

High-energy physics with particles carrying non-zero orbital angular momentum

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Outline

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- 2 Single-twisted cross section
- 3 Double-twisted cross section
- 4 Conclusions

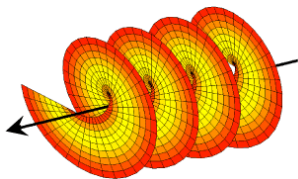
Based on I.P.I., PRD83, 093001 (2011)

Twisted states: scalar case

Orbital angular momentum (OAM) projection operator $L_z = -i\partial/\partial\phi$.

A plane wave propagating along axis z : zero OAM.

A non-plane wave configuration with ϕ -dependence $\exp(im\phi)$ carries OAM $= m$. This **twisted state** has helical wave fronts.



Bessel states in the coordinate space

Plane wave solutions of the wave equation with definite ω , k_z and \mathbf{k} :

$$|PW(\mathbf{k})\rangle = e^{-i\omega t + ik_z z} \cdot e^{i\mathbf{k}\mathbf{r}}.$$

Another type of solution in cylindrical coordinates: twisted state $|\kappa, m\rangle$:

$$|\kappa, m\rangle = e^{-i\omega t + ik_z z} \cdot \psi_{\kappa m}(\mathbf{r}),$$

where

$$\psi_{\kappa m}(\mathbf{r}) = \frac{e^{im\phi_r}}{\sqrt{2\pi}} \sqrt{\kappa} J_m(\kappa r).$$

Here $r = |\mathbf{r}|$ and κ is the conical momentum spread.

Momentum space

The twisted state $|\kappa, m\rangle$ is **superposition of various plane waves** $|PW(\mathbf{k})\rangle$ with fixed $|\mathbf{k}| = \kappa$ and all ϕ_k :

$$|\kappa, m\rangle = \int \frac{d^2\mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) |PW(\mathbf{k})\rangle,$$

where

$$a_{\kappa m}(\mathbf{k}) = (-i)^m e^{im\phi_k} \sqrt{2\pi} \frac{\delta(|\mathbf{k}| - \kappa)}{\sqrt{\kappa}}.$$

Twisted photons

For the photons consider a similar non-plane wave state:

$$A_{\kappa m \Lambda}^{\mu}(\mathbf{r}) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) e_{\Lambda}^{\mu}(\mathbf{k}) |PW(\mathbf{k})\rangle,$$

- The polarization vector **does not factorize** because it depends on \mathbf{k} .
- $e_{\Lambda}^{\mu}(\mathbf{k})$ corresponding to different PWs inside a twisted state don't even lie in the same plane.
- Twisted photons are described by a **polarization field** instead of a polarization vector.

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Experimental situation

Laser beams with non-zero OAM are well-known in the optics.

Recently, it was suggested to use Compton backscattering to generate high-energy photons carrying OAM [*Jentschura, Serbo, PRL106, 013001 (2011)*]:

$$e + \gamma_{\text{twisted}}^{\text{optical}} \rightarrow e + \gamma_{\text{twisted}}^{\text{high-energy}} .$$

In the last months: impressive progress in creating twisted electron states with $E = 300$ keV and $m \sim 100$ were observed.

Twisted particles enter high-energy physics.

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Single twisted scattering

Now consider the same process but with one twisted particle:

$$|\kappa, m\rangle + |PW(\mathbf{p})\rangle \rightarrow X.$$

The scattering matrix is

$$S_{tw} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) S_{PW}(\mathbf{k}, \mathbf{p}).$$

Its square is

$$\begin{aligned} |S_{tw}|^2 &= \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a_{\kappa m}^*(\mathbf{k}') S_{PW}(\mathbf{k}, \mathbf{p}) S_{PW}^*(\mathbf{k}', \mathbf{p}) \\ &\propto \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a_{\kappa m}^*(\mathbf{k}') \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X) \delta^{(2)}(\mathbf{k}' + \mathbf{p} - \mathbf{p}_X) \mathcal{M}(\mathbf{k}, \mathbf{p}) \mathcal{M}^*(\mathbf{k}', \mathbf{p}) \\ &= \int \frac{d^2\mathbf{k}}{(2\pi)^4} a_{\kappa m}(\mathbf{k}) a_{\kappa m}^*(\mathbf{k}) \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X) |\mathcal{M}(\mathbf{k}, \mathbf{p})|^2. \end{aligned}$$

