ETH zürich

Hadronic Jets:
flavour and substructure

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COMETA WG3
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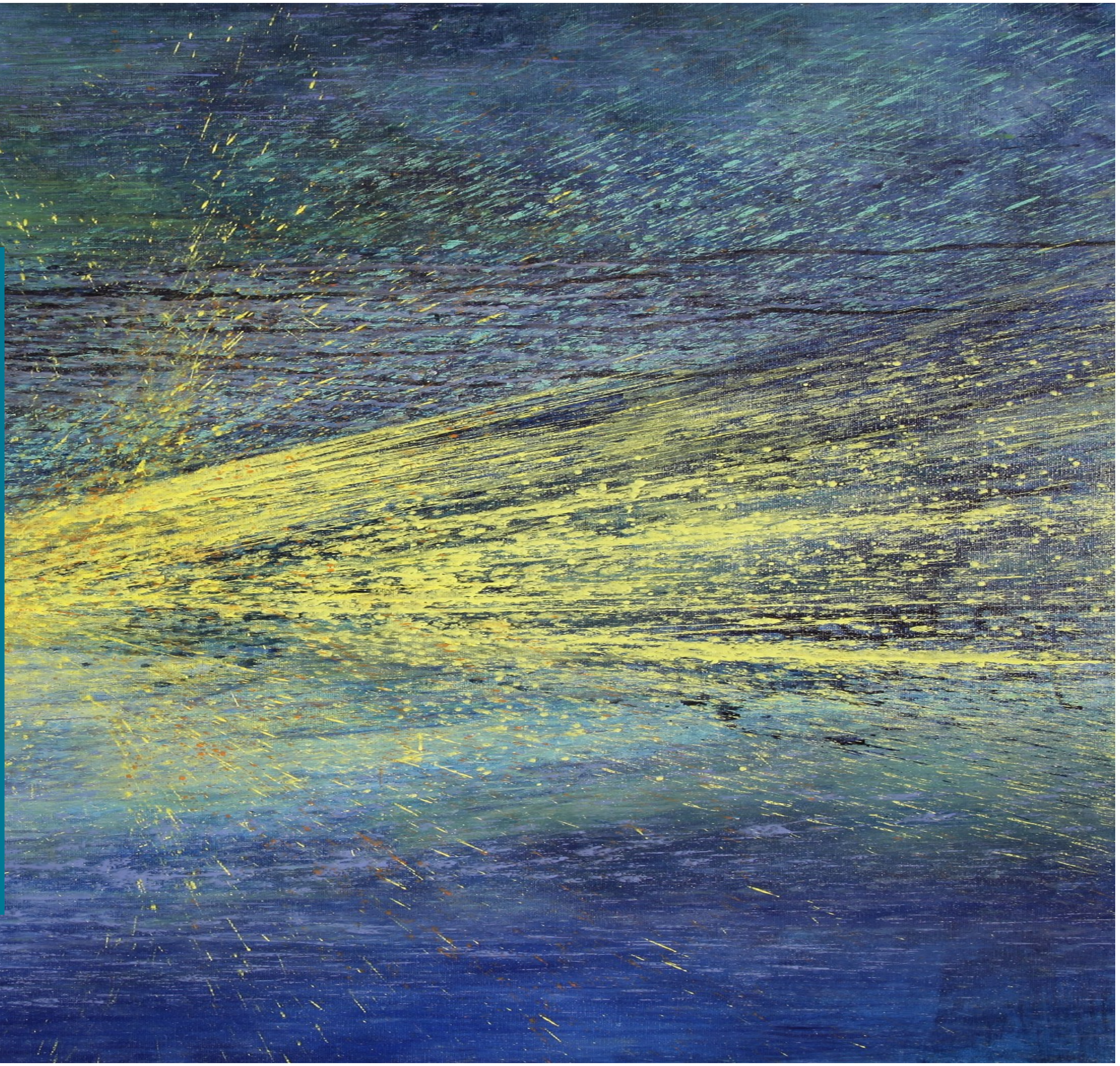
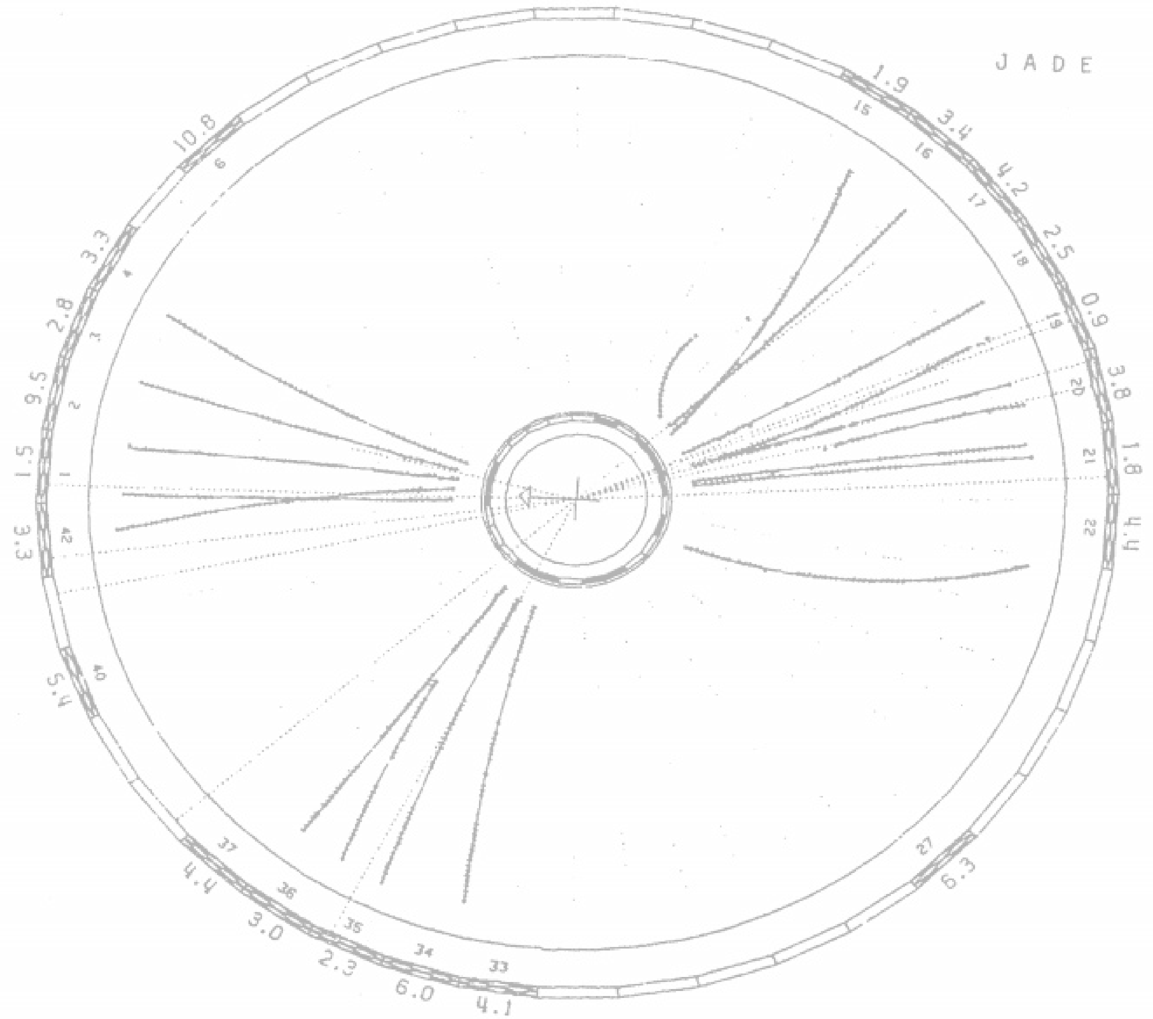


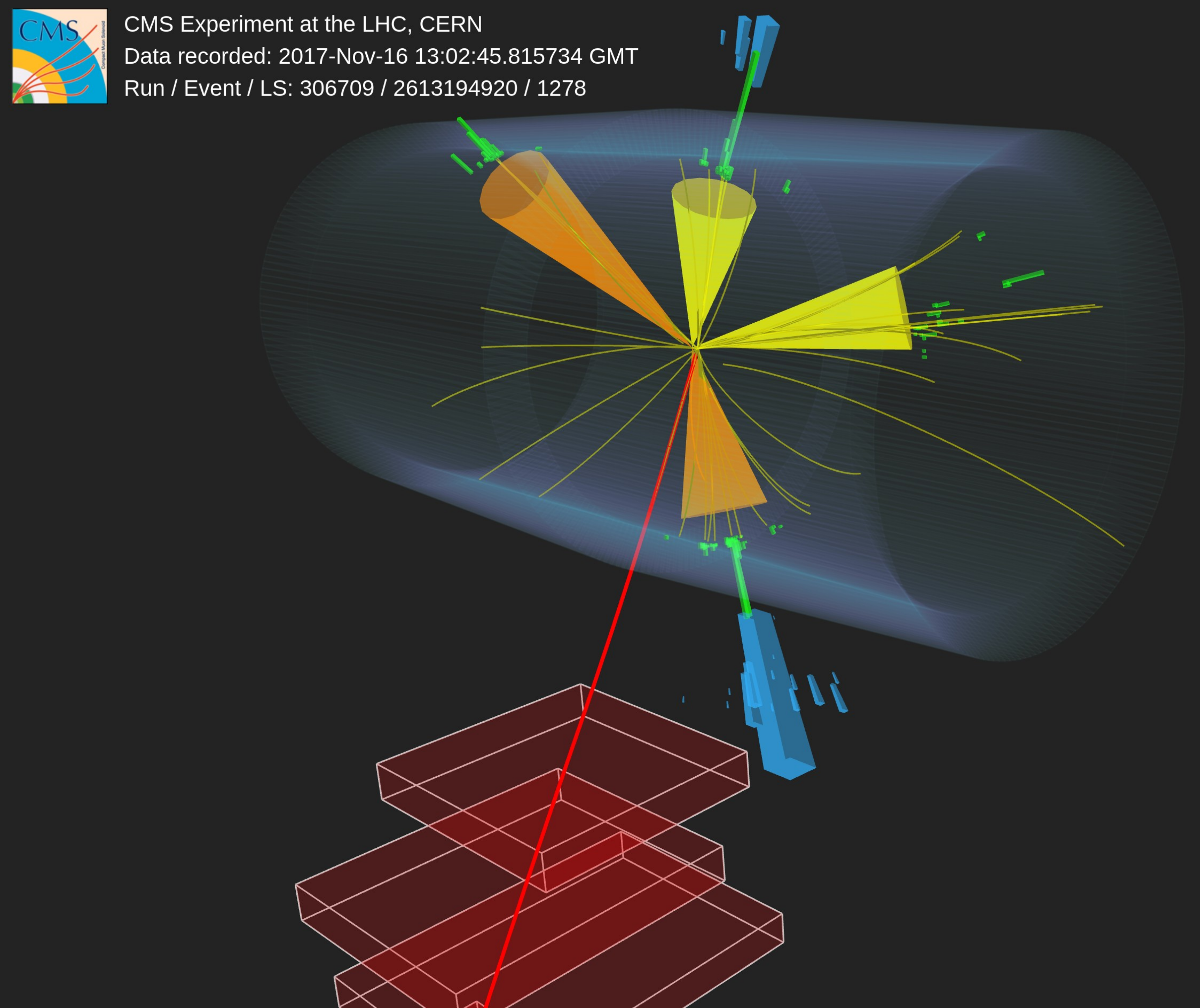
Outline



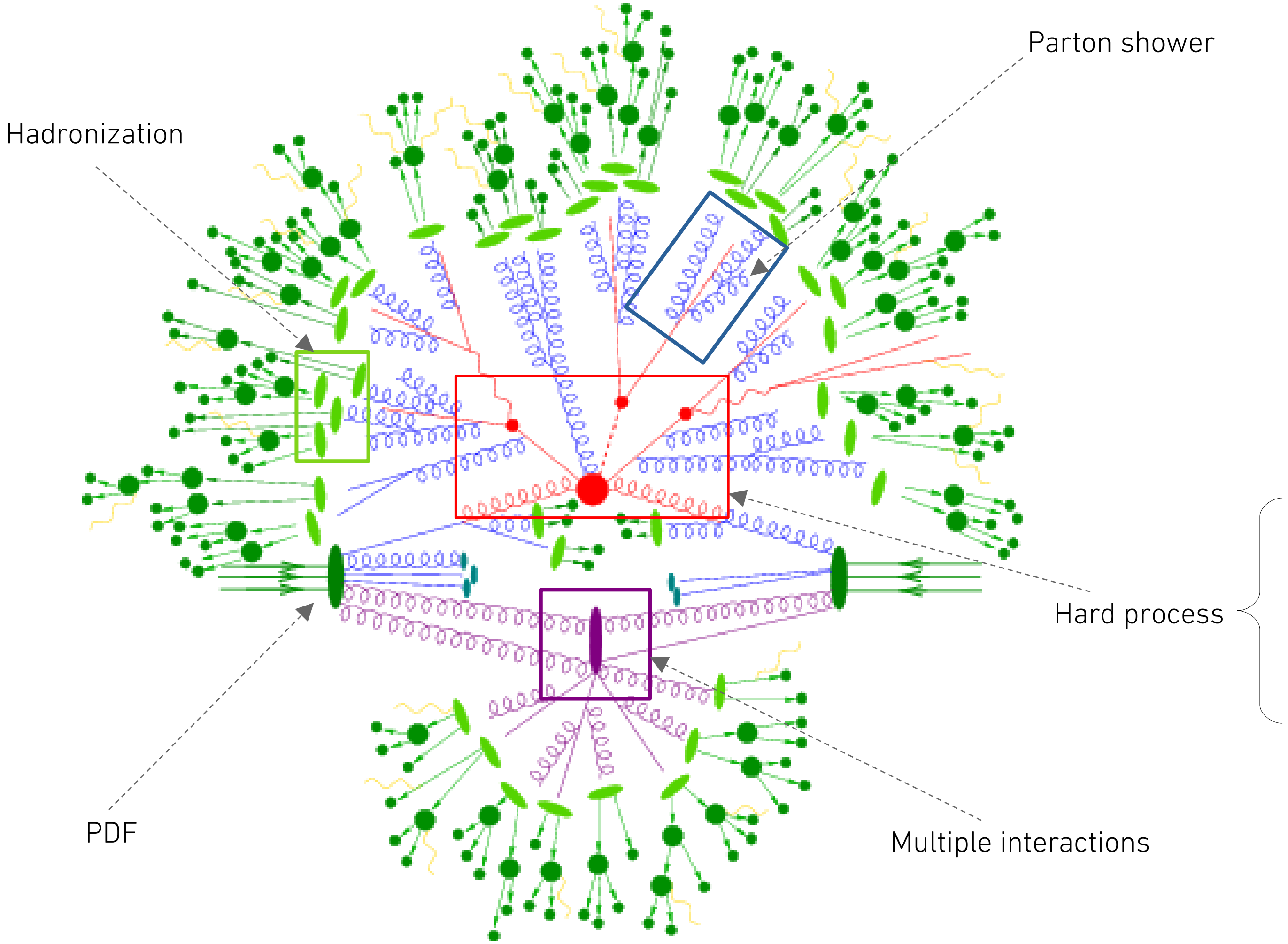
I. Introduction:
- IRC safety, clustering algorithms, grooming

II. Substructure: jet angularities
- fixed order and resummation, lund plane

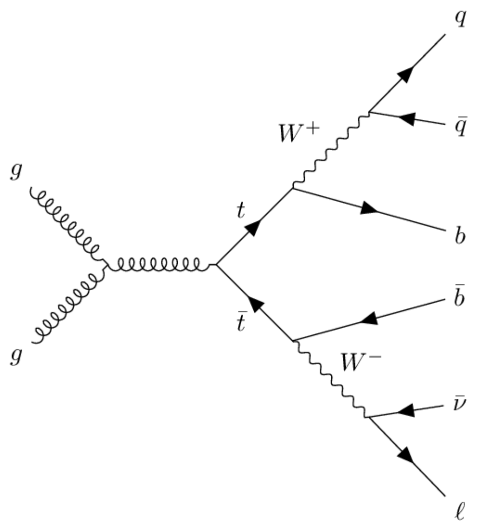
III. Jet flavour
- flavour problem and solutions, measurements for massive quarks


CMS
CMS Experiment at the LHC, CERN
Data recorded: 2017-Nov-16 13:02:45.815734 GMT
Run / Event / LS: 306709 / 2613194920 / 1278

Structure of LHC events

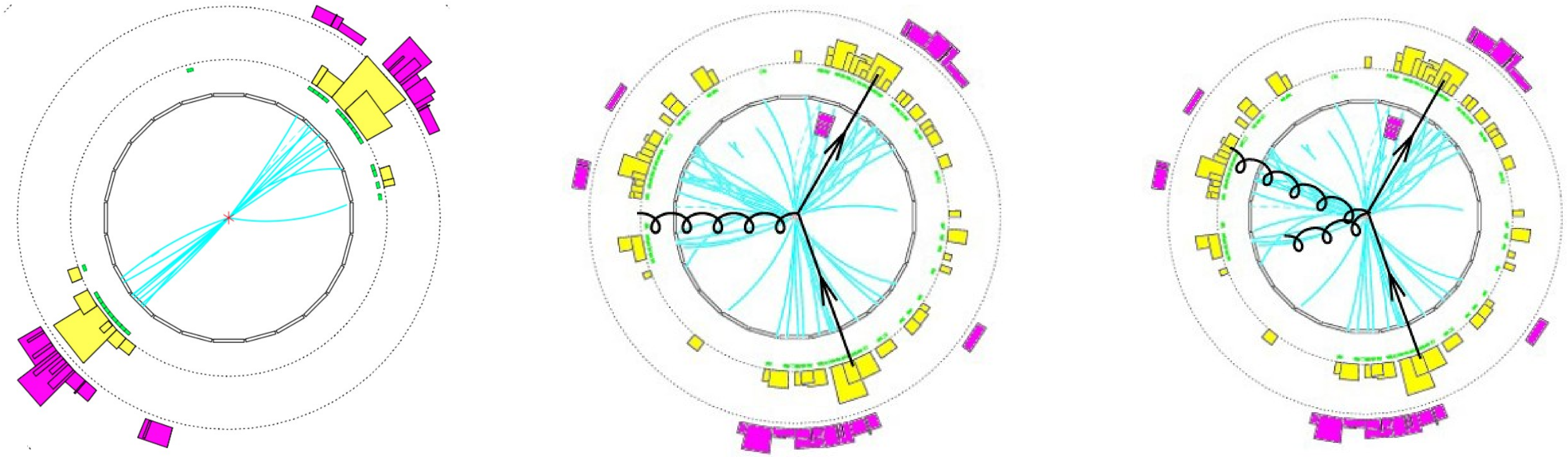


e. g.



Gleisberg et al. (2009)

Jets are probe of the underlying process



But we need to define what we call “jet”.

