \bar{Q}_x \bar{Q}_y - \tfrac{1}{9}\delta_a^b \epsilon_{akl} Q^k Q^l \epsilon^{bxy} \bar{Q}_x \bar{Q}_y)$$

$$[QQ]_{\mathbf{6}}[\bar{Q}\bar{Q}]_{\bar{\mathbf{6}}} = S^{ij} S_{kl} = \frac{1}{9}\delta_n^m \delta_p^o S^{np} S_{mo} + (S^{ij} S_{kl} - \frac{1}{9}\delta_n^m \delta_p^o S^{np} S_{mo})$$

$$[Q\bar{Q}]_{\mathbf{8}}[Q\bar{Q}]_{\mathbf{8}} = O_i^{ab} O_j^{cd} = \tfrac{1}{3}\delta_{ij} O_n^{ab} O_n^{cd} + (O_i^{ab} O_j^{cd} - \tfrac{1}{3}\delta_{ij} O_n^{ab} O_n^{cd})$$

The currents can be transposed into each other.

Synset diagrams

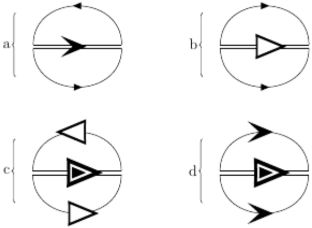


Fig.: Sunset diagrams for a) meson b) diquark c-d) tetraquark

Sunset diagrams can help us calculate self energy of multi-particle states.

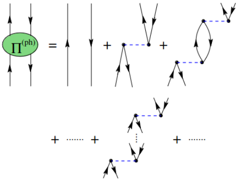


Fig.: Annihilation diagram.
Source: J. Aichelin 2024

Similar diagrams can be done for polarisation loops.

Mott effect

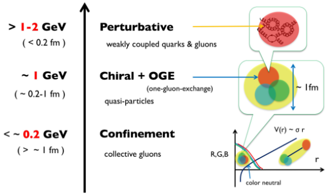


Fig.: Energy Scale in QCD. Source: GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt/Germany

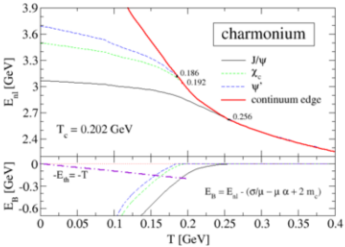


Fig.: Mott temperature for charmonia. Źródło: Source: http://dx.doi.org/10.1016/j.nuclphysbps.2011.03.073

Flip-flop effect

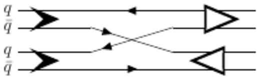


Fig.: Transition between the two possible tetraquark configuration.

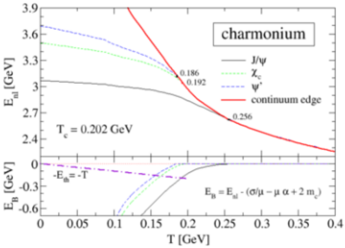


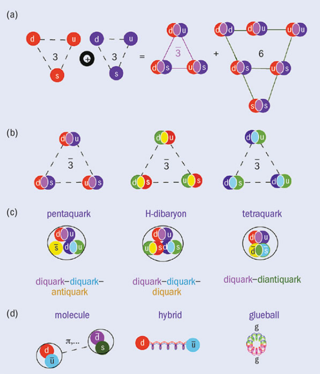
Fig.: Mott temperature for charmonia. Źródło: Source: http://dx.doi.org/10.1016/j.nuclphysbps.2011.03.073

Mr Pauli is blocking

Simple diquark propagator: $= \frac{\delta_{i_1 i_2}\delta_{\mu_1\mu_2}}{z_\mu - E_{i\mu}}$
We do not know how the Pauli Blocking works within the tetraquark and simple Green's function approach needs to argumented.

We need to consider the symmetry of the wave function:

- $|X^c_{6\bar{6}}\rangle = |X^c_6\rangle\langle X^c_{\bar{6}}|$



Thank you very much!