Dijet Production at Large Rapidity Separation in $\mathcal{N} = 4$ SYM

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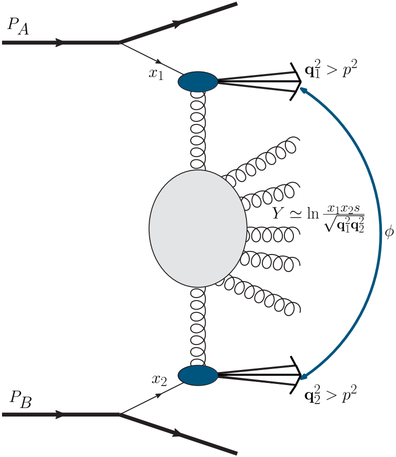
Outline

- NLO BFKL Dijet Production
- Ratios of Angular Correlations as Optimal BFKL Observables
- BLM Procedure & Azimuthal-Angle-Dependent Observables
- SL(2, $\mathbb{C}$) Invariance of the Theory and Comparison with MSYM
- Conclusions

Mueller-Navelet Jets



Mueller & Navelet '87
Schwennsen & Sabio Vera '06
Marquet & Royon '06, '08
Colferai, Schwennsen, Szymanowsky & Wallon '10

- Two Forward Jets
- Need to Resum $[\alpha_s \ln(s/p^2)]^n$
- Dijet Cross Section in terms of BFKL Green's Function

$$\frac{d\hat{\sigma}}{d^2\mathbf{q}_1^2 d^2\mathbf{q}_2^2} = \frac{\pi^2\bar{\alpha}_s^2}{2}\frac{f(\mathbf{q}_1,\mathbf{q}_2,Y)}{\mathbf{q}_1^2\mathbf{q}_2^2},$$

$$f(\mathbf{q}_1,\mathbf{q}_2,Y) = \int \frac{d\omega}{2\pi i} e^{\omega Y} f(\mathbf{q}_1,\mathbf{q}_2,\omega)$$

NLL BFKL Kernel in QCD and MSYM

Fadin & Lipatov '98
Ciafaloni & Camici '98
Kotikov & Lipatov '00

NLL BFKL Equation

$$\omega f(\mathbf{q}_1^2,\mathbf{q}_2^2,\omega)=\delta^2(\mathbf{q}_1^2-\mathbf{q}_2^2)+\int d^2\boldsymbol{\kappa}\,\mathcal{K}_{\rm NLL}(\mathbf{q}_1,\boldsymbol{\kappa})f(\boldsymbol{\kappa},\mathbf{q}_2,\omega)$$

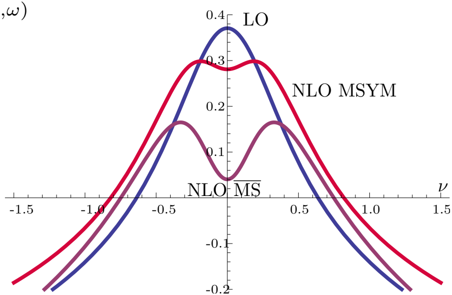
(LL) Eigenfunctions

$$\langle\mathbf{q}|n,\nu\rangle=\frac{1}{\pi\sqrt{2}}(\mathbf{q}^2)^{i\nu-\frac{1}{2}}e^{in\vartheta}$$

NLL 'Eigenvalues'

