Neff: SM prediction and BSM implications

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based on: M.E.A. 1812.05605 & 2001.04466 [JCAP]
with Cielo, Mangano & Pisanti 2306.05460 [PRD]
with Hooper, Krnjaic & Pierre 1901.02010 [JHEP]
with Sabti, Alvey, Fairbairn & Blas 1910.01649 [JCAP]

3rd CAGE BSM workshop
Annecy
21-05-2024

Motivation

Precision Cosmology

Today:

BBN $N_{\text{eff}}^{\text{BBN}} = 2.86 \pm 0.28$ — **Pisanti et al. 2011.11537**, **Yeh et al. 2207.13133**

Planck+BAO $N_{\text{eff}}^{\text{CMB}} = 2.99 \pm 0.17$ — **Planck 2018, 1807.06209**

circa 2010

BBN $N_{\text{eff}}^{\text{BBN}} \simeq 3 \pm 1$

WMAP+H0++ $N_{\text{eff}}^{\text{CMB}} \simeq 4 \pm 1$

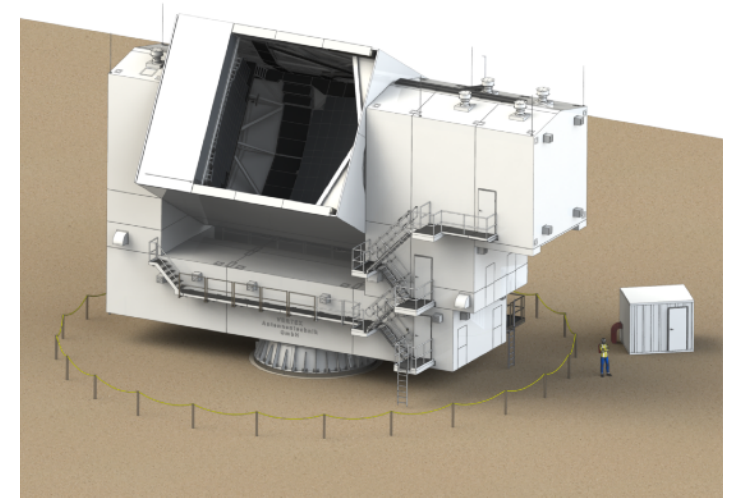
Next years:

Simons Observatory:

$\sigma(N_{\text{eff}}) = 0.05$ **~2029**

under construction and fully funded

CMB-S4:



$\sigma(N_{\text{eff}}) = 0.03$ **~2035?**

main recommendation of the US P5 report

Outline/Motivation

Understand how to maximize the BSM potential of these measurements

1) The Physics of Neutrino Decoupling (Tools)

a) Simplified approach to solve neutrino decoupling M.E.A. 1812.05605 & 2001.04466

b) Comparison with traditional SM evaluations

Understand novel physical aspects of Neff in the SM and explore the cosmological implications of well motivated scenarios

2) Applications:

a) SM : Neff at NLO with Cielo, Mangano & Pisanti 2306.05460

b) Constraints on MeV-scale thermal dark matter with Sabti, Alvey, Fairbairn & Blas 1910.01649

c) Constraints on a light $L_\mu - L_\tau$ gauge boson with Hooper, Krnjaic & Pierre 1901.02010

3) Conclusions

Please, feel free to stop me at any time! 😃

The Process of Neutrino Decoupling

The origin of the Cosmic Neutrino Background

t ~ 0.1s

T > 2 MeV

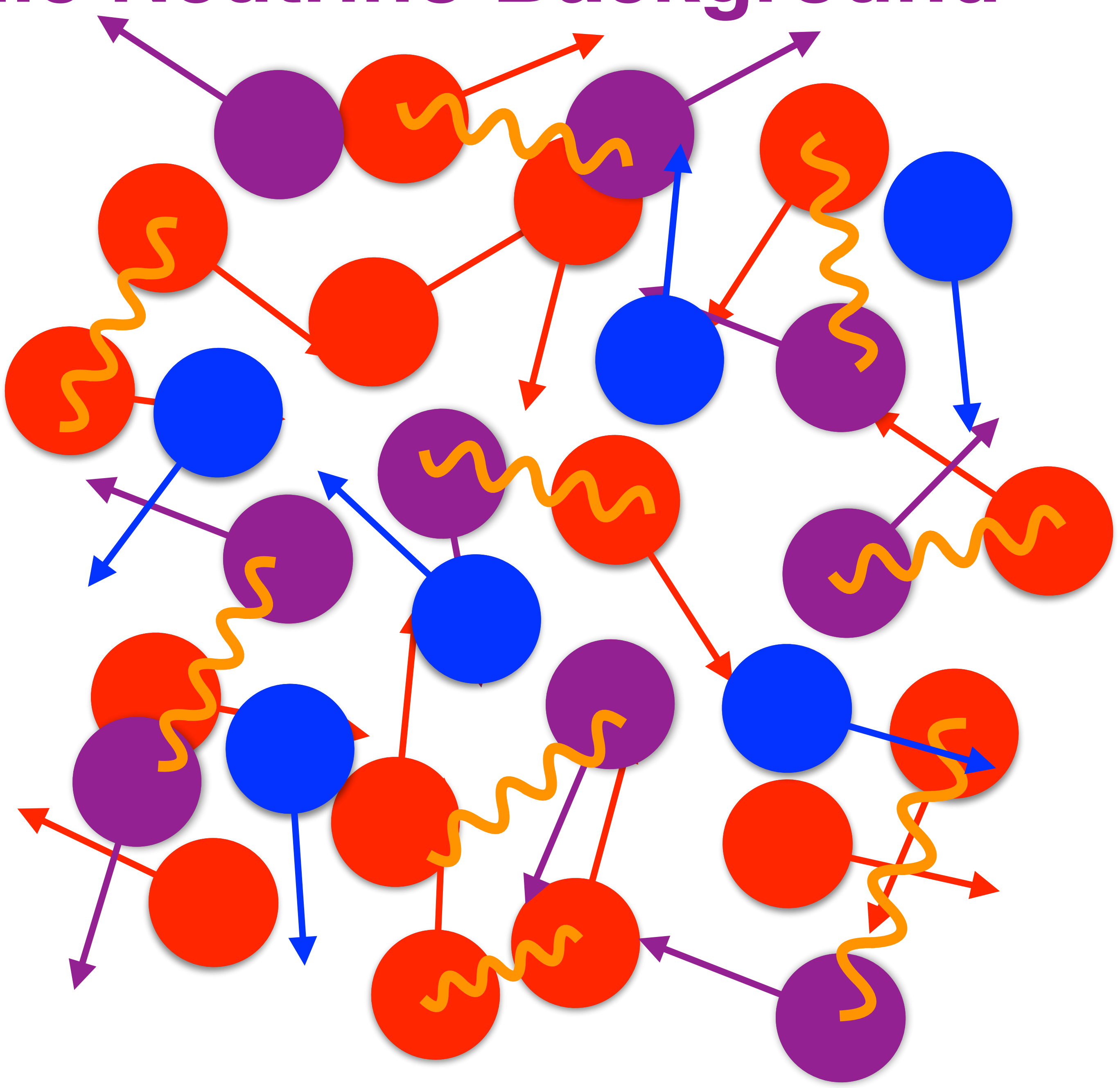
Highly Efficient Processes

$$e^+e^- \leftrightarrow \gamma\gamma$$

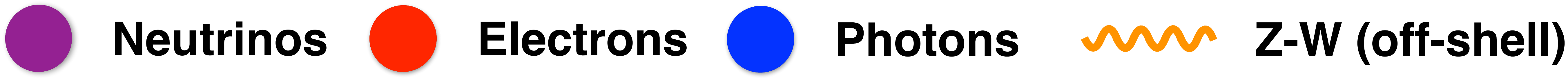
$$e^\pm\gamma \leftrightarrow e^\pm\gamma$$

