

NLO Photon Impact Factor for DIS at small- x

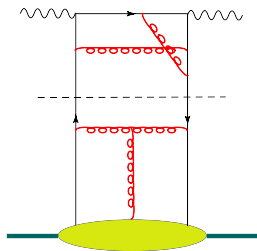
Giovanni Antonio Chirilli

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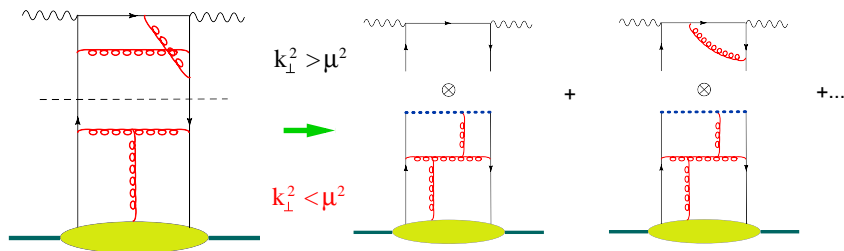
Santiago de Compostela, Low- x meeting 03-07 June 2011

- OPE at high-energy.
- NLO Photon Impact Factor: analytic result.
- LO and NLO BK equation.
- Conclusions.

Light-cone expansion and DGLAP evolution in the NLO



Light-cone expansion and DGLAP evolution in the NLO

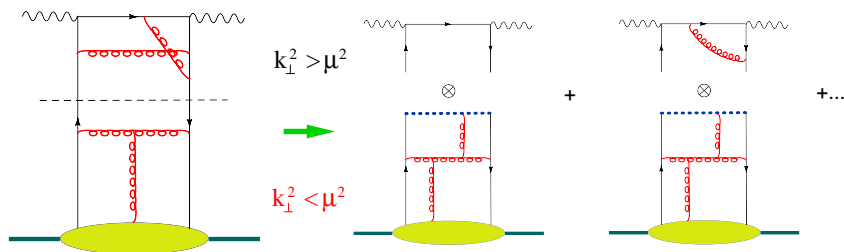


μ^2 - factorization scale (normalization point)

$k_{\perp}^2 > \mu^2$ - coefficient functions

$k_{\perp}^2 < \mu^2$ - matrix elements of light-ray operators (normalized at μ^2)

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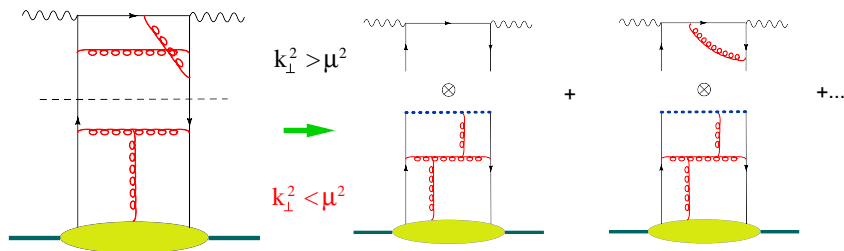
OPE in light-ray operators

$(x - y)^2 \rightarrow 0$

$$T\{j_{\mu}(x)j_{\nu}(y)\} = \frac{(x - y)_{\xi}}{2\pi^2(x - y)^4} \left[1 + \frac{\alpha_s}{\pi} (\ln(x - y)^2 \mu^2 + C) \right] \bar{\psi}(x) \gamma_{\mu} \gamma^{\xi} \gamma_{\nu} [x, y] \psi(y)$$

$[x, y] \equiv P e^{ig \int_0^1 du (x-y)^{\mu} A_{\mu}(ux+(1-u)y)}$ - gauge link

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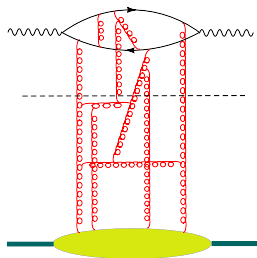
Renorm-group equation for light-ray operators \Rightarrow DGLAP evolution of

parton densities

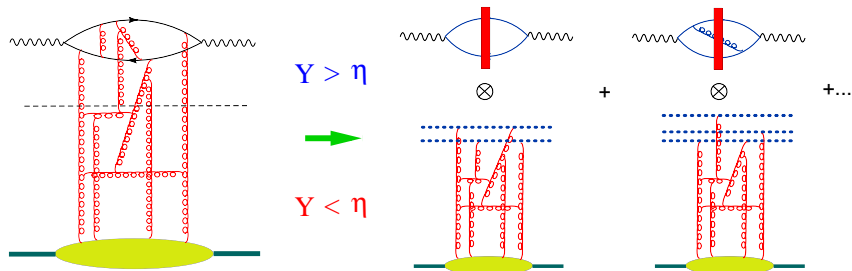
$$(x - y)^2 = 0$$

$$\mu^2 \frac{d}{d\mu^2} \bar{\psi}(x)[x, y]\psi(y) = K_{\text{LO}} \bar{\psi}(x)[x, y]\psi(y) + \alpha_s K_{\text{NLO}} \bar{\psi}(x)[x, y]\psi(y)$$