$$\langle n,\nu|\mathcal{K}|\nu',n'\rangle$$

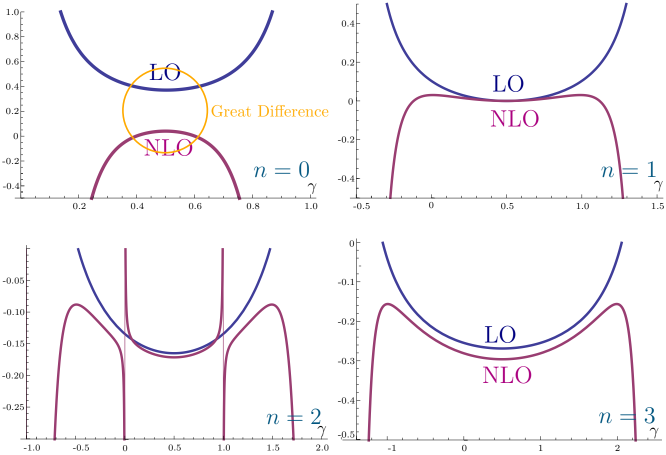
$$=\bar\alpha_{s,\overline{\rm MS}}\Big[\chi_0\Big(|n'|,\frac{1}{2}+i\nu'\Big)+\bar\alpha_{s,\overline{\rm MS}}\chi_1\Big(|n'|,\frac{1}{2}+i\nu'\Big)$$
$$-\frac{\bar\alpha_{s,\overline{\rm MS}}\beta_0}{8N_c}\chi_0\Big(|n'|,\tfrac{1}{2}+i\nu'\Big)\Big\{-i\frac{\partial}{\partial\nu'}+i\frac{\partial}{\partial\nu}-2\ln\mu^2\Big\}$$
$$+i\frac{\bar\alpha_{s,\overline{\rm MS}}\beta_0}{8N_c}\frac{\chi_0\Big(|n'|,\frac{1}{2}+i\nu'\Big)}{\partial\nu'}\Big]\delta_{n,n'}\delta(\nu-\nu'),$$



$$\text{LO}:\bar\alpha_s\chi_0(n=0,\nu);\qquad \bar\alpha_s\equiv\frac{\alpha_s N_c}{\pi}$$

$$\text{NLO}:\bar\alpha_s\chi_0+\bar\alpha_s^2\chi_1(n=0,\nu)$$

NLL BFKL Kernel in QCD and MSYM



$$\omega(n,\gamma) = \bar{\alpha}_s \chi_0(|n|,\gamma) + \bar{\alpha}_s^2 \chi_1(|n|,\gamma)$$

Eigenvalues of the Scale Invariant Sector of the BFKL Kernel

For $n = 0$

- Large and Negative NLO Corrections
- Oscillatory Behaviour of Cross Section. Need of an All-Orders Collinear Resummation

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In the MSYM Case

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While for Conformal Spins $n \geq 1$

- Asymptotically ($\gamma \sim \frac{1}{2}$) Hardly Sensitive to Radiative Corrections
- Suggests to Look for Observables Insensitive to $n = 0$

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Angular Coefficients and Ratios

Del Duca '94
Stirling '94
Sabio Vera & Schwennsen '07

Dijet Partonic Differential Cross Section

$\phi = \vartheta_1 - \vartheta_2 - \pi$, ϑ_i Angles of the Tagged Jets

$$\frac{d\hat{\sigma}(\bar{\alpha}_s, Y, p_{1,2}^2)}{d\phi} = \frac{\pi^2 \bar{\alpha}_s^2}{4\sqrt{p_1^2 p_2^2}} \sum_{n=-\infty}^{\infty} e^{in\phi} \mathcal{C}_n(Y)$$

$$\mathcal{C}_n^{\mathrm{QCD}}(Y) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \frac{e^{\bar{\alpha}_s(p^2)Y\left(\chi_0(|n|,\nu)+\bar{\alpha}_s(p^2)\left(\chi_1(|n|,\nu)-\frac{\beta_0}{8N_c}\frac{\chi_0(|n|,\nu)}{\left(\frac{1}{4}+\nu^2\right)}\right)\right)}}{\left(\frac{1}{4}+\nu^2\right)}$$

$$\mathcal{C}_n^{\mathrm{SUSY}}(Y) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \frac{e^{aY\left[\chi_0(|n|,\nu)+a\chi_1^{\mathrm{SUSY}}(|n|,\nu)\right]}}{\left(\frac{1}{4}+\nu^2\right)} \quad (\beta_0 = 0)$$

Total Cross Section Only Involves $n = 0$ as It Averages Over ϕ

$$\hat{\sigma}(\alpha_s, Y, p_{1,2}^2) = \frac{\pi^3 \bar{\alpha}_s^2}{2\sqrt{p_1^2 p_2^2}} \mathcal{C}_0(Y)$$

The Averages $\langle \cos(n\phi) \rangle$, $n \in \mathbb{Z}$ Project Out the Contribution of Higher n Angular Components

$$\langle \cos(m\phi) \rangle = \frac{\mathcal{C}_m(Y)}{\mathcal{C}_0(Y)}$$

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Excellent Perturbative Convergence

