Application of stochastic model to the calculation of the all-loop RFT amplitudes.

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RFT

The theory of Reggeon exchanges is known to be very successfull phenomenologically. The elastic amplitude is written as:

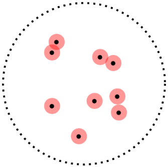
$$T = \sum_{n,m} V_n \otimes G_{nm} \otimes V_m$$

Green functions G_{mn} are obtained within the effective field theory

$$\mathcal{L} = \frac{1}{2}\phi^\dagger(\overleftarrow{\partial_y} - \overrightarrow{\partial_y})\phi - \alpha'(\nabla_{\mathbf{b}}\phi^\dagger)(\nabla_{\mathbf{b}}\phi) + \Delta\phi^\dagger\phi + \mathcal{L}_{int}.$$

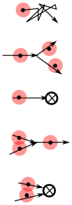
For $\mathcal{L}_{int} = i\, r_{3P}\phi^\dagger\phi(\phi^\dagger + \phi) + \chi{\phi^\dagger}^2\phi^2$
it is possible to use reaction-diffusion (or “stochastic”) models for obtaining the Green functions with account of all loops.
[Grassberger&Sundermeyer'78; Boreskov'01]

The stochastic model.



Consider a system of classic "partons" in the transverse plane with:

- Diffusion (chaotical movement) D;
- Splitting (λ – prob. per unit time)
- Death (m_1)
- Fusion ($\sigma_\nu \equiv \int d^2b\, p_\nu(b)$)
- Annihilation ($\sigma_{m_2} \equiv \int d^2b\, p_{m_2}(b)$)



Parton number and positions are described in terms of **probability densities** $\rho_N(y, \mathcal{B}_N)$ ($N = 0, 1, ...; \mathcal{B}_N \equiv \{b_1, \ldots, b_N\}$) with normalization $p_N(y) \equiv \frac{1}{N!} \int \rho_N(y, \mathcal{B}_N) \prod d\mathcal{B}_N$; $\quad \sum_0^\infty p_N = 1$.

Inclusive distributions

S-parton inclusive distributions:

$$f_s(y;\mathcal{Z}_s) = \sum_N \frac{1}{(N-s)!}\int d\mathcal{B}_N\ \rho_N(y;\mathcal{B}_N)\prod_{i=1}^{s}\delta(\mathbf{z}_i - \mathbf{b}_i);$$

$\int d\mathcal{Z}_s f_s(y;\mathcal{Z}_s) = \sum \frac{N!}{(N-s)!}\, p_N(y) \equiv \mu_s(y)$. – factorial moments.

Example: Start with a single parton with only diffusion and splitting allowed.

$$f_1^{1\ \mathrm{parton}}(y,b) = \frac{\exp(\lambda y)\exp(-b^2/4Dy)}{4\pi D y}.$$

– imaginary part of the bare Pomeron propagator.

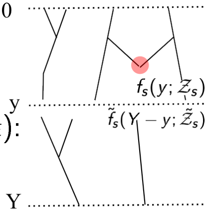
The set of evolution equations for $f_s(\mathcal{Z}_s)$, $(s=1,\ldots)$ coincides with the set of equations for the Green functions of the RFT.

The amplitude.

Green functions:

$f_s(y;\mathcal{Z}_s) \propto \sum_m \int d\mathcal{X}_m\, V_m(\mathcal{X}_m) G_{mn}(0;\mathcal{X}_m|y;\mathcal{Z}_n)$;

$f_m(y = 0, \mathcal{X}_m) \propto V_m(\mathcal{X}_m)$ – particle–mPomeron vertices



The amplitude ($g(b)$ assumed narrow; $\int g(b) d^2 b \equiv \epsilon$):