$$e^+e^- \leftrightarrow \bar{\nu}_i\nu_i$$

$$e^\pm\nu_i \leftrightarrow e^\pm\nu_i$$



In comoving coordinates

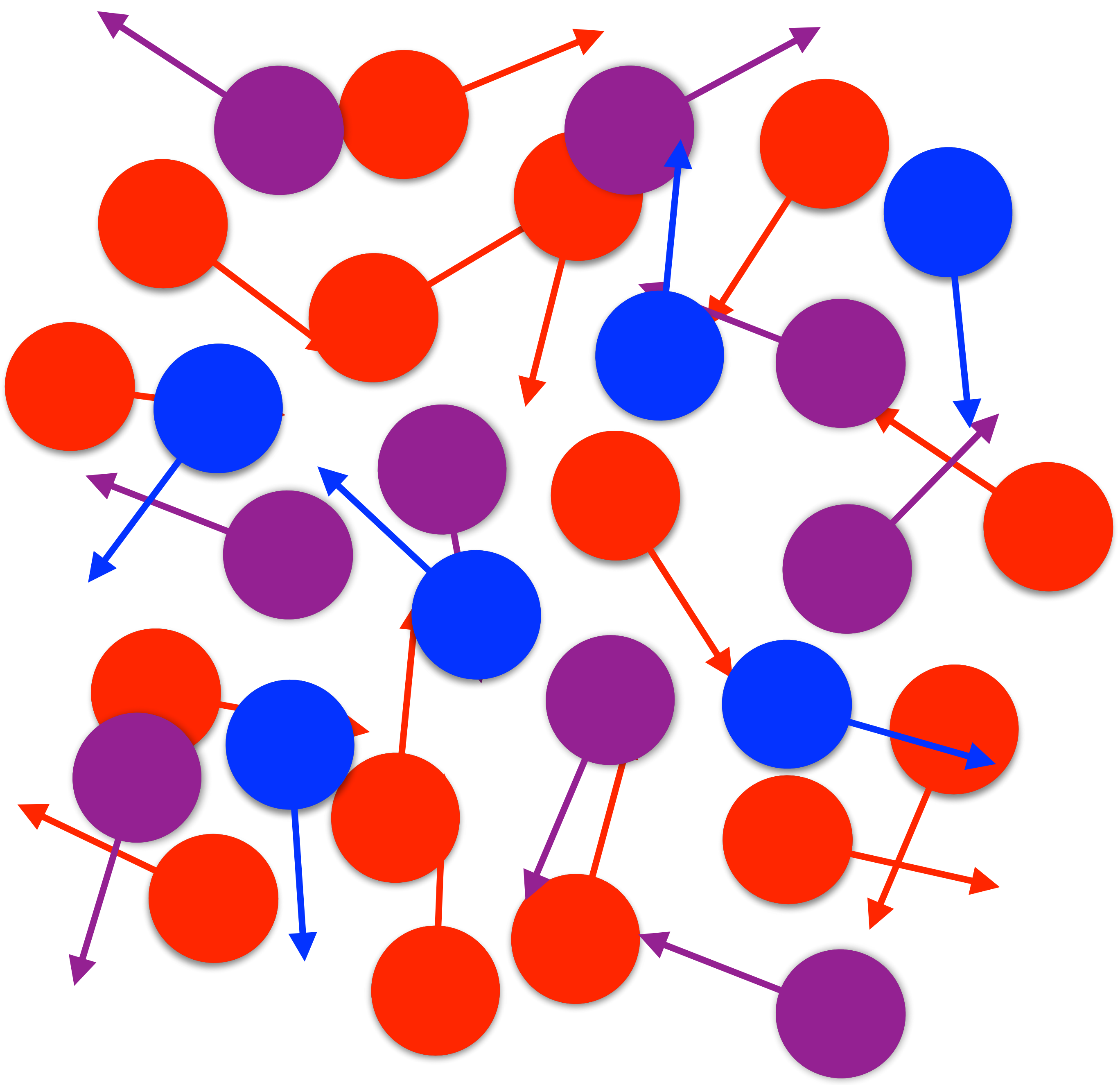


The Process of Neutrino Decoupling

$m_e < T < 2$ MeV

Highly Efficient Processes

$$e^+e^- \leftrightarrow \gamma\gamma$$
$$e^\pm\gamma \leftrightarrow e^\pm\gamma$$



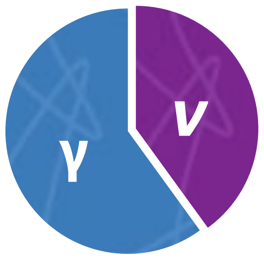
In comoving coordinates

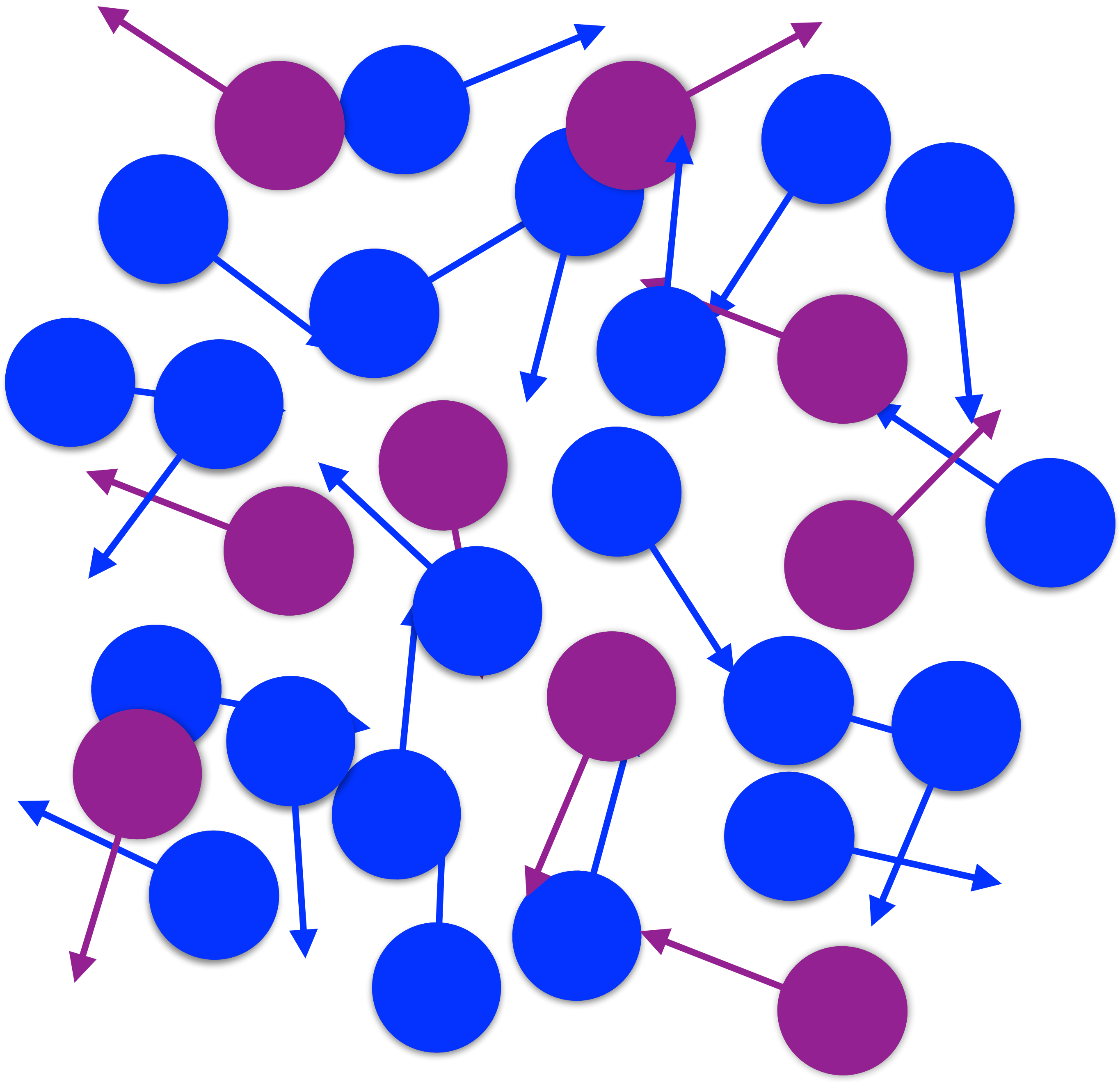
Neutrinos **Electrons** **Photons** **Z-W (off-shell)**

The Process of Neutrino Decoupling

$T_\gamma < m_e/10$

- **Black Body Photon Radiation**
- **Only Neutrinos and Photons**
- $T_\gamma/T_\nu = 1.4$
- $\rho_\gamma/(\rho_\nu + \rho_\gamma) = 0.6$

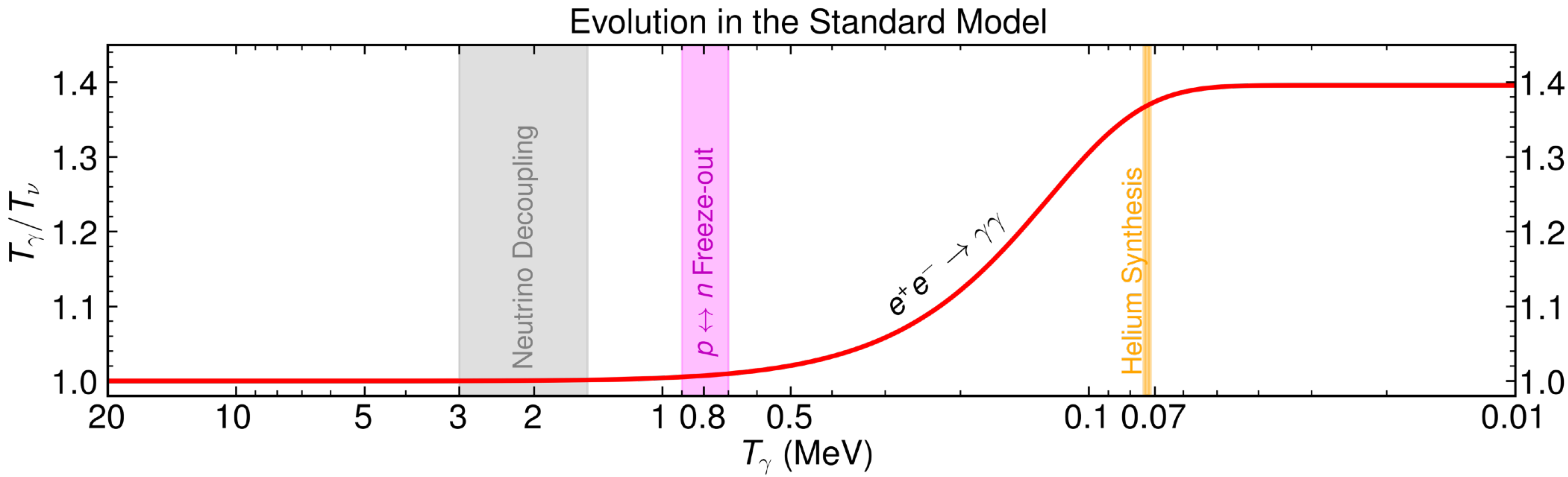




In comoving coordinates

Neutrinos **Electrons** **Photons** **Z-W (off-shell)**

Cosmic Neutrino Background



- $$N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\rm rad}-\rho_\gamma}{\rho_\gamma}\right)$$

$$N_{\rm eff} = 3\left(\frac{1.4 T_\nu}{T_\gamma}\right)^4$$

- $N_{\rm eff}^{\rm SM} = 3.044(1)$

Mangano et al. hep-ph/0506164
de Salas & Pastor 1606.06986
Bennett, Buldgen, Drewes & Wong 1911.04504
Escudero Abenza 2001.04466
Akita & Yamaguchi 2005.07047
Froustey, Pitrou & Volpe 2008.01074
Gariazzo, de Salas, Pastor et al. 2012.02726
Hansen, Shalgar & Tamborra 2012.03948

Why is it not exactly 3?