- Cone algorithms
- Sequential recombination algorithms

Gen-k_t recombination algorithms

- Take the particles in the events as our initial list of objects.
- From this list build the *inter-particle distance* as

$$d_{ij} = \min\left(p_{T,i}^{2p}, p_{T,j}^{2p}\right) \Delta_{ij}^2$$

where we introduced

$$\Delta_{ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i + \eta_j)^2}$$

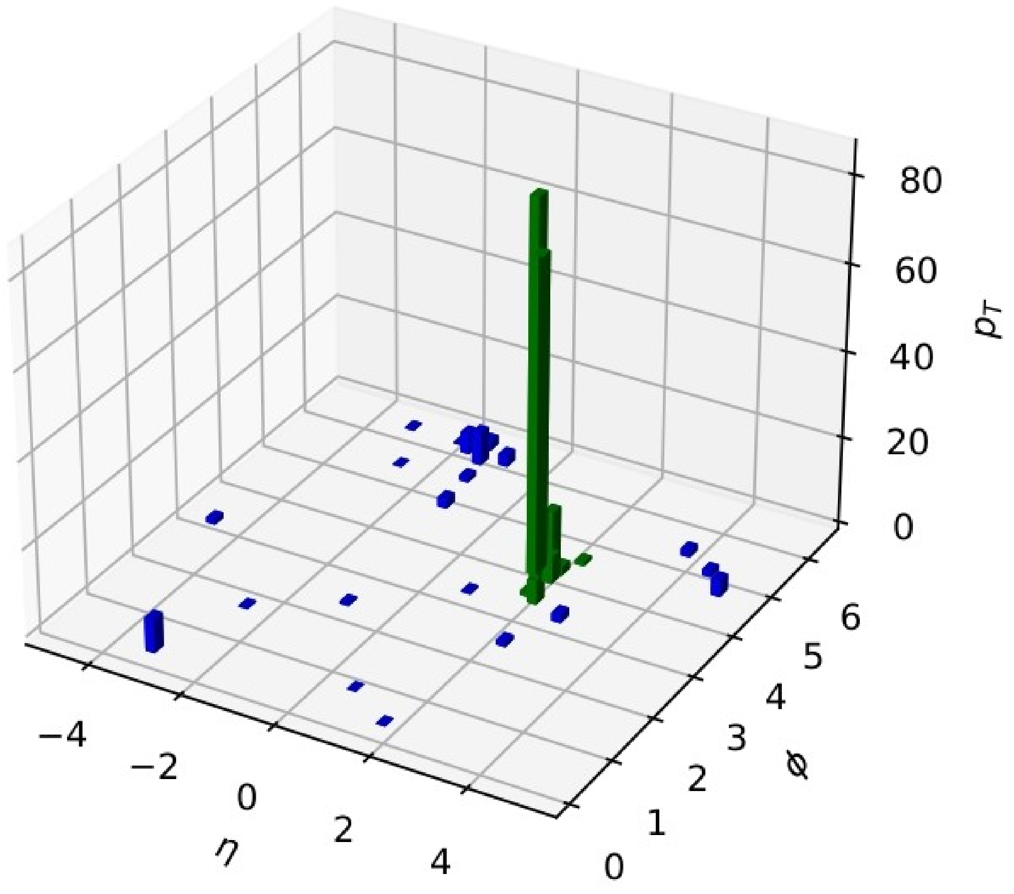
and the *beam distance* as

$$d_{B,i} = p_{T,i}^{2p} R^2$$

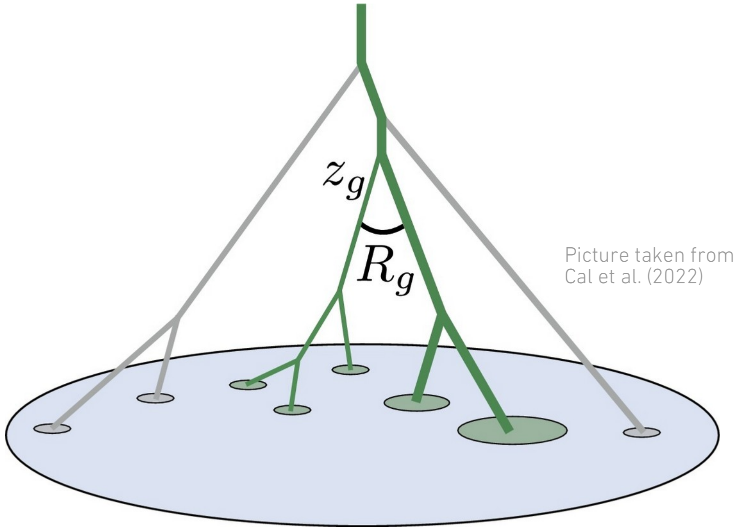
with R the jet radius.

- Iteratively find the smallest among all the two distances:
  - If $d_{ij} < d_{B,i}$ then remove i and j and recombine them into a new object k which is added to the new list.
  - If $d_{B,i} < d_{ij}$ then it is called a *jet* and removed from the list.

while(! list is empty)



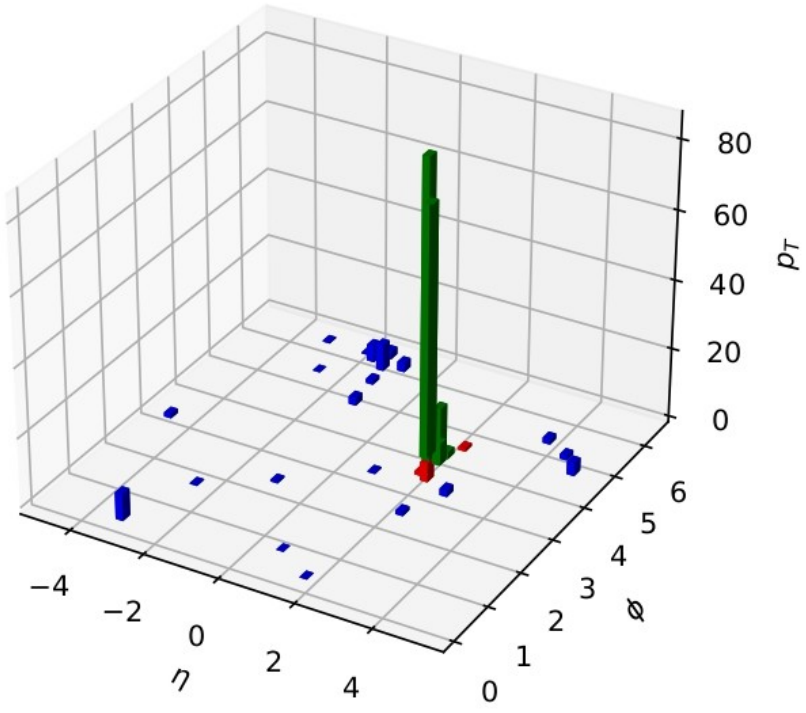
Grooming: Soft Drop



Picture taken from
Cal et al. (2022)

Larkoski et al. (2014)

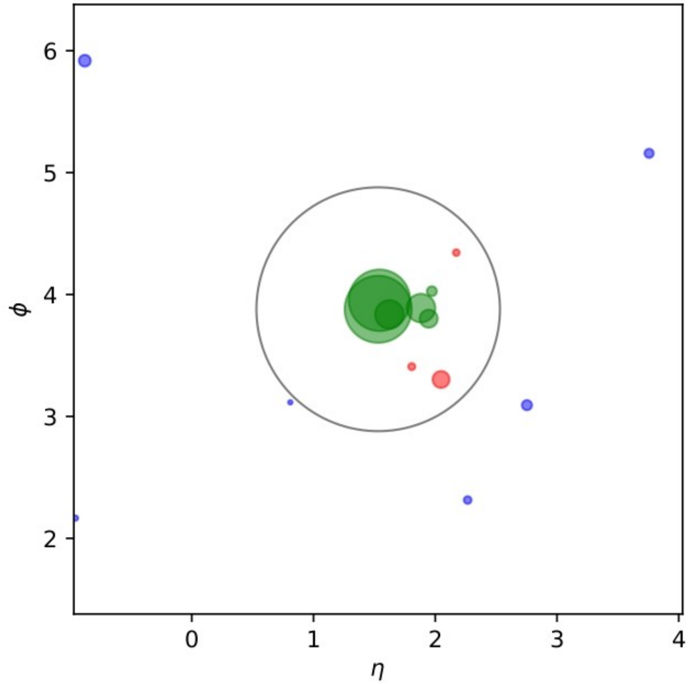
$$\frac{\min(p_{T,i}, p_{T,j})}{p_{T,i} + p_{T,j}} > z_{\mathrm{cut}} \left(\frac{\Delta_{ij}}{R}\right)^{\beta}$$



1. Break the jet j into two subjets by undoing the last stage of C/A clustering and label them as j1 and j2.
2. If j1 and j2 pass the SD condition then deem j to be the final soft-drop jet.
3. Else: redefine

$$j = \max_{p_T}[j_1, j_2]$$

while(! SD)



IRC safety

We start considering the e+e- → qqg process

$$\frac{d^2\sigma}{dx_q dx_{\bar{q}}} = \sigma_B \frac{\alpha_S}{2\pi} C_F \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$$, where $$x_i = \frac{2E_i}{\sqrt{s}}$$

The differential cross section diverges in the soft and/or collinear limit. Since

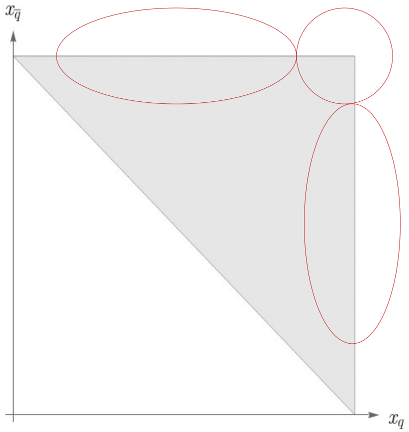
$$1 - x_{\bar{q}} = \frac{1}{2} x_q x_g (1 - \cos\theta_{qg})$$

$$1 - x_q = \frac{1}{2} x_{\bar{q}} x_g (1 - \cos\theta_{\bar{q}g})$$

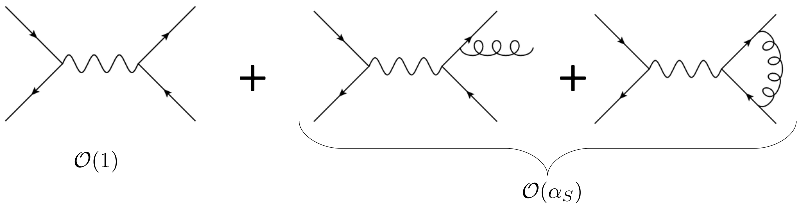
divergences correspond to

$$\theta_{qg} \quad \text{or} \quad \theta_{\bar{q}g} \to 0$$

$$E_g \to 0$$



Computing e+e-→ qq @ NLO we can see the cancellation of IRC singularity between real and virtual contributions



and we can handle the divergences analytically using, for example, dimreg. We obtain

$$\sigma^{real} = \sigma_{q\bar{q}} \frac{C_F}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \left(\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} - \pi^2 + \frac{19}{2} + \mathcal{O}(\varepsilon)\right)$$

$$\sigma^{virtual} = \sigma_{q\bar{q}} \frac{C_F}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \pi^2 - 8 + \mathcal{O}(\varepsilon)\right)$$

We might want to compute other observables than total cross sections. We can do introducing a corresponding measurement function

$$\sigma = \int d\Phi_2 |\mathcal{M}_{Born} + \mathcal{M}_{virt.}|^2 J_r(k_1, k_2) + \int d\Phi_3 |\mathcal{M}_{real}|^2 J_r(k_1, k_2, k_3)$$

$$J_r(k_1, k_2, k_3) \stackrel{k_3 \to 0}{\longrightarrow} J_r(k_1, k_2)$$

which has not to spoil the IRC cancellation

⇨**IRC safety**

Jet angularities

ANGULARITY

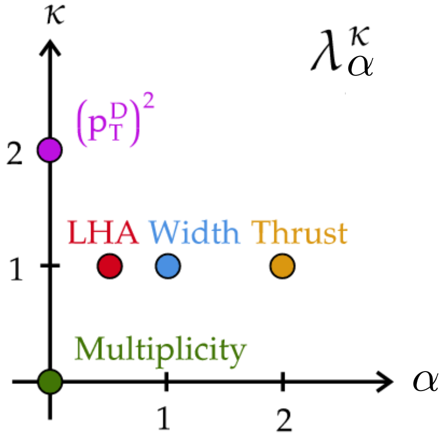
$$\lambda_\alpha^\kappa = \sum_{j\in\text{Jet}} \left(\frac{p_{T,j}}{\sum_{j\in\text{Jet}} p_{T,j}}\right)^\kappa \left(\frac{\Delta_j}{R}\right)^\alpha \simeq \sum_{j\in\text{Jet}} z_j^\kappa \theta_j^\alpha$$

In the soft limit, the contribution O(as) to the cumulant distribution is given by

$$\alpha_S \Sigma^{(1)}(v) = \frac{2\alpha_S C_F}{\pi}\int_{-1}^{+1} d\cos\theta \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^Q \frac{\omega}{\omega}\frac{1}{(1-\cos\theta)(1+\cos\theta)} \times\left[\Theta_{\text{in-jet}}\Theta\left(\frac{2\omega}{Q}(1-\cos\theta)^{\alpha/2} < v\right) + \Theta_{\text{out-jet}} - 1\right]$$

$$= -\frac{\alpha_S C_F}{2\pi}\frac{\alpha}{2}\left[\log^2\left(2\left(\frac{2}{v}\right)^{2/\alpha}\tan^2\frac{R}{2}\right) - \log^2\left(\cos^2\frac{R}{2}\right) - \left(\frac{\alpha^2-4}{2\alpha}\right)^2\log^2\frac{2}{v} - 2\sin^2\left(\frac{R}{2}\right)\right] + \mathcal{O}(v)$$

$$\approx -\frac{\alpha_S C_F}{2\pi}\log^2\left(\frac{R^2}{v}\right)$$ in the small R limit and with α = 2



In numbers: $0.118L^2 \sim 1$ ⇒ $v \sim 0.035$ for $R = 0.8$

We need to resum these logs, i.e.

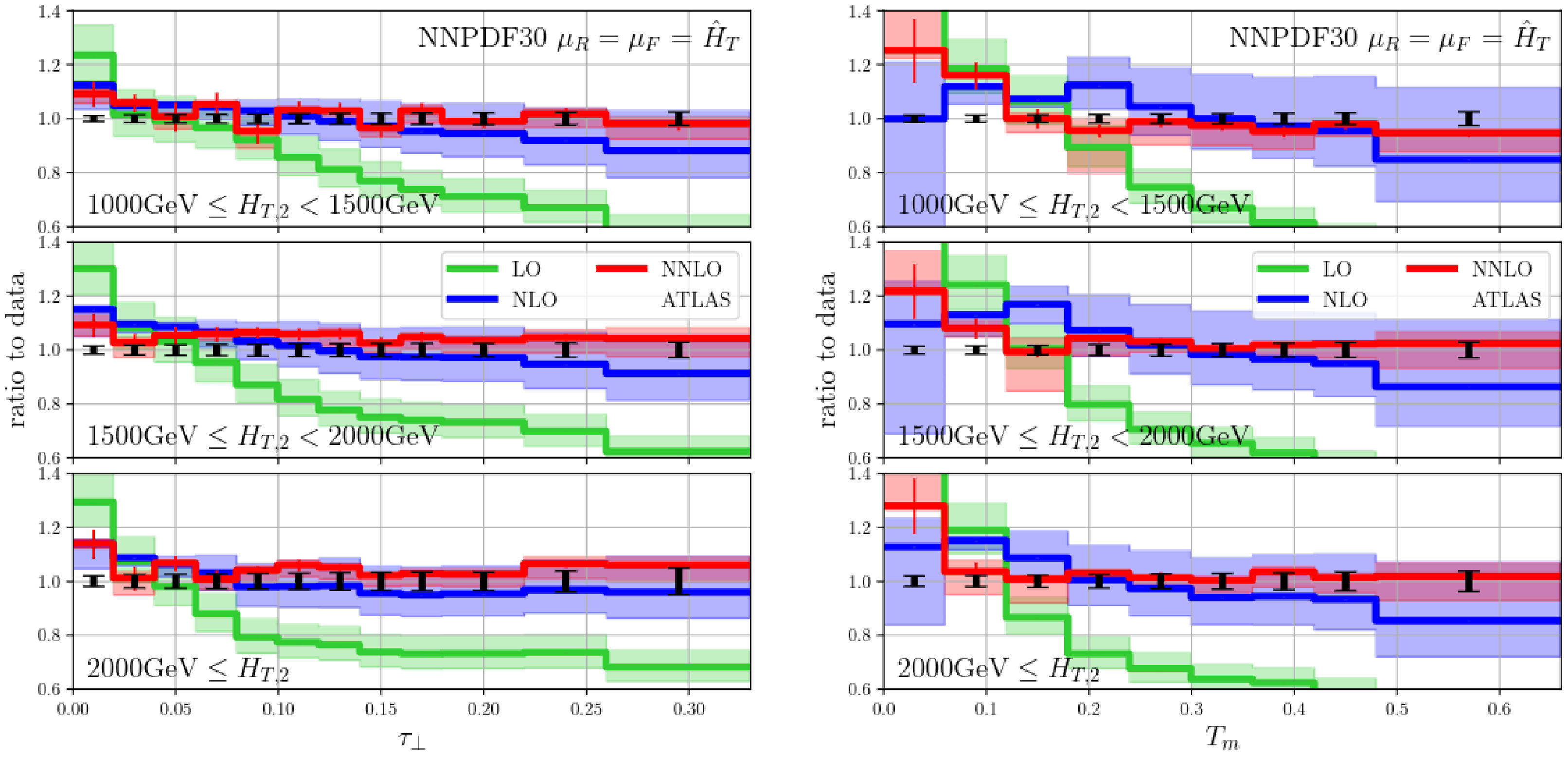
$$\Sigma(v) = 1 + \alpha_S\left(c_{12}L^2 + c_{11}L + \cdots\right) + \alpha_S^2\left(c_{24}L^4 + c_{23}L^3 + \cdots\right) + \mathcal{O}(\alpha_S^n L^{2n})$$

⇒

$$\Sigma(v) \propto \left(1 + C(\alpha_S)\right)\exp\left[Lg_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) + \cdots\right]$$

NNLO event shapes

Taken from Alvarez et al . (2023)



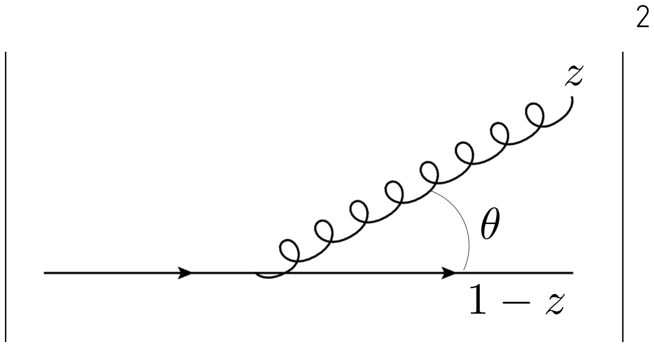
where

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}$$

$$T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}$$

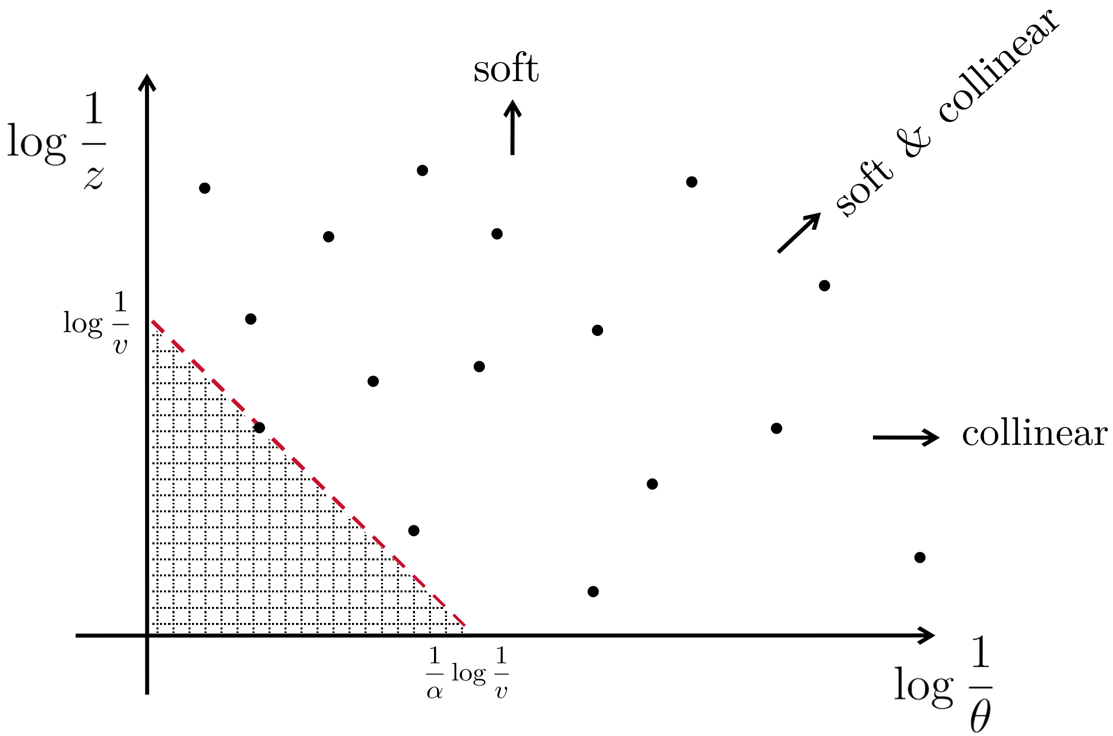
- One can improve the accuracy of the perturbative expansion. For many processes and some observables we have NNLO.
- This requires NNLO subtraction techniques and their implementation in MC event generators.

