An implementation of

The Three Site Model

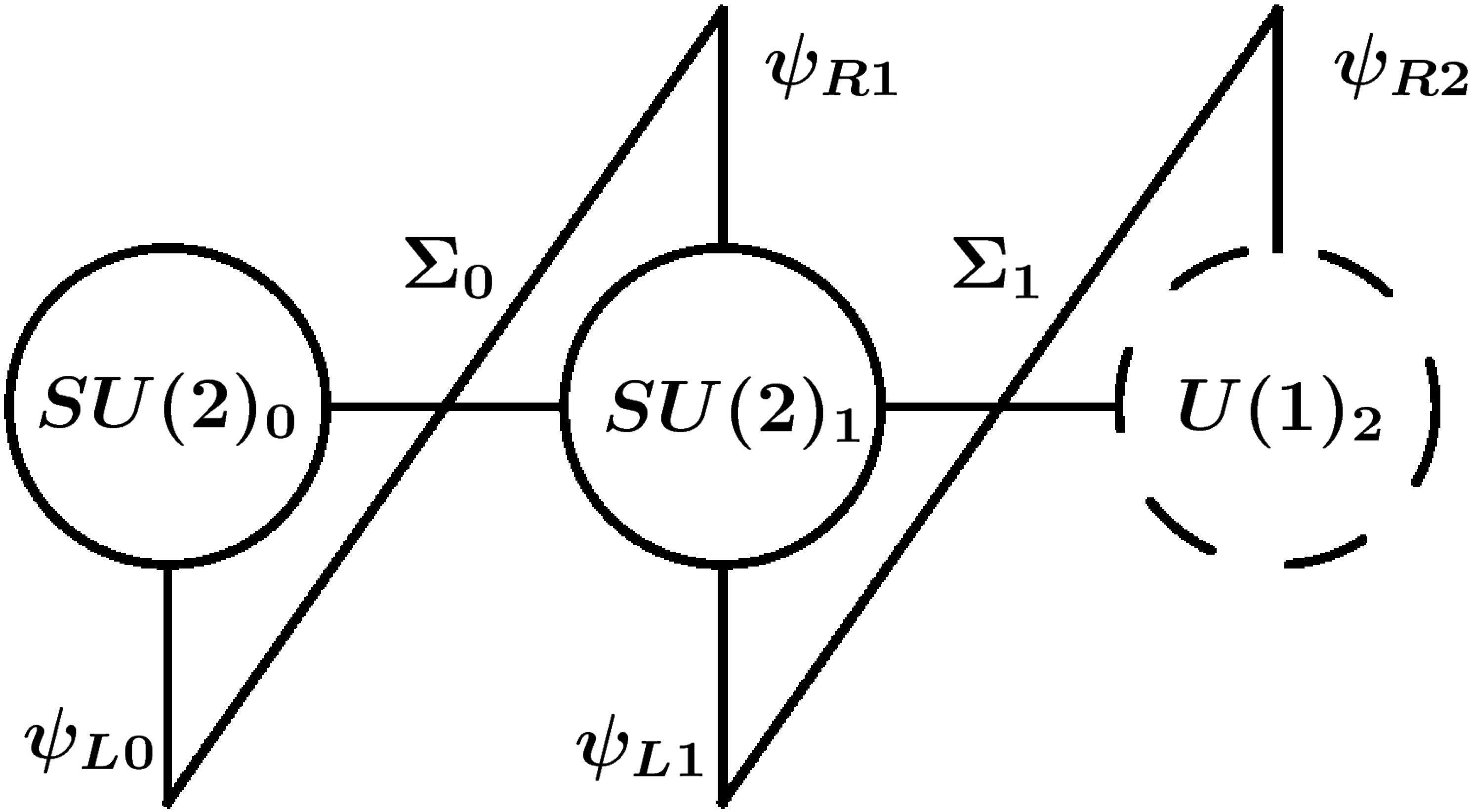
in CalcHEP

Alexander Belyaev and Neil Christensen

In collaboration with
S. Chivukula and E. Simmons

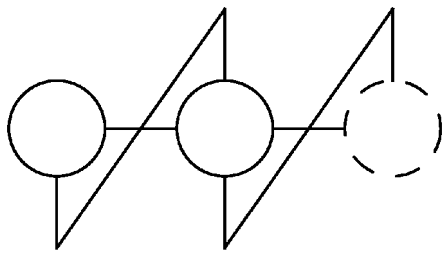
Michigan State University

The Three Site Model



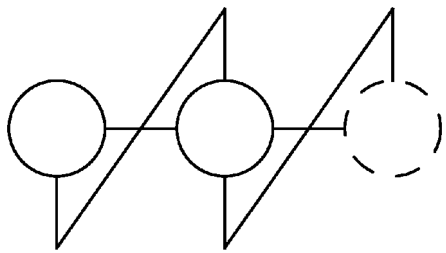
Chivukula, Coleppa, Di Chiara, Simmons
PRD **74**, 075011 (2006)

Representative of a Higgsless Extra Dimension



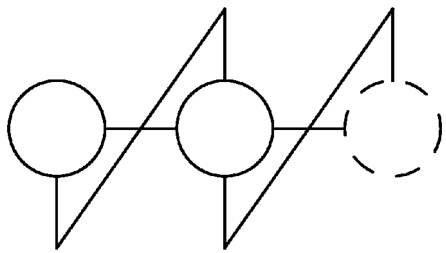
- Low energy phenomenology of a Higgsless ED is dominated by the 1st KK mode.
- The Three Site Model consistently implements the 1st KK mode in a gauge invariant way.

Representative of Dynamical EWSB



- Warped Higgsless ED is conjectured to be dual to a walking technicolor theory.
- The Three Site Model consistently implements the vector resonances (ρ_{TC}) in a gauge invariant way.
- Satisfies precision electroweak measurements (S=0).

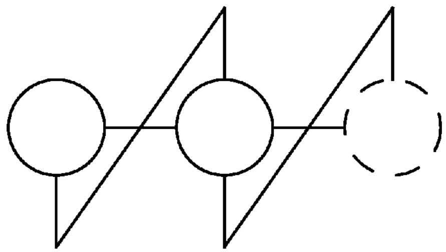
The Three Site Model is testable



The parameter space is:

- Simple.
- Bounded
  - from below by experiment.
  - from above by unitarity.
- Potentially covered at the LHC
  - work in progress.

Gauge Sector



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left[F_0^2 + F_1^2 + F_2^2\right]$$

where

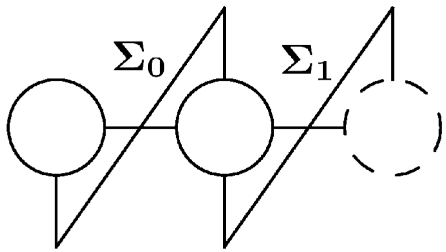
$$F_j^{\mu\nu} = \partial^\mu W_j^\mu - \partial^\nu W_j^\mu + i g_j \left[W_j^\mu, W_j^\nu\right]$$

$$W_j = \begin{pmatrix} \frac{1}{2}W_j^0 & \frac{1}{\sqrt{2}}W_j^+ \\ \frac{1}{\sqrt{2}}W_j^- & -\frac{1}{2}W_j^0 \end{pmatrix}$$

where *j=0,1*

$$W_2 = \begin{pmatrix} \frac{1}{2}W_2^0 & 0 \\ 0 & -\frac{1}{2}W_2^0 \end{pmatrix}$$

Casalbuoni, De Curtis, Dominici, Gatto
(BESS) Phys. Lett. B155 (1985) 95



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

Gauge - Goldstone Sector

$$\mathcal{L}_{D\Sigma} = \frac{f^2}{2}\mathrm{Tr}\Big[\,(D_\mu\Sigma_0)^\dagger\, D^\mu\Sigma_0 + (D_\mu\Sigma_1)^\dagger\, D^\mu\Sigma_1\Big]$$

$$\Sigma_j = e^{i\frac{2\pi_j}{f}}$$

$$\pi_j = \begin{pmatrix} \frac{1}{2}\pi_j^0 & \frac{1}{\sqrt{2}}\pi_j^+ \\ \frac{1}{\sqrt{2}}\pi_j^- & -\frac{1}{2}\pi_j^0 \end{pmatrix}$$