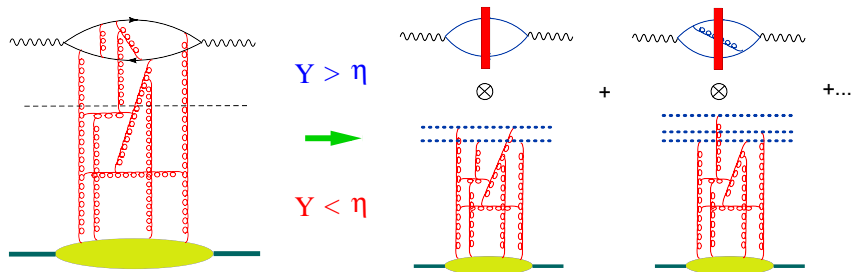
High-energy expansion in color dipoles in the NLO



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η - rapidity factorization scale

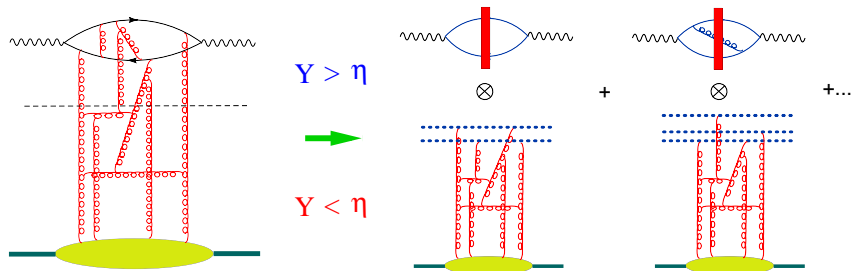
Rapidity $Y > \eta$ - coefficient function (“impact factor”)

Rapidity $Y < \eta$ - matrix elements of (light-like) Wilson lines with rapidity divergence cut by η

$$U_x^\eta = \text{Pexp} \left[ig \int_{-\infty}^{\infty} dx^+ A_+^\eta(x_+, x_\perp) \right]$$

$$A_\mu^\eta(x) = \int \frac{d^4 k}{(2\pi)^4} \theta(e^\eta - |\alpha_k|) e^{-ik \cdot x} A_\mu(k)$$

High-energy expansion in color dipoles in the NLO



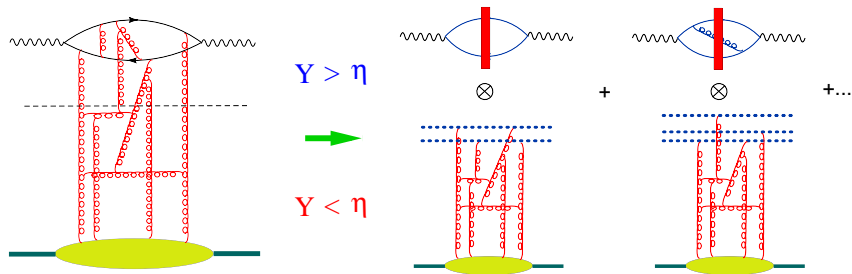
The high-energy operator expansion is

$$\begin{aligned}
 T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} &= \int d^2z_1 d^2z_2 I_{\mu\nu}^{\text{LO}}(z_1, z_2, x, y) \text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\} \\
 &+ \int d^2z_1 d^2z_2 d^2z_3 I_{\mu\nu}^{\text{NLO}}(z_1, z_2, z_3, x, y) [\text{tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_3}^{\dagger\eta}\} \text{tr}\{\hat{U}_{z_3}^\eta \hat{U}_{z_2}^{\dagger\eta}\} - N_c \text{tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}]
 \end{aligned}$$

In the leading order the impact factor is Möbius invariant

In the NLO one should also expect conf. invariance since $I_{\mu\nu}^{\text{NLO}}$ is given by tree diagrams

High-energy expansion in color dipoles in the NLO

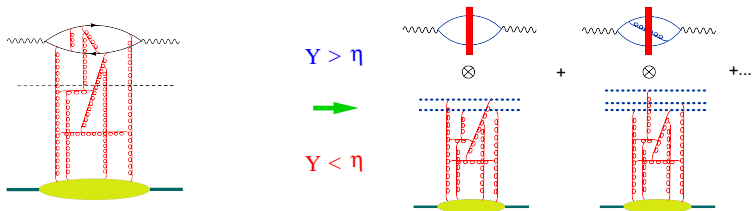


η - rapidity factorization scale

BK equation: Ian Balitsky (1996), Yu. Kovchegov (1999)

$$\frac{d}{d\eta} \hat{U}(x, y) = \frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2 z (x-y)^2}{(x-z)^2 (y-z)^2} \left\{ \hat{U}(x, z) + \hat{U}(z, y) - \hat{U}(x, y) - \hat{U}(x, z) \hat{U}(z, y) \right\}$$

