Tetraquarks at large N: an explicit construction

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Introduction

Increasing experimental evidence for states with four valence quarks

- First candidate: X(3872), possibly a $c\bar{c}q\bar{q}$ state — Belle 2003
- Several more candidates since then
- More recently first state with two heavy quark rather than with heavy $Q\bar{Q}$ pair: $T_{cc}^{+}(3875)$ $(cc\bar{u}\bar{d})$ — LHCb 2021

Introduction

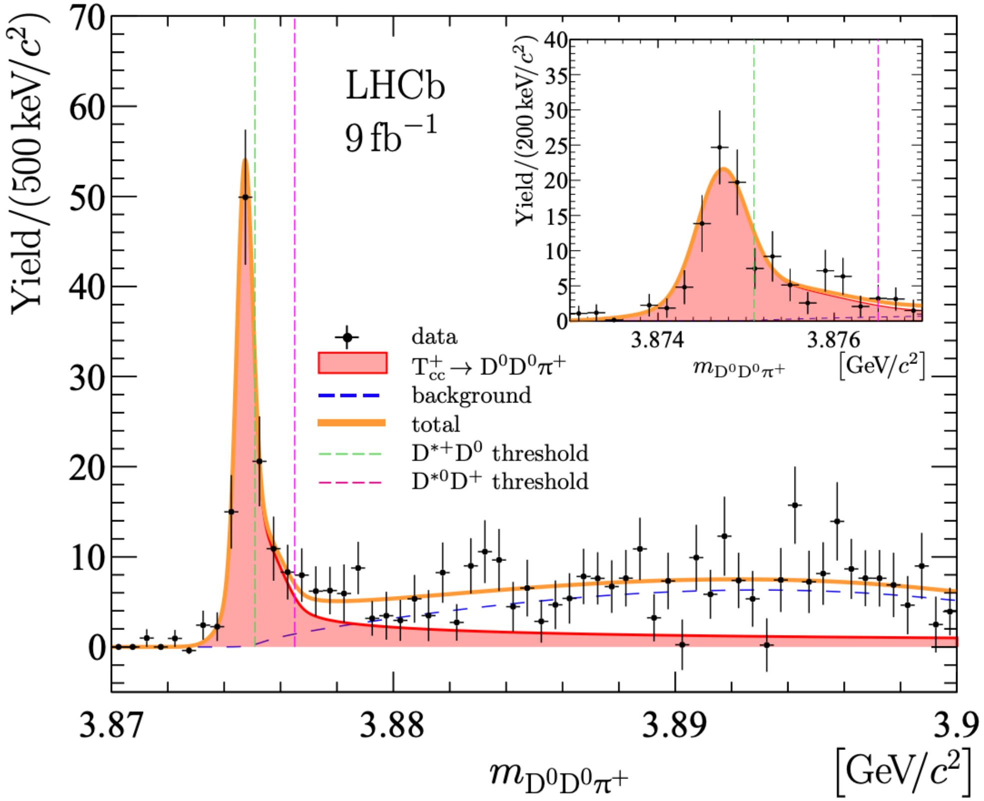
Closeness to threshold

- All candidates systematically within ~10 MeV of the corresponding two-meson thresholds
- Some much closer:

$X(3872)$ within ~ 120 keV of $D_0\bar{D}^{*0}$

$T_{cc}^{+}(3875)$ within 400 keV of $D_0 D^{*+}$

- Competing explanations as compact tetraquark states of hadronic molecules
- Binding energy of ~ Λ_{QCD} expected for compact tetraquarks and ~ $\frac{\Lambda_{QCD}^2}{M}$ for the molecule , both need tuning



$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0}\ \text{keV}/c^2,$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\ \text{keV},$$

LHCb 2021

Introduction- Tetraquarks at large N

Debate on the existence of narrow tetraquark states for large N

- Argument for non-existence originally given by Witten and presented by Coleman:

  Large N two-point functions of tetraquark operators dominated by disconnected diagrams, propagation of free mesons only

Witten 1979
Coleman 1985

- Later Weinberg points out a loophole in the argument:

  Tetraquarks may still appear as narrow resonances/poles in the connected diagrams even if subleading in 1/N.

Weinberg 2013

- Still does not conclude that narrow tetraquark states must exist in the large N limit.

Expansion parameters

- This work: explicitly construct the possible tetraquark states in a theoretically controlled regime
- Expansion parameters:
  - 1/N
  - α (for heavy quark masses, $m \gg \Lambda_{QCD}$)
  - m/M ratio of quark masses

Outline

- Hamiltonian
- Born-Oppenheimer approximation
- Two types of $QQ\bar{q}\bar{q}$ tetraquarks
- Real-world QCD tetraquarks

The single-gluon-exchange Hamiltonian

$$H = \sum_i m_i + \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} \alpha_s(r_{ij}) \frac{T^a_{(i)} T^a_{(j)}}{r_{ij}} \qquad r_{ij} \ll \Lambda_{\mathrm{QCD}}^{-1}.$$

- SU(N) –singlet subspace of $qq\bar{q}\bar{q}$ system is 2 dimensional:

  2 ways to contract SU(N) indices:

$$q^i_{(1)} q^j_{(2)}\, \bar{q}^{(3)}{}_i\, \bar{q}^{(4)}{}_j \qquad\qquad q^i_{(1)} q^j_{(2)}\, \bar{q}^{(3)}{}_j\, \bar{q}^{(4)}{}_i$$

Choice of basis

SU(N) –singlet subspace of $qq\bar{q}\bar{q}$ system 2 dimensional

- Symmetric-Antisymmetric: $q^i q^j$ in the color-symmetric or color-antisymmetric representation and $\bar{q}_i \bar{q}_j$ in the corresponding conjugate representation

- Singlet-Adjoint: $q^i_{(1)}\, \bar{q}^{(3)}{}_i\, q^j_{(2)}\, \bar{q}^{(4)}{}_j$ (13)(24)-singlet

  $q^i_{(1)}\, T^{a}{}^k_i\, \bar{q}^{(3)}{}_k\, q^j_{(2)} T^{a}{}^l_j\, \bar{q}^{(4)}{}_l$ (13)(24)-Adjoint

The single-gluon-exchange Hamiltonian

In the Symmetric-Asymmetric basis:

$$V_{SS} = -\frac{\alpha}{2}\left(\frac{1}{r_{1\bar{3}}} + \frac{1}{r_{1\bar{4}}} + \frac{1}{r_{2\bar{3}}} + \frac{1}{r_{2\bar{4}}}\right)$$
$$\boxed{+\frac{\alpha}{2N}\left(\frac{2}{r_{12}}\right.} + \frac{2}{r_{\bar{3}\bar{4}}} - \frac{1}{r_{1\bar{3}}} - \frac{1}{r_{1\bar{4}}} - \frac{1}{r_{2\bar{3}}} - \left.\frac{1}{r_{2\bar{4}}}\right) + \mathcal{O}\left(\frac{1}{N^2}\right),$$
$$V_{SA} = V_{AS} = -\frac{\alpha}{2}\left(\frac{1}{r_{1\bar{3}}} + \frac{1}{r_{2\bar{4}}} - \frac{1}{r_{1\bar{4}}} - \frac{1}{r_{2\bar{3}}}\right) + \mathcal{O}\left(\frac{1}{N^2}\right),$$
$$V_{AA} = -\frac{\alpha}{2}\left(\frac{1}{r_{1\bar{3}}} + \frac{1}{r_{1\bar{4}}} + \frac{1}{r_{2\bar{3}}} + \frac{1}{r_{2\bar{4}}}\right)$$
$$\boxed{-\frac{\alpha}{2N}\left(\frac{2}{r_{12}}\right.} + \frac{2}{r_{\bar{3}\bar{4}}} - \frac{1}{r_{1\bar{3}}} - \frac{1}{r_{1\bar{4}}} - \frac{1}{r_{2\bar{3}}} - \left.\frac{1}{r_{2\bar{4}}}\right) + \mathcal{O}\left(\frac{1}{N^2}\right).$$