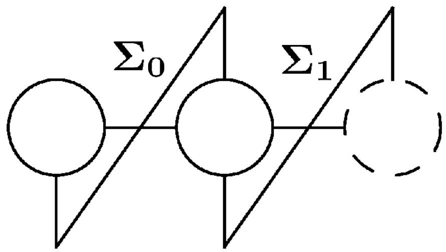
where

$$D_\mu\Sigma_j = \partial_\mu\Sigma_j + ig_jW_j\Sigma_j - ig_{j+1}\Sigma_jW_{j+1}$$

This gives the gauge boson mass matrices:

$$M_\pm^2 = \frac{f^2}{4}\begin{pmatrix} g_0^2 & -g_0g_1 \\ -g_0g_1 & 2g_1^2 \end{pmatrix}$$

$$M_n^2 = \frac{f^2}{4}\begin{pmatrix} g_0^2 & -g_0g_1 & 0 \\ -g_0g_1 & 2g_1^2 & -g_1g_2 \\ 0 & -g_1g_2 & g_2^2 \end{pmatrix}$$



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

Independent parameters (M_W , M_Z , e , $M_{W'}$)

Dependent parameters (g_0 , g_1 , g_2 , f)

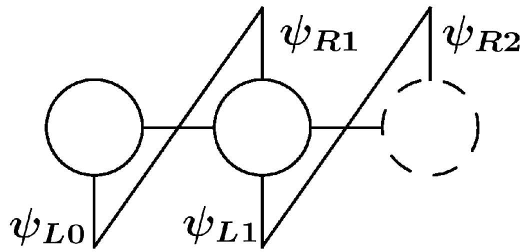
$x = \frac{g_0}{g_1} \qquad t = \frac{g_2}{g_0}$

$$\frac{M_W^2}{M_{W'}^2} = \frac{2+x^2-\sqrt{4+x^4}}{2+x^2+\sqrt{4+x^4}}$$

$$M_W = g_1 f \frac{\sqrt{2+x^2-\sqrt{4+x^4}}}{2\sqrt{2}}$$

$$\frac{M_W^2}{M_Z^2} = \frac{2+x^2-\sqrt{4+x^4}}{2+x^2(1+t^2)-\sqrt{4+x^4(1-t^2)^2}}$$

$$\frac{1}{e^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{1}{g_2^2}$$



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

Fermion - Gauge Sector

$$\mathcal{L}_{D\psi} = \bar{\psi}_{L0} \not{D}\, \psi_{L0} + \bar{\psi}_1 \not{D}\, \psi_1 + \bar{\psi}_{R2} \not{D}\, \psi_{R2}$$

$Y_{0,1Q} = 1/6 \quad Y_{0,1L} = -1/2$

$Y_{2u} = 2/3$

$Y_{2d} = -1/3 \quad Y_{2e} = -1$

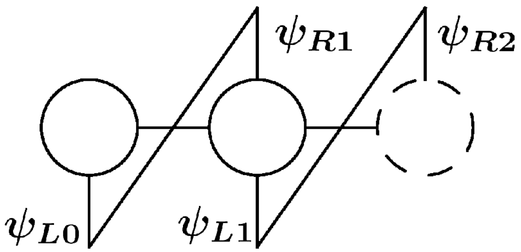
where

$$D_\mu \psi_j = \partial_\mu \psi_j + i g_j W_j \psi_j + i g_2 Y_{jf} W_2 \psi_j$$

for *j=1,2*

and

$$D_\mu \psi_2 = \partial_\mu \psi_2 + i g_2 Y_{2f} W_2 \psi_2$$



$$SU(2)_0 \times SU(2)_1 \times U(1)_2$$

Fermion - Goldstone Sector

$$\mathcal{L}_{\Sigma\psi} = -M_F\left(\epsilon_L \bar{\psi}_{L0}\Sigma_0\psi_{R1} + \bar{\psi}_{L1}\psi_{R1} + \bar{\psi}_{L1}\Sigma_1\epsilon_R\psi_{R2}\right)$$

e.g.

$$\epsilon_R = \begin{pmatrix} \epsilon_{Rt} & 0 \\ 0 & \epsilon_{Rb} \end{pmatrix}$$

This gives the fermion mass matrix:

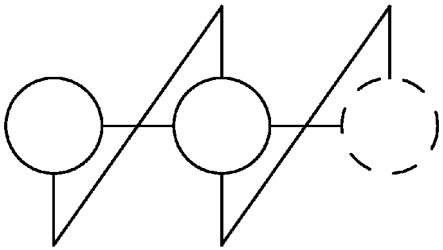
$$M = M_F \begin{pmatrix} \epsilon_L & 0 \\ 1 & \epsilon_{Rf} \end{pmatrix}$$

Indpendent parameters (M_f , M_F , M_W , M_W')
Dependent parameters (ϵ_L , ϵ_R)

$$\epsilon_L^2 = \frac{2x^2}{2 - x^2 + \sqrt{4 + x^4}}$$

$$M_f = M_F \frac{\sqrt{1+\epsilon_L^2+\epsilon_R^2-\sqrt{(1+\epsilon_L^2+\epsilon_R^2)^2-4\epsilon_L^2\epsilon_R^2}}}{\sqrt{2}}$$

(see next slide)



Ideal Delocalization

$$g_i v^i_{Le} v^i_{L\nu} = g_{W_{SM}} v^i_W$$

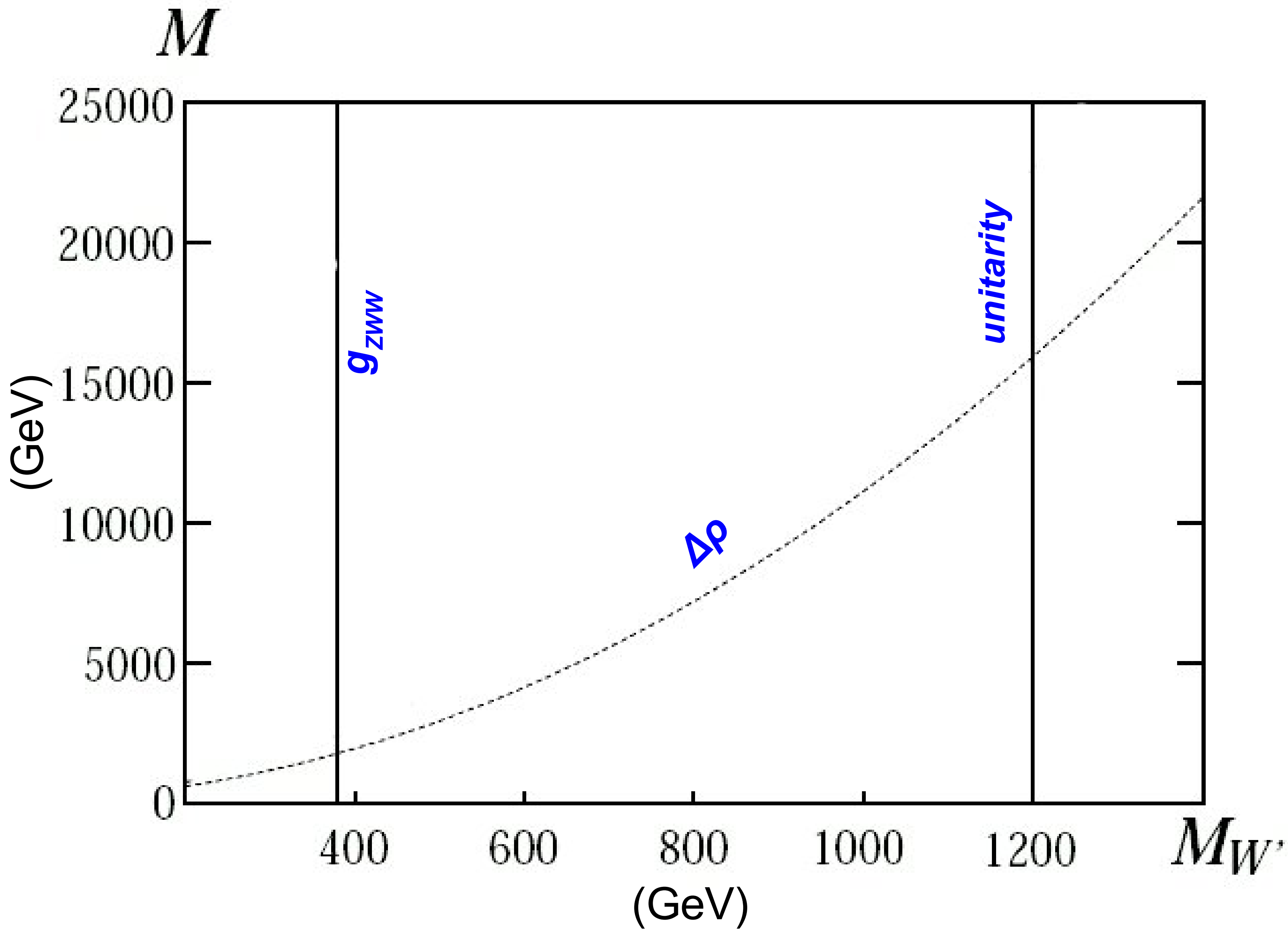
$$\epsilon_L^2 = \frac{2x^2}{2 - x^2 + \sqrt{4 + x^4}}$$

