

***AAMQS: A non-linear QCD analysis
of new HERA data at small- x***

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IPhT CEA/Saclay

Low- x Workshop
June 2-7 2011 Santiago de Compostela, Galicia, Spain

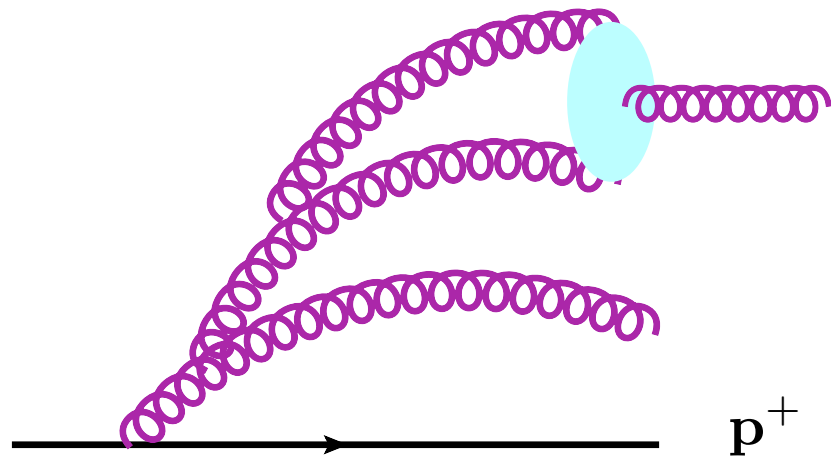
OUTLINE

- ⇒ Motivation. Dipole model of DIS
- ⇒ Running coupling corrections to BK evolution
- ⇒ Fits to DIS inclusive structure function at small- x
- ⇒ Inclusion of heavy quarks
- ⇒ Delineating the saturation line (by G Milhano, P Quiroga and J Rojo)
- ⇒ Conclusions/Outlook

Based on:

- JLA, N. Armesto, J.G. Milhano P. Quiroga and C. Salgado (arXiv 1012.4408 [hep-ph])
- JLA, N. Armesto, J.G. Milhano and C. Salgado (arXiv 1209.1112 [hep-ph])
- JLA PRL99:262301
- JLA and Y. Kovchegov PRD75:125021

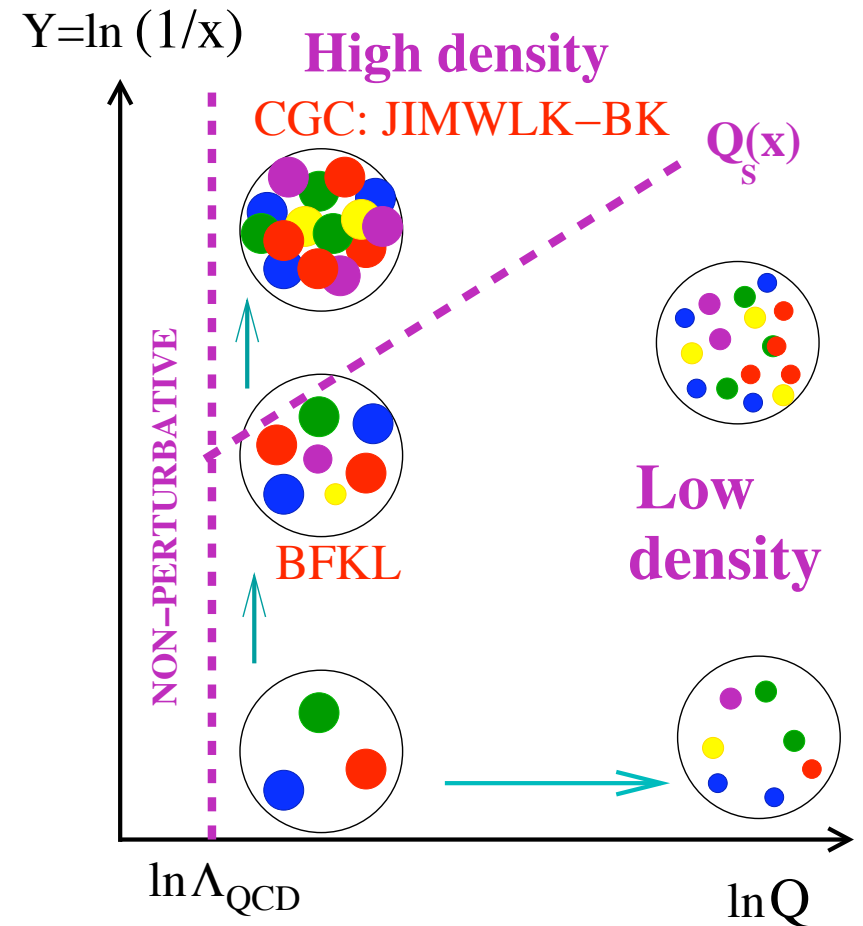
⇒ **Saturation:** At small Bjorken- x the hadron wave function gets dense and non-linear processes become a relevant dynamical ingredient



“BK-JIMWLK”

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k}_t)}{\partial \ln(\mathbf{x}_0/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k}_t) - \phi(\mathbf{x}, \mathbf{k}_t)^2$$

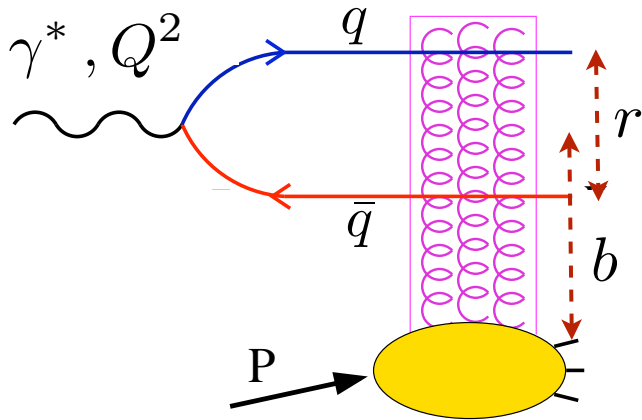
Non-linear *recombination* corrections are demanded by UNITARITY



⇒ To what extent are such effects present in available $e+p$ data?

Dipole model of DIS

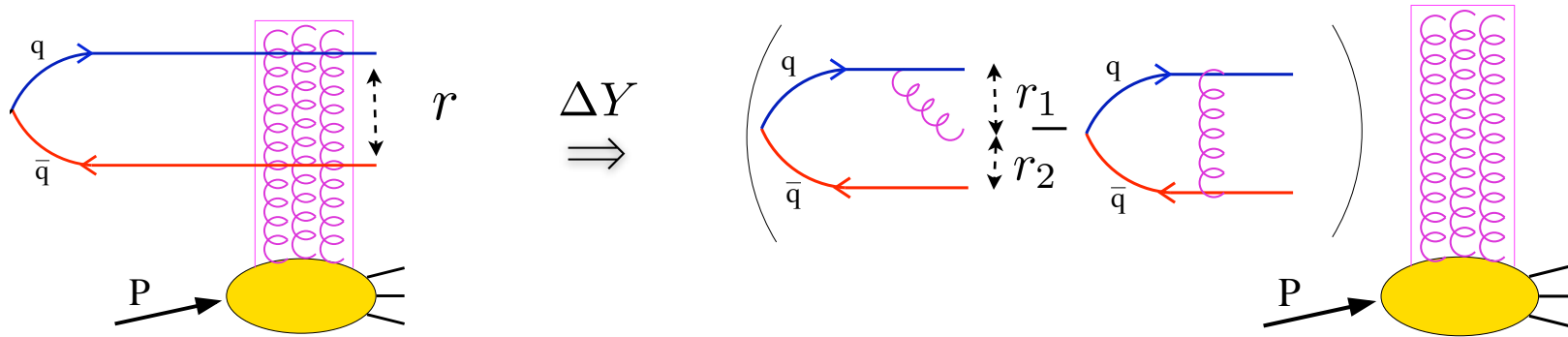
- ⇒ Dipole models including saturation describe a large amount of HERA data (inclusive and longitudinal structure functions, diffraction, DVCS, VM, geometric scaling..).
- ⇒ They provide insight in the region “forbidden” to DGLAP ($Q^2 < 2 \text{ GeV}^2$).



$$\sigma_{T,L}^{\gamma^* P}(x, Q^2) = \int_0^1 dz \int d^2 \mathbf{r} \left| \Psi_{T,L}^{\gamma^* \rightarrow q\bar{q}}(z, Q, r) \right|^2 \sigma^{dip}(x, r)$$

$$\sigma^{dip}(x, r) = 2 \int d^2 b \mathcal{N}(x, b, r) \rightarrow \text{Dipole cross section. Strong interactions and x-dependence are here}$$

⇒ **pQCD tools:** The non-linear **Balitsky-Kovchegov eqn.** describes the small- x evolution of the dipole scattering amplitude at leading order in $\alpha_s \ln(1/x)$