$T(Y) = \langle A|T|\tilde{A}\rangle =$

$$= \sum_{s=1}^{\infty} \frac{(-1)^{s-1}}{s!} \int d\mathcal{Z}_s d\tilde{\mathcal{Z}}_s f_s(y;\mathcal{Z}_s) \tilde{f}_s(Y-y;\tilde{\mathcal{Z}}_s) \prod_{i=1}^{s} g(\mathbf{z}_i - \tilde{\mathbf{z}}_i - \mathbf{b}).$$

It does not depend on the linkage point y (“boost invariance”) **if**

$$\lambda \int g(b) db = \int p_{m_2}(b) db + \frac{1}{2} \int p_\nu b db \ ,$$

Correspondence RFT–Stochastic model

We use the simplest form of $g(b)$, $p_{m_2}(b)$ and $p_\nu(b)$:

$$p_{m_2}(\mathbf{b}) = m_2\ \theta(a - |\mathbf{b}|); \quad p_\nu(\mathbf{b}) = \nu\ \theta(a - |\mathbf{b}|);$$
$$g(\mathbf{b}) = \theta(a - |\mathbf{b}|);.$$

with a – some small scale; $\epsilon \equiv \pi a^2$.

RFT	stochastic model
Rapidity y	Evolution time y
Slope α'	Diffusion coefficient D
$\Delta = \alpha(0) - 1$	$\lambda - m_1$
Splitting vertex r_{3P}	$\lambda\sqrt{\epsilon}$
Fusion vertex r_{3P}	$(m_2 + \frac{1}{2}\nu)\sqrt{\epsilon}$
Quartic coupling χ	$\frac{1}{2}(m_2 + \nu)\epsilon$

Boost invariance ($\lambda = m_2 + \frac{\nu}{2}$) $\Leftrightarrow$ equality of fusion and splitting vertices.

Calculation method

Taking an explicit note of the initial parton distributions

$$T = \sum_{n,s,k} \frac{(-1)^{s-1}}{s!} \underbrace{P_n(\mathcal{X}) \otimes f_{ns}(\mathcal{X}|\mathcal{Z})}_{f_s(y,\mathcal{Z})} \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{Z}}) \otimes \underbrace{\tilde{f}_{ks}(\tilde{\mathcal{X}}|\tilde{\mathcal{Z}}) \otimes \tilde{P}_k(\tilde{\mathcal{X}})}_{\tilde{f}_s(Y-y,\tilde{Z})}$$

Calculation method

Taking an explicit note of the initial parton distributions

$$T = \sum_{n,k} P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{Z}}) \otimes \tilde{f}_{ks}(\tilde{\mathcal{X}}|\tilde{\mathcal{Z}})}_{T_{sample}} \otimes \tilde{P}_k(\tilde{\mathcal{X}}).$$

Main idea: simulate a sample of 2 parton sets which correspond to f_s and $\tilde{f}_s$ on the average, compute T_{sample} and make its MC average. For N partons with fixed positions

$$f_s(\mathcal{Z}_s) = \sum_{\{\hat{\mathbf{x}}_{i_1},\dots,\hat{\mathbf{x}}_{i_s}\}\in\hat{\mathcal{X}}_N} \delta(\mathbf{z}_1 - \hat{\mathbf{x}}_{i_1})\dots\delta(\mathbf{z}_s - \hat{\mathbf{x}}_{i_s})$$

$$T_{\rm sample} = \sum_{s=1}^{N_{min}} (-1)^{s-1} \sum_{i_1<i_2\dots<i_s} \sum_{j_1<\dots<j_s} g_{i_1 j_1}\cdots g_{i_s j_s}.$$

- expansion of T_{sample} in the number of **P** exchanges s;
- works for any position of the linkage point y.

