Hard inclusive production of a pair of rapidity-separated identified hadrons in proton collisions

A. Papa

Università della Calabria & INFN - Cosenza

in collaboration with
D.Yu. Ivanov (Sobolev Institute of Mathematics, Novosibirsk)

Low-x Meeting 2011
Santiago de Compostela, June 3-7, 2011

Outline

Outline

Kinematics

The process under consideration is

$$\mathrm{proton}(\mathrm{p}_1) \; + \; \mathrm{proton}(\mathrm{p}_2) \; \to \; \mathrm{hadron}_1(\mathrm{k}_1) \; + \; \mathrm{hadron}_2(\mathrm{k}_2) \; + \; \mathrm{X}$$

Sudakov decomposition for the identified hadron momentum:

$$k_h = \alpha_h p_1 + \frac{\vec{k}_h^2}{\alpha_h s} p_2 + k_{h\perp} \; , \qquad k_{h\perp}^2 = -\vec{k}_h^2 \; , \qquad s = 2 p_1 \cdot p_2$$

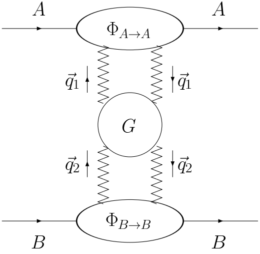
- large hadrons' transverse momenta: $\vec{k}_1^{\,2} \sim \vec{k}_2^{\,2} \gg \Lambda_{\mathrm{QCD}}^2$
  $\longrightarrow$ perturbative QCD applicable
- large energy: $s = 2 p_1 \cdot p_2 \gg \vec{k}_{1,2}^{\,2}$
  $\longrightarrow$ BFKL resummation

Outline

BFKL “survival kit”

Total cross section $A + B \to X$, via the optical theorem, $\sigma = \frac{\mathrm{Im}_s \mathcal{A}}{s}$

- Regge limit ($s \to \infty$)
  $\Rightarrow$ BFKL factorization for $\mathrm{Im}_s \mathcal{A}$:
  convolution of the Green’s function of two interacting Reggeized gluons and of the impact factors of the colliding particles.



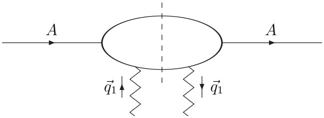
- Valid both in
  LLA (resummation of all terms $(\alpha_s \ln s)^n$)
  NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

$$\mathrm{Im}_s \mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

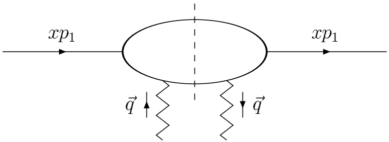
- The Green’s function is process-independent and is determined through the BFKL equation. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)\,.$$

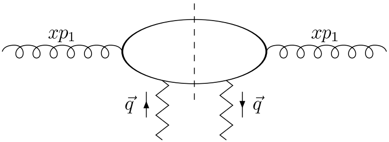
- Impact factors are process-dependent;
  only very few have been calculated in the NLA.



For the process under consideration, the starting point is provided by the impact factors for colliding partons. [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]



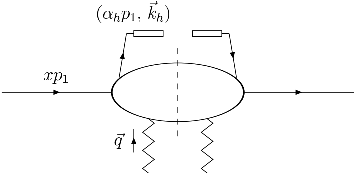
quark impact factor



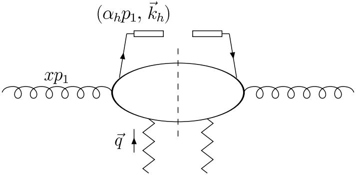
gluon impact factor

- LLA $\longrightarrow$ leading-order (LO) impact factor $\longrightarrow$ one-particle intermediate state
- NLA $\longrightarrow$ next-to-LO (NLO) impact factor:
  virtual corrections $\longrightarrow$ one-particle intermediate state
  real particle production $\longrightarrow$ two-particle intermediate state

