Axion dark matter from inflation-driven quantum phase transition

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based on: **2402.08716+work in progress**

in collaboration with **Ameen Ismail** and **Seung J. Lee**

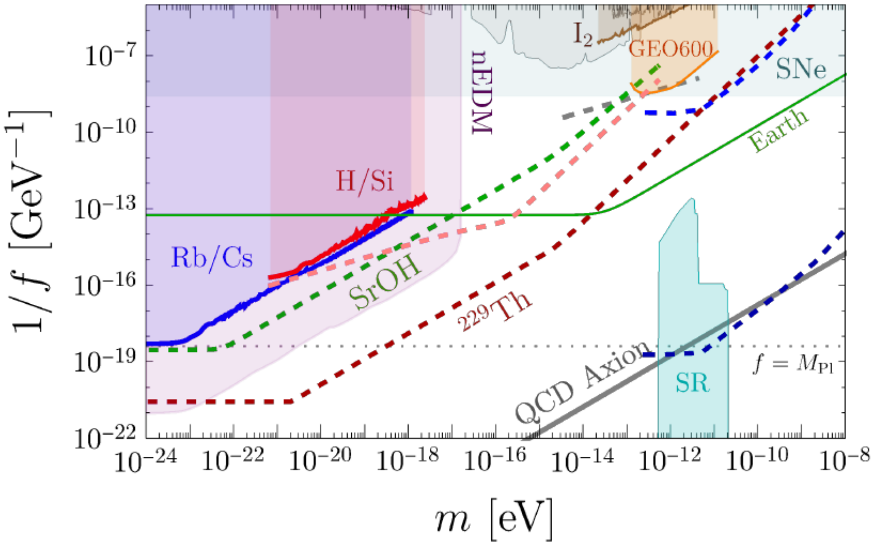
Motivation

Ultralight dark matter

- **Ultralight** DM: $10^{-22}\,\text{eV} < m < \text{eV}$ **wave-like, oscillatory**

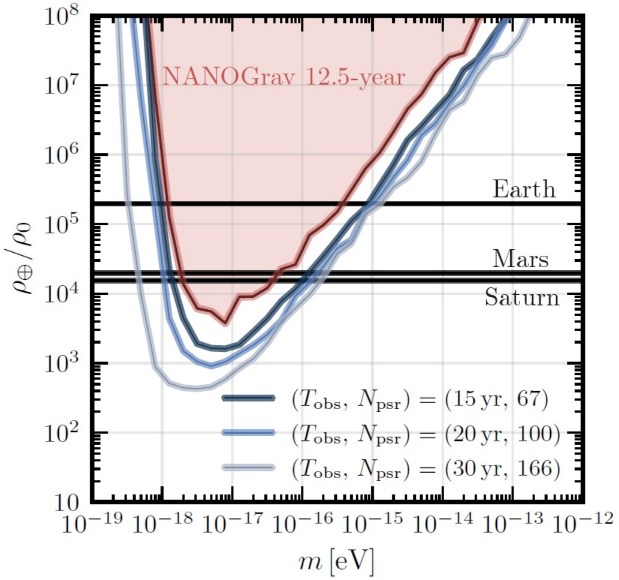
Future atomic-/astro-physics experiments: $m < 10^{-10}\text{eV}$

Quantum Sensor



Kim, Perez, 22'

Pulsar Timing Arrays



Kim, Mitridate, 23'

See also talks by Konstantin and Wolfram

Axion from misalignment mechanism

- Axion: well-motivated ultralight DM (protected by shift symmetry)

$$\ddot{\eta} + 3H\dot{\eta} + m_\eta^2 \eta = 0 \qquad V(\eta) = \Lambda_\eta^4 \left[1 - \cos\left(\eta/f_\eta\right)\right] \Rightarrow m_\eta = \Lambda_\eta^2/f_\eta$$

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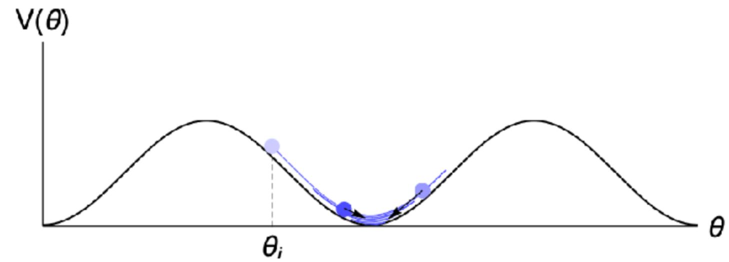
- Misalignment mechanism: axion starts to oscillate when $H \sim m_\eta$, and behaves as matter after then, $\rho_\eta \sim a^{-3}$

$$\left(\frac{\Omega_\eta h^2}{0.12}\right)_{\text{ALP, mis.}} \sim \left(\frac{m_\eta}{10^{-10}\ \text{eV}}\right)^{1/2} \left(\frac{f_\eta}{10^{14}\ \text{GeV}}\right)^2$$

$$\left(\frac{\Omega_\eta h^2}{0.12}\right)_{\text{QCD axion, mis.}} \sim \left(\frac{10^{-6}\ \text{eV}}{m_\eta}\right)^{3/2}$$

misalignment angle (no fine-tuning)

$$\theta_i = \eta/f_\eta \sim \mathcal{O}(1)$$



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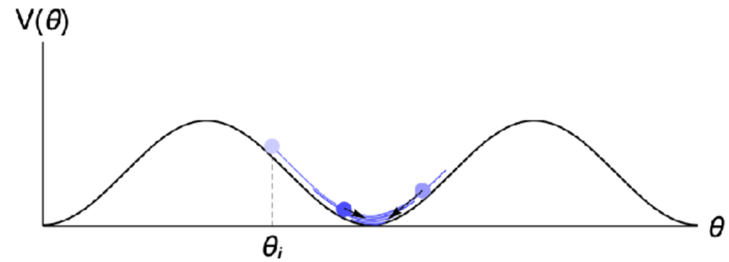

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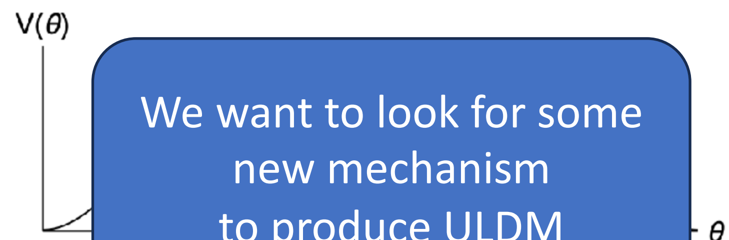


We want to look for some new mechanism to produce ULDM

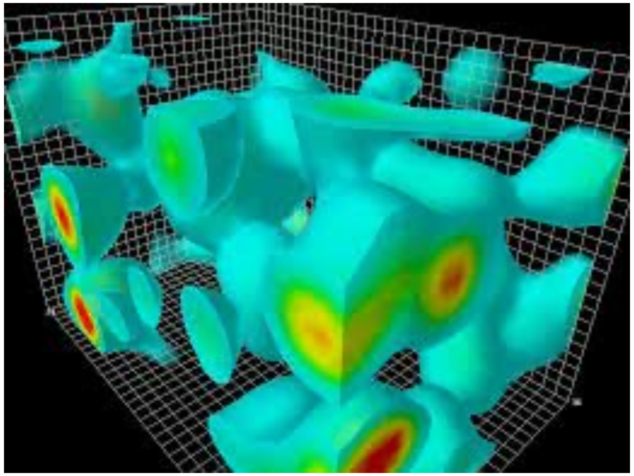
- For QCD axion DM: $m_\eta \sim 10^{-6}$ eV

  For ALP DM: $f_\eta > 10^{14}\text{GeV}$ if $m_\eta < 10^{-10}$ eV

How to reduce f_η for better experimental sensitivity?