Recently reviewed by Akita & Yamaguchi, 2210.10307, see also the nice review by Dolgov hep-ph/0202122

1) Neutrino Decoupling is not instantaneous

$$\sigma \sim G_F^2 E_\nu^2$$

2) Weak Interactions freeze out at T = 2-3 MeV hence, some heating from e^+e^- annihilation

$$n \langle \sigma v \rangle \simeq G_F^2 T^5 \simeq H$$

$$\Delta N_{\rm eff} \simeq +0.03$$

Kolb et al. '82
Dolgov et al. '97

3) Finite Temperature QED corrections

$$\delta m_e^2(T),\ \delta m_\gamma^2(T)$$

$$\Delta N_{\rm eff} \simeq +0.01$$

Heckler '94
Bennet et al. '21

4) Neutrino oscillations are active at T < 10 MeV

$$\Delta N_{\rm eff} \simeq +0.0007$$

Mangano et al. '05
de Salas & Pastor '16

$$t_\nu^{\rm os} \sim \frac{12\,T}{\Delta m^2} \qquad t_{\rm exp} = \frac{1}{2H} \sim \frac{m_{Pl}}{3.44\sqrt{10.75}T^2} \qquad t_\nu^{\rm scat} \sim \frac{1}{G_F^2\,T^5}$$

Standard Model prediction as of 2021: $N_{\rm eff}^{\rm SM} = 3.0440(2)$

Akita & Yamaguchi 2005.07047
Froustey, Pitrou & Volpe 2008.01074
Bennett, Buldgen, de Salas, Drewes, Gariazzo, Pastor & Wong 2012.02726

CMB-S4 $\delta N_{\rm eff} \simeq 0.03$

Solving for Neutrino Decoupling

1) Traditional SM approach

Dodelson & Turner, Hannestad & Madsen,
Dolgov et al. '90, de Salas and Pastor, 1606.06986
Mangano et al. hep-ph/0506164

Work in terms of the Liouville equation:

$$\frac{\partial f_\nu}{\partial t} - Hp\frac{\partial f_\nu}{\partial p} = C[f_\nu]$$

Accounts for: QED corrections + non thermal effects + neutrino oscillations

System of ~200 *STIFF* coupled diff equations: computationally very expensive

see nice code: FortEPiaNO Gariazzo, Pastor & de Salas

2) Traditional BSM approach

Boehm, Dolan and McCabe 1207.0497, 1303.6270
Serpico and Raffelt astro-ph/0403417
Kolb, Turner and Walker PRD 34 (1986) 2197

Assume neutrinos decouple instantaneously and use entropy conservation

Fast and good as a first approximation

Does not account for the neutrino interactions

Neutrino Decoupling

Simplified approach: Escudero '18-'21

check code at
https://github.com/MiguelEA/nudec_BSM/

Taking advantage of the fact that neutrinos were in thermal equilibrium

Assume that neutrinos follow Fermi-Dirac distributions parametrized by a dynamical temperature T_ν and simply write down simple ODEs for them

$$\frac{\partial f_\nu}{\partial t} - Hp\frac{\partial f_\nu}{\partial p} = C[f_\nu] \quad \xrightarrow{\int [] p^3 dp} \quad \frac{d\rho}{dt} + 3H(\rho + p) = \frac{\delta \rho}{\delta t} = \int g\, E\, \frac{d^3p}{(2\pi)^3}\, \mathcal{C}[f]$$

Results in: **2-3 simple coupled differential equations for** $T_\gamma,\, T_\nu$

$$\frac{dT_\gamma}{dt} = -\frac{4H\rho_\gamma + 3H\left(\rho_e + p_e\right) + \frac{\delta \rho_{\nu_e}}{\delta t} + 2\frac{\delta \rho_{\nu_\mu}}{\delta t}}{\frac{\partial \rho_\gamma}{\partial T_\gamma} + \frac{\partial \rho_e}{\partial T_\gamma}} \qquad \frac{dT_\nu}{dt} = -H\,T_\nu + \frac{\frac{\delta \rho_{\nu_e}}{\delta t} + 2\frac{\delta \rho_{\nu_\mu}}{\delta t}}{3\,\frac{\partial \rho_\nu}{\partial T_\nu}}$$

Analytical expressions for the SM energy transfer rates:

As a result of a 12 dimensional integral!

$$\left.\frac{\delta \rho_\nu}{\delta t}\right|_{\rm SM}^{\rm MB} = \frac{G_F^2}{\pi^5}\left[4\left(g_{eL}^2 + g_{eR}^2\right)\right]\left[32\left(T_\gamma^9 - T_\nu^9\right) + 56\, T_\gamma^4 T_\nu^4 (T_\gamma - T_\nu)\right]$$

Limit
$m_e = 0$
$f = f_{\rm MB}$

One can account for the effect of a non-zero electron mass and Pauli blocking with tabulated rates

Neutrino Decoupling

Results:

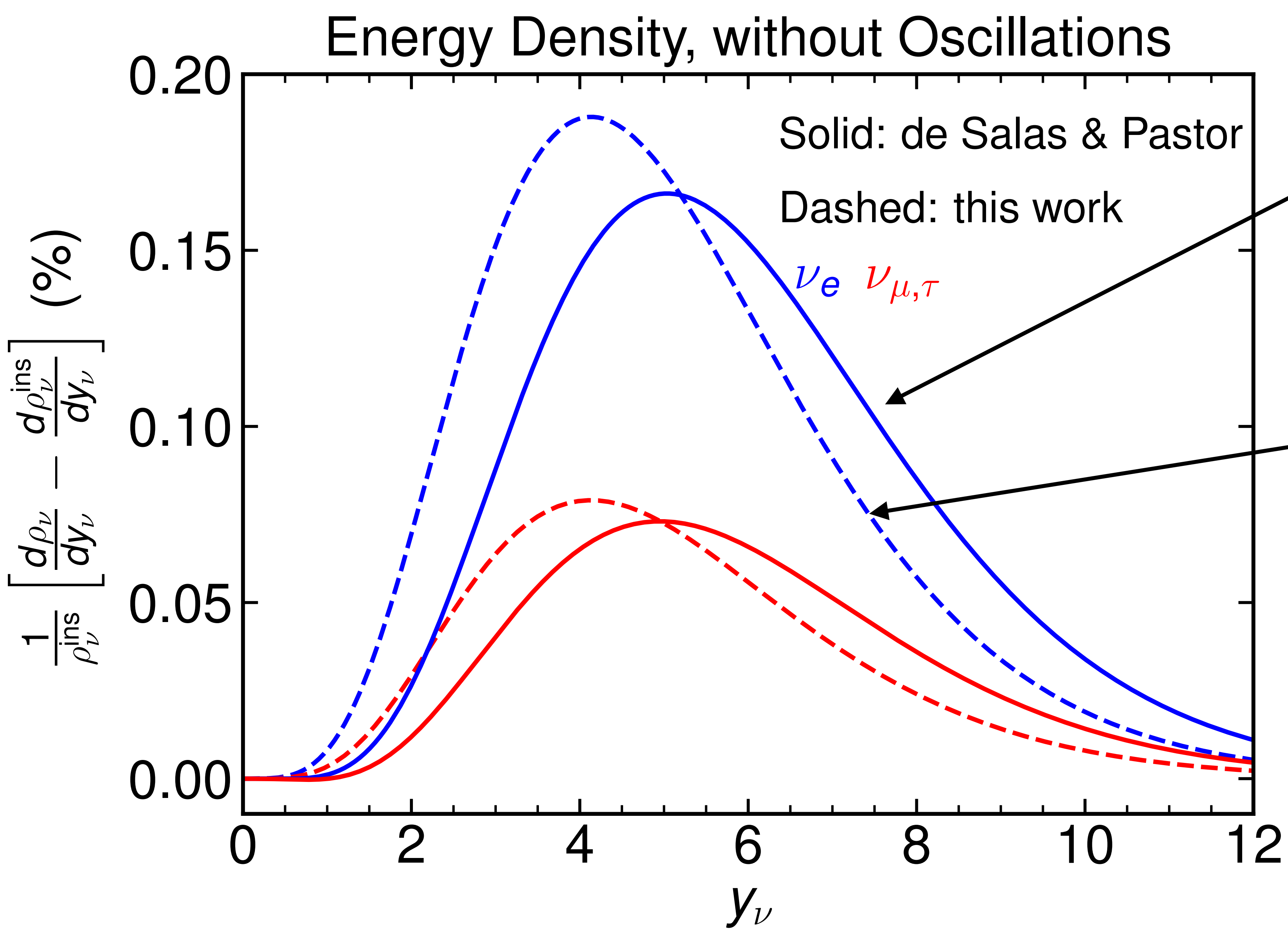
check code at
https://github.com/MiguelEA/nudec_BSM/

Neutrino Decoupling in the SM	$T_{\nu_e} = T_{\nu_\mu}$		$T_{\nu_e} \neq T_{\nu_{\mu},\nu_\tau}$		
Scenario	T_γ/T_ν	$N_{\rm eff}$	T_γ/T_{ν_e}	T_γ/T_{ν_μ}	$N_{\rm eff}$
Instantaneous decoupling	1.4010	3	1.4010	1.4010	3
Instantaneous decoupling + QED	1.3998	3.011	1.3998	1.3998	3.011
MB collision term + QED	1.3949	3.053	1.3935	1.3958	3.052
FD collision term + QED	1.3954	3.049	1.3940	1.3962	3.048
FD+m_e collision term + QED	1.3957	**3.046**	1.3946	1.3965	**3.045**

Neutrino Decoupling in the Standard Model: Key Parameters and Observables					
Parameter	$N_{\rm eff}$	$Y_{\rm P}$	$\rm D/H\vert_P$	$g_{\star s}$	$\sum m_\nu/\Omega_\nu h^2$
This work	3.045	-	-	3.931	93.05 eV
Difference w.r.t. instantaneous ν-dec	1.5 %	0.1 %	0.4 %	0.6 %	1.2 %
Difference w.r.t. [28, 29, 58, 59]	0.03 %	0.008 %	0.08 %	0.05 %	0.09 %
Current precision [3, 10]	5-6 %	1.2 %	1.1 %	-	-
Future precision [5, 6, 11, 12]	1-2 %	< 1 %	0.1? %	-	-

Agreement at better than the per-mille level for any cosmological observable

Comparison of the distribution function



Liouville

simplified approach

Agreement at better than the per-mille level even at the level of the energy density spectrum!