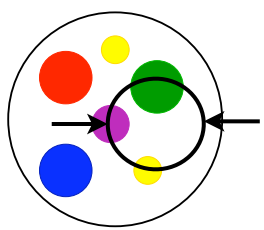


$$\frac{\partial \mathcal{N}(x, r)}{\partial \ln(x_0/x)} = \int d^2 r_1 K^{LO}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) [\mathcal{N}(x, r_1) + \mathcal{N}(x, r_2) - \mathcal{N}(x, r) - \mathcal{N}(x, r_1)\mathcal{N}(x, r_2)]$$

The LL kernel:
$$K^{LO}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \frac{\alpha_s N_c}{2\pi^2} \frac{r^2}{r_1^2 r_2^2}$$

↑
Non-linear term

⇒ **However,** at LL accuracy (fixed coupling) the BK equation is not compatible with data



$$\left\{ \begin{array}{l} Q_s^2(Y) = Q_0^2 \exp \lambda Y \\ \lambda = \frac{d \ln Q_s^2(Y)}{dY} \end{array} \right.$$

**Fits to HERA
and RHIC data**

$$\lambda \sim 0.2 \div 0.3$$

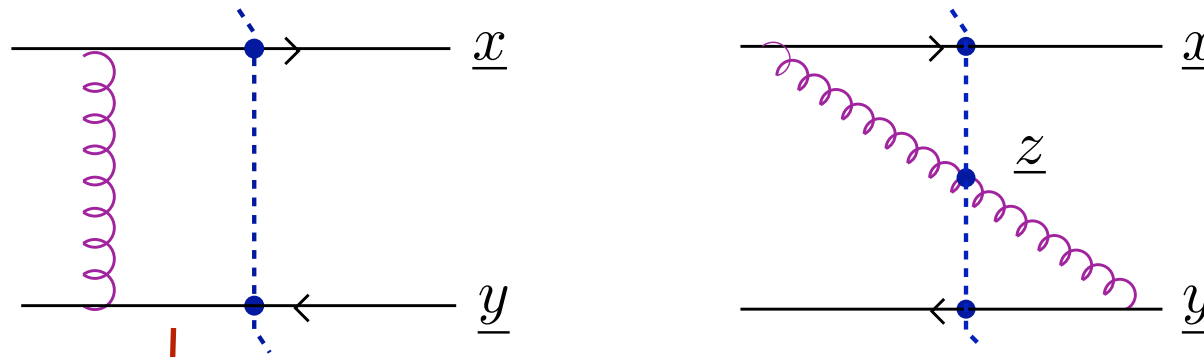
**LL-BK
(fixed coupling)**

$$\lambda^{LL} \sim 4.8 \alpha_s$$

Running coupling corrections (Kovchegov-Weigert, Balitsky, Gardi et al)

Strategy: resummation of quark loops to all orders, plus $N_f \longrightarrow -6\pi\beta$

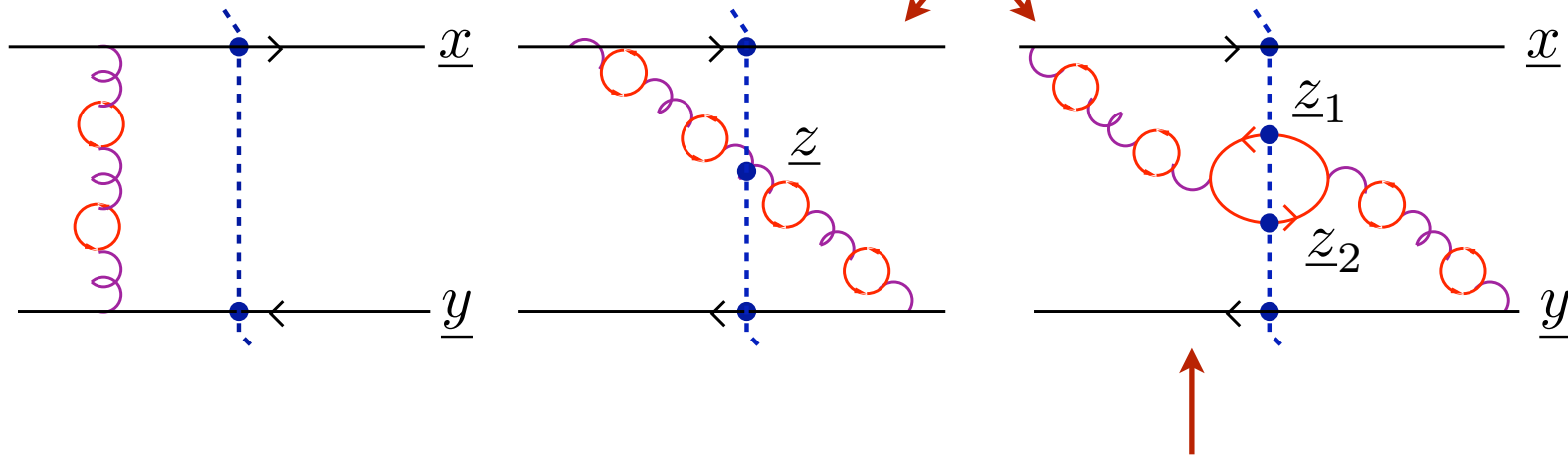
⇒ Leading log
(fixed coupling)



⇒ All orders in $\alpha_s N_f$

$$N_f \longrightarrow -6\pi\beta$$

(running coupling)



New physical channels: quark-antiquark pairs in the final state.
They contain UV divergencies that contribute to the running of the coupling

Fixed vs Running

⇒ The **running of the coupling** reduces the speed of the evolution down to values compatible with experimental data (JLA PRL 99 262301 (07)):

$$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$

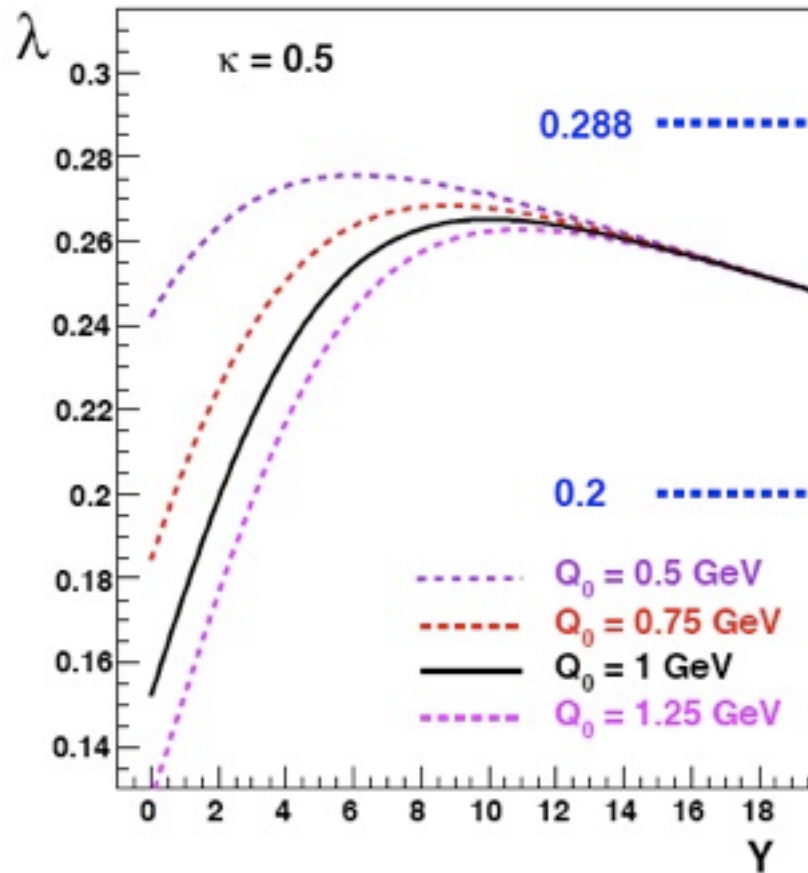
$$\lambda = \frac{d \ln Q_s^2(Y)}{dY}$$

LL evolution:

$$\lambda^{LL} \approx 4.8 \alpha_s$$

DIS data:

$$\lambda^{DIS} \approx 0.288$$



⇒ Fits to inclusive DIS e+p structure functions & reduced x-section

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} (\sigma_T + \sigma_L) \quad \sigma_r(y, x, Q^2) = F_2(x, Q^2) - \frac{y^2}{1 + (1-y)^2} F_L(x, Q^2).$$

⇒ x-dependence: translational invariant (no b-dependence) running coupling BK using Balitsky's prescription

$$\frac{\partial \mathcal{N}(x, r)}{\partial \ln(x_0/x)} = \int d^2 r_1 K^{Bal}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) [\mathcal{N}(x, r_1) + \mathcal{N}(x, r_2) - \mathcal{N}(x, r) - \mathcal{N}(x, r_1)\mathcal{N}(x, r_2)]$$

$$K^{Bal}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

⇒ Regularization of the coupling: We freeze to a constant, $\alpha_{fr}=0.7$ in the IR:

$$\alpha_s(r^2) = \frac{12\pi}{(11N_c - 2N_f) \ln\left(\frac{4C^2}{r^2 \Lambda_{QCD}}\right)} \quad \text{for } r < r_{fr}, \quad \text{with } \alpha_s(r_{fr}^2) \equiv \alpha_{fr} = 0.7$$

$$\alpha_s(r^2) = \alpha_{fr} = 0.7 \quad \text{for } r > r_{fr} \quad \Lambda_{QCD} = 0.241 \text{ GeV}$$