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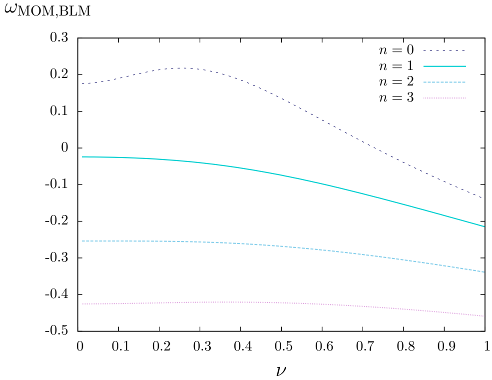
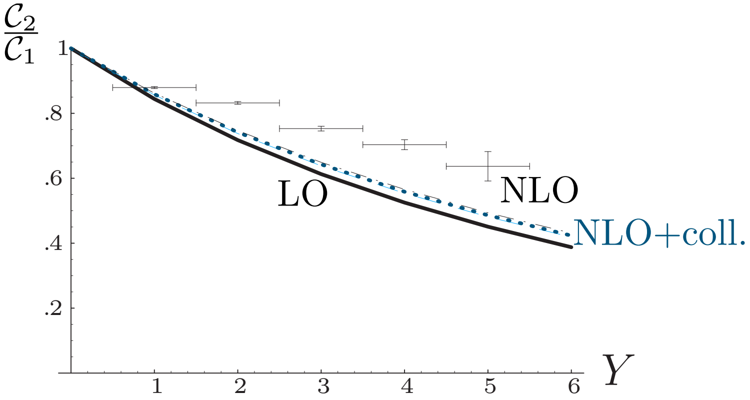
(QCD)



Intercept at $\nu = 0$: Divergence of $\mathcal{C}_0$; Convergence of $\mathcal{C}_n$, $n \geq 1$

Angular Coefficients and Ratios

$$\frac{\mathcal{C}_2}{\mathcal{C}_1} = \frac{\langle \cos 2\phi \rangle}{\langle \cos \phi \rangle}$$



The BLM Idea

Brodsky, Lepage & Mackenzie '83

- Renormalization Ambiguities Remain for Any Fixed-Order Computation
- Choice of Renormalization Scheme & Scale Can Render Small Perturbative Coefficients:
  First Orders Meaningful & Comparable with Physical Predictions
- Finite Renormalization $\leftrightarrow$ Redefinition of the Coupling

$$\alpha_s \to \alpha_s \left[1 + T\frac{\alpha_s}{\pi}\right]$$

- To NLO, Redefinition of the Coupling = Rescaling of the Scale

$$\mu \to \bar{\mu} = \mu \exp\left(-\frac{T}{2\beta_0}\right)$$

(Inverse Rescaling of Corresponding Landau Pole)

- BLM Prescription: Set the Scale by Redefining the Coupling to Absorb the Corrections Coming from Charge Renormalization.
- Perturbative Coefficients with the BLM Prescription Identical to Those of the Conformal Theory with $\beta_0 = 0$

Application to the Pomeron Intercept

Brodsky, Fadin, Kim, Lipatov & Pivovarov '99

- Original Computation of NLO BFKL Eigenvalues Gives

$$\omega_{\overline{\mathrm{MS}}}(\mathbf{q}^2,n,\nu)=N_c\chi_0(n,\nu)\,\frac{\alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^2)}{\pi}\left[1+r_{\overline{\mathrm{MS}}}(n,\nu)\,\frac{\alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^2)}{\pi}\right];\quad r_{\overline{\mathrm{MS}}}=r^{\beta}_{\overline{\mathrm{MS}}}+r^{\mathrm{conf}}_{\overline{\mathrm{MS}}}$$

$$r^{\beta}_{\overline{\mathrm{MS}}}(n,\nu)=-\frac{\beta_0}{4}\left[\frac{\chi_0(n,\nu)}{2}-\frac{5}{3}\right],$$

$$\begin{aligned} r^{\mathrm{conf}}_{\overline{\mathrm{MS}}}(n,\nu)=\frac{N_c}{4\chi_0(n,\nu)}\Bigg[&\frac{\pi^2-4}{3}\chi_0(n,\nu)-6\zeta(3)-\left(\psi''\left(\frac{n+1}{2}+i\nu\right)+\psi''\left(\frac{n+1}{2}-i\nu\right)\right.\\ &\left.-2\Phi\left(n,\tfrac{1}{2}+i\nu\right)-2\Phi\left(n,\tfrac{1}{2}-i\nu\right)\right)+\frac{\pi^2}{2\nu}\,\mathrm{sech}(\pi\nu)\tanh(\pi\nu)\\ &\times\left\{\left[3+\left(1+\frac{N_f}{N_c^3}\right)\left(\frac{3}{4}-\frac{1}{16(1+\nu^2)}\right)\right]\delta_n^0-\left(1+\frac{N_f}{N_c^3}\right)\left(\frac{1}{8}-\frac{3}{32(1+\nu^2)}\right)\delta_n^2\right\}\Bigg]. \end{aligned}$$

