UHE neutrinos: $\sigma^{\nu N}$, saturation.

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Low x 2011

Cosmic ν and $\sigma^{\nu N}$

a lot of papers on

$$\sigma^{\nu N}(E_\nu)$$

with E_ν up to

$$E_\nu \sim 10^{12} \text{ GeV}$$

Existence of $\boldsymbol{\nu}$ with $E_\nu \sim 10^{12}$ GeV - big question!

but IceCube $\boldsymbol{\nu}$-detector operates – $\sigma^{\nu N}$ is called upon.

demand drives supply and a lot of papers on $\sigma^{\nu N}(E_\nu)$.

$\nu N \to \mu X$: scales

$$\sigma^{\nu N} \propto \int dQ^2 \left(\frac{m_W^2}{m_W^2 + Q^2} \right)^2 \exp \sqrt{C \log(1/x) \log \log Q^2}$$

W-propagator introduces cut-off. Then the scale is

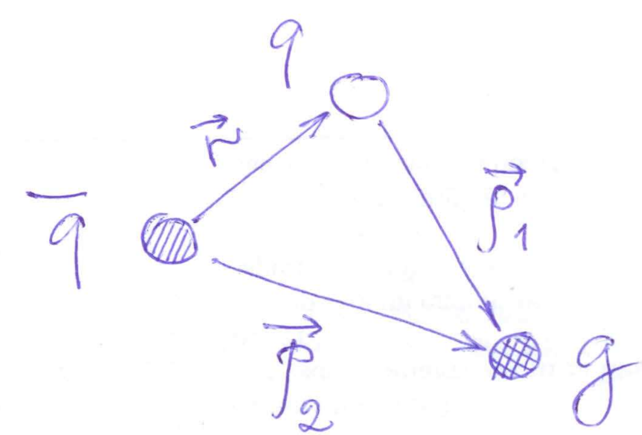
$$Q^2 \lesssim m_W^2$$

Characteristic x, for $E_\nu \sim 10^{12}$ GeV, is low enough to be presented at Low-x workshop,

$$x \sim \frac{m_W^2}{2mE_\nu} \sim 10^{-8}$$

to advance to $x \sim 10^{-8}$

The BFKL equation for the dipole cross section $\sigma(x, r)$ (Nikolaev, Zakharov and VRZ, 1994):



$$\frac{\partial \sigma(x,r)}{\partial \log(1/x)} = \mathcal{K} \otimes \sigma(x,r),$$

The kernel $\mathcal{K}$ is related to the LCWF of qg and $\bar{q}g$ dipoles

$$\psi(\vec{\rho}) \sim \boldsymbol{g}_S(\rho) \frac{\vec{\rho}}{\rho^2} \times (\text{screening factor}).$$

- running color charge - $\boldsymbol{g}_S(\rho)$
- infrared regularization:
  Debye screening radius – $R_c \simeq 0.3$ fm

BFKL spectrum and solutions

The spectrum of the running BFKL equation is a series of moving poles in the complex j-plane (Lipatov 1986). Hence, the BFKL-Regge expansion:

$$\sigma(x,r) = \sigma_0(r)x^{-\Delta_0} + \sigma_1(r)x^{-\Delta_1} + \sigma_2(r)x^{-\Delta_2} + ...$$

Eigenfunctions:

$$\sigma_n(r)$$

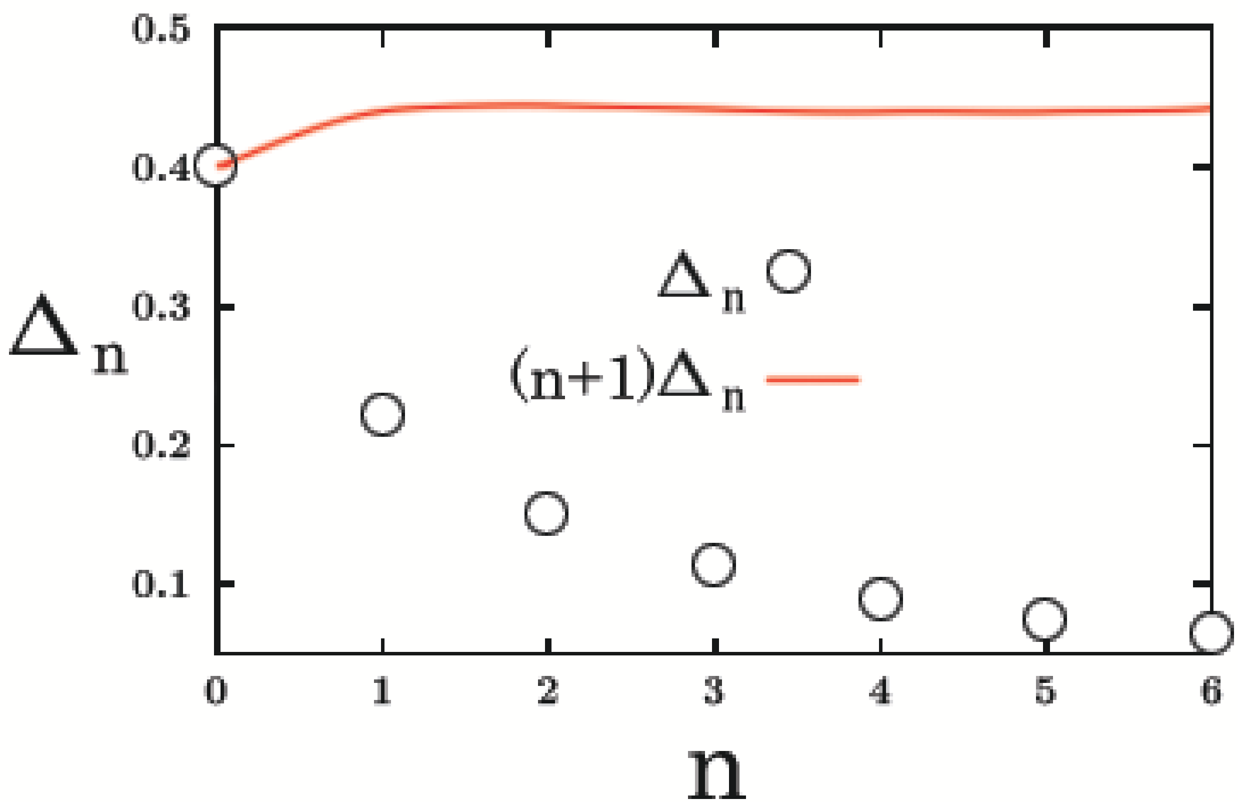
a solution of the eigenvalue problem

$$\mathcal{K} \otimes \sigma_n = \Delta_n \sigma_n(r).$$

with the pomeron intercept Δ_n as the eigenvalue

$$\Delta_n = j_n - 1$$

Δ_n vs. quasiclassical approx.



Quasiclassical approximation:

$$\Delta_n \approx \frac{\Delta_0}{(n+1)}, \; n \gg 1$$

findings:

- leading pomeron singularity – moving pole in j-plane

$$j = \alpha_{I\!P}(t) = 1 + \Delta_{I\!P} + \alpha'_{I\!P} t$$

- dimensionful $\alpha'_{I\!P}$ connected to the non-perturbative infrared parameter R_c (Nikolaev, Zakharov and VRZ '96):

$$\alpha'_{I\!P} \sim \frac{3}{16\pi} \alpha_S(R_c) R_c^2 \sim 0.1\,\mathrm{GeV}^{-2}$$

- Regge radius of the interaction region growing as

$$R^2 \sim \alpha'_{I\!P} \log(1/x)$$

Saturation. Milestones

pre-QCD patron model

- 1973 – O.V. Kancheli, *JETP Lett.* **18**, 274(cited 33 times)
- 1975 – N.N. Nikolaev and V.I. Zakharov, *Phys. Lett.* B **55**, 397