⇒ **Initial Conditions.** Inspired in the GBW and MV models:

$$\text{A) } \mathcal{N}^{GBW}(r, x_0 = 10^{-2}) = 1 - \exp \left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^\gamma \right]$$

$$\text{B) } \mathcal{N}^{MV}(r, x_0 = 10^{-2}) = 1 - \exp \left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^\gamma \ln \left(\frac{1}{r \Lambda_{QCD}} \right) \right]$$

$$\text{C) "scaling" } \mathcal{N}(r, Y \gg 1) \rightarrow \mathcal{N}^{scal}(\tau = r Q_s(Y)). \quad r Q_s(Y) \rightarrow r Q_{s0}$$

Free parameters: proton saturation scale at $x_0=10^{-2}$, Q_{s0}^2 , and anomalous dimension, γ

⇒ **Overall normalization:**

$$2 \int d\mathbf{b} \rightarrow \sigma_0$$

kinematic shift: $\tilde{x} = x \left(1 + \frac{4 m_f^2}{Q^2} \right)$

⇒ **3 (4) free parameters:** Normalization, σ_0 , initial saturation scale, Q_{s0}^2

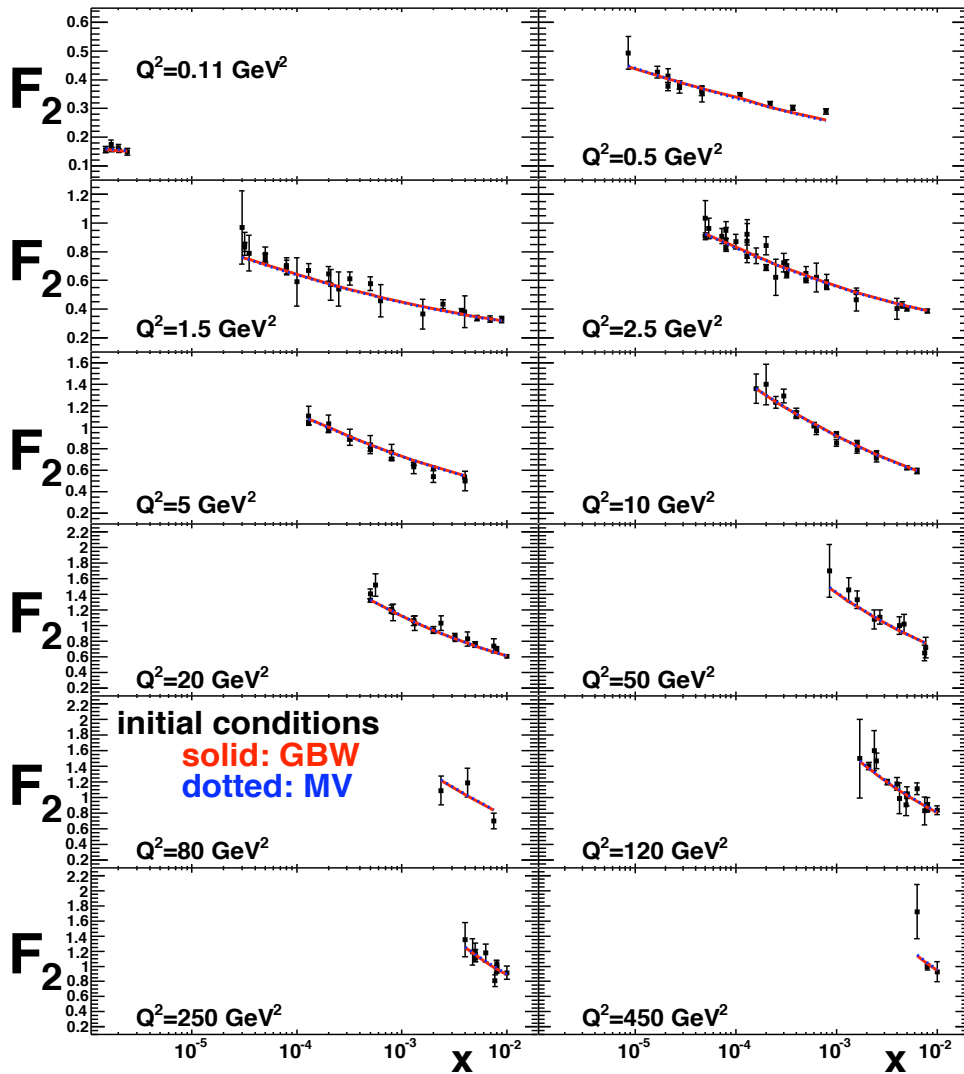
IR parameter, C^2 (anomalous dimension of the i.c. γ)

⇒ **Experimental data:** New ZEUS+HI combined analysis (HERA), NMC (CERN-SPS) and E665 (Fermilab) coll.

$$x \leq 10^{-2} \quad 0.045 < Q^2 < 50 \text{ GeV}^2$$

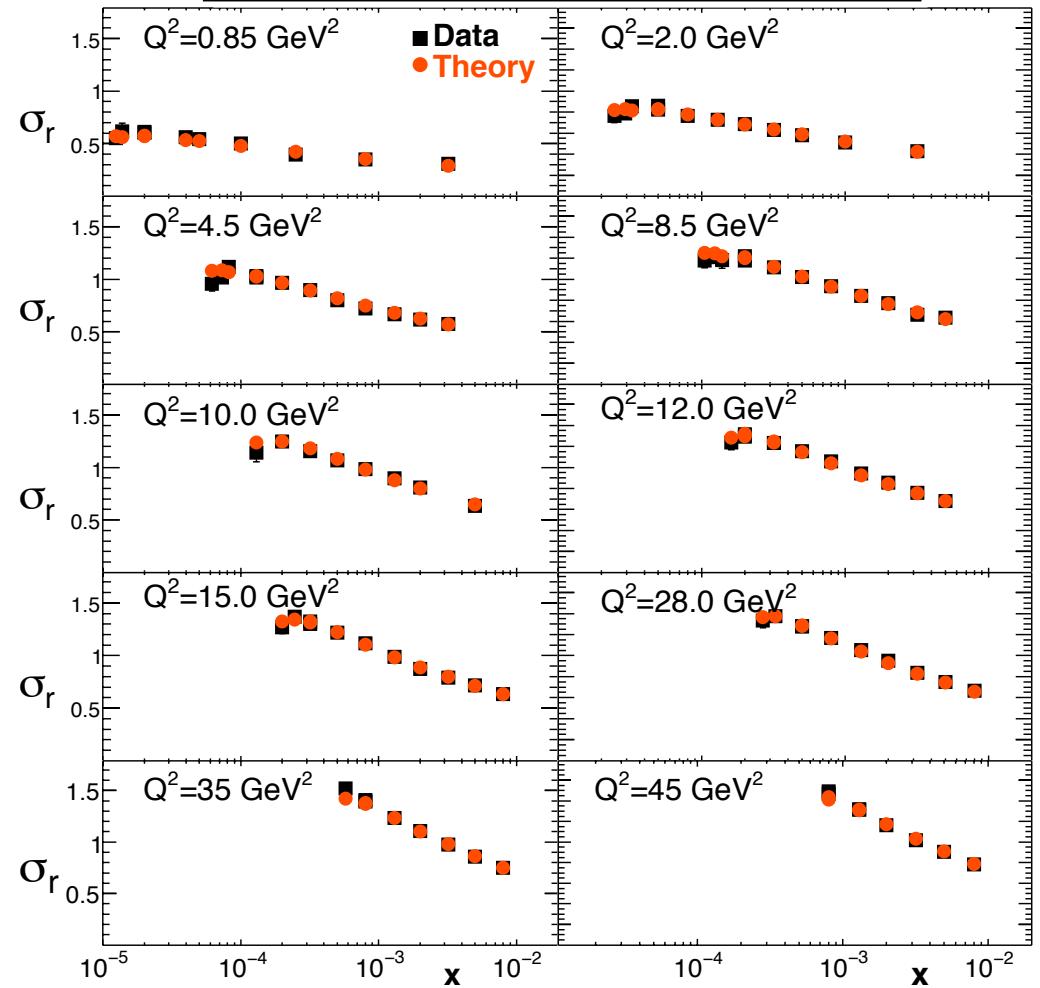
⇒ Fit results

Fits to "OLD" HERA data



Combined H1 + ZEUS data

Fit with only light quarks



JLA, N. Armesto, J.G. Milhano, C. Salgado
 Phys.Rev.D80:034031,2009;

JLA, N. Armesto, J.G. Milhano, P Quiroga
 and C. Salgado arXiv 1012.4408 [hep-ph]

