

# PARAMETER ESTIMATION FOR **INFLATIONARY** GRAVITATIONAL WAVE BACKGROUNDS WITH **LISA**

BASED ON PROJECT 15 OF THE COSWG

**11TH LISA COSWG MEETING**

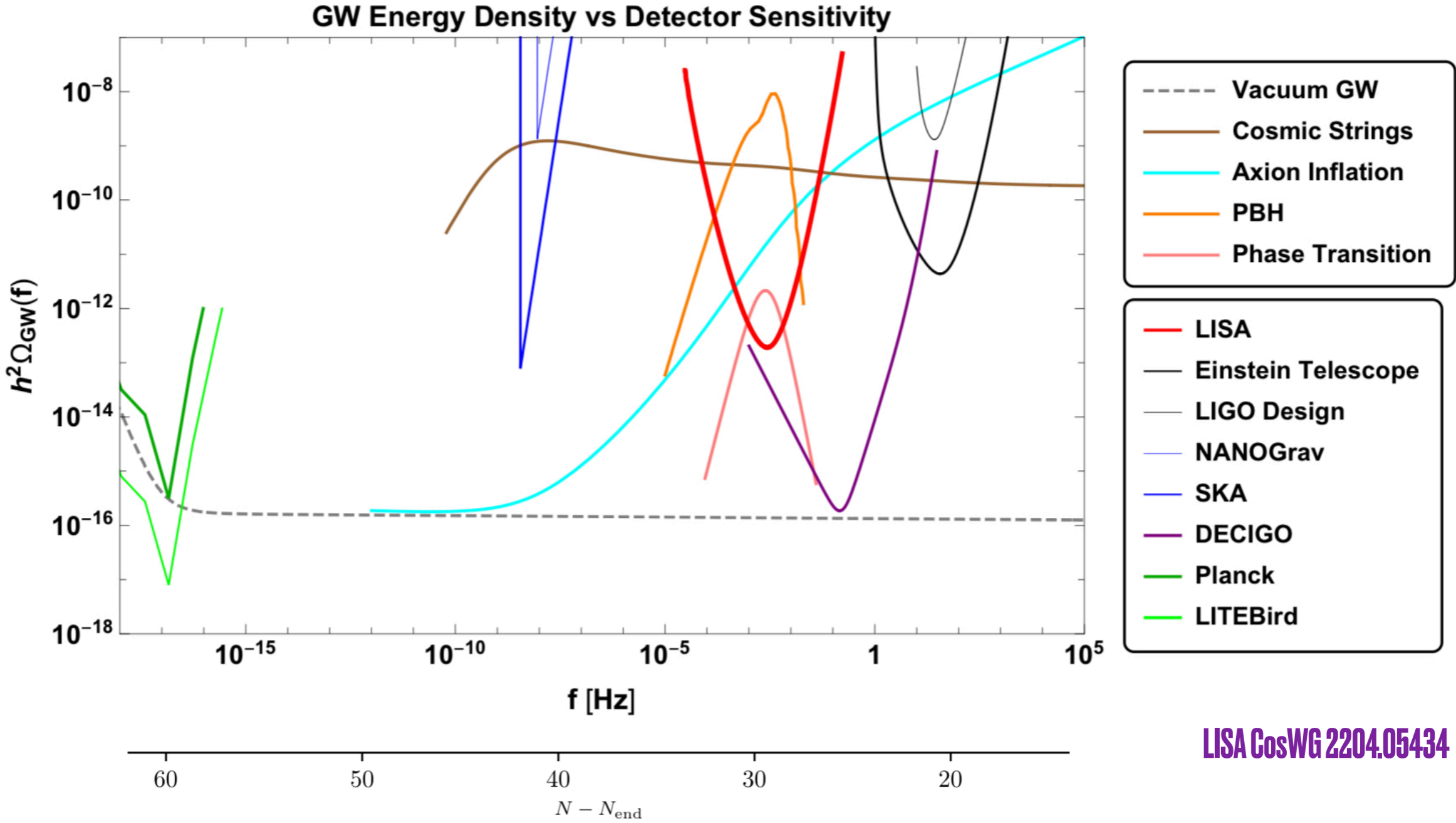
**JUN 17 2024**

**MATTEO BRAGLIA**

**NEW YORK  
UNIVERSITY**



# STOCHASTIC GRAVITATIONAL WAVE BACKGROUNDS



**SURVEY INFLATIONARY MODELS**



**GROUP THEM ACCORDING TO THE SPECTRAL SHAPE OF  $\Omega_{\text{GW}}(f)$**



**BUILD A TEMPLATE BANK**



**FORECAST CONSTRAINTS ON TEMPLATES**



**DRAW CONCLUSIONS ABOUT EARLY UNIVERSE PHYSICS**

# DATA & ANALYSIS PIPELINE

# MEASURING A BACKGROUND WITH LISA

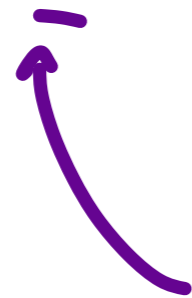
**Data:**

$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} d_i(t) e^{-2\pi i f t} dt$$

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**Channel  $i$**

# MEASURING A BACKGROUND WITH LISA

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Observation time

# MEASURING A BACKGROUND WITH LISA

**Data:**

$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} \underline{d_i(t)} e^{-2\pi i f t} dt$$




**Time domain data stream**



# MEASURING A BACKGROUND WITH LISA

**Data:**

$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} d_i(t) e^{-2\pi i f t} dt = \sum_{\nu} \tilde{n}_i^{\nu}(f) + \sum_{\sigma} \tilde{s}_i^{\sigma}(f)$$


**Assumption 1: only stochastic components.**

**Transients, deterministic sources and glitches  
are removed**

# MEASURING A BACKGROUND WITH LISA

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$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} d_i(t) e^{-2\pi i f t} dt = \sum_{\nu} \tilde{n}_i^{\nu}(f) + \sum_{\sigma} \tilde{s}_i^{\sigma}(f)$$

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Instrumental noise

GW signal

Assumptions: only stochastic

# MEASURING A BACKGROUND WITH LISA

**Data:** 
$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} d_i(t) e^{-2\pi i f t} dt = \sum_{\nu} \tilde{n}_i^{\nu}(f) + \sum_{\sigma} \tilde{s}_i^{\sigma}(f)$$

$i = 1, 2, 3$   $\longrightarrow$

$X, Y, Z$   $\longrightarrow$

**TDI variables**

$$A = \frac{Z - X}{\sqrt{2}},$$

$$E = \frac{X - 2Y + Z}{\sqrt{6}},$$

$$T = \frac{X + Y + Z}{\sqrt{3}}$$

**Orthogonal channels**

Assumptions: only stochastic

# MEASURING A BACKGROUND WITH LISA

**Data:**

$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} d_i(t) e^{-2\pi i f t} dt = \sum_{\nu} \tilde{n}_i^{\nu}(f) + \sum_{\sigma} \tilde{s}_i^{\sigma}(f)$$

**(Implicit) assumption 2: equilateral geometry and equal noise in each spacecraft. (See Hartwig et al 2303.15929 for a paper relaxing these assumptions)**

Assumptions: only stochastic

# MEASURING A BACKGROUND WITH LISA

Data:

$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} d_i(t) e^{-2\pi i f t} dt = \sum_{\nu} \tilde{n}_i^{\nu}(f) + \sum_{\sigma} \tilde{s}_i^{\sigma}(f)$$

Properties:

Assumption 3: noise and signals are

**Gaussian** and **stationary**.

The signal is also assumed to be **isotropic**  
and **non-chiral**.

Assumptions: only stochastic, equilateral geometry

# MEASURING A BACKGROUND WITH LISA

**Data:** 
$$\tilde{d}_i(f) = \int_{-T/2}^{T/2} d_i(t) e^{-2\pi i f t} dt = \sum_{\nu} \tilde{n}_i^{\nu}(f) + \sum_{\sigma} \tilde{s}_i^{\sigma}(f)$$

**Properties:** 
$$\langle \tilde{d}_i(f) \rangle = 0 ,$$

$$\langle \tilde{d}_i(f) \tilde{d}_j^*(f') \rangle = \frac{\delta(f - f')}{2} \left[ \sum_{\nu} P_{N,ij}^{\nu}(f) + \sum_{\sigma} P_{S,ij}^{\sigma}(f) \right]$$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.

# INSTRUMENTAL NOISE

$$P_{N,ij}(f) \equiv \sum_{\nu} P_{N,ij}^{\nu}(f) = [T_{ij,lk}^{\text{TM}}(f)S_{lk}^{\text{TM}}(f) + T_{ij,lk}^{\text{OMS}}(f)S_{lk}^{\text{OMS}}(f)]$$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.



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**Transfer functions**

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.

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$$P_{N,ij}(f) \equiv \sum_{\nu} P_{N,ij}^{\nu}(f) = [T_{ij,lk}^{\text{TM}}(f)S_{lk}^{\text{TM}}(f) + T_{ij,lk}^{\text{OMS}}(f)S_{lk}^{\text{OMS}}(f)]$$

**Test mass:** 
$$S_{lk}^{\text{TM}}(f) = A_{lk}^2 \left( 1 + \left( \frac{0.4\text{mHz}}{f} \right)^2 \right) \left( 1 + \left( \frac{f}{8\text{mHz}} \right)^4 \right) \left( \frac{1}{2\pi f c} \right)^2 \left( \frac{\text{fm}^2}{\text{s}^3} \right)$$

**Optical metrology system:** 
$$S_{lk}^{\text{OMS}}(f) = P_{lk}^2 \left( 1 + \left( \frac{2 \times 10^{-3}\text{Hz}}{f} \right)^4 \right) \times \left( \frac{2\pi f}{c} \right)^2 \times \left( \frac{\text{pm}^2}{\text{Hz}} \right)$$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.

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Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.

# SIGNAL

$$h^2\Omega_{\text{GW}}(f) \equiv \frac{4\pi^2 f^3}{3H_0^2} S_h(f)$$

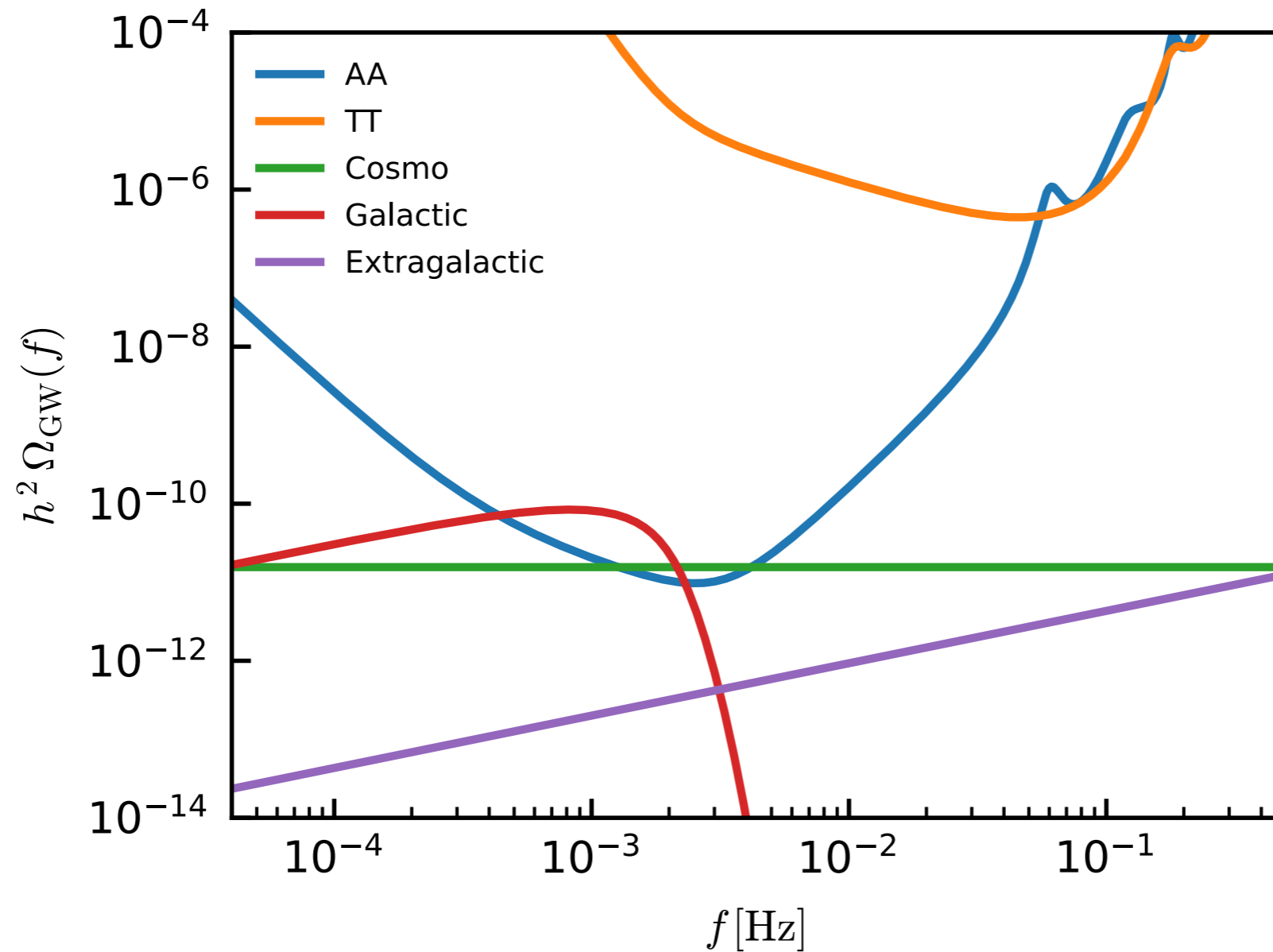
$$P_{S,ij}(f) \equiv \sum_{\sigma} P_{S,ij}^{\sigma}(f) = \mathcal{R}_{ij}(f) [S_{\text{Gal}}(f) + S_{\text{Ext}}(f) + S_{\text{Cosmo}}(f)]$$

**Galactic:** 
$$S_{\text{Gal}}(f) = A_{\text{Gal}} \left( \frac{f}{1 \text{ Hz}} \right)^{-\frac{7}{3}} \times e^{-(f/f_1)^{\alpha}} \times \frac{1}{2} \left[ 1 + \tanh \frac{f_{\text{knee}} - f}{f_2} \right]$$

**Extragalactic:** 
$$h^2\Omega_{\text{Ext}} = 10^{\log_{10}(h^2\Omega_{\text{Ext}})} \left( \frac{f}{0.001 \text{ Hz}} \right)^{2/3}$$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.

# SIGNAL AND NOISE IN THE LISA BAND



Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.

# LIKELIHOOD

Divide full data stream into  $N_d$  segments, each lasting  $T_{\text{obs}}/N_d$ .

$$T_{\text{obs}}/N_d \sim 11.5 \text{ days} \mapsto \Delta f \sim 10^{-6} \text{ Hz}$$

Assumption 4:  $T_{\text{obs}} = 4 \text{ years}$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality.

# LIKELIHOOD

**Divide full data stream into  $N_d$  segments, each lasting  $T_{\text{obs}}/N_d$ .**

**Generate  $N_d$  Gaussian realizations of noise and data.**

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality, 4 years data.

# LIKELIHOOD

**Divide full data stream into  $N_d$  segments, each lasting  $T_{\text{obs}}/N_d$ .**

**Generate  $N_d$  Gaussian realizations of noise and data.**

**Average over realizations**  $\bar{D}_{ij}^k \equiv \sum_{s=1}^{N_d} \tilde{d}_i^s(f_k) \tilde{d}_j^s(f_k) / N_d$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality, 4 years data.



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**Down sample data by coarse-graining**  $\bar{D}_{ij}^k \mapsto D_{ij}^k$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality, 4 years data.

# LIKELIHOOD

$$\ln \mathcal{L}(\vec{\theta}) = \frac{1}{3} \ln \mathcal{L}_G(\vec{\theta} | D_{ij}^k) + \frac{2}{3} \ln \mathcal{L}_{LN}(\vec{\theta} | D_{ij}^k)$$

$$\ln \mathcal{L}_G(\vec{\theta} | D_{ij}^k) = -\frac{N_d}{2} \sum_k \sum_{i,j} w_{ij}^k \left[ 1 - D_{ij}^k / D_{ij}^{Th}(f_{ij}^k, \vec{\theta}) \right]^2$$

$$\ln \mathcal{L}_{LN}(\vec{\theta} | D_{ij}^k) = -\frac{N_d}{2} \sum_k \sum_{i,j} w_{ij}^k \ln^2 \left[ D_{ij}^{Th}(f_{ij}^k, \vec{\theta}) / D_{ij}^k \right]$$

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$$\vec{\theta} = \{A, P, \Omega_{gal}, \Omega_{ext}, \vec{\theta}_{Inf}\}$$

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality, 4 years data.

