Two dijet production at LHC and Tevatron in QCD.

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(based on joint work with Yu. Dokshitzer, L. Frankfurt, M. Strikman)

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Introduction

The subject of this talk: the two dijet production in four to four collisions of partons at Tevatron and LHC. The conventional mechanism of multijet production at high energy hadron—hadron collisions is the two to two mechanism. In this mechanism a parton from one nucleon interact with a parton in another nucleon producing two o more jets. However, recent experiments by CDF (1997) and D0 (2009) collaborations show that there exists a kinematic region where a more complicated, four to four mechanism, where two partons from one nucleon interact with two partons from another nucleon, is dominant. Such kinematic domain corresponds to the case when the axes of two dijets in a transverse plane are oriented according to angle $\pi/2$

relative to each other. Indeed, such events can be easily separated from two to four processes. In the latter the multijet production is basically isotropic in the asimuthal angle. The source of dijets are the processes two to two hard parton collision plus a bremsstralling of two additional partons. In such processes the dijets are oriented parallel to each other. Such separation persists even when the pQCD corrections to these processes are taken into account. (details of pQCD corrections are in B.Blok,Yu. Dokshitzer, L. Frankfurt,M. Strikman (in preparation).

The aim of the talk: calculation of 4 jet cross section directly in QCD.

1) We express the four to four cross-section through new geometric objects-generalised two-particle distributions GPDs, in momentum representation.

$$D_a(x_1, x_2, k_1^2, k_2^2, \vec{\Delta}).$$

in particular we express the parameter sigma effective in terms of these GPD

2) we argue that enhancement of cross section relative to independent parton approximation is due to short range nonperturbative correlations. We stress that the enhancement of four to four processes relative to the independent particle approximation is the first confirmation of an existence of short range nonperturbative correlations in hadron (such correlations naturally arise in constituent quark model and in string model).

3)We derive from Feynman diagrams the geometric picture in impact parameter space used to study these collisions before (Paver and Treleani, Mekhfi, Calucci and Treleani, de Fabbro and Treleani, Domdey Pirner and Wiedemann, Pythia, HERWIG, Diehl)

Let us stress the importance of the subject.

1) Such processes serve a main background for some channels of standard

Model Higgs production. (De Fabbro and Treleani 1999) for example we can take:

$$p+p \to WH+X, \text{ with } W \to l\nu_l,\ H \to b\bar{b}, \text{ where } l=e,\mu.$$

2) Such processes for the first time permit us to study the internal structure of the nucleon, since the cross-section is determined by the radius of short range correlations in the nucleon. In particular there must be nonperturbative scales in the hadron smaller than the gluon radius of the nucleon.

3) Collider physics (see e.g. talks at recent MPI 2010 meeting in DESY, with detailed reasoning and suggestions.

Basically:MPI rates are slightly above Higgs in early LHC data, extra energy jets complicate all hard process, pileup rejection depends upon track based quantaties-poor resolution-soft jets matter