$$\begin{aligned} g_{W_{TSM}} &= g_0 v^0_{Le} v^0_{L\nu} v^0_W + g_1 v^1_{Le} v^1_{L\nu} v^1_W \\ &= g_{W_{SM}} \left(v^0_W v^0_W + v^1_W v^1_W \right) \\ &= g_{W_{SM}} \end{aligned}$$

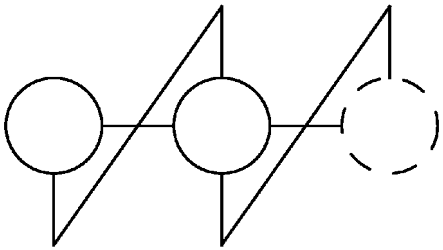
$$\begin{aligned} g_{W'_{TSM}} &= g_0 v^0_{Le} v^0_{L\nu} v^0_{W'} + g_1 v^1_{Le} v^1_{L\nu} v^1_{W'} \\ &= g_{W_{SM}} \left(v^0_W v^0_{W'} + v^1_W v^1_{W'} \right) \\ &= 0 \end{aligned}$$

Chivukula, Simmons, He, Kurachi, Tanabashi: PRD 72, 015008 (2005)
Casalbuoni, Deandrea, De Curtis, Dominici, Gatto, Grazzini, : PRD 53, 5201 (1996)

Bounds



Chivukula, Coleppa, Di Chiara, Simmons: PRD **74**, 075011 (2006)



Particle Content

$$\gamma, G \qquad Z, W^{\pm} \qquad Z', W'^{\pm}$$

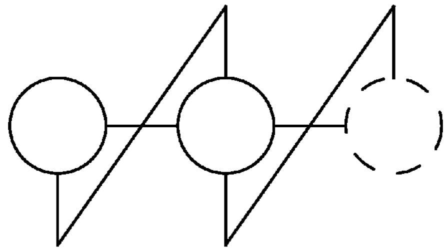
$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{pmatrix} u' \\ d' \end{pmatrix} \begin{pmatrix} c' \\ s' \end{pmatrix} \begin{pmatrix} t' \\ b' \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \qquad \begin{pmatrix} \nu'_e \\ e' \end{pmatrix} \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix} \begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}$$

Independent Parameters: $M_{W'}$, M_F

CalcHEP (by Alexander Pukhov)

- **User friendly graphical interface.**
  - **Batch mode also available.**
- **Easy implementation of new models.**
  - **Especially using LanHEP (by Andrei Semenov).**
- **Feynman gauge and unitary gauge.**
  - **Important cross check.**
- **Interface with Pythia**
- **Many other features.**
  - **See Sasha Belyaev's talk.**



LanHEP (by Andrei Semenov)

- ***Automatic Feynman rules from Lagrangian.***

- ***Has checks for***
  - ***Hermiticity.***
  - ***BRST invariance.***
  - ***EM charge conservation.***
  - ***Particle mixings, mass terms, and mass matrices.***

Example of Implementation with LanHEP

LanHEP

$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left(F_0^2 + F_1^2 + F_2^2\right) \quad \textbf{where} \quad F_j^{\mu\nu} = \partial^\mu W_j^\mu - \partial^\nu W_j^\mu + i g_j \left[W_j^\mu, W_j^\nu\right]$$

```
%%%%%%%%%%  Kinetic and self interaction Lagrangian terms.
lterm  -F**2/4  where  F=deriv^mu*W23^nu-deriv^nu*W23^mu.
lterm  -F**2/4  where  F=deriv^mu*W0^nu^a-deriv^nu*W0^mu^a-g*eps^a^b^c*W0^mu^b*W0^nu^c.
lterm  -F**2/4  where  F=deriv^mu*W1^nu^a-deriv^nu*W1^mu^a-g/x*eps^a^b^c*W1^mu^b*W1^nu^c.
```

(gauge kinetic term as an example)

lhep 3-site.mdl

CalcHEP

Lagrangian

P1	P2	P3	P4	> Factor
A	W+	W-		-g*v0g
A	~W+	~W-		-g*v0g
W+	W-	Z		-g/x
W+	W-	~Z		-g/x
W+	Z	~W-		-g/x
W+	~W-	~Z		-g/x
W-	Z	~W+		-g/x
W-	~W+	~Z		-g/x
Z	~W+	~W-		-g/x
~W+	~W-	~Z		-g/x
A	A	W+	W-	-g**2*v0g**2
A	A	~W+	~W-	-g**2*v0g**2
A	W+	W-	Z	-g**2*v0g/x
A	W+	W-	~Z	-g**2*v0g/x
A	W+	Z	~W-	-g**2*v0g/x
A	W+	~W-	~Z	-g**2*v0g/x
A	W-	Z	~W+	-g**2*v0g/x
A	W-	~W+	~Z	-g**2*v0g/x
A	Z	~W+	~W-	-g**2*v0g/x
A	~W+	~W-	~Z	-g**2*v0g/x

W+	W+	W-	W-	g**2/x**2
W+	W+	W-	~W-	g**2/x**2
W+	W+	~W-	~W-	g**2/x**2
W+	W-	W-	~W+	g**2/x**2
W+	W-	Z	Z	-g**2/x**2
W+	W-	Z	~Z	-g**2/x**2
W+	W-	~W+	~W-	g**2/x**2
W+	W-	~Z	~Z	-g**2/x**2
W+	Z	Z	~W-	-g**2/x**2
W+	Z	~W-	~Z	-g**2/x**2
W+	~W+	~W-	~W-	g**2/x**2
W+	~W-	~Z	~Z	-g**2/x**2
W-	W-	~W+	~W+	g**2/x**2
W-	Z	Z	~W+	-g**2/x**2
W-	Z	~W+	~Z	-g**2/x**2
W-	~W+	~W+	~W-	g**2/x**2
W-	~W+	~Z	~Z	-g**2/x**2
Z	Z	~W+	~W-	-g**2/x**2
Z	~W+	~W-	~Z	-g**2/x**2
~W+	~W+	~W-	~W-	g**2/x**2
~W+	~W-	~Z	~Z	-g**2/x**2