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$$\vec{\theta} = \{A, P, \Omega_{\text{gal}}, \Omega_{\text{ext}}, \vec{\theta}_{\text{Inf}}\}$$


**Assumption 5: Gaussian priors on noise and foregrounds**

Assumptions: only stochastic, equilateral geometry, Gaussianity, stationarity, isotropy, no chirality, 4 years data.

# **TEMPLATE BANK FOR INFLATIONARY MODELS & FORECASTS**

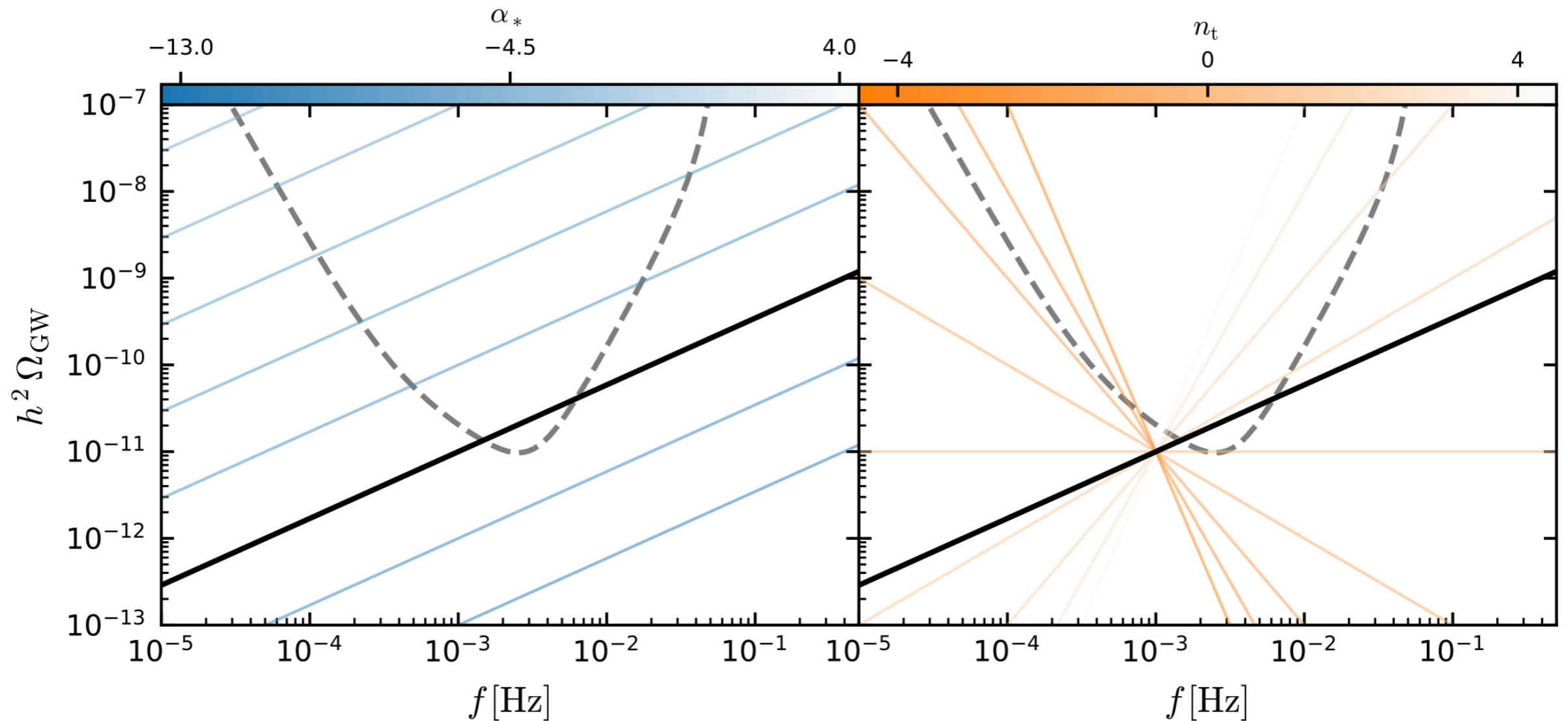
# **TEMPLATE BANK FOR INFLATIONARY MODELS & FORECASTS**

- 1. POWER LAW**
- 2. LOGNORMAL BUMP**
- 3. BROKEN POWER LAW**
- 4. DOUBLE PEAK**
- 5. EXCITED STATES**
- 6. LINEAR OSCILLATIONS**
- 7. LOGARITHMIC OSCILLATIONS**

# POWER LAW

# TEMPLATE DEFINITION

$$h^2 \Omega_{\text{GW}}^{\text{PL}}(f, \vec{p}) = 10^{\alpha_*} \left( \frac{f}{f_*} \right)^{n_t} \quad \vec{p} = \{\alpha_*, f_*, n_t\}$$





# INFLATIONARY MODELS PRODUCING THE SIGNAL

## Axion inflation:

Barnaby & Peloso 1011.1500, Sorbo 1101.1525

The inflaton is an axion coupled to a gauge field through an axial interaction. The rolling axion strongly amplifies the gauge field, which in turn produces a strong SGWB.

$$h^2 \Omega_* \simeq 1.5 \times 10^{-13} \frac{H_*^4}{M_{\text{Pl}}^4} \frac{e^{4\pi\xi_*}}{\xi_*^6}$$

$$n_t \simeq -4\epsilon_* + (4\pi\xi_* - 6)(\epsilon_* - \eta_*)$$

## Broken space diffeomorphisms:

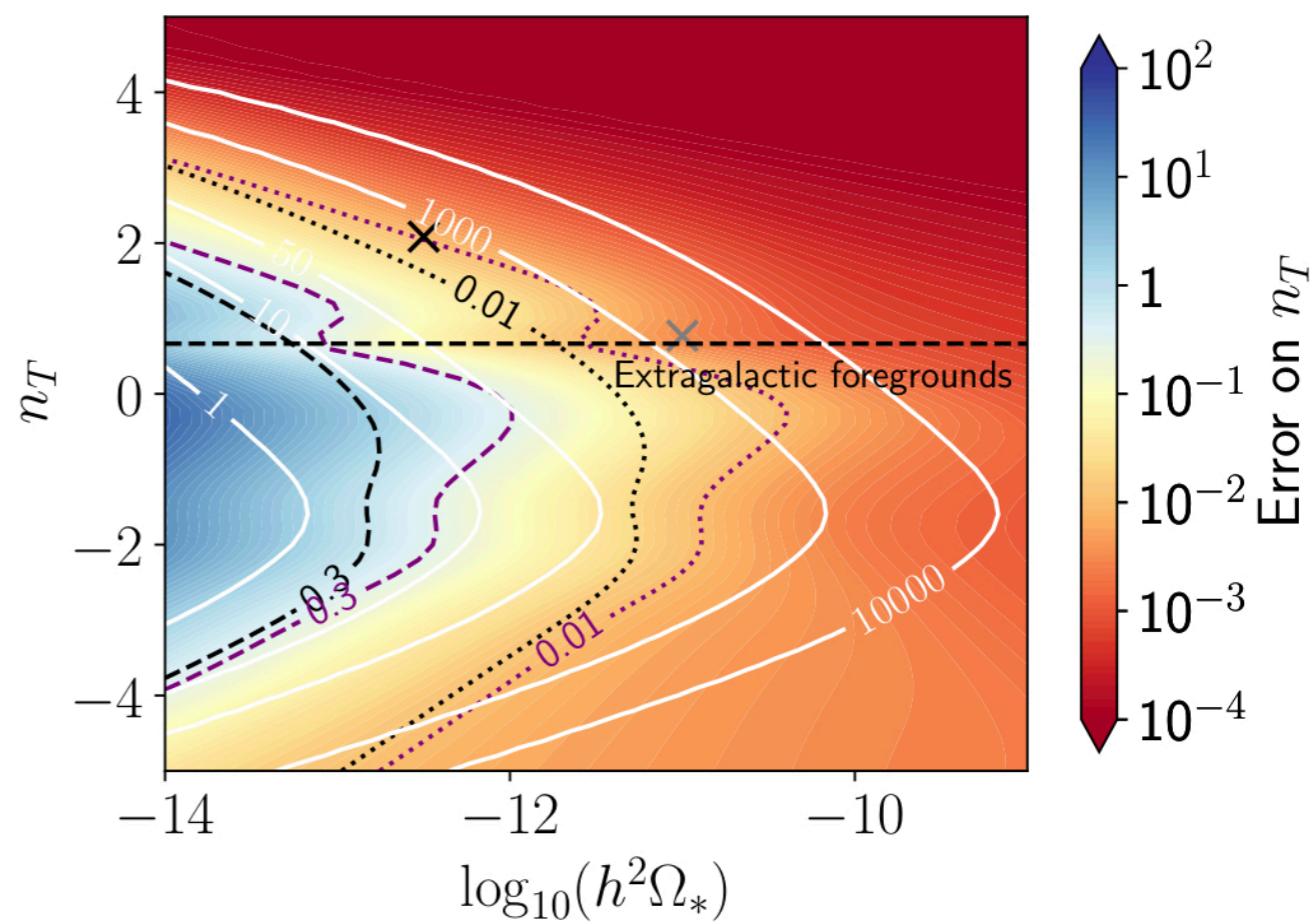
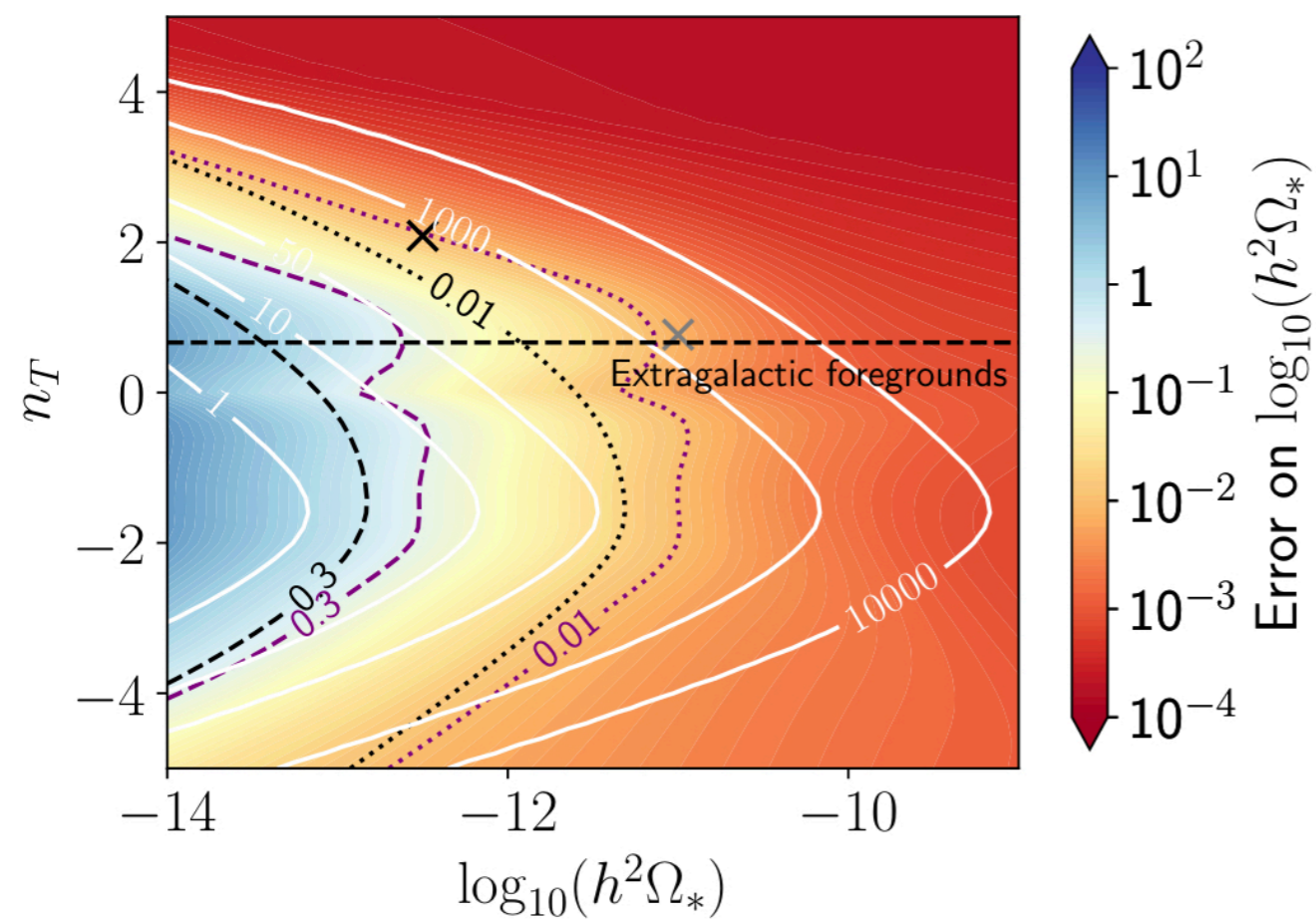
Ricciardone & Tasinato 1611.04516, 1711.02635, Fujita et al 1808.02381

The breaking of space diffeomorphisms can give rise to a massive graviton during the inflationary epoch which tilts the SGWB spectrum towards the blue

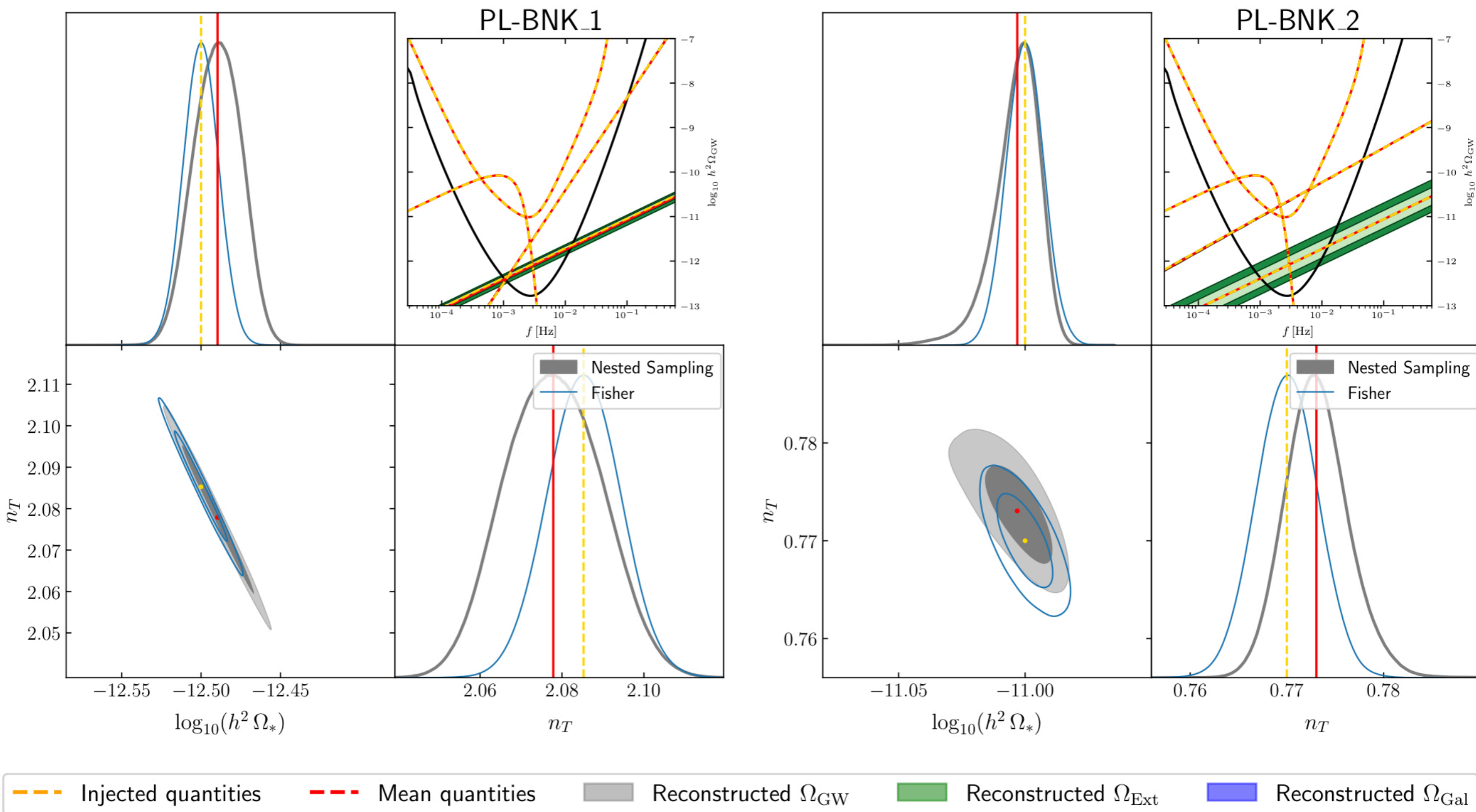
$$h^2 \Omega_* = \text{Model dependent}$$

$$n_t \simeq \frac{2}{3} \frac{m_h^2}{H_*^2} > 0$$

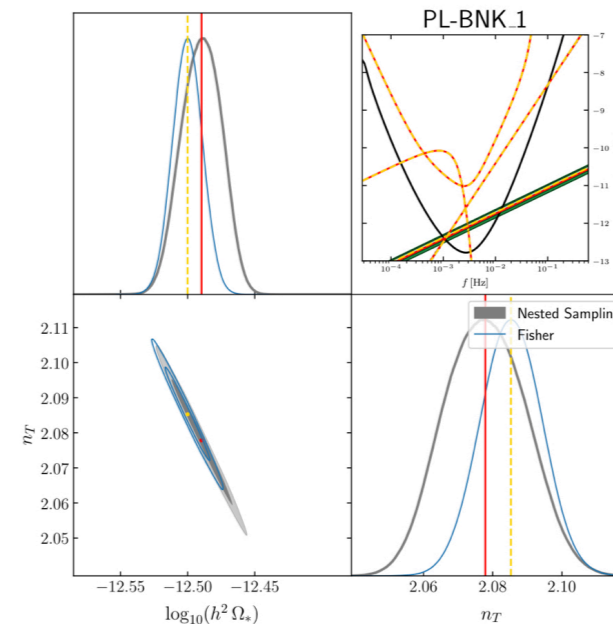
# FORECASTS (FISHER)