Particle production from inflationary quantum fluctuations

- ➢ Particle production in expanding universe: **Parker, 68'**



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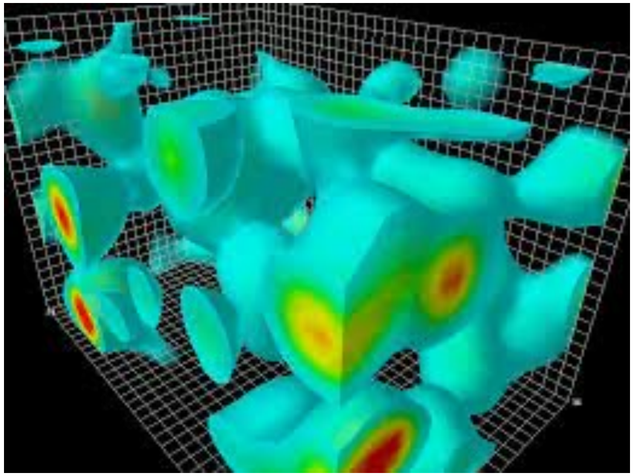

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  1) mass term (suppressed for light particles)

  2) coupling to gravity (significant during inflation)

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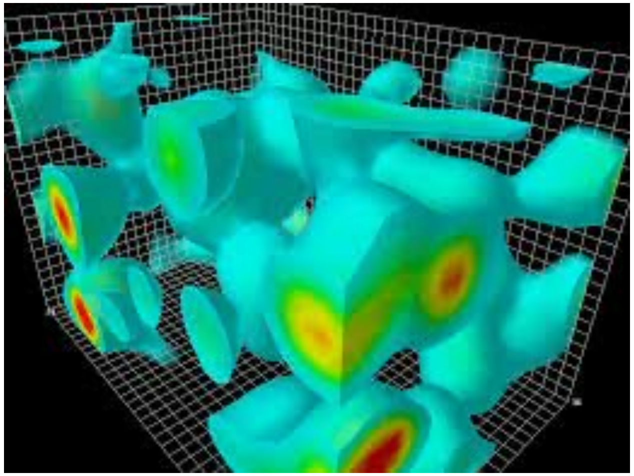

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- Typical scale of inflationary quantum fluctuation:

$$T_{\rm GH} = H_{\rm inf}/2\pi$$ **Gibbons, Hawking, 77'**

$$H_{\rm inf} \lesssim 10^{14}\ {\rm GeV}$$ (constrained by tensor-to-scalar ratio)

scalar curvature in de Sitter space

$$\mathcal{R} = 12 H_{\rm inf}^2$$

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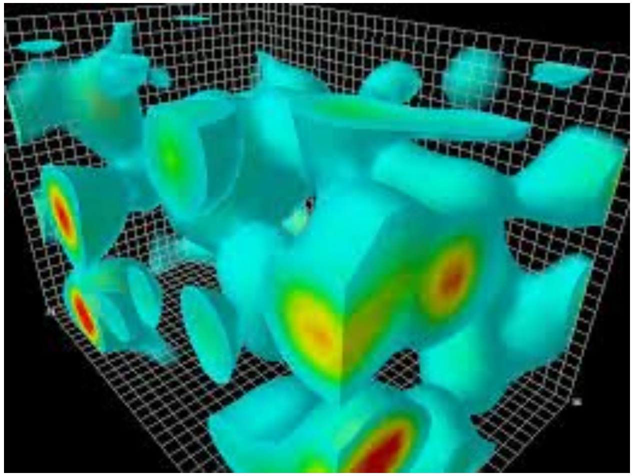

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$$\frac{\Omega_\eta h^2}{0.12} \sim \left(\frac{m_\eta}{10^{-5}\ {\rm eV}}\right)^{1/2} \left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^2$$

Purely inflationary quantum fluctuation is *not* enough to produce axions lighter than 10^{-5}eV

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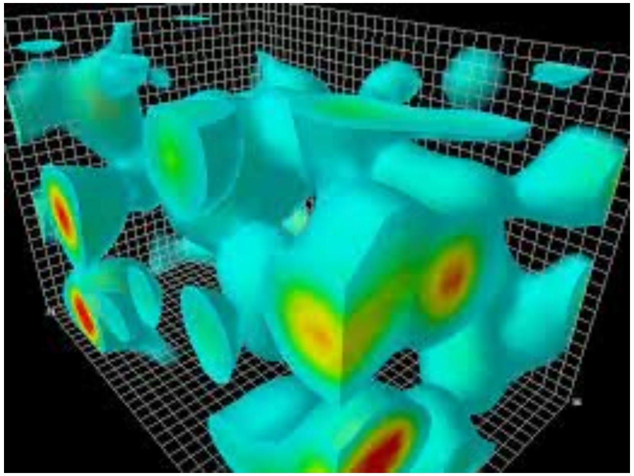

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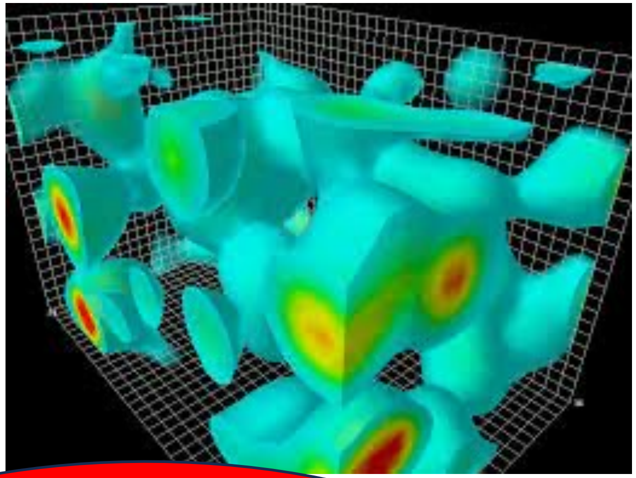
Difficulties to have lighter DM:

1) mass suppression to relic abundance
2) kinematic suppression, $p_{\rm e} \sim H_{\rm inf} \gg m_\eta$
   DM is ultra-relativistic by the end of inflation

Particle production from inflationary quantum fluctuations



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Key of our mechanism: change the kinematics of axion by the end of inflation!

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Framework

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- We assume the PQ symmetry has broken during inflation, $f_\eta > H_{\rm inf}/2\pi$. Axion is effectively massless during inflation if $m_\eta/K < H_{\rm inf}$

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}K^2(\phi)g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta\right]$$

ϕ: inflaton

η: axion

$K(\phi)$ dynamically reduces to unit after inflation

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- Flat FLRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(\tau)\left(-\mathrm{d}\tau^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right)$$

conformal time: $\mathrm{d}\tau \equiv \mathrm{d}t/a$

de Sitter background: $a = -1/(H\tau)$

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- EOM of axion:

$f \equiv aK\eta$

$$f'' - \nabla^2 f - \left(\frac{a''}{a} + \frac{K''}{K} + 2\frac{a'}{a}\frac{K'}{K}\right)f = 0$$

$f' \equiv \mathrm{d}f/\mathrm{d}\tau$

$$f(\tau,\mathbf{k}) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left[f_k(\tau)\hat{a}_{\mathbf{k}}e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + f_k^*(\tau)\hat{a}_{\mathbf{k}}^\dagger e^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\right]$$

Mode functions f_k depend on time, so particles can be produced

Effective curvature

- Parametrization: $\kappa \equiv \tau^2 \frac{K''}{K} - 2\tau \frac{K'}{K}$

  slow-roll approximation: $\kappa \approx M_{\rm Pl}^2 \left(2\epsilon_V \frac{K_{\phi\phi}}{K} - 3 \frac{K_\phi}{K} \frac{V_\phi}{V} \right)$

$V_\phi \equiv \partial V / \partial \phi$

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- Geometric meaning of κ:

$$f'' - \nabla^2 f - \frac{1}{6} a^2 \left(\mathcal{R} + 6\kappa H_{\rm inf}^2 \right) f = 0$$

scalar curvature in dS space

$$\mathcal{R} = 12 H_{\rm inf}^2$$

Scalar curvature is responsible for light particle production in dS space (negative mass term)

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- κ also serves as an order parameter to break the scale-invariant axion spectrum

κ drives a phase transition from CFT conserving phase to broken phase

Axion power spectrum

$$f_k(\tau) = \frac{\sqrt{\pi}}{2}\sqrt{-\tau} H_\nu^{(1)}(-k\tau)$$

$$\nu \equiv \sqrt{9/4 + \kappa}$$

- Two-point correlation function:

$$\langle f_k f_k^* \rangle = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} |f_k|^2 = \int \mathrm{d}\log k \, \frac{k^3}{2\pi^2} |f_k|^2$$

- Power spectrum: $P_k \equiv \frac{1}{a^2} \frac{k^3}{2\pi^3} |f_k|^2$

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For superhorizon modes ($x \ll 1$): $|f_k|^2 \approx \frac{2^{2\nu}}{4\pi k}\Gamma^2(\nu)\,x^{1-2\nu}$ $\quad x \equiv k/(aH_{\rm inf})$