Resummation in pictures



$$\sim 2\frac{\alpha_S C_F}{\pi}\frac{dz}{z}\frac{d\theta}{\theta}$$

$$\sim 2\frac{\alpha_S C_F}{\pi}d\log\left(\frac{1}{z}\right)d\log\left(\frac{1}{\theta}\right)$$



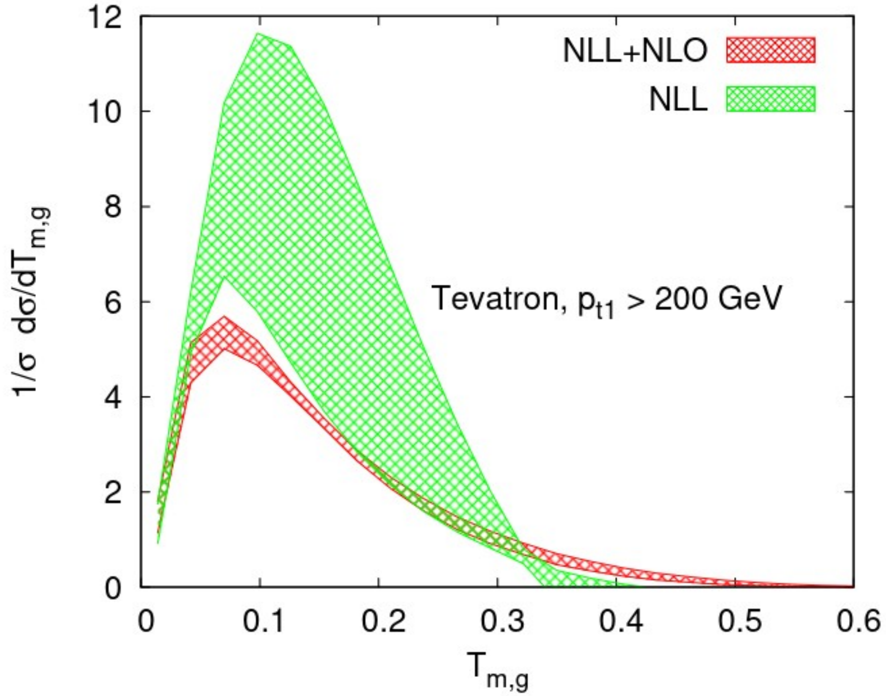
- Emissions are uniformly distributed among the Lund plane.
- We consider the strongly ordering condition, i.e. one emission dominates the others
- The angularity is a line on this log plane
$$\log\frac{1}{z} = \log\frac{1}{v} - \alpha\log\frac{1}{\theta}$$
- For observable whose LL exponentiate, the running of the coupling does not change the hierarchy of the logs
$$\alpha_S(k_t) = \alpha_S(Q^2)\Big(1 - 2\beta_0\alpha_S(Q^2)\log\frac{Q}{k_t} + \cdots\Big)$$

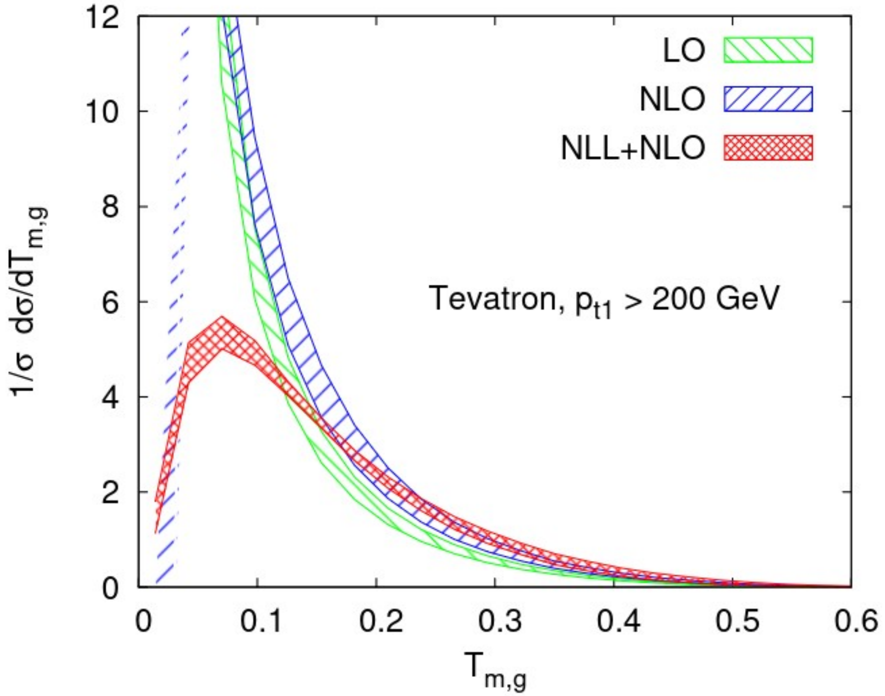
Probability for no emission $= 1 - \frac{2\alpha_S}{\pi}C_F\frac{\triangle}{N}$, with $\triangle = \frac{1}{\alpha}\log^2 v$

$$P(\lambda_\alpha < v) = \lim_{N\to\infty}\left(1 - \frac{2\alpha_S}{\pi}C_F\frac{\triangle}{N}\right)^N = e^{-\frac{2\alpha_S}{\pi}C_F\triangle}$$

Matching

Taken from Banfi, Salam and Zanderighi (2010)



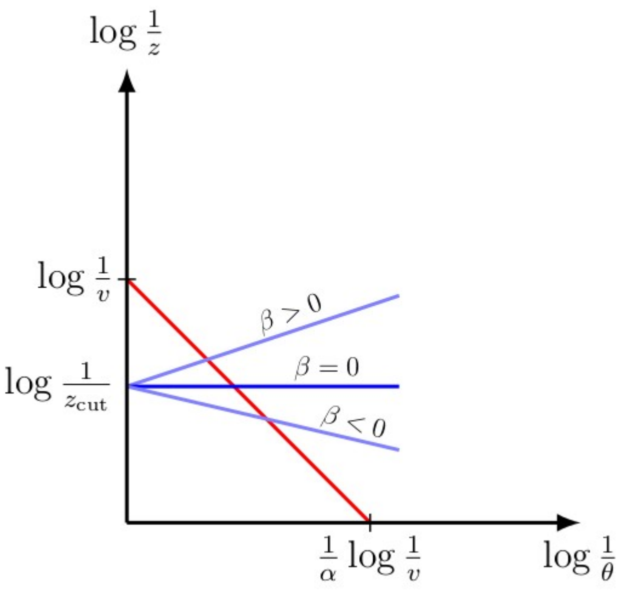


Directly global thrust minor:

$$T_{m,g} = \frac{\sum_i |\vec{q}_i \times \vec{n}_T|}{\sum_i q_{\perp,i}}$$

- Poor level of agreement between NLL and matched distribution, even at fairly small values of Tm,g
- LO essentially never agrees with the matched distribution
- At small Tm,g values, fixed order underestimates the effect of higher order corrections

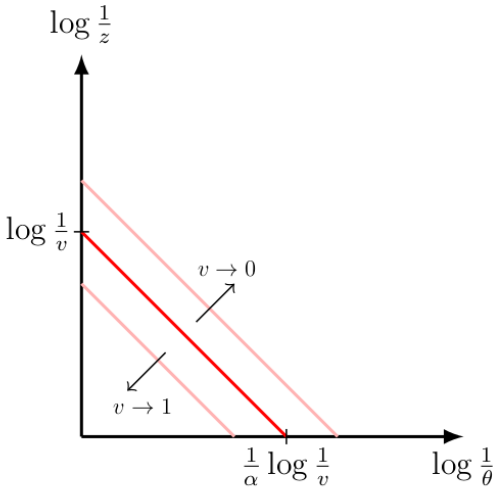
Lund plane geography

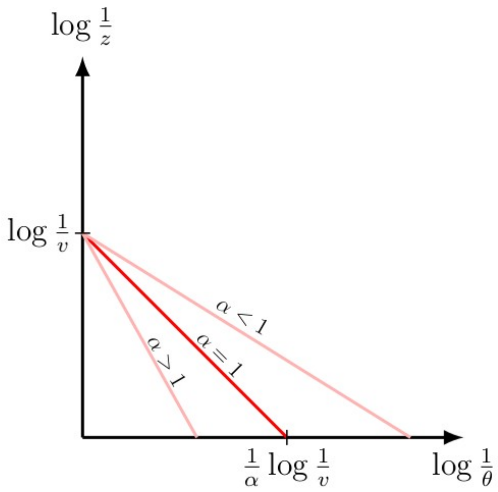


Angularities:

$\lambda_\alpha \simeq z\theta^\alpha$, in the soft and collinear limit.

$$z\theta^\alpha < v \quad \Rightarrow \quad \log\frac{1}{z} < -\alpha\log\frac{1}{\theta} + \log\frac{1}{v}$$





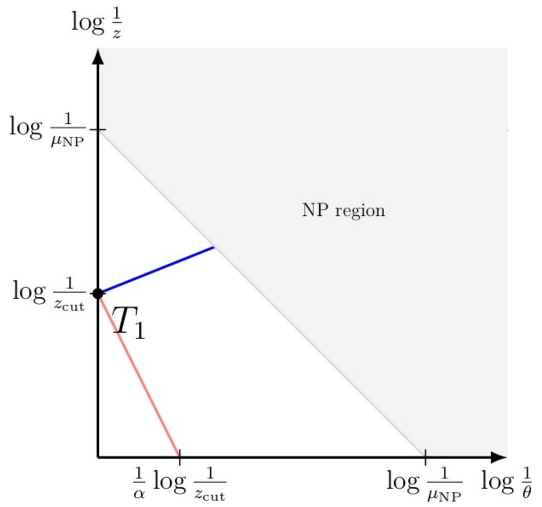
Soft Drop:

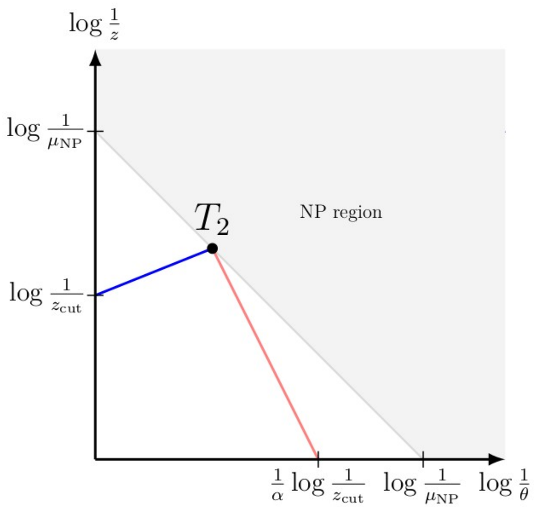
$$z > z_{\rm cut}\theta^\beta$$

$$\Downarrow$$

$$\log\frac{1}{z} < \beta\log\frac{1}{\theta} + \log\frac{1}{z_{\rm cut}}$$

Lund plane geography

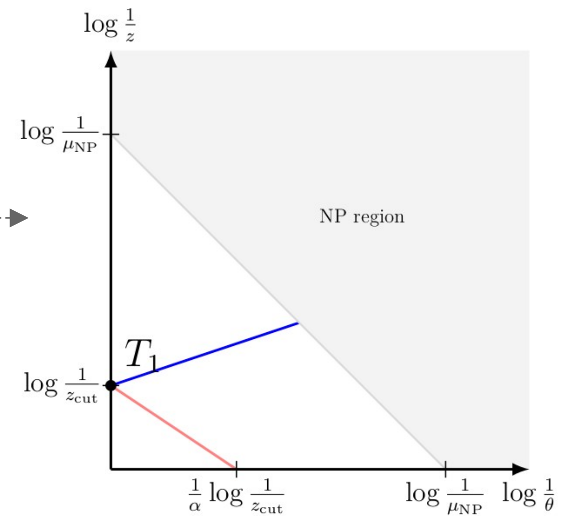


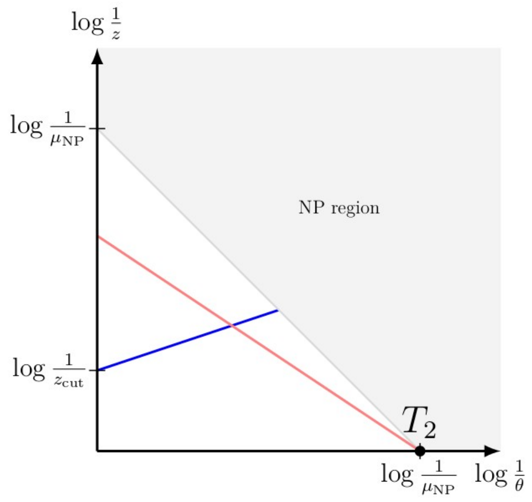


- T_1: SD transition point. No SD effects for $v>z_{cut}$.
- T_2: NP transition point
- SD extends perturbative domain for $\alpha>1$.

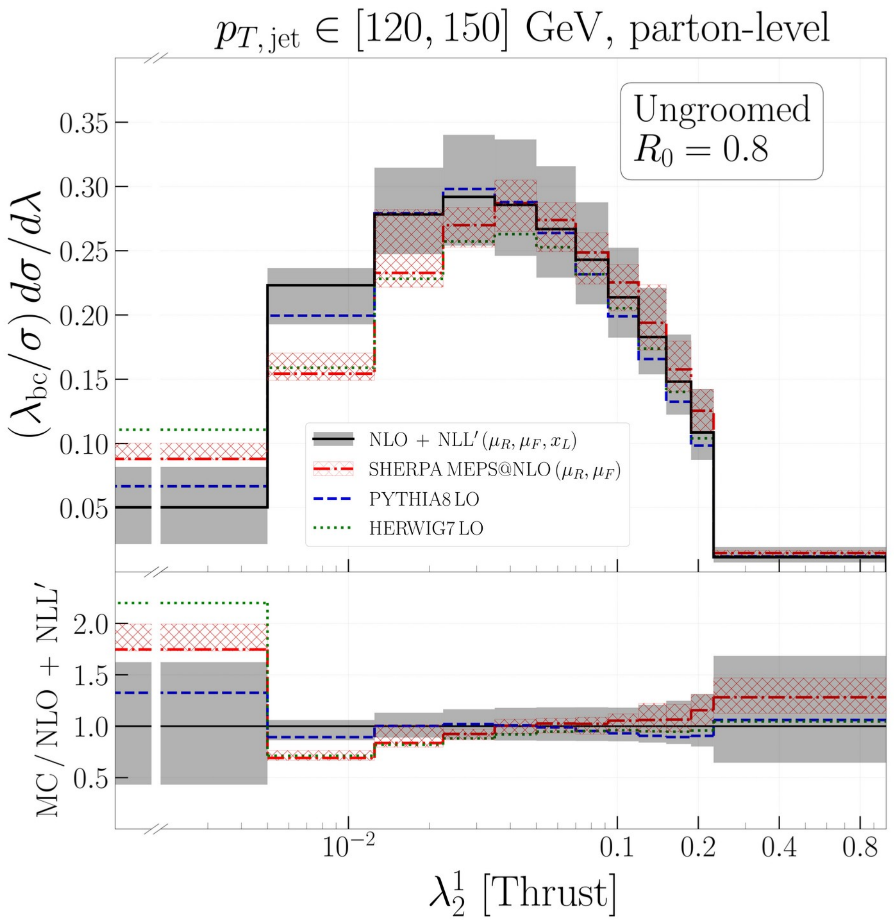
$$\begin{cases} \alpha > 1 \\ v > \mu_{NP}\left(\dfrac{\mu_{NP}}{z_{cut}}\right)^{\frac{\alpha-1}{\beta+1}} \end{cases} \qquad \begin{cases} \alpha \leq 1 \text{ or ungroomed} \\ v > \mu_{NP}^{\min[\alpha,1]} \end{cases}$$