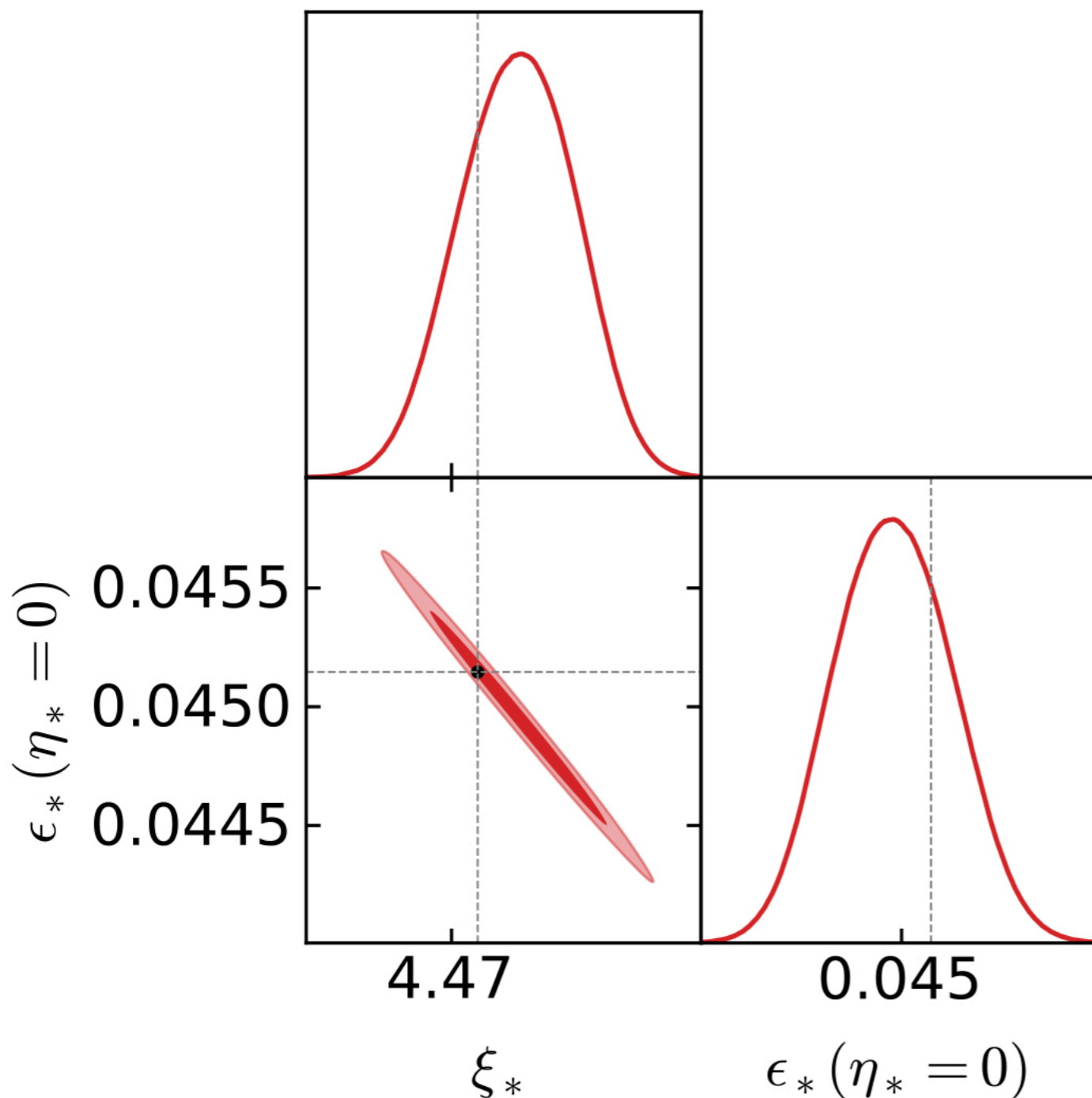
# FORECASTS (NESTED SAMPLING)



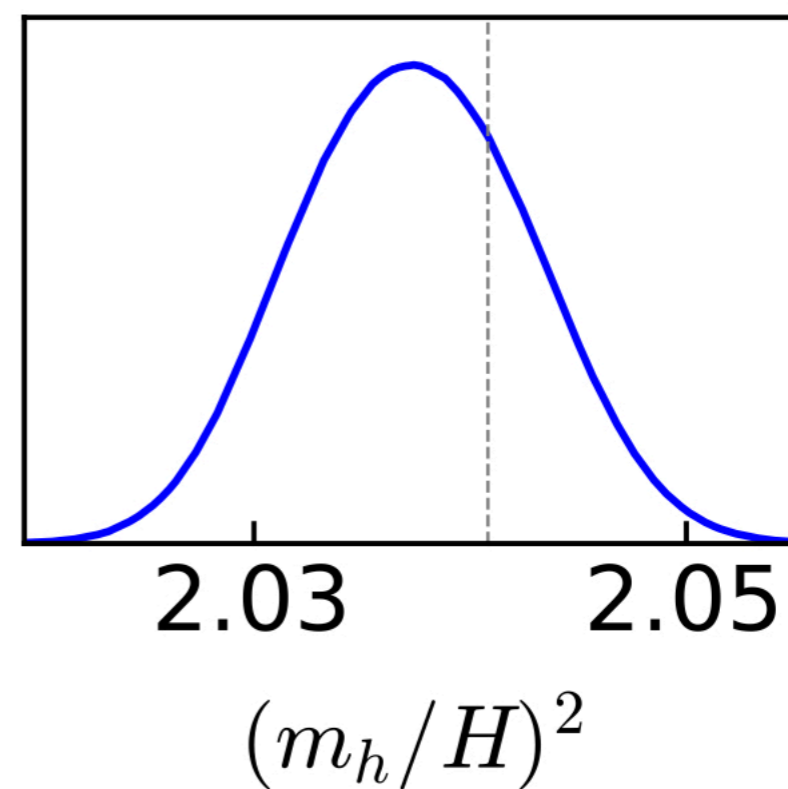
# FORECASTS (BNK 1)



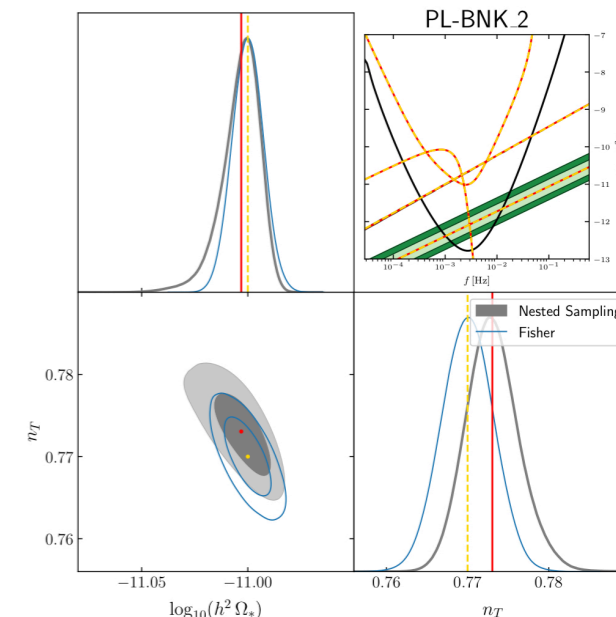
Ax Inf, PL-BNK\_1



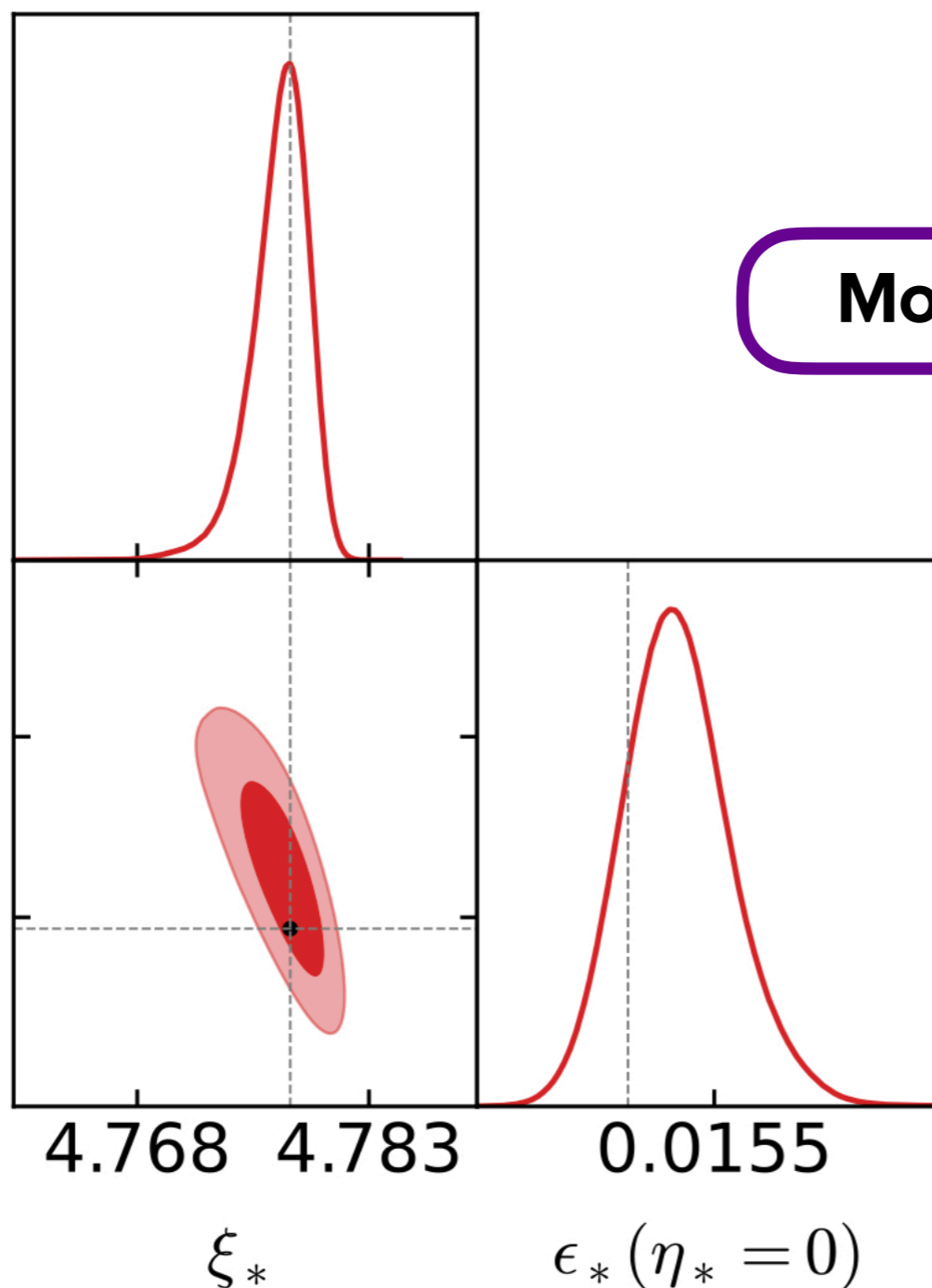
Massive Grav. PL-BNK\_1



# FORECASTS (BNK 2)



Ax Inf, PL-BNK\_2



$\epsilon_*(\eta_* = 0)$

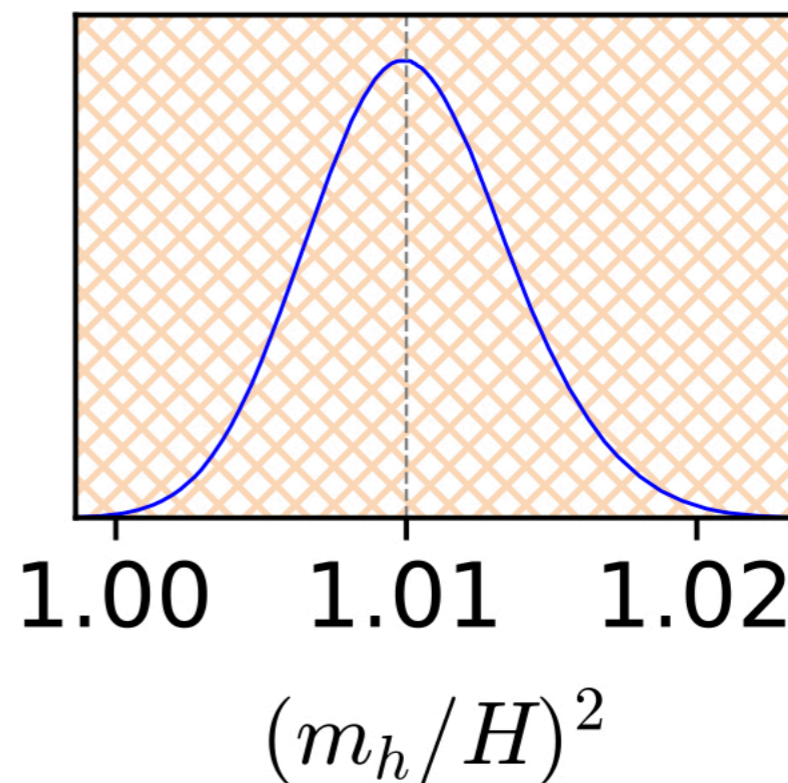
$\xi_*$

$\epsilon_*(\eta_* = 0)$

**Model 2 is excluded**



Massive Grav. PL-BNK\_2

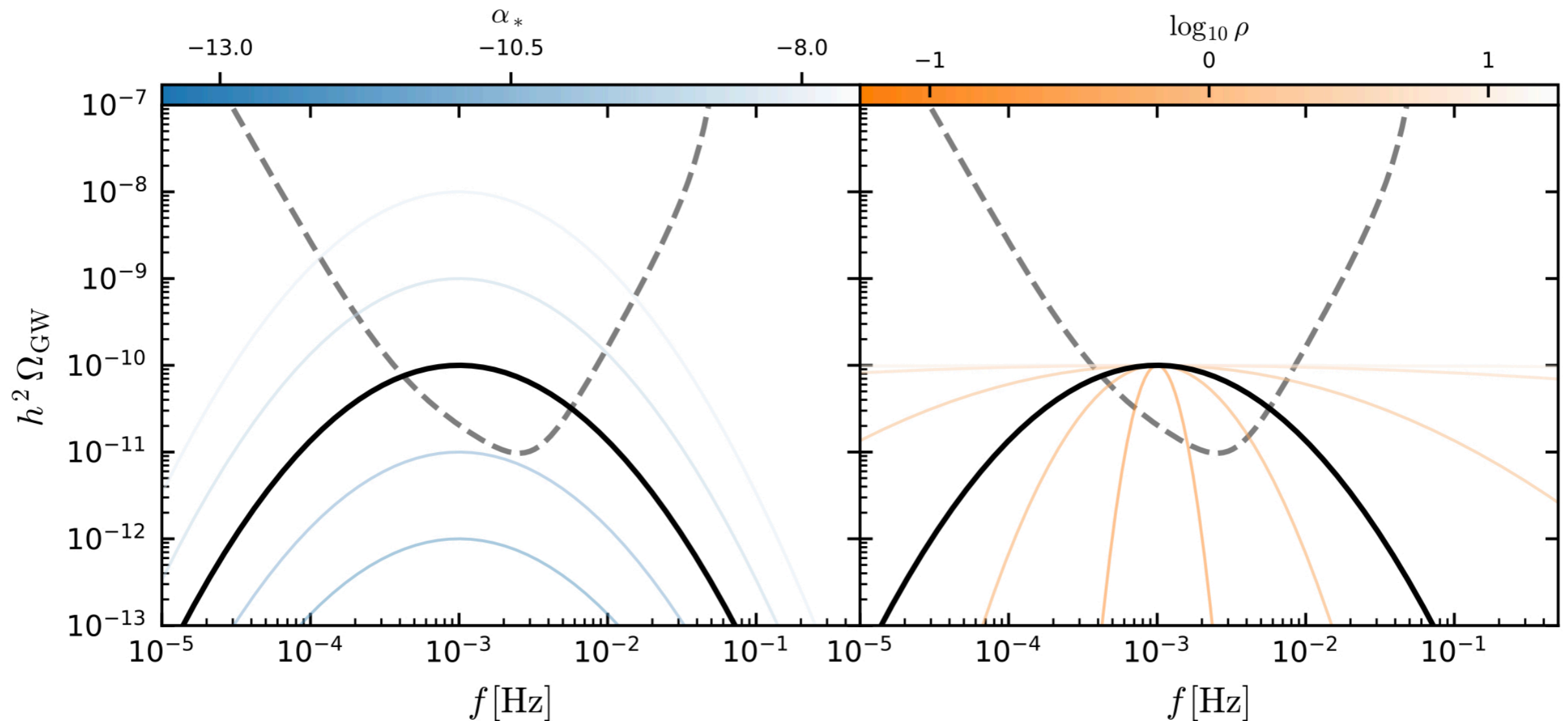


$(m_h/H)^2$

**LOGNORMAL BUMP**

# TEMPLATE DEFINITION

$$h^2 \Omega_{\text{GW}}^{\text{LBp}}(f, \vec{p}) = 10^{\alpha_*} \exp \left[ -\frac{1}{2\rho^2} \log_{10}^2 \left( \frac{f}{f_*} \right) \right] \quad \vec{p} = \{ \alpha_*, f_*, \rho \}$$



# INFLATIONARY MODELS PRODUCING THE SIGNAL

## Axion spectator:

Namba et al 1509.07521

The axion coupled to a gauge field is now a spectator field rolling only for a short time  $\Delta N$ .