So the power spectrum for superhorizon modes becomes:

$$P_k = \frac{2^{2\nu}}{2\pi}\Gamma^2(\nu)\left(\frac{H_{\rm inf}}{2\pi}\right)^2\left(\frac{1}{x}\right)^{2\nu-3} \approx \boxed{\left(\frac{H_{\rm inf}}{2\pi}\right)^2\left(\frac{1}{x}\right)^{2\kappa/3}}$$

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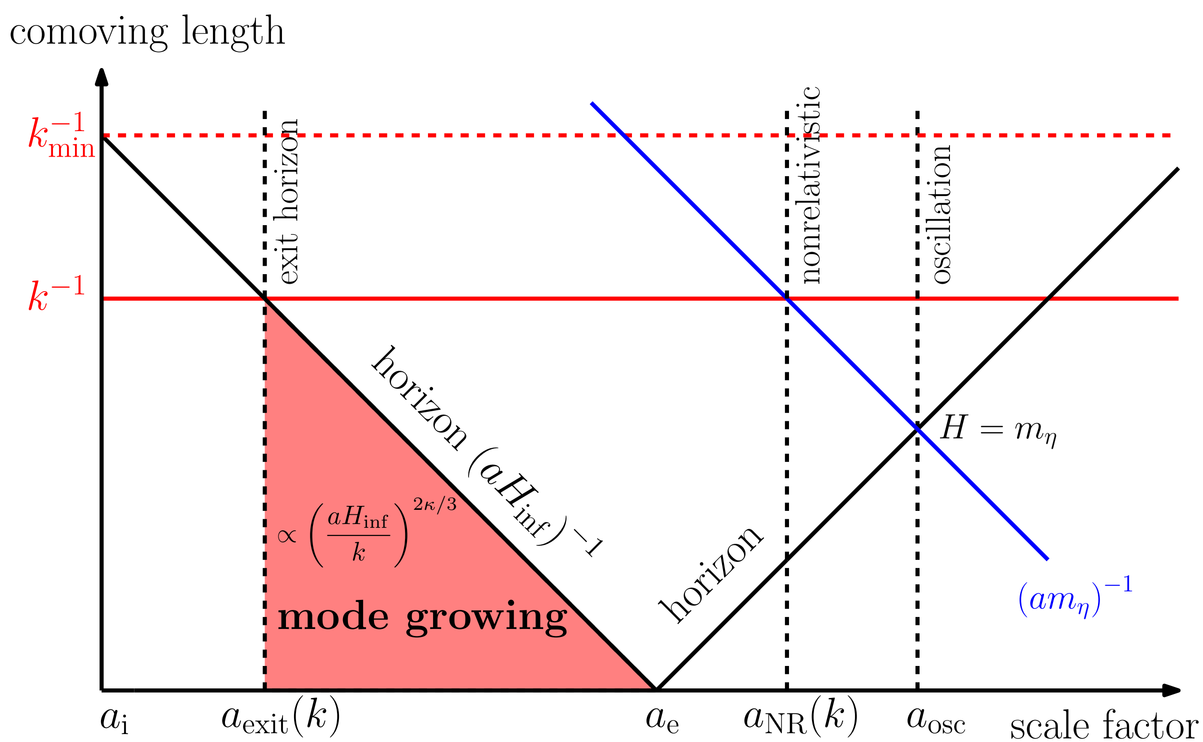
$\kappa = 0$: critical point (scale invariant)

$\kappa > 0$: red tilt (exponential enhancement)

$\kappa < 0$: blue tilt (no enhancement)

Quantum Phase Transition (QPT) modulated by κ

Axion power spectrum



minimal mode receives largest enhancement

$$k_{\rm min} \propto a_{\rm i} H_{\rm inf}$$

$$P_{k_{\rm min}} \propto e^{2\kappa N/3}$$

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comoving horizon exponentially shrinks during inflation

$\Rightarrow$ **power spectrum exponentially grows if** $\kappa > 0$

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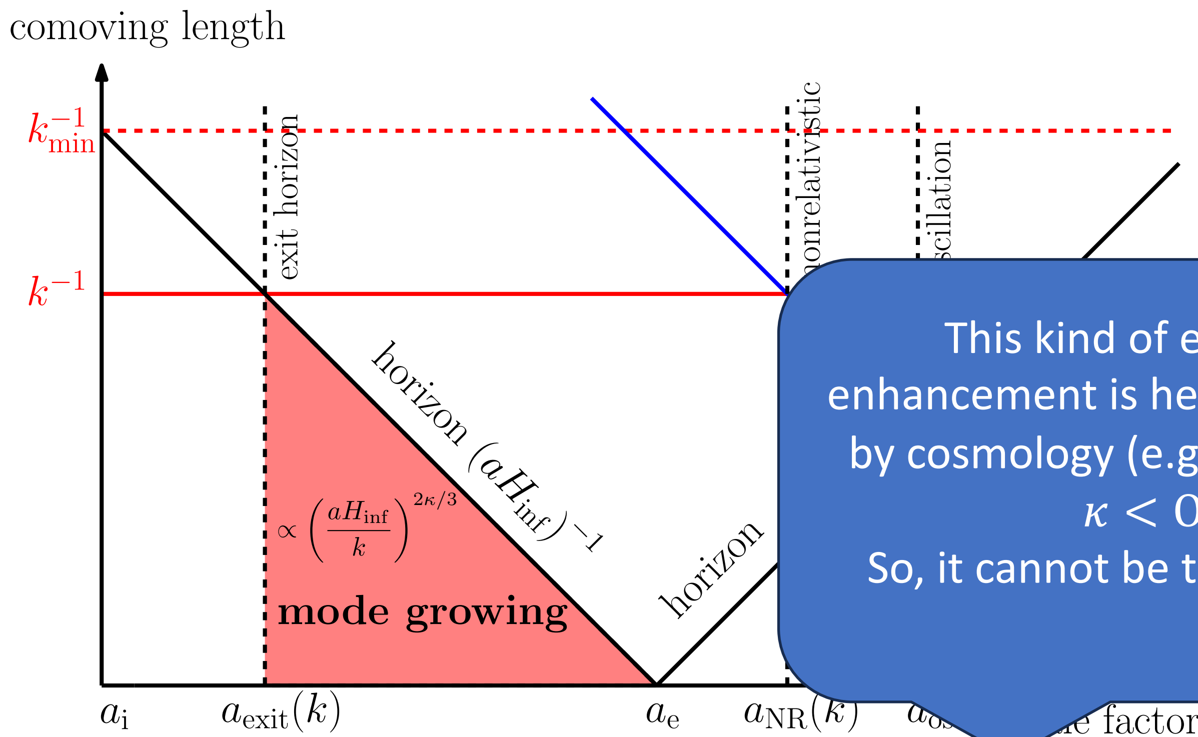

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This kind of exponential enhancement is heavily constrained by cosmology (e.g., isocurvature):
$\kappa < O(1)$
So, it cannot be the whole story

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Axion energy density

- Energy density from inflationary fluctuations:

$$\langle \rho_\eta(\tau) \rangle = \frac{1}{2a^4} \left(\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left| f_k' + \frac{1+\kappa/3}{\tau} f_k \right|^2 + k^2 \left| f_k \right|^2 \right)$$

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second term is suppressed after exiting the horizon

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But we want superhorizon modes (small k) to dominate the energy

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- However, if we have a nonzero κ:

$$\left| f_k' + \frac{1+\kappa/3}{\tau} f_k \right|^2 \approx a^2 H_{\mathrm{inf}}^2 \left(\frac{3}{2} + \frac{\kappa}{3} - \nu \right)^2 |f_k|^2 \overset{\kappa \ll 1}{\approx} \frac{\kappa^4}{729} a^2 H_{\mathrm{inf}}^2 |f_k|^2 \qquad \nu \equiv \sqrt{9/4 + \kappa}$$

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- However, if we have a nonzero κ:

$$\left| f_k' + \frac{1+\kappa/3}{\tau} f_k \right|^2 \approx a^2 H_{\mathrm{inf}}^2 \left(\frac{3}{2} + \frac{\kappa}{3} - \nu \right)^2 |f_k|^2 \overset{\kappa \ll 1}{\approx} \frac{\kappa^4}{729} a^2 H_{\mathrm{inf}}^2 |f_k|^2 \qquad \nu \equiv \sqrt{9/4 + \kappa}$$

$$\langle \rho_\eta(\tau) \rangle \approx \frac{H_{\mathrm{inf}}^4}{16\pi^3} \int_{-k_{\mathrm{min}}\tau}^{\mathcal{O}(1)} \frac{\mathrm{d}x}{x} \left\{ 2^{2(\nu-1)} \Gamma^2(\nu-1)\, x^{7-2\nu} + 2^{2\nu} \left[\left(\frac{\kappa}{3} + \frac{3}{2} - \nu \right)^2 + x^2 \right] \Gamma^2(\nu)\, x^{3-2\nu} \right.$$
$$\left. + 2^{2\nu} \left(\frac{\kappa}{3} + \frac{3}{2} - \nu \right) \Gamma(\nu-1)\, \Gamma(\nu)\, x^{5-2\nu} \right\}$$

$\kappa \neq 0$ gives the non-vanishing leading term for superhorizon modes!