Expansion of $F_2(x)$ in color dipoles in the next-to-leading order

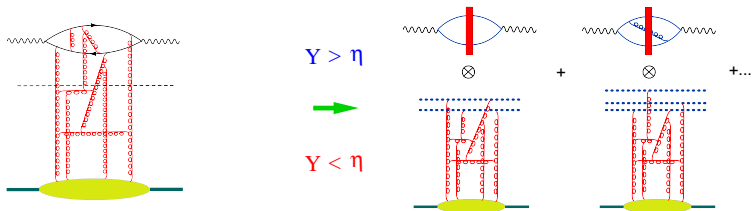


$$F_2(x_B) \simeq \int d^2 z_1 d^2 z_2 I^{LO}(z_1, z_2) \langle \text{tr} \{ U_{z_1}^\eta U_{z_2}^\dagger \eta \} \rangle$$

$$+ \frac{\alpha_s}{\pi} \int d^2 z_1 d^2 z_2 d^2 z_3 I^{NLO}(z_1, z_2, z_3) \left[\langle \text{tr} \{ U_{z_1}^\eta U_{z_3}^\dagger \eta \} \text{tr} \{ U_{z_3} U_{z_2}^\dagger \eta \} \rangle - N_c \text{tr} \{ U_{z_1} U_{z_2}^\dagger \} \right]$$

$$\eta = \ln \frac{1}{x_B}$$

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$\eta = \ln \frac{1}{x_B}$

plan

- Calculate the NLO photon impact factor.
- Calculate the NLO evolution of color dipole.
- Convolute the solution with the initial conditions for the evolution and get the amplitude.

Propagation in the shock wave: Wilson line (Spectator frame)



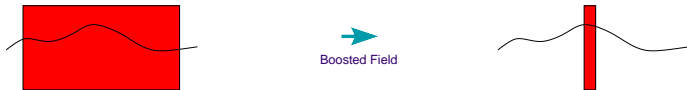
Each path is weighted with the gauge factor $P e^{ig \int dx_\mu A^\mu}$. Since the external field exists only within the infinitely thin wall, quarks and gluons do not have time to deviate in the transverse direction \Rightarrow we can replace the gauge factor along the actual path with the one along the straight-line path.



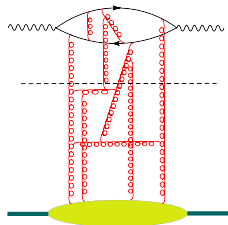
$$U_z = [\infty p_1 + z_\perp, -\infty p_1 + z_\perp]$$

$$[x, y] = P e^{ig \int_0^1 du (x-y)^\mu A_\mu (ux + (1-u)y)} \quad p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

Propagation in the shock wave: Wilson line (Spectator frame)



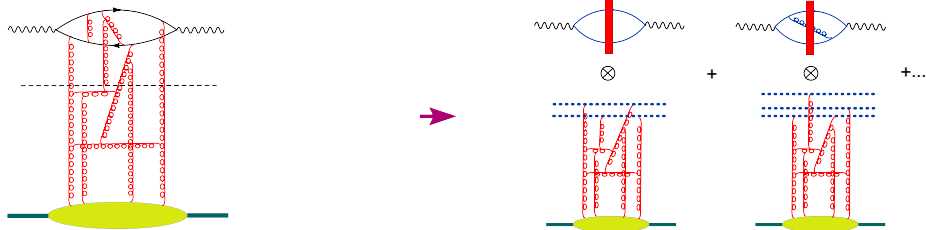
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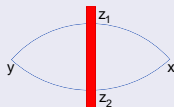


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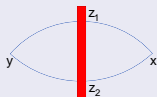
LO Impact Factor diagram: I^{LO}



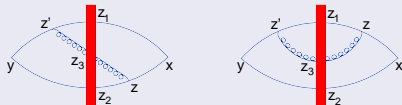
LO and NLO Impact Factor

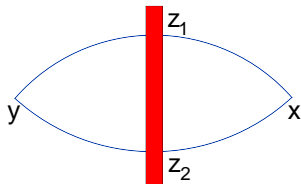
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LO Impact Factor diagram: I^{LO}



NLO Impact Factor diagrams: I^{NLO}





Conformal vectors:

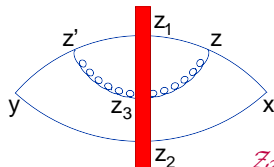
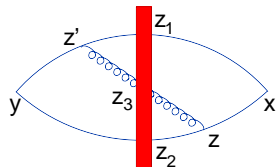
$$\kappa = \frac{\sqrt{s}}{2x_*} \left(\frac{p_1}{s} - x^2 p_2 + x_\perp \right) - \frac{\sqrt{s}}{2y_*} \left(\frac{p_1}{s} - y^2 p_2 + y_\perp \right)$$

$$\zeta_1 = \left(\frac{p_1}{s} + z_{1\perp}^2 p_2 + z_{1\perp} \right), \quad \zeta_2 = \left(\frac{p_1}{s} + z_{2\perp}^2 p_2 + z_{2\perp} \right)$$