Example of Implementation with LanHEP

LanHEP

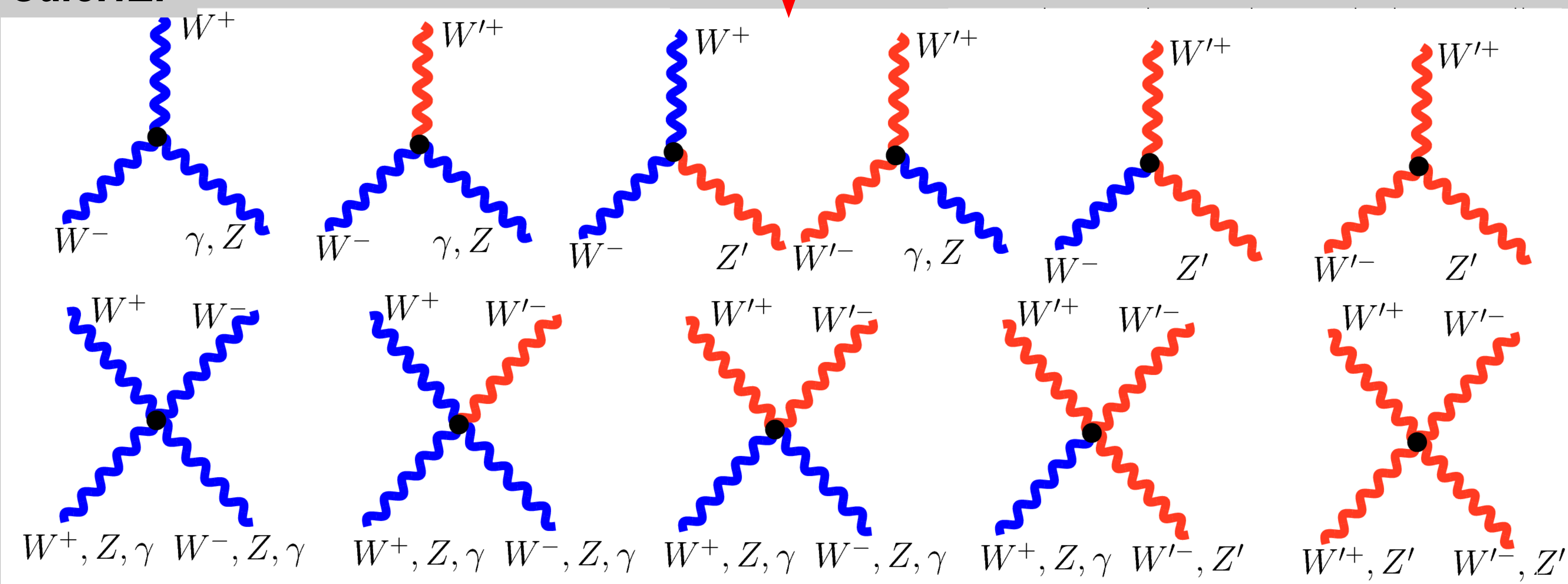
$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left(F_0^2 + F_1^2 + F_2^2\right)$$ **where** $$F_j^{\mu\nu} = \partial^\mu W_j^\mu - \partial^\nu W_j^\mu + i g_j \left[W_j^\mu, W_j^\nu\right]$$

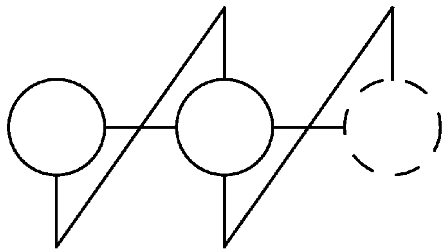
```
%%%%%%%%%%  Kinetic and self interaction Lagrangian terms.
lterm  -F**2/4  where  F=deriv^mu*W23^nu-deriv^nu*W23^mu.
lterm  -F**2/4  where  F=deriv^mu*W0^nu^a-deriv^nu*W0^mu^a-g*eps^a^b^c*W0^mu^b*W0^nu^c.
lterm  -F**2/4  where  F=deriv^mu*W1^nu^a-deriv^nu*W1^mu^a-g/x*eps^a^b^c*W1^mu^b*W1^nu^c.
```

(gauge kinetic term as an example)