Axion energy density

- For $\kappa > 0$, axion energy is dominated by superhorizon modes:

$$\langle \rho_\eta(\tau) \rangle \approx \frac{H_{\rm inf}^4}{16\pi^3} \, 2^{2\nu} \left(\frac{\kappa}{3} + \frac{3}{2} - \nu \right)^2 \Gamma^2(\nu) \int_{-k_{\rm min}\tau}^{\mathcal{O}(1)} {\rm d}x x^{2-2\nu}$$

minimal mode is set by the initial horizon:

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There are two kinds of enhancements:

1) Fluctuation enhancement $\sim e^{2\kappa N/3}$ from mode expansion during inflation

2) Kinematic enhancement $\sim \kappa^3 \, e^N$ due to less redshift received from minimal mode after inflation

$$p_{\rm e} = k_{\rm min}/a_{\rm e} \sim e^{-N} H_{\rm inf}$$

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main contribution to relic abundance to compensate the suppression from small mass

$$p_{\rm e} = k_{\rm min}/a_{\rm e} \sim e^{-N} H_{\rm inf}$$

Relic abundance

- Axion momentum by the end of inflation: $p_\mathrm{e} = k_\mathrm{min}/a_\mathrm{e} \sim e^{-N} H_\mathrm{inf}$

  Compared with the usual case, the axion momentum is suppressed by e^{-N}, so it will become nonrelativistic earlier and the energy density is redshifted less

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$$\langle \rho_\eta(\tau_0) \rangle = \langle \rho_\eta(\tau_{\mathrm{e}}) \rangle \left(\frac{a_{\mathrm{e}}}{a_{\mathrm{NR}}} \right)^4 \left(\frac{a_{\mathrm{NR}}}{a_0} \right)^3 \sim \langle \rho_\eta(\tau_{\mathrm{e}}) \rangle \left(\frac{T_0}{T_{\mathrm{reh}}} \right)^3 \frac{m_\eta}{H_{\inf}}\, e^N$$

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kinematic enhancement due to less redshift

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- Relic abundance:

$$\Omega_\eta = \frac{g_{*0} g_{*{\rm reh}}^{-1/4}}{48\pi^3} \left(\frac{\pi^2}{90} \right)^{3/4} \frac{m_\eta T_0^3 H_{\rm inf}^{3/2}}{M_{\rm Pl}^{7/2} H_0^2} \frac{\mathcal{F}(\kappa)}{\sqrt{\kappa}} e^N e^{N(2\nu-3)}$$

$$\nu \equiv \sqrt{9/4+\kappa}$$

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$e^{N(2\nu-3)}$ comes from enhancement to inflationary fluctuations

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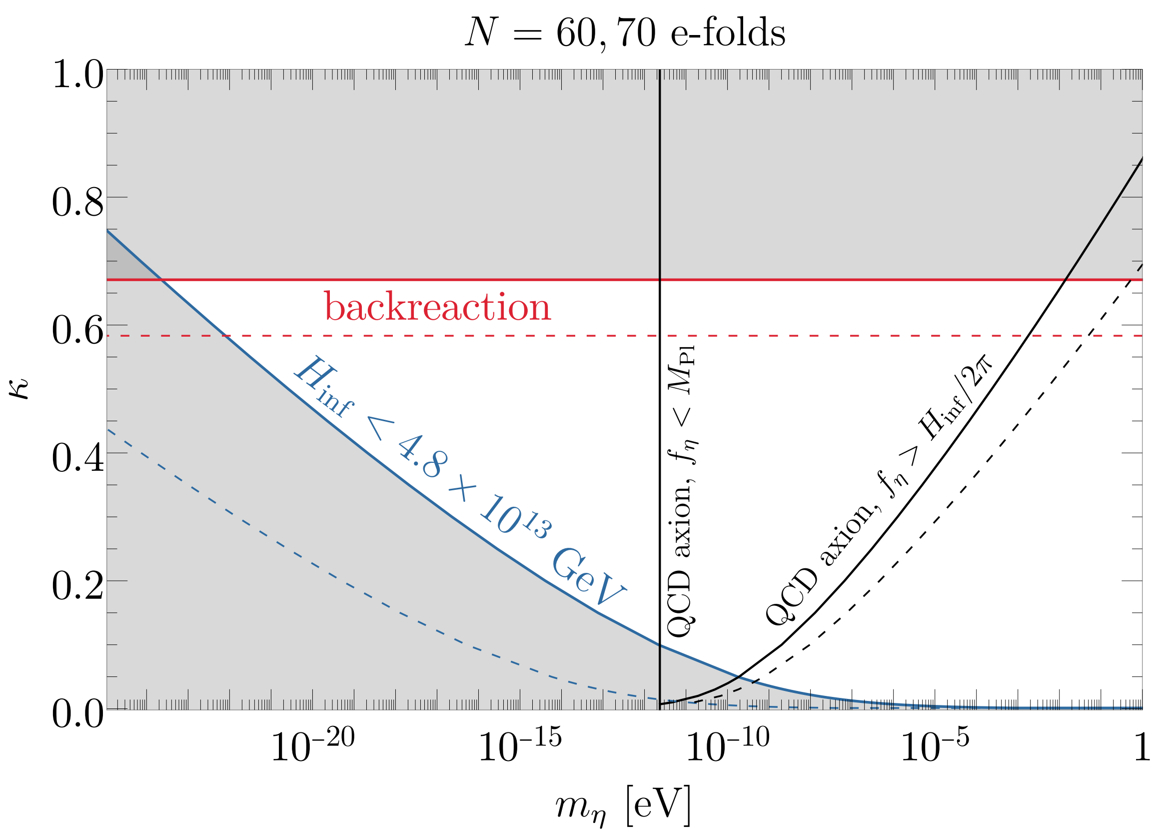
For example, for $N = 60$ e-folds, $\kappa = 0.5$ $\quad \frac{\mathcal{F}(\kappa)}{\sqrt{\kappa}} e^N e^{N(2\nu-3)} = 0.6 \times 10^{32}$ **well compensate the mass suppression!**

Results

Parameter space for effective curvature



For $N = 60$ e-folds:

m_η can reach 10^{-24} eV

QCD axion is further bounded below 10^{-2} eV

relaxed with larger e-folds

$H_{\rm inf} = 2\pi M_{\rm pl}\sqrt{A_{\rm s} r_{\rm T}/8}$

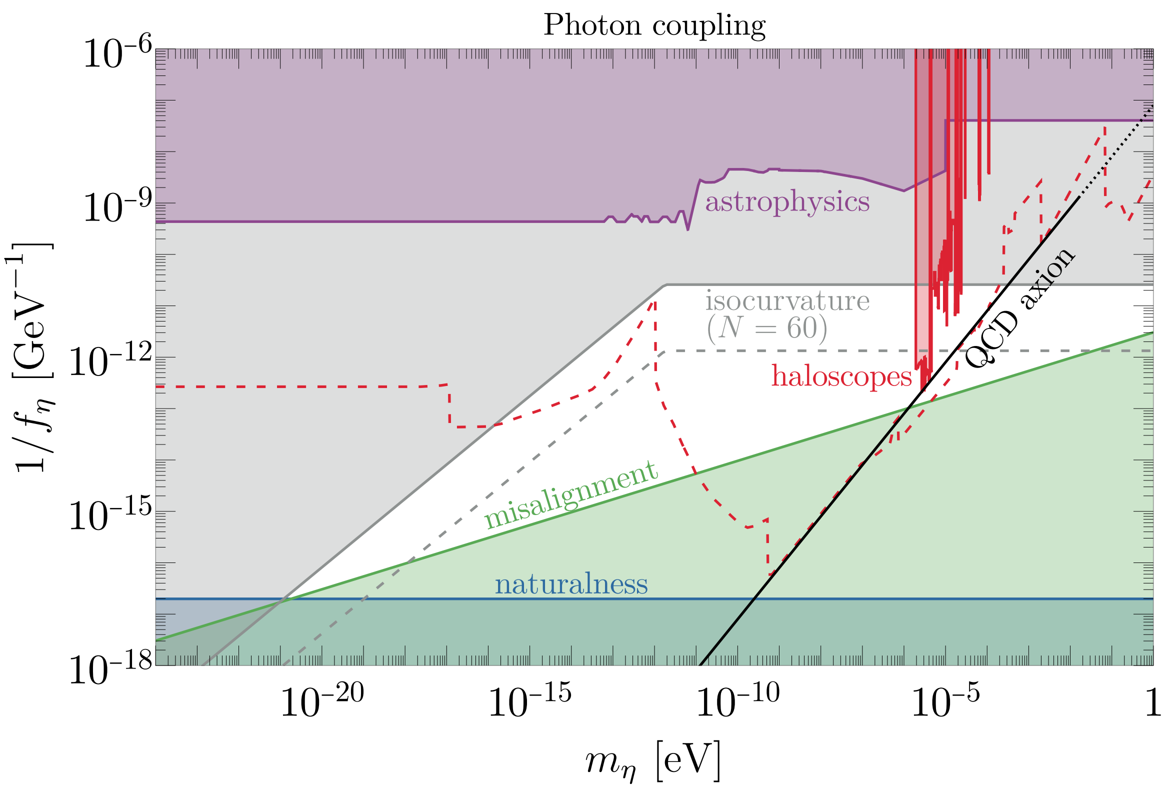
$A_{\rm s} = 2.1 \times 10^{-9} \quad r_{\rm T} < 0.036$