The single-gluon-exchange Hamiltonian

- A change of basis that diagonalizes the potential at leading order in N:

$$\Psi_+ = \frac{1}{\sqrt{2}}(\Psi_S + \Psi_A), \qquad \Psi_- = \frac{1}{\sqrt{2}}(\Psi_S - \Psi_A).$$

- Potential:

$$V_{++} = -\frac{\alpha}{r_{1\bar{3}}} - \frac{\alpha}{r_{2\bar{4}}} + \mathcal{O}\left(\frac{1}{N^2}\right),$$
$$V_{+-} = V_{-+} = \frac{\alpha}{2N}\left(\frac{2}{r_{12}} + \frac{2}{r_{\bar{3}\bar{4}}} - \frac{1}{r_{1\bar{3}}} - \frac{1}{r_{1\bar{4}}} - \frac{1}{r_{2\bar{3}}} - \frac{1}{r_{2\bar{4}}}\right) + \mathcal{O}\left(\frac{1}{N^2}\right),$$
$$V_{--} = -\frac{\alpha}{r_{1\bar{4}}} - \frac{\alpha}{r_{2\bar{3}}} + \mathcal{O}\left(\frac{1}{N^2}\right).$$

- For $N \to \infty$:

$$\Psi_+ \to (1\bar{3})_{\text{singlet}}(2\bar{4})_{\text{singlet}},$$
$$\Psi_- \to (1\bar{4})_{\text{singlet}}(2\bar{3})_{\text{singlet}},$$

Leading order in 1/N: free mesons

- Potential:

$$V = -\alpha \begin{pmatrix} \frac{1}{r_{13}} + \frac{1}{r_{24}} & 0 \\ 0 & \frac{1}{r_{14}} + \frac{1}{r_{23}} \end{pmatrix} + \mathcal{O}\left(\frac{1}{\mathrm{N}}\right)$$

- Exactly solvable, two decoupled “Hydrogen atom” problems in each sector

sector + : $(1\bar{3})$ $(2\bar{4})$ mesons

sector - : $(1\bar{4})$ $(2\bar{3})$ mesons

Subleading in 1/N

- To include the effect of subleading in 1/N interactions, we allow also for a hierarchy of masses $m/M << 1$

$$H = \frac{P_i^2}{2M_i} + \underbrace{\frac{p_i^2}{2m_i} - \alpha \begin{pmatrix} \frac{1}{r_{13}} + \frac{1}{r_{24}} & 0 \\ 0 & \frac{1}{r_{14}} + \frac{1}{r_{23}} \end{pmatrix}}_{\text{Leading in } 1/N \text{ and } m/M} + \frac{\alpha}{2N}\sigma_1 \left(\frac{2}{r_{12}} + \frac{2}{r_{34}} - \frac{1}{r_{13}} - \frac{1}{r_{23}} - \frac{1}{r_{24}}\right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

- For $\frac{m}{M} \ll \frac{1}{N}$: include first the 1/N terms and then consider the $\frac{1}{M}$ suppressed kinetic terms (Born-Oppenheimer approximation)

Born-Oppenheimer approximation

The mass hierarchy leads to separation of scales, simplifying the problem:

$$H = \frac{P_i^2}{2M_i} + \frac{p_i^2}{2m_i} + \mathrm{V}(\{\mathrm{r}\}, \{\mathrm{R}\})$$

↳ Ignore kinetic terms of heavy particles, treat coordinates of heavy particles $\{\mathrm{R}\}$ as parameters

$$H_{\mathrm{red}} = \frac{p_i^2}{2m_i} + \mathrm{V}(\{\mathrm{r}\}; \{\mathrm{R}\})$$

↳ Solve the reduced problem for the light particles as a function of heavy particle coordinates

$$V_{\mathrm{BO}}(\{R\}) = E_{\mathrm{red}}\,(\{R\})$$

↳ The $\{\mathrm{R}\}$-dependent eigenstate energies provide BO potentials for the heavy particles

$$H_{\mathrm{eff}} = \frac{P_i^2}{2M_i} + V_{\mathrm{BO}}(\{R\})$$

↳ Consider the kinetic terms of heavy particles with the BO potential and solve the effective problem for the heavy coordinates

Born-Oppenheimer for $QQ\bar{q}\bar{q}$ tetraquarks

- First ignore Kinetic terms of heavy quarks
- Solve the problem of light quarks
- At leading order in 1/N: two decoupled system of mesons

  (Hydrogen atoms)

Born-Oppenheimer for $QQ\bar{q}\bar{q}$ tetraquarks

- Free mesons at leading order in 1/N - two set of states:

  sector + : $(1\bar{3})$ $(2\bar{4})$ mesons
  sector - : $(1\bar{4})$ $(2\bar{3})$ mesons

$$V = -\alpha \begin{pmatrix} \frac{1}{r_{13}} + \frac{1}{r_{24}} & 0 \\ 0 & \frac{1}{r_{14}} + \frac{1}{r_{23}} \end{pmatrix} + \mathcal{O}\left(\frac{1}{\mathrm{N}}\right)$$

- Considering only ground states for now
  - ➢ Two-fold degenerate at leading order in 1/N

$$E_0 = -\mathcal{E}_3 - \mathcal{E}_4$$
$$= -\frac{1}{2}m_3\alpha^2 - \frac{1}{2}m_4\alpha^2$$