⇒ Fit results

- Fits parameters are stable w.r.t to the fits to older data

	fit	$\frac{\chi^2}{d.o.f}$	Q_{s0}^2	σ_0	γ	C	m_l^2
	GBW						
a	$\alpha_{fr} = 0.7$	1.226	0.241	32.357	0.971	2.46	fixed
a'	$\alpha_{fr} = 0.7 (\Lambda_{m_\tau})$	1.235	0.240	32.569	0.959	2.507	fixed
b	$\alpha_{fr} = 0.7$	1.264	0.2633	30.325	0.968	2.246	1.74E-2
c	$\alpha_{fr} = 1$	1.279	0.254	31.906	0.981	2.378	fixed
c'	$\alpha_{fr} = 1 (\Lambda_{m_\tau})$	1.244	0.2329	33.608	0.9612	2.451	fixed
d	$\alpha_{fr} = 1$	1.248	0.239	33.761	0.980	2.656	2.212E-2
	MV						
e	$\alpha_{fr} = 0.7$	1.171	0.165	32.895	1.135	2.52	fixed
f	$\alpha_{fr} = 0.7$	1.161	0.164	32.324	1.123	2.48	1.823E-2
g	$\alpha_{fr} = 1$	1.140	0.1557	33.696	1.113	2.56	fixed
h	$\alpha_{fr} = 1$	1.117	0.1597	33.105	1.118	2.47	1.845E-2
h'	$\alpha_{fr} = 1 (\Lambda_{m_\tau})$	1.104	0.168	30.265	1.119	1.715	1.463E-2

NOTE: Statistical and systematic errors added in quadrature

⇒ Including heavy quarks

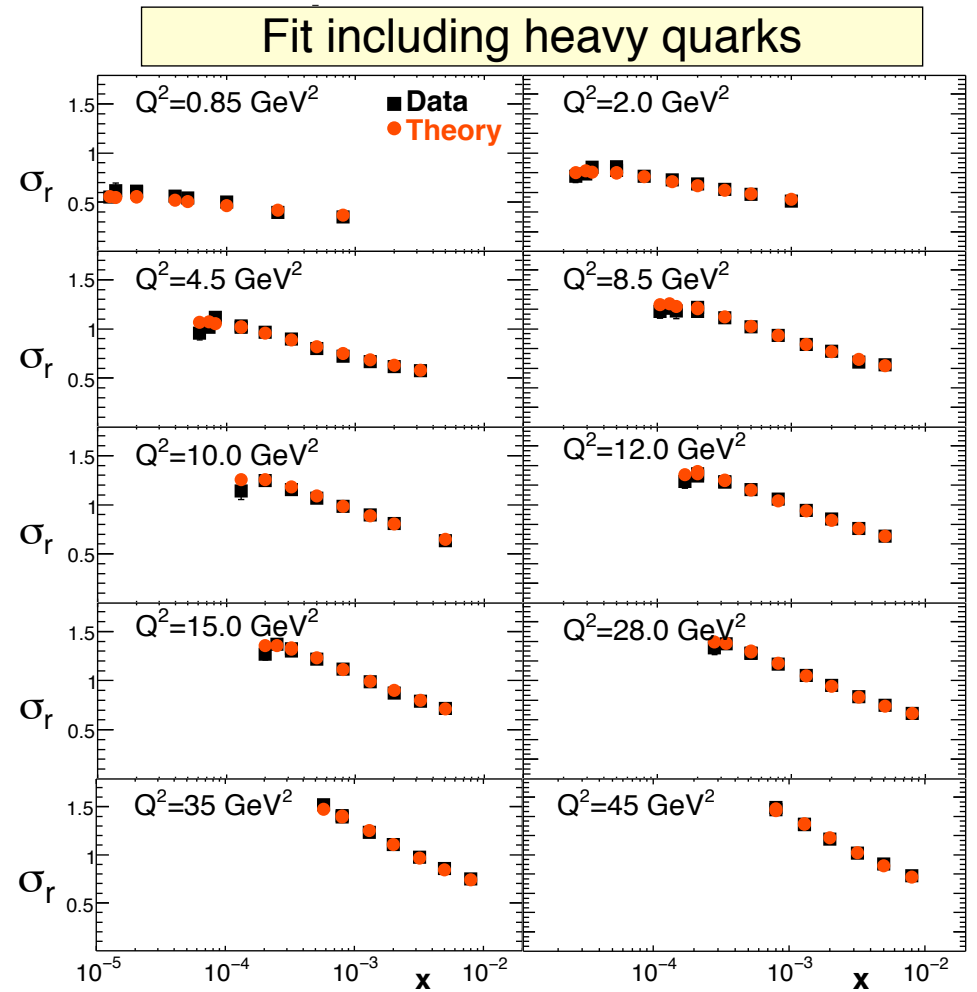
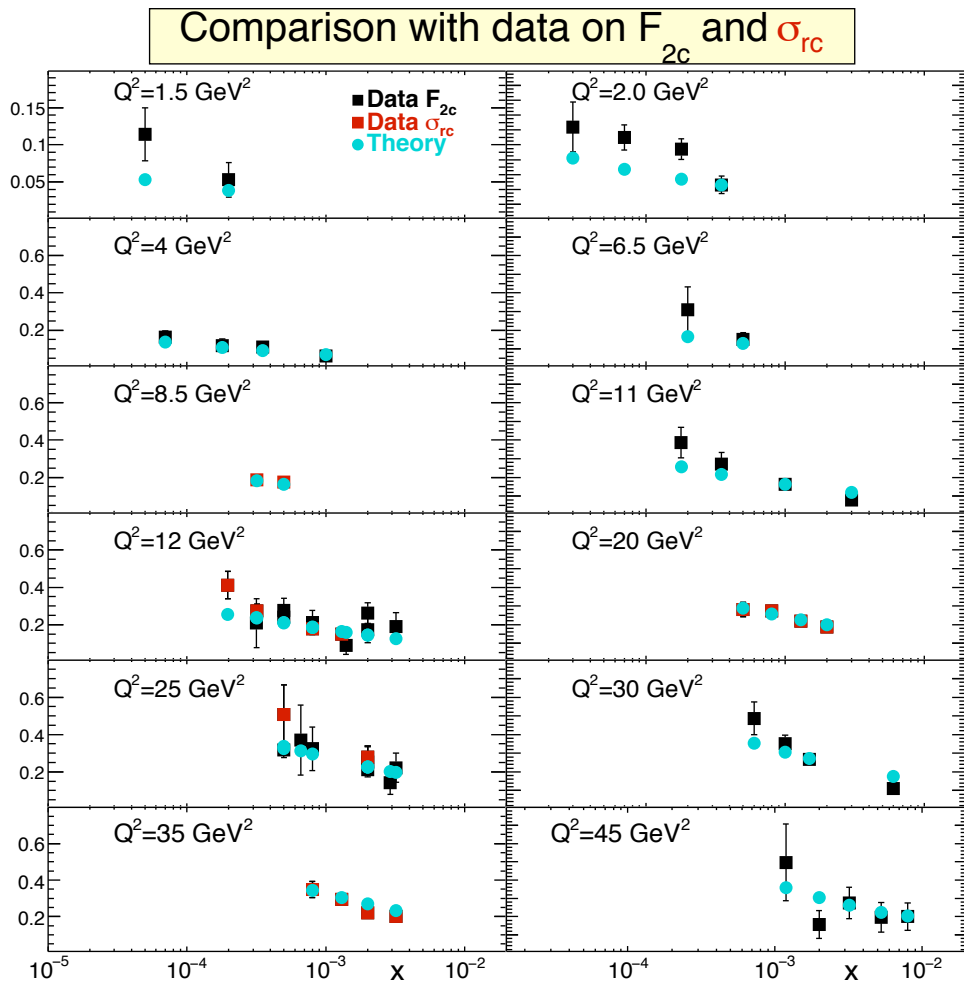
- Extend the sum to heavy flavors (b and c) in the dipole model
- Allow for different parameters for the heavy quark contribution and initial conditions

$$\begin{aligned}\sigma_{T,L}(x, Q^2) &= \sigma_0 \sum_{f=u,d,s} \int_0^1 dz d\mathbf{r} |\Psi_{T,L}^f(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}^{light}(\mathbf{r}, x) \\ &+ \sigma_0^{heavy} \sum_{f=c,b} \int_0^1 dz d\mathbf{r} |\Psi_{T,L}^f(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}^{heavy}(\mathbf{r}, x).\end{aligned}$$

- For consistency, we consider a variable flavor number scheme for the running of the coupling

$$\alpha_{n_f}(r^2) = \frac{4\pi}{\beta_{0,n_f} \ln\left(\frac{4C^2}{r^2 \Lambda_{n_f}^2}\right)} \quad \alpha_{S,n_f-1}(r_\star^2) = \alpha_{S,n_f}(r_\star^2) \quad r_\star^2 = 4C^2/m_f^2$$

⇒ Fits with heavy quarks



- No constraints to b contribution from present data...