$\Rightarrow H_{\rm inf} < 4.8 \times 10^{13}$ GeV

DM relic abundance does not depend on the breaking scale directly

Axion-photon coupling

generic lower bound from Ly-α forest: $m_\eta > 10^{-21}$eV (not included in figure)



$$\mathcal{L} \supset \frac{\alpha_{\rm EM}}{8\pi f_\eta}\eta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Future haloscopes (red dashed line):

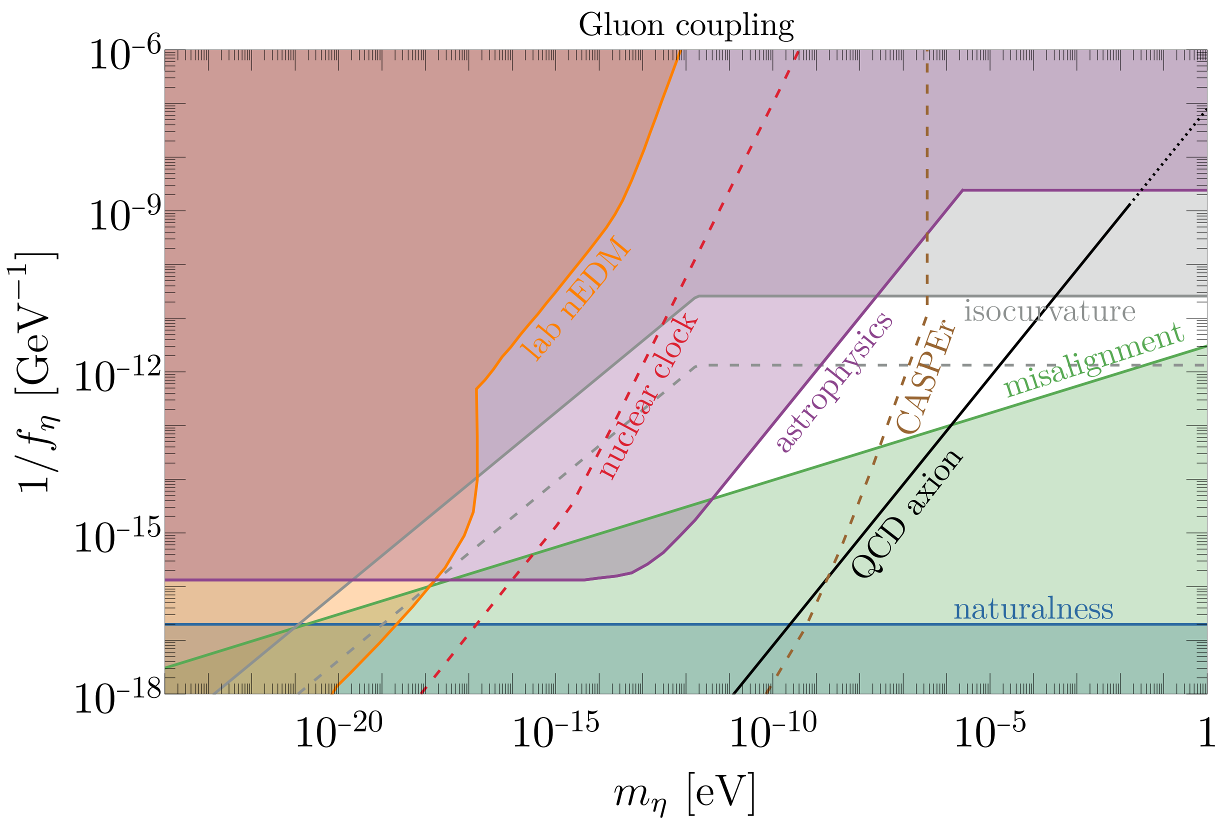
DANCE, SRF, DM-Radio, etc.

Future CMB, 21 cm telescopes (gray dashed line):

CMB-S4, SKA2

Compared with misalignment prediction, our mechanism allows lower axion decay constant and therefore larger couplings to SM particles

Axion-gluon coupling



$$\mathcal{L} \supset \frac{\alpha_s}{8\pi f_\eta} \eta G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

Future nuclear clock (red dashed line):

Thorium-229

Future CMB, 21 cm telescopes (gray dashed line):

CMB-S4, SKA2

CASPEr (brown dashed line)

$\eta G\tilde{G}$ is heavily constrained by experiments for ALPs lighter than 10^{-10} eV

But this operator is not predicted in our mechanism and can simply be turned off

The generic axion-gluon coupling can be induced by gravity

Possible origin of kinetic coupling

A. EFT operators

- Exponential enhancement could be realized by some effective operator:

$$K(\phi) = 1 + \frac{C_6}{M_{\rm Pl}^2}\phi^2 \qquad\qquad \left|C_6\phi^2\right| < M_{\rm Pl}^2$$

Effective curvature: $\kappa \approx M_{\rm Pl}^2\left(2\epsilon_V \frac{K_{\phi\phi}}{K} - 3\frac{K_\phi}{K}\frac{V_\phi}{V}\right)$ $\qquad V(\phi) = m_\phi^2\phi^2/2$

$$\boxed{\kappa \approx -4C_6\left(3-\epsilon_V\right) \approx -12C_6}$$

Wilson coefficient plays the role of effective curvature

$$\kappa > 0 \Leftrightarrow C_6 < 0$$

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Wilson coefficient plays the role of effective curvature

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- Resuming form also works:

$$K(\phi) \sim e^{-\beta\phi/M_{\rm Pl}}$$

$$\boxed{\kappa \approx 3\beta\sqrt{2\epsilon_V}} \qquad \kappa > 0 \Leftrightarrow \beta > 0$$

Such coupling form can come from string-theory compactification

Possible origin of kinetic coupling

B. UV completion

- Noncanonical kinetic term can be realized in the supergravity framework

$$\mathcal{L}_{KE} = (\partial_\mu \phi^*, \partial_\mu T^*) \left(\frac{3}{(T + T^* - |\phi|^2/3)^2} \right) \begin{pmatrix} (T+T^*)/3 & -\phi/3 \\ -\phi^*/3 & 1 \end{pmatrix} \begin{pmatrix} \partial^\mu \phi \\ \partial^\mu T \end{pmatrix},$$

Ellis *et al.*, 84'
Ellis *et al.*, 13'

ϕ: inflaton $\quad T$: modulus

Kinetic coupling is determined by Kähler potential, but one needs to check whether it gives positive κ

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- Radial mode as inflaton

Linde, 91'
Fairbairn, Hogan, Marsh, 14'

$$\chi = \rho\, e^{i\eta/f_\eta}/\sqrt{2}$$

$$|\partial_\mu \chi|^2 = \frac{1}{2}\left[(\partial_\mu \rho)^2 + \frac{\rho^2}{f_\eta^2}(\partial_\mu \eta)^2\right]$$