Calculation method

Setting the linkage point to full rapidity interval $y = Y$ simplifies the calculation: $\tilde{f}_s(y = 0, \mathcal{Z}_s) = N_s(\mathcal{Z}_s)/\epsilon^{s/2}$ and the MC average involves evolution from only one side:

$$T = \sum_n P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{X}}) \otimes \tilde{P}_s(\tilde{\mathcal{X}}).}_{T_{sample}}$$

Peculiarities of the stochastic approach to the RFT:

- Presence of the $2 \longrightarrow 2$ coupling
- Regularization scale (equivalent to the cutoff or the Pomeron size in RFT) enters via functions $g(b)$, $p_{m_2}(b)$ and p_ν
- Neglect of the real part of the amplitude.

Realization features:

- We do the explicit parton sets evolution starting from initial configuration generated in accord with the vertices
- The realization can be used for both $0D$ and $2D$ RFT

Applications

- The full Pomeron propagator
  - Difference between $0D$ and $2D$ RFT
  - Role of the $2 \rightarrow 2$ coupling
- Amplitude in the quasieikonal approximation
  - Role of the $2 \rightarrow 2$ coupling and regularization scale

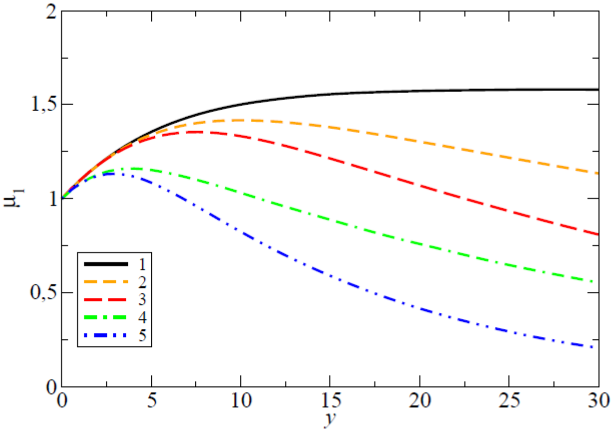
The full propagator

The propagator coincides with 1-parton inclusive distribution with a single parton at the start of the evolution. We use parameter sets which differ by the values of the couplings only.

Set	λ	ν	m_1	m_2	Δ	r_{3P} $(0D)$	χ $(0D)$
1	0.1	0.2	0	0	0.1	0.1	0.1
2	0.1	0.1	0	0.05	0.1	0.1	0.075
3	0.1	0	0	0.1	0.1	0.1	0.05
4	0.15	0.3	0.05	0	0.1	0.15	0.15
5	0.15	0	0.05	0.15	0.1	0.15	0.075

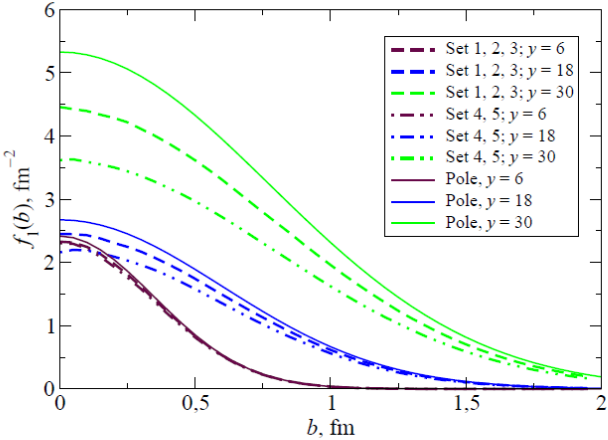
For the 2D case we additionaly introduce the partonic interaction distance $a = 0.05$ fm and the diffusion coefficient $D = 0.01$ fm^{-2}

The full propagator, 0D case



- The 2 $\rightarrow$ 2 coupling is crucial for the asymptotic behavior (in accord with preceding works)
- Non-zero asymptotic behavior needs a special relation btw r_{3P}, Δ and χ.

The full propagator, 2D case



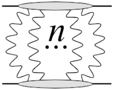
- The role of $2 \to 2$ coupling is negligible.
- $f_1(y, b = fix)$ grows with y, growth is defined exclusively by Δ and r_{3P}.