QCD

- 1983 – L.V. Gribov, E.M. Levin and M.G. Ryskin *Phys. Rep.* **100**, 1
- 1986 – A.H. Mueller and J. Qiu, *Nucl. Phys.* B **286**, 427
- 1996 – I. Balitsly, *Nucl. Phys.* B **463**, 99
- 1999 – Yu.V. Kovchegov, *Phys. Rev.* D **60**, 034008
- 2000 – M. Braun, *Eur. Phys. J.* C **16**, 337

Saturation. Prescription:

1) take linear BFKL (Γ – profile function)

$$\frac{\partial \Gamma(x,r,\mathbf{b})}{\partial \log(1/x)} = \frac{N_c^2}{N_c^2-1} \int d^2\boldsymbol{\rho}_1 \left|\psi(\boldsymbol{\rho}_1) - \psi(\boldsymbol{\rho}_2)\right|^2$$

$$\times \left[\Gamma(x,\rho_1,\mathbf{b}+\frac{1}{2}\boldsymbol{\rho}_2) + \Gamma(x,\rho_2,\mathbf{b}+\frac{1}{2}\boldsymbol{\rho}_1) - \Gamma(x,r,\mathbf{b})\right],$$

(Nikolaev, Zakharov and VRZ ’94)

2) add non-linear term

$$-\Gamma(x,\rho_2,\mathbf{b}+\frac{1}{2}\boldsymbol{\rho}_1)\Gamma(x,\rho_1,\mathbf{b}+\frac{1}{2}\boldsymbol{\rho}_2), \quad N_c \to \infty$$

(Balitsky ’96,’99, Kovchegov ’99)

3) compare linear and non-linear terms

$\Gamma(b)$ – area where gluons live

The profile function in b-space used by many

$$\Gamma(b) \sim e^{-b^2/R^2}$$

confines gluons within the area $S \sim R^2$

Different analyses use R^2 that varies
from $R^2 = 16$ Gev^{-2} down to $R^2 = 3.1$ Gev^{-2}