- Step 1: “open” one of the integrations over the phase space of the intermediate state to allow one parton to fragment into a given hadron



quark vertex



gluon vertex

- Step 2: use QCD collinear factorization

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes D_a^h \quad + \quad f_g \otimes (\text{gluon vertex}) \otimes D_g^h$$

- Step 3: project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer to the (ν, n)-representation

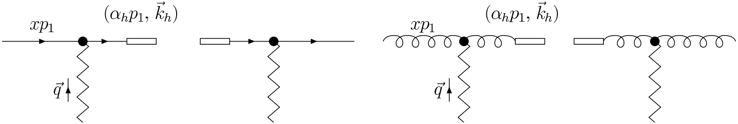
$$\Phi(\nu, n) = \int d^2\vec{q}\, \frac{\Phi(\vec{q})}{\vec{q}^{\,2}} \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{\gamma - \frac{n}{2}} \left(\vec{q}\cdot\vec{l}\,\right)^{n}\,, \qquad \gamma = i\nu - \frac{1}{2}\,, \qquad \vec{l}^{\,2} = 0$$

The impact factor in the LLA

The starting point is given by "inclusive" LO parton impact factors:

$$\Phi_q = g^2 \frac{\sqrt{N^2-1}}{2N}\ , \qquad \Phi_g = \frac{C_A}{C_F}\Phi_q$$

- Step 1: "open" the integration over the one-particle intermediate state, i.e. introduce a delta function



- Step 2: use QCD collinear factorization ($D = 4 + 2\epsilon$)

$$\frac{d\Phi^h}{\vec{q}^{\,2}} = \Phi_q \, d\alpha_h \frac{d^{2+2\epsilon}\vec{k}}{\vec{k}^{\,2}} \int_{\alpha_h}^{1} \frac{dx}{x}\, \delta^{(2+2\epsilon)}\left(\vec{k}-\vec{q}\right)$$

$$\times \left(\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_h}{x}\right) + \sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_h}{x}\right) \right)$$

Outline

Collinear and QCD coupling counterterms

The collinear singularities which will arise in the NLA calculation are to be removed by the renormalization of PDFs and FFs:

$$f_q(x) = f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[P_{qq}(z)f_q(\tfrac{x}{z},\mu_F) + P_{qg}(z)f_g(\tfrac{x}{z},\mu_F)\right]$$

$$f_g(x) = f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[P_{gq}(z)f_q(\tfrac{x}{z},\mu_F) + P_{gg}(z)f_g(\tfrac{x}{z},\mu_F)\right]$$

$$D_q^h(x) = D_q^h(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[D_q^h(\tfrac{x}{z},\mu_F)P_{qq}(z) + D_g^h(\tfrac{x}{z},\mu_F)P_{gq}(z)\right]$$

$$D_g^h(x) = D_g^h(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[D_q^h(\tfrac{x}{z},\mu_F)P_{qg}(z) + D_g^h(\tfrac{x}{z},\mu_F)P_{gg}(z))\right]$$

$$\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^{\epsilon}}$$

- Collinear counterterm (in the (ν, n)-representation):

$$\frac{\pi\sqrt{2}\,\vec{k}^{\,2}}{\Phi_q}\frac{d\Phi^h(\nu,n)|_{\text{coll. c.t.}}}{d\alpha_h d^{2+2\epsilon}\vec{k}} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}}+\ln\frac{\mu_F^2}{\mu^2}\right)\int_{\alpha_h}^{1}\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^{1}\frac{dz}{z}\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n$$

$$\times\Bigg[(1+z^{-2\gamma})P_{qq}(z)\sum_{a=q,\bar{q}}f_a(x)D_a^h\left(\frac{\alpha_h}{xz}\right)+\left(\frac{C_A}{C_F}+z^{-2\gamma}\right)P_{gq}(z)\sum_{a=q,\bar{q}}f_a(x)D_g^h\left(\frac{\alpha_h}{xz}\right)$$

$$+(1+z^{-2\gamma})\frac{C_A}{C_F}P_{gg}(z)f_g(x)D_g^h\left(\frac{\alpha_h}{xz}\right)+\frac{C_A}{C_F}\left(\frac{C_F}{C_A}+z^{-2\gamma}\right)P_{qg}(z)f_g(x)\sum_{a=q,\bar{q}}D_a^h\left(\frac{\alpha_h}{xz}\right)\Bigg]$$