The quaieikonal approximation

We estimate the role of loop corrections by comparing the full calculation to the quasieikonal fit [Ter-Martirosyan'86] to the experimental data on pp cross sections.

The starting point :

$$T(Y,\mathbf{b}) = \sum_{n=1}^{\infty} \frac{(-C)^{n-1}}{n!} \left(T_P(Y,\mathbf{b})\right)^n = \sum$$



$$T_P(Y,\mathbf{b}) = \frac{g_0^2 \exp(\Delta Y)}{R_P^2 + \alpha' Y} \exp[-\tfrac{1}{4} b^2/(R_P^2 + \alpha' Y)].$$

$$g_0^2 = 2.14 \text{ GeV}^{-2} \approx 0.083\text{fm}^2 \text{ , } R_P^2 = 3.30\text{GeV}^{-2} \approx 0.128\text{fm}^2 \text{ ,}$$
$$\alpha'_P = 0.22 \text{ GeV}^{-2} \approx 0.0085\text{fm}^2 \text{ , } \Delta = 0.12 \text{ , } \quad C = 1.5 \text{ .}$$

We take
- the same p–nP vertices (Gaussian)
- triple coupling $r_{3P} = 0.087\text{GeV}^{-1}$ following[Kaidalov'79].

The parameter sets

Quasieikonal vertices $\Leftrightarrow$ "quasipoissonian" distribution in the # of partons: $P_n = C^{(n-1)/2}\frac{\bar{N}^n}{n!}e^{-C\bar{N}}, n = 1, \ldots, \infty;$

$$p_p(\mathbf{b}) = \frac{1}{4\pi R^2}\exp\left[-\frac{b^2}{2R^2}\right]$$

The parameter sets:

Set	a, fm	λ	m_1	m_2	ν	$\bar{N}$
1	0.018	0.54722	0.42722	0	1.09488	32.02
2	0.018	0.54722	0.42722	0.54722	0	32.02
3	0.036	0.27361	0.15361	0	0.54722	16.01
4	0.036	0.27361	0.15361	0.27361	0	16.01

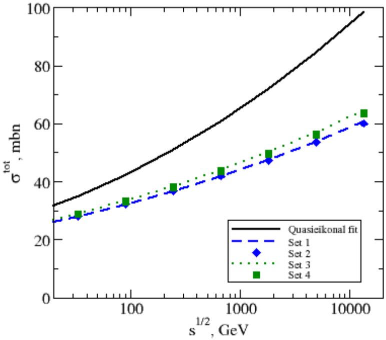
with $D = \alpha'_P = 0.0085$ fm^2 and $R = R_P = 0.36$ fm.

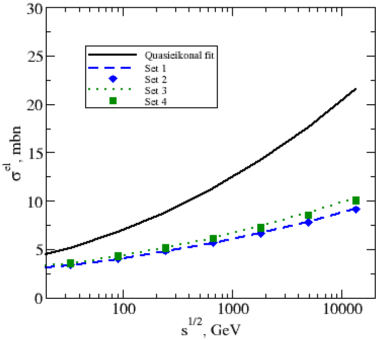
$$\chi_1 = \chi_4 = 0.00056\ \text{fm}^2,\ \chi_1 = 2\chi_2,\ \chi_3 = 2\chi_4.$$

$r_{3P} = 0.017$ fm for all sets.