Here $x^2 = -x_\perp^2$, $x_* \equiv x_\mu p_2^\mu$ (similarly for y); $\mathcal{R} = \frac{\kappa^2 (\zeta_1 \cdot \zeta_2)}{2(\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2)}$

$$I_{\mu\nu}^{\text{LO}}(z_1, z_2) = \frac{\mathcal{R}^2}{\pi^6 (\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2)} \frac{\partial^2}{\partial x^\mu \partial y^\nu} \left[(\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2) - \frac{1}{2} \kappa^2 (\zeta_1 \cdot \zeta_2) \right]$$

NLO Impact Factor

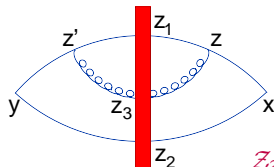
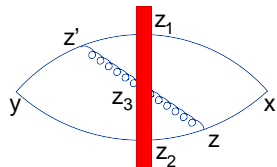


$$\mathcal{Z}_3 \equiv \frac{(x-z_3)_1^2}{x^+} - \frac{(y-z_3)_1^2}{y^+}$$

$$I_{\mu\nu}^{\text{NLO}}(x, y; z_1, z_2, z_3; \eta) = -I_{\mu\nu}^{\text{LO}} \times \frac{\alpha_s}{2\pi} \frac{z_{13}^2}{z_{12}^2 z_{23}^2} \ln \frac{\sigma s}{4} \mathcal{Z}_3 + \text{conf.}$$

The NLO impact factor is not Möbius invariant \Rightarrow the color dipole with the cutoff $\eta = \ln \sigma$ is not invariant.

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However, if we define a composite operator (a - analog of μ^{-2} for usual OPE)

$$\begin{aligned} [\text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}]^{\text{conf}} &= \text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\} \\ &+ \frac{\alpha_s}{4\pi} \int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[\frac{1}{N_c} \text{tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_3}^{\dagger\eta}\} \text{tr}\{\hat{U}_{z_3}^\eta \hat{U}_{z_2}^{\dagger\eta}\} - \text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\} \right] \ln \frac{az_{12}^2}{z_{13}^2 z_{23}^2} + O(\alpha_s^2) \end{aligned}$$

the impact factor becomes conformal in the NLO.

Analogy:

When the UV cutoff does not respect the symmetry of a local operator, the composite local renormalized operator must be corrected by finite counter-terms order by order in perturbation theory.

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2 I_{\mu\nu}^{\text{LO}}(z_1, z_2, x, y) \text{tr}\{\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}\}^{\text{conf}} \\ + \int d^2z_1 d^2z_2 d^2z_3 I_{\mu\nu}^{\text{NLO}}(z_1, z_2, z_3, x, y) \left[\frac{1}{N_c} \text{tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_3}^{\dagger\eta}\} \text{tr}\{\hat{U}_{z_3}^\eta \hat{U}_{z_2}^{\dagger\eta}\} - \text{tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\} \right]$$

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$$I_{\mu\nu}^{\text{NLO}} = - I_{\mu\nu}^{\text{LO}} \frac{\alpha_s N_c}{4\pi} \int dz_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \ln \frac{z_{12}^2 e^{2\eta} a s^2}{z_{13}^2 z_{23}^2} \mathcal{Z}_3^2 + \text{conf.}$$

The new NLO impact factor is conformally invariant.

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The new NLO impact factor is conformally invariant.

In conformal $\mathcal{N} = 4$ SYM theory one can construct the composite conformal dipole operator order by order in perturbation theory.

$$\Delta \equiv (x - y), \quad x_* = x^+ \sqrt{s/2}, \quad y_* = y^+ \sqrt{s/2}, \quad R \equiv \frac{\Delta^2 z_{12\perp}^2}{x_* y_* z_1 z_2}$$

$$\begin{aligned}
 I_{\mu\nu}^{NLO}(x, y) = & \frac{\alpha_s}{4\pi^7 \Delta^4} \frac{\partial \kappa^\alpha}{\partial x^\mu} \frac{\partial \kappa^\beta}{\partial y^\nu} \int \frac{dz_1 dz_2}{z_{12}^4} \mathcal{U}(z_1, z_2) R^2 \left\{ -\frac{2}{\kappa^2} \left(g^{\alpha\beta} - 2 \frac{\kappa^\alpha \kappa^\beta}{\kappa^2} \right) \right. \\
 & + \frac{\zeta_1^\alpha \zeta_2^\beta + \zeta_1 \leftrightarrow \zeta_2}{(\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2)} \left[4\text{Li}_2(1 - R) - \frac{2\pi^2}{3} + \frac{2 \ln R}{1 - R} + \frac{\ln R}{R} - 4 \ln R + \frac{1}{2R} - 2 - 4C - \frac{2C}{R} \right. \\
 & + 2 \left(\ln \frac{1}{R} + \frac{1}{R} - 2 \right) \left(\ln \frac{1}{R} + 2C \right) \left. \right] + \left(\frac{\zeta_1^\alpha \zeta_1^\beta}{(\kappa \cdot \zeta_1)^2} + \zeta_1 \leftrightarrow \zeta_2 \right) \left[\frac{\ln R}{R} - \frac{2C}{R} + 2 \frac{\ln R}{1 - R} - \frac{1}{2R} \right] \\
 & + \left[-2 \frac{\ln R}{1 - R} - \frac{\ln R}{R} + \ln R - \frac{3}{2R} + \frac{5}{2} + 2C + \frac{2C}{R} \right] \left[\frac{\zeta_1^\alpha \kappa^\beta + \zeta_1^\beta \kappa^\alpha}{(\kappa \cdot \zeta_1) \kappa^2} + \zeta_1 \leftrightarrow \zeta_2 \right] \\
 & + \frac{g^{\alpha\beta} (\zeta_1 \cdot \zeta_2)}{(\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2)} \left[\frac{2\pi^2}{3} - 4\text{Li}_2(1 - R) - 2 \left(\ln \frac{1}{R} + \frac{1}{R} + \frac{1}{2R^2} - 3 \right) \left(\ln \frac{1}{R} + 2C \right) \right. \\
 & \left. \left. + 6 \ln R - \frac{2}{R} + 2 + \frac{3}{2R^2} \right] \right\}
 \end{aligned}$$