the strength of the saturation effect is estimated as

$$\kappa \sim \frac{Q^{-2}\alpha_S(Q^2)G(x,Q^2)}{\pi R^2}$$

- ! Need for certainty ! and growing $R^2 \sim \alpha'_{\mathbb{P}} \log(1/x)$

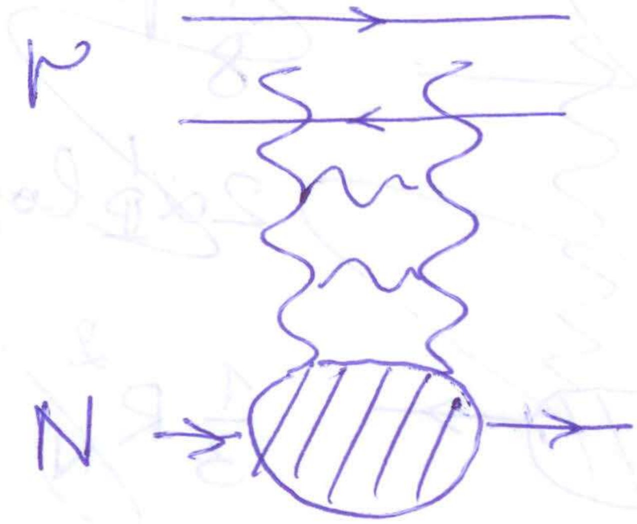
$\Gamma(b)$ – standard definition

Restrictions on $\Gamma(b)$ imposed by

$$\sigma(x,r) = 2\int d^2\mathbf{b}\,\Gamma(x,r,\mathbf{b})$$

and

$$\frac{d\sigma_{el}}{dt} \sim e^{Bt}$$



give the partial-wave amplitude

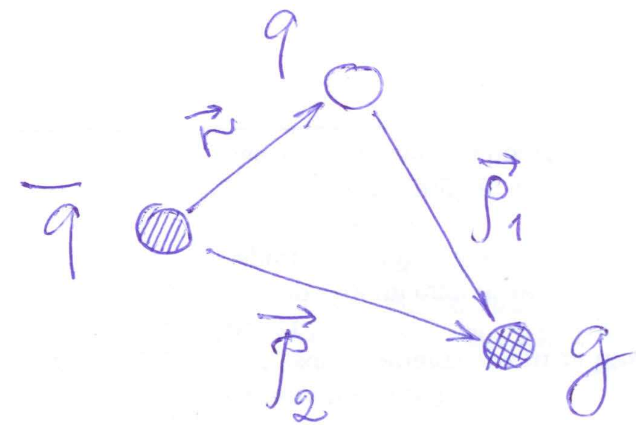
$$\Gamma(x,r,\mathbf{b}) = \frac{\sigma(x,r)}{4\pi B(x,r)} \exp\left[-\frac{b^2}{2B(x,r)}\right]$$

- $$B(x,r) = \frac{1}{8}r^2 + \frac{1}{3}R_N^2 + 2\alpha'_{\mathbb{P}}\log(1/x)$$

from Γ back to σ

impact parameter integration

$$\int d^2\mathbf{b}\,\Gamma\cdot\Gamma$$



yields the non-linear term in BFKL eq.

$$-\frac{\sigma(x,\rho_1)\sigma(x,\rho_2)}{4\pi(B_1+B_2)}\exp\left[-\frac{\mathbf{r}^2}{8(B_1+B_2)}\right]$$

$$B_{1,2}=B(x,\rho_{1,2})=\frac{1}{8}\rho_{1,2}^2+\frac{1}{3}R_N^2+2\alpha'_{\mathbb{P}}\log(1/x)$$

magnitude of effect

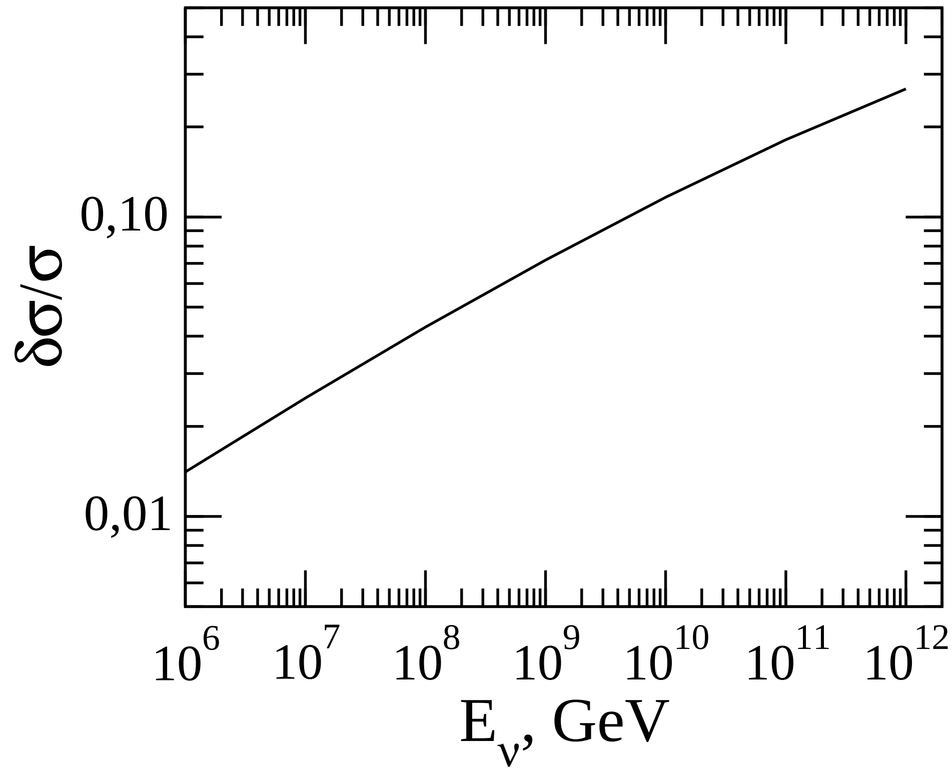
$$\frac{\delta\sigma}{\sigma} = 1 - \frac{\sigma^{\nu N}_{non-linear}}{\sigma^{\nu N}_{linear}}$$