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2} R \left(1 + \xi \frac{\rho^2}{M_{\rm Pl}^2}\right) - \frac{1}{2} g^{\mu\nu}\partial_\mu \rho \partial_\nu \rho - \frac{1}{2}\frac{\rho^2}{f_\eta^2} g^{\mu\nu}\partial_\mu \eta \partial_\nu \eta - \frac{\lambda}{4}\left(\rho^2 - f_\eta^2\right)^2\right]$$

$$\kappa \approx -4q^4 \left[3\xi^2(6\xi+1)^2 + \left(24\xi^2 + 8\xi + 3\right) q^4 + 2\xi\left(24\xi^2 + 22\xi + 3\right) q^2\right] / \left(6\xi^2 + \xi + q^2\right)^3$$

$$q \equiv M_{\rm Pl}/\rho$$

$\kappa > 0 \quad \Rightarrow \quad \xi$ should satisfy $-1/6 < \xi < 0$

Summary

- Inflationary quantum fluctuations + Quantum Phase Transition
  $\Rightarrow$ sufficient production of axion as ultralight DM

- This new mechanism predicts much larger couplings to SM particles and a wider range of allowed couplings than misalignment mechanism

- Much of the parameter space will be probed by near-future axion experiments

- It covers a large range of DM masses, from sub-eV down to fuzzy DM range

- It works for both QCD axion and ALPs. We expect it can also be applicable to other bosonic ultralight DM scenarios (e.g., dilaton, majoron, dark photon)

Backup slides

Backreaction constraint

- EOM of inflaton:

$$\ddot{\phi} + 3H_{\rm inf}\dot{\phi} + V_\phi + KK_\phi g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta = 0$$

- Axion should not affect inflaton dynamics (single-field inflation):

$$\left|KK_\phi\langle g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta\rangle\right| \ll \left|3H_{\rm inf}\dot{\phi}\right|$$

$$\langle\rho_\eta\rangle \ll 3M_{\rm Pl}^2 H_{\rm inf}^2$$

- Upper bound on the effective curvature κ:

$$\boxed{\kappa\,\mathcal{F}(\kappa)\,e^{N(2\nu-3)} \ll 18\pi/A_{\rm s}}$$

$N = 50 \Rightarrow \kappa < 0.79$

$N = 60 \Rightarrow \kappa < 0.67$

$N = 70 \Rightarrow \kappa < 0.58$

$$\nu \equiv \sqrt{9/4+\kappa}$$

$$A_{\rm s} \equiv H_{\rm inf}^2/\left(8\pi^2\epsilon M_{\rm Pl}^2\right) = 2.1\times10^{-9}$$

$$\mathcal{F}(\kappa) \equiv \frac{2^{2\nu}\left(\kappa/3+3/2-\nu\right)^2\Gamma^2(\nu)}{2\nu-3}\left(\frac{1}{\kappa}\right)^{\nu-3/2}$$

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Relic abundance: ALP

$$\mathcal{F}(\kappa) \equiv \frac{2^{2\nu}\left(\kappa/3+3/2-\nu\right)^2\Gamma^2(\nu)}{2\nu-3}\left(\frac{1}{\kappa}\right)^{\nu-3/2}$$

- For lighter ALP with $m_\eta < p_\mathrm{e}$: $\langle\rho_\eta(\tau_0)\rangle = \langle\rho_\eta(\tau_\mathrm{e})\rangle\left(a_\mathrm{e}/a_\mathrm{NR}\right)^4\left(a_\mathrm{NR}/a_0\right)^3$

$$\Omega_\eta = \frac{g_{*0}g_{*\mathrm{reh}}^{-1/4}}{48\pi^3}\left(\frac{\pi^2}{90}\right)^{3/4}\frac{m_\eta T_0^3 H_\mathrm{inf}^{3/2}}{M_\mathrm{Pl}^{7/2}H_0^2}\frac{\mathcal{F}(\kappa)}{\sqrt{\kappa}}e^N e^{N(2\nu-3)}$$

$$g_{*0} = 2$$

$$g_{*\mathrm{reh}} = 106.75$$

$$\frac{\Omega_\eta}{\Omega_\mathrm{cdm}} = 2.7\times10^{-34}\times\frac{\mathcal{F}(\kappa)}{\sqrt{\kappa}}e^{N(2\nu-2)}\times\left(\frac{m_\eta}{10^{-22}\ \mathrm{eV}}\right)\left(\frac{H_\mathrm{inf}}{10^{13}\ \mathrm{GeV}}\right)^{3/2}$$

- For heavier ALP with $m_\eta > p_\mathrm{e}$: $\langle\rho_\eta(\tau_0)\rangle = \langle\rho_\eta(\tau_\mathrm{e})\rangle\left(a_\mathrm{e}/a_0\right)^3$

$$\Omega_\eta = \frac{g_{*0}g_{*\mathrm{reh}}^{-1/4}}{48\pi^3}\left(\frac{\pi^2}{90}\right)^{3/4}\frac{T_0^3 H_\mathrm{inf}^{5/2}}{M_\mathrm{Pl}^{7/2}H_0^2}\mathcal{F}(\kappa)\,e^{N(2\nu-3)}$$

$$\frac{\Omega_\eta}{\Omega_\mathrm{cdm}} = 2.6\,\mathcal{F}(\kappa)e^{N(2\nu-3)}\left(\frac{H_\mathrm{inf}}{10^9\ \mathrm{GeV}}\right)^{5/2}$$

Relic abundance: QCD axion

- Assuming $T_{\rm reh} \gg \Lambda_{\rm QCD}$, axion is relativistic when produced, and becomes nonrelativistic when $p_{\rm e} < m_\eta$

$$m_\eta(T) = \beta m_\eta \left(\frac{\Lambda_{\rm QCD}}{T}\right)^\gamma \qquad \beta \sim 10^{-2} \quad \gamma \approx 4$$

- Relic abundance of QCD axion:

$$\frac{\Omega_\eta}{\Omega_{\rm cdm}} = 10^{-3}\mathcal{F}(\kappa)e^{N(2\nu-3)}\left(\frac{H_{\rm inf}}{10^{13}\ {\rm GeV}}\right)^2\left(\frac{T_{\rm NR}}{10^2\ {\rm GeV}}\right)$$

$$T_{\rm NR} = \left(\frac{\beta}{\sqrt{\kappa}}e^N\frac{m_\eta}{H_{\rm inf}}T_{\rm reh}\Lambda_{\rm QCD}^4\right)^{\frac{1}{5}} \sim 100\ {\rm GeV}\left(\frac{m_\eta}{10^{-6}\ {\rm eV}}\right)^{\frac{1}{5}}\left(\frac{H_{\rm inf}}{10^{10}\ {\rm GeV}}\right)^{-\frac{1}{10}}.$$

- Upper bound on QCD axion mass:

PQ symmetry broken during inflation: $f_\eta > H_{\rm inf}/2\pi \Rightarrow m_\eta < 2\pi\Lambda_{\rm QCD}^2/H_{\rm inf}$

For $N = 60$ e-folds, we have $m_\eta < 10^{-2}$ eV $\qquad f_\eta m_\eta \approx \Lambda_{\rm QCD}^2$

It can be further relaxed with a larger number of e-folds

Isocurvature bound

- The isocurvature (entropy) mode measures the deviation from the adiabatic mode of single-field inflation, parametrized by:

$A_{\rm s}$ = scalar amplitude

$A_{\rm iso}$ = isocurvature perturbation

$$\beta_{\rm iso} \equiv A_{\rm iso}/\left(A_{\rm s}+A_{\rm iso}\right) \approx A_{\rm iso}/A_{\rm s}$$

$$\left\langle \delta\eta^2(\tau,k)\right\rangle = \frac{k^3}{2\pi^2}\int {\rm d}^3x\, e^{-{\rm i}\vec{k}\cdot\vec{x}}\eta\left(\tau,\vec{x}\right)\eta\left(\tau,0\right)$$

$$\left\langle \delta\eta^2(\tau,k)\right\rangle = \frac{2^{2\nu}}{2\pi}\Gamma^2(\nu)\left(\frac{H_{\rm inf}}{2\pi}\right)^2\left(\frac{1}{-k\tau}\right)^{2\nu-3}$$

- For our mechanism, isocurvature perturbation is dominated by $k_{\rm min}$

$$\left\langle \delta\eta^2(\tau_*,k_{\rm min})\right\rangle = \frac{2^{2\nu}}{2\pi}\Gamma^2(\nu)\left(\frac{H_{\rm inf}}{2\pi}\right)^2\left(\frac{k_*}{k_{\rm min}}\right)^{2\nu-3} \qquad \tau_* = -1/k_*$$

$$\begin{aligned} A_{\rm iso}\left(k_*,k_{\rm min}\right) &= 4\left\langle \delta\eta^2(\tau_*,k_{\rm min})\right\rangle/\eta^2 \\ &= \frac{2^{2\nu}}{2\pi}\Gamma^2(\nu)\left(\frac{k_*}{k_{\rm min}}\right)^{2\nu-3}\left(\frac{H_{\rm inf}}{\pi f_\eta\theta_{\rm i}}\right)^2 \end{aligned}$$