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$$\begin{aligned}
 I_{\mu\nu}^{NLO}(x, y) = & \frac{\alpha_s}{4\pi^7 \Delta^4} \frac{\partial \kappa^\alpha}{\partial x^\mu} \frac{\partial \kappa^\beta}{\partial y^\nu} \int \frac{dz_1 dz_2}{z_{12}^4} \mathcal{U}(z_1, z_2) R^2 \left\{ -\frac{2}{\kappa^2} \left(g^{\alpha\beta} - 2 \frac{\kappa^\alpha \kappa^\beta}{\kappa^2} \right) \right. \\
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 & \left. + 2 \left(\ln \frac{1}{R} + \frac{1}{R} - 2 \right) \left(\ln \frac{1}{R} + 2C \right) \right] + \left(\frac{\zeta_1^\alpha \zeta_1^\beta}{(\kappa \cdot \zeta_1)^2} + \zeta_1 \leftrightarrow \zeta_2 \right) \left[\frac{\ln R}{R} - \frac{2C}{R} + 2 \frac{\ln R}{1-R} - \frac{1}{2R} \right] \\
 & + \left[-2 \frac{\ln R}{1-R} - \frac{\ln R}{R} + \ln R - \frac{3}{2R} + \frac{5}{2} + 2C + \frac{2C}{R} \right] \left[\frac{\zeta_1^\alpha \kappa^\beta + \zeta_1^\beta \kappa^\alpha}{(\kappa \cdot \zeta_1) \kappa^2} + \zeta_1 \leftrightarrow \zeta_2 \right] \\
 & + \frac{g^{\alpha\beta} (\zeta_1 \cdot \zeta_2)}{(\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2)} \left[\frac{2\pi^2}{3} - 4\text{Li}_2(1-R) - 2 \left(\ln \frac{1}{R} + \frac{1}{R} + \frac{1}{2R^2} - 3 \right) \left(\ln \frac{1}{R} + 2C \right) \right. \\
 & \left. \left. + 6 \ln R - \frac{2}{R} + 2 + \frac{3}{2R^2} \right] \right\}
 \end{aligned}$$

Conformal vectors

$$\begin{aligned}\kappa^\mu &= \frac{\sqrt{s}}{2x_*} \left(\frac{p_1^\mu}{s} - x^2 p_2^\mu + x_\perp^\mu \right) - \frac{\sqrt{s}}{2y_*} \left(\frac{p_1^\mu}{s} - y^2 p_2^\mu + y_\perp^\mu \right) \\ \zeta_1^\mu &= \left(\frac{p_1^\mu}{s} + z_{1\perp}^2 p_2^\mu + z_{1\perp}^\mu \right), \quad \zeta_2^\mu = \left(\frac{p_1^\mu}{s} + z_{2\perp}^2 p_2^\mu + z_{2\perp}^\mu \right)\end{aligned}$$

Conformal vectors

$$\kappa^\mu = \frac{\sqrt{s}}{2x_*} \left(\frac{p_1^\mu}{s} - x^2 p_2^\mu + x_\perp^\mu \right) - \frac{\sqrt{s}}{2y_*} \left(\frac{p_1^\mu}{s} - y^2 p_2^\mu + y_\perp^\mu \right)$$

$$\zeta_1^\mu = \left(\frac{p_1^\mu}{s} + z_{1\perp}^2 p_2^\mu + z_{1\perp}^\mu \right), \quad \zeta_2^\mu = \left(\frac{p_1^\mu}{s} + z_{2\perp}^2 p_2^\mu + z_{2\perp}^\mu \right)$$