- Subleading 1/N interaction breaks the degeneracy and

  provides a BO potential

The Born Oppenheimer potential

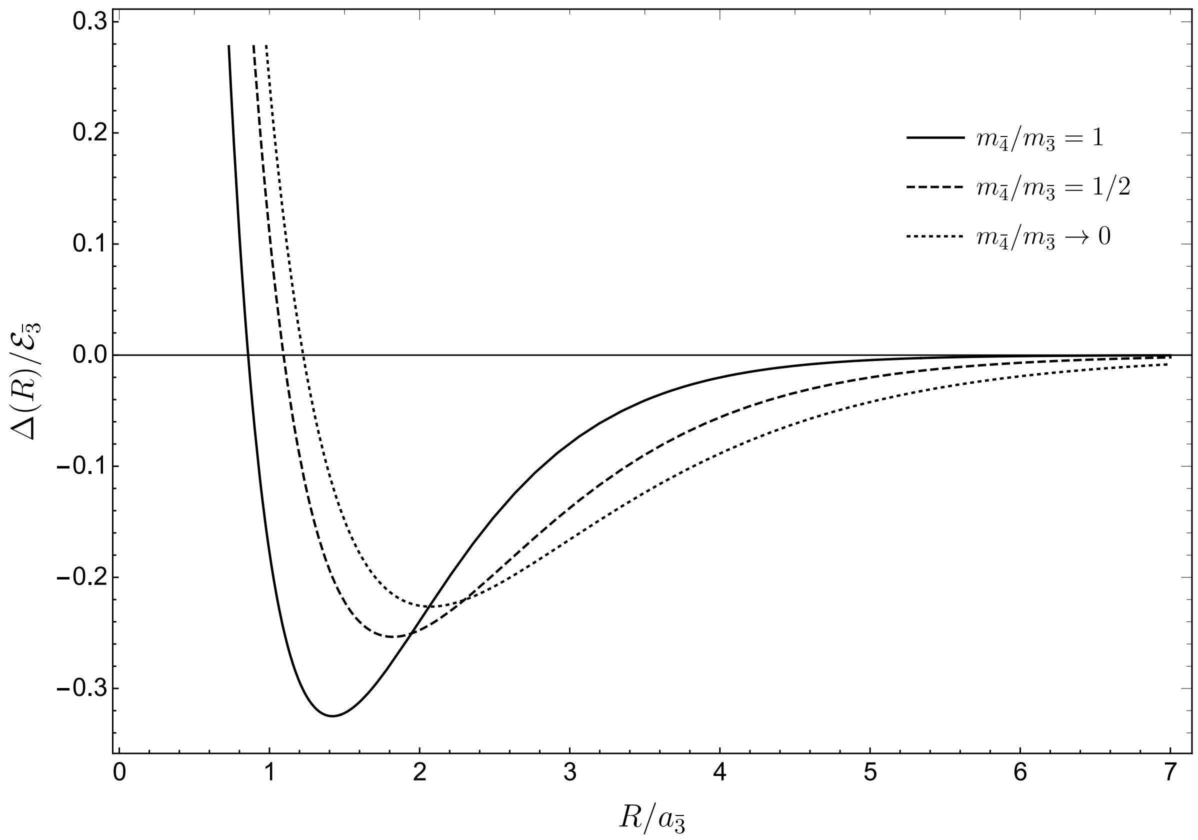
$$\frac{1}{\mathrm{N}}\Delta(R) = {}_{+}\langle\,(1\bar{3})\,(2\bar{4})|\,V\,|(1\bar{4})\,(2\bar{3})\rangle_{-}$$

$$E(R) = E_0 \pm \frac{1}{N}\Delta(R)$$

$$V_{BO} = \pm\frac{1}{\mathrm{N}}\Delta(R)$$

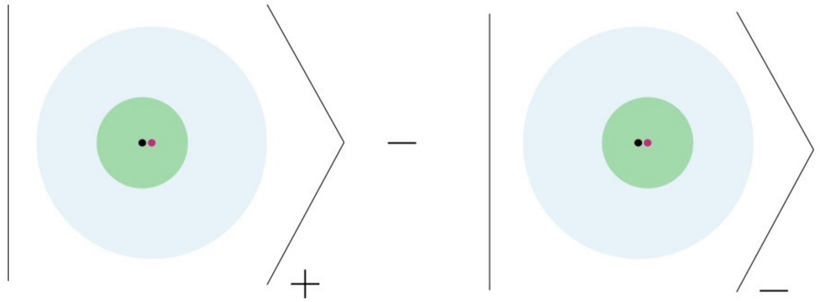
- For $\frac{m_4}{m_3} \ll 1$:

$$V_{BO} = \pm\frac{2}{N}\mathcal{E}_3\ \mathrm{e}^{-\frac{\mathrm{R}}{\mathrm{a}_3}}\left(\frac{\mathrm{a}_3}{\mathrm{R}} - \frac{2}{3}\frac{\mathrm{R}}{\mathrm{a}_3}\right)$$

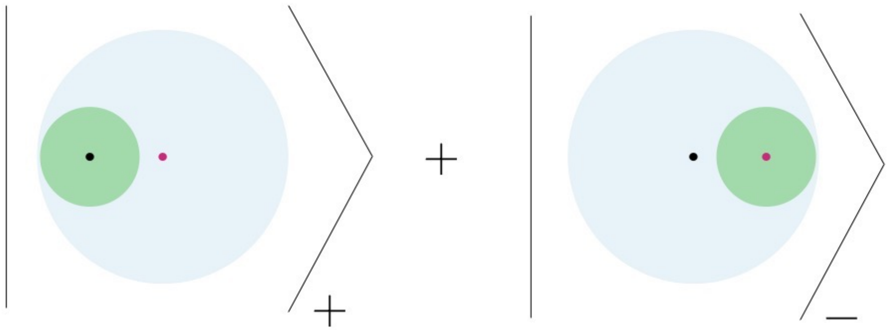


Two types of tetraquarks

- Type-I: $R \ll a_3$



- Type-II: $R \sim a_3$



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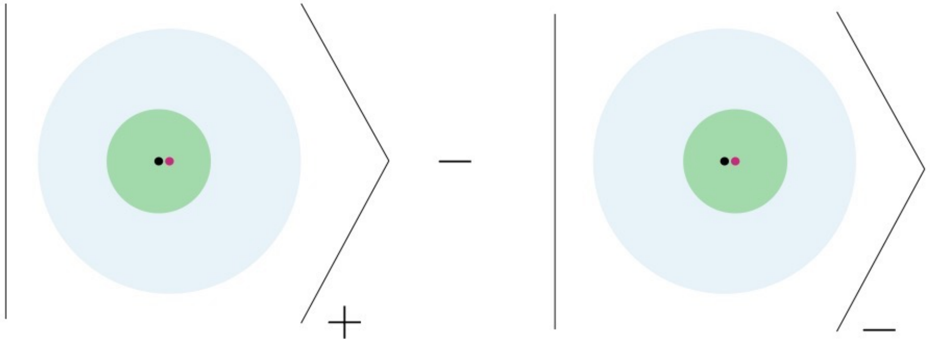