lhep 3-site.mdl

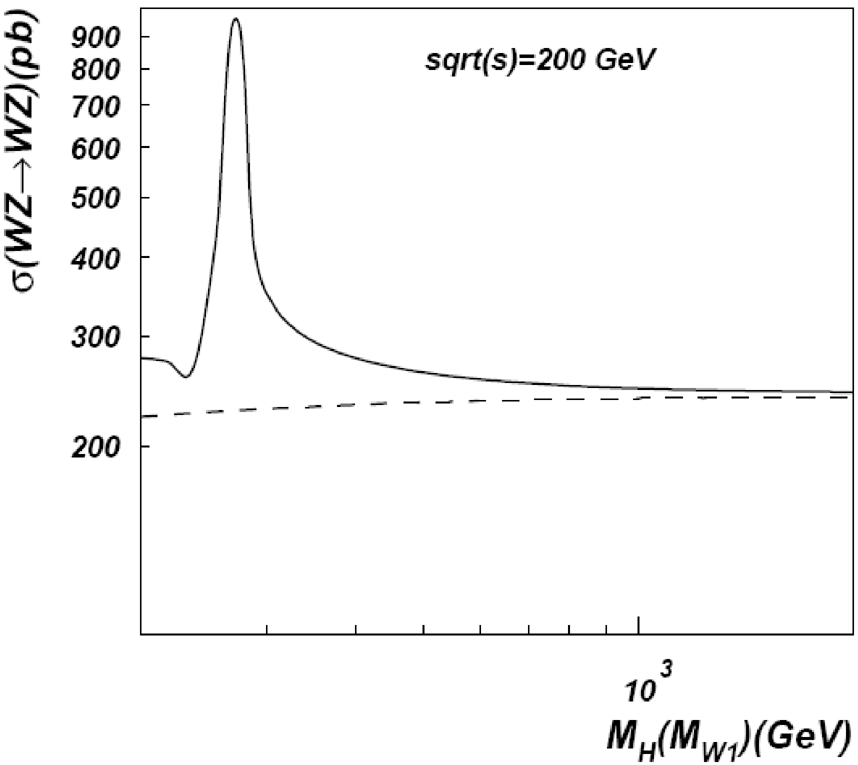
CalcHEP



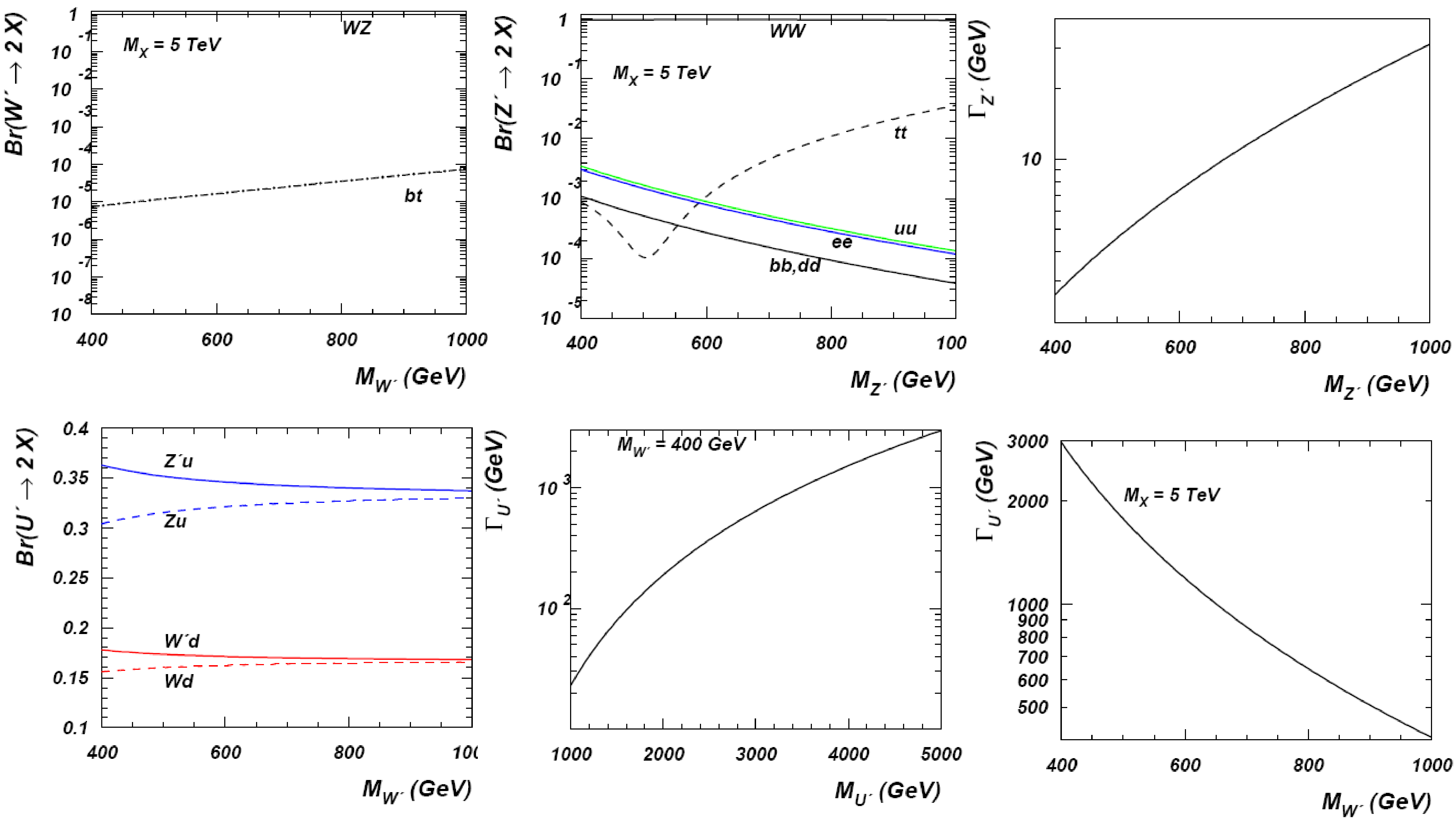


Checks:

- Feynman vs. Unitary gauge.
- Decoupling of heavy fields.
- Masses and mixings **(LanHEP)**.
- Hermiticity **(LanHEP)**.