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The cross section can then be written as

$$d\sigma_{tw} = \int \frac{d\phi_k}{2\pi} d\sigma_{PW}(\mathbf{k}) \cdot \frac{j_{PW}(\mathbf{k})}{j_{tw}}.$$

For paraxial scattering the ratio of fluxes is very close to 1.

- An unusual quantity: $d\sigma_{PW}(\mathbf{k})$ averaged over **initial** angle and fixed final momenta.
- The single-twisted cross section is ***m*-independent**: no smallness associated with OAM.
- **No smallness associated with small κ** : can be studied experimentally with today's technology.
- $d\sigma_{tw}$ is an **incoherent sum** of $d\sigma_{PW}(\mathbf{k})$. Initial coherence is lost via non-interfering final states.

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Twisted state in superposition of different m

Example: $a|\kappa, m\rangle + a'|\kappa, m'\rangle$, with $|a|^2 + |a'|^2 = 1$.

$$d\sigma = d\sigma_{tw} + 2|aa'|d\sigma_{tw}^{\Delta m}.$$

where

$$d\sigma_{tw}^{\Delta m} = \int \frac{d\phi_k}{2\pi} \cos(\Delta m \phi_k + \alpha) d\sigma_{PW}(\mathbf{k}) \cdot \frac{j_{PW}(\mathbf{k})}{j_{tw}},$$

with $\Delta m = m - m'$ and α is the relative phase between the two complex coefficients a and a' .

One gets a new tool: **Fourier-analyzer of the plane wave cross section** w.r.t. initial azimuthal angle ϕ_k .

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A novel way to probe composite particles?

Generic example: elastic scattering of a probe particle by a target particle.
 $k + p \rightarrow k' + p'$; transverse momentum transfer $\mathbf{q} = \mathbf{k}' - \mathbf{k}$.

The target particle has an internal structure (\rightarrow **formfactor**) and can have transverse polarization (\rightarrow **preferred direction \mathbf{n}**).

A toy model for the cross-section dependence on \mathbf{q} :

$$d\sigma(\mathbf{k}, \mathbf{k}') \propto A + B \cdot \mathbf{q}^2 + C \cdot (\mathbf{q}\mathbf{n})^2,$$

$$\mathbf{q}^2 = \mathbf{k}^2 + \mathbf{k}'^2 - 2kk' \cos(\phi_k - \phi'_k),$$

$$(\mathbf{q}\mathbf{n}) = k' \cos(\phi'_k - \phi_n) - k \cos(\phi_k - \phi_n).$$

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Usual way: partial wave analysis requires measuring x-section at different \mathbf{k}' , fitting the angular distribution and extracting different spherical harmonics.

Using twisted states with adjustable Δm : Fourier analysis w.r.t. ϕ_k can be performed at fixed \mathbf{k}' by comparing several initial states.

Different requirements for the detectors \rightarrow different systematics \rightarrow complementary tools.

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Colliding two twisted particles

$$|\kappa, m\rangle + |\eta, n\rangle \rightarrow X.$$

The scattering matrix is

$$S_{2tw} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{p}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a_{\eta n}(\mathbf{p}) S_{PW}(\mathbf{k}, \mathbf{p}).$$