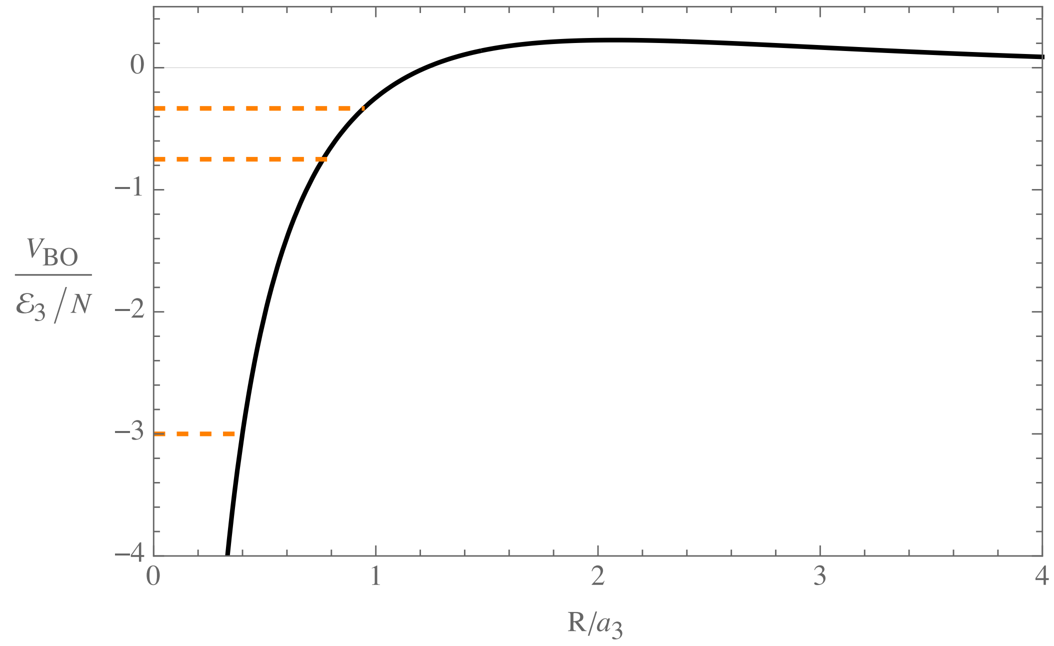

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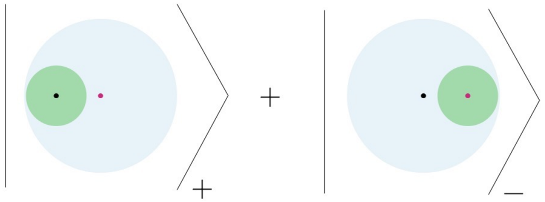

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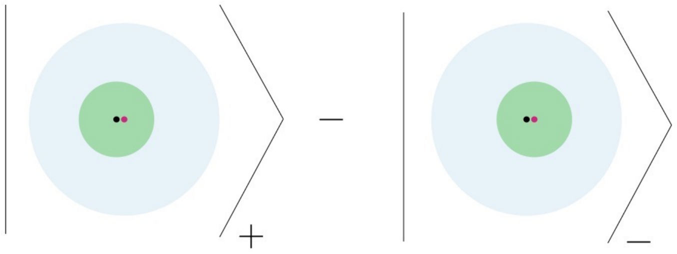
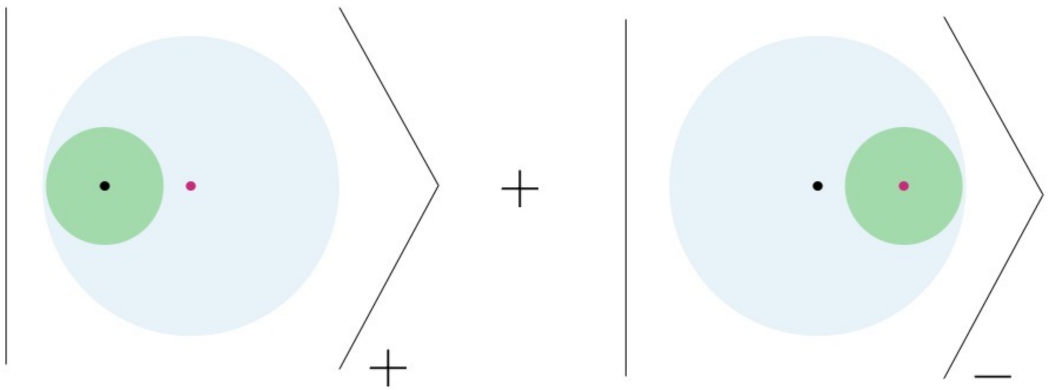
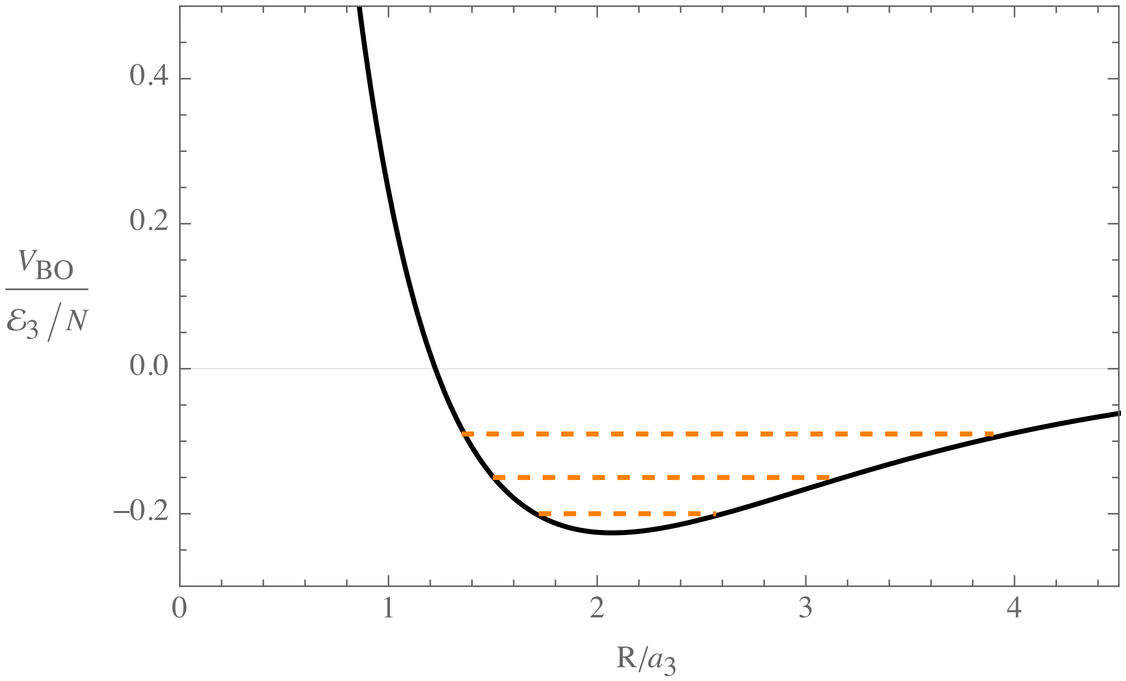


- Type-II: $R \sim a_3$

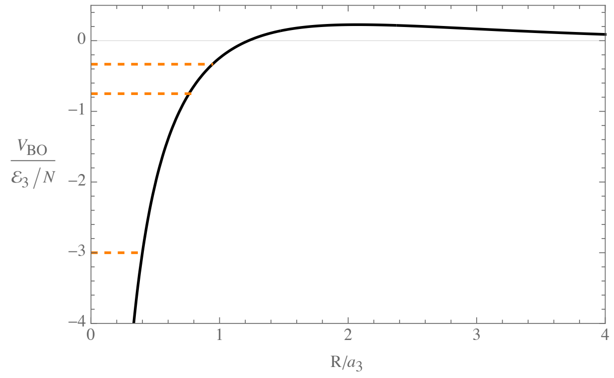




Condition for formation of bound states

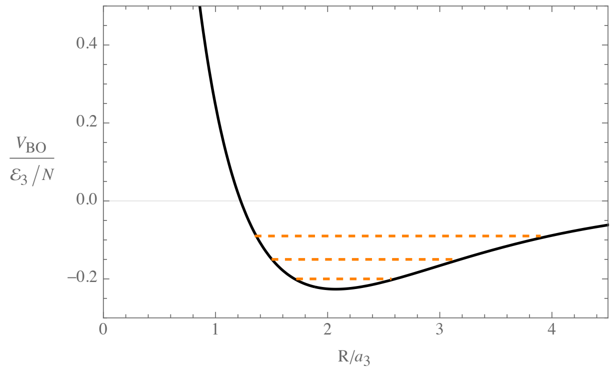
- Type-I: $R \ll a_3 \Rightarrow \dfrac{1}{M(\alpha/N)} \ll \dfrac{1}{m\alpha}$