⇒ Fits with heavy quarks

	fit	$\frac{\chi^2}{d.o.f}$	Q_{s0}^2	σ_0	γ	Q_{s0c}^2	σ_{0c}	γ_c	C	m_t^2
	GBW									
a	$\alpha_{f_r} = 0.7$	1.269	0.2294	36.953	1.259	0.2289	18.962	0.881	4.363	fixed
a'	$\alpha_{f_r} = 0.7 (\Lambda_{m_\tau})$	1.302	0.2341	36.362	1.241	0.2249	20.380	0.919	7.858	fixed
b	$\alpha_{f_r} = 0.7$	1.231	0.2386	35.465	1.263	0.2329	18.430	0.883	3.902	1.458E-2
c	$\alpha_{f_r} = 1$	1.356	0.2373	35.861	1.270	0.2360	13.717	0.789	2.442	fixed
d	$\alpha_{f_r} = 1$	1.221	0.2295	35.037	1.195	0.2274	20.262	0.924	3.725	1.351E-2
	MV									
e	$\alpha_{f_r} = 0.7$	1.395	0.1673	36.032	1.355	0.1650	18.740	1.099	3.813	fixed
f	$\alpha_{f_r} = 0.7$	1.244	0.1687	35.449	1.369	0.1417	19.066	1.035	4.079	1.445E-2
g	$\alpha_{f_r} = 1$	1.325	0.1481	40.216	1.362	0.1378	13.577	0.914	4.850	fixed
h	$\alpha_{f_r} = 1$	1.298	0.156	37.003	1.319	0.147	19.774	1.074	4.355	1.692E-2

- Larger transverse “size” of the light contribution $\sigma_0^{light} > \sigma_0^{charm}$
- χ^2/dof improve significantly if charm data excluded in its calculation

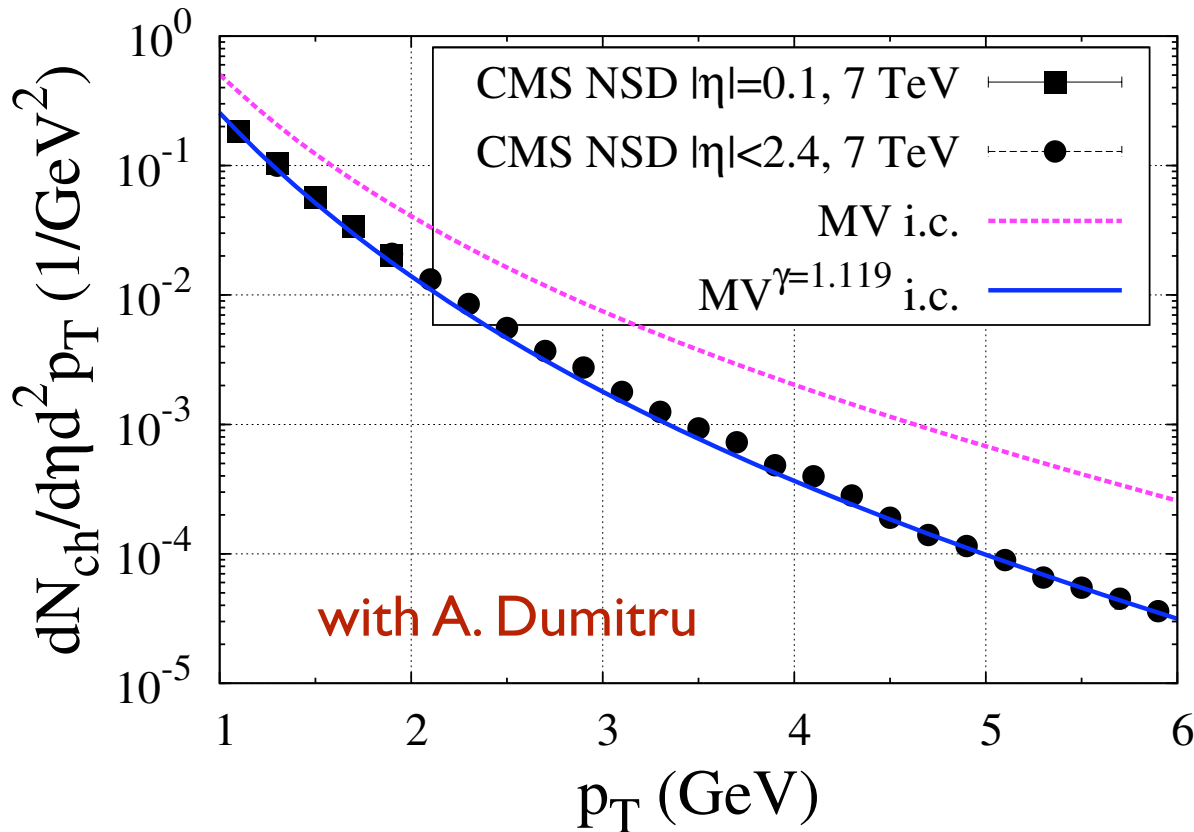
- Steeper than MV (i.e $\gamma > 1$) preferred by the fits are needed to describe the **p+p spectra measured at the LHC**. kt-factorization+KKP fragmentation

$$\mathcal{N}^{MV}(r, x_0 = 10^{-2}) = 1 - \exp \left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^\gamma \ln \left(\frac{1}{r \Lambda_{QCD}} \right) \right] \quad \gamma = 1.119$$

$$\frac{d\sigma^{A+B \rightarrow g}}{dy d^2p_t d^2R} = \kappa \frac{2}{C_F} \frac{1}{p_t^2} \int^{p_t} \frac{d^2k_t}{4} \int d^2b \alpha_s(Q) \varphi\left(\frac{|p_t + k_t|}{2}, x_1; b\right) \varphi\left(\frac{|p_t - k_t|}{2}, x_2; R - b\right)$$

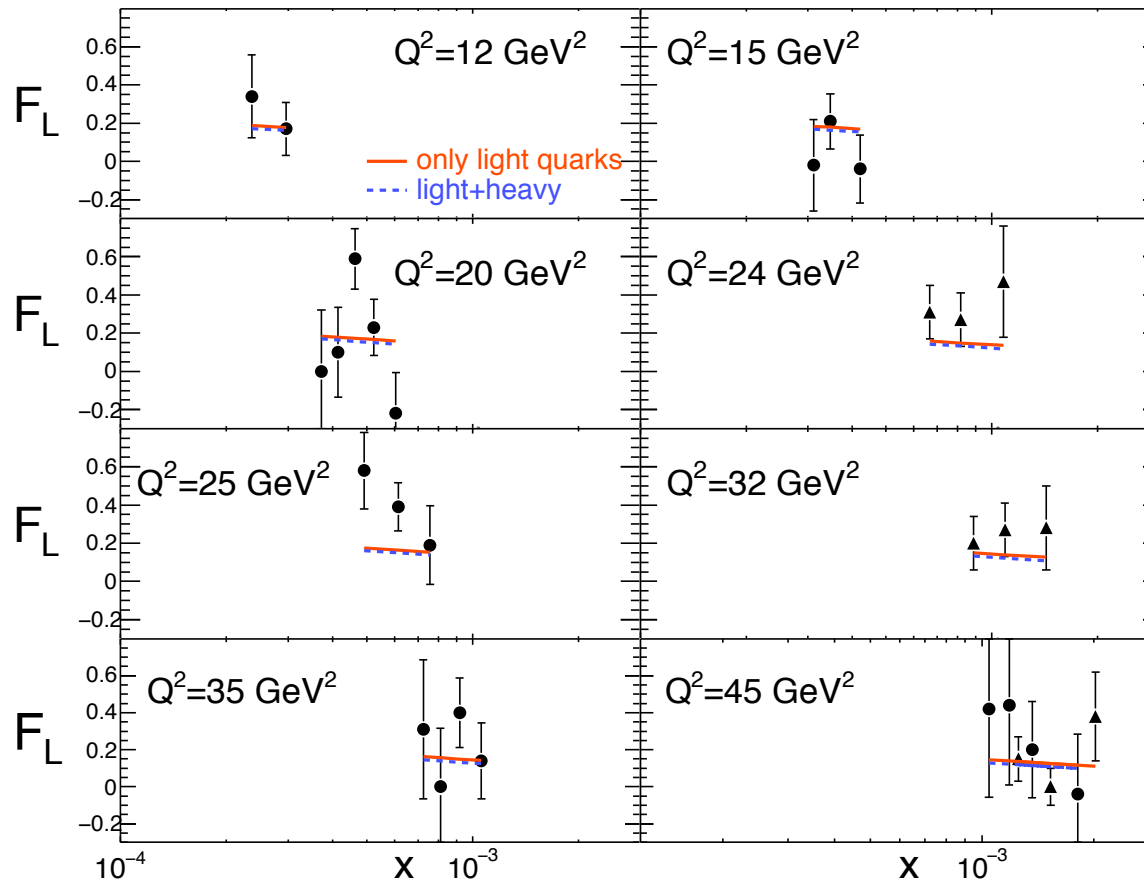
unintegrated gluon
distributions

$$\varphi(k, x, b) = \frac{C_F}{\alpha_s(k) (2\pi)^3} \int d^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \nabla_{\mathbf{r}}^2 \mathcal{N}_G(r, Y = \ln(x_0/x), b)$$