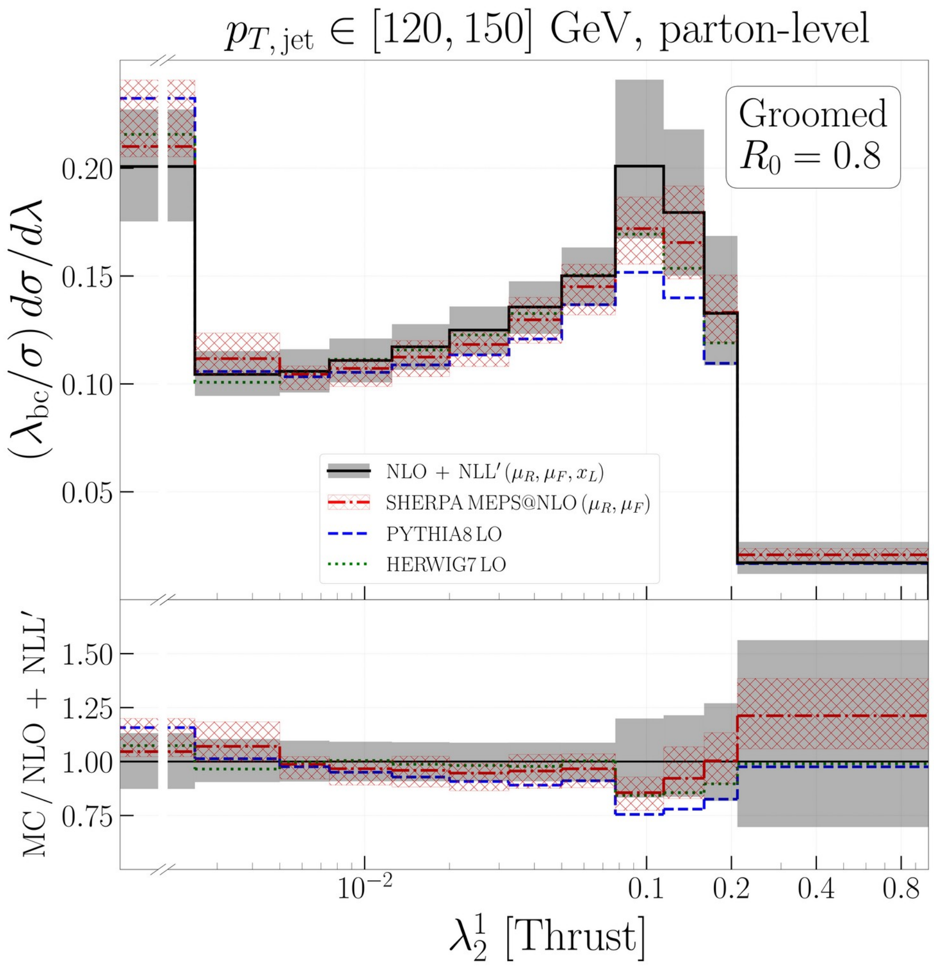
perturbative domain





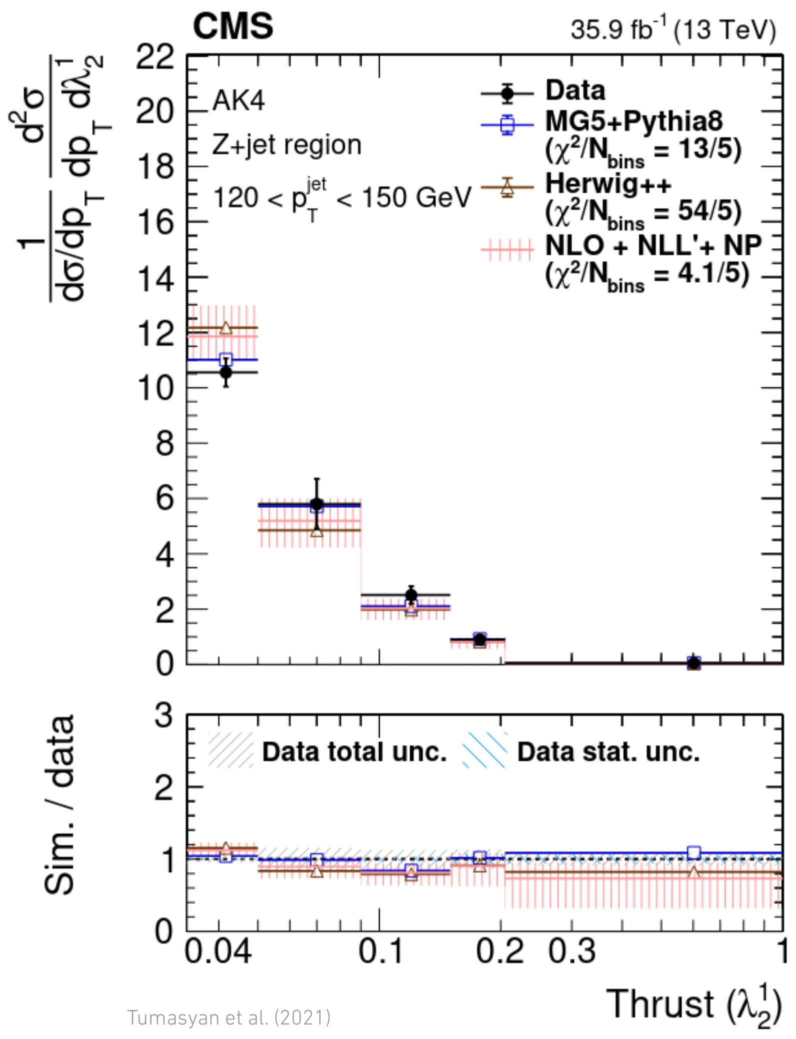
Ang. Distributions (PL) in Z+j



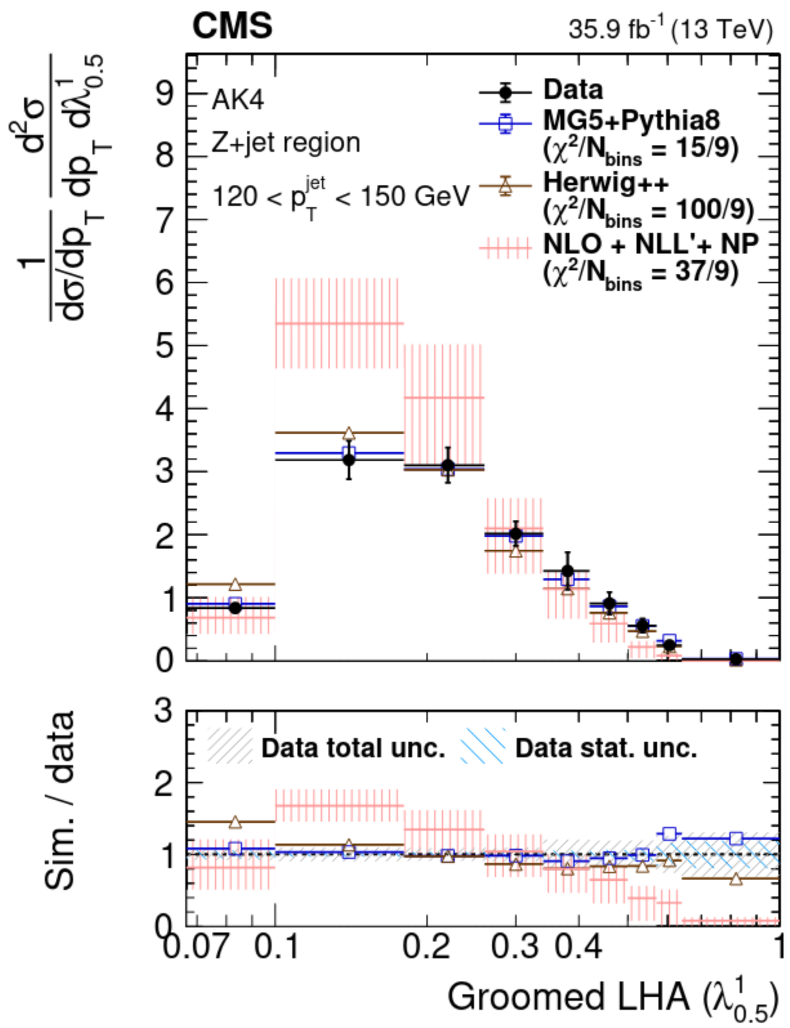


- Good agreement between MC and analytical predictions.
- Visible SD transition point (right).
- Visible NP transition point.
- SD extends the perturbative domain.
- Calculations are automatized in the SHERPA framework through an external plugin.

Comparison with CMS



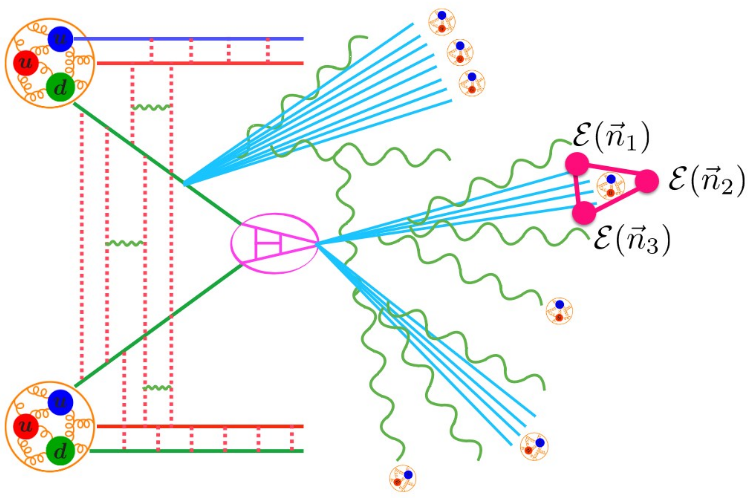
Tumasyan et al. (2021)



- Results for the LHA (i.e. α=0.5) are not in good agreement in the prediction. Intuituvely, we would expect so, at least in case of grooming.
- Have been done both prediction and measurement also for the dijet process.
- In these plots hadronization correction have been taken into account.

Other remarkable results

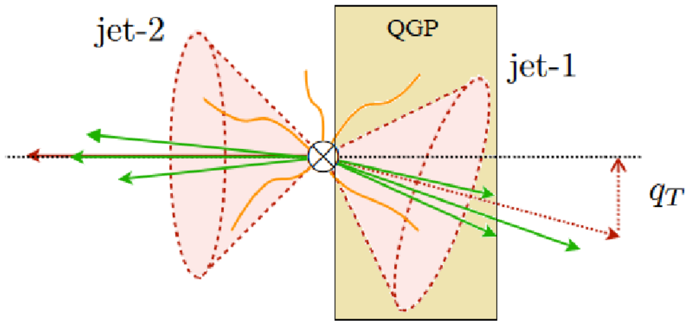
Taken from Chen et al. (2020)



Different type of observables, like the EEC based on the Energy flow operator

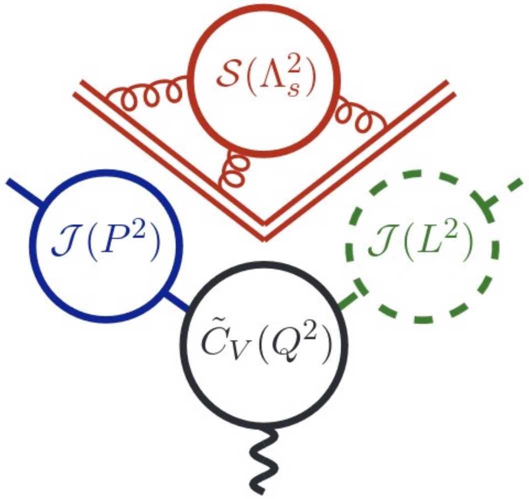
$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 T_{0i}(t, r\vec{n})$$

Vaidya (2021)



How QGP modifies JSS in heavy ion collisions

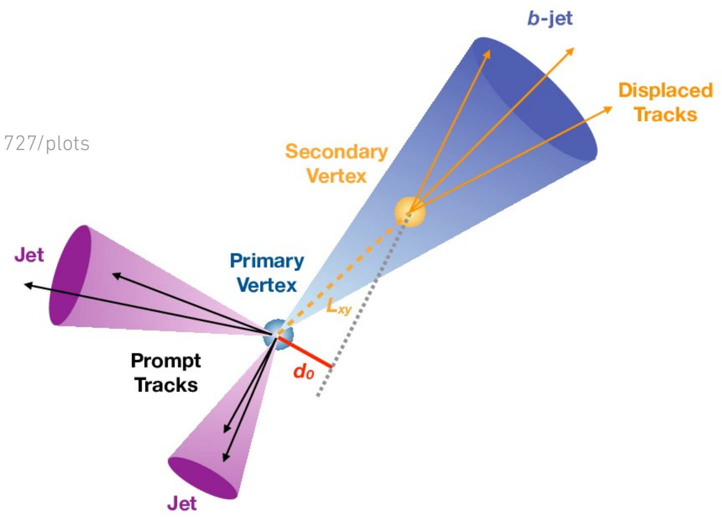
Take from Becher (2020)

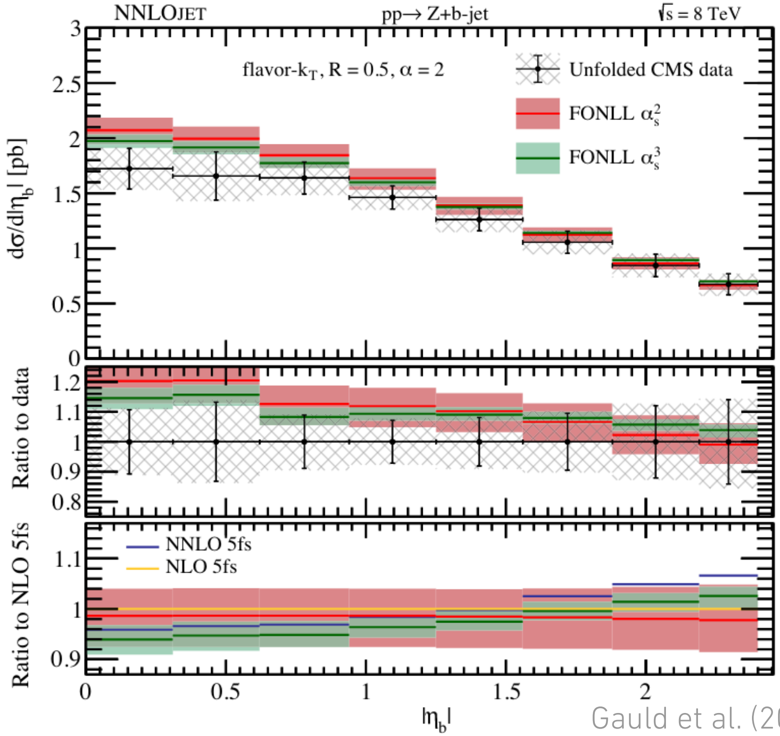


Different techniques applied to JSS, like SCET effective field theory

Experiment VS theory

https://cds.cern.ch/record/2771727/plots





Gauld et al. (2020)

- Heavy-quark-initiated jets are experimentally identified exploiting B hadron lifetime, i.e. the display vertex.
- Jets are defined with **anti-k_t**.

- High accuracy (NNLO) QCD calculations Z+b (W+c) jet have been performed in the last years.
- Jets were defined following the **BSZ flavour algorithm**.

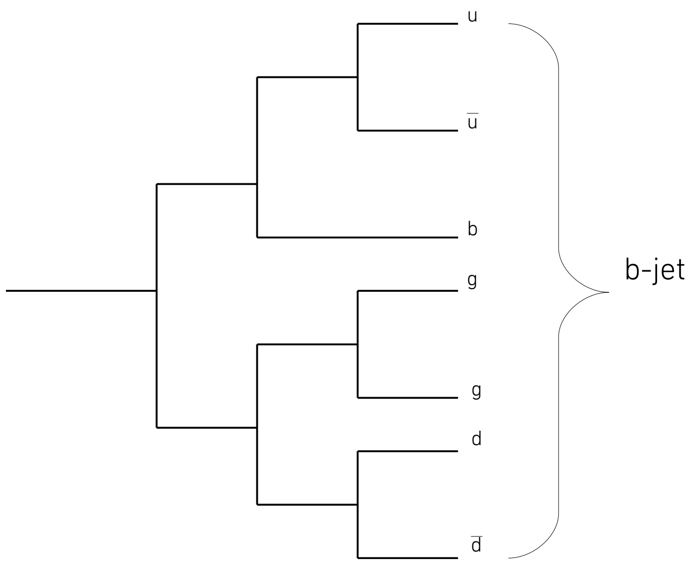
An extra unfolding procedure is required to compare the two.

Gauld et al. (2023)

> A comparison between these predictions with data will require an alignment of a flavour-tagging procedure in theory and experiment that is infrared and collinear safe.

The “naive” anti-k_t flavour

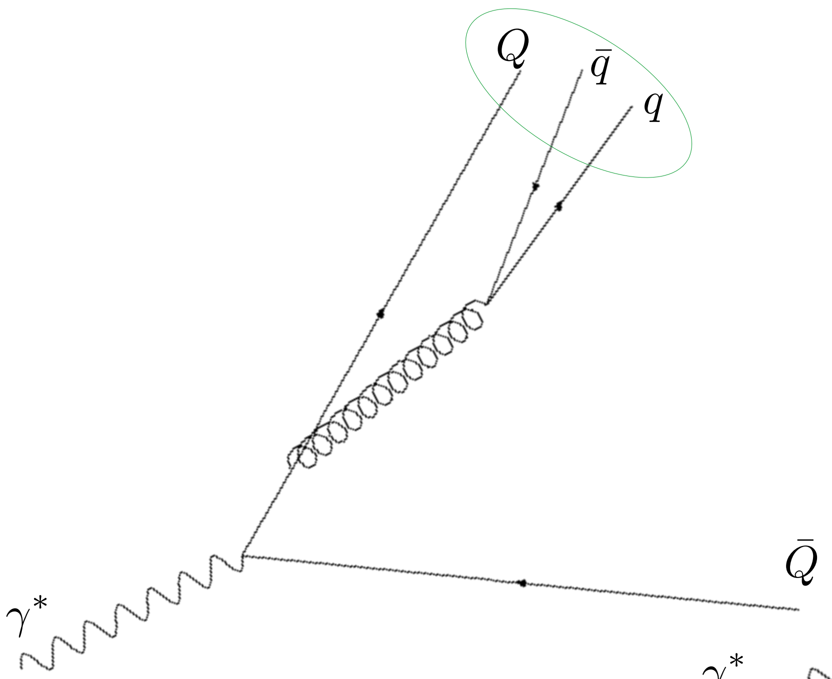
In short: the net flavour of anti-kt jets is **not** IRC safe beyond NLO.



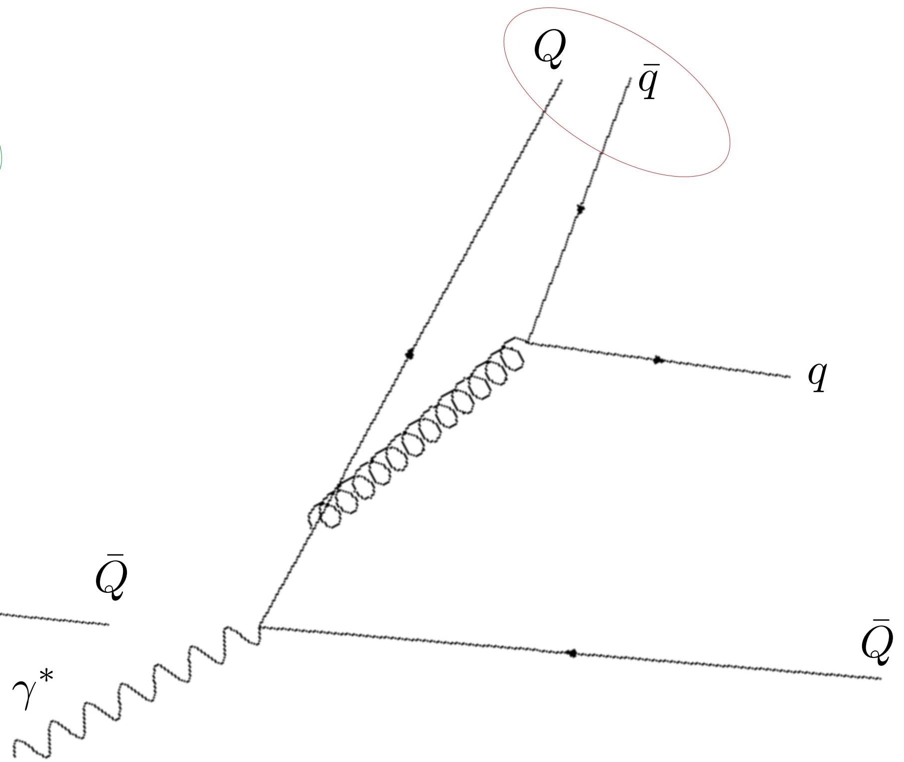
$$\text{net flavor} = \sum_i \#q_i - \#\bar{q}_i$$

Example: $e^+e^- \to 2\,\text{jet}$

$\mathcal{O}(\alpha_S^2)$ with a quark-antiquark collinear pair: any IRC safe algorithm is OK

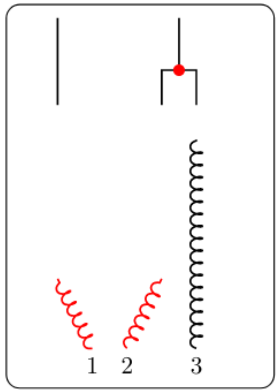


$\mathcal{O}(\alpha_S^2)$ with a quark-antiquark soft pair: large-angle polluting issue



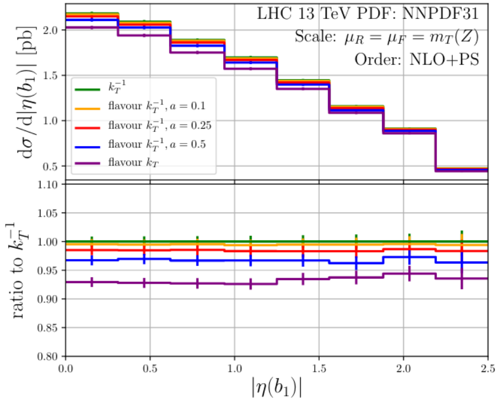
The “jet flavour gate”

- Introduce a neutralization distance



Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (2023)

- Define a flavor algorithm that resembles anti-k_T

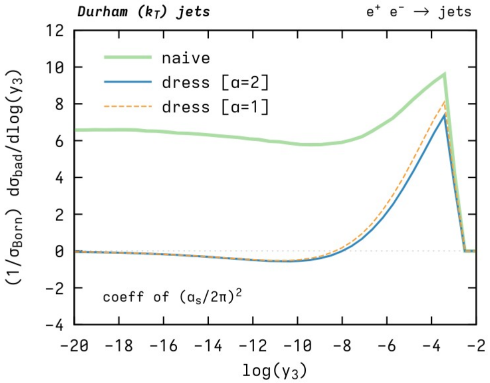


$$d_{ij}^{\mathrm{CMP}} = d_{ij} \times \begin{cases} \mathcal{S}_{ij}\,, & \text{if both } i \text{ and } j \text{ have non-zero flavor of opposite sign} \\ 1\,, & \text{otherwise} \end{cases}$$

Czakon, Mitov, Poncelet (2022)

Applied to W+c-jet and Wbb production

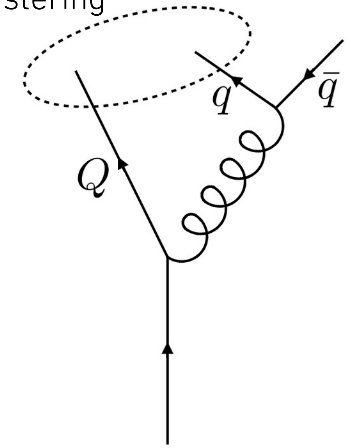
- Construct a flavor dressing and associate the flavour to a given jet



Gauld, Huss, Stagnitto (2022)