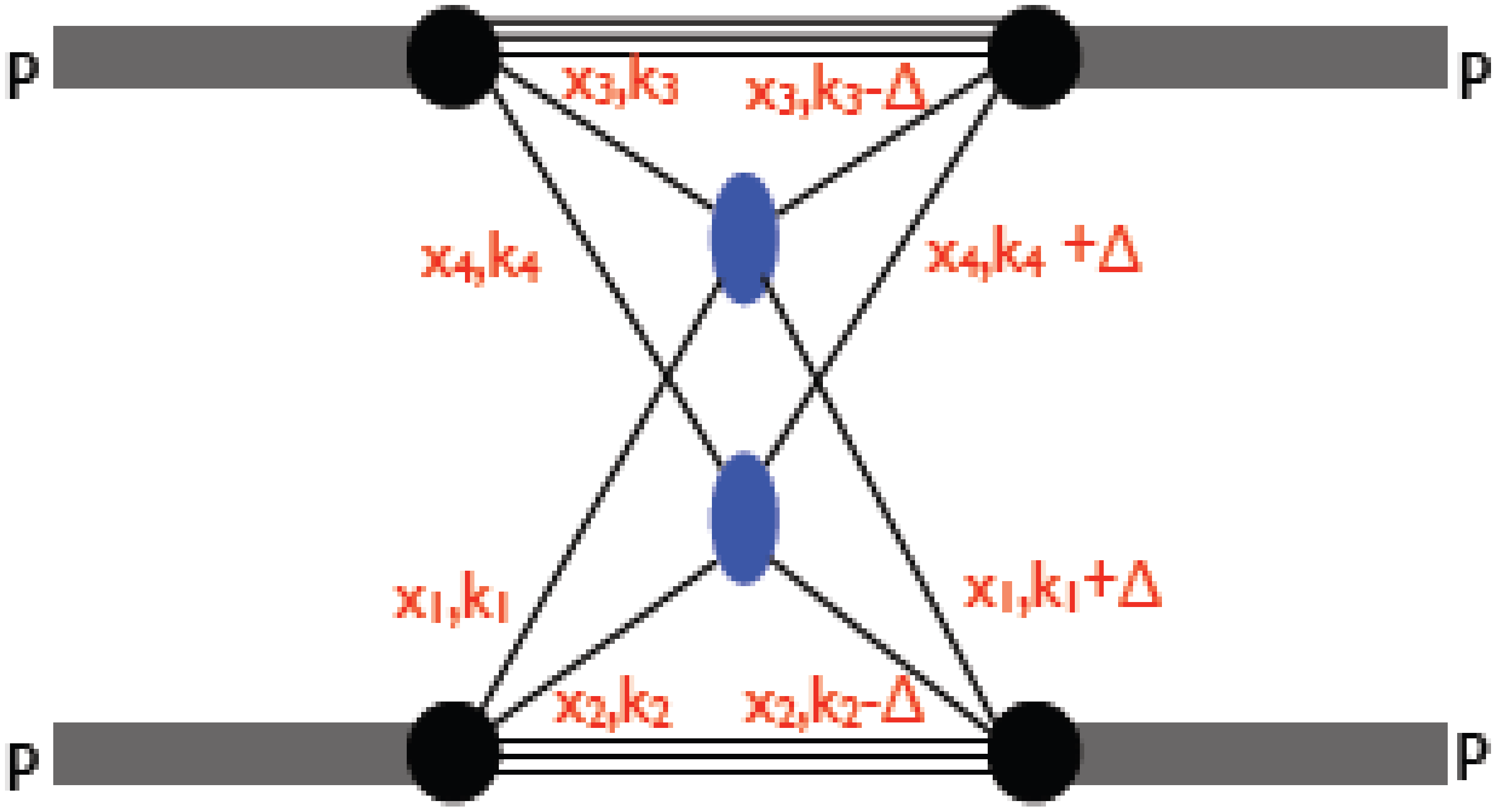
The four jet cross-section.

The four jet cross-section can be directly calculated in momentum space (see Figure 1) and is given by the formula:

$$\sigma_4 = \int \frac{d^2\Delta}{(2\pi)^2} \int dx_1 \int dx_2 \int dx_3 \int dx_4$$

$$\times \quad D_a(x_i, x_2, \vec{\Delta}) D_b(x_3, x_4, -\vec{\Delta}) \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} d\hat{t}_1 d\hat{t}_2$$

In this formula a,b are the indices of colliding hadrons, and we introduced new GPDs denoted by symbol D and calculated at virtualities p_1^2 and p_2^2 where p_1^2 and $p^2{}_2$ are the transverse momenta of the individual jets inside a dijet. We always assume a kinematics $p^2 >> \delta^2$ where δ is the dijet momentum (a sum of transverse momenta of two individual jets). In practice it is of order 5 GeV (at Tevatron), while p is at least 20-30 GeV.



p
p
x3,k3
x3,k3-Δ
x4,k4
x4,k4 +Δ
x1,k1
x1,k1+Δ
x2,k2
x2,k2-Δ
p
p

Note that new functions D that we introduced depend on the external transverse vector $\vec{\Delta}$, that describes the difference of momenta in the final and initial states of parton pair. In particular it takes into account that while the sum of the momenta is conserved between initial and final states is conserved, the difference does not