$$\Rightarrow \frac{m}{M} \ll \frac{1}{N}$$



- Type-II: $\Delta R \ll a_3 \Rightarrow \dfrac{1}{(M\,k\,)^{1/4}} \ll a_3 \quad k \sim \dfrac{\mathcal{E}_3}{N\,a_3^2} \sim \dfrac{m}{N a_3^4}$

$$\Rightarrow \frac{m}{M} \ll \frac{1}{N}$$



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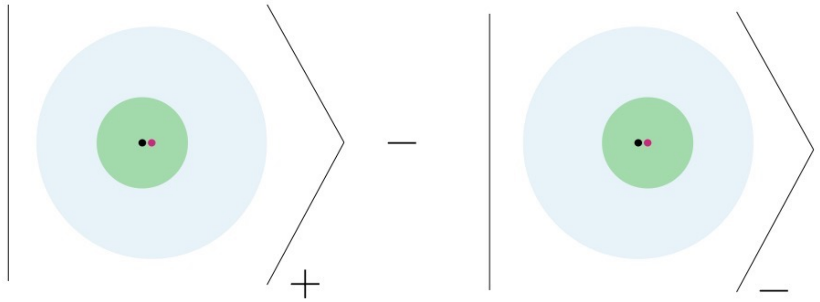
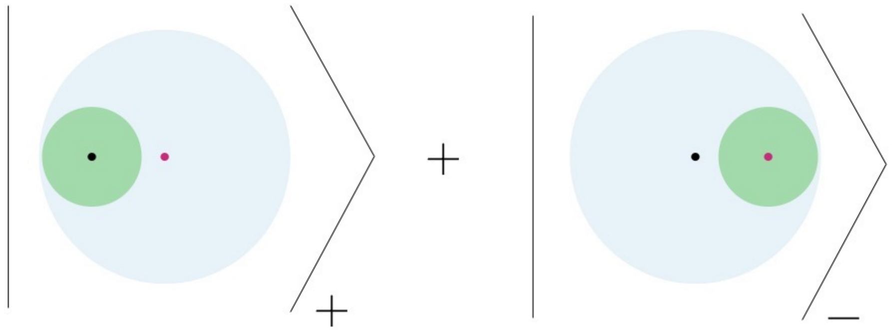


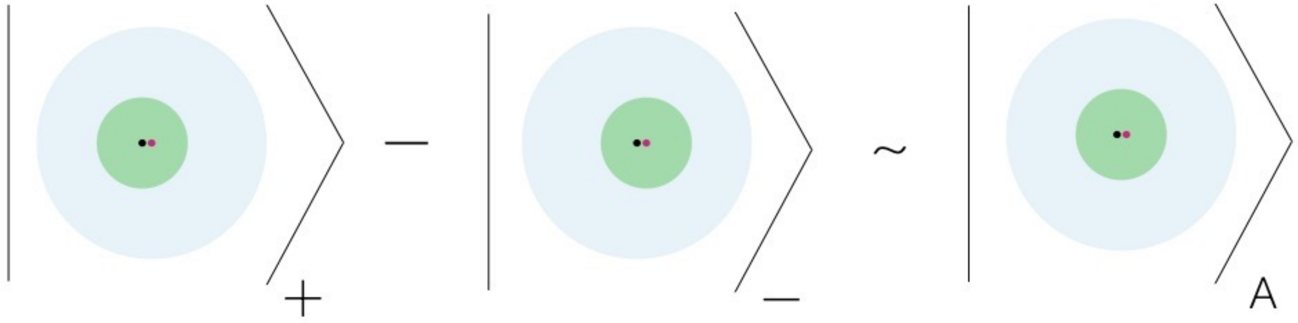
- Type-II: $R \sim a_3$



Two types of tetraquarks

Type-I

- In the limit of $R \to 0$, the heavy quarks in the color-antisymmetric state



- Can consider as a compact heavy diquark forming a bound state with light (anti)quarks
- For SU(3): A “baryon” with the diquark (in color $\bar{3}$) instead of one of the quarks

[Manohar & Wise 1993]

Closeness to the threshold and tuning-Type II states

- Consider a potential $V(X) = V_0 e^{-X} \left(\frac{1}{X} - \epsilon\, X\right)$

$$V_{min} \sim V_0 e^{-\mathcal{O}\left(\frac{1}{\sqrt{\epsilon}}\right)} \qquad (\epsilon \ll 1)$$

- $V_{BO} = \pm \frac{2}{N} \mathcal{E}_3\ \mathrm{e}^{-\frac{\mathrm{R}}{\mathrm{a}_3}} \left(\frac{\mathrm{a}_3}{\mathrm{R}} - \frac{2}{3} \frac{\mathrm{R}}{\mathrm{a}_3}\right)$ $\qquad V_{min} \sim \frac{\mathcal{E}_3}{N}\, \mathrm{e}^{-2.07} \sim \frac{0.1}{N}\, \mathcal{E}_3$

Closeness to the threshold and tuning Type II states

- $V_{BO} = \pm \frac{2}{N} \mathcal{E}_3 \, \mathrm{e}^{-\frac{\mathrm{R}}{\mathrm{a}_3}} \left(\frac{\mathrm{a}_3}{\mathrm{R}} - \frac{2}{3} \frac{\mathrm{R}}{\mathrm{a}_3} \right)$ $\quad V_{min} \sim \frac{\mathcal{E}_3}{N} \mathrm{e}^{-2.07} \sim \frac{0.1}{N} \mathcal{E}_3$
- Close to threshold by 1/N and the exponentially suppressed overlap
- For $N = 3$, a binding energy of $E_{\text{binding}} \sim 10^{-3} \, \mathcal{E}_3$ needs tuning m_3/M_2 only to within $\sim 20\%$ of the critical ratio

Closeness to the threshold and tuning Type I states

- $E_{\text{binding}} \sim \frac{1}{N^2} M\alpha^2$
- Generally independent of $\mathcal{E}_3$, but for $\frac{1}{N^2} \ll \frac{m}{M} \ll \frac{1}{N}$:

$$\frac{E_{\text{binding}}}{\mathcal{E}_3} \sim \frac{M}{mN^2} \ll 1$$

➢ Parametrically close to threshold compared to the binding energy of mesons

Beyond Born-Oppenheimer

$$H = \frac{P_i^2}{2M_i} + \frac{p_i^2}{2m_i} - \alpha \begin{pmatrix} \frac{1}{r_{13}} + \frac{1}{r_{24}} & 0 \\ 0 & \frac{1}{r_{14}} + \frac{1}{r_{23}} \end{pmatrix} + \frac{\alpha}{2N} \sigma_1 \left(\frac{2}{r_{12}} + \frac{2}{r_{34}} - \frac{1}{r_{13}} - \frac{1}{r_{23}} - \frac{1}{r_{24}} \right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

- If $\frac{m}{M} \gg \frac{1}{N}$ the B.O. approximation does not apply
- But can include the heavy quark kinetic terms first: free mesons
- With a variational argument can show that the $1/N$ suppressed terms are too small to lead to bound states of mesons below the two-meson threshold

Real-world tetraquarks:
Lessons and speculations

All-heavy tetraquarks

- States of tt$\bar{q}\bar{q}'$ would exist and be stable within QCD itself but t decays due to weak interactions with a decay width larger than Λ_{QCD} ($\Gamma_t = 1.4 \text{ GeV}$)