- $\overline{\mathrm{MS}}$ + Arbitrary Scale → MOM Scheme + Optimal BLM Scale

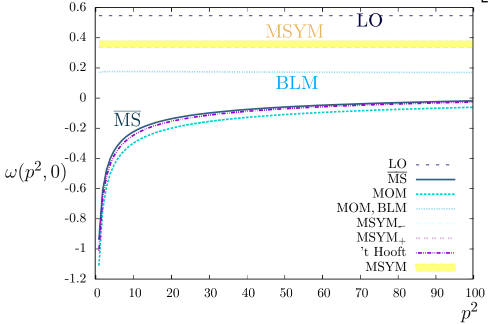
We Pass to MOM Scheme $\alpha_{\mathrm{MOM}} = \alpha_{\overline{\mathrm{MS}}}\left[1 + T_{\mathrm{MOM}}\frac{\alpha_{\overline{\mathrm{MS}}}}{\pi}\right]$

$$\omega^{\mathrm{MOM}}(\mathbf{q}^{2\,\mathrm{MOM}}_{\ \mathrm{BLM}}, n, \nu) = \chi_0(n,\nu)\frac{\alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^{2\,\mathrm{MOM}}_{\ \mathrm{BLM}})}{\pi}\left[1 + r^{\mathrm{MOM}}(n,\nu)\frac{\alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^{2\,\mathrm{MOM}}_{\ \mathrm{BLM}})}{\pi}\right];$$

$$r_{\mathrm{MOM}}(n,\nu) = r_{\overline{\mathrm{MS}}}(n,\nu) + T_{\mathrm{MOM}}$$

and Then Fix the Scale With BLM

$$\mathbf{q}^{2\,\mathrm{MOM}}_{\ \mathrm{BLM}}(n,\nu) = \mathbf{q}^2 \exp\left[-\frac{4r^{\beta}_{\mathrm{MOM}}(n,\nu)}{\beta_0}\right].$$



- More Sensible Result for Pomeron Intercept
- BLM Intercept Has Very Weak Dependence on the Energy Scale p^2:
  1. In Agreement With Regge Theory
  2. Nearly Insensitive to Non-Perturbative Effects
  3. Approach to Conformal Behaviour

Angular Coefficients in QCD+BLM & MSYM

Angioni, Chachamis, JDM & Sabio Vera '11

* Apply the Same Procedure for Our Angular Coefficients and Ratios
* Compare to Those Obtained in MSYM.
* MSYM Coupling Going from $a = \bar{\alpha}_s(p^2/4)$ [MSYM$_-$] to $a = \bar{\alpha}_s(4\,p^2)$ [MSYM$_+$].

- Predictions for $\langle \cos m\phi \rangle$ Are 'Spoiled' By Bad Convergence of the $n = 0$ Contribution.

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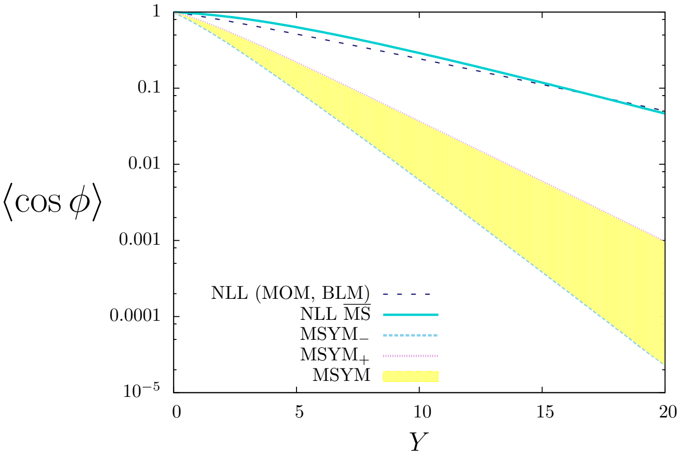
Angular Coefficients with BLM Setting and Physical Renormalization Prescription