- QCD renormalization counterterm (in the (ν, n)-representation):

$$\frac{\pi\sqrt{2}\,\vec{k}^{\,2}}{\Phi_q}\frac{d\Phi(\nu,n)|_{\text{charge c.t.}}}{d\alpha d^{2+2\epsilon}\vec{k}} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}}+\ln\frac{\mu_R^2}{\mu^2}\right)\left(\frac{11C_A}{6}-\frac{n_f}{3}\right)$$

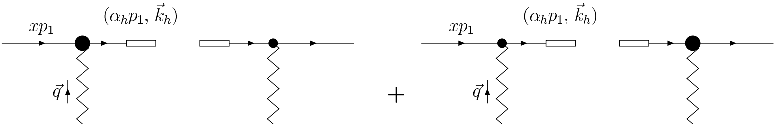
$$\times\int_{\alpha_h}^{1}\frac{dx}{x}\left(\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_h}{x}\right)+\sum_{a=q,\bar{q}}f_a(x)D_a^h\left(\frac{\alpha_h}{x}\right)\right)\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n$$

In the following

$$\frac{\pi\sqrt{2}\,\vec{k}^{\,2}}{\Phi_q}\frac{d\Phi^h(\nu,n)}{d\alpha_h d^{2+2\epsilon}\vec{k}} \equiv I$$

Outline

Virtual corrections



$$I_q^V = -\frac{\alpha_s}{2\pi}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{1}{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int_{\alpha_h}^{1}\frac{dx}{x}\sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_h}{x}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n$$

$$\times\left\{C_F\left(\frac{2}{\epsilon}-3\right)-\frac{n_f}{3}+C_A\left(\ln\frac{s_0}{\vec{k}^2}+\frac{11}{6}\right)\right\}+\text{finite terms}$$

“Real” corrections: quark-gluon intermediate state

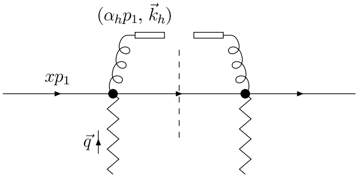
Starting point

$$\Phi^{\{QG\}} = \Phi_q g^2 \vec{q}^{\,2} \frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}} \frac{d\beta_1}{\beta_1} \frac{[1+\beta_2^2+\epsilon\beta_1^2]}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}(\vec{k}_2\beta_1-\vec{k}_1\beta_2)^2} \left\{ C_F\beta_1^2\vec{k}_2^{\,2} + C_A\beta_2\left(\vec{k}_1^{\,2}-\beta_1\vec{k}_1\cdot\vec{q}\right)\right\}$$

$\beta_{1,2}$ and $\vec{k}_{1,2}$: relative longitudinal and transverse momenta of the gluon(quark)

$$\beta_1+\beta_2=1, \quad \vec{k}_1+\vec{k}_2=\vec{q}$$

- gluon fragmentation
  “parent” parton variables: $\vec{k}=\vec{k}_1$, $\zeta=\beta_1$



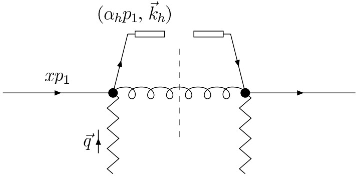
$$I^R_{q,g} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\int_{\alpha_h}^1\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a(x) D_g^h\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n$$

$$\times P_{gq}(\zeta)\left[\frac{C_A}{C_F}+\zeta^{-2\gamma}\right] + \text{finite terms}$$

The divergence cancels the corresponding term of the collinear counterterm.