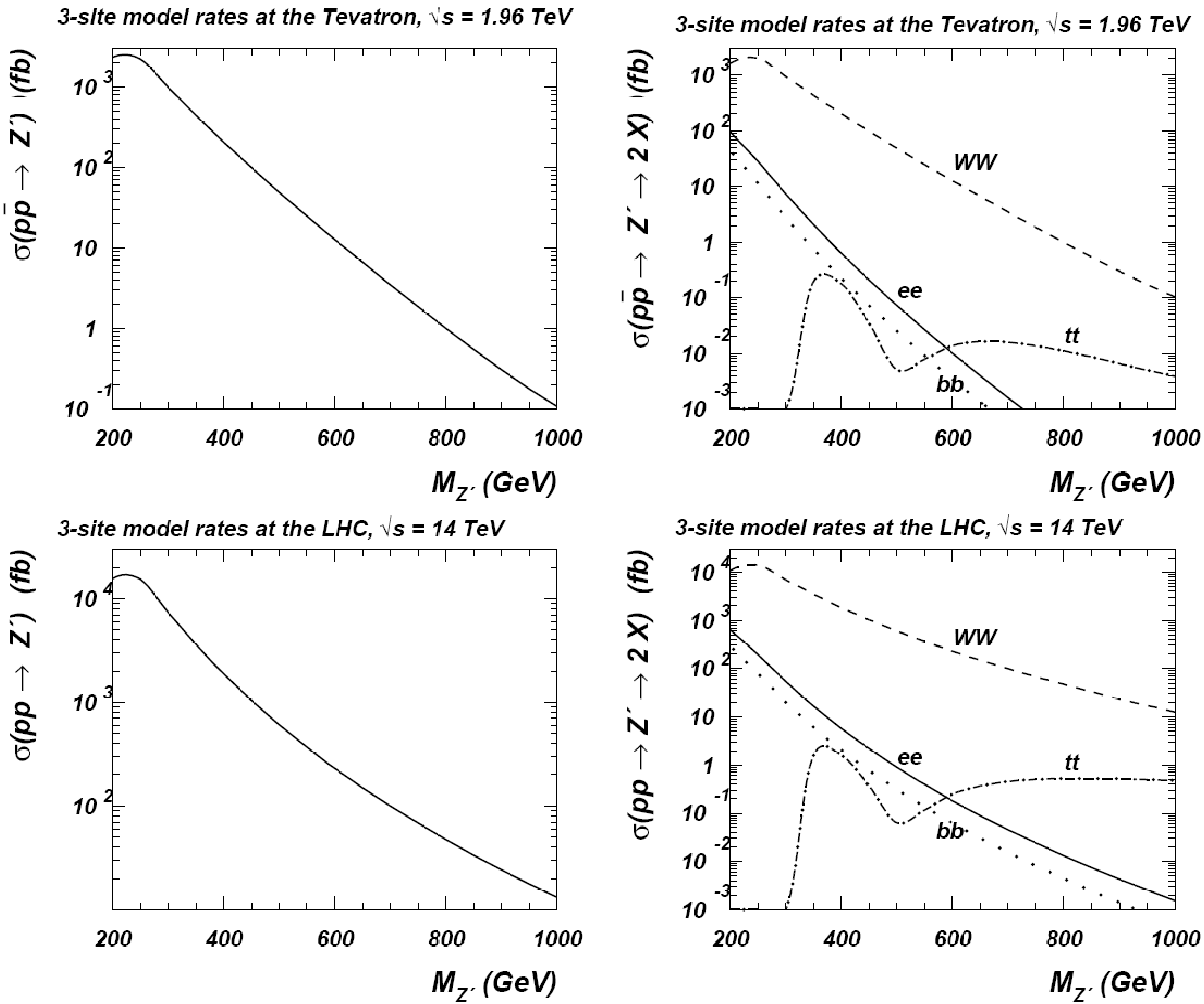


Widths

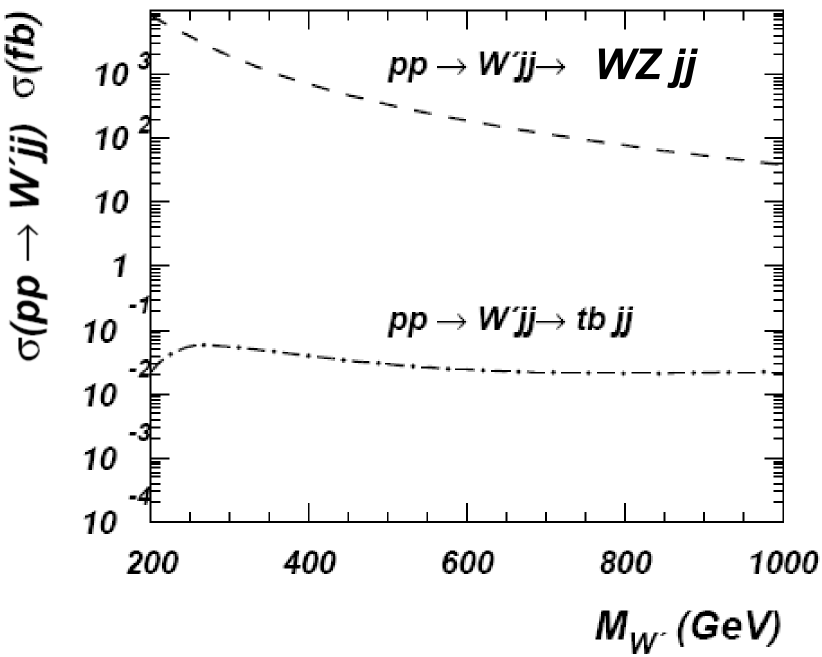


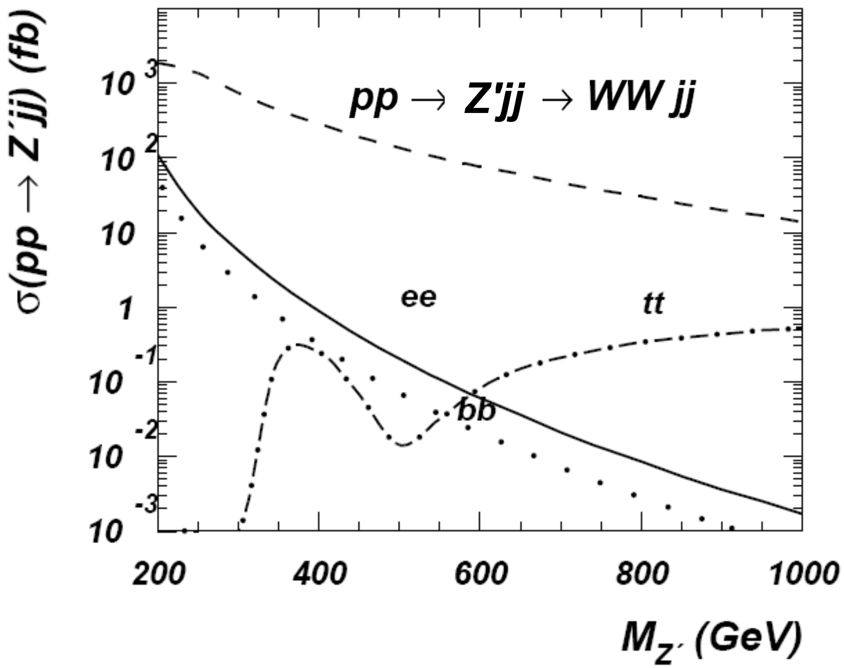
Br(W′ → 2 X)
M_X = 5 TeV
WZ
bt
M_W′ (GeV)
Br(Z′ → 2 X)
M_X = 5 TeV
WW
tt
uu
ee
bb,dd
M_Z′ (GeV)
Γ_Z′ (GeV)
M_Z′ (GeV)
Br(U′ → 2 X)
Z′u
Zu
W′d
Wd
M_W′ (GeV)
Γ_U′ (GeV)
M_W′ = 400 GeV
M_U′ (GeV)
Γ_U′ (GeV)
M_X = 5 TeV
M_W′ (GeV)