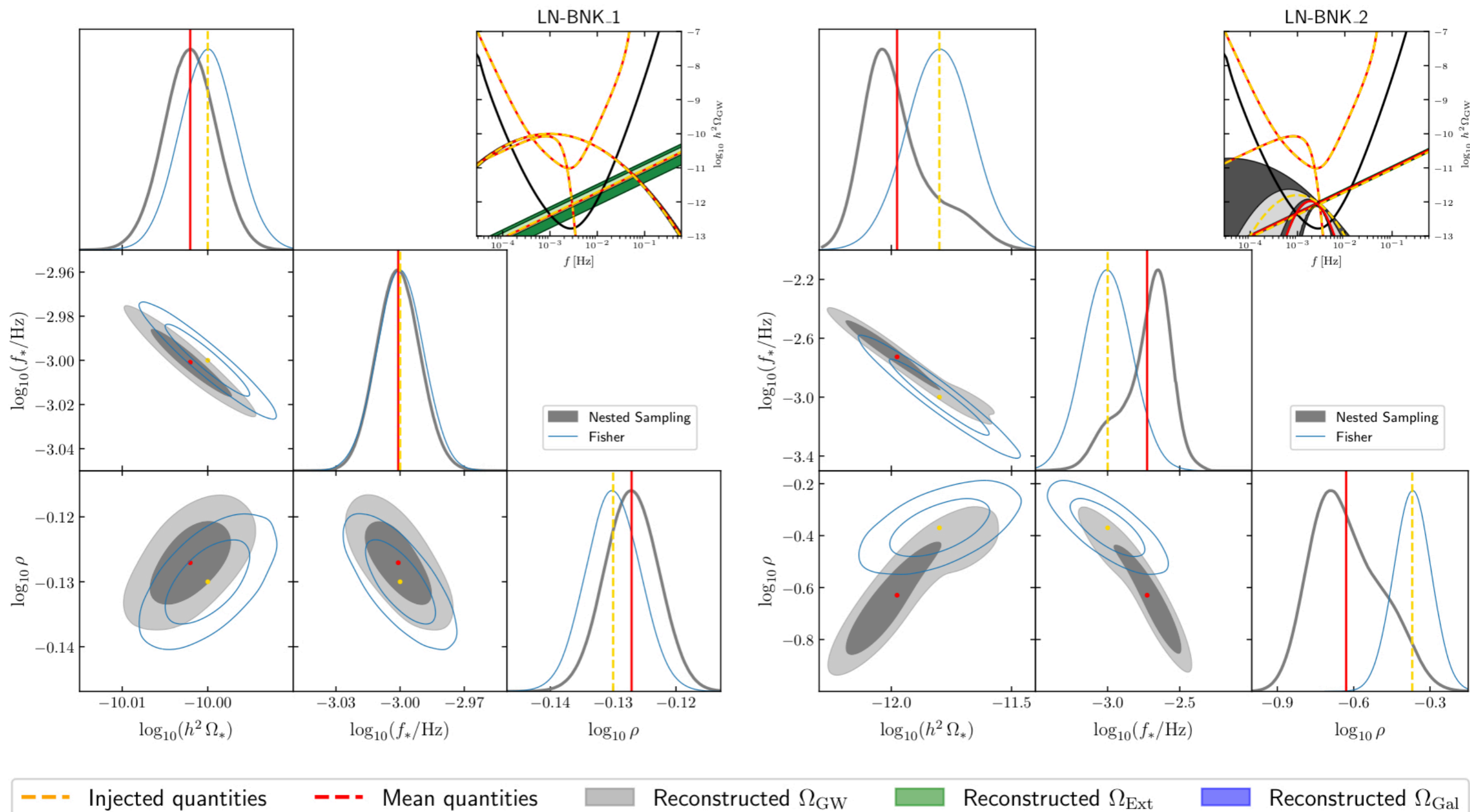
$$h^2 \Omega_* \simeq 1.5 \times 10^{-13} \frac{H_*^4}{M_{\text{Pl}}^4} \frac{e^{4\pi\xi_*}}{\xi_*^6}$$

$$\rho \propto \Delta N$$

$$f_* \propto a(N_*)H(N_*)$$



# FORECASTS (NESTED SAMPLING)



# INTERMEZZO: SCALAR-INDUCED GRAVITATIONAL WAVES

Large scalar perturbations source gravitational waves at 2nd order in perturbation theory when they re-enter the horizon during radiation era

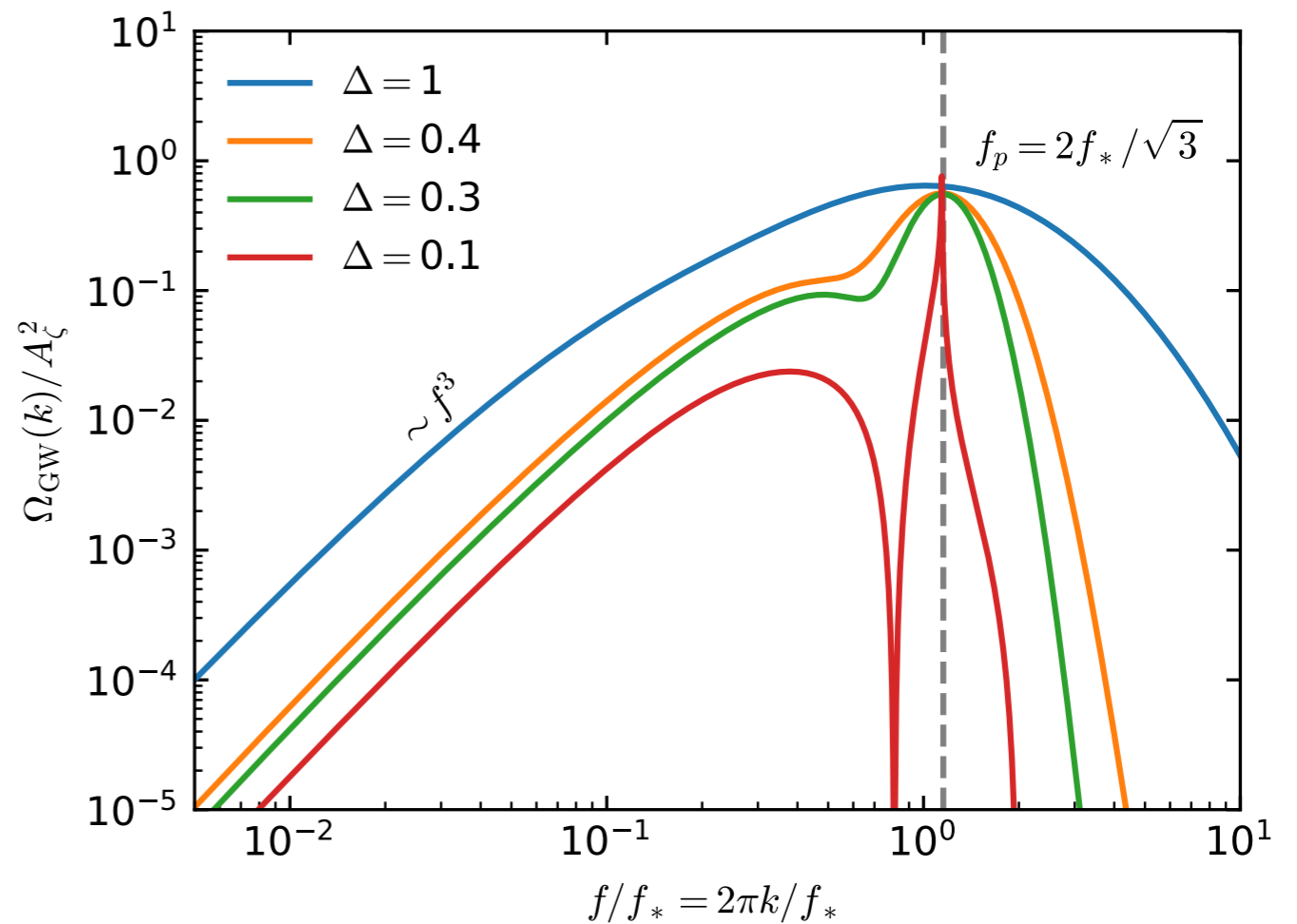
$$\Omega_{\text{ind}}(k) = 0.387 \Omega_{\text{R}} \left( \frac{g_{*,s}^4 g_*^{-3}}{106.75} \right)^{-\frac{1}{3}} \frac{1}{6} \int_{-1}^1 dx \int_1^{\infty} dy \mathcal{P} \left( \frac{y-x}{2} k \right) \mathcal{P} \left( \frac{x+y}{2} k \right) F(x, y)$$

see Robert's talk and Domenech  
2109.01398 for a review

# SIGW FROM A LOGNORMAL $\mathcal{P}(k)$

Bumps in  $\mathcal{P}(k)$  are often modeled using a Lognormal template

$$\mathcal{P}(k) = \frac{A_\zeta}{\sqrt{2\pi}\Delta} \exp\left[-\frac{\ln^2 k/k_*}{2\Delta^2}\right]$$

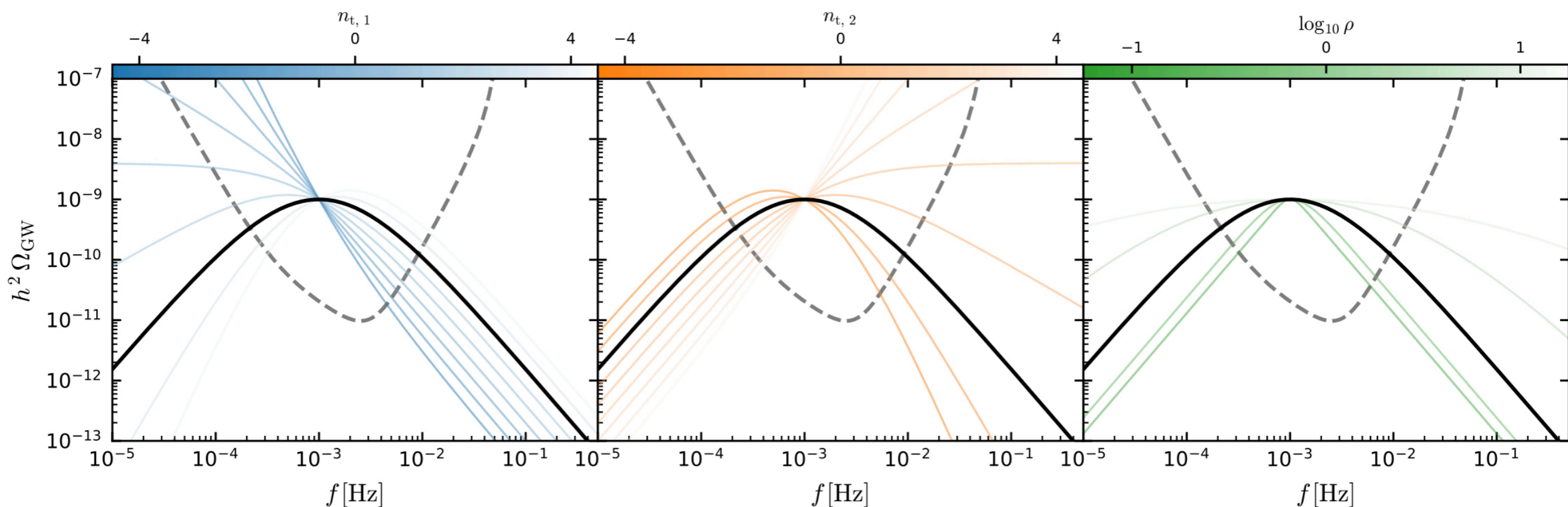


See [Pi & Sasaki 2005.12306](#) for analytical solutions for  $\Omega_{\text{GW}}(f)$

**BROKEN POWER LAW**

# TEMPLATE DEFINITION

$$h^2 \Omega_{\text{GW}}^{\text{BPL}}(f, \vec{p}) = 10^{\alpha_*} \frac{\left(\frac{f}{f_*}\right)^{n_{t,1}}}{\left\{ \frac{1}{2} \left[ 1 + \left(\frac{f}{f_*}\right)^{1/\delta} \right] \right\}^{(n_{t,1} - n_{t,2})\delta}} \quad \vec{p} = \{\alpha_*, f_*, n_{t,1}, n_{t,2}, \delta\}$$