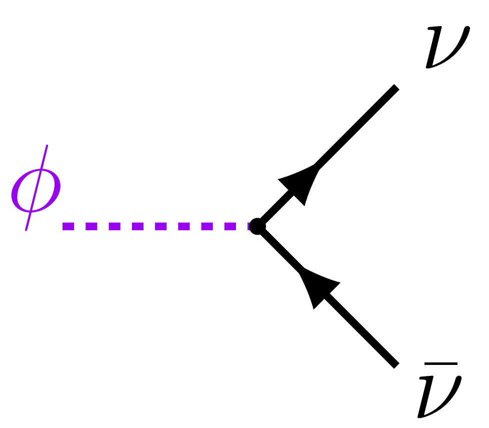
BSM accuracy?

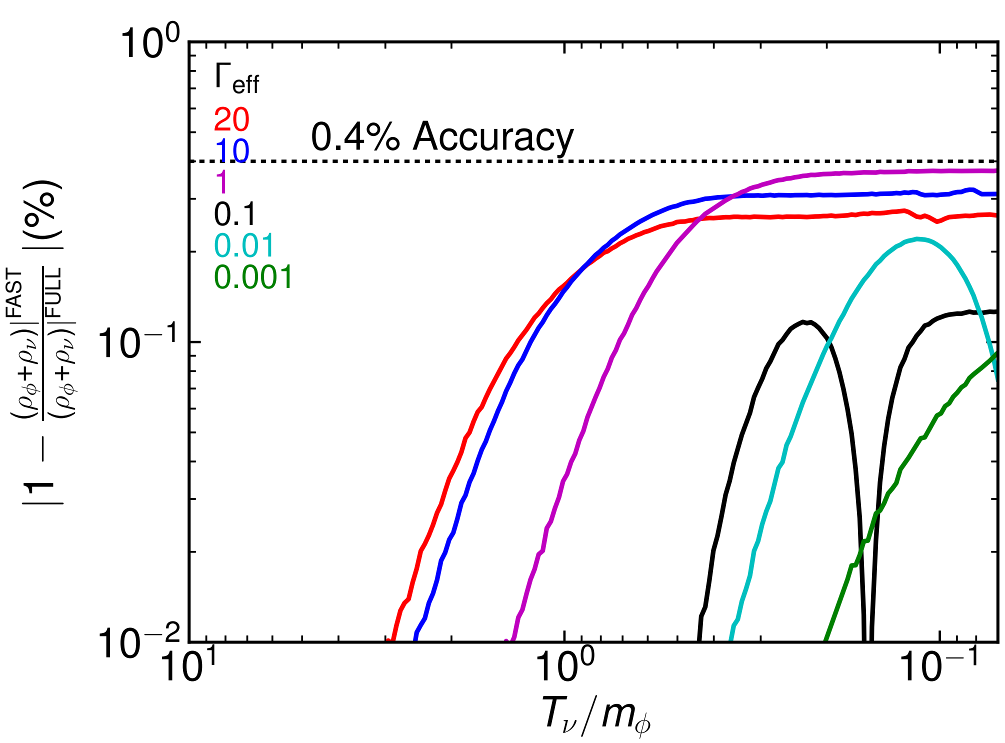
The precision should be even higher in BSM scenarios with highly interacting particles

SM: $\Gamma_{\rm SM}/H = (T/2{\rm MeV})^3$

BSM: $\Gamma_{\rm SM}/H \simeq 10^{10}$ **for** $m_{Z'} = 5\,{\rm MeV}$ **for** $g_{\rm SM} = 10^{-5}$

Demonstrated high precision even in scenarios with siezable departures from thermal equilibrium





$0.4\,\%$ **precision for** $\Gamma/H\,|_{T=m_\phi} \gtrsim 10^{-3}$

Summary & Applications

- **It turns out that the Cosmic Neutrino Background follows a blackbody spectrum with at most $\delta\rho/\rho \lesssim 5\times 10^{-5}$ distortions**
- **This means that we can solve the process of neutrino decoupling just by tracking the temperature of neutrinos and photons/electrons**

(very similar to what we do for WIMPs freeze-out)

Advantages BSM:

1) Simple: it is easy to add light BSM states and their interactions

2) Fast: it takes < 10 seconds to run **(but see Giovanetti, Lisanti, Liu & Ruderman 2109.03246)**

3) Accurate: reproduces all thermodynamic quantities with 0.1% acc

Part 2 Applications:

a) SM : Neff at NLO

b) Constraints on MeV-scale thermal dark matter

c) Constraints on a light $L_\mu - L_\tau$ gauge boson

Radiative Corrections to Interaction Rates

The prediction of Neff in the Standard Model is so precise that one wonders about other effects:

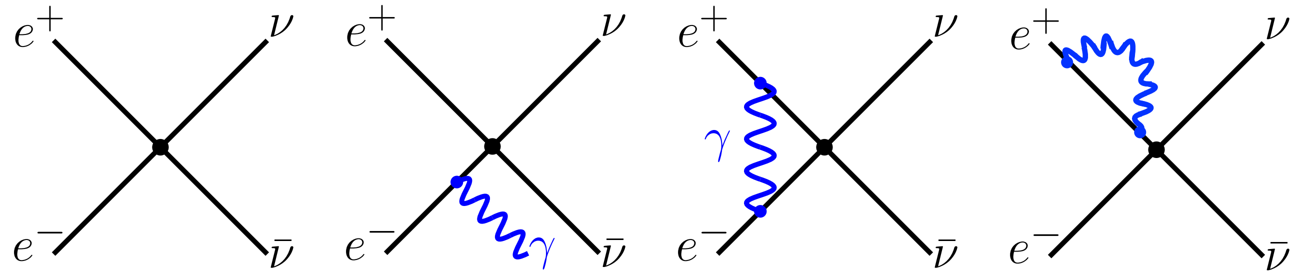
Radiative corrections to neutrino interaction rates

Why?

BBN: Radiative corrections for $p \leftrightarrow n$ interaction rates are crucial to predict accurately the primordial Helium abundance (1% error) Dicus et al. '82

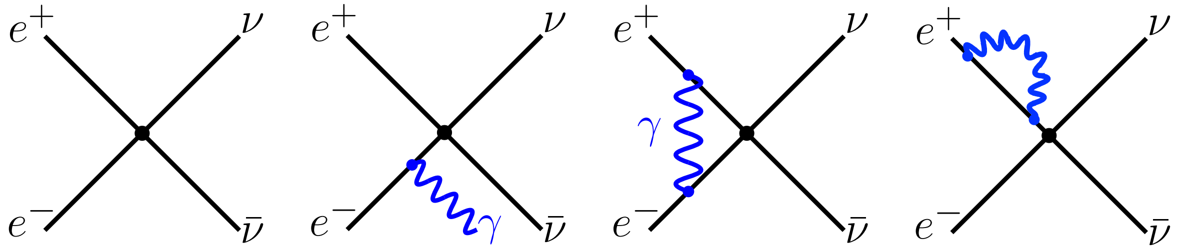
The Lab: Radiative corrections to $\nu e \to \nu e$ processes can be as large as 5% for solar neutrinos (MeV energies) Bahcall, Kamionkowski & Sirlin [astro-ph/9502003]

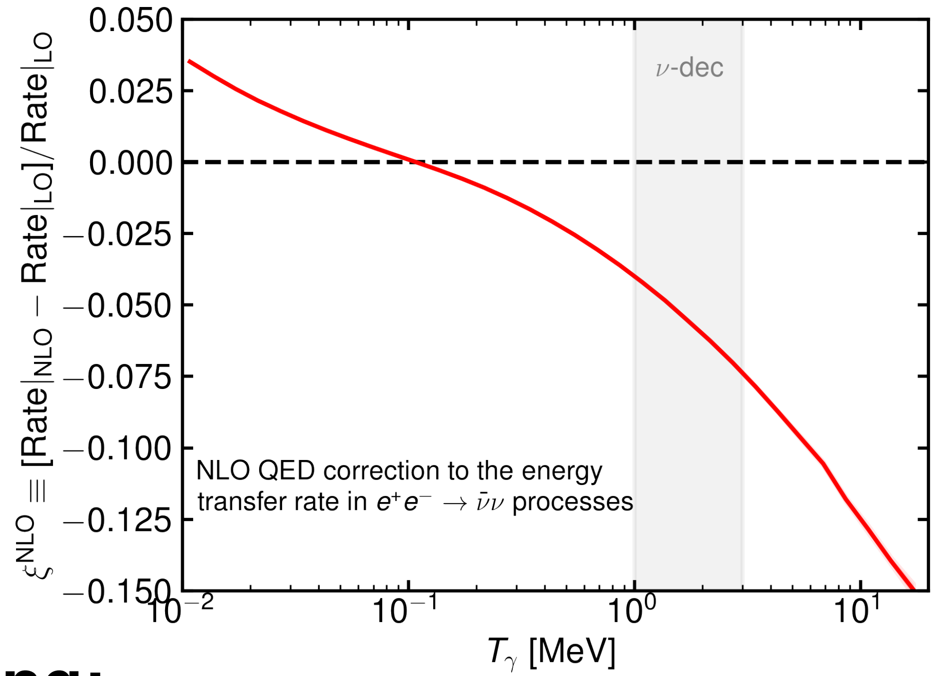
Together with Gianpiero Mangano, Ofelia Pisanti and Mattia Cielo we have for the first time accounted for the correction to the energy transfer rates which is ~ -4% at T = 1 MeV



Neff at NLO

1) Calculation performed following the real time formalism in thermal field theory





See Esposito, Mangano, Miele, Picard & Pisanti astro-ph/0301438 & astro-ph/0112384 for the thermal corrections and Passera for the radiative corrections in vacuum [hep-ph/0011190]

2) Solve for the process of neutrino decoupling:

Solving the Liouville equation in the presence of an additional particle is simply unfeasible!

$$\frac{dT_\nu}{dt} = -H\,T_\nu + \frac{\frac{\delta\rho_{\nu_e}}{\delta t} + 2\frac{\delta\rho_{\nu_\mu}}{\delta t}}{3\,\frac{\partial\rho_\nu}{\partial T_\nu}}$$

Add here the correction!