The formula for 4 jet cross section is often written in the form:

$$\sigma_4 = \sigma_1 \sigma_2 / \pi R_{\text{int}}^2$$

Here σ_1 *and* σ_2 *are the individual two jets cross-sections, and* πR_{int}^2 *characterises the transverse area occupied by the partons participating in the interaction. (Note that some authors denote this area as* σ_{eff} *We disagree with this notation since this implies that this factor is some kind of fitting parameter, while we see that it can be calculated in principle directly from the theory in a model independent way.*

It follows from the discussion above that the area $\pi R_{\rm int}^2$ can be written explicitly in terms of these new two particle GPDs as

$$\frac{1}{\pi R_{\rm int}^2} = \int \frac{d^2\Delta}{(2\pi)^2} D(x_1, x_2, \Delta) D(x_3, x_4, -\vec{\Delta})$$

This formula is valid for inclusive dijet production. When the momentum fraction are different, the exclusive production DDT formula can be easily obtained. This formula expresses the interaction area in the model independent way as the single integral over the transverse momenta.

The new GPDs can be explicitly expressed through the light cone wave functions of the hadron as

$$D\ (x_1, x_2, p_1^2, p_2^2, \overrightarrow{\Delta}) = \sum_{n=3}^{\infty} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \theta(p_1^2 - k_1^2)$$

$$\theta\ (p_2^2 - k_2^2) \int \prod_{i \neq 1,2} \frac{d^2k_i}{(2\pi)^2} \int_0^1 \prod_{i \neq 1,2} dx_i$$

$$\times\ (\psi(x_1, \vec{k}_1, x_2, \vec{k}_2, .., \vec{k}_i, x_i..)\psi^+(x_1, \overrightarrow{k_1} + \overrightarrow{\Delta}, x_2, \overrightarrow{k_2}$$

$$-\ \overrightarrow{\Delta}, x_3, \vec{k}_3, ...)$$

$$+\ h.c.)(2\pi)^3\delta(\sum_{i=1}^{i=n} x_i - 1)\delta(\sum_{i=1}^{i=n} \vec{k}_i). \qquad (4$$

Here psi are the light cone wave functions of the nucleon in the initial and final states.

These GPDs describe a four jet production. Four multijet production with more than four jets one can introduce a N paricle GPDs in a similar way, by shifting

the arguments of first N k_t

by $\vec{\Delta}_t$ subject to the constraint $\sum \vec{\Delta}_t = 0$.

We imply here the summation over color and Lorentz indices in a relevant way. In princliple a number of different GPD-s can be written depending on their color and Lorentz structure. In our kinematics the relevant GPD-s will be those without spin and color spin flip.

These GPDs can be also written in the operator form, for example for the most important gluon case we have

$$
\begin{aligned}
D(\Delta) &= < N| \int d^4x_1 d^4x_2 d^4x_3 \\
&\times G^a_{i+}(x_1) G^b_{j+}(x_2) G^a_{i+}(x_3) G^b_{j+}(x_4) \\
&\times \exp(ip_1^+(x_1 - x_3)^- + ip_2^+(x_2 - x_4)^- \\
&+ i\vec{\Delta}_t(\vec{x}_4 - \vec{x}_3)_t)|N >,
\end{aligned}
$$

(we do not write explicitly here the Wilson line factors that appear due to gauge invariance)

Such GPD will probe N dijets production

$$\sigma_{2N} \propto \int \prod_{i=1}^{i=N} \frac{d\vec{\Delta}_i}{(2\pi)^2} D_a(\vec{\Delta}_1, ...\vec{\Delta}_N)$$

$$\times D_b(\vec{\Delta}_1, ...\vec{\Delta}_N) \delta(\sum_{i=1}^{i=N} \vec{\Delta}_i).$$

Note that very similar distributions arise in DIS scattering where they were denoted as quasipartonic operators (Bukhvostov,Frolov, Lipatov and Kuraev 1985). Thus we can use their classification. There however important differences: our distributions are diagonal in longitudinal momenta and vector $\vec{\Delta}$

Is transverse (and no integration over $\vec{\Delta}$

Is implied)

Very similar formula for the cross-section but in mixed representation was known before (Mekhfi, Treleani and Paver) but we see that working purely in momentum representation has advantage for a large number of applications).