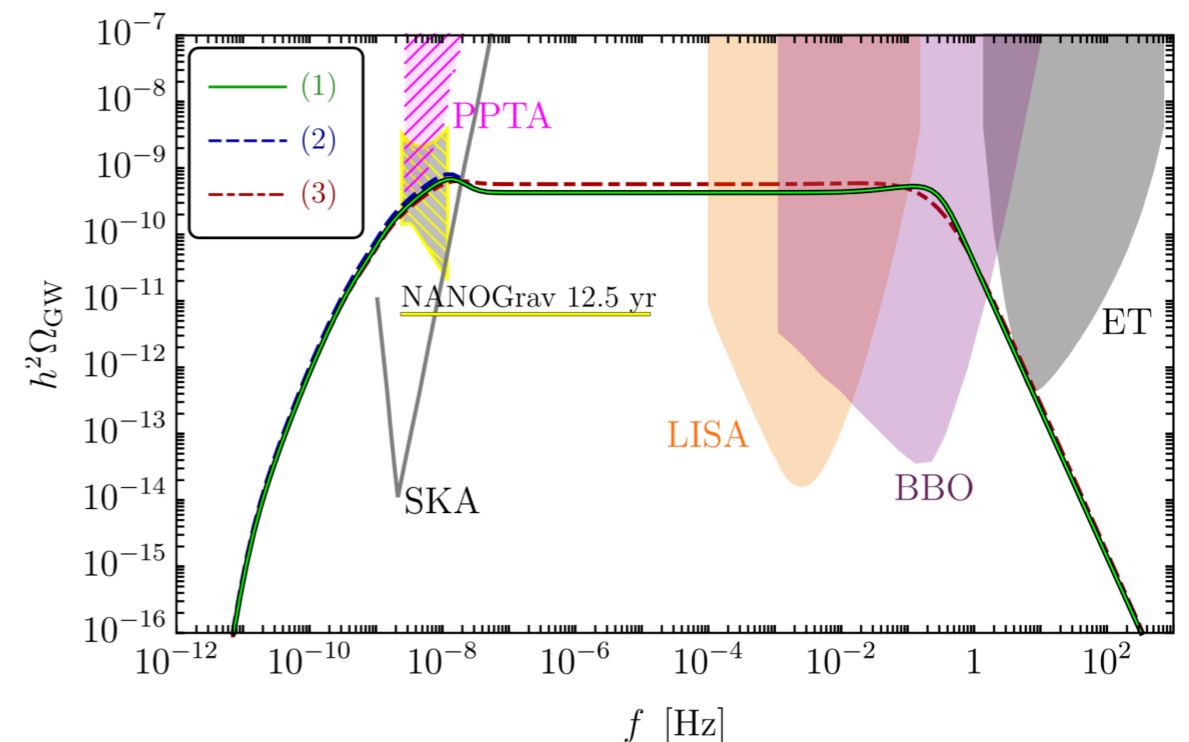
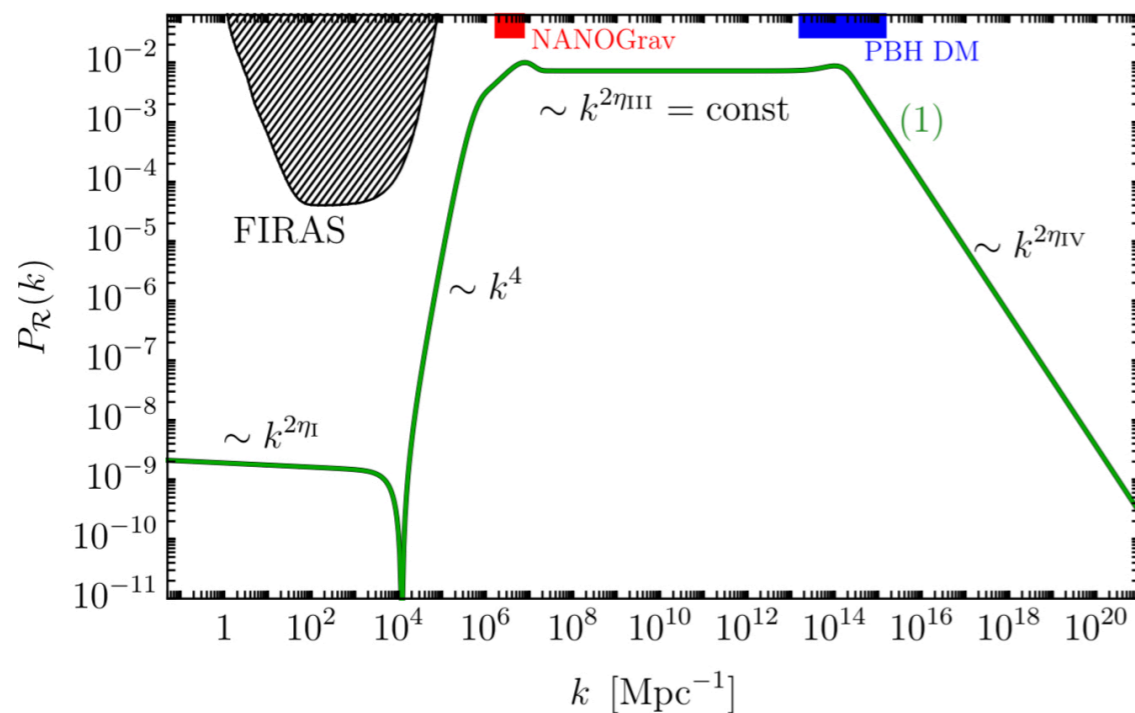
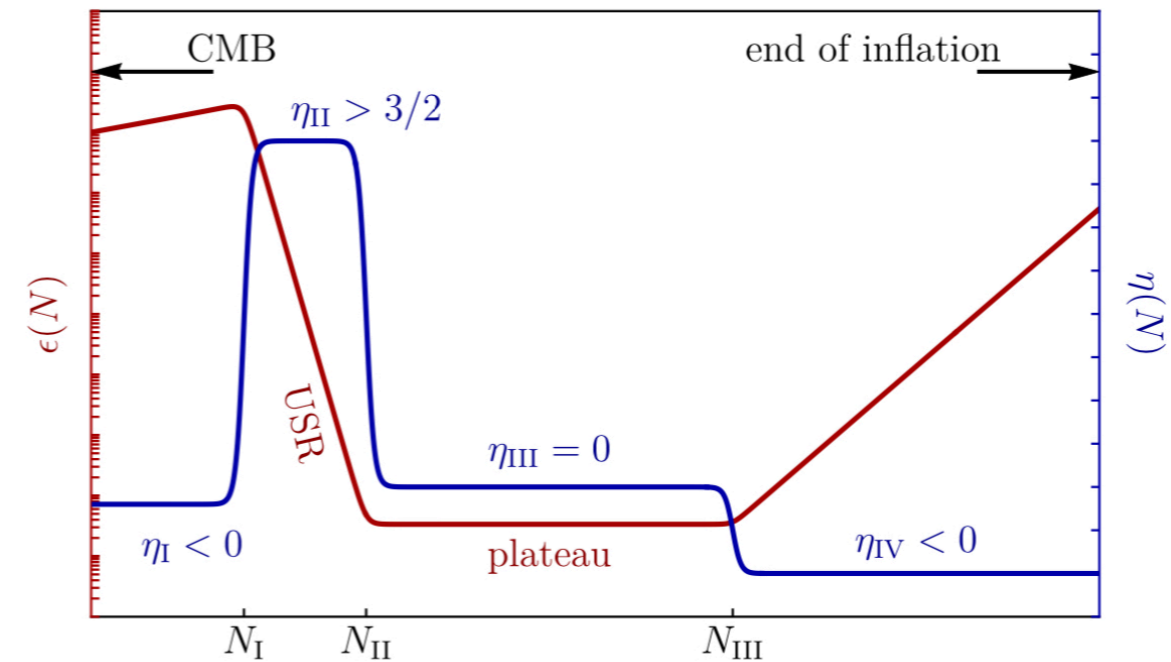


# INFLATIONARY MODELS PRODUCING THE SIGNAL MODEL 1

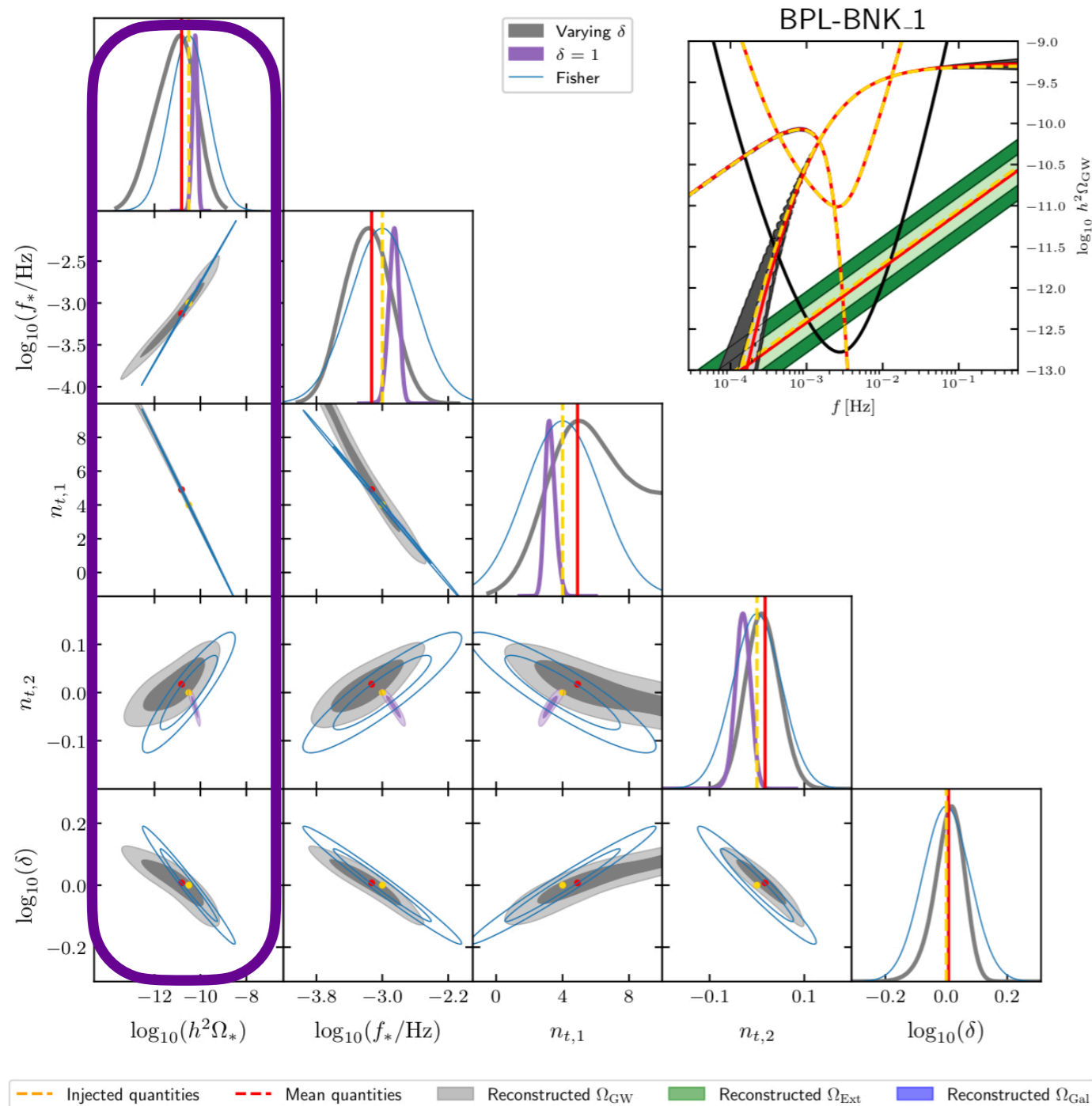
## Second slow-roll stage:

Franciolini & Urbano 2207.10056

An ultra-slow-roll phase amplifies primordial scalar perturbations and is followed by a second slow-roll regime generating a plateau.

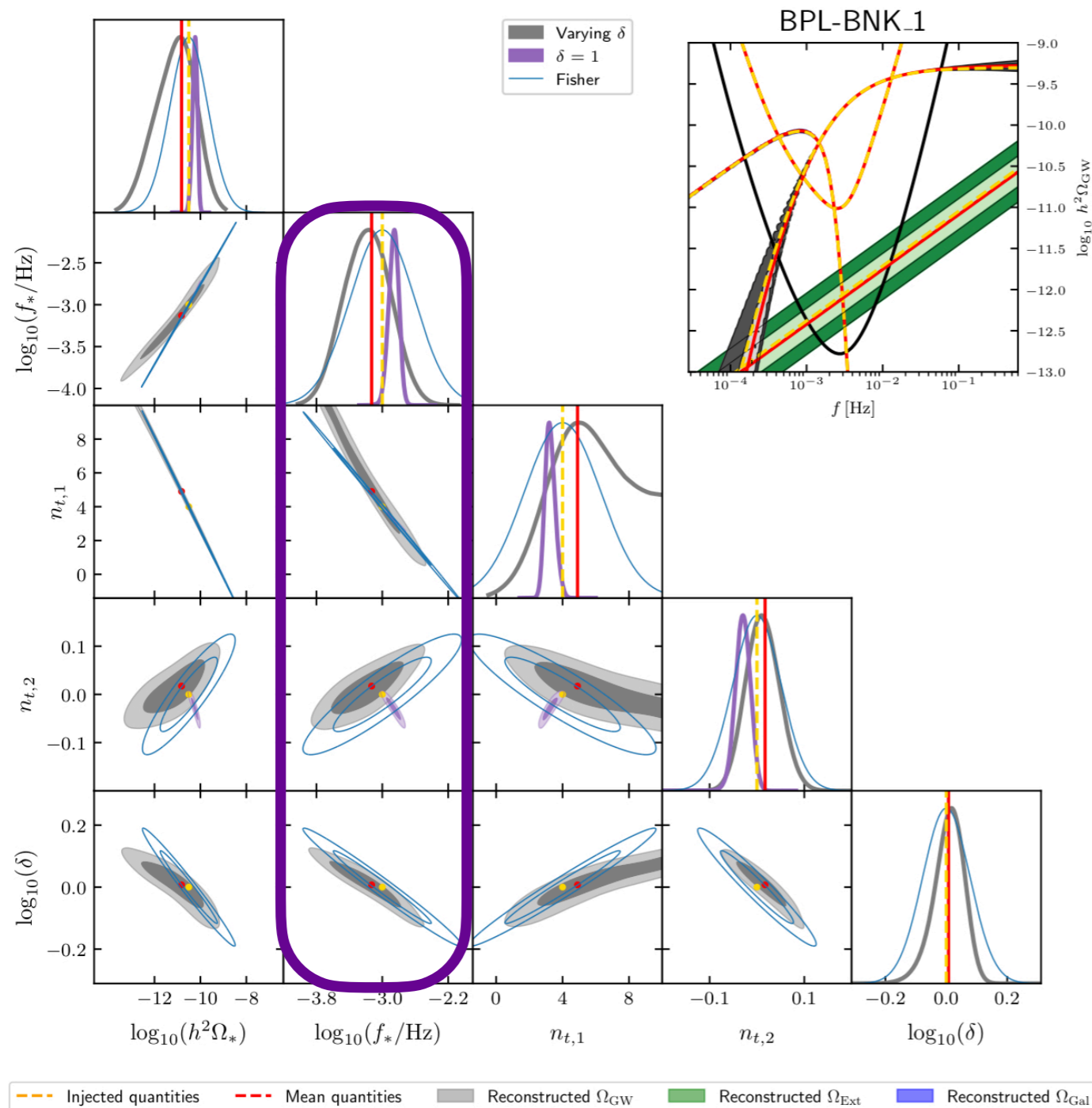


# FORECAST. MODEL 1



**Amplitude:** ratio  $H^2/\epsilon$  during the second SR stage, abundance of Primordial Black Holes  $f_{\text{PBH}}$ .

# FORECAST. MODEL 1

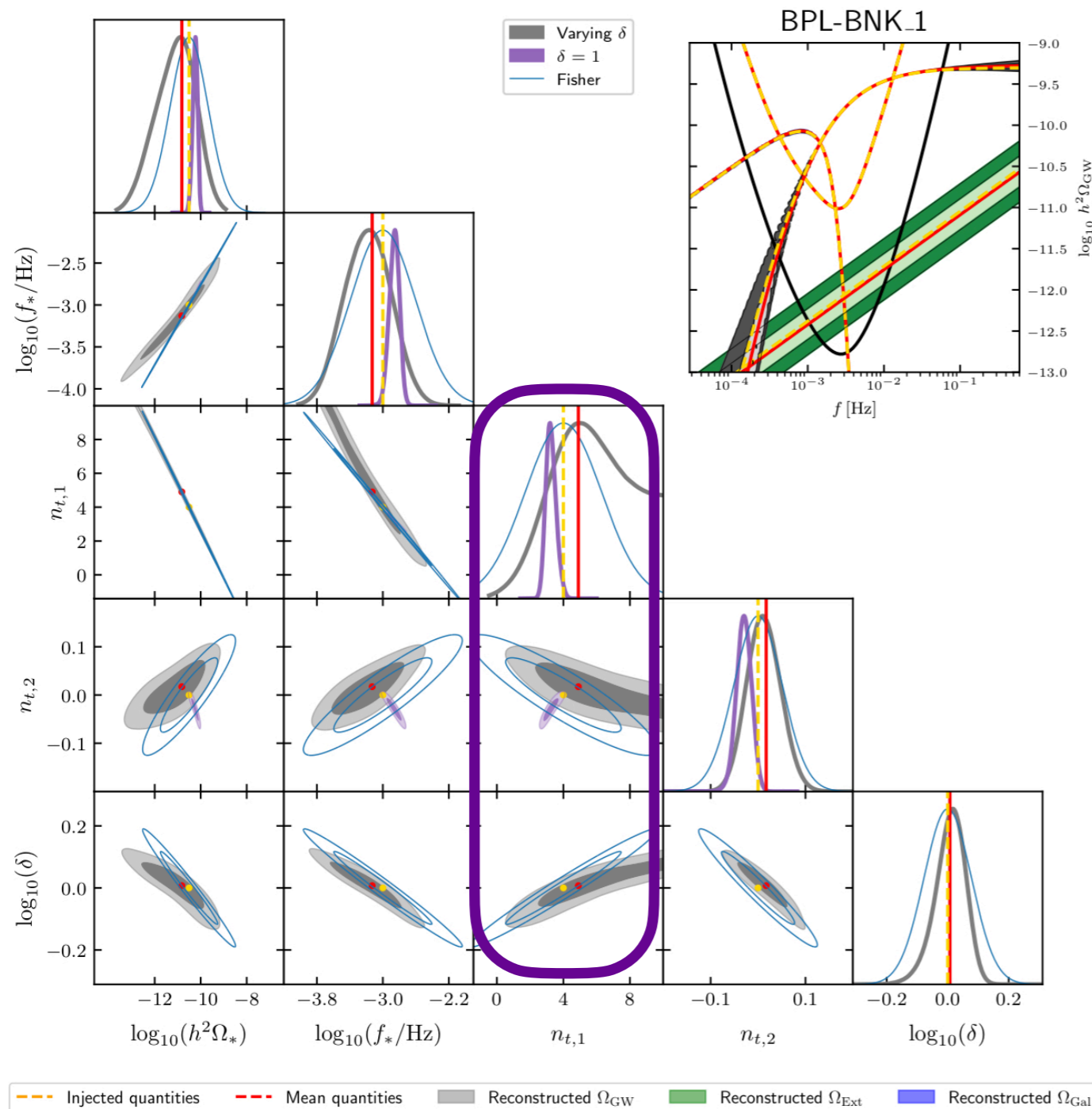


**Amplitude:** ratio  $H^2/\epsilon$  during the second SR stage, abundance of Primordial Black Holes  $f_{\text{PBH}}$ .