Applied to W+c-jet production

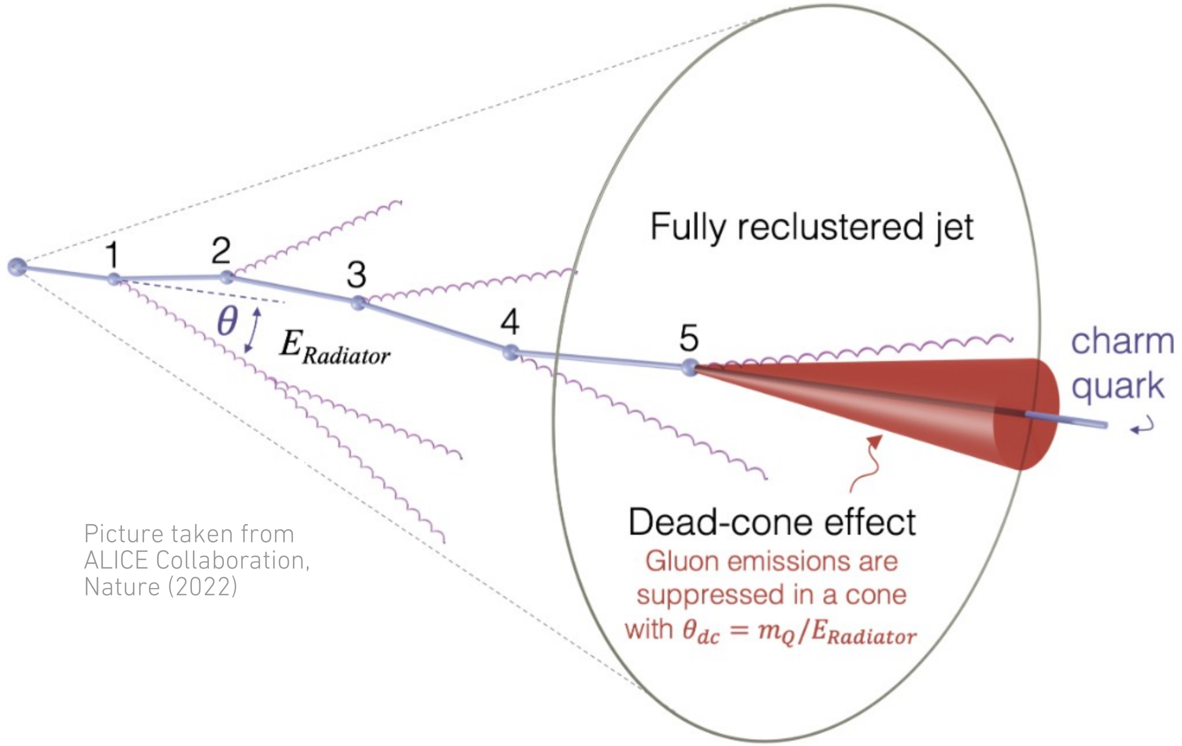
- Groom jets with SD after JADE reclustering



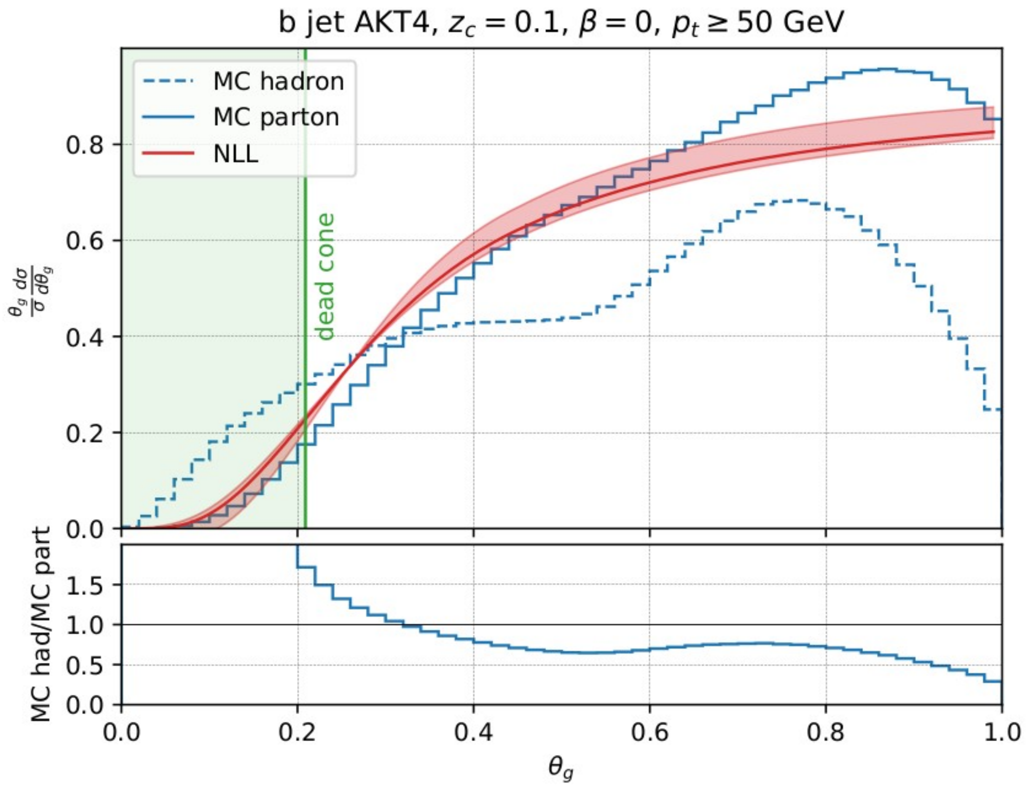
Caletti, Reichelt, Larkoski, Marzani (2022)

- All of them implemented as a contrib in Fastjet and soon available (goal of the LH23 workshop)

JSS for HQ jets



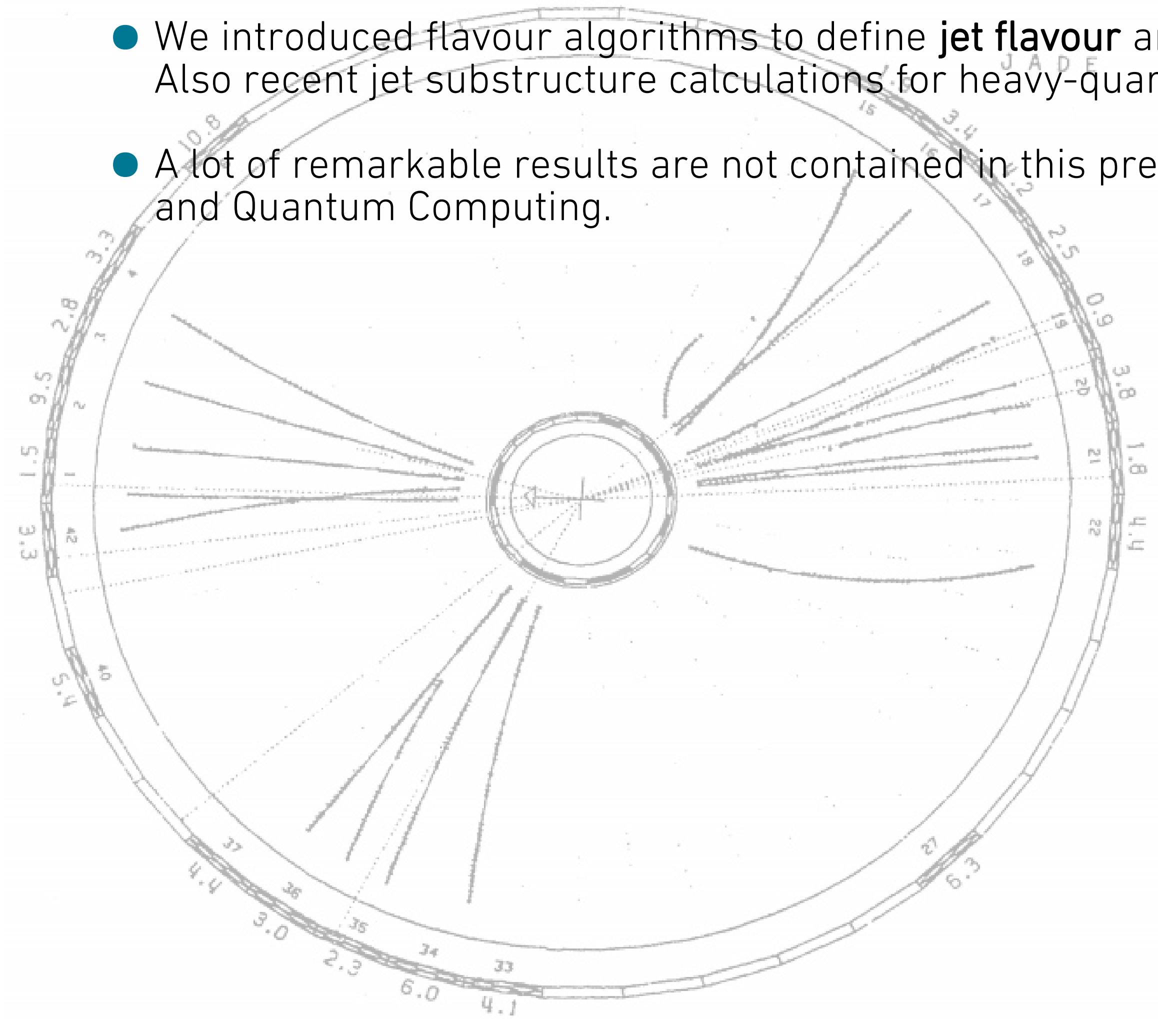
Picture taken from ALICE Collaboration, Nature (2022)



- Heavy quark jets allow to explore the dead-cone effect and to study heavy-quark fragmentation.
- First jet substructure resummed calculations with mass effects are available. Still to work on automatization, matching, etc. in order to provide full phenomenology.
- It would be interesting to compare heavy quark jet measurements with in-jet hadron fragmentation.

Conclusions

- We discussed the origin of the concept of **hadronic jet** and how to study their substructure, with fixed order and resummed calculations.
- We introduced flavour algorithms to define **jet flavour** and we motivated recent theoretical development on this topic. Also recent jet substructure calculations for heavy-quark jets have been discussed.
- A lot of remarkable results are not contained in this presentation, including recent work involving Machine Learning and Quantum Computing.



Thank you for your attention

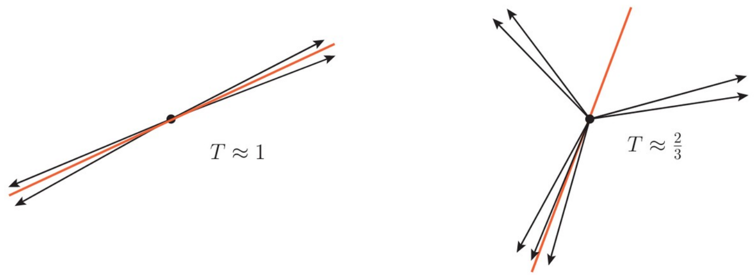
BACKUP SLIDES

Example: event thrust

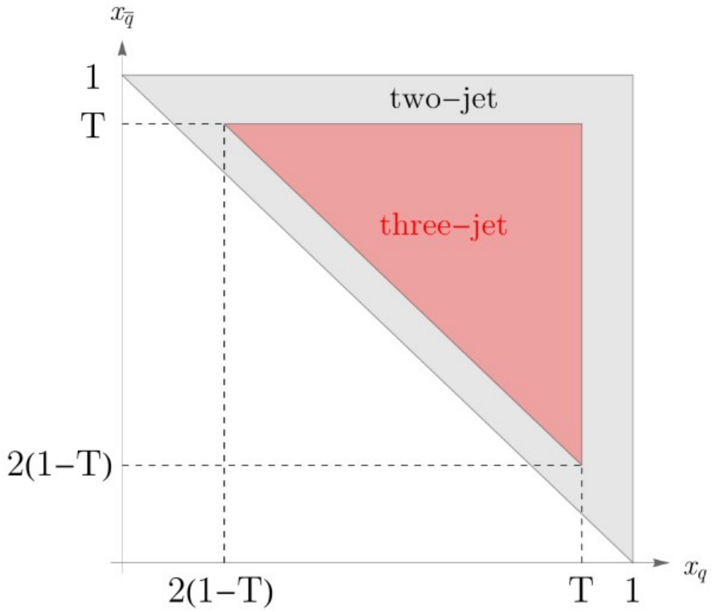
To distinguish between "2-jet" and "3-jet" events we might introduce the **event thrust**, defined as

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

and it is an IRC safe quantity.



Requiring for example that $max[x_q, x_{\bar{q}}, x_g] < 0.9 \equiv T$ corresponds to integrate over



Doing this obtain the following results

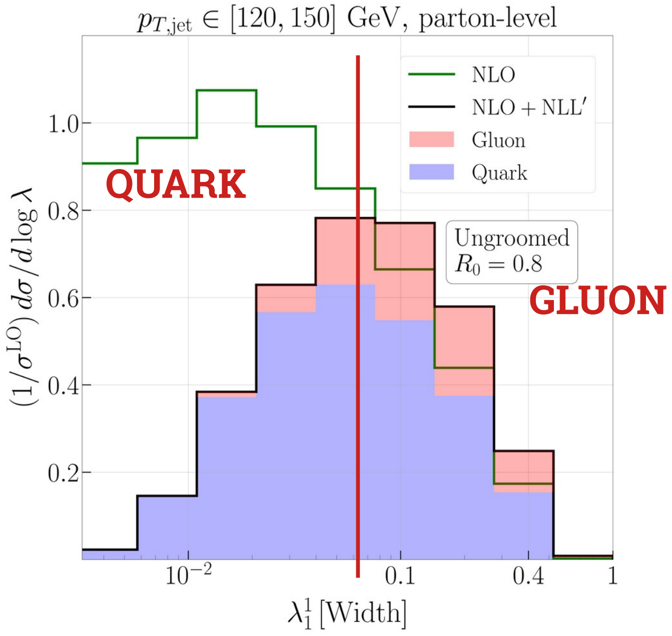
$$f_3 = C_F \frac{\alpha_S}{2\pi}\left[(3-6y)\log\left(\frac{y}{1-2y}\right) + 2\log^2\left(\frac{y}{1-y}\right) + \frac{5}{2} - 6y - \frac{9}{2}y^2 + 4\frac{y}{1-y} - \frac{\pi^2}{3}\right]$$

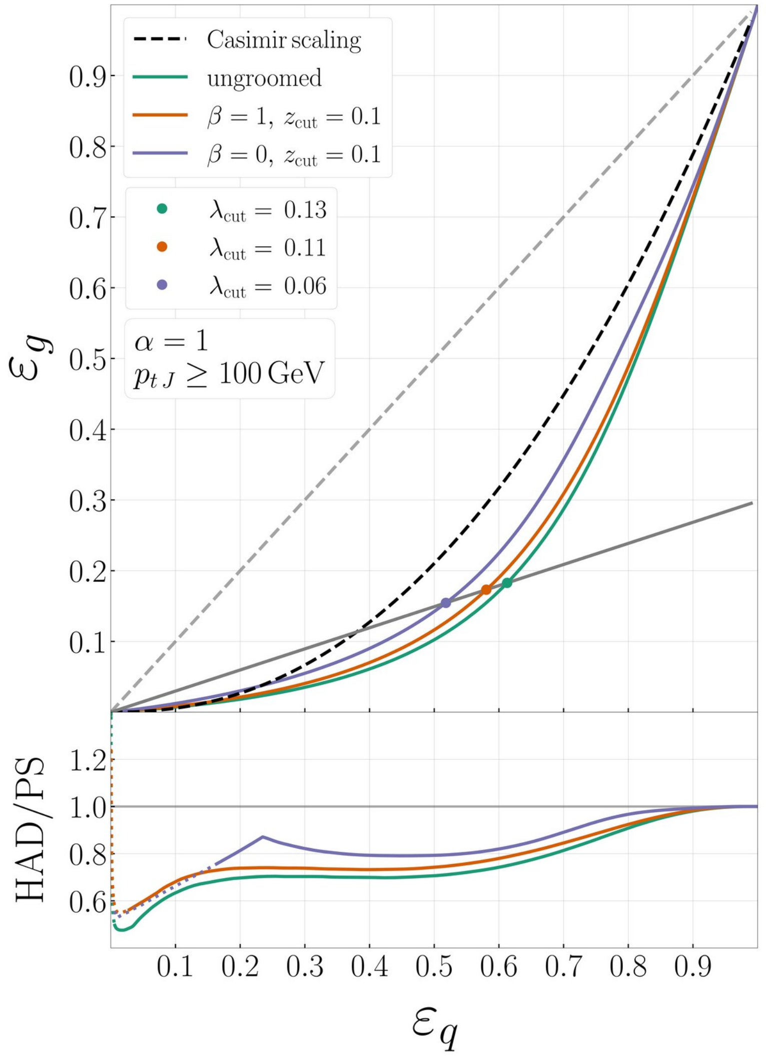
$$f_2 = 1 - f_3$$

, with $y = 1 - T$

Flavour tagging with JSS

- Z+jet process can be used to probe the gluon PDF.
- Idea: use angularities to distinguish between quark-initiated and gluon-initiated jets.
- Find the λ_{cut} using the ROC curve.

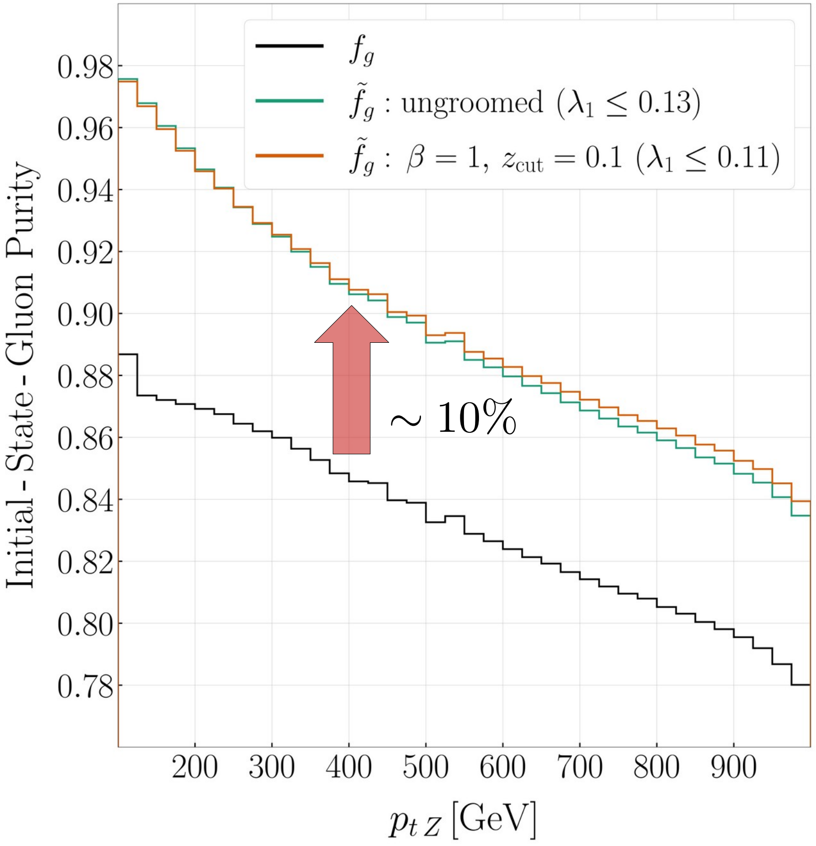


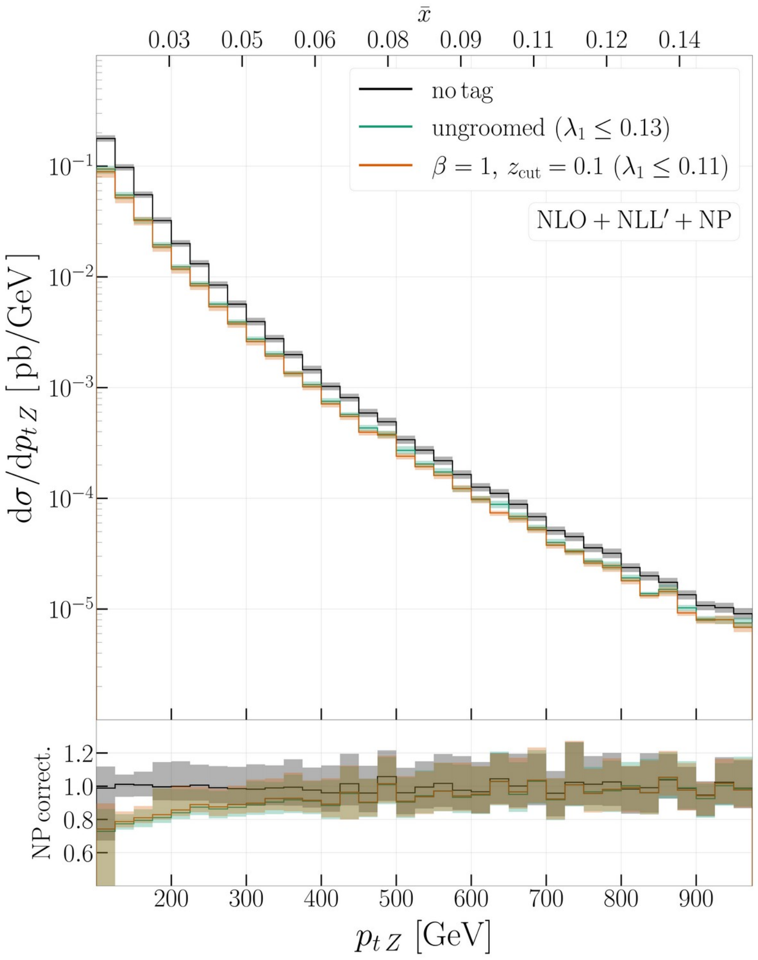


$$\varepsilon_k = \frac{\Sigma_{ij}(\lambda_{\mathrm{cut}})}{\Sigma_{ij}(1)} = \frac{1}{\sigma_{ij}} \int_0^{\lambda_{\mathrm{cut}}} \frac{d\sigma_{ij}}{d\lambda} d\lambda$$

$$\tilde{f}_g = \frac{\varepsilon_q \sigma_{qg}}{\varepsilon \sigma_{qq} + \varepsilon_q \sigma_{qg}} = \frac{\varepsilon_q f_g}{\varepsilon_g (1 - f_g) + \varepsilon_q f_g}$$