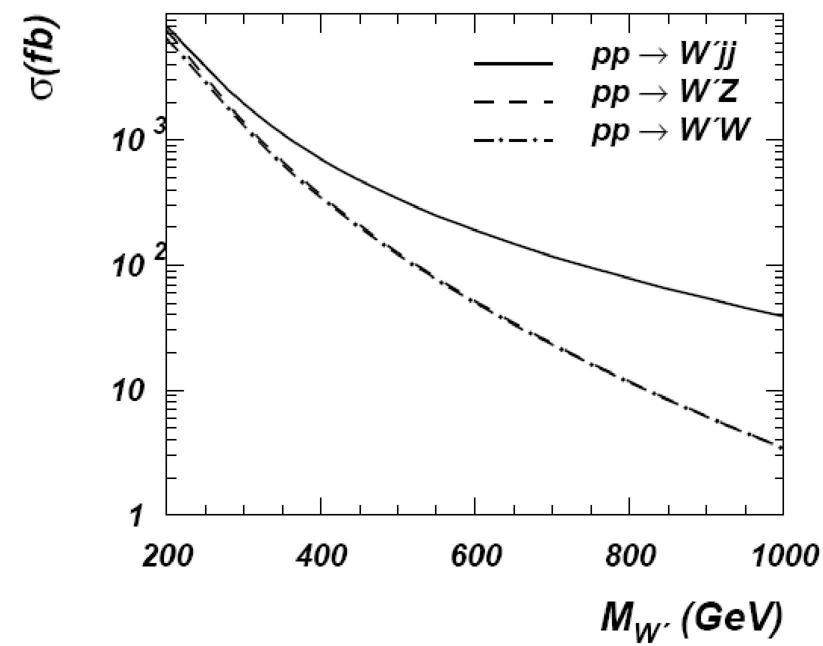
Z' Production



WW → Z' and WZ → W' Fusion

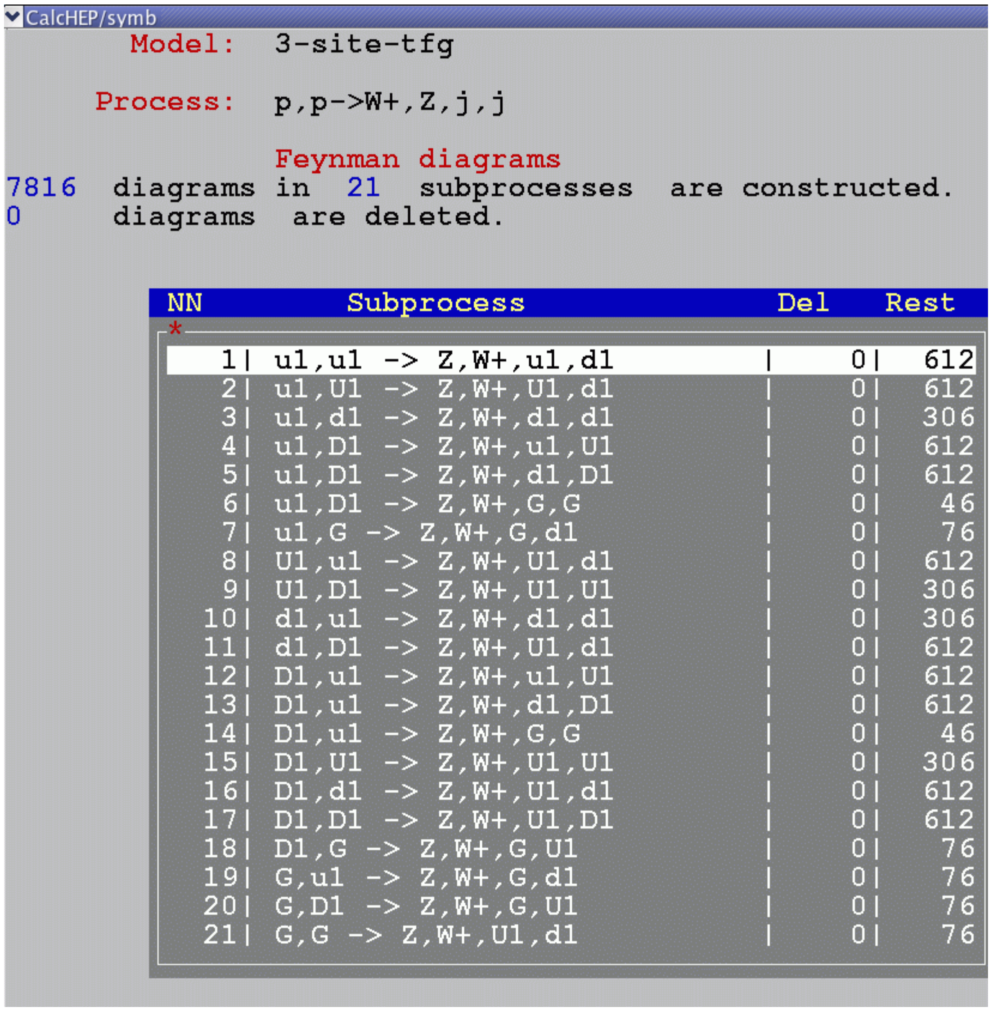


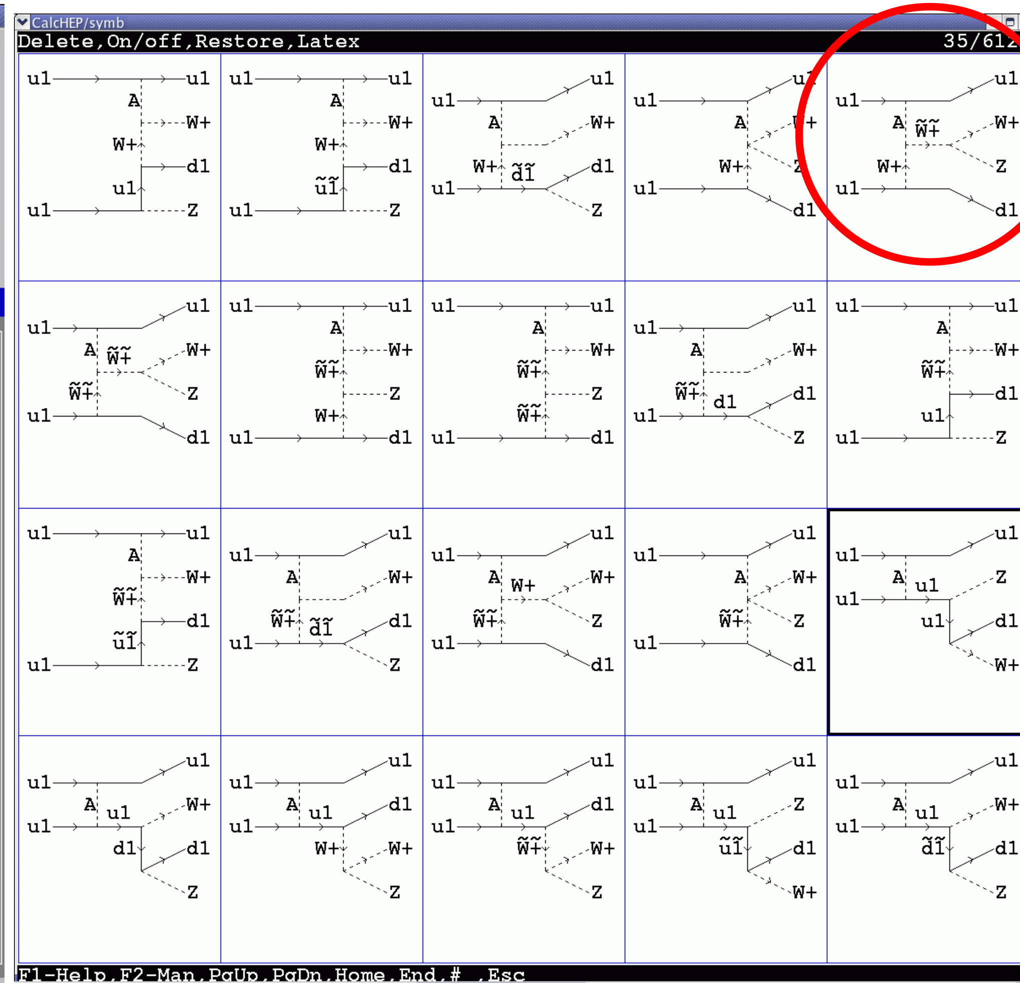




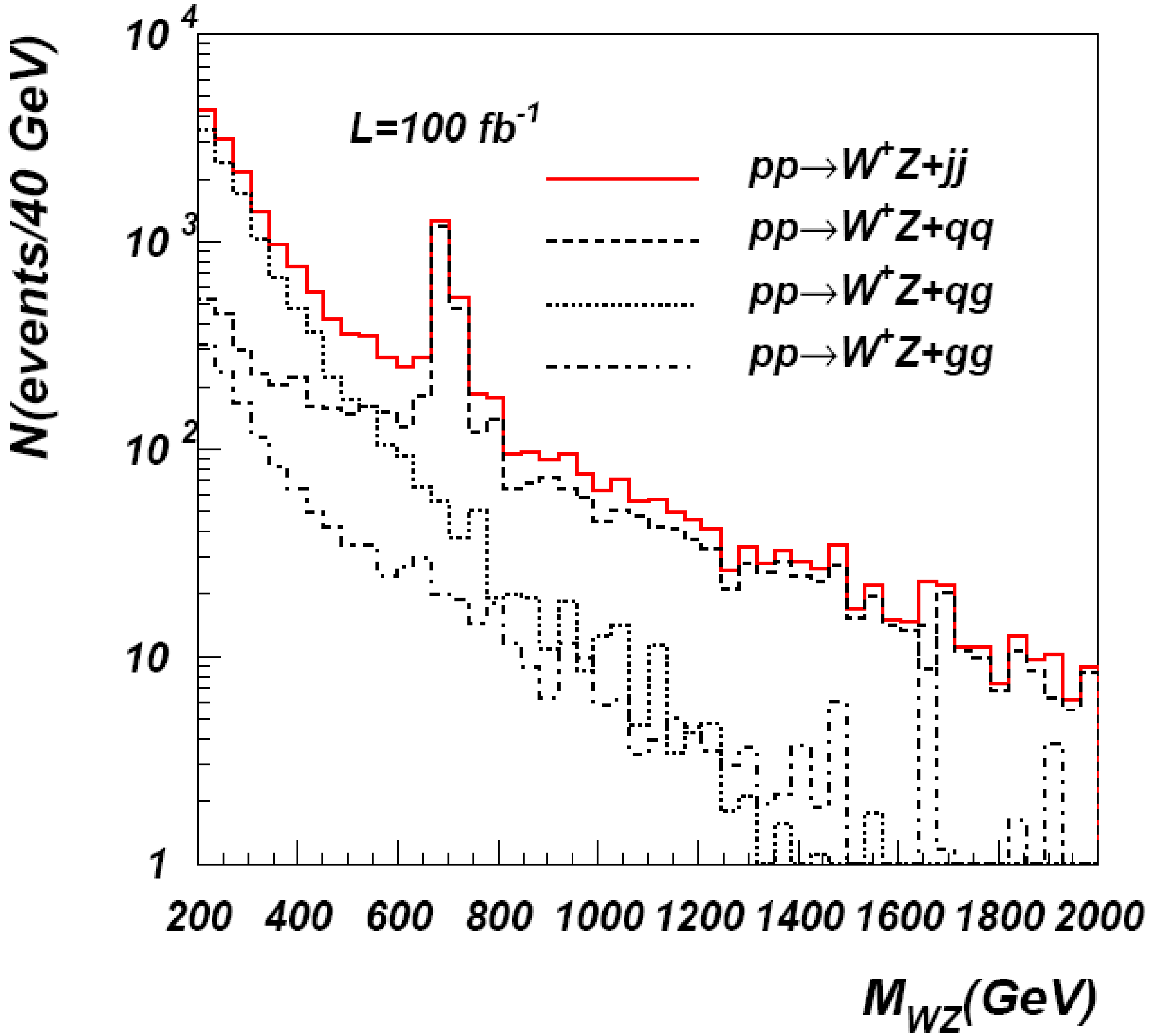
$pp \to W^+Zjj$

- ***No effective WZ approximation.***
- ***Complete set of signal and background diagrams including interference.***





Preliminary $pp \to W^+Zjj$



$p_T^j > 30$ GeV

$2 < |\eta^j| < 4.5$

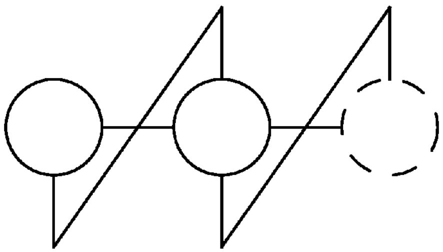
$E^j > 300$ GeV

$E^{W,Z} > 200$ GeV

$\Delta Rjj > 0.5.$

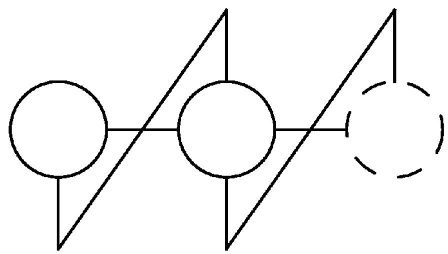
To be compared with Birkedal, Matchev, Perelstein: PRL 94, 191803 (2005).