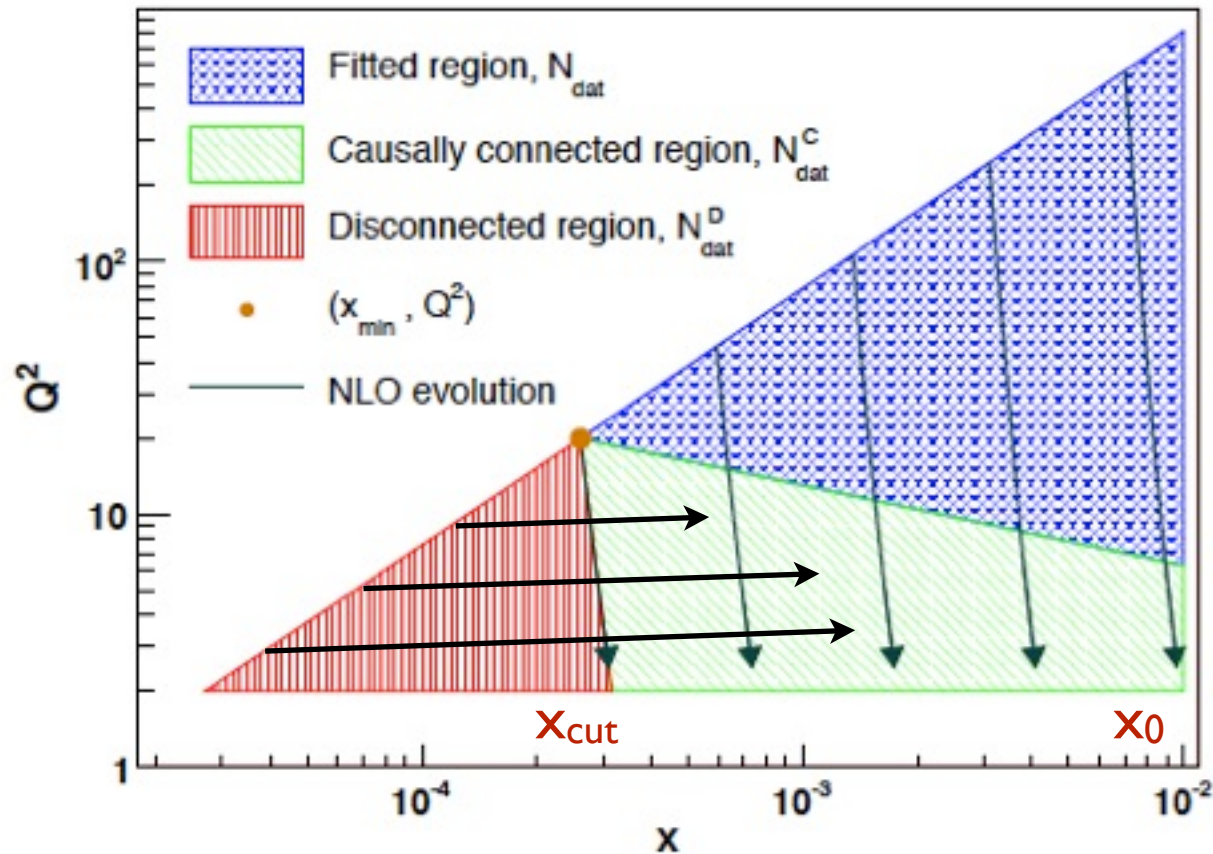
⇒ Fits with heavy quarks

- In both cases, i.e. only light or light+heavy quarks, a good description of FL data (not included in the fits) is obtained



⇒ Delineating the saturation boundary (G Milhano, P. Quiroga and J Rojo):

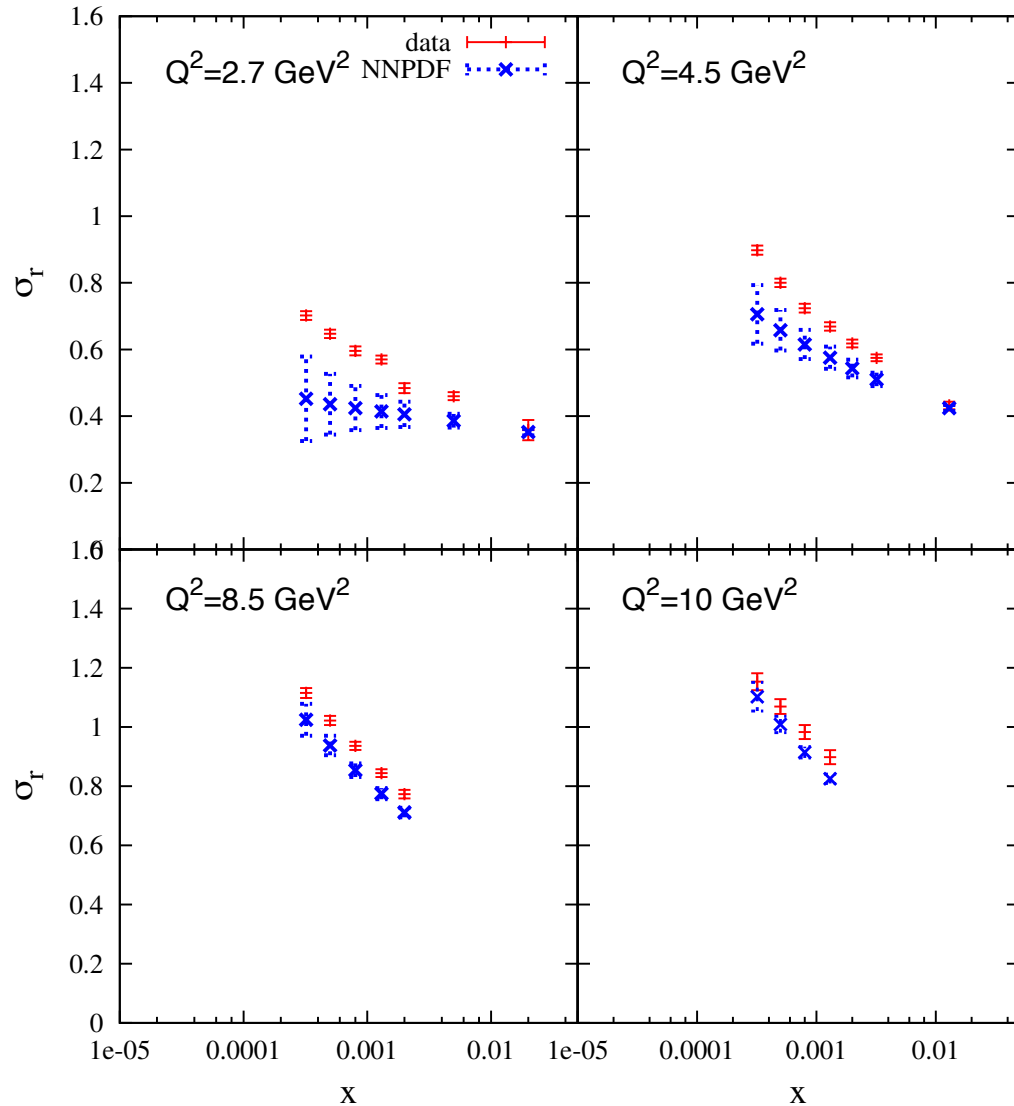
- NLO DGLAP analysis exhibit deviations after systematic exclusion of low- Q^2 regions (“saturation cuts”) from the fits (Caola, Forte, Rojo)
- Analogous exercise with rcBK:
 - Systematically exclude high- x regions from the fits ($x > x_{\text{cut}} > x_0 = 10^{-2}$)
 - Compare with fits including the region ($x > x_{\text{cut}} = 10^{-2}$)