- quark fragmentation

  "parent" parton variables: $\vec{k} = \vec{k}_2$, $\zeta = \beta_2$



$$\left(I^R_{q,q}\right)^{C_F} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int_{\alpha_h}^{1}\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^{1}\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n$$

$$\times\left\{C_F\left(\frac{2}{\epsilon}-3\right)\delta(1-\zeta) + P_{qq}(\zeta)\left(1+\zeta^{-2\gamma}\right) + \text{finite terms}\right\}$$

The first divergence cancels the corresponding term in the virtual contribution I_q^V;
the second divergence cancels the corresponding term of the collinear counterterm.

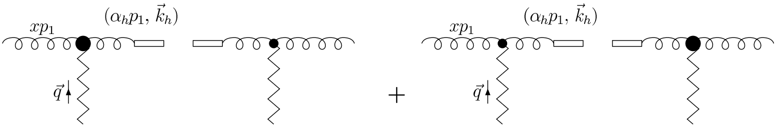
$$\left(I^R_{q,q}\right)^{C_A} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\int_{\alpha_h}^{1}\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^{1}\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n$$

$$\times C_A\,\delta(1-\zeta)\ln\frac{s_0}{\vec{k}^2} + \text{finite terms}$$

The divergence cancels a part of the virtual contribution I_q^V; the remaining divergence in I_q^V vanishes after the charge renormalization.

Outline

Virtual corrections



$$I_g^V = -\frac{\alpha_s}{2\pi}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{1}{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int_{\alpha_h}^{1}\frac{dx}{x} f_g(x) D_g^h\left(\frac{\alpha_h}{x}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n \frac{C_A}{C_F}$$

$$\times\left\{C_A\left(\ln\frac{s_0}{\vec{k}^2}+\frac{2}{\epsilon}-\frac{11}{6}\right)+\frac{n_f}{3}\right\} + \text{finite terms}$$

“Real” corrections: quark-antiquark intermediate state

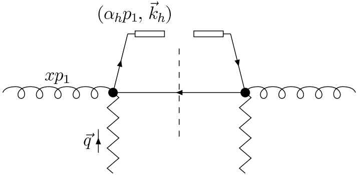
Starting point ($T_R = 1/2$)

$$\Phi^{\{Q\bar{Q}\}} = \Phi_g g^2 \vec{q}^{\,2} \frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}} d\beta_1 T_R \left(1 - \frac{2\beta_1\beta_2}{1+\epsilon}\right) \left\{ \frac{C_F}{C_A}\frac{1}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}} + \beta_1\beta_2 \frac{\vec{k}_1\cdot\vec{k}_2}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}(\vec{k}_2\beta_1 - \vec{k}_1\beta_2)^2} \right\}$$

$\beta_{1,2}$ and $\vec{k}_{1,2}$: relative longitudinal and transverse momenta of the quark(antiquark)

$$\beta_1 + \beta_2 = 1, \quad \vec{k}_1 + \vec{k}_2 = \vec{q}$$

- quark fragmentation
  (antiquark fragmentation goes similarly)
  “parent” parton variables: $\vec{k} = \vec{k}_1$, $\zeta = \beta_1$



$$I^R_{g,q} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\int_{\alpha_h}^{1}\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^{1}\frac{d\zeta}{\zeta} f_g(x)\sum_{a=q,\bar{q}} D_a^h\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n \frac{C_A}{C_F}$$

$$\times P_{qg}(\zeta)\left[\frac{C_F}{C_A} + \zeta^{-2\gamma}\right] + \text{finite terms}$$

The divergence cancels the corresponding term of the collinear counterterm.