PQCD effects

An important problem are the processes where the two dijets are created not from two pairs of independent partons, but from two pairs of partons such that each pair in turn is created as a result of a splitting of perturbative partons at some hard scale. It is possible to prove however that for the back to back kinematics we are interested here such processes are strongly suppressed. The suppression is seen already in the Born approximation. The cross section of two to four processes is proportional to $\delta(\delta_1 + \delta_2)$, i.e. has one delta function, while the cross section of four to four processes is proportional to $\delta(\delta_1)\delta(\delta_2)$, the vectors δ_1 and δ_2 are transverse momentum disbalances of dijets (sums of transverse momenta of two individual jets in a dijet). This suppression can be directly extended to Leading Logarithmic Approximation in pQCD.

Cuts and phase space.

The theoretical discussion above was actually given for inclusive dijet cross-section, i.e. we integrated over the dijet disbalances from small virtualities of order GeV to the transverse momenta of order k_t -the momentum of individual jet. In practice however this is not a problem that we need to solve at experiment. What we really need to do, we have study the differential cross-section $d^4\sigma / dk_{1t}^2\, d\delta^2_1\, dk^2_{2t}\, d\delta^2_2$

Where the deltas are the dijet disbalances. In fact the integration over delta-s goes till the cuts much smaller than k_t of individual jets, and the answers will be strongly dependent on the nature of these cuts. The practical scale of these delta-s at the Tevatron is around 5 GeV.

The detailed formulae will be published elsewhere thus giving the extension of the DDT formula to two dijet processes.

The approximation of independent particles.

Suppose the multiparton wave faction factorise, i.e. we neglect possible interparton correlations and recoil effects. Then it's straightforward to see that the two particle GPDs factorise and acquire a form:

$$D(x_1, x_2, \vec{\Delta}) = G(x_1, \vec{\Delta}) G(x_2, \vec{\Delta})$$

The one-particle GPD-s are conventionally written in the dipole form:

$$G(x, Q^2)\Delta) = G(x, Q^2) F(\Delta)$$

$$F(\Delta) = 1/(\Delta^2/m_g^2 + 1)^2$$

Then for the πR^2_{int} factor we immediately obtain

$$\frac{1}{\pi R^2_{\rm int}} = \int \frac{d^2\Delta}{(2\pi)^2} F^4_{2g}(\Delta) = \frac{m_g^2}{28\pi}.$$

This result corresponds with the one by Frankfurt,Strikman and Weiss (2003) calculating complicated six dimensional integral. Here the parameter m_g is determined by the gluon radius of the nucleon:

The gluon radius of a hadron is given by

$$r_g^2/4 = dF_{2g}(t)/dt_{t=0}.$$

Then we immediately obtain

$$R^2_{\rm int} = 7/2 r_g^2$$

Let us note that this result coincides with the one obtained in a geometric picture (Frankfurt,Strikman and Weiss 2003)

However the latter computation involved a complicated 6 dimensional integral that potentially could lead to large numerical uncertainties.

Most of the experimental information on the nucleon's two–gluon form factor comes from J/ψ photoproduction, which probes the gluon GPD at an effective scale $Q^2 \approx 3\,\text{GeV}^2$, determined by the average transverse size of the $c\bar{c}$ pair during its interaction with the target, and momentum fractions of the order $x \sim M^2_{J/\psi}/W^2$

(Frankfurt, Strikman and Weiss (2003). The latter work gives also estimate of the gluon radius

$$\langle \rho^2 \rangle \approx 0.28\,\text{fm}^2 \qquad (x \gtrsim 0.01).$$

The dependence of r^2_g on Q^2 and x is given by the approximate formula that

Takes into account the DGLAP evolution:

$$\langle\rho^2\rangle(x,Q^2) \;=\; \langle\rho^2\rangle(x,Q_0^2)\,\left(1 + A\,\ln\frac{Q^2}{Q_0^2}\right)^{-a},$$

$$A \;=\; 1.5, \qquad a \;=\; 0.0090\,\ln\frac{1}{x}.$$

Here $r_g\,(Q_0^2)$ is given by equation in the previous slide. Thus we can obtain very accurate value of r_g This value remains true after the recent experimental update (Frankfurt,Strikman,Weiss, ***arXiv:1009.2559*** *).*

Substituting this value of the gluon radius leads t $34mb.$

area .