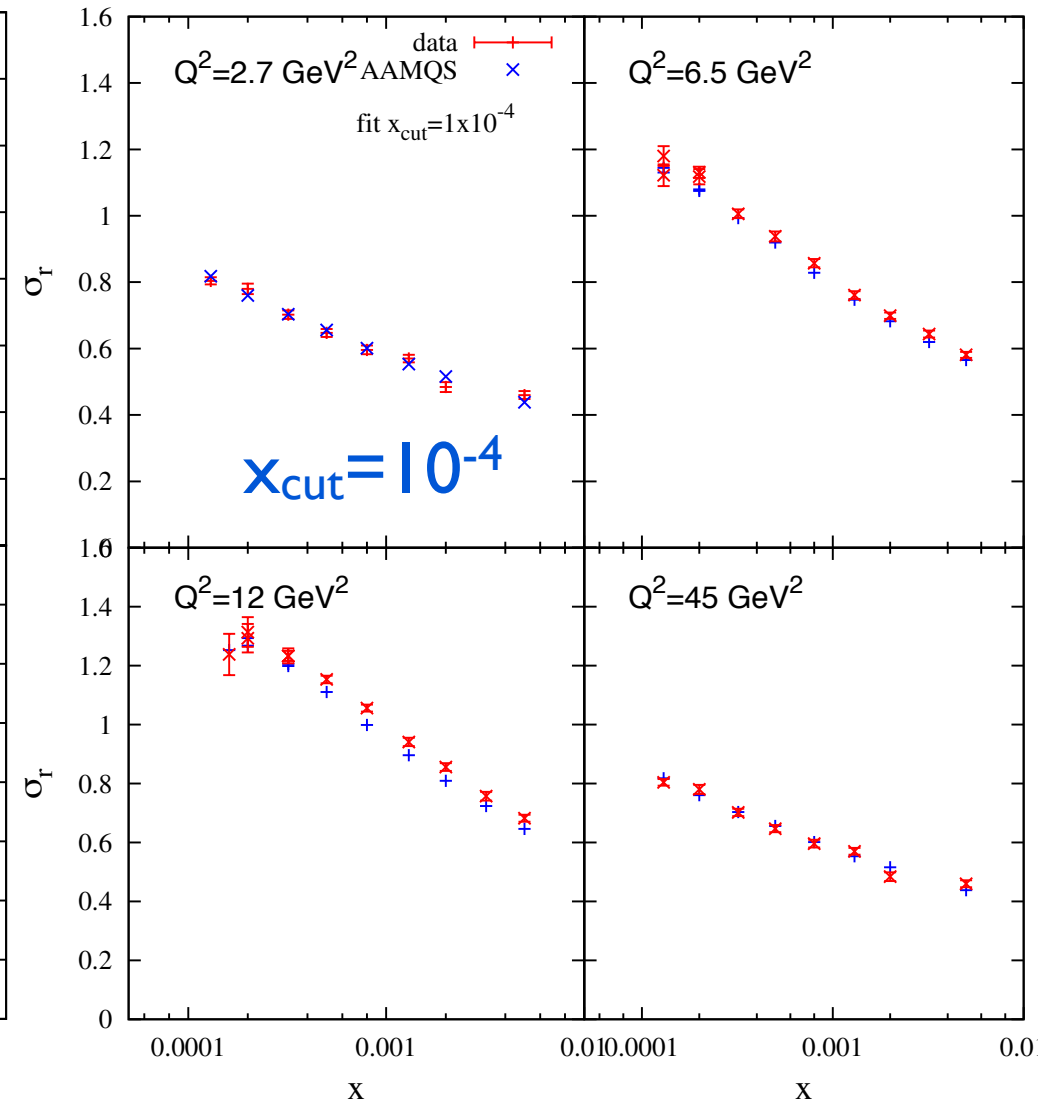
⇒ Delineating the saturation boundary (G Milhano, P. Quiroga and J Rojo):

- NNPDF and AAMQS comparison to **same** σ_r data in the unfitted region

NNPDF (NLO DGLAP)

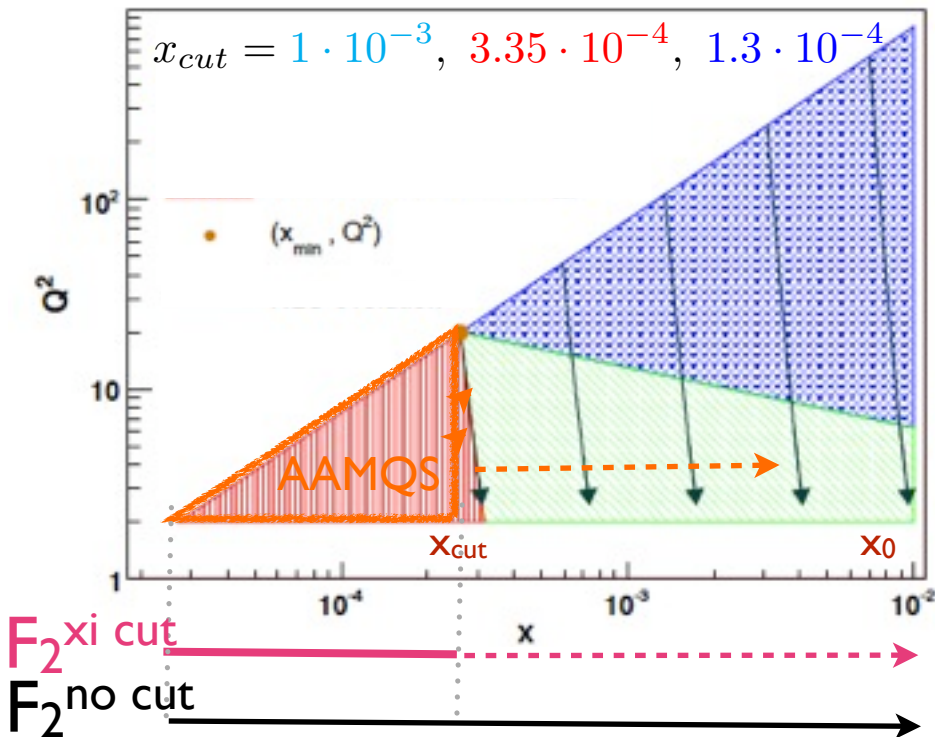
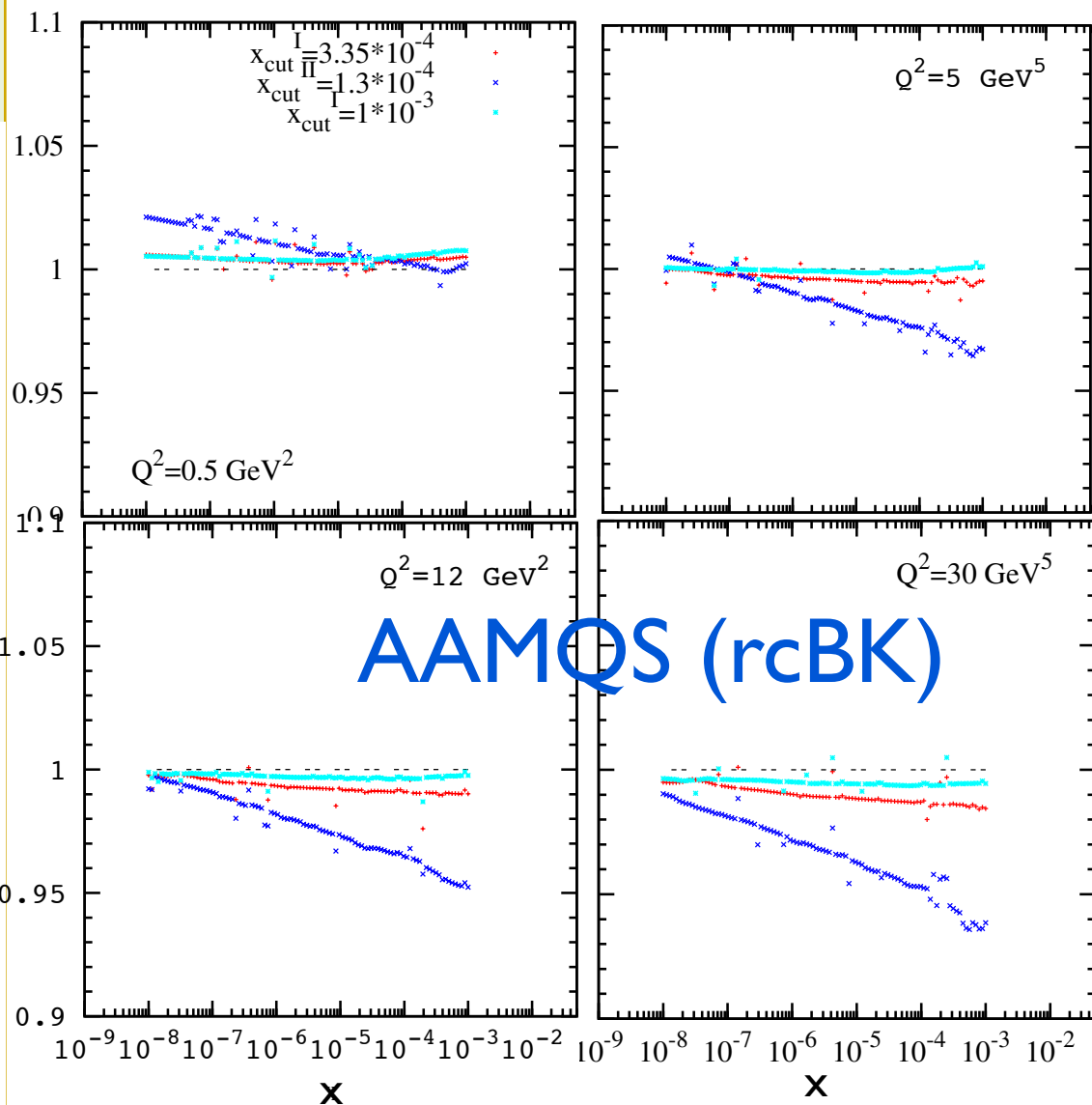


AAMQS (rcBK)



⇒ Delineating the saturation boundary (G. Milhano, P. Quiroga and J Rojo):

$$F_2^{x_{cut}^i} / F_2^{no\ cut}$$



$F_2^{x_{cut}}$
 $F_2^{no\ cut}$

- Small deviations are found: other relevant physics [DGLAP, ...?] not included in AAMQS is relevant in such region

- Deviations increase with decreasing x_{cut} and increasing Q^2 . MAKES PERFECT SENSE

Summary

- Running coupling BK evolution successfully describes new combined H1+ZEUS data on reduced cross sections at small- x
- Fit parameters are stable after the inclusion of the new data
- Charm contribution to the cross section can be accounted for, albeit allowing a smaller radius for the charm distribution in the proton than for light ones
- Systematic exploration of the saturation boundary ongoing
- Precision studies indicate deviations of DGLAP approaches in the low- x , low- Q^2 data, while rcBK yields more stable results
- Next: analogous global fits for nuclear data, include NLO photon impact factor, realistic b -dependence...