The effect of loops

Calculations with $\Delta = 0.12$:

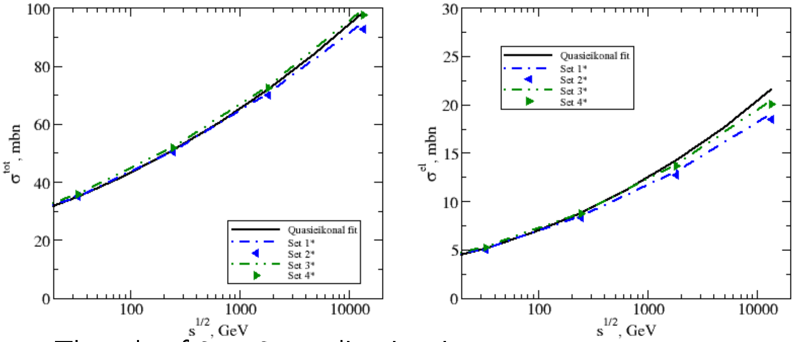




- The growth with $\sqrt{s}$ is suppressed compared to the eikonal.
- The role of $2 \to 2$ coupling is minor.

The effect of loops

Full calculation with $\Delta = 0.165$ and the same couplings vs the quasieikonal fit.



- The role of $2 \rightarrow 2$ coupling is minor.
- The contribution of loops can be imitated via Δ renormalization

Conclusions

- Our numerical realization of the RFT allows to obtain the scattering amplitude with all loops taken into account.
- The approach is capable of giving the amplitude as an expansion in the number of Pomeron exchanges at given rapidity y.

On the basis of numerical calculations we conclude:

- The intercept is effectively reduced as a result of the full account of the Pomeron interactions.
- $2 \to 2$ coupling may be neglected for $\alpha' \neq 0$ at the available energies.

Backup

Backup slides

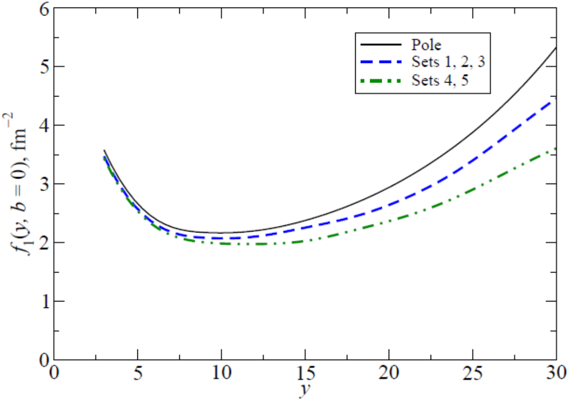
Ryser formula

Simplification in the expression for the amplitude after employing the Ryser formula

$$T_{\mathrm{sample}} = \sum_{s_1 \subseteq \{1,\dots,N\}} \sum_{\substack{s_2 \subseteq \{1,\dots,\tilde{N}\},\\ |s_2|<|s_1|}} (-1)^{|s_2|-1} \mathrm{C}^{|s_1|-|s_2|}_{\tilde{N}-|s_2|} \prod_{i \in s_1} \left(\sum_{j \in s_2} g_{ij} \right) \quad ;$$

The estimated number of operations is $O(4^N)$.

2D propagator at $b = 0$



The bare propagator $D_{bare}(y, b = 0) \propto \exp(\Delta y)/y$

Cross sections definitions

$$\sigma^{\rm tot}(Y) = 2\,{\rm Im} M(Y, \mathbf{q}=0), \quad \sigma^{\rm el} = \int \frac{d^2q}{(2\pi)^2}\, |M(Y,\mathbf{q})|^2 \;,$$

$$f(Y,\mathbf{b}) = \frac{1}{(2\pi)^2}\int d^2q\, e^{-i\mathbf{q}\mathbf{b}} M(Y,\mathbf{q}) \;.$$

$$\sigma^{\rm tot}(Y) = 2\int d^2b\, {\rm Im} f(Y,\mathbf{b}) \;, \quad \sigma^{\rm el} = \int d^2b\, |f(Y,\mathbf{b})|^2.$$

$$f(Y,\mathbf{b}) \simeq iT(Y,\mathbf{b}), \quad T \equiv {\rm Im} f$$

Quasieikonal within the stochastic model

- Forbid fusion and annihillation
- Each connected component plays in f_s^{sample} only once