3) Result at NLO: $\Delta N_{\rm eff} \simeq -0.0007$ $\quad N_{\rm eff}^{\rm SM} = 3.0432(2) = 3.043$

Cielo, Escudero, Mangano & Pisanti 2306.05460

But, Jackson and Laine [2312.07015] have recently calculated a related quantity at NLO and they find smaller corrections and with different sing to the one we use.
Also Drewes et al. [2402.18481] have performed a partial calculation.

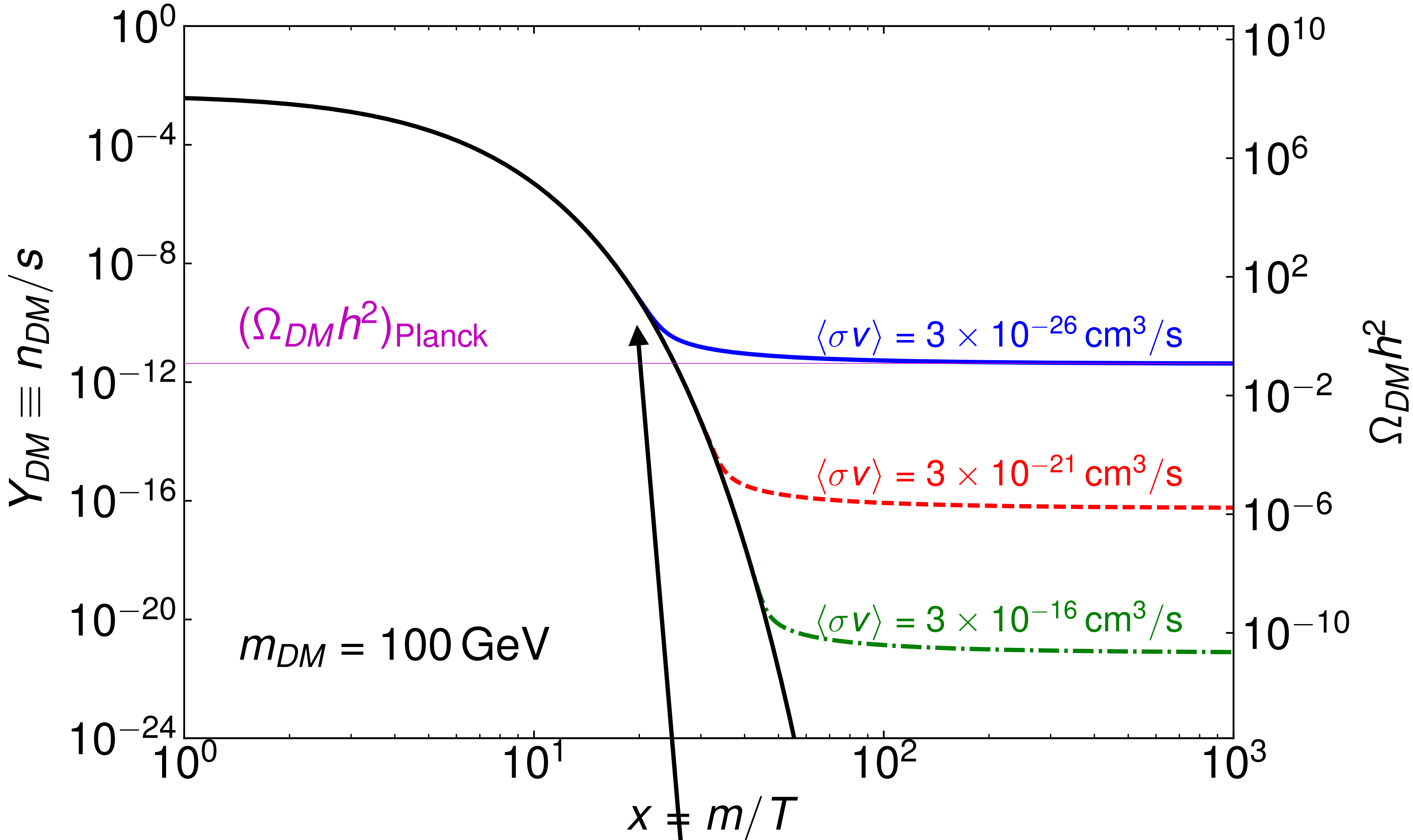
$$N_{\rm eff}^{\rm SM} = 3.043(1)$$

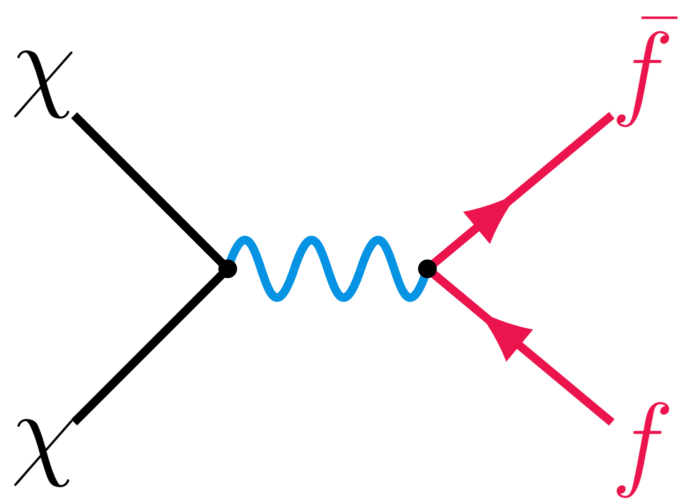
Under investigation!

CMB-S4 $\delta N_{\rm eff} \simeq 0.03$

Application N2: WIMPS

Weakly Interacting Massive Particles





- **The WIMP Miracle:**

  Stable particle with $\langle \sigma v \rangle \simeq 3 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$ **gives the observed abundance**

Key: these particles should be in thermal equilibrium with the Standard Model plasma at least up to $T = m/20$

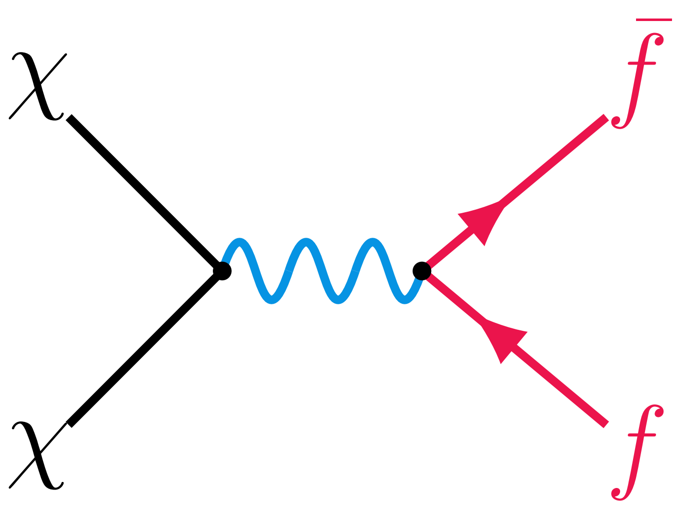
Impact of WIMPs on neutrino decoupling

BBN and the CMB give a lower bound on the thermal dark matter mass!