$$\delta\sigma \sim \frac{\langle\sigma^2\rangle}{8\pi B}$$

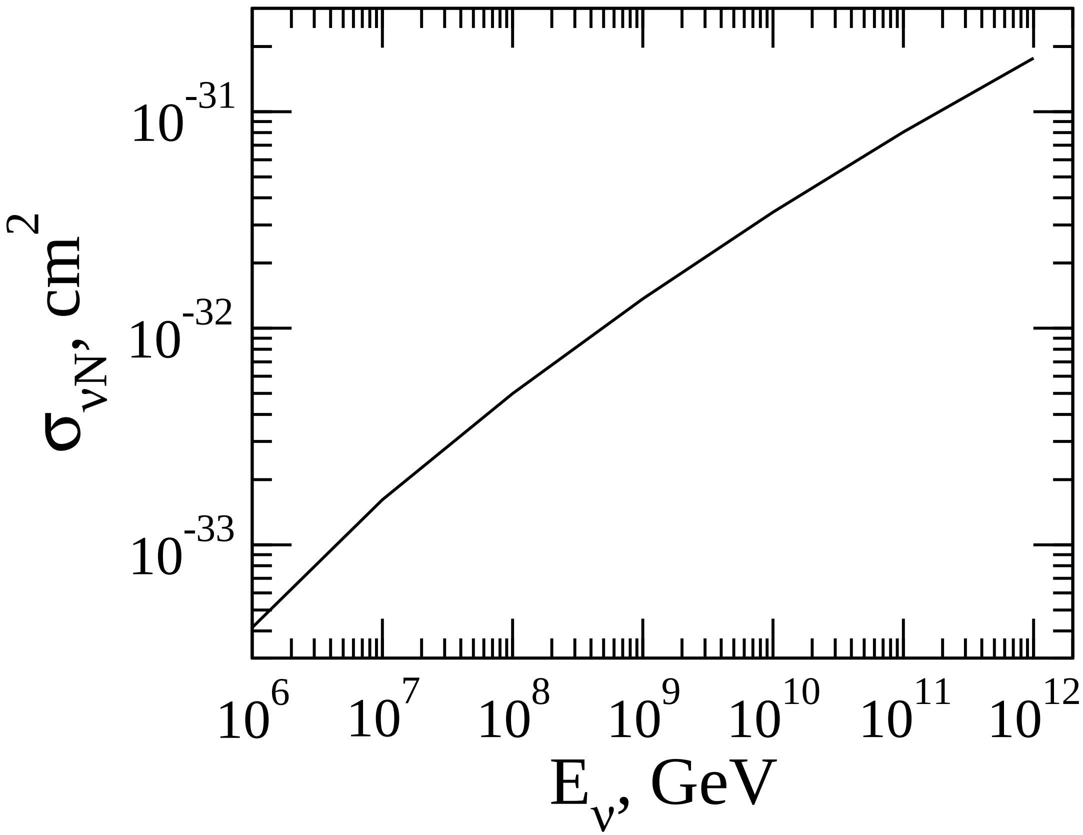
with $B \sim 2\alpha'_{\mathbb{P}} \log(1/x)$ is, of course, smaller than friquently used

$$\frac{\langle\sigma^2\rangle}{\pi R^2}$$

with arbitrary $R^2 = const$.



$\sigma^{\nu N}(E_\nu)$



Conclusions

- Unitarity corrections to $\sigma^{\nu N}$ for UHE neutrinos are estimated.
- The gluon fusion $\propto \langle\sigma^2\rangle$ is the large-distance effect.
- BFKL diffusion with $\alpha'_{\mathbb{P}} \neq 0$ dilutes the flux of gluons and diminishes the fusion effect.
- Saturation effect amounts to the 25% correction to $\sigma^{\nu N}$ at $E_\nu = 10^{12}$ GeV.