**Frequency of the turn:** time of the onset of the second SR stage, mass of Primordial Black Holes  $M/M_{\odot}$ .



# FORECAST. MODEL 1

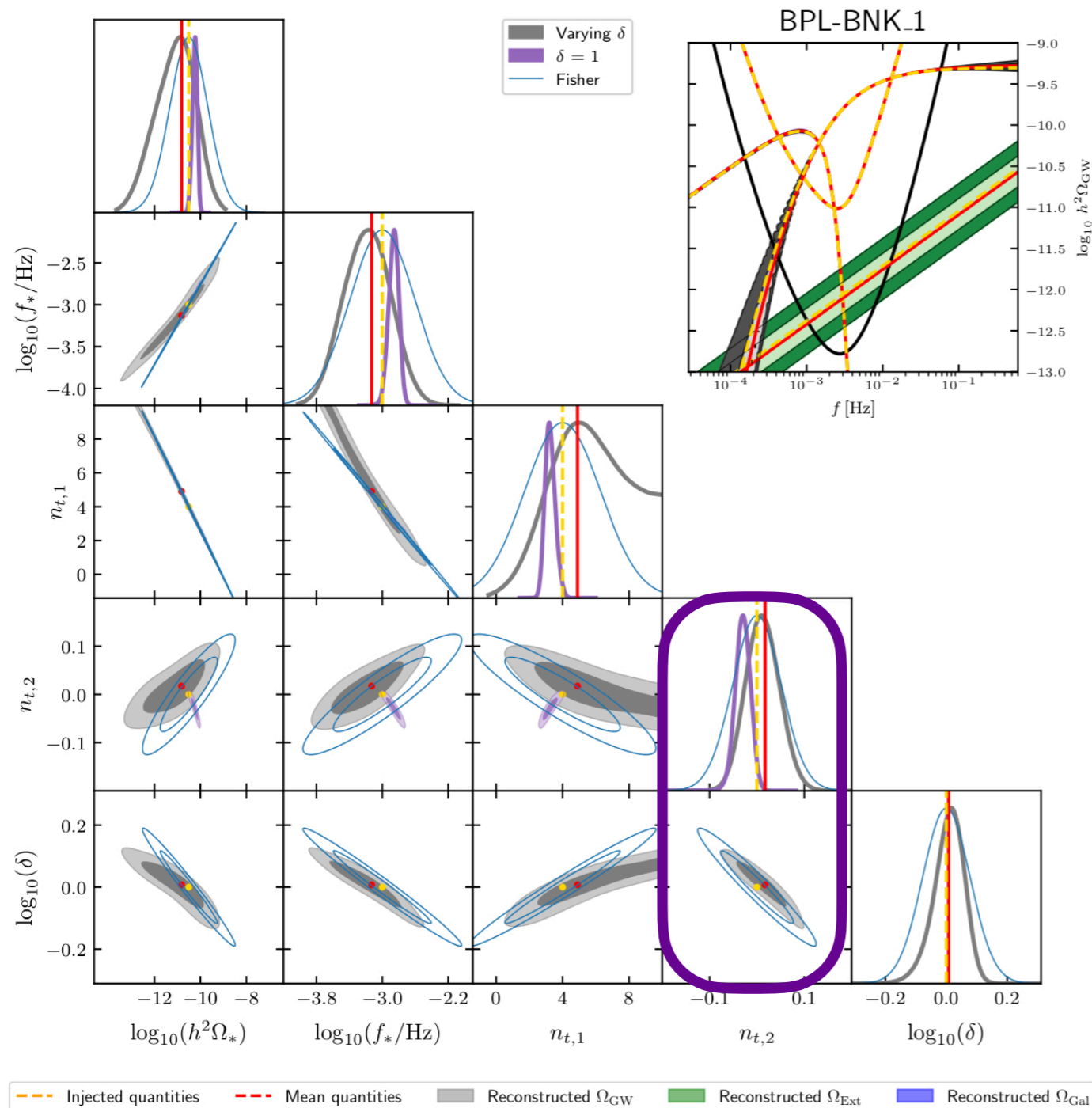


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**IR spectral index:** related to the scalar IR spectral index.

# FORECAST. MODEL 1



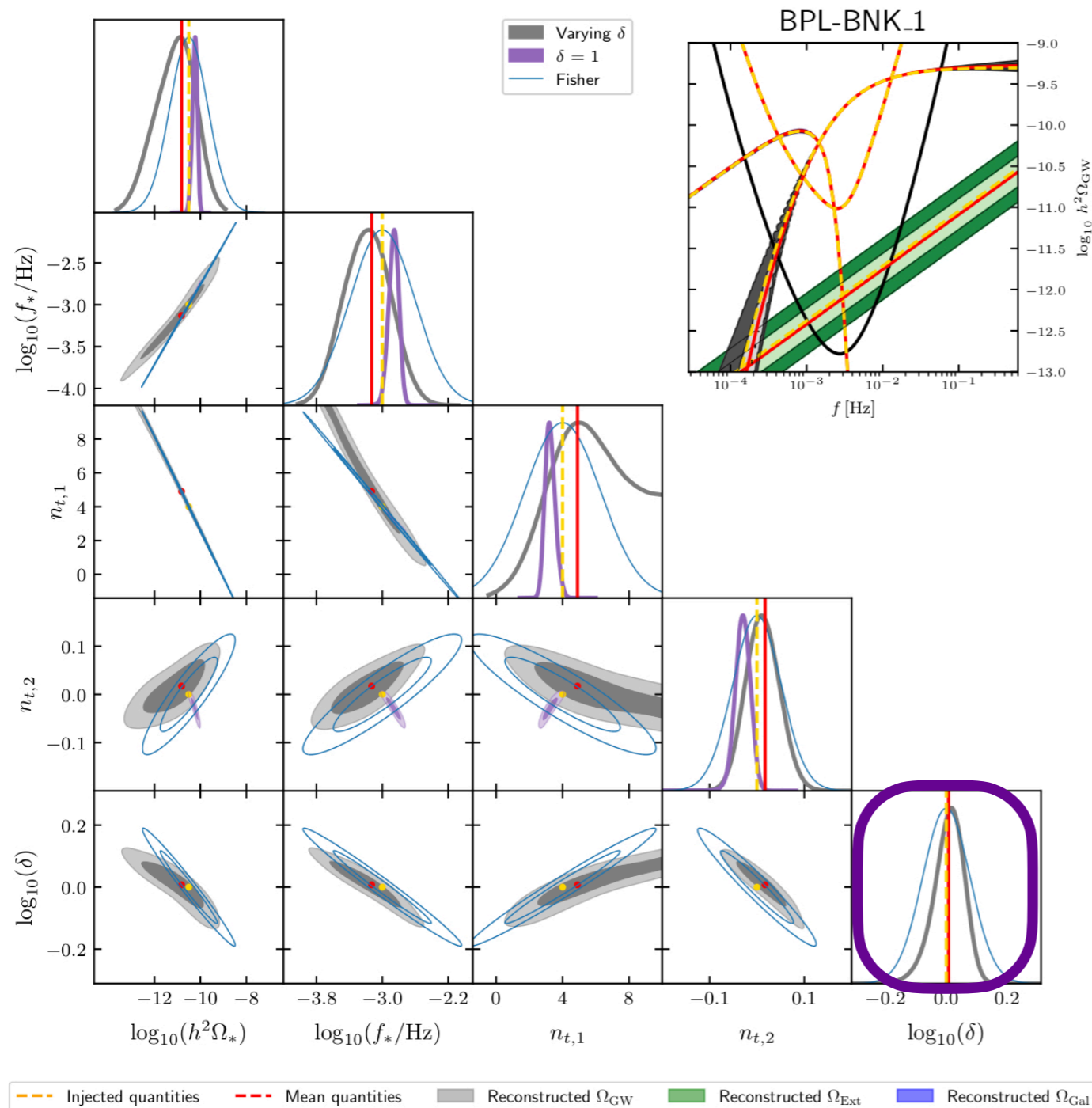
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**Frequency of the turn:** time of the onset of the second SR stage, mass of Primordial Black Holes  $M/M_{\odot}$ .

**IR spectral index:** related to the scalar IR spectral index.

**UV spectral index:** predicted to be flat in this model

# FORECAST MODEL 1



**Amplitude:** ratio  $H^2/\epsilon$  during the second SR stage, abundance of Primordial Black Holes  $f_{\text{PBH}}$ .

**Frequency of the turn:** time of the onset of the second SR stage, mass of Primordial Black Holes  $M/M_{\odot}$ .

**IR spectral index:** related to the scalar IR spectral index.

**UV spectral index:** predicted to be flat in this model.

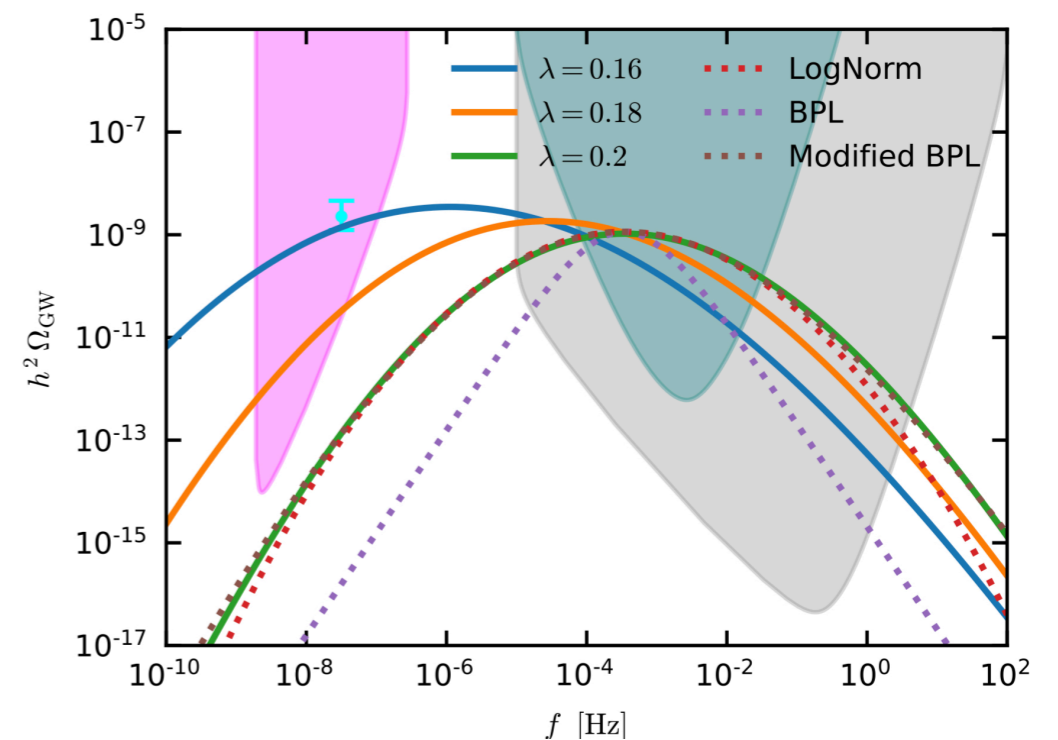
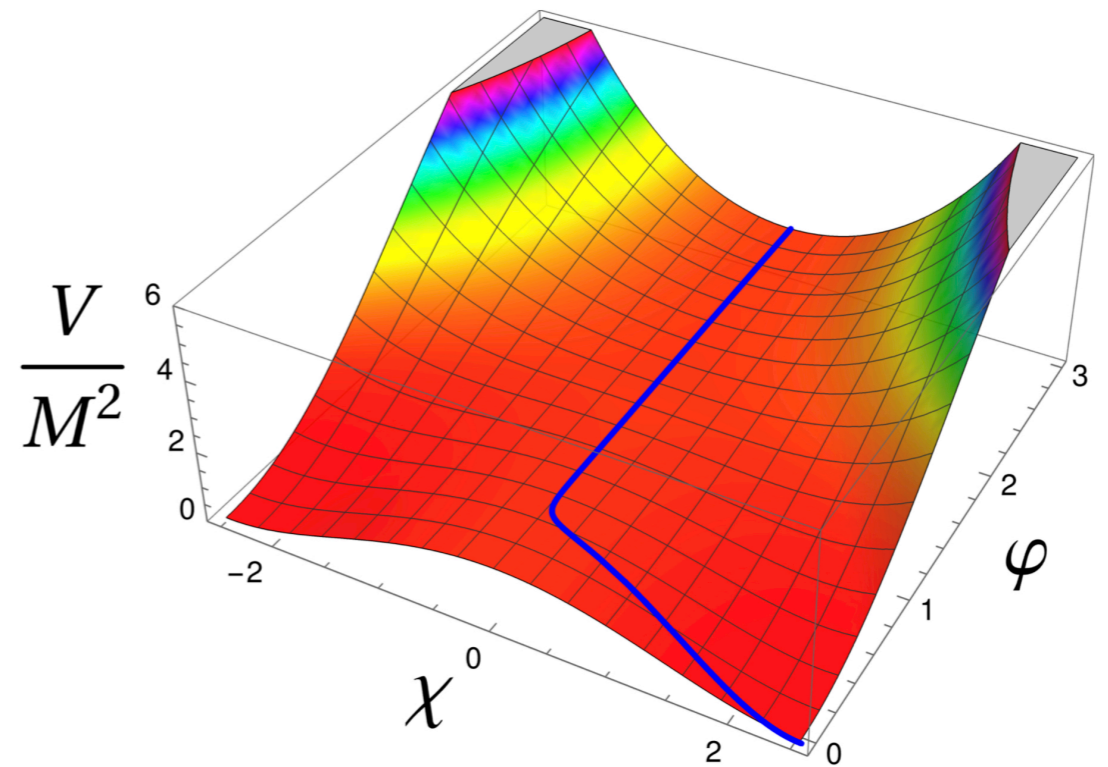
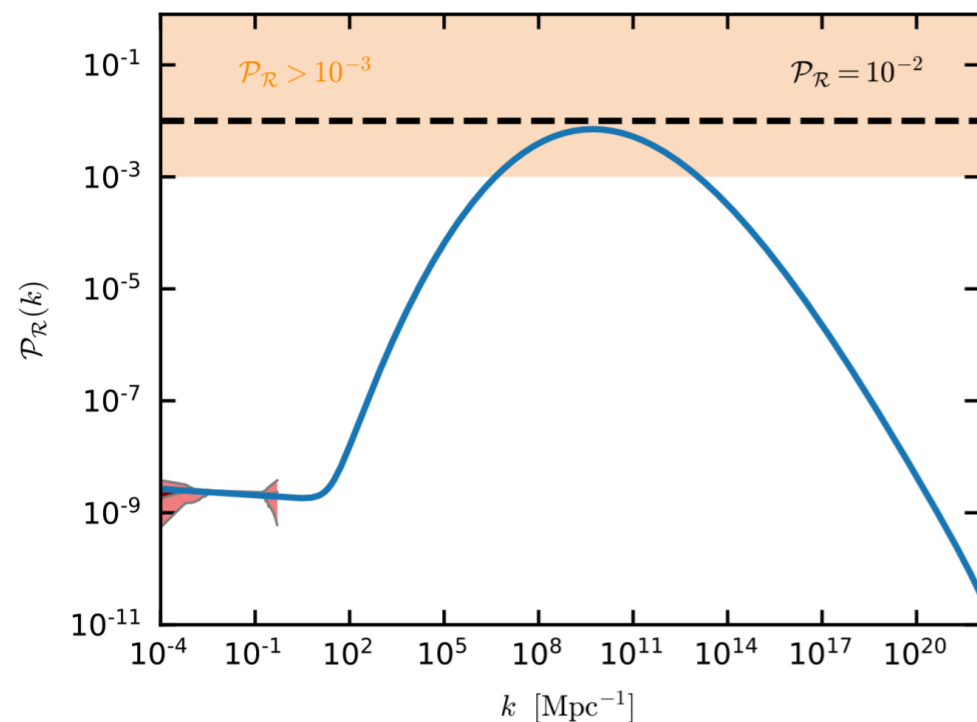
**Smoothing parameter:** related to the sharpness of the USR-SR transition.