Flavour tagging with JSS



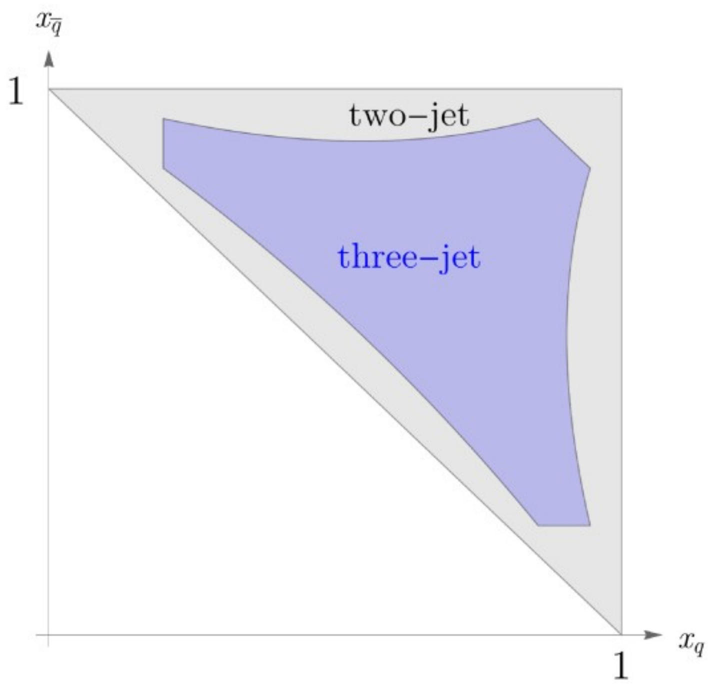


- Gluon enhancement in the initial state.
- The p_t distribution might be employed for gluon PDF fitting.

Another example: SW jets

The first intuitive notion of jets has been given by Sterman and Weinberg (1977) and for “two-jet” events in e+e-→ qq at O(as) is defined by the following measurement function

$$J_{\varepsilon,\delta}(k_1,k_2,k_3) = \Theta(\min[\theta_{12},\theta_{13},\theta_{23}] < 2\delta)$$
$$+\Theta(\min[\theta_{12},\theta_{13},\theta_{23}] > 2\delta)\Theta(\min[E_1,E_2,E_3] < \varepsilon E)$$



that at this order satisfies the IRC safety condition.

This corresponds to modify the phase space according to the following condition

$$1-\cos(2\delta) \le \frac{2(1-x_g)}{x_q x_{\bar{q}}}, \quad 1-\cos(2\delta) \le \frac{2(1-x_q)}{x_{\bar{q}} x_g}, \quad 1-\cos(2\delta) \le \frac{2(1-x_{\bar{q}})}{x_g x_q}$$

plus the energy cuts.
Thus the two-jet fraction is given by

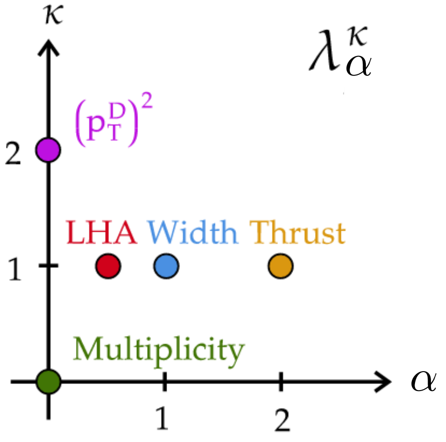
$$f_2 = 1 - 2C_F\frac{\alpha_S}{2\pi}\left\{\log\delta\left[3+4\log(2\varepsilon)\right] + \frac{\pi^2}{3} - \frac{7}{4}\right\}$$

modulo terms proportional to ε and δ.

Resummation

ANGULARITY

$$\lambda_\alpha^\kappa = \sum_{j\in \text{Jet}} \left(\frac{p_{T,j}}{\sum_{j\in\text{Jet}} p_{T,j}} \right)^\kappa \left(\frac{\Delta_j}{R} \right)^\alpha \simeq \sum_{j\in\text{Jet}} z_j^\kappa \theta_j^\alpha$$



To do that, we consider n extra (uncorrelated) emissions in the soft and collinear limit

$$\Sigma(v)^{\text{LL}} = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \frac{2\alpha_S C_F}{\pi} \int \frac{d\theta_i}{\theta_i} \int \frac{dz_i}{z_i} \Theta_{i\in J} \Theta \left(\sum_{j=0}^{n} \lambda_j < v \right)$$

$$+ \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \frac{2\alpha_S C_F}{\pi} \int \frac{d\theta_i}{\theta_i} \int \frac{dz_i}{z_i} \left[\Theta_{i\notin J} - 1 \right]$$

At the LL it is enough to consider only to have **strongly ordered** emissions, i.e.

$$\Theta \left(\sum_{i=1}^{n} \lambda_i < v \right) \simeq \Theta(\max_i \lambda_i < v) = \prod_{i=1}^{n} \Theta(\lambda_i < v)$$

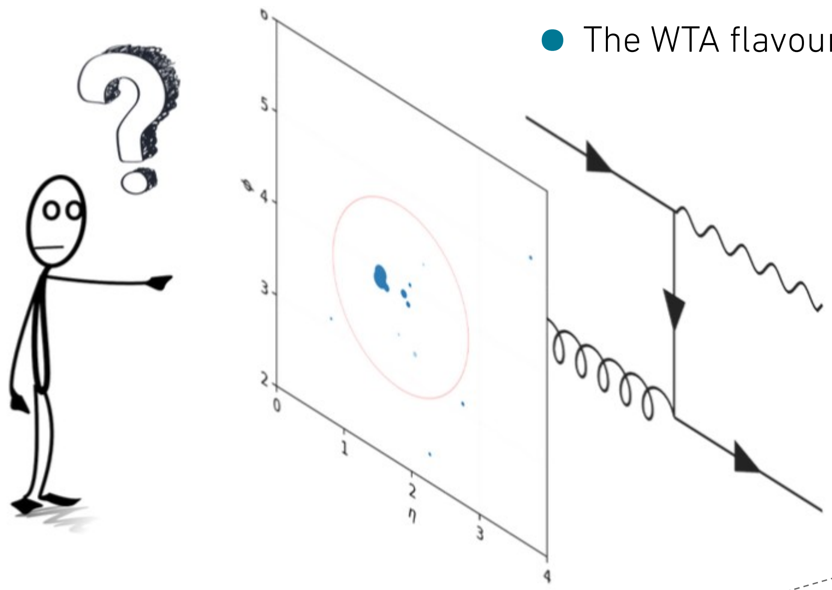
$$\Sigma(v)^{\text{LL}} = -\sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \frac{2\alpha_S C_F}{\pi} \int \frac{d\theta_i}{\theta_i} \int \frac{dz_i}{z_i} \Theta \left(z_i \theta_i^2 > v \right) \Theta_{i\in J}$$

$$= \exp \left[-\frac{2\alpha_S C_F}{\pi} \int_0^R \frac{d\theta}{\theta} \int_0^1 \frac{dz}{z} \Theta \left(z_i \theta_i^2 > v \right) \right]$$

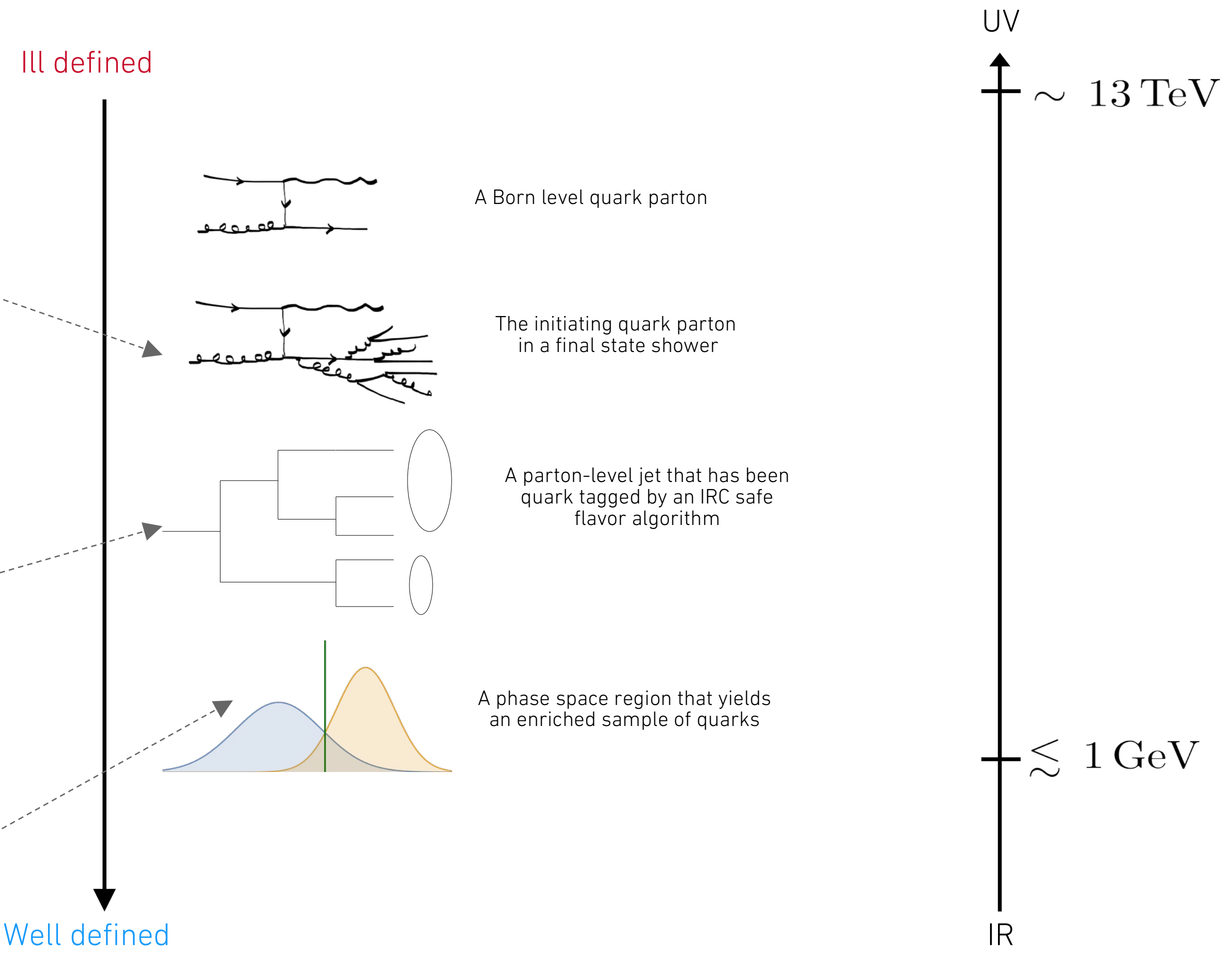
$$= \exp \left[-\frac{\alpha_S C_F}{\pi} \frac{1}{2} \log^2 v \right] = \exp \left[-\mathcal{R}(v) \right]$$

also called **radiator,** it represents the non emission probability

The jet flavour problem



- The WTA flavour
- The SoftDrop flavour
  (and the other flav. alg.)
- Using JSS beyond Casimir scaling



Adapted from Gras et al. (2017)

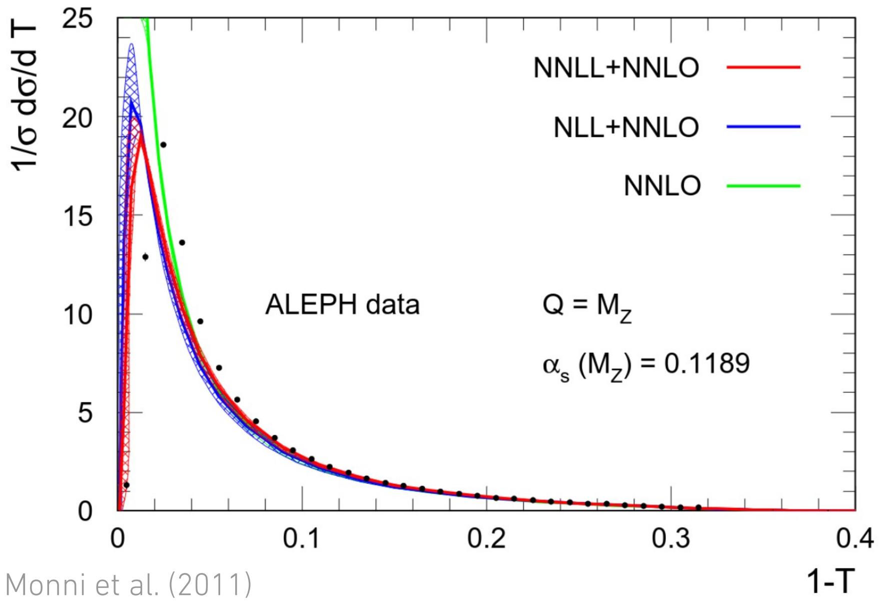
Toward NLL...

To go NLL we have to consider:
- the running coupling @ 2-loops
- the complete splitting function (hard-collinear limit) @ 1-loop and its soft limit @ 2-loops

The last one, corresponds to the 2-loops cusp anomalous dimension and can be reabsorbed in the running coupling introducing the CMW scheme

$$\frac{\alpha_S^{\rm CMW}(\mu^2)}{2\pi} = \frac{\alpha_S(\mu^2)}{2\pi} + K\left(\frac{\alpha_S(\mu^2)}{2\pi}\right)^2$$

where $K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}n_f$



Monni et al. (2011)

Also the strong ordering approximation is no longer valid and we have to deal with the factorization spoiling measurement function.
However, in the conjugate space we have

$$\Theta\left(v - \sum_{i=1}^{n} V(k_i)\right) = \int \frac{d\nu}{2\pi i\nu} e^{\nu\tau} \prod_{i=1}^{n} e^{-\nu V(k_i)}$$

Putting together real and virtual, the all-order cumulative distribution is given by

$$\Sigma(v) = \int \frac{d\nu}{2\pi i\nu} e^{\nu\tau} \exp\left[\int dz \frac{dk_t^2}{k_t^2}\frac{d\phi}{2\pi} 2C_F \frac{\alpha_S(k_t)}{2\pi} P_{gq}(z)\left(e^{-\nu V(k)} - 1\right)\right]$$

At NLL we are allowed to use the following approximation

$$e^{-\nu V(k)} - 1 \approx -\Theta\left(V(k) - e^{\gamma_E}\nu^{-1}\right)$$

Catani, Trentadue (1989)

At the end, we come back to physical space performing the inverse Mellin transform.
For the thrust this can be done in a closed form

$$\Sigma^{\rm NLL}(v) = \mathcal{M} e^{-\mathcal{R}} \qquad \text{with} \qquad \mathcal{M}(v) = \frac{e^{-\gamma_E \mathcal{R}'(v)}}{\Gamma(1+\mathcal{R}'(v))}$$

...with CAESAR

NLL Radiator
- Running coupling (CMW scheme)
- Hard collinear

Kinematic cuts
- Ensures well separated legs with extra (IRC safe) cuts
- Includes the observable's measurement function

Master formula

$$\Sigma^{\delta}_{res}(v) = \int d\Phi_{\mathcal{B}^\delta} \frac{d\sigma_\delta}{d\Phi_{\mathcal{B}^\delta}} \exp\Big[-\sum_{l\in\mathcal{B}^\delta} R_l^{\mathcal{B}^\delta}(L)\Big] \mathcal{S}^{\mathcal{B}^\delta}(L)\mathcal{P}^{\mathcal{B}^\delta}(L)\mathcal{F}^{\mathcal{B}^\delta}(L)\Theta^{\mathcal{B}^\delta}_{hard}$$

$$\Sigma_{res}(v) = \sum_{\delta} \Sigma^{\delta}_{res}(v)$$

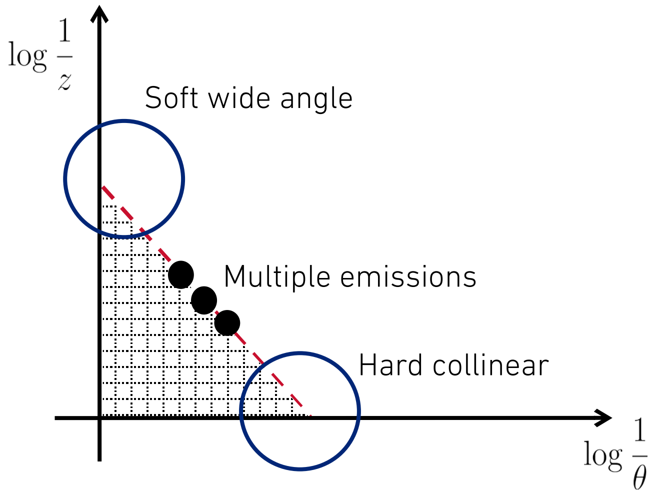
Banfi et al. (2005)

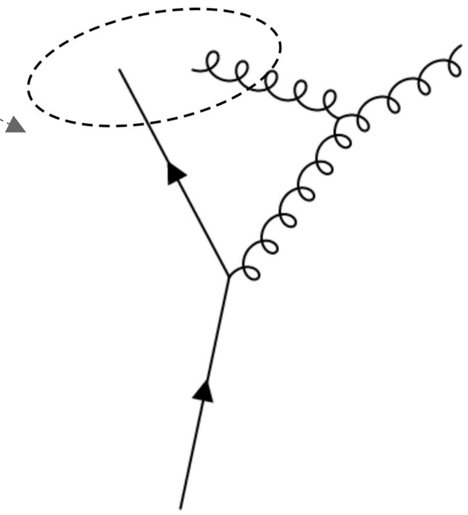
Multiple emissions
- known behaviour for a large category of observables, including angularities.

Sum over flavour configurations

Soft function
- Colour structure
- NGL for jet shapes

PDF ratio
- initial state emissions
- P = 1 for jet shapes





Transfer matrix

A simple way to include NP correction due to hadronization to theoretical prediction would be to extract them from MC event generator, i.e.