For early studies see:

Boehm, Dolan and McCabe 1207.0497, 1303.6270
Serpico and Raffelt astro-ph/0403417
Kolb, Turner and Walker PRD 34 (1986) 2197

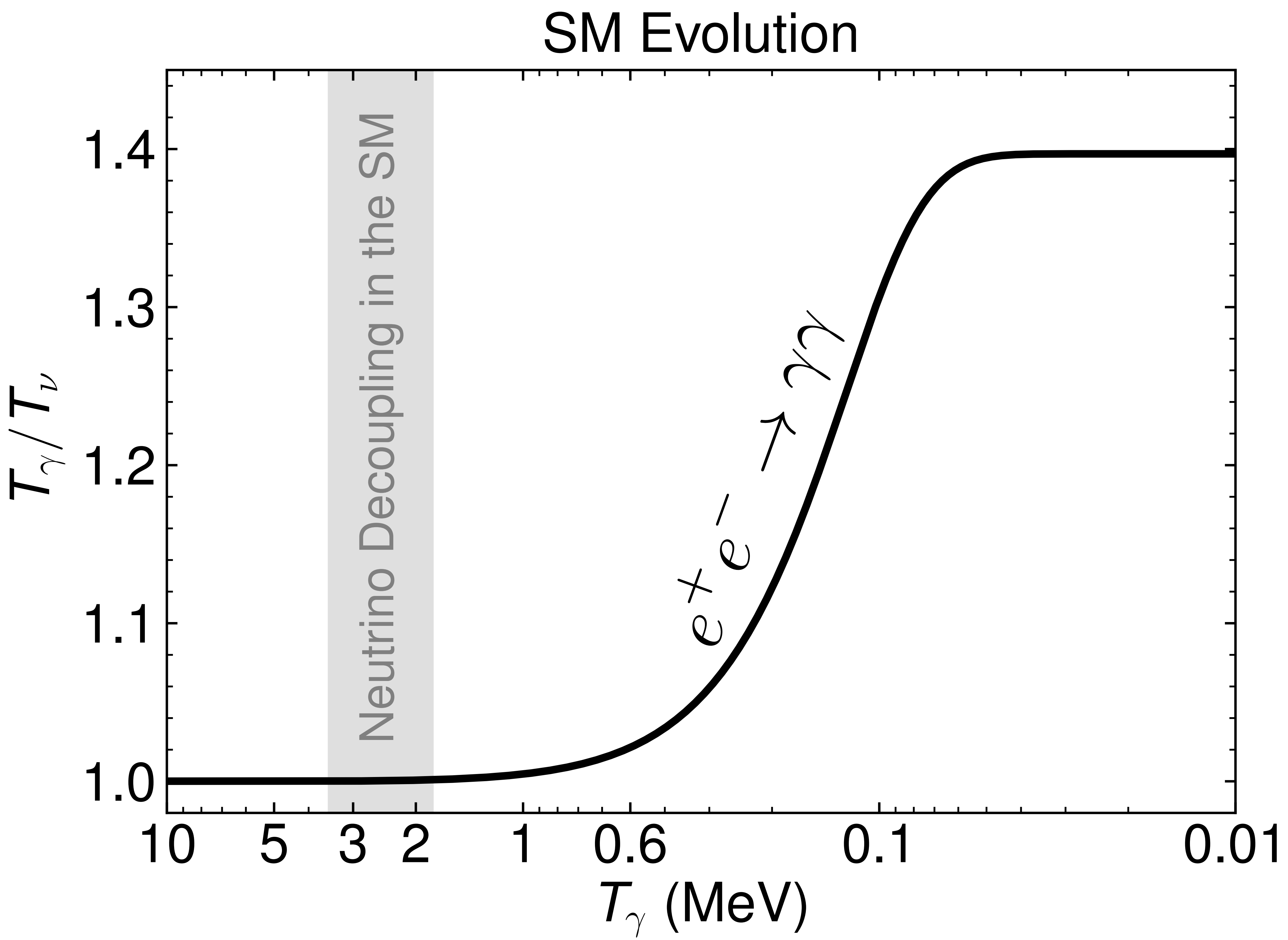
Effects on neutrino decoupling and BBN:



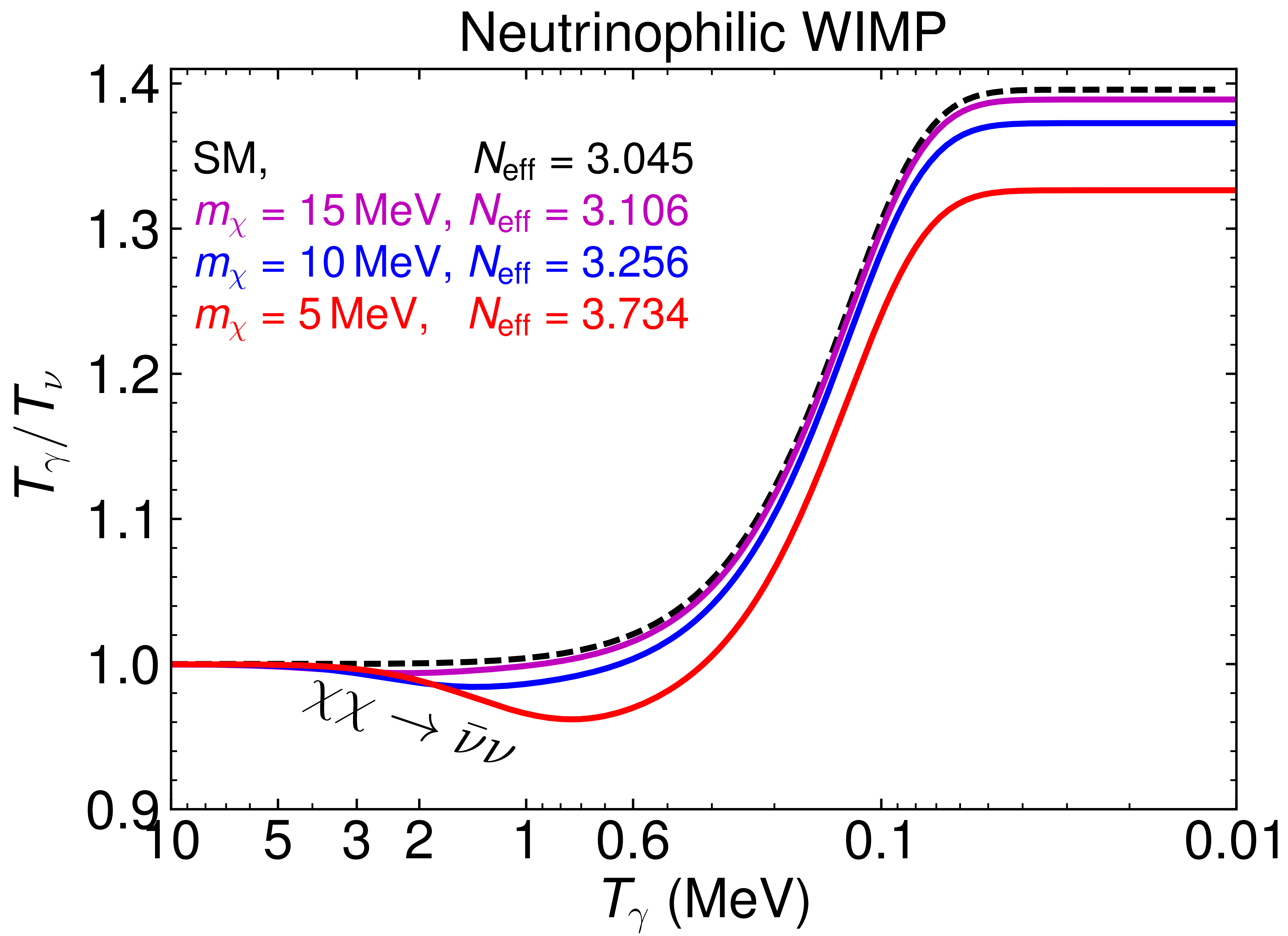
1) WIMPs will damp energy/entropy into the system — enhance the expansion history of the Universe and modify number densities

2) If WIMPs interact with electrons and neutrinos, they will delay the process of neutrino decoupling — typically leading to weaker constraints