- $bb\bar{c}\bar{c}$ tetraquarks? $\frac{m_b}{Nm_c}$ close to 1

- If we extrapolate our leading order results, need:

  - $\frac{m_b}{Nm_c} > 4.8$ for existence of type-I states

  - $\frac{m_b}{Nm_c} > 3$ for existence of type-II states

- $bb\bar{c}\bar{c}$ states not expected, but this is near the regime of validity of approximations, conclusion can change

Tetraquarks with one light (anti-)quark

- Our analysis is valid also if one of the (anti)quarks is lighter than Λ_{QCD}
- Critical $\frac{m_b}{Nm_c}$ for $bb\bar{c}\bar{q}$ states with q=u,d,s expected to be lower than for $bb\bar{c}\bar{c}$:
- If we extrapolate our leading order results, need:
  - $\frac{m_b}{Nm_c} > 3.4$ for existence of type-I states
  - $\frac{m_b}{Nm_c} > 1.8$ for existence of type-II states
- $bb\bar{c}\bar{q}$ states more likely than $bb\bar{c}\bar{c}$ states, but can't draw any robust conclusions

Doubly heavy tetraquarks

- In the heavy quark limit the existence of type-I states only relies on the short distance QQ interaction
- Parametric condition for existence of type-I states below two meson threshold becomes $M \gg N\,\Lambda_{\rm QCD}$
- This limit for SU(3) studied also using heavy quark-diquark symmetry and HQET
  - Symmetry relates $\mathrm{QQ}\bar{q}\bar{q}$ states to $\mathrm{Q}\bar{q}\bar{q}$ baryons
  - Expansion in control for $\mathrm{bb}\bar{q}\bar{q}$ states: predicts a QCD-stable T_{bb} ground state
  - Have been also applied for T_{cc} : predictions don't match the observed state (and likely not in control)

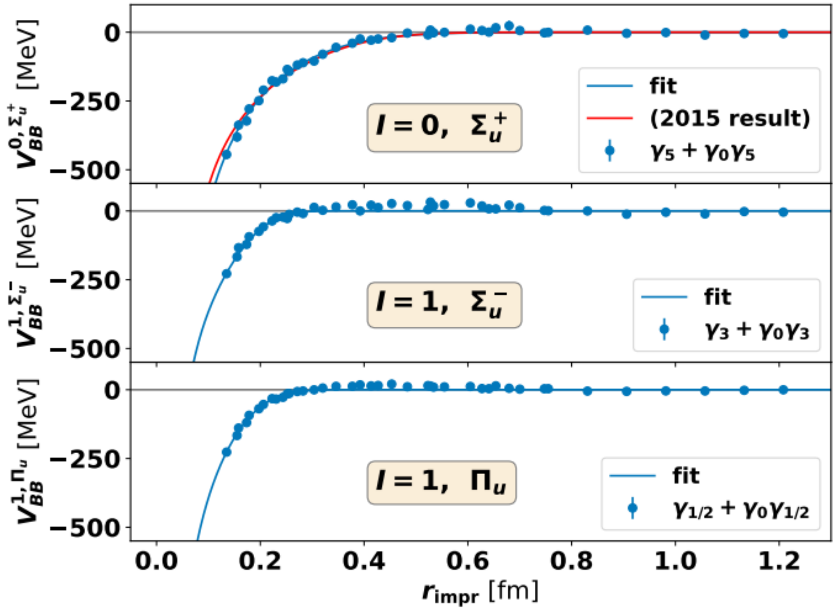
Savage & Wise 1990
Hu & Mehen 2006

Mehen 2017
Eichten & Quigg 2017
Braaten, He & Mohapatra 2020

An & Wise 2018

Doubly heavy tetraquarks on Lattice

- Lattice QCD can compute the B.O. potential
- Other lattice methods also developed for studying doubly heavy tetraquarks
- Increasing agreement on existence of QCD-stable T_{bb} states



B.O. potential for $bb\bar{u}\bar{d}$
[arXiv: 2312.17060]

Doubly heavy tetraquarks on Lattice

- Lattice QCD can compute the B.O. potential
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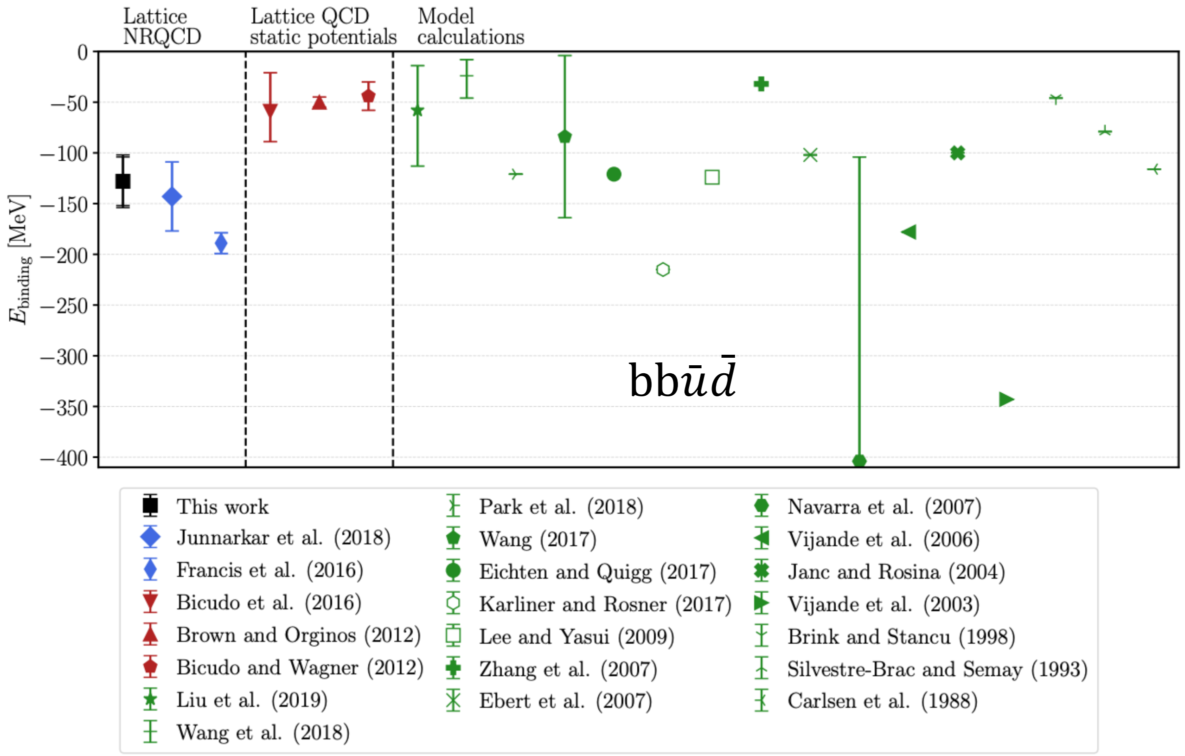
- Increasing agreement on existence of QCD-stable T_{bb} states
- Less clear for T_{bc} states



[arXiv: 2312.17060]