# INFLATIONARY MODELS PRODUCING THE SIGNAL. MODEL 2

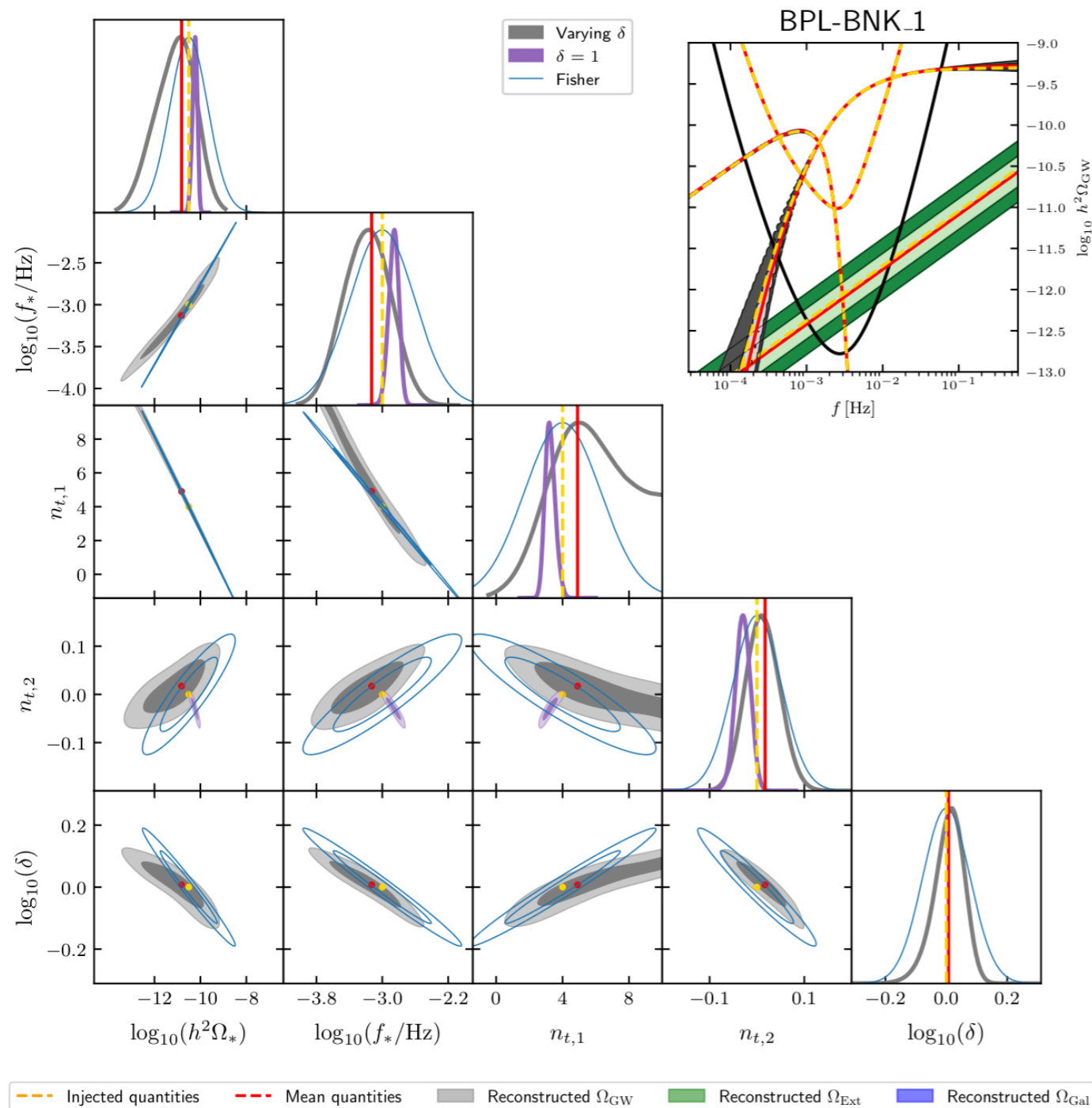
## Hybrid inflation:

Braglia, Linde, Kallosh, Finelli 2211.14262

Primordial scalar perturbations are sourced by tachyonically amplified isocurvature perturbations during a waterfall phase producing a very broad bump.



# FORECAST. MODEL 2



The signal is **well reconstructed**, but affected by **strong degeneracies**. The parameters are not well constrained.

**Suboptimal parameterization.**

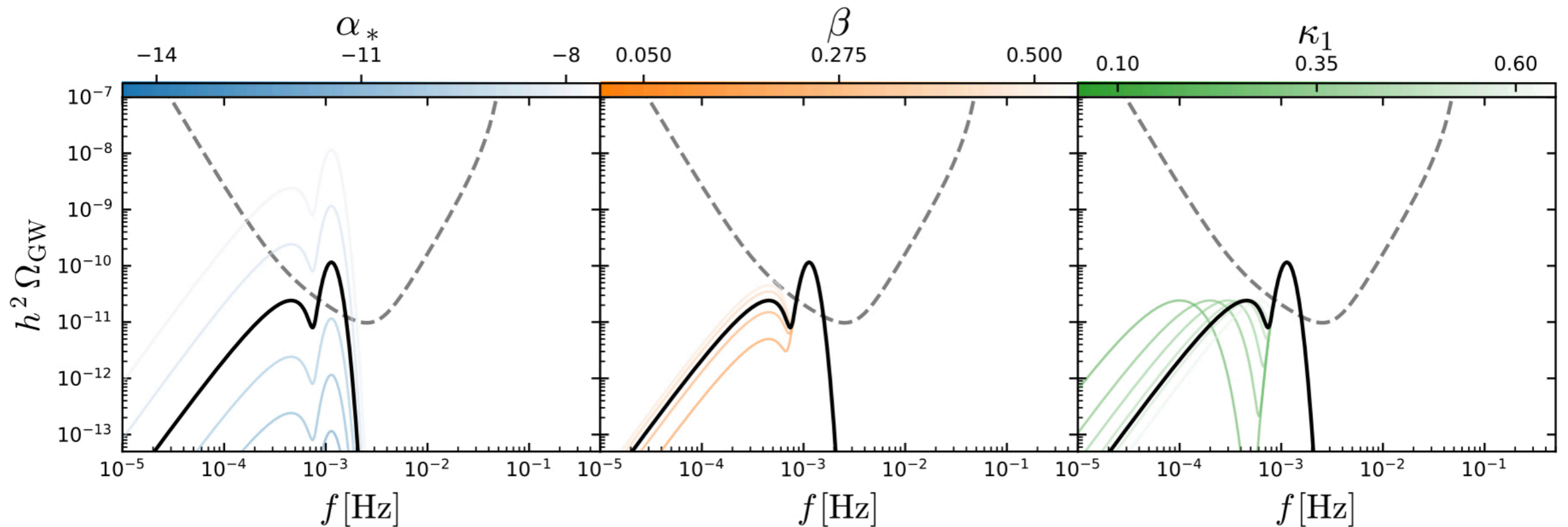
LISA **cannot measure** moderately **asymmetries** in very broad bumps, due to its limited bandwidth.

**DOUBLE PEAK**

# TEMPLATE DEFINITION

$$\vec{p} = \{\alpha_*, f_*, \beta, \kappa_1, \kappa_2, \rho, \gamma\}$$

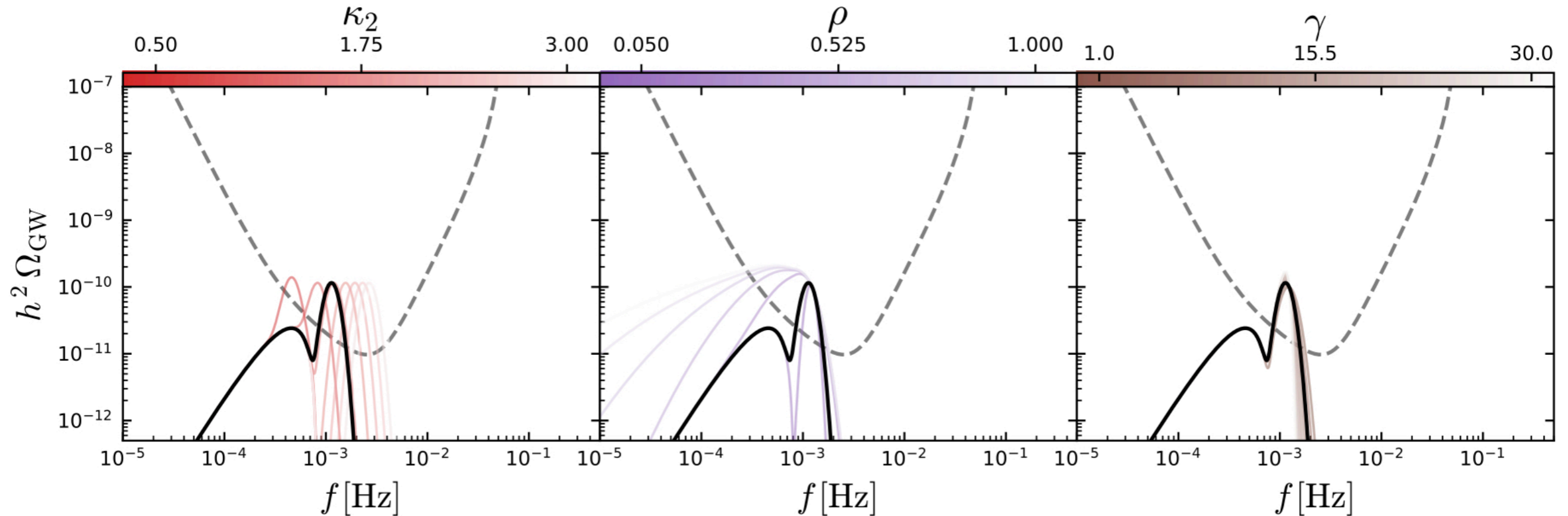
$$h^2 \Omega_{\text{GW}}^{\text{DP}}(f, \vec{p}) = 10^{\alpha_*} \left[ \beta \left( \frac{f}{\kappa_1 f_*} \right)^{n_p} \left[ \frac{c_1 - f/f_*}{c_1 - \kappa_1} \right]^{\frac{n_p}{\kappa_1} (c_1 - \kappa_1)} \Theta \left( c_1 - \frac{f}{f_*} \right) + \exp \left[ -\frac{1}{2\rho^2} \log_{10}^2 \left( \frac{f}{\kappa_2 f_*} \right) \right] \left\{ 1 + \text{erf} \left[ -\gamma \log_{10} \left( \frac{f}{\kappa_2 f_*} \right) \right] \right\} \right]$$



# TEMPLATE DEFINITION

$$\vec{p} = \{\alpha_*, f_*, \beta, \kappa_1, \kappa_2, \rho, \gamma\}$$

$$h^2 \Omega_{\text{GW}}^{\text{DP}}(f, \vec{p}) = 10^{\alpha_*} \left[ \beta \left( \frac{f}{\kappa_1 f_*} \right)^{n_p} \left[ \frac{c_1 - f/f_*}{c_1 - \kappa_1} \right]^{\frac{n_p}{\kappa_1} (c_1 - \kappa_1)} \Theta \left( c_1 - \frac{f}{f_*} \right) + \exp \left[ -\frac{1}{2\rho^2} \log_{10}^2 \left( \frac{f}{\kappa_2 f_*} \right) \right] \left\{ 1 + \text{erf} \left[ -\gamma \log_{10} \left( \frac{f}{\kappa_2 f_*} \right) \right] \right\} \right]$$





# INFLATIONARY MODELS PRODUCING THE SIGNAL

Broken power law  $\mathcal{P}_\zeta(k)$ :

$$\mathcal{P}_\zeta^{\text{bpl}}(k) = \frac{\mathcal{A}_s(p_1 + p_2)}{\left[ p_2 \left( \frac{k}{k_*} \right)^{-p_1} + p_1 \left( \frac{k}{k_*} \right)^{p_2} \right]}$$

Lognormal  $\mathcal{P}_\zeta(k)$ :

$$\mathcal{P}_\zeta^{\text{ln}}(k) = \mathcal{A}_s \exp \left[ -\frac{1}{2\Delta^2} \ln^2 \left( \frac{k}{k_*} \right) \right]$$

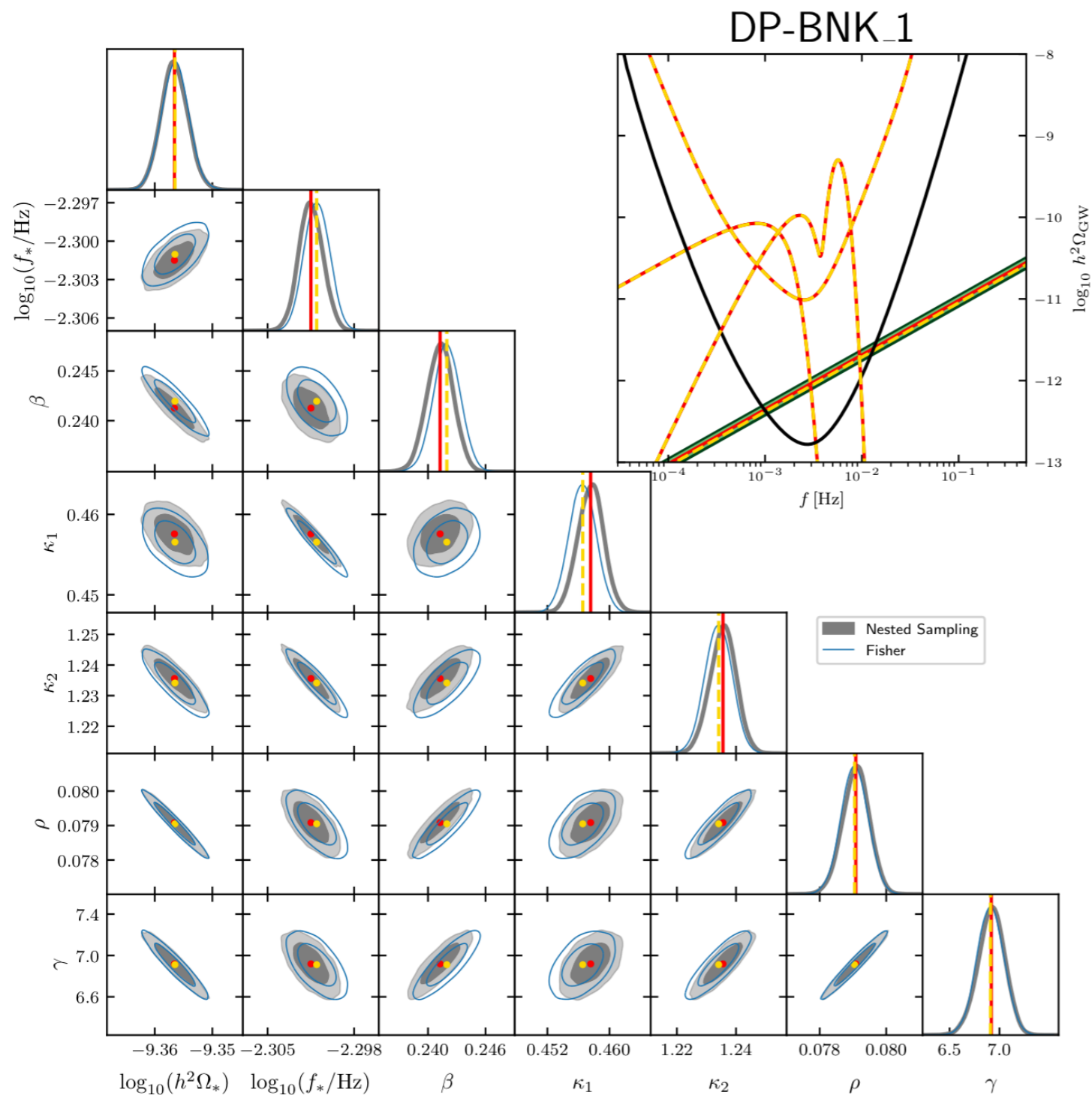
Prior on the 7 model

parameters chosen so as to reproduce the GW background from these models.

More efficient approach: start from the parameters for  $\mathcal{P}_\zeta(k)$  instead. See Robert's talk.

See Ozsoy & Tasinato 2301.03600 for a review of models producing a peak in the scalar power spectrum

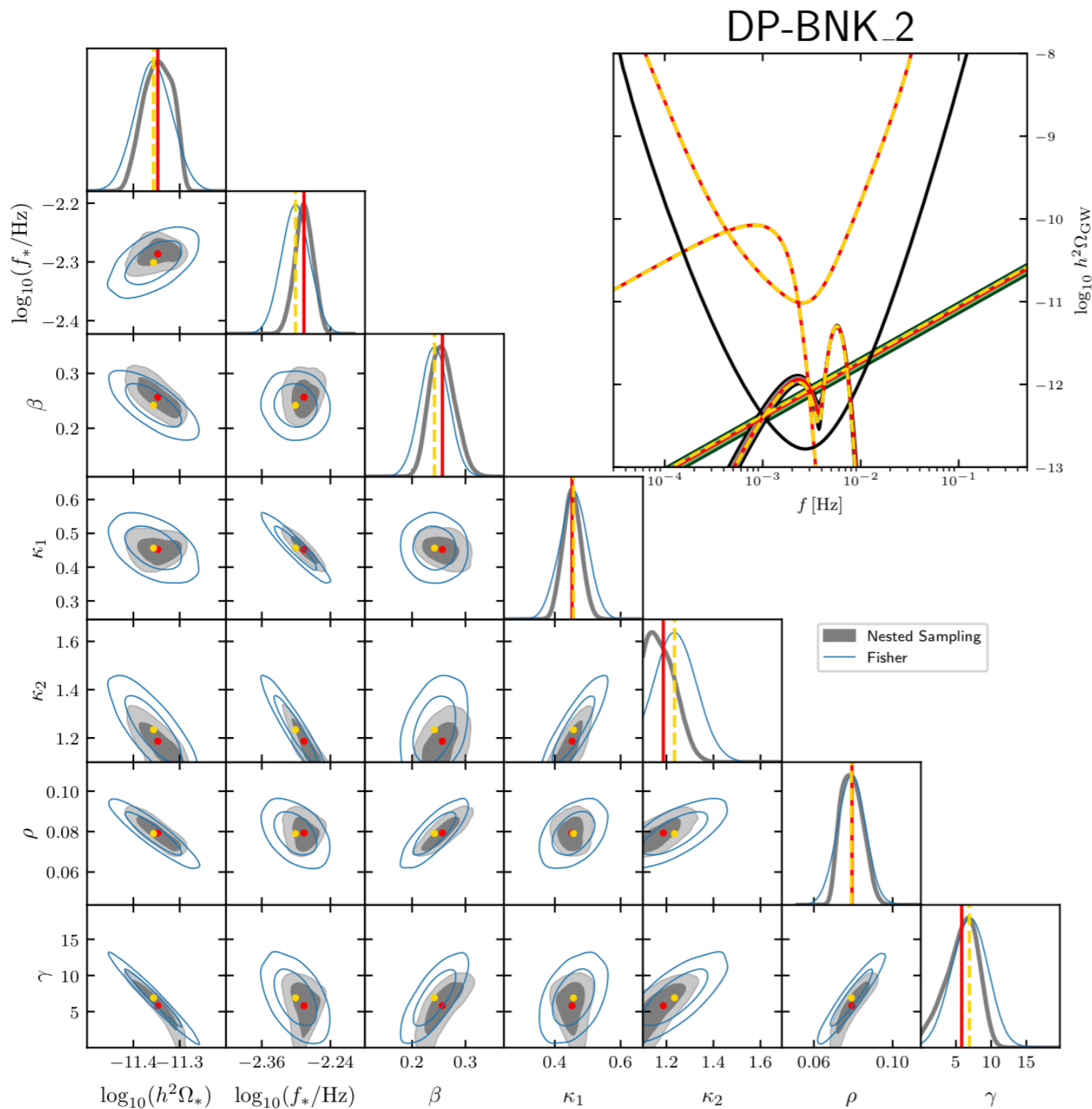
# FORECAST. BNK1



Very loud signal:

tight constraints on all parameters.