The experimental result is 15 mb, while the use of the electromagnetic radius of the nucleon leads to this area

being 60 mb.

The similar analysis for quark sea leads to slightly bigger transverse area (Strikman and Weiss 2009). Recoil may be important for large x_i but also leads to smaller total cross section, i.e. to larger R_{int}

Thus we see that the approximation of independent particles leads to the cross section two times smaller than the experimental one (Frankfurt,Strikman and Weiss 2004),

The important point here is that the range of inter parton correlations is measured by an effective range of integration over Δ

This indicates an important role of non perturbative short range correlations. The perturbative correlations are suppressed, as we see already in the representation of independent particles (the detailed analysis of soft hard interplay will be given elsewhere).

Note that such correlations naturally arise in a constituent quark model and in a string model (Lund) This is a first such indication on important role these correlations paly.

The Geometric picture.

The simplest way to derive the geometric picture is to introduce slightly more general GPDs:

$$
\begin{aligned}
D\ &(x_1, x_2, Q_1^2, Q_2^2, \vec{\Delta}_1, \vec{\Delta}_2) = \int d^2k_1 \int d_2k_2 \theta(Q_1^2 - k_1^2) \\
\theta\ &(Q_2^2 - k_2^2) \int \prod_{i \neq 1,2} dx_i d^2k_i \\
\times\ &\psi(x_1, x_2, k_1, k_2, ..k_i, x_i..)\psi^*(k_1 + \vec{\Delta}_1 \\
+\ &x_1(\vec{\Delta}_2 - \vec{\Delta}_1), k_2 - (\vec{\Delta}_1 - \vec{\Delta}_2)x_2, k_3 \\
+\ &(\vec{\Delta}_1 - \vec{\Delta}_2)x_3, ...) + h.c.)\delta(\sum x_i - 1)\delta(\sum \vec{k}_i). \qquad (7)
\end{aligned}
$$

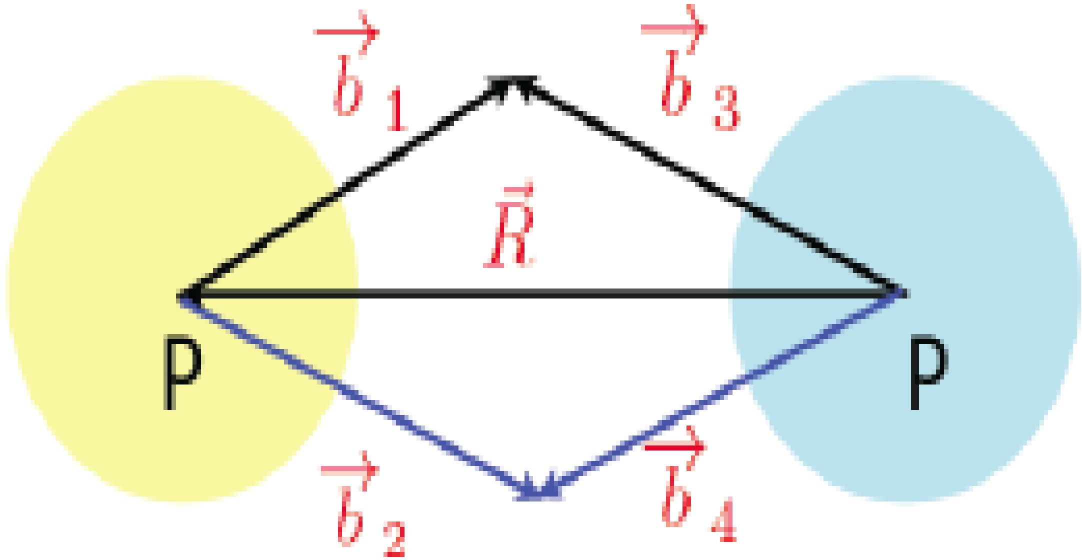


FIG. 2: Geometry of two hard collisions in impact parameter picture.