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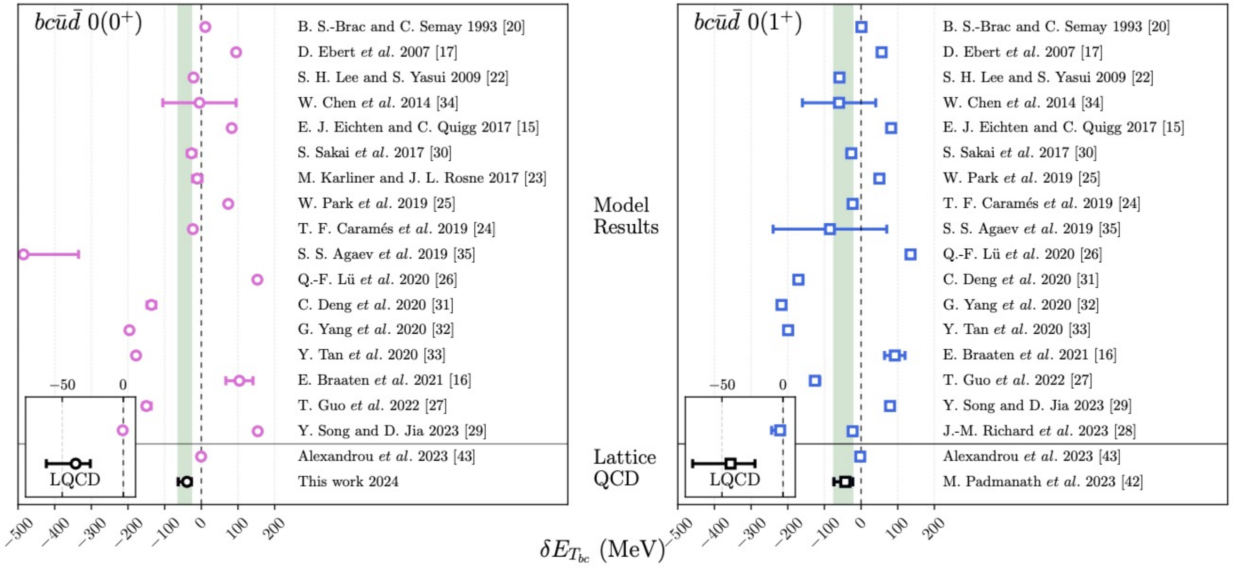
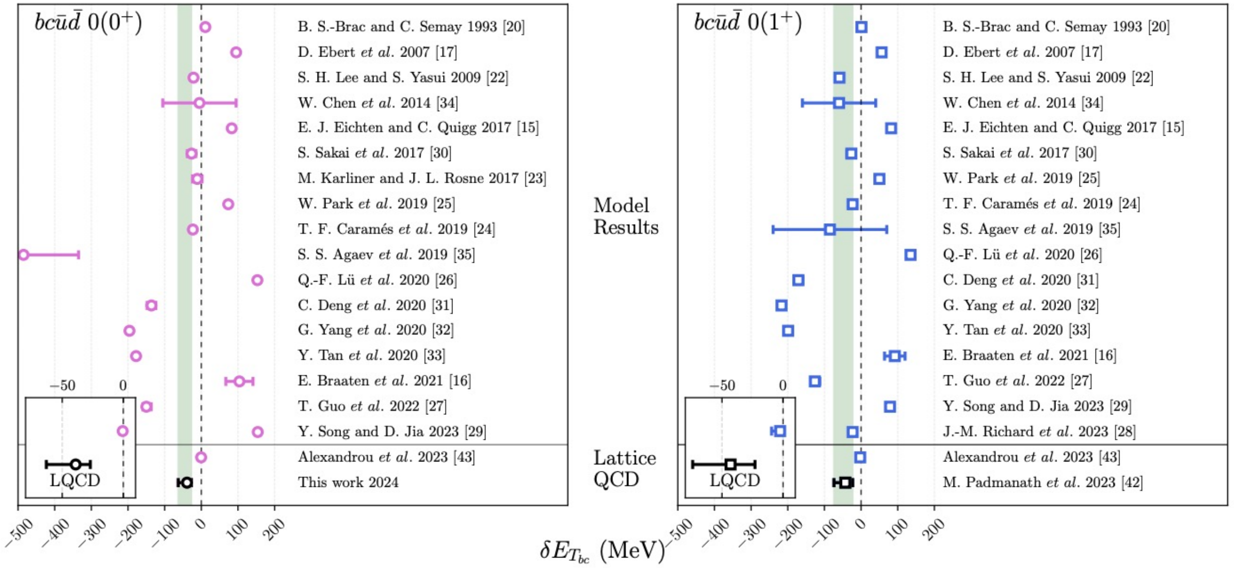


[arXiv: 2404.08109]

Doubly heavy tetraquarks on Lattice

- Lattice QCD can compute the B.O. potential
- Other lattice methods also developed for studying doubly heavy tetraquarks
- Increasing agreement on existence of QCD-stable T_{bb} states
- Less clear for T_{bc} states:
- Question: are there any type-II states in the heavy quark limit?



[arXiv: 2404.08109]

Summary and conclusions

- Stable $QQ\bar{q}\bar{q}$ tetraquarks exist in large N QCD for a hierarchy of masses larger than N
- Free meson pairs at leading order, but 1/N correction provides Born-Oppenheimer potential

Two types of tetraquarks

- Type-I states: QQ pair distance much smaller than $Q\bar{q}$ meson size
  - Continuously connected to states with a color-antisymmetric diquark core as M/m increases
  - Exist also if the lighter quarks are below the confinement scale for $M \gg N\, \Lambda_{\mathrm{QCD}}$
- Type-II states: QQ pair localized at a distance comparable to $Q\bar{q}$ meson size
  - Closeness to threshold by $1/N$ as well by an exponential wavefunction-overlap suppression

Thank you!