The square $|S_{2tw}|^2$ contains

$$\int \frac{d^2\mathbf{k} d^2\mathbf{p} d^2\mathbf{k}' d^2\mathbf{p}'}{(2\pi)^8} a_{\kappa m}(\mathbf{k}) a_{\eta n}(\mathbf{p}) a_{\kappa m}^*(\mathbf{k}') a_{\eta n}^*(\mathbf{p}') \\ \times \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X) \delta^{(2)}(\mathbf{k}' + \mathbf{p}' - \mathbf{p}_X) \mathcal{M}(\mathbf{k}, \mathbf{p}) \mathcal{M}^*(\mathbf{k}', \mathbf{p}').$$

Kinematical restrictions: $|\mathbf{k}| = \kappa = |\mathbf{k}'|$ and $|\mathbf{p}| = \eta = |\mathbf{p}'|$; as well as $\mathbf{k} + \mathbf{p} = \mathbf{p}_X = \mathbf{k}' + \mathbf{p}'$.

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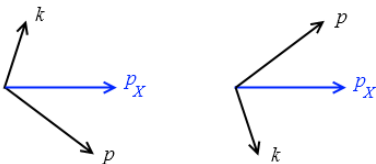
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For each \mathbf{p}_X there are **two kinematical configurations**:



In the product $\mathcal{M}(\mathbf{k}, \mathbf{p})\mathcal{M}^*(\mathbf{k}', \mathbf{p}')$ one can have either $\mathbf{k}' = \mathbf{k}, \mathbf{p}' = \mathbf{p}$, or

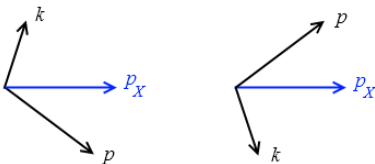
$$\mathbf{k}' = \mathbf{k}^* \equiv -\mathbf{k} + 2(\mathbf{k}\mathbf{n}_X)\mathbf{n}_X, \quad \mathbf{p}' = \mathbf{p}^* \equiv -\mathbf{p} + 2(\mathbf{p}\mathbf{n}_X)\mathbf{n}_X,$$

where $\mathbf{n}_X \equiv \mathbf{p}_X/|\mathbf{p}_X|$.

These two possibilities interfere. Therefore, the cross section will depend not only on $|\mathcal{M}(\mathbf{k}, \mathbf{p})|^2$ but also on $\mathcal{M}(\mathbf{k}, \mathbf{p})\mathcal{M}^*(\mathbf{k}^*, \mathbf{p}^*)$.

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The cross section is

$$d\sigma_{2tw} = \frac{1}{8\pi \sin(\delta_k + \delta_p)} \int d\phi_k d\phi_p \frac{j_{PW}(\mathbf{k}, \mathbf{p})}{j_{2tw}} [d\sigma_{PW}(\mathbf{k}, \mathbf{p}) + d\sigma'_{PW}(\mathbf{k}, \mathbf{p})],$$

where

$$d\sigma'_{PW}(\mathbf{k}, \mathbf{p}) = \frac{(2\pi)^4 \delta(E_i - E_f) \delta(p_{zi} - p_{zf}) \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X)}{4E_p \omega j_{PW}} \\ \times \operatorname{Re} \left[e^{2im(\phi_k - \phi_X) + 2in(\phi_p - \phi_X)} \mathcal{M}(\mathbf{k}, \mathbf{p}) \mathcal{M}^*(\mathbf{k}^*, \mathbf{p}^*) \right] d\Gamma_X.$$

and

$$\delta_k = \arccos \left(\frac{\mathbf{p}_X^2 + \kappa^2 - \eta^2}{2|\mathbf{p}_X| \kappa} \right), \quad \delta_p = \arccos \left(\frac{\mathbf{p}_X^2 - \kappa^2 + \eta^2}{2|\mathbf{p}_X| \eta} \right).$$

The double-twisted cross section is *m, n*-dependent and stays finite at small κ, η .

Conclusions

- Some effects are observable with **today's technology**.
- **Fourier transform w.r.t. initial azimuthal angle** might emerge as a new tool, complementary to the standard PWA.
- Double-twisted x-section is sensitive to the **autocorrelation function**, which is inaccessible in plane wave collisions.
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