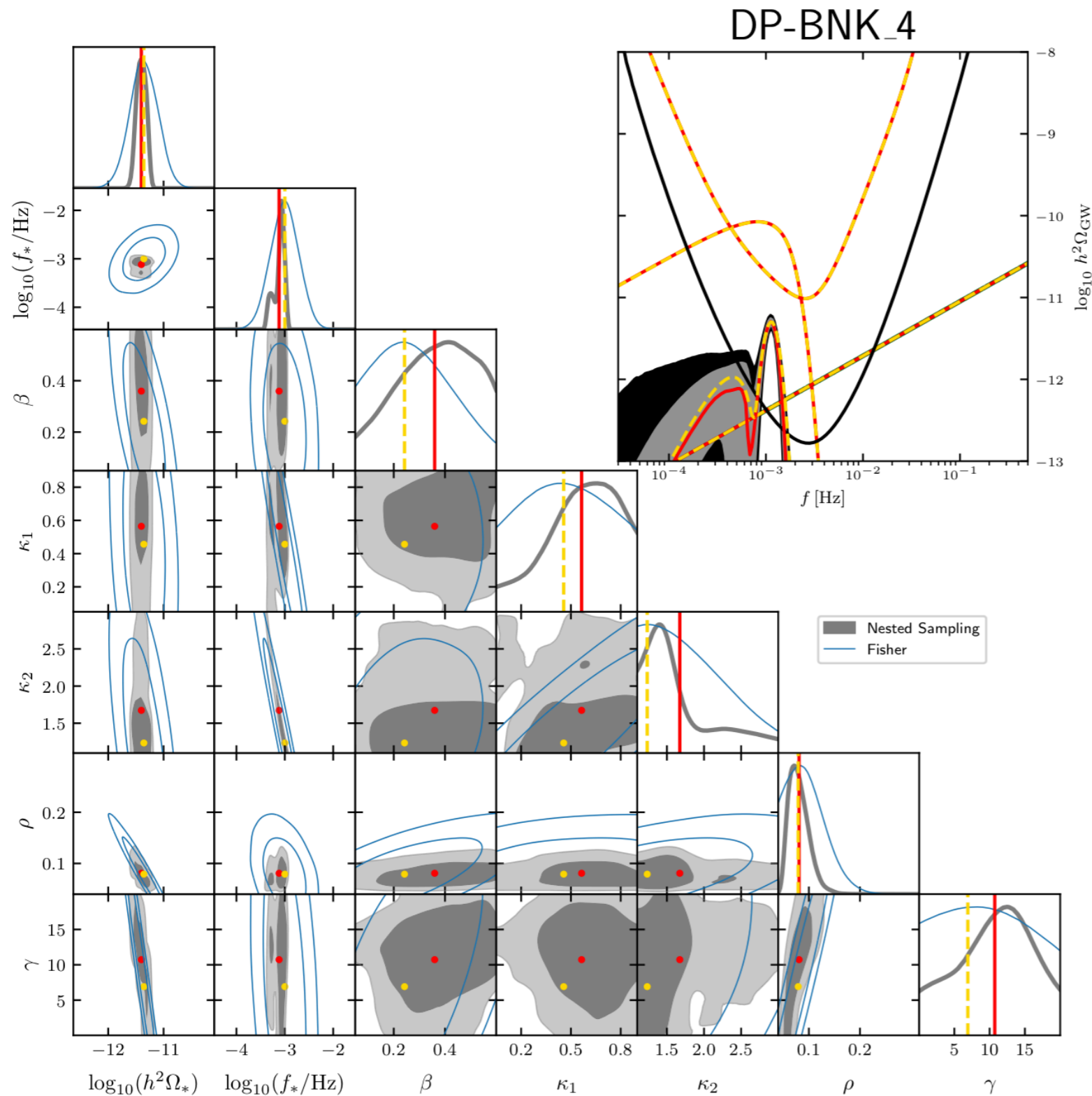
# FORECAST.BNK2



**Moderately loud signal:**

some parameters are not constrained, but signal is reconstructed quite well.

# FORECAST.BNK3



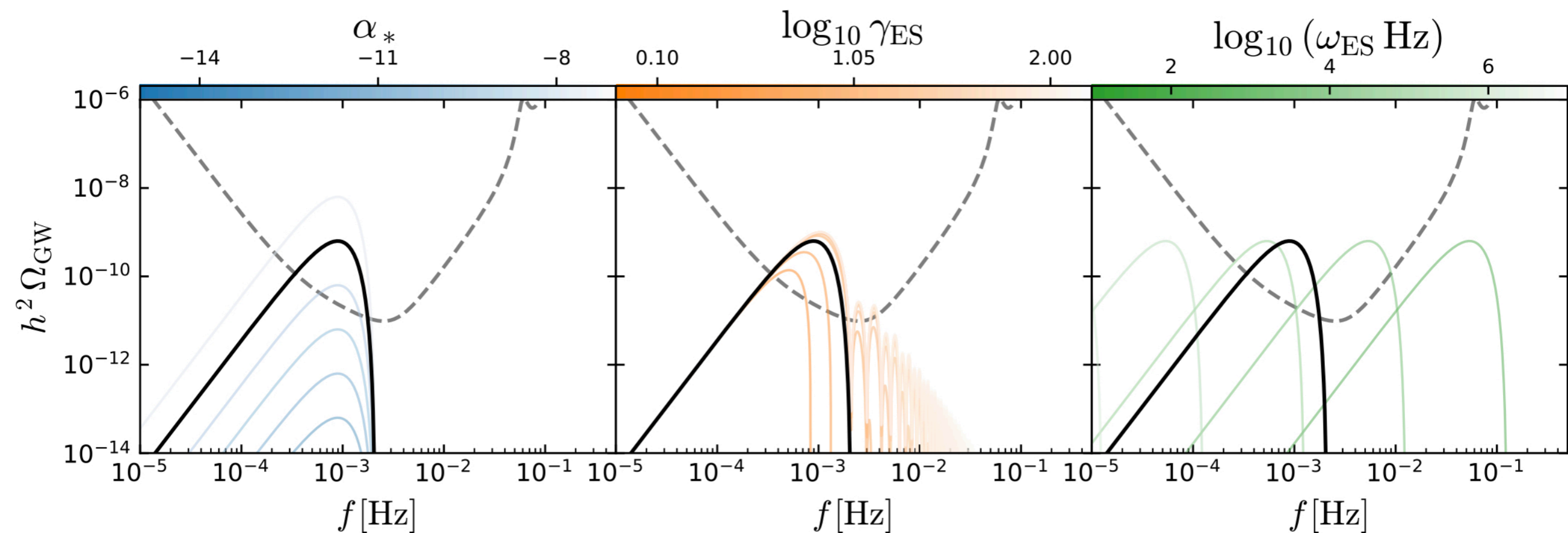
**Faint signal:** only certain features of the signal are constrained.

**EXCITED STATES**

# TEMPLATE DEFINITION

$$h^2 \Omega_{\text{GW}}^{\text{ES}}(f, \vec{p}) = \frac{10^{\alpha_*}}{0.052} \frac{1}{x^3} \left[ 1 - \frac{x^2}{4\gamma_{\text{ES}}^2} \right]^2 \left[ \sin(x) - 2 \frac{1 - \cos(x)}{x} \right]^2 \Theta(x_{\text{cut}} - x)$$

$$x \equiv (f \omega_{\text{ES}})/2 \quad \vec{p} = \{ \alpha_*, \gamma_{\text{ES}}, \omega_{\text{ES}} \}$$



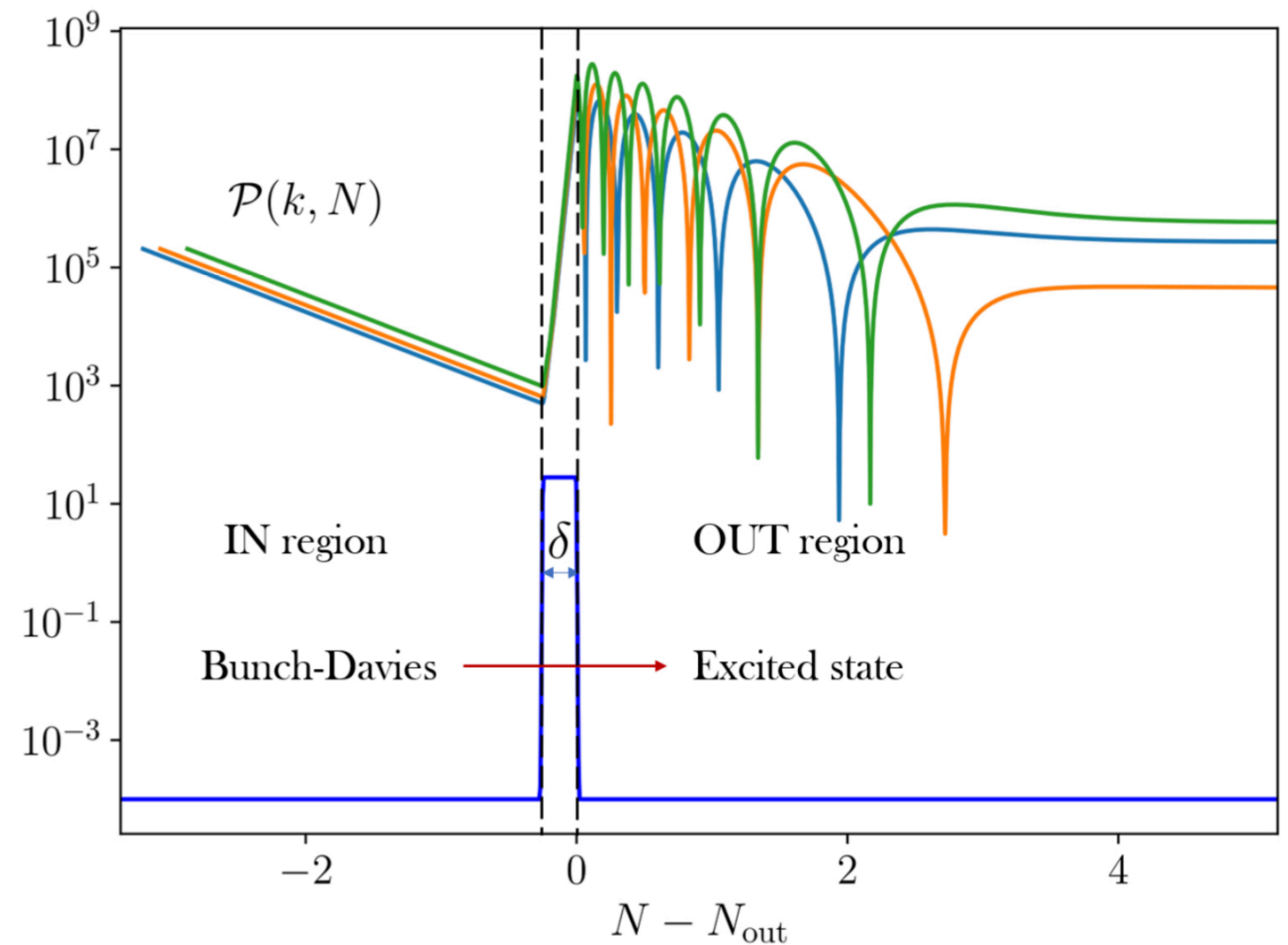
# INFLATIONARY MODELS PRODUCING THE SIGNAL

## Scalar- induced GWs during inflation:

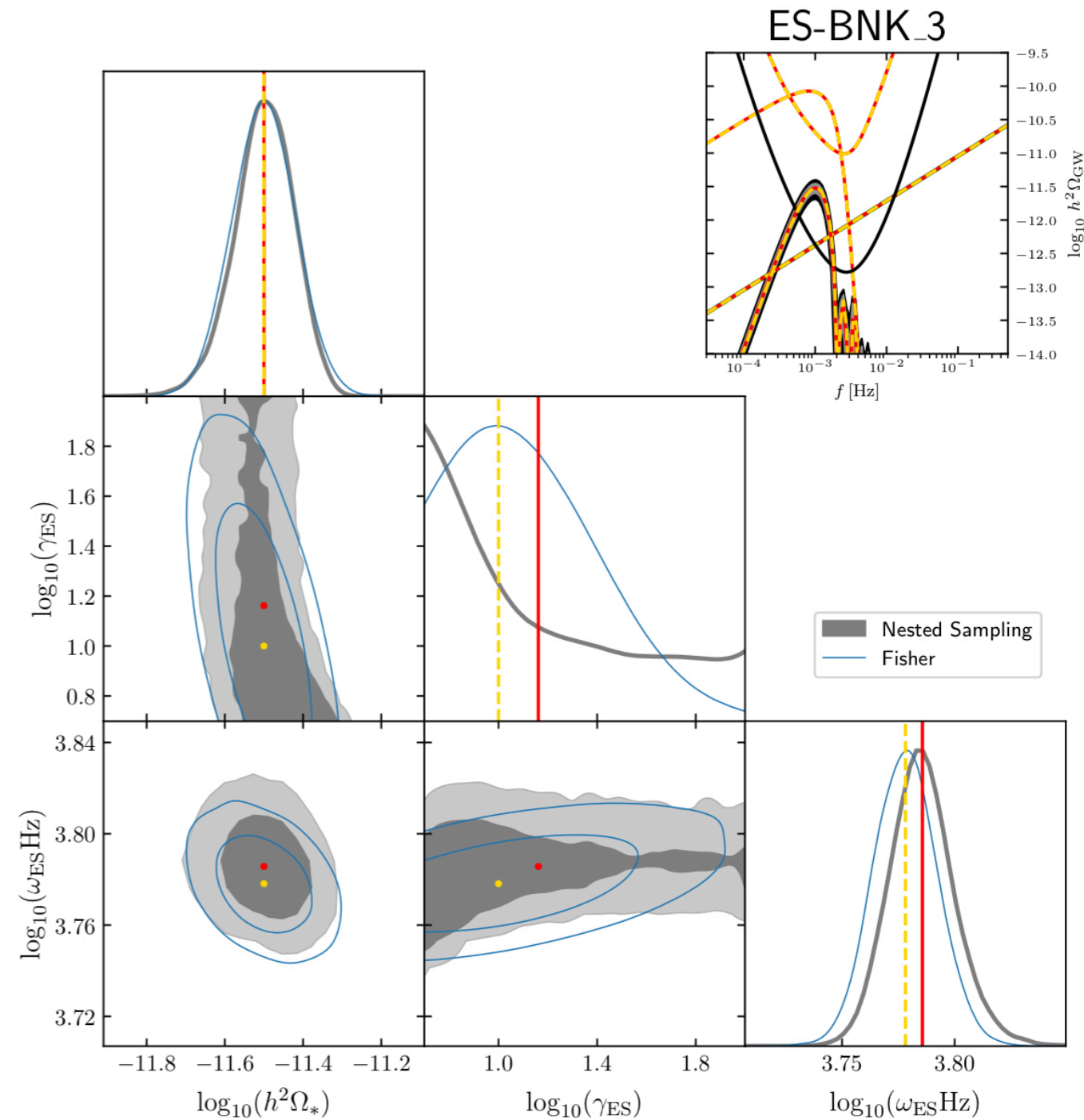