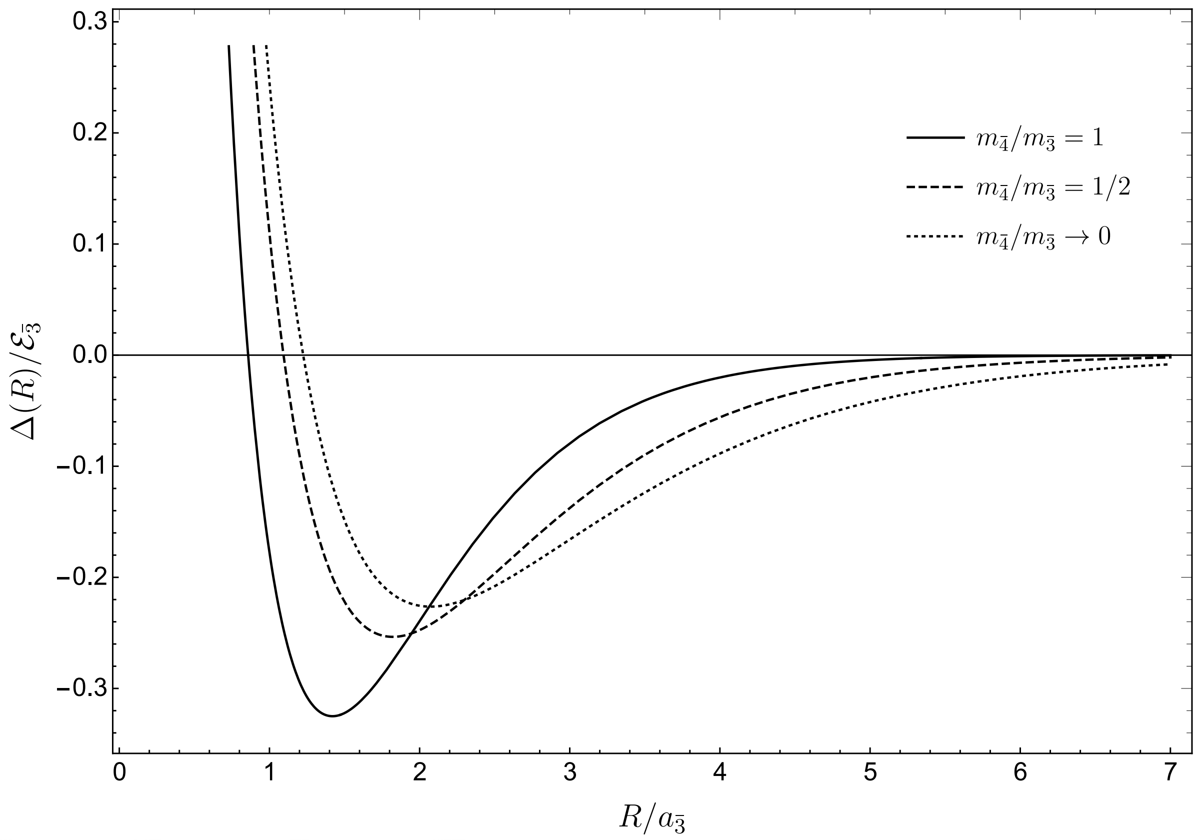
Extra Slides

The Born Oppenheimer potential

- $V_{BO} = \pm \frac{1}{\mathrm{N}} \Delta(R)$

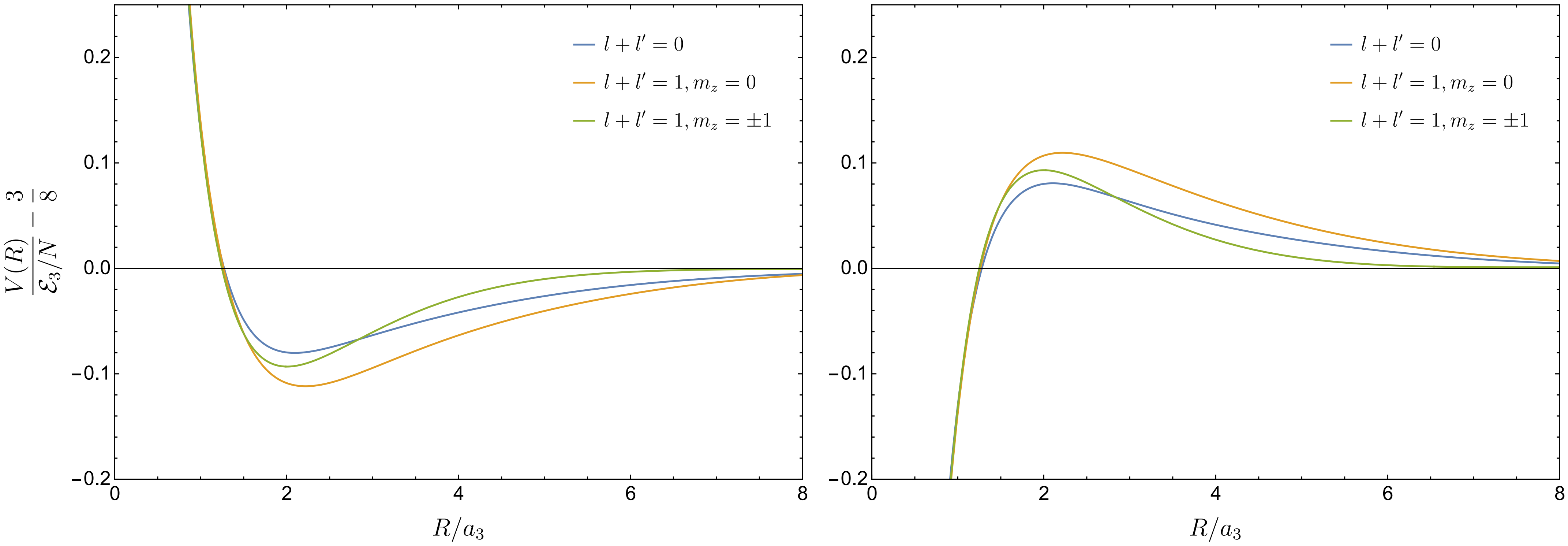
- For $\frac{m_4}{m_3} \ll 1$:

$$V_{BO} = \pm \frac{2}{N} \mathcal{E}_3 \ e^{-\frac{R}{a_3}} \left(\frac{a_3}{R} - \frac{2}{3} \frac{R}{a_3} \right)$$



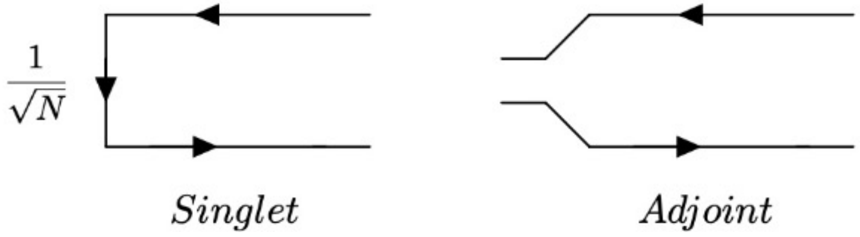
$$\frac{\Delta(R)}{\mathcal{E}_{\bar{3}}} = 2e^{-\frac{R}{a_{\bar{3}}}} \left(\frac{a_{\bar{3}}}{R} - \frac{2}{3} \frac{R}{a_{\bar{3}}} - \frac{1}{2} \left(\frac{m_{\bar{4}}}{m_{\bar{3}}} \right)^2 \frac{R}{a_{\bar{3}}} + \frac{1}{9} \left(\frac{m_{\bar{4}}}{m_{\bar{3}}} \right)^2 \left(\frac{R}{a_{\bar{3}}} \right)^3 \right) + \mathcal{O}\left(\left(\frac{m_{\bar{4}}}{m_{\bar{3}}} \right)^3 \right).$$

Excited states



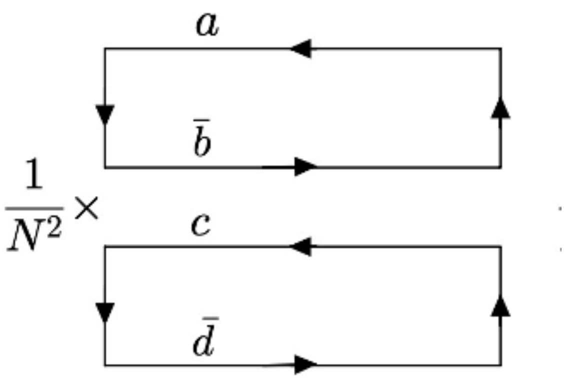
Higher order α corrections-diagrammatic representation

- Representation of states



- With the diagrammatic representation chosen for the states, problem can be mapped to the usual diagrammatic of mesons at large N

- Leading diagrams of diagonal elements



Higher order corrections in α

- Higher order corrections in α don't change the picture, only refine it
- The leading 1/N interactions to arbitrary order in α lead only to free mesons
- The possibility of formation of tetraquarks only considering subleading 1/N interactions