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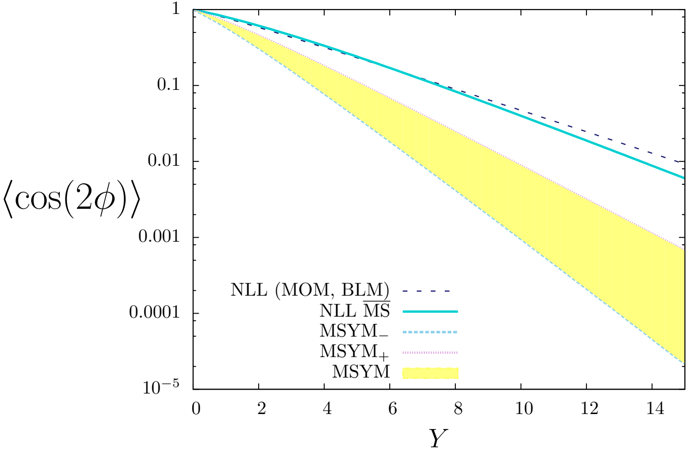

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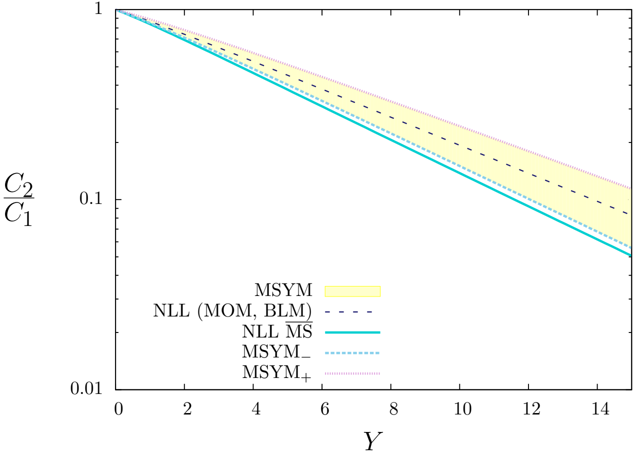

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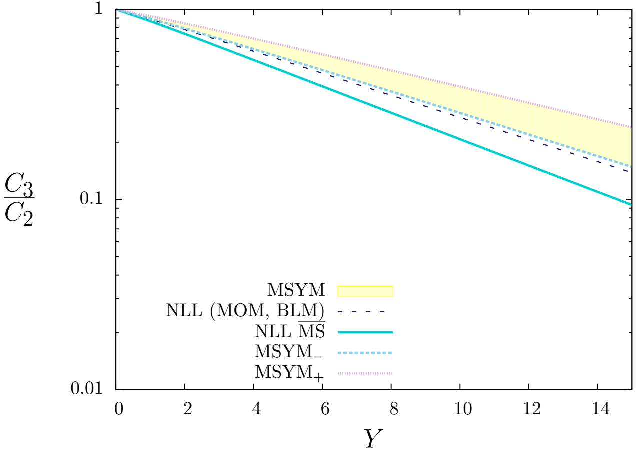

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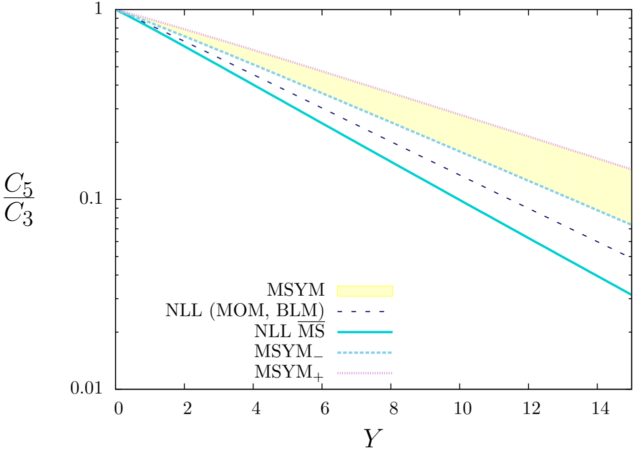

Conclusions

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