DIS photon impact factor is a linear combination of the following tensor basis

$$\mathcal{I}_1^{\mu\nu} = g^{\mu\nu} \quad \mathcal{I}_2^{\mu\nu} = \frac{\kappa^\mu \kappa^\nu}{\kappa^2}$$

$$\mathcal{I}_3^{\mu\nu} = \frac{\kappa^\mu \zeta_1^\nu + \kappa^\nu \zeta_1^\mu}{\kappa \cdot \zeta_1} + \frac{\kappa^\mu \zeta_2^\nu + \kappa^\nu \zeta_2^\mu}{\kappa \cdot \zeta_2}$$

$$\mathcal{I}_4^{\mu\nu} = \frac{\kappa^2 \zeta_1^\mu \zeta_1^\nu}{(\kappa \cdot \zeta_1)^2} + \frac{\kappa^2 \zeta_2^\mu \zeta_2^\nu}{(\kappa \cdot \zeta_2)^2} \quad \mathcal{I}_5^{\mu\nu} = \frac{\zeta_1^\mu \zeta_2^\nu + \zeta_2^\mu \zeta_1^\nu}{\zeta_1 \cdot \zeta_2}$$

Cornalba, Costa, Penedones (2010)

$$\begin{aligned}
 & \int \frac{d^2 z_1 d^2 z_2}{z_{12}^2} I_{LO}^{\mu\nu}(z_1, z_2) \left(\frac{\kappa^2}{(\kappa \cdot \zeta_0)^2} \right)^\gamma = B(1 - \gamma)\Gamma(\gamma + 2)\Gamma(3 - \gamma) \\
 & \times \left\{ \frac{\gamma(1 - \gamma)D_1^{\mu\nu}}{12(1 + \gamma)(2 - \gamma)} + \frac{D_2^{\mu\nu}}{2(1 + \gamma)(2 - \gamma)} - \frac{D_3^{\mu\nu}}{8(1 + \gamma)(2 - \gamma)} \right. \\
 & \left. - \frac{\gamma(1 - \gamma)D_4^{\mu\nu}}{16(1 + 2\gamma)(3 - 2\gamma)(1 + \gamma)(2 - \gamma)} - \frac{D_1^{\mu\nu} + D_2^{\mu\nu}}{8} \right\}_{\mu\nu} \left(\frac{\kappa^2}{(\kappa \cdot \zeta_0)^2} \right)^\gamma
 \end{aligned}$$

Projection of the LO impact factor on the eigenfunctions

$$\int \frac{d^2 z_1 d^2 z_2}{z_{12}^2} I_{LO}^{\mu\nu}(z_1, z_2) \left(\frac{\kappa^2}{(\kappa \cdot \zeta_0)^2} \right)^\gamma = B(1-\gamma)\Gamma(\gamma+2)\Gamma(3-\gamma)$$

$$\times \left\{ \frac{\gamma(1-\gamma)D_1^{\mu\nu}}{12(1+\gamma)(2-\gamma)} + \frac{D_2^{\mu\nu}}{2(1+\gamma)(2-\gamma)} - \frac{D_3^{\mu\nu}}{8(1+\gamma)(2-\gamma)} \right.$$

$$\left. - \frac{\gamma(1-\gamma)D_4^{\mu\nu}}{16(1+2\gamma)(3-2\gamma)(1+\gamma)(2-\gamma)} - \frac{D_1^{\mu\nu} + D_2^{\mu\nu}}{8} \right\}_{\mu\nu} \left(\frac{\kappa^2}{(\kappa \cdot \zeta_0)^2} \right)^\gamma$$

where

$$(D_1 + D_2)^{\mu\nu} = -2\Delta^2 x_* y_* \kappa^{-2} \partial_x^\mu \partial_y^\nu \kappa^2$$

$$D_2^{\mu\nu} = -\Delta^2 x_* y_* \partial_x^\mu (\ln \kappa^2) \partial_y^\nu \ln \kappa^2$$

$$D_3^{\mu\nu} = 4\gamma \Delta^2 x_* y_* [(\partial_x^\mu \ln \kappa^2) \partial_y^\nu \ln(\kappa \cdot \zeta_0) + (\partial_y^\nu \ln \kappa^2) \partial_x^\mu \ln(\kappa \cdot \zeta_0) - (\partial_x^\mu \ln \kappa^2) \partial_y^\nu \ln \kappa^2]$$

$$D_4^{\mu\nu} = 4\gamma(1+2\gamma)\Delta^2 x_* y_* \left[-\frac{1}{3} \partial_x^\mu \partial_y^\nu \ln \kappa^2 - \partial_x^\mu (\ln \kappa^2) \partial_y^\nu \ln \kappa^2 \right.$$

$$\left. + (\partial_x^\mu \ln \kappa^2) \partial_y^\nu \ln(\kappa \cdot \zeta_0) + (\partial_y^\nu \ln \kappa^2) \partial_x^\mu \ln(\kappa \cdot \zeta_0) - 2\partial_x^\mu \ln(\kappa \cdot \zeta_0) \partial_y^\nu \ln(\kappa \cdot \zeta_0) \right]$$

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, C = -\psi(1) \text{ is the Euler constant, and } \psi'(a) = \frac{d}{da} \ln \Gamma(a)$$