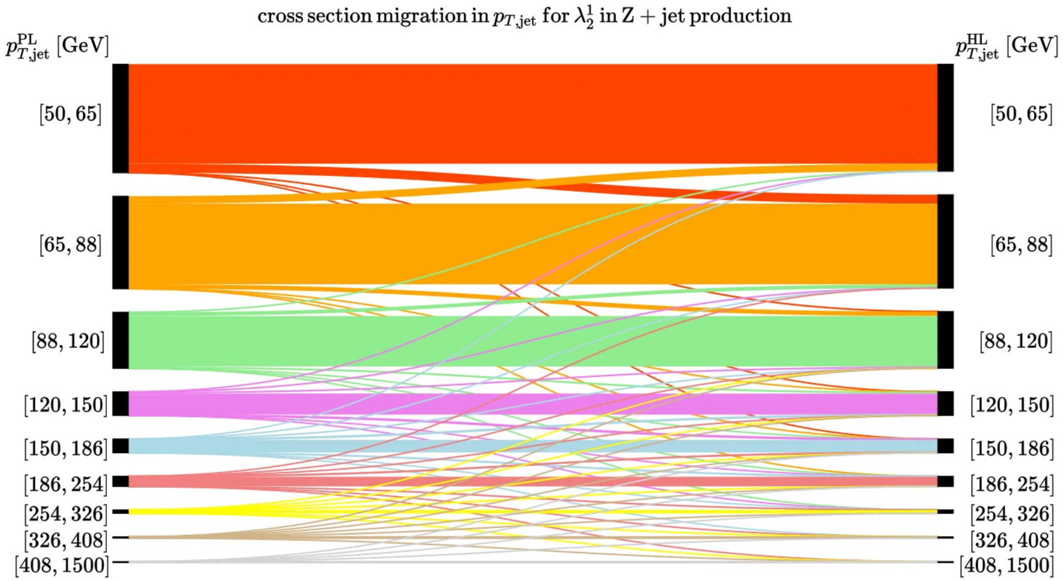
$$\text{Parton level semi-analytic prediction} \times \frac{\text{Hadron level MC simulation}}{\text{Parton level MC simulation}} = \text{"Hadron level" semi-analytic prediction}$$

- Equivalent to multiply the PL distribution by a n_bin x n_bin diagonal matrix extracted from MC

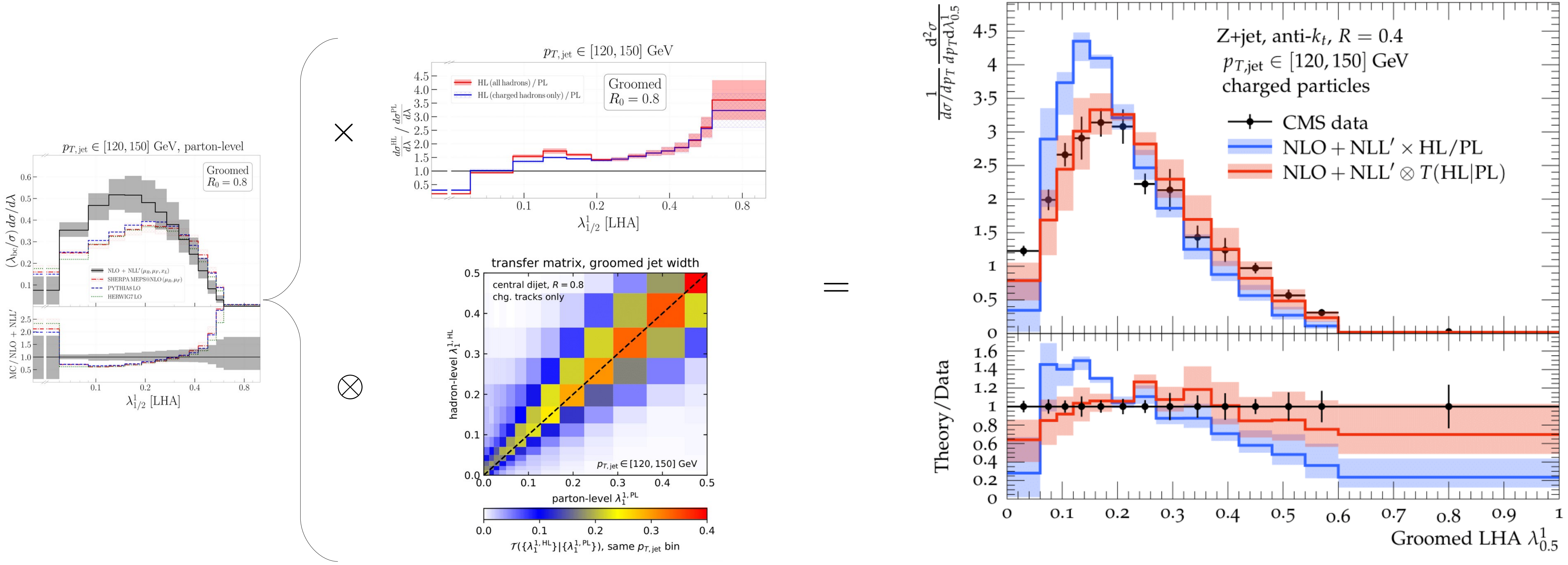
Otherwise, we might allow this matrix to be non-diagonal. This means, we consider possible bin migration effects between PL and HL.

$$\mathcal{T}(\vec{v}_h|\vec{v}_p) = \frac{\int d\mathcal{P} \frac{d\sigma}{d\mathcal{P}} \delta^{(m)}\left(\vec{v}_p - \vec{V}(\mathcal{P})\right) \delta^{(n)}\left(\vec{v}_h - \vec{V}(\mathcal{H}(\mathcal{P}))\right)}{\int d\mathcal{P} \frac{d\sigma}{d\mathcal{P}} \delta^{(m)}\left(\vec{v}_p - \vec{V}(\mathcal{P})\right)}$$

$$\frac{d^m\sigma^{\mathrm{HL}}}{dv_{h,1}\dots dv_{h,m}} = \int d^m\vec{v}_p \mathcal{T}(\vec{v}_h|\vec{v}_p) \frac{d^m\sigma^{\mathrm{PL}}}{dv_{p,1}\dots dv_{p,m}}$$

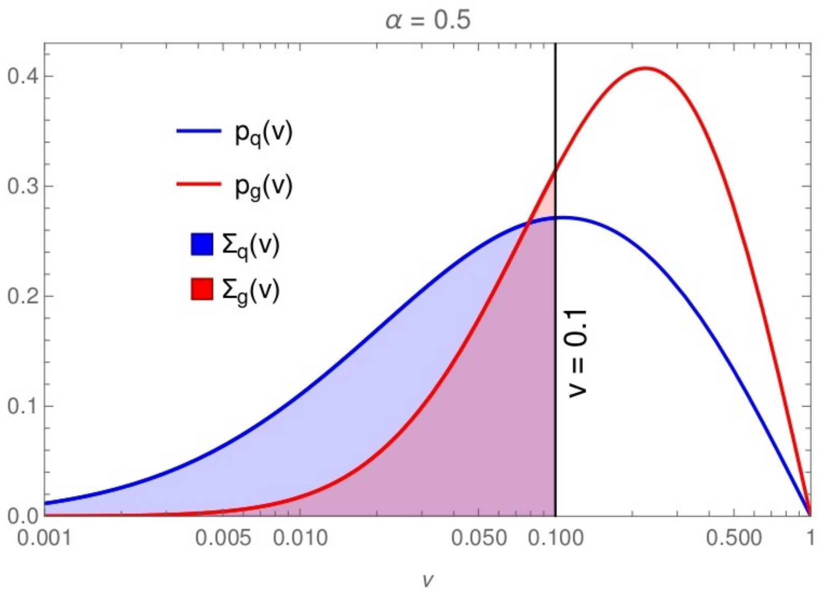


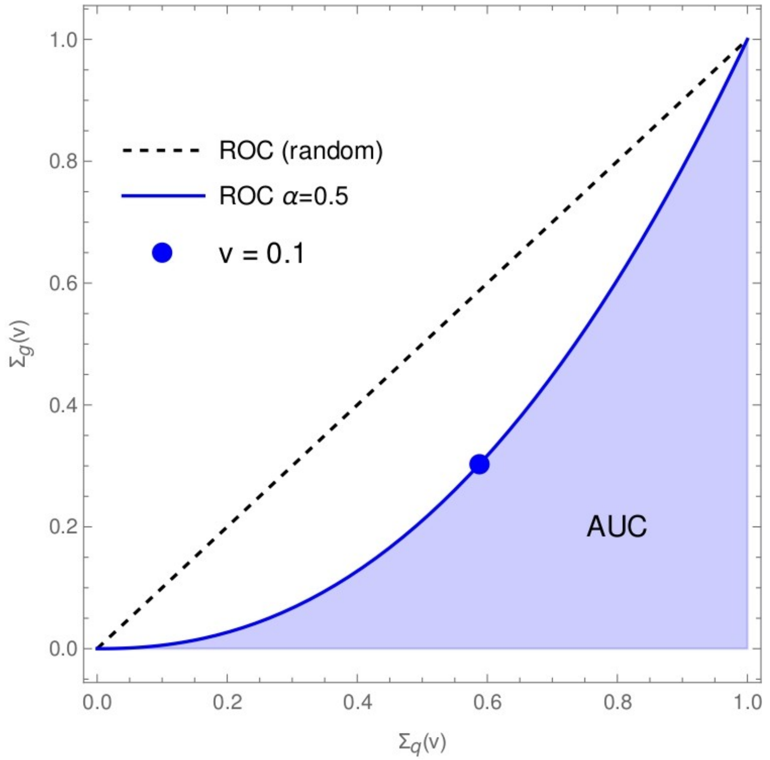
Hadronization corrections



Casimir scaling

The difference between the quark and gluon LL distributions is only contained in the colour factors, i.e. the casimir





$$\Sigma_q = P(\lambda_\alpha < v) = e^{-\frac{2\alpha_S}{\pi} C_F}$$

$$\Sigma_g = P(\lambda_\alpha < v) = e^{-\frac{2\alpha_S}{\pi} C_A}$$

$$\mathrm{ROC}(x) = \Sigma_g\left(\Sigma_q^{-1}(x)\right)$$
$$= x^{C_A/C_F}$$

Going beyond LL we start seeing differences in the distributions beyond the Casimir scaling.

The BSZ solution

- Similar to standard reclustering algorithm but with a flavour-sensitive metric.
- The metric reflects the absence of soft quark singularities. It is IRC safe to all orders because it tends to recombine problematic soft $q\bar{q}$ pairs.
- However, the use of BSZ in experimental analyses is far from straightforward:
  - ☞ Obviously, it's not anti-k_T
  - ☞ It requires knowledge of the flavor at each step of the clustering

$$d_{ij}^{\mathrm{BSZ}} = \left(\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2\right) \times \begin{cases} \max\left(k_{ti}^2, k_{tj}^2\right) , & \text{if the softer of } i,\ j \text{ is flavored,} \\ \min\left(k_{ti}^2, k_{tj}^2\right) , & \text{if the softer of } i,\ j \text{ is flavorless.} \end{cases}$$

Banfi et al. (2006)

Soft Drop flavour @ NLO

- Through NLO even anti-k_T is IRC safe. Is there any subtlety with Soft Drop flavor?
- We concentrate on quark vs gluon, in e^+e^-
- At LO is trivial:

$$P_q = 1 + \mathcal{O}(\alpha_S)$$
$$P_g = 0 + \mathcal{O}(\alpha_S)$$

- At NLO there are q→qg splitting to consider. We assign a gluon flavor if:

(a) The quark and the gluon are not recombined in the same jet && the gluon is the most energetic.

$$P_g^{(a)} = \frac{\alpha_S C_F}{2\pi}\int_{R^2}^{1}\frac{d\theta^2}{\theta^2}\int_{1/2}^{1} dz\frac{1+(1-z)^2}{z} = \frac{\alpha_S C_F}{2\pi}\log R^2\left(\frac{5}{8} - 2\log 2\right)$$

(b) The two partons are recombined in the same jet && the quark fails the SD condition (so it is groomed away).

$$P_g^{(b)} = \frac{\alpha_S C_F}{2\pi}\int_{0}^{R^2}\frac{d\theta^2}{\theta^2}\int_{0}^{1} dz\frac{1+(1-z)^2}{z} \times\Theta\left(z_{\text{cut}}\left(\frac{\theta^2}{R^2}\right)^{\beta} > (1-z)\right) = \frac{\alpha_S C_F}{2\pi}\frac{z_{\text{cut}}}{\beta}$$

- Thus, at NLO:

$$P_g = P_g^{(a)} + P_g^{(b)}$$
$$P_q = 1 - P_g$$

- So even at NLO, we must use Soft Drop with β > 0

Q→Qq̄ in the soft & coll. limit

- Triple collinear splitting function for the Q→Qq̄ splitting:

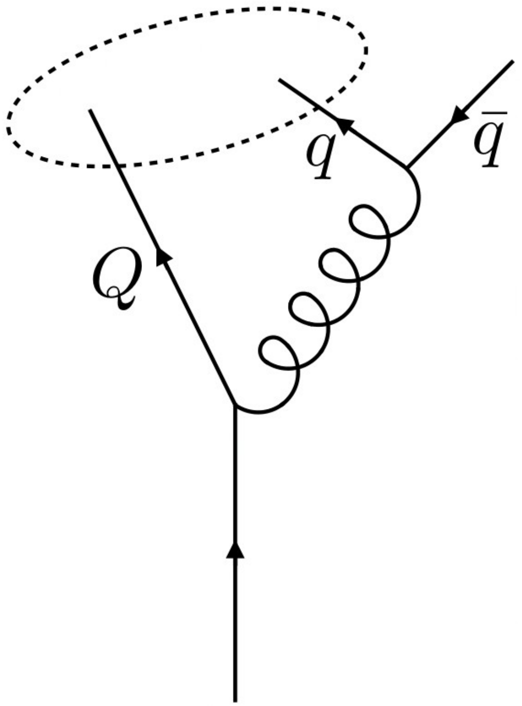
$$|\mathcal{M}(z_q, z_{\bar{q}})|^2 \propto \frac{z_q z_{\bar{q}}\left(\theta_{q\bar{q}}^2(\theta_{Qq}^2+\theta_{Q\bar{q}}^2)-(\theta_{Qq}^2-\theta_{Q\bar{q}}^2)^2\right)+\theta_{q\bar{q}}^2(z_q^2\theta_{Qq}^2+z_{\bar{q}}^2\theta_{Q\bar{q}}^2)}{z_q z_{\bar{q}}\theta_{q\bar{q}}^4(z_q+z_{\bar{q}})^2\left(z_q\theta_{Qq}^2+z_{\bar{q}}\theta_{Q\bar{q}}^2\right)^2}$$

- Triple collinear phase space:

$$d\Pi_3 \propto \frac{z_q z_{\bar{q}}\, dz_q\, dz_{\bar{q}}\, d\theta_{q\bar{q}}^2\, d\theta_{Qq}^2\, d\theta_{Q\bar{q}}^2}{\sqrt{2\theta_{q\bar{q}}^2\theta_{Qq}^2+2\theta_{q\bar{q}}^2\theta_{Q\bar{q}}^2+2\theta_{Qq}^2\theta_{Q\bar{q}}^2-\theta_{q\bar{q}}^4-\theta_{Qq}^4-\theta_{Q\bar{q}}^4}}$$

- The splitting function diverges in the $\theta_{q\bar{q}} \to 0$ limit.

- $d\Pi_3|\mathcal{M}(z_q, z_{\bar{q}})|^2$ is invariant under the $z_q \to \lambda z_q\,, z_{\bar{q}} \to \lambda z_{\bar{q}}$ transformations.



Soft Drop flavour @ NNLO

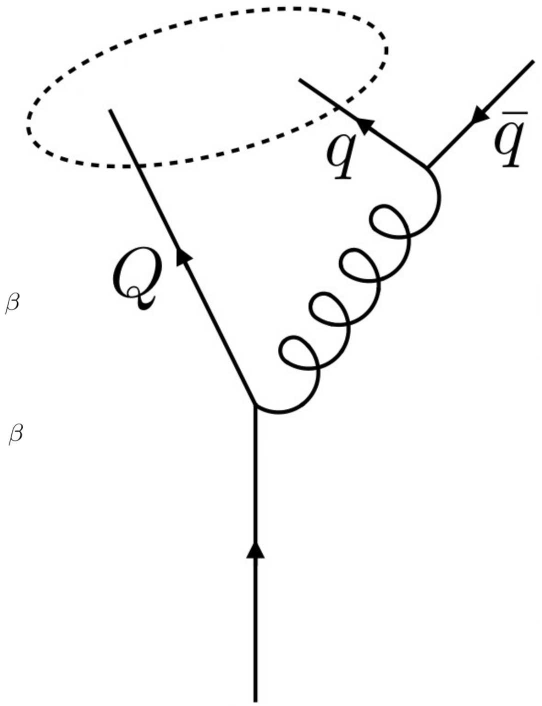
- If the dashed oval represents the jet boundary, then Soft Drop screens the singularity.
- If the dashed oval represents the effective grooming boundary, then Soft Drop fails to screen the singularity.

...why is that?

$$\Theta_{\mathrm{SD}}^{\mathrm{C/A}} = \overbrace{\Theta(\theta^2_{Q\bar{q}} > \theta^2_{Qq})\Theta(\theta^2_{q\bar{q}} > \theta^2_{Qq})}^{\text{C/A reclustering}} \times$$
$$\times \underbrace{\Theta\left(z_q > z_{\mathrm{cut}}\left(\frac{\theta^2_{Qq}}{R^2}\right)^{\beta}\right)}_{q\text{ passes SD}} \underbrace{\Theta\left(z_{\mathrm{cut}}\left(\frac{\theta^2_{q\bar{q}}}{R^2}\right)^{\beta} > z_{\bar{q}}\right)}_{\bar{q}\text{ fails SD}}$$

Rescaling:

$$z_q = x_q z_{\mathrm{cut}}\left(\frac{\theta^2_{Qq}}{R^2}\right)^{\beta}$$
$$z_{\bar{q}} = x_{\bar{q}} z_{\mathrm{cut}}\left(\frac{\theta^2_{Q\bar{q}}}{R^2}\right)^{\beta}$$

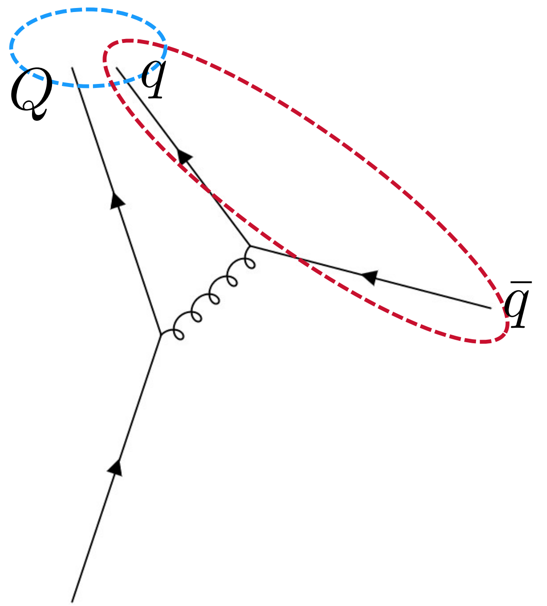


$$d\Pi_3|\mathcal{M}(z_q, z_{\bar{q}})|^2\Theta_{\mathrm{SD}}^{\mathrm{C/A}} \simeq d\Pi_3|\mathcal{M}(x_q, x_{\bar{q}})|^2\Theta(\theta^2_{Q\bar{q}} > \theta^2_{Qq})\Theta(\theta^2_{q\bar{q}} > \theta^2_{Qq})\Theta(x_q > 1)\Theta(1 > x_{\bar{q}})$$

- The constraints do nothing to regulate the collinear singularity.