“Real” corrections: two-gluon intermediate state

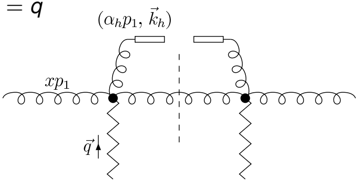
Starting point

$$\Phi^{\{GG\}} = \Phi_g g^2 \vec{q}^{\,2} \frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}} d\beta_1 \frac{C_A}{2}\left[\frac{1}{\beta_1}+\frac{1}{\beta_2}-2+\beta_1\beta_2\right]\left\{\frac{1}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}}+\frac{\beta_1^2}{\vec{k}_1^{\,2}(\vec{k}_2\beta_1-\vec{k}_1\beta_2)^2}+\frac{\beta_2^2}{\vec{k}_2^{\,2}(\vec{k}_2\beta_1-\vec{k}_1\beta_2)^2}\right\}$$

$\beta_{1,2}$ and $\vec{k}_{1,2}$: relative longitudinal and transverse momenta of the two gluons

$$\beta_1+\beta_2=1, \quad \vec{k}_1+\vec{k}_2=\vec{q}$$

- gluon fragmentation (times 2)



$$I^R_{g,g} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\int_{\alpha_h}^{1}\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^{1}\frac{d\zeta}{\zeta} f_g(x) D_g^h\left(\frac{\alpha_h}{x\zeta}\right)\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n \frac{C_A}{C_F}$$

$$\times\left\{P_{gg}(\zeta)\left(1+\zeta^{-2\gamma}\right)+\delta(1-\zeta)\left[C_A\left(\ln\frac{s_0}{\vec{k}^2}+\frac{2}{\epsilon}-\frac{11}{3}\right)+\frac{2n_f}{3}\right]\right\}+\text{finite terms}$$

The first divergence cancels the corresponding term of the collinear counterterm; the second divergence cancels the corresponding term in the virtual contribution I_g^V; the remaining divergence in I_g^V vanishes after the charge renormalization.

Final result

All the IR and UV divergences canceled!

$$
\vec{k}_h^{\,2}\,\frac{d\Phi^h(\nu,n)}{d\alpha_h d^2\vec{k}_h} = 2\,\alpha_s(\mu_R)\sqrt{\frac{C_F}{C_A}}\,\left(\vec{k}_h^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}_h\cdot\vec{l}\,\right)^n
$$

$$
\times\left\{\int_{\alpha_h}^{1}\frac{dx}{x}\left(\frac{x}{\alpha_h}\right)^{2\gamma}\left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_h}{x}\right)+\sum_{a=q,\bar{q}}f_a(x)D_a^h\left(\frac{\alpha_h}{x}\right)\right]\right.
$$

$$
+\frac{\alpha_s\left(\mu_R\right)}{2\pi}\int_{\alpha_h}^{1}\frac{dx}{x}\int_{\frac{\alpha_h}{x}}^{1}\frac{d\zeta}{\zeta}\left(\frac{x\,\zeta}{\alpha_h}\right)^{2\gamma}\left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_h}{x\zeta}\right)C_{gg}\left(x,\zeta\right)+\sum_{a=q,\bar{q}}f_a(x)D_a^h\left(\frac{\alpha_h}{x\zeta}\right)C_{qq}\left(x,\zeta\right)\right.
$$

$$
\left.\left.+\sum_{a=q,\bar{q}}f_a(x)D_g^h\left(\frac{\alpha_h}{x\zeta}\right)C_{qg}\left(x,\zeta\right)+\frac{C_A}{C_F}f_g(x)\sum_{a=q,\bar{q}}D_a^h\left(\frac{\alpha_h}{x\zeta}\right)C_{gq}\left(x,\zeta\right)\right]\right\}\,.
$$

Conclusions and Outlook

- The missing ingredient for the study of the hard inclusive production of a pair of rapidity-separated identified hadrons in proton collisions at large energies has been calculated.

- It has been explicitly shown that it is free of IR and UV divergences.
  An essential role in the cancellation of IR divergences has been played by the projection onto LO BFKL eigenfunctions.

- After the convolution with the BFKL Green's function, definite predictions can be built to be compared with Tevatron and LHC data.