Projection of the NLO impact factor on the eigenfunctions

Conformal spin 0: NLO impact factor for the unpolarized forward structure functions

$$\begin{aligned}
 & \int \frac{d^2 z_1 d^2 z_2}{z_{12}^4} I_{NLO}^{\mu\nu}(z_1, z_2) \left(\frac{\kappa^2}{(\kappa \cdot \zeta_0)^2} \right)^\gamma = \alpha_s \frac{B(1-\gamma)\Gamma(3-\gamma)\Gamma(2+\gamma)}{(2-\gamma)(1+\gamma)} \times \\
 & \left\{ \frac{D_1^{\mu\nu}}{3 \sin^2(\gamma\pi)} \left[(1 - \cos(2\gamma\pi)) \left(\chi - 1 - \gamma(1-\gamma) \left(C\chi - \frac{1}{2} \right) \right) - \gamma(1-\gamma) \frac{\pi^2}{3} (5 + \cos(2\gamma\pi)) \right] \right. \\
 & + D_2^{\mu\nu} \left[-\frac{3}{\gamma(1-\gamma)} + 2\chi \left(\frac{1}{\gamma(1-\gamma)} - 2C + 1 \right) + \frac{4}{3}\pi^2 \left(1 - \frac{3}{\sin^2(\gamma\pi)} \right) \right] \\
 & + D_3^{\mu\nu} \left[C\chi - \frac{1}{2} - \frac{1}{\gamma(1-\gamma)} - \frac{\chi}{4} \left(1 + \frac{2}{\gamma(1-\gamma)} \right) - \frac{\pi^2}{3} \left(1 - \frac{3}{\sin^2(\gamma\pi)} \right) \right] \\
 & + \frac{D_4^{\mu\nu}}{4[3 + 4\gamma(1-\gamma)]} \left[\frac{15}{\gamma(1-\gamma)} + 10 + \gamma(1-\gamma) - \chi - 2\gamma(1-\gamma) \left(C\chi - \frac{\pi^2}{3} + \frac{\pi^2}{\sin^2(\gamma\pi)} \right) \right] \\
 & + \frac{D_1^{\mu\nu} + D_2^{\mu\nu}}{[2 + \gamma(1-\gamma)]^{-1}} \left[-\frac{1}{2} - \frac{\pi^2}{3} + \frac{\pi^2}{\sin^2 \pi\gamma} + \frac{4\gamma(1-\gamma) + 3}{2\gamma(1-\gamma)(1+\gamma)(2-\gamma)} \right. \\
 & \left. + C\chi(\gamma) - \frac{1 + 2\gamma(1-\gamma)}{\gamma(1-\gamma)(1+\gamma)(2-\gamma)} \chi(\gamma) \right] \left. \right\} \quad \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)
 \end{aligned}$$

Projection of the NLO impact factor on the eigenfunctions

Conformal spin 2: NLO impact factor for the polarized forward structure functions

$$\int \frac{d^2 z_1 d^2 z_2}{z_{12}^4} I_{NLO}^{\mu\nu}(z_1, z_2) \left(\frac{\kappa^2}{(\kappa \cdot \zeta_0)^2} \right)^\gamma$$
$$= \alpha_s \Delta^2 \left(\partial_x^\mu \partial_{\bar{x}} \frac{\Delta^2}{\Delta_*} \right) \left(\partial_y^\nu \partial_{\bar{y}} \frac{\Delta^2}{\Delta_*} \right) (-\Delta^2 \bar{u}u)^\gamma \Gamma(3 - \gamma) \Gamma(\gamma + 2) B(2 - \gamma) \left\{ \frac{4\pi^2}{\sin^2 \pi\gamma} - \frac{4}{(1 - \gamma)^2} - \frac{4}{\gamma^2} \right.$$
$$\left. + \frac{20}{3} - \frac{4\pi^2}{3} - \frac{6\chi(2, \gamma)}{(1 + \gamma)(2 - \gamma)} + 6 + 4C\chi(2, \gamma) - \frac{6C}{(2 - \gamma)(1 + \gamma)} - \frac{6}{(1 + \gamma)(2 - \gamma)} \right\}$$