The Three Site Model:



- Is simple, yet consistently implements the 1st KK mode of a Higgsless ED
- Is representative of Higgsless models and their duals – dynamical symmetry breaking models
- Is consistent with precision electroweak observables (ideal deloc.)
- Has a simple parameter space (M_F , $M_{W'}$) that can be probed at the LHC and is possibly entirely testable at the LHC (work in progress).

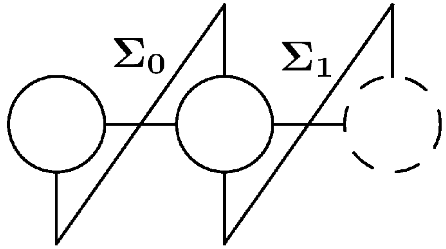
Our implementation of the Three Site Model in CalcHEP:



- *Is complete:*
  - *No effective WZ approximation*
  - *Both signal and background are included in calculations*
  - *Keeps all orders in $M_W/M_{W'}$.*
- *Has been tested:*
  - *Gauge invariant*

    (Feynman gauge = Unitary gauge)
  - *Masses and mixings (LanHEP)*
  - *Hermitian (LanHEP)*
- *Can be found on our website:*
  - hep.pa.msu.edu/people/belyaev/public/3-site/
- *Is currently producing exciting results!*
  - *Watch for our paper on the arXiv.*

Appendix



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

Gauge Fixing Sector

$$\mathcal{L}_{GF} = -\mathrm{Tr}\left[G_0^2 + G_1^2 + G_2^2\right]$$

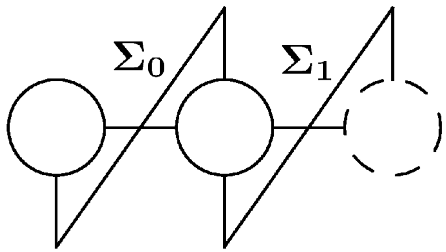
where

$$G_0 = \partial \cdot W_0 - \tfrac{1}{2} g_0 f \left(\pi_0 \quad\right)$$

$$G_1 = \partial \cdot W_1 - \tfrac{1}{2} g_1 f \left(\pi_1 - \pi_0\right)$$

$$G_2 = \partial \cdot W_2 - \tfrac{1}{2} g_2 f \left(\quad - \pi_1^{ns}\right)$$

$$\pi_1^{ns} = \begin{pmatrix} \frac{1}{2}\pi_j^0 & 0 \\ 0 & -\frac{1}{2}\pi_j^0 \end{pmatrix}$$



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

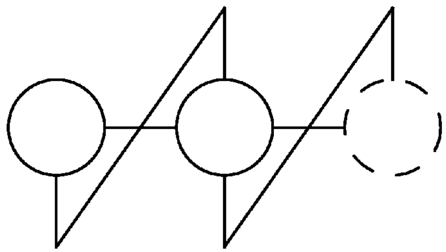
Ghost Sector

$$\mathcal{L}_{\bar{c}c} = -\mathrm{Tr}\Big[\bar{c}_0 \delta_{BRST} G_0 + \bar{c}_1 \delta_{BRST} G_1 + \bar{c}_2 \delta_{BRST} G_2\Big]$$

where

$$\delta_{BRST} W_{\mu j} = -\Big(\partial_\mu c_j + i g_j \,[\, W_{\mu j} \,,\, c_j \,]\Big)$$

$$\delta_{BRST} \pi_j = \tfrac{1}{2} f \big(g_j c_j - g_{j+1} c_{j+1}\big) + \tfrac{i}{2} [\, g_j c_j + g_{j+1} c_{j+1} \,,\, \pi_j \,]$$
$$- \tfrac{1}{6f} [\, \pi_j \,,\, [\, \pi_j \,,\, g_j c_j - g_{j+1} c_{j+1} \,] \,] + \cdots$$



Extra dimension:

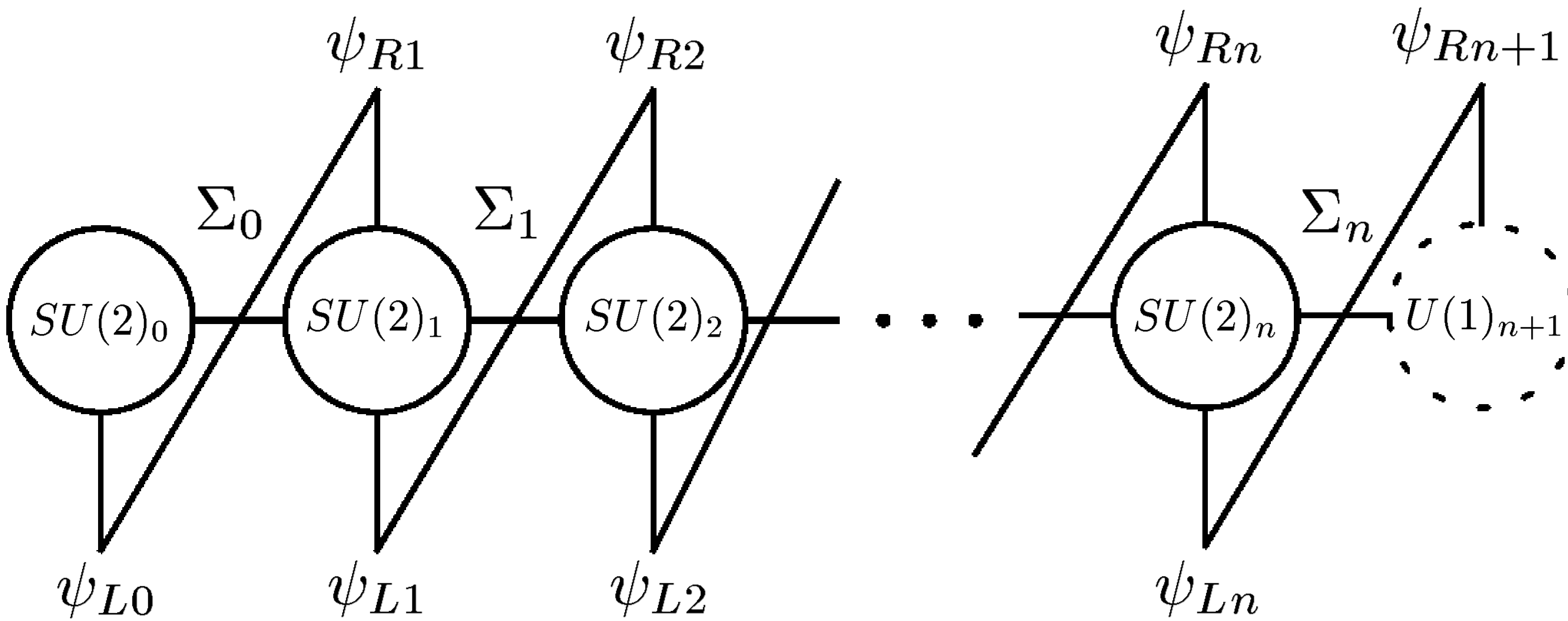
- Low energy phenomenology is dominated by 1st KK mode.
- The Three Site Model is a simple consistent way of implementing just the lowest KK mode.

Dynamical EWSB:

- Warped extra dimension dual to walking technicolor.
- The Three Site Model is a simple consistent way of implementing the first vector resonance (ρ_{TC}) of a walking technicolor model.

Testability:

- Phase space can be covered at future colliders.


ψ_{R1}
ψ_{R2}
ψ_{Rn}
ψ_{Rn+1}
Σ_0
Σ_1
Σ_n
$SU(2)_0$
$SU(2)_1$
$SU(2)_2$
$SU(2)_n$
$U(1)_{n+1}$
ψ_{L0}
ψ_{L1}
ψ_{L2}
ψ_{Ln}