- Note that the 2 particle GPDs defined this way is connected to one particle GPDs as

$$G(x_1,\Delta_1) = \int dx_2 G(x_1,x_2,\Delta_1,\Delta_2=0).$$

- The GPDs defined above are obviously connected with these two argument GPDs as

$$D(\Delta) \equiv D(\Delta,-\Delta).$$

- In the Impact parameter space we define:

$$\begin{aligned} D\ \ (x_1,x_2,\vec{b}_1,\vec{b}_2) &= \int \frac{d^2\vec{\Delta}_1}{(2\pi)^2}\frac{d^2\vec{\Delta}_2}{(2\pi)^2} \exp\left[i(\vec{\Delta}_1\right. \\ &+ \left.(\vec{\Delta}_2-\vec{\Delta}_1)x_1\vec{b}_1\right]\exp\left[i\vec{b}_2(\vec{\Delta}_2-(\vec{\Delta}_1-\vec{\Delta}_2)x_2\right] \\ &\times D(x_1,x_2,\Delta_1,\Delta_2). \end{aligned}$$

In the impact parameter space we have

$$\sigma_4 = \int d^2Rd^2b_1d^2b_2d^2b_3d^2b_4D(b_1,b_2)D(b_3,b_4)$$

$$\times\ \delta(-b_1-(R-b_3))\delta(-b_2-(R-b_4)) =$$

$$= \int d^2Rd^2b_1d^2b_2D(b_1,b_2)D(-R+b_1,-R+b_2).$$

The delta functions express the fact that the partons interact at the same point with the acuuracy 1/p where p is a hard scale.

We now use the standard expression for delta functions:

$$\delta(-b_1-(R-b_3)) = \int \frac{d^2\Delta}{(2\pi)^2}\exp\left[i\Delta(-b_1-(R-b_3)\right]$$

To obtain for the cross-section:

$$\sigma_4 = \int d^2R \frac{d^2\Delta_1}{(2\pi)^2}\frac{d^2\Delta_2}{(2\pi)^2} d^2b_1 d^2b_2 d^2b_3 d^2b_4 D(b_1,b_2)$$

$$D\ \ (b_3,b_4)\exp\left[i\Delta_1(b_1-(b_3-R)+i\Delta_2(b_2-(b_4-R)\right].$$

We then write $D(b_1,b_2)$ and $D(b_3,b_4)$ using inverse Fourie transform in terms of GPDs in the momentum space $D(q_1,q_2)$ and $D(q_3,q_4)$ respectively.

Then integrating over the impact parameters b_1,b_2,b_3,b_4 we obtain

$$q_1 = q_3 = \Delta_1 = -q_2 = -q_4.$$

leading to the representation for the cross section

$$\sigma_4 = \int \frac{d^2\Delta}{(2\pi)^2} D(x_1,x_2,\Delta)D(x_3,x_4,\Delta)......$$

Let us note that similar relation between the representation of cross section in momentum and impact parameter spaces can be written also for the two to two collisions.

Conclusions

1) We have argued that there exists a kinematic region where four to four collisions are a dominant mechanism for four jet production.

2) The four jet cross section in this kinematic domain can be written in terms of new GPDs that can be explicitly expressed through light cone wave functions and via matrix elements of operator products.

3) We see that this picture permits one to easily visualise the key role of nonperturbative interpartonic correlations with scale smaller than the gluon radius of the hadron. The scale of these correlations corresponds to the range of integration over the transverse vector Δ

Recall that in the approximation of independent particles we obtain the result two times smaller than the experimental one.

4) We were able to derive the intuitive geometric picture widely used before and reduce the corresponding 6 dimensional integrals to the one dimensional integral in momentum space.

5) The detailed analysis of pQCD corrections is in B.Blok, Yu.Dokshitzer,M. Strikman,L. Frankfurt in preparation

In particular, there are corrections to factorisation at small x whose numerical scale is currently under investigation

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