Isocurvature bound

- It is more intuitive to rewrite the ratio $k_*/k_{\rm min}$ in terms of e-folds:

$$N \equiv \log\left(\frac{a_{\rm e}}{a_{\rm i}}\right)$$

total number of e-folds

$$N_* \equiv \log\left(\frac{a_{\rm e}}{a_*}\right)$$

number of e-folds between the time when k_* exits the horizon until the end of inflation

$$A_{\rm iso}\left(k_*, k_{\rm min}\right) = \frac{2^{2\nu}}{2\pi}\Gamma^2(\nu)\left(\frac{1}{\kappa}\right)^{\nu-3/2} e^{(N-N_*)(2\nu-3)}\left(\frac{H_{\rm inf}}{\pi f_\eta \theta_{\rm i}}\right)^2 \equiv \mathcal{G}\left(\kappa, k_*\right)\left(\frac{H_{\rm inf}}{\pi f_\eta \theta_{\rm i}}\right)^2$$

$$\beta_{\rm iso}(k_*) = \mathcal{G}\left(\kappa, k_*\right)\frac{\Omega_\eta}{\Omega_{\rm cdm}}\frac{1}{A_{\rm s}}\left(\frac{H_{\rm inf}}{\pi f_\eta \theta_{\rm i}}\right)^2$$

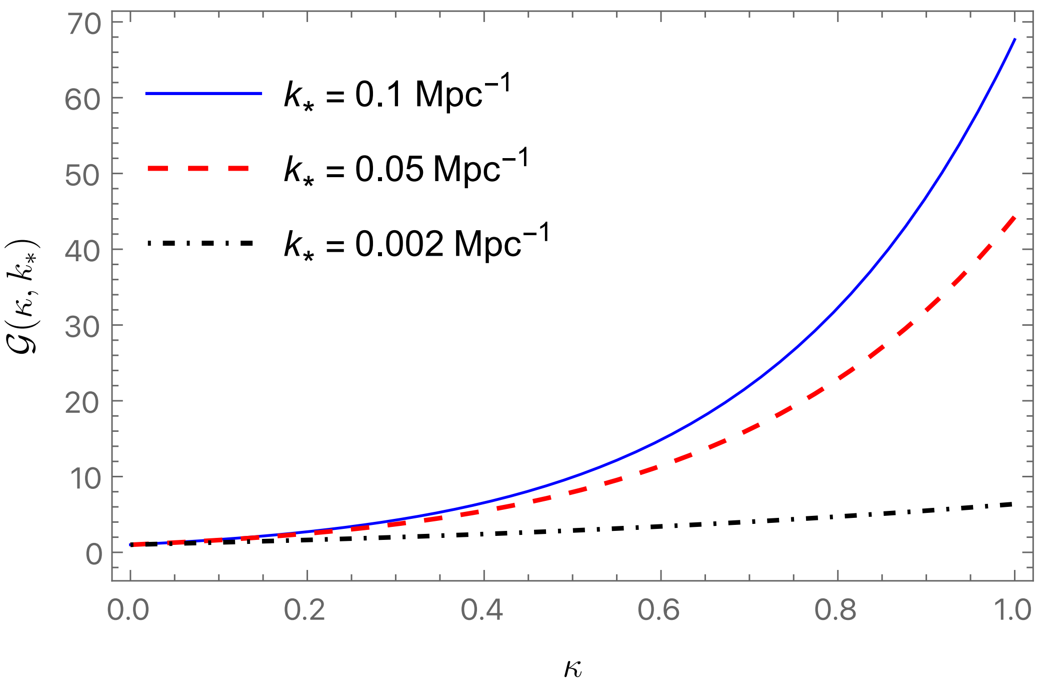
- Upper bounds on $\beta_{\rm iso}$ from Planck

upper bound on $\beta_{\rm iso}$	pivot scale $k_*/{\rm Mpc}^{-1}$	effective e-folds $N - N_*$
0.035	0.002	2.2
0.038	0.05	5.4
0.039	0.1	6.1

Isocurvature bound

- Compared with the usual pre-inflationary scenario, the isocurvature perturbation in our case is enhanced by $\mathcal{G}$

$$\mathcal{G}\left(\kappa, k_*\right) = \frac{2^{2\nu}}{2\pi}\Gamma^2(\nu)\left(\frac{1}{\kappa}\right)^{\nu-3/2} e^{(N-N_*)(2\nu-3)}\left(\frac{H_{\rm inf}}{\pi f_\eta \theta_{\rm i}}\right)^2$$



Planck measurement gives

$$\frac{f_\eta \theta_{\rm i}}{H_{\rm inf}} > 3.5 \times 10^4 \sqrt{\mathcal{G}}$$

As long as the backreaction bound is satisfied, $\kappa < O(1)$, we have $\sqrt{G} < O(10)$, so only a mild enhancement

Isocurvature bound

- Some numerical values of enhancement with different values of κ

κ	enhancement to ultralight DM relic abundance $e^{N(2\nu-2)}$	enhancement to inflationary quantum fluctuation $e^{N(2\nu-3)}$	enhancement to axion isocurvature perturbation $e^{(N-N_*)(2\nu-3)}$
0.1	6.0×10^{27}	52	1.5
0.2	2.9×10^{29}	2.5×10^{3}	2.2
0.5	2.0×10^{34}	1.8×10^{8}	6.9
1.0	6.9×10^{41}	6.0×10^{15}	40

$N = 60$

$N - N_* = 6.1$

We can realize a large enhancement to the relic abundance with only O(1) enhancement to the isocurvature

Axion EOM and solution during inflation

- Axion EOM reduces to:

$$f_k'' + \left(k^2 - \frac{2+\kappa}{\tau^2}\right) f_k = 0$$

- Bunch-Davis initial condition:

$$\lim_{k\tau \to -\infty} f_k(\tau) = \frac{1}{\sqrt{2k}} e^{-\mathrm{i}k\tau}$$

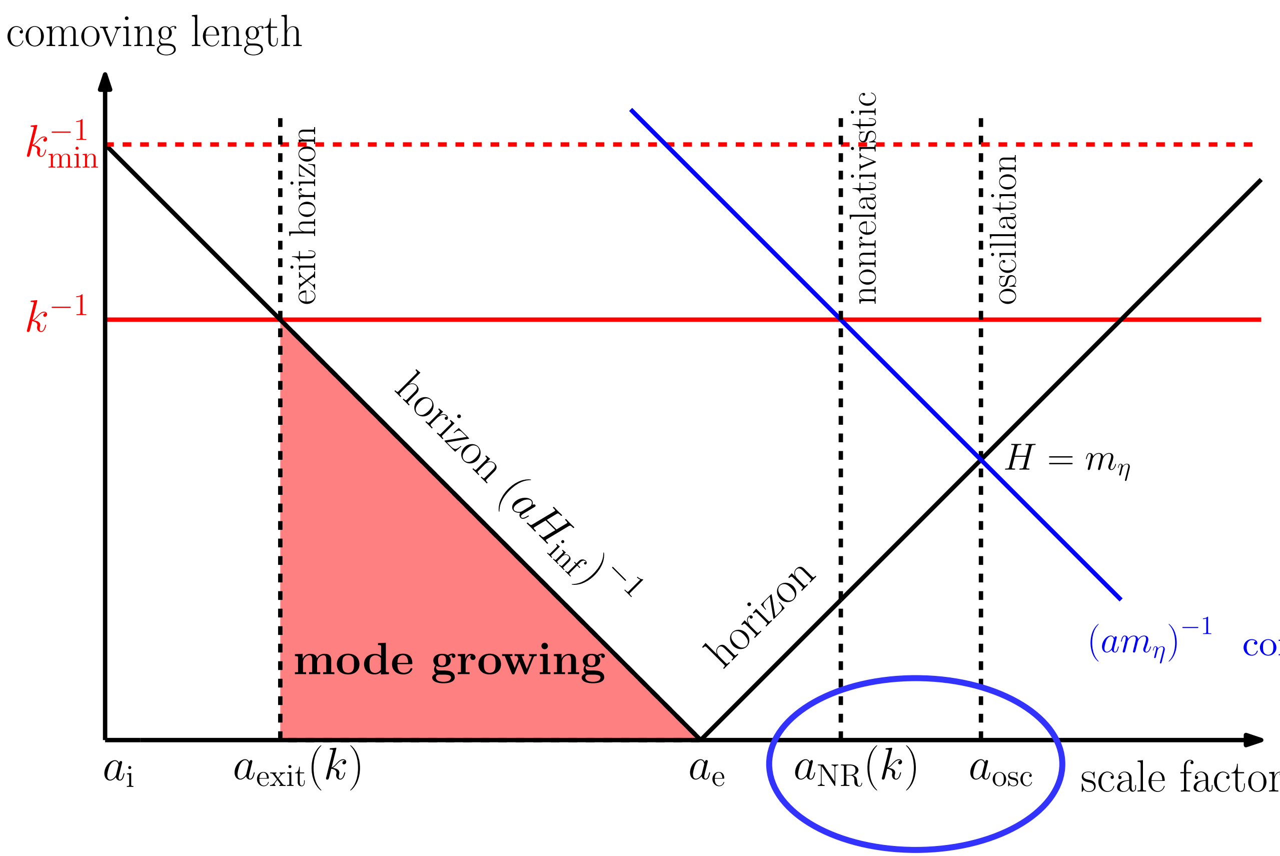
- Solution of axion field during inflation:

$$\boxed{f_k(\tau) = \frac{\sqrt{\pi}}{2}\sqrt{-\tau}H_\nu^{(1)}(-k\tau)}$$

$\nu \equiv \sqrt{9/4+\kappa}$

small κ: $\nu \approx 3/2 + \kappa/3$

Axion evolution after inflation



comoving Compton wavelength

$$a_{\rm NR}(k) = k/m_\eta$$

$$H(a_{\rm osc}) \sim m_\eta$$

$a > a_{\rm NR}$: axion becomes nonrelativistic

$a > a_{\rm osc}$: axion starts coherent oscillation

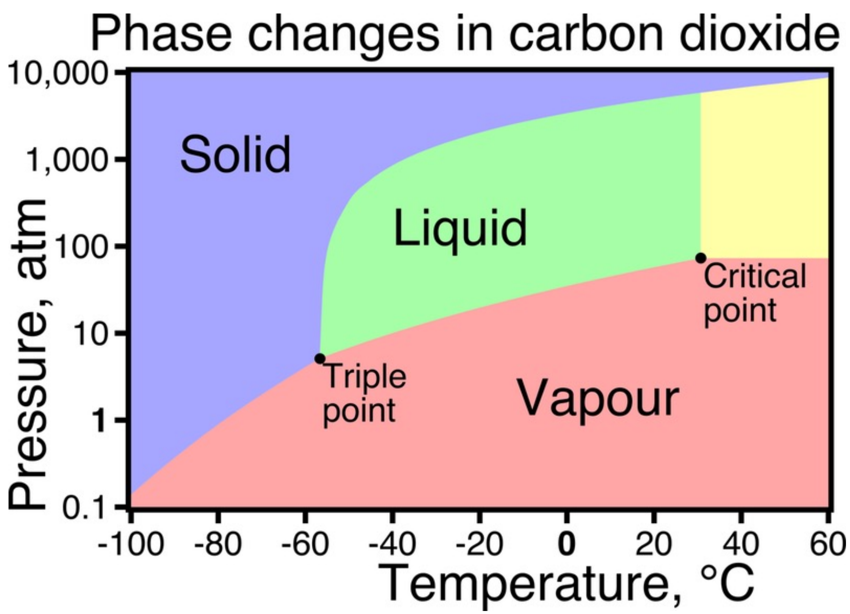
for the minimal mode:

$$k_{\rm min} \propto e^{-N} H_{\rm inf}$$

$$a_{\rm NR}(k_{\rm min}) \ll a_{\rm osc}$$

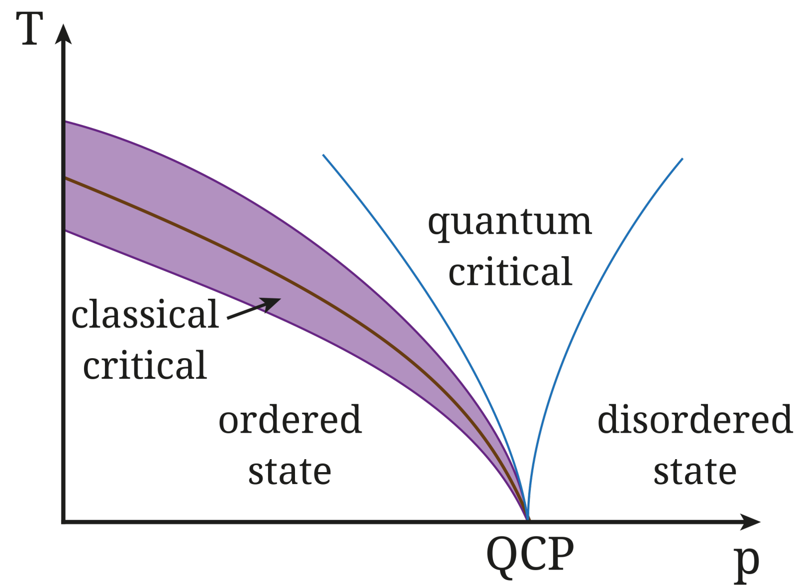
turn NR much earlier than oscillation

Classical & Quantum phase transition



Classical Phase Transition (CPT)

driven by **thermal fluctuations** $k_B T$



Quantum Phase Transition (QPT)
driven by **quantum fluctuations** $\hbar\omega$
(zero temperature)

Analogy to condensed matter system

- In some condensed matter systems, scale invariance is restored when the parameter approaches the critical point

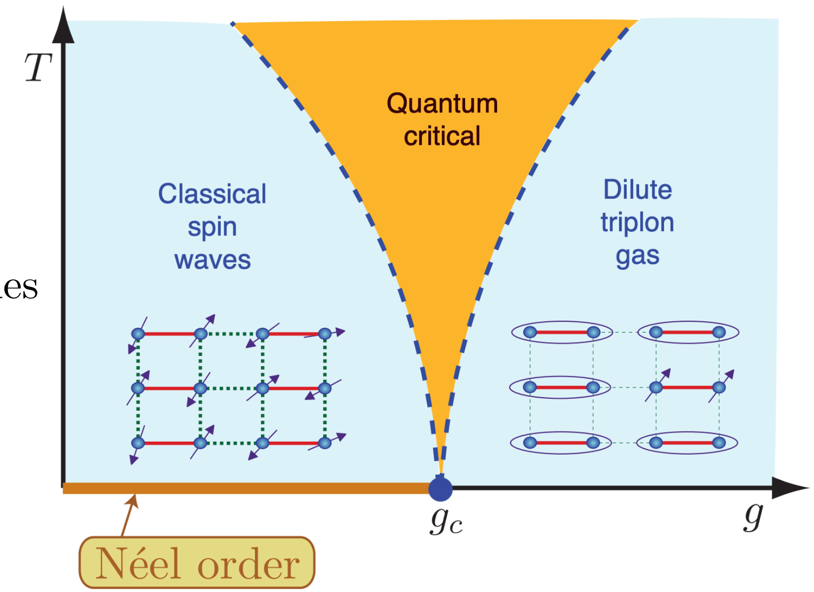
$$\langle \phi(x)\phi(0)\rangle \sim e^{-|x|/\xi} \quad \textbf{correlation function}$$

$$\xi \sim (g-g_c)^{-\alpha} \quad \textbf{correlation length}$$

as $g \to g_c$, $\xi \to \infty$, $\boxed{\langle \phi(x)\phi(0)\rangle \sim 1/|x|^{d-2+\gamma}}$

2nd order QPT happens, the theory becomes scale invariant

Sachdev & Keimer, 1102.4628



- In our axion case, QPT is modulated by the effective curvature κ

$$\boxed{P_k \sim \langle f_k f_k^*\rangle \sim (H_{\rm inf}/2\pi)^2\,(1/x)^{2\kappa/3}} \qquad \langle \rho(\tau_{\rm e})\rangle \propto H_{\rm inf}^4 \kappa^3 e^{2\kappa N/3}$$

$1/x = aH_{\rm inf}/k$

$\kappa \to 0$, $P_k \sim (H_{\rm inf}/2\pi)^2$, scale-invariant spectrum

$\kappa > 0$, exponential enhancement to axion abundance