$$\chi(2, \gamma) = \chi(\gamma) - \frac{1}{\gamma(1 - \gamma)}$$

$$\begin{aligned}
 & [\text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}]^{\text{conf}} = \text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\} \\
 & + \frac{\alpha_s}{4\pi} \int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[\frac{1}{N_c} \text{tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_3}^{\dagger\eta}\} \text{tr}\{\hat{U}_{z_3}^\eta \hat{U}_{z_2}^{\dagger\eta}\} - \text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\} \right] \ln \frac{az_{12}^2}{z_{13}^2 z_{23}^2} + O(\alpha_s^2)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{d}{d\eta} [\text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}]^{\text{conf}} \\
 & = \frac{\alpha_s}{\pi^2} \int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[1 - \frac{\alpha_s N_c}{4\pi} \frac{\pi^2}{3} \right] [\text{Tr}\{T^a \hat{U}_{z_1}^\eta \hat{U}_{z_3}^{\dagger\eta} T^a \hat{U}_{z_3}^\eta \hat{U}_{z_2}^{\dagger\eta}\} - N_c \text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}]^{\text{conf}} \\
 & - \frac{\alpha_s^2}{4\pi^4} \int d^2 z_3 d^2 z_4 \frac{z_{12}^2}{z_{13}^2 z_{24}^2 z_{34}^2} \left\{ 2 \ln \frac{z_{12}^2 z_{34}^2}{z_{14}^2 z_{23}^2} + \left[1 + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2} \right] \ln \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} \right\} \\
 & \times \text{Tr}\{[T^a, T^b] \hat{U}_{z_1}^\eta T^{a'} T^{b'} \hat{U}_{z_2}^{\dagger\eta} + T^b T^a \hat{U}_{z_1}^\eta [T^{b'}, T^{a'}] \hat{U}_{z_2}^{\dagger\eta}\} [(\hat{U}_{z_3}^\eta)^{aa'} (\hat{U}_{z_4}^\eta)^{bb'} - (z_4 \rightarrow z_3)]
 \end{aligned}$$

Now Möbius invariant!

Agrees with Kotikov and Lipatov (2000)

NLO evolution of composite “conformal” dipoles in QCD

$$\begin{aligned}
 \frac{d}{d\eta} [\text{tr}\{\hat{U}_{z_1} U_{z_2}^\dagger\}]^{\text{conf}} &= \frac{\alpha_s}{2\pi^2} \int d^2 z_3 \left([\text{tr}\{\hat{U}_{z_1} \hat{U}_{z_3}^\dagger\} \text{tr}\{\hat{U}_{z_3} \hat{U}_{z_2}^\dagger\} - N_c \text{tr}\{\hat{U}_{z_1} \hat{U}_{z_2}^\dagger\}]^{\text{conf}} \right. \\
 &\times \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[1 + \frac{\alpha_s N_c}{4\pi} \left(b \ln z_{12}^2 \mu^2 + b \frac{z_{13}^2 - z_{23}^2}{z_{13}^2 z_{23}^2} \ln \frac{z_{13}^2}{z_{23}^2} + \frac{67}{9} - \frac{\pi^2}{3} \right) \right] \\
 &+ \frac{\alpha_s}{4\pi^2} \int \frac{d^2 z_4}{z_{34}^4} \left\{ \left[-2 + \frac{z_{14}^2 z_{23}^2 + z_{24}^2 z_{13}^2 - 4z_{12}^2 z_{34}^2}{2(z_{14}^2 z_{23}^2 - z_{24}^2 z_{13}^2)} \ln \frac{z_{14}^2 z_{23}^2}{z_{24}^2 z_{13}^2} \right] \right. \\
 &\times [\text{tr}\{\hat{U}_{z_1} \hat{U}_{z_3}^\dagger\} \text{tr}\{\hat{U}_{z_3} \hat{U}_{z_4}^\dagger\} \{\hat{U}_{z_4} \hat{U}_{z_2}^\dagger\} - \text{tr}\{\hat{U}_{z_1} \hat{U}_{z_3}^\dagger \hat{U}_{z_4} \hat{U}_{z_2}^\dagger \hat{U}_{z_3} \hat{U}_{z_4}^\dagger\} - (z_4 \rightarrow z_3)] \\
 &+ \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2} \left[2 \ln \frac{z_{12}^2 z_{34}^2}{z_{14}^2 z_{23}^2} + \left(1 + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2} \right) \ln \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} \right] \\
 &\times [\text{tr}\{\hat{U}_{z_1} \hat{U}_{z_3}^\dagger\} \text{tr}\{\hat{U}_{z_3} \hat{U}_{z_4}^\dagger\} \text{tr}\{\hat{U}_{z_4} \hat{U}_{z_2}^\dagger\} - \text{tr}\{\hat{U}_{z_1} \hat{U}_{z_4}^\dagger \hat{U}_{z_3} \hat{U}_{z_2}^\dagger \hat{U}_{z_4} \hat{U}_{z_3}^\dagger\} - (z_4 \rightarrow z_3)] \left. \right\}
 \end{aligned}$$

$$b = \frac{11}{3} N_c - \frac{2}{3} n_f$$

I. Balitsky and G.A.C

$K_{\text{NLO BK}}$ = Running coupling part + Conformal "non-analytic" (in j) part
 + Conformal analytic ($\mathcal{N} = 4$) part

Linearized $K_{\text{NLO BK}}$ reproduces the known result for the forward NLO BFKL kernel **Fadin and Lipatov (1998)**.

- High-energy operator expansion in color dipoles works at the NLO level.
- The analytic NLO photon impact factor in coordinate space and in the Mellin space for forward unpolarized and polarized structure functions has been calculated: the result is conformal.
- The NLO BK kernel in QCD and $\mathcal{N} = 4$ SYM agrees with NLO BFKL.
- The NLO BK kernel in QCD is a sum of the running-coupling part and conformal part.