JADE Soft Drop

- Can we modify Soft Drop to save the day?
- gen-k_T algorithms do not cluster two soft particles together, if there is a hard particle around at smaller angle, but JADE does.
- Idea: we can change the algorithm use for reclustering.
- Let's look at the problematic configuration with JADE reclustering



$$\Theta_{\rm SD}^{\rm JADE} = \overbrace{\Theta(m_{Q\bar q}^2 > m_{Qq}^2)\Theta(m_{q\bar q}^2 > m_{Qq}^2)}^{\rm JADE\ reclustering}\overbrace{\Theta\left(z_q > z_{\rm cut}\left(\frac{\theta_{Qq}^2}{R^2}\right)^\beta\right)}^{q\ \rm passes\ SD}\overbrace{\Theta\left(z_{\rm cut}\left(\frac{\theta_{Q\bar q}^2}{R^2}\right)^\beta > z_{\bar q}\right)}^{\bar q\ \rm fails\ SD}$$

$$= \Theta\left(x_{\bar q}\theta_{Q\bar q}^{2(\beta+1)} > x_q\theta_{Qq}^{2(\beta+1)}\right)\Theta\left(x_{\bar q}z_{\rm cut}\left(\frac{\theta_{Q\bar q}^2}{R^2}\right)^\beta\theta_{q\bar q}^2 > \theta_{Qq}^2\right)\Theta(x_q > 1)\Theta(1 > x_{\bar q})$$

Rescaling:

$$z_q = x_q z_{\rm cut}\left(\frac{\theta_{Qq}^2}{R^2}\right)^\beta$$

$$z_{\bar q} = x_{\bar q} z_{\rm cut}\left(\frac{\theta_{Q\bar q}^2}{R^2}\right)^\beta$$

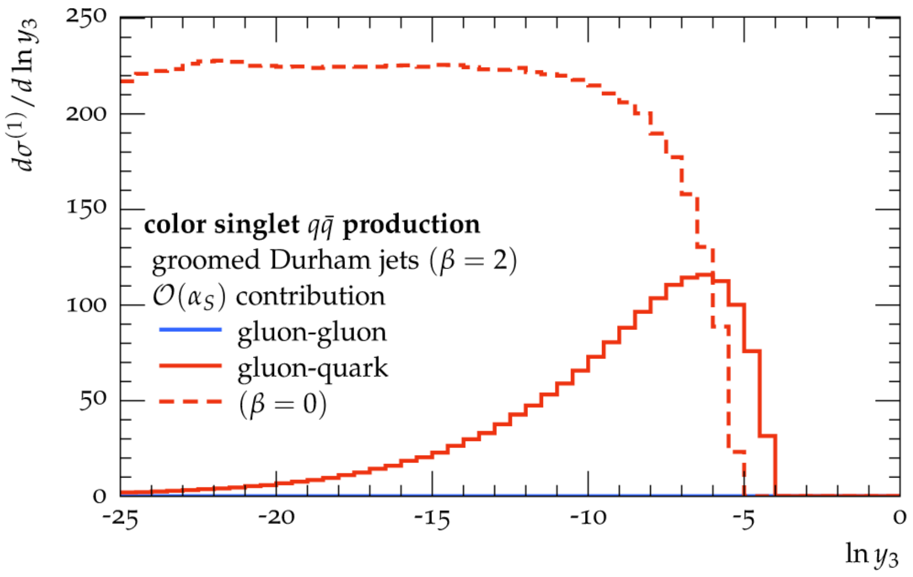
- With JADE reclustering energies and angles are coupled even after rescaling: the singularity is successfully screened!

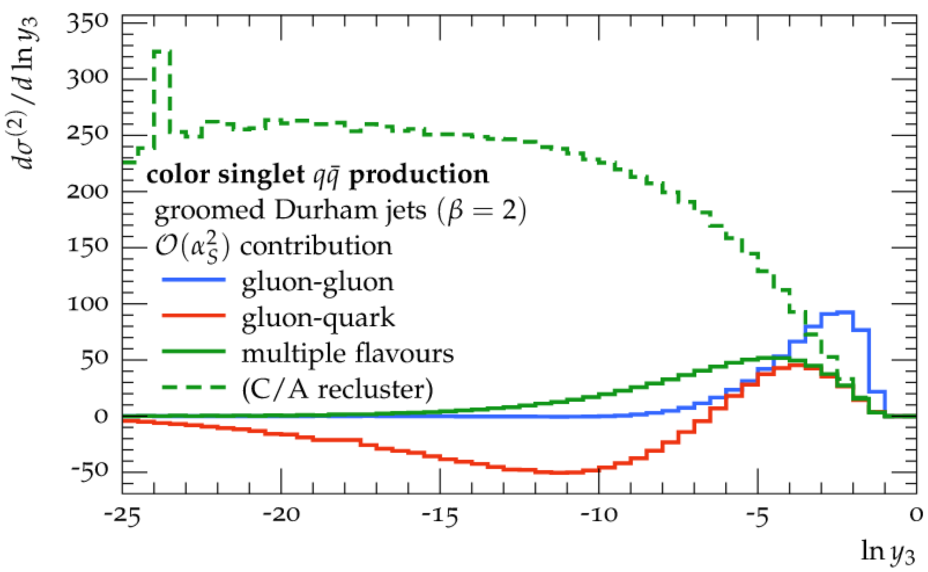
Numerical checks

- Introduce a resolution parameter to separate

$$\sigma^{\rm NNLO} = \int_0^{y_3} dy_3' \frac{d\sigma}{dy_3'} + \int_{y_3}^{y_{\rm max}} dy_3' \frac{d\sigma}{dy_3'}$$

- Following BSZ, we use the 3-jet resolution parameter
- Distributions of non-Born configurations should vanish in the $y_3 \to 0$ limit





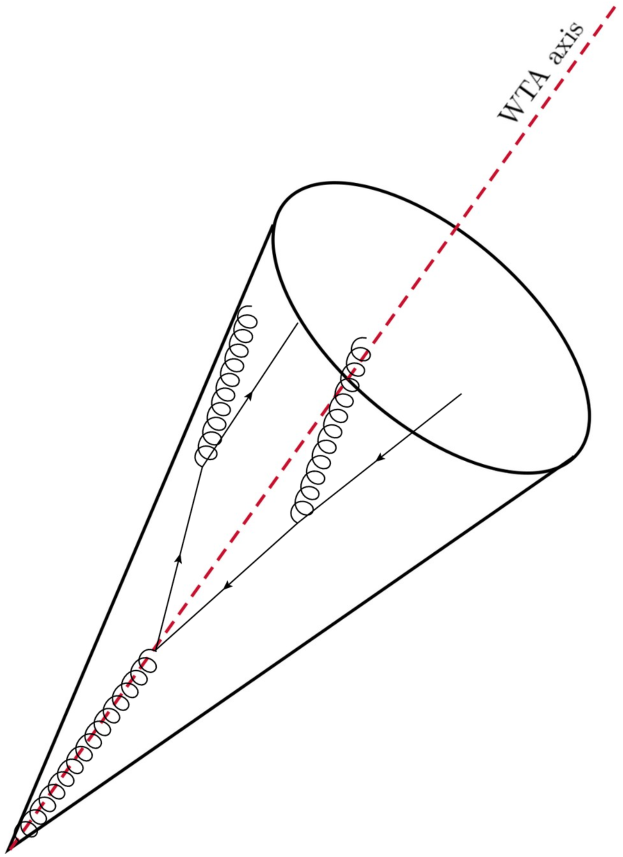
The WTA flavour

- Idea: by asymptotic freedom, jet flavour is unambiguous in deep UV.
- Define the flavor of the jet to be the flavor of the particle(s) lying along the WTA axis.
- WTA flavor is soft safe, but not collinear safe: thus, we introduce WTA fragmentation functions.
- Unlike the micro-jet approach (Dasgupta et al. 2014), the evolution equations are DGLAP-like, hence, linear

upper limits take into account the WTA condition

$$Q^2 \frac{df_q(x,Q^2)}{dQ^2} = \frac{\alpha_S}{2\pi} \int_x^{\min[1,2x]} \frac{dz}{z} \left[P_{qg\leftarrow q}\left(\frac{x}{z}\right) f_q(z,Q^2) + P_{q\bar{q}\leftarrow g}\left(\frac{x}{z}\right) f_g(z,Q^2) \right]$$

$$Q^2 \frac{df_g(x,Q^2)}{dQ^2} = \frac{\alpha_S}{2\pi} \int_x^{\min[1,2x]} \frac{dz}{z} \left[P_{gq\leftarrow q}\left(\frac{x}{z}\right) f_q(z,Q^2) + P_{gg\leftarrow g}\left(\frac{x}{z}\right) f_g(z,Q^2) \right]$$



- Interesting properties, e.g. IR fixed point.

Solution for an IR gluon

The differential equation can be equivalently expressed in terms of the β-function, where $\beta(\alpha_S) \equiv Q\frac{d\alpha_S}{dQ}$

$$Q^2\frac{df_g(\alpha_S)}{d\alpha_S} = -\frac{2}{\beta_0}\left[C_F\left(2\log 2-\frac{5}{8}\right)-\left(C_F\left(2\log 2-\frac{5}{8}\right)+\frac{2}{3}n_f T_R\right)f_g(\alpha_S)\right]$$

Using the lowest order β-function the solution is

$$f_g(Q^2) = \frac{C_F(2\log 2-\frac{5}{8})}{C_F(2\log 2-\frac{5}{8})+\frac{2}{3}n_f T_R}+$$
$$+\left(f_g(Q_0^2)-\frac{C_F(2\log 2-\frac{5}{8})}{C_F(2\log 2-\frac{5}{8})+\frac{2}{3}n_f T_R}\right)\left(\frac{\alpha_S(Q_0^2)}{\alpha_S(Q^2)}\right)^{\frac{2}{\beta_0}\left(C_F(2\log 2-\frac{5}{8})+\frac{2}{3}n_f T_R\right)}$$

with: $Q_0^2 > Q^2$

$Q^2 \sim 1\,\mathrm{GeV}$

Note that there is an IR fixed point for

$$\lim_{Q_0^2\to\infty} f_g(Q^2) = \frac{C_F(2\log 2-\frac{5}{8})}{C_F(2\log 2-\frac{5}{8})+\frac{2}{3}n_f T_R} \equiv \bar{f}_g$$

$$\frac{2}{\beta_0}\left(C_F\left(2\log 2-\frac{5}{8}\right)+\frac{2}{3}n_f T_R\right) \simeq 0.7$$

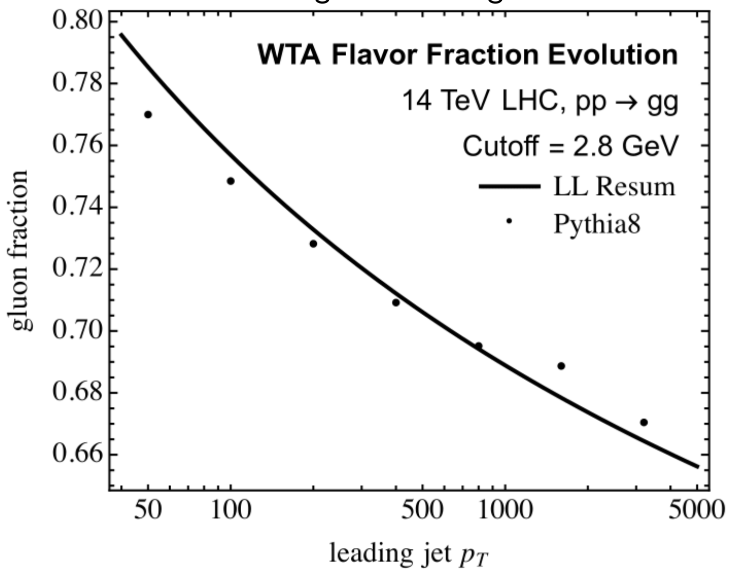
$$\frac{\alpha_S(5\,\mathrm{TeV})}{\alpha_S(1\,\mathrm{GeV})} \simeq 0.22$$

$$0.22^{0.7} \simeq 0.35$$

Comparison with parton shower

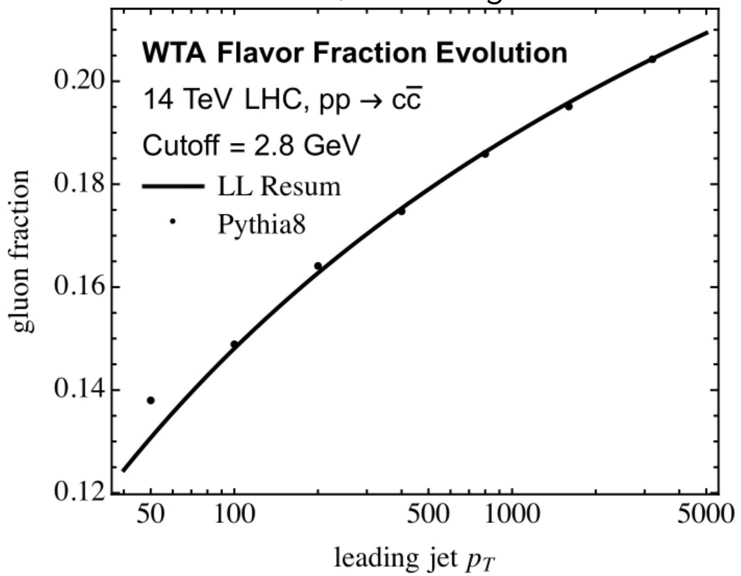
There is a good agreement between the LL resummation and Pythia parton shower

g_{UV} into a g_{IR}



$$f_g(Q_{UV}) = 1$$
$$f_q(Q_{UV}) = 0 \quad \forall\, q$$

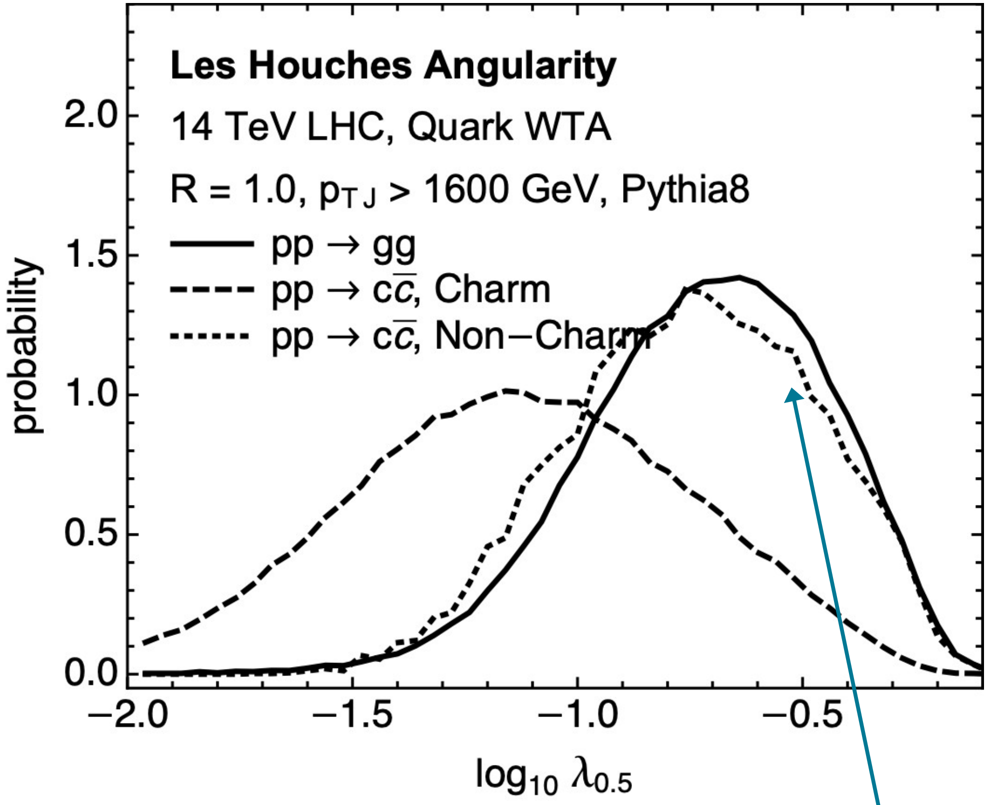
c_{UV} into a g_{IR}

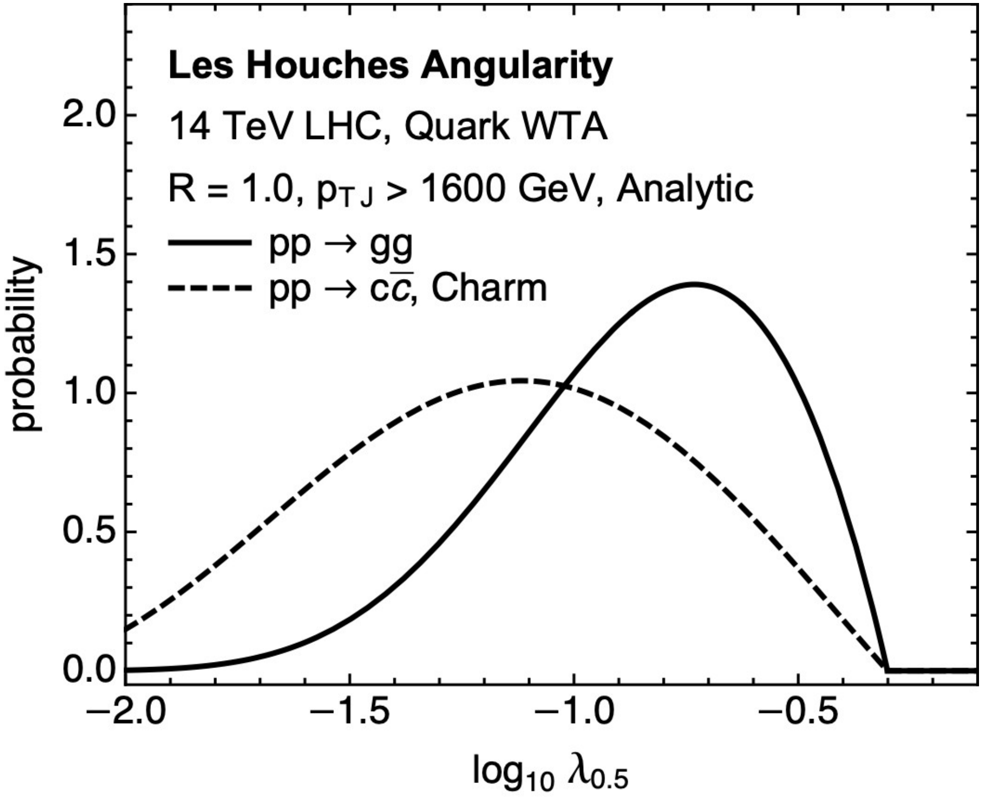


$$f_c(Q_{UV}) = 1$$
$$f_g(Q_{UV}) = f_q(Q_{UV}) = 0 \quad \forall\, q \neq c$$

Flavoured observables

Using WTA flavour we can also compute flavoured observables, e.g. angularities, and we find a good qualitative agreement with MC





Flavour change starts at order α_S^2

Matching to fixed order

$$\Sigma_{\text{match, mult}}(\lambda_\alpha) = \sum_\delta \Sigma^\delta_{\text{match, mult}}(\lambda_\alpha)$$

$$\Sigma^\delta_{\text{match, mult}}(\lambda_\alpha) = \Sigma^\delta_{\text{res}}(\lambda_\alpha)\Bigg[1 + \frac{\Sigma^{\delta,(1)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)}{\sigma^{\delta,(0)}} + \\ + \frac{1}{\sigma^{\delta,(0)}}\left(-\bar{\Sigma}^{\delta,(2)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(2)}_{\text{res}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)\frac{\Sigma^{\delta,(1)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)}{\sigma^{\delta,(0)}}\right)\Bigg]$$

$$\begin{cases} \sigma = \Sigma(1) \\ \Sigma^{(k)} \propto \alpha^2_{\text{EW}}\alpha^{1+k}_{\text{S}} \\ \bar{\Sigma} = \sigma - \Sigma \end{cases}$$

$$\frac{\alpha_S}{2\pi}C^{\delta,(1)} \equiv \lim_{\lambda\to 0}\frac{\Sigma^{\delta,(1)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)}{\sigma^{\delta,(0)}}$$

Numerical checks (gluon-gluon)

- Introduce a resolution parameter to separate

$$\sigma^{\text{NNLO}} = \int_0^{y_3} dy_3' \frac{d\sigma}{dy_3'} + \int_{y_3}^{y_{\max}} dy_3' \frac{d\sigma}{dy_3'}$$

- The process is $\mu^+\mu^- \to H \to g\,g$ (where a top loop induces the effective coupling between the gluons and the Higgs)