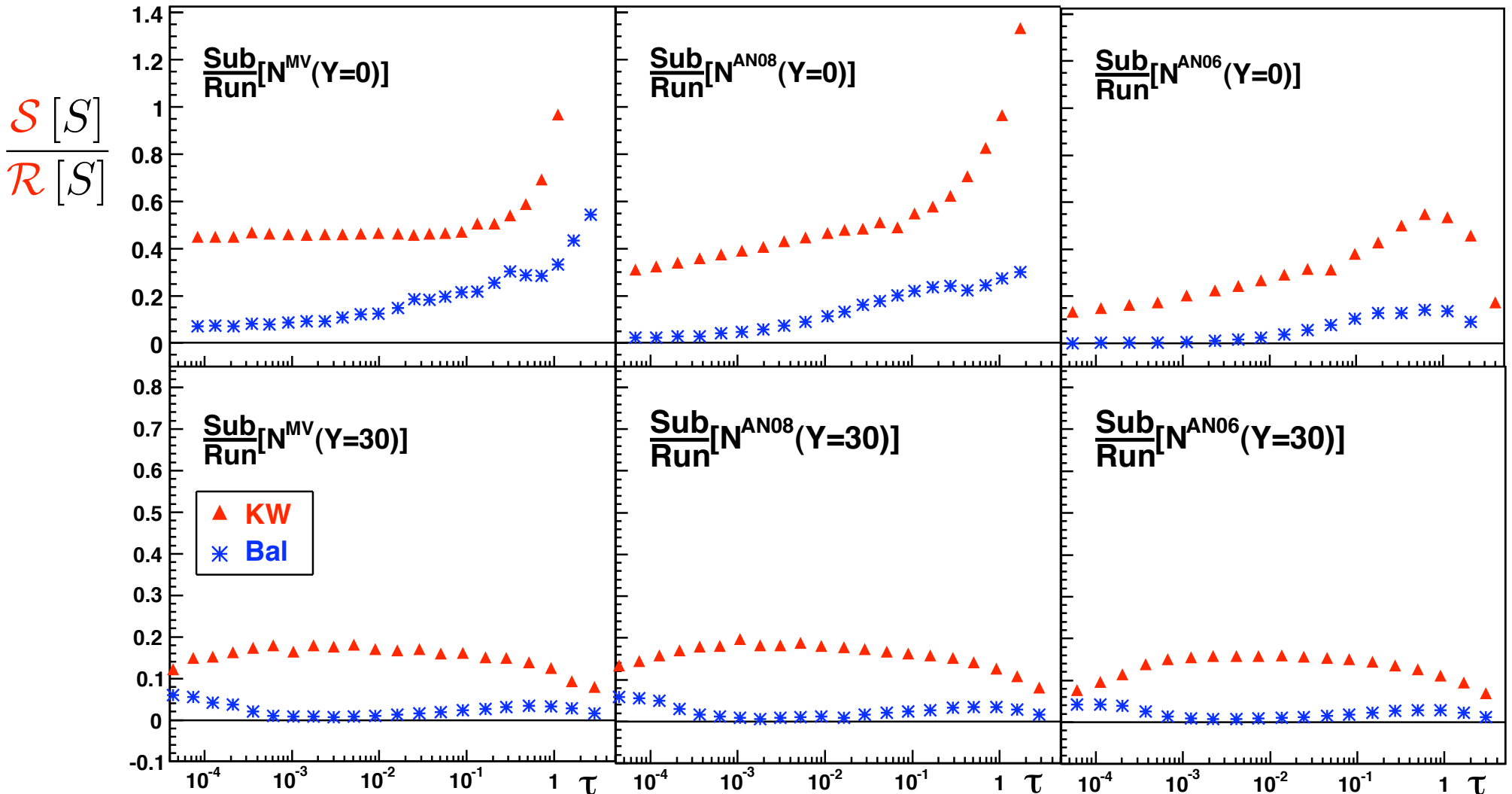
Parametrizations of the proton-dipole amplitude available at
<http://www-fp.usc.es/phenom/software.html>

Thanks!

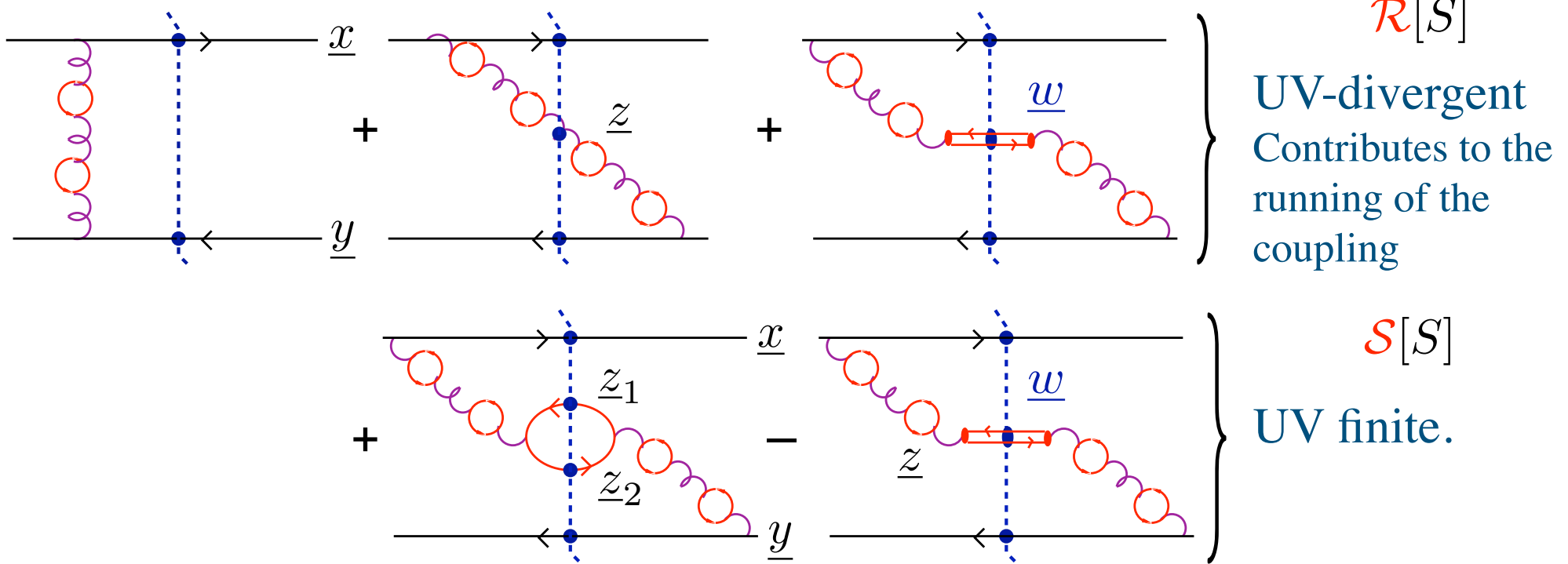
BACK UP SLIDES

- The dominant contribution to the evolution is given by the **running** term
- Balitsky's separation scheme minimizes the role of the subtraction term w.r.t. to KW's one

$$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$



$$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$



⇒ **Running term:** $\mathcal{R}[S] = \int d^2 z \tilde{K}(\underline{x}, \underline{z}, \underline{y}) [S(\underline{x}, \underline{z})S(\underline{z}, \underline{y}) - S(\underline{x}, \underline{y})]$

⇒ **Subtraction term:** $\mathcal{S}[S] = \int d^2 z_1 d^2 z_2 K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2) [S(\underline{x}, \underline{w})S(\underline{w}, \underline{y}) - S(\underline{x}, \underline{z}_1)S(\underline{z}_2, \underline{y})]$

Two different separation schemes: Balitsky's (BAL) and Kovchegov-Weigert's (KW)

- Steeper than MV (i.e $\gamma > 1$) preferred by the fits are needed to describe the **p+p spectra measured at the LHC**. kt-factorization+KKP fragmentation

$$\mathcal{N}^{MV}(r, x_0 = 10^{-2}) = 1 - \exp \left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^\gamma \ln \left(\frac{1}{r \Lambda_{QCD}} \right) \right] \quad \gamma = 1.119$$

$$\frac{d\sigma^{A+B \rightarrow g}}{dy d^2p_t d^2R} = \kappa \frac{2}{C_F} \frac{1}{p_t^2} \int^{p_t} \frac{d^2k_t}{4} \int d^2b \alpha_s(Q) \varphi\left(\frac{|p_t + k_t|}{2}, x_1; b\right) \varphi\left(\frac{|p_t - k_t|}{2}, x_2; R - b\right)$$

unintegrated gluon
distributions

$$\varphi(k, x, b) = \frac{C_F}{\alpha_s(k) (2\pi)^3} \int d^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \nabla_{\mathbf{r}}^2 \mathcal{N}_G(r, Y = \ln(x_0/x), b)$$