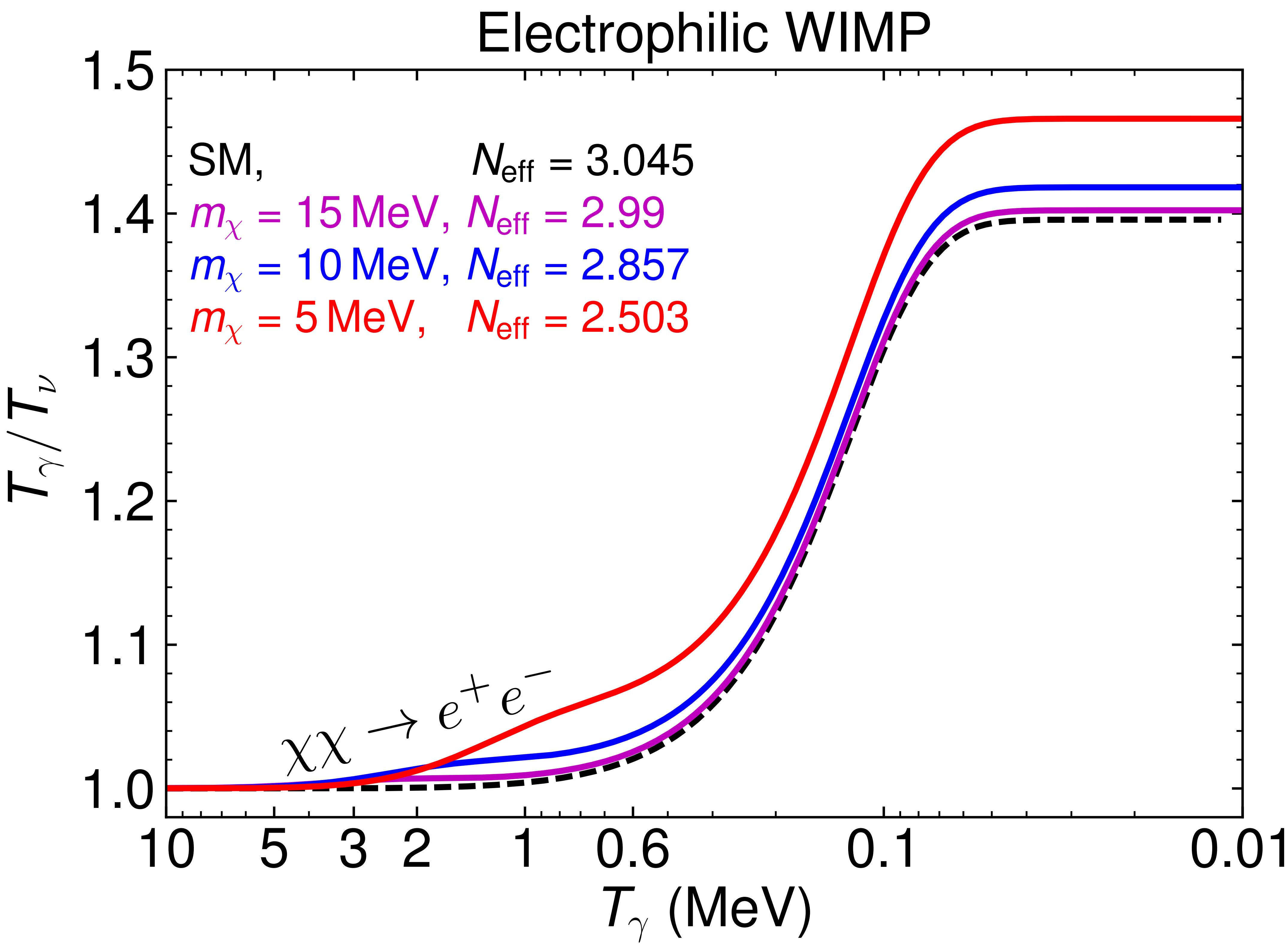
Standard Model



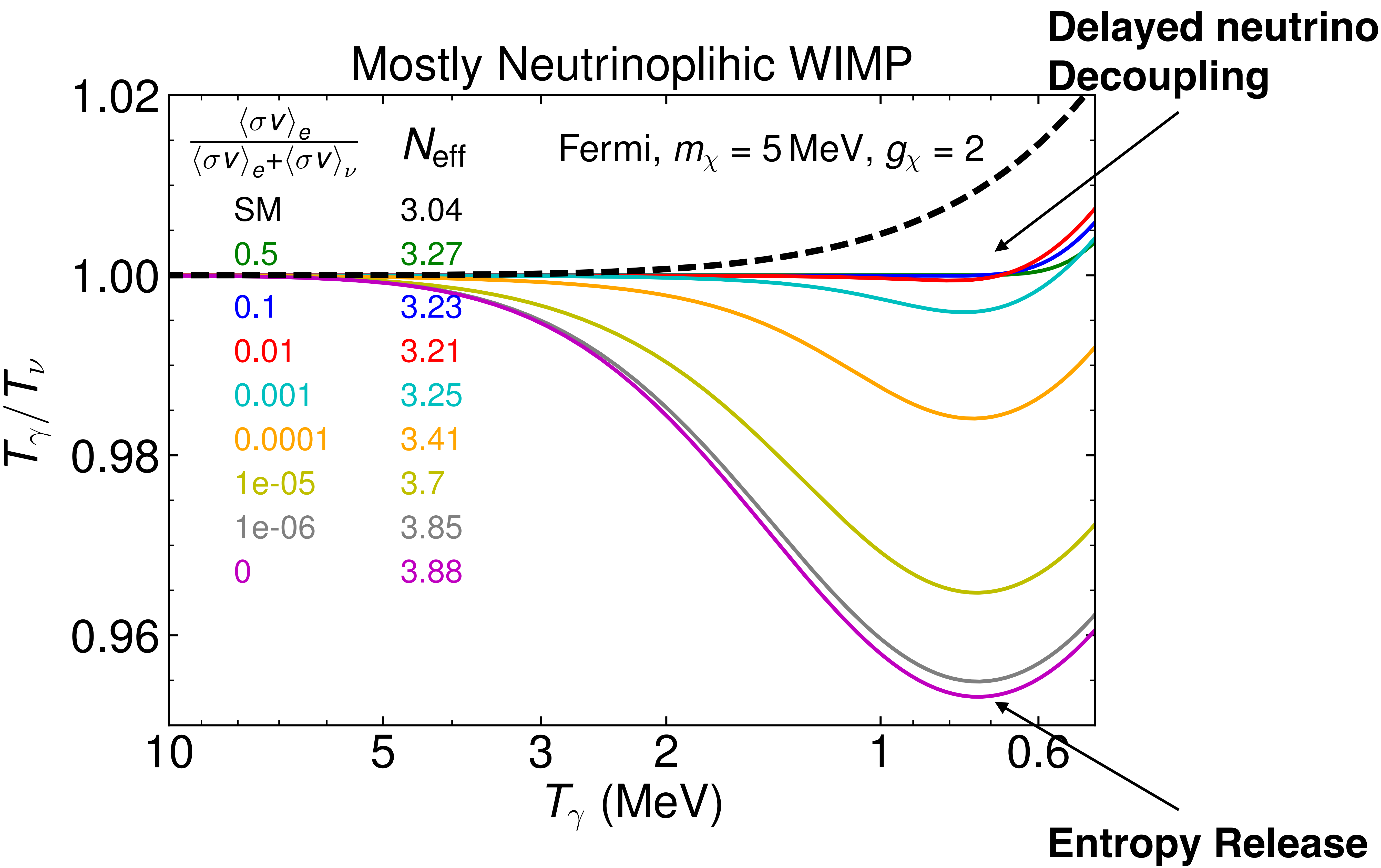
Neutrinophilic Relic: $N_{eff} > 3.043$



Electrophilic Relic: $N_{eff} < 3.043$



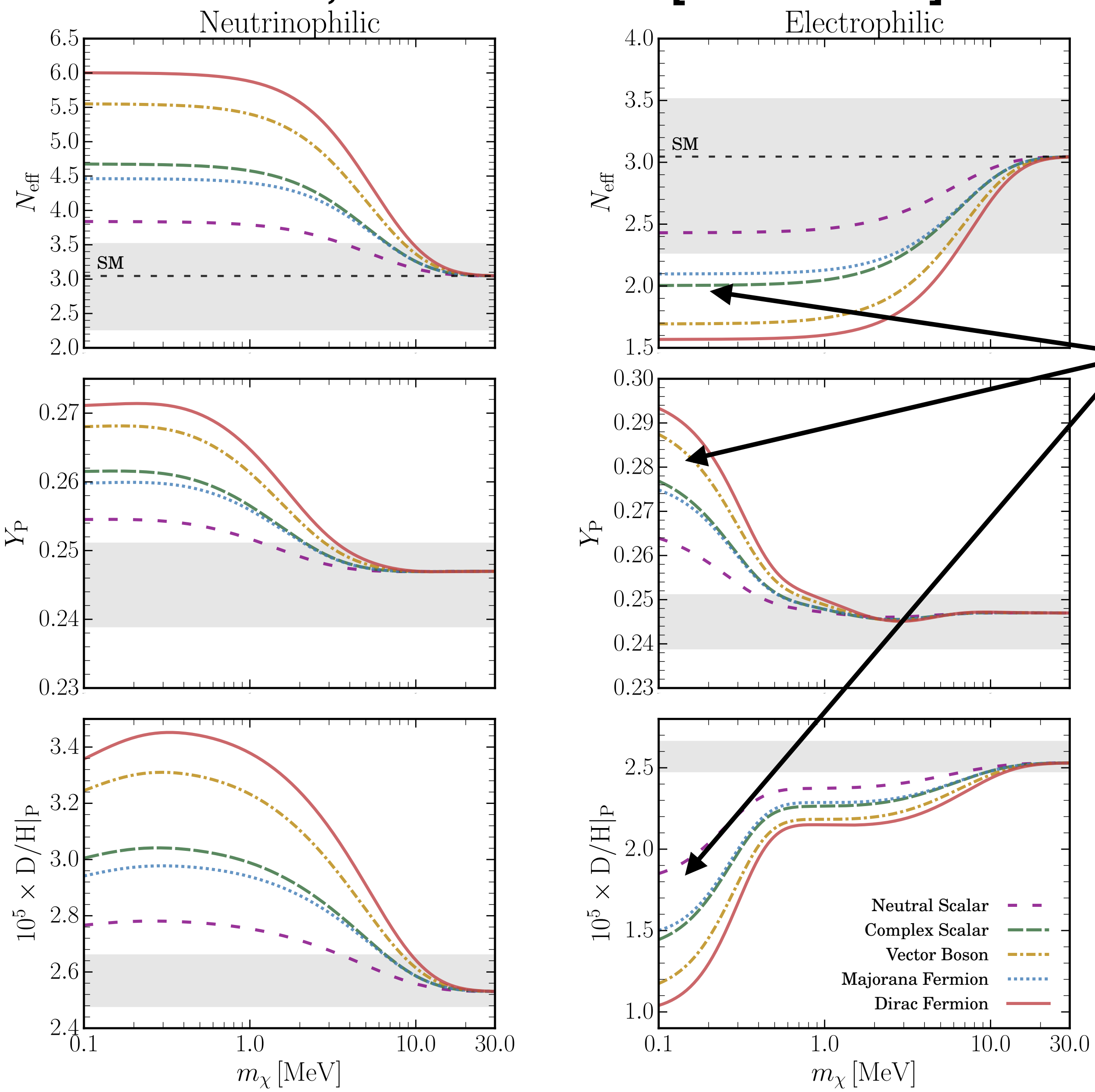
Generic Thermal Dark Matter



Lower bound on thermal dark matter

By solving for the thermodynamics of light dark sectors and running the BBN code PRIMAT (Pitrou et al.) we were able to obtain strong cosmological constraints on MeV-scale thermal dark matter

with Sabti, Alvey, Fairbairn and Blas 1910.01649
see also Chu, Kuo & Pradler [2205.05714]



BBN bounds depend upon the specific properties of the WIMP and used nuclear reaction rates but globally one finds that

$$m_{\rm DM}^{\rm thermal} \gtrsim 10\,{\rm MeV}$$ **at 95% CL**

but constraints for lower masses are very strong!

Type	BSM Particle: Particle	BSM Particle: g-Spin	Current Constraints: BBN	BBN+$\Omega_{\rm b}h^2$	Planck	Planck+H_0	BBN+Planck
Neutrinophilic	Majorana	2-F	2.2	3.5 3.0 3.0 2.9	8.4	4.9	8.4 8.4 7.1 6.8
	Dirac	4-F	3.7	6.4 5.6 5.8 5.7	11.3	8.0	11.2 11.2 10.0 9.7
	Scalar	1-B	1.3	1.7 1.5 1.5 1.4	5.6	1.6	5.6 5.5 4.3 4.0
	Complex Scalar	2-B	2.3	3.7 3.2 3.2 3.1	8.5	5.1	8.5 8.4 7.2 6.9
	Vector	3-B	3.1	5.3 4.6 4.7 4.6	10.1	6.8	10.1 10.1 8.9 8.6
Electrophilic	Majorana	2-F	0.5	0.7 0.7 2.9 3.3	4.4	9.2	5.0 4.7 7.1 7.7
	Dirac	4-F	0.7	4.2 3.5 6.3 6.6	7.4	12.0	8.0 7.8 10.0 10.5
	Scalar	1-B	0.4	0.4 0.4 0.5 0.6	2.4*	6.4	1.6 1.2 4.2 4.8
	Complex Scalar	2-B	0.5	0.9 0.8 3.2 3.6	4.6	9.2	5.1 4.9 7.2 7.8
	Vector	3-B	0.6	3.0 2.3 5.1 5.4	6.3	10.9	6.9 6.6 8.8 9.4

Table 1. Lower bounds at 95.4% CL on the masses of various thermal BSM particles in MeV. The columns correspond to analyses across different data sets, and the colors indicate our resulting constraints from taking the nuclear reaction rates used by Pisanti et al. '21 [4], Yeh et al. '21 [5], Pitrou et al. '21 [3] or Pitrou et al. '18 [10]. The bounds for the BBN, Planck and Planck+H_0 analyses are insensitive to the choice of nuclear reaction rates between these groups. In Ref. [9], we used the rates from Pitrou et al. '18 and refer the reader to this reference for a detailed description of the data set used in each case. *This bound is only at 86% CL.

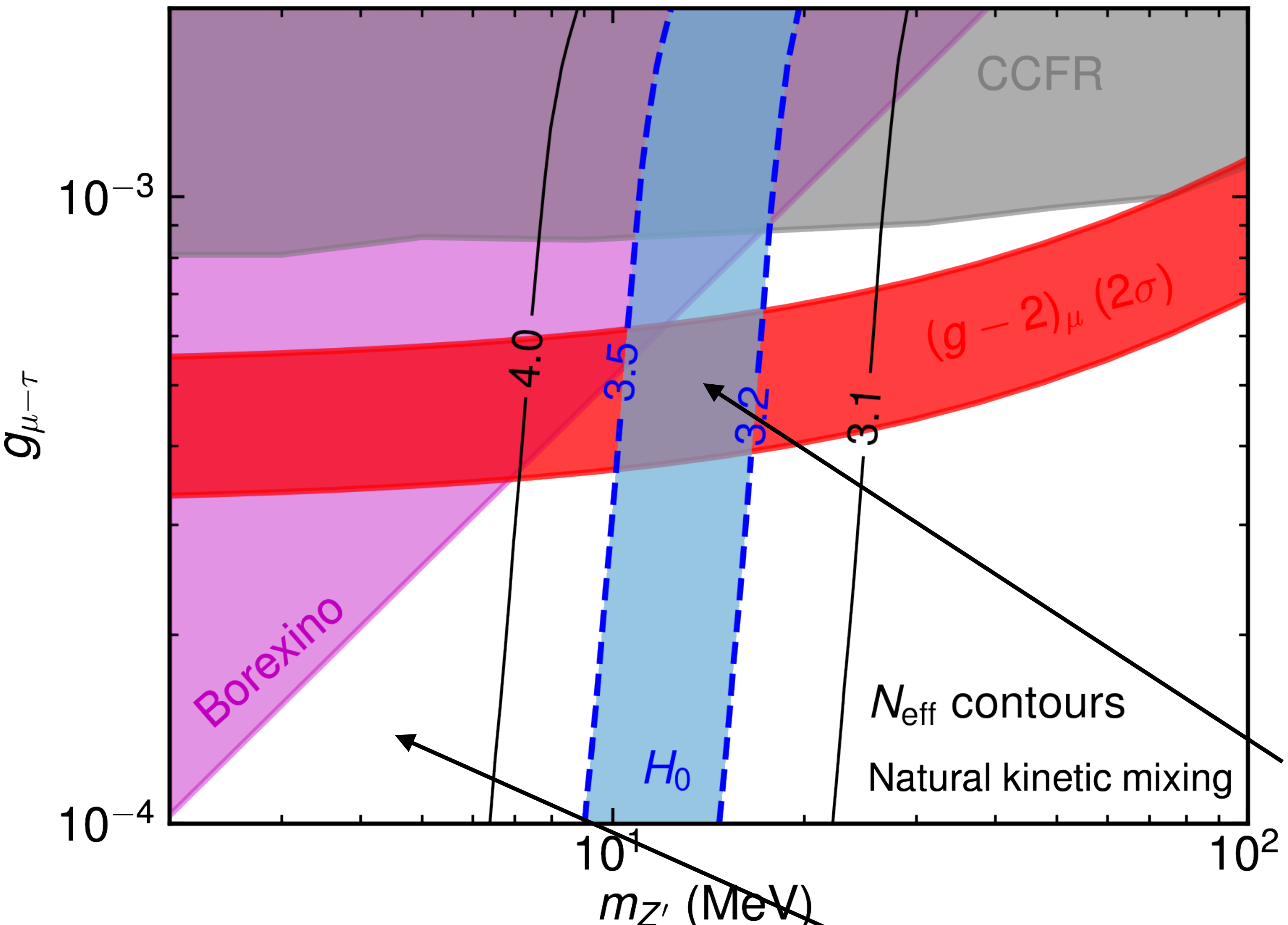
2107.11232

Light Z’s

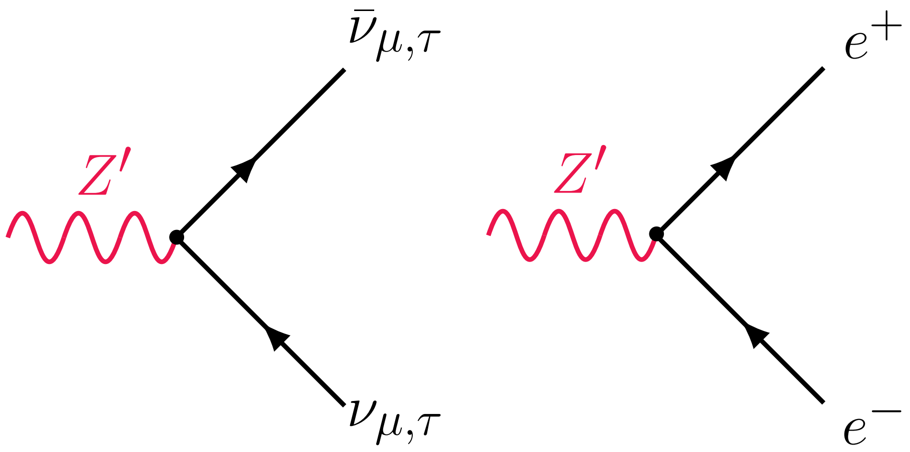
A Light $U(1)_{L_\mu - L_\tau}$ gauge boson: $\mathcal{L}_{\rm int} = g_{\mu-\tau} Z'_\alpha (\bar{\mu}\gamma^\alpha \mu + \bar{\nu}_\mu \gamma^\alpha P_L \nu_\mu - \bar{\tau}\gamma^\alpha \tau - \bar{\nu}_\tau \gamma^\alpha P_L \nu_\tau)$

see also Kamada & Yu [1504.00711]
Kamada, Kaneta, Yanagi & Yu [1805.00651]

Escudero, Hooper, Krnjaic, Pierre 1901.02010



New relevant processes:



These processes can be easily included

The code still takes < 1min to run!

Potential connection between g-2 and a large Neff

Excluded by BBN and the CMB!

Conclusions

- **The Cosmic Neutrino Background appears to be a blackbody spectrum with distortions of at most** $\delta\rho/\rho \lesssim 4 \times 10^{-5}$
- **Simplified method to solve for neutrino decoupling:**

  Just solve for the neutrino and photon temperatures!

  Key features: Simple (i.e. BSM friendly), Fast (< 10s), Open & Accurate!

Results:

- **The effect of NLO QED corrections is small:** $N_{\rm eff} = 3.043(1)$
- **Cosmological bounds on Neff from the CMB and BBN are very stringent for MeV-scale states:**
  - **Thermal dark matter should have** $m \gtrsim 10\,\text{MeV}$
  - **Strong constraints on light bosons, e.g.** $L_\mu - L_\tau$ **also** $m \gtrsim 10\,\text{MeV}$

Tools

- **Simplified code to solve for neutrino decoupling**

  https://github.com/MiguelEA/nudec_BSM/ — Python or Mathematica

- **Full code to solve for neutrino decoupling (in the SM + sterile neutrinos)**

  https://bitbucket.org/ahep_cosmo/fortepiano_public — Fortran

  Gariazzo, Pastor & de Salas

- **Public BBN codes:**

Parthenope	https://parthenope.na.infn.it/	Pisanti et al.	Fortran
PRIMAT	https://www2.iap.fr/users/pitrou/primat.htm	Pitrou et al.	Mathematica
AlterBBN	https://alterbbn.hepforge.org/	Arbey et al.	C

- **Combination of codes:**

PRyMordial	https://github.com/vallima/PRyMordial	Burns, Tait & Valli	Python

- **Including BSM thermodynamics in CLASS**

  https://github.com/stefanmarinus/CLASS_neutrinophilic — Sandner — C

Time for Questions and Comments

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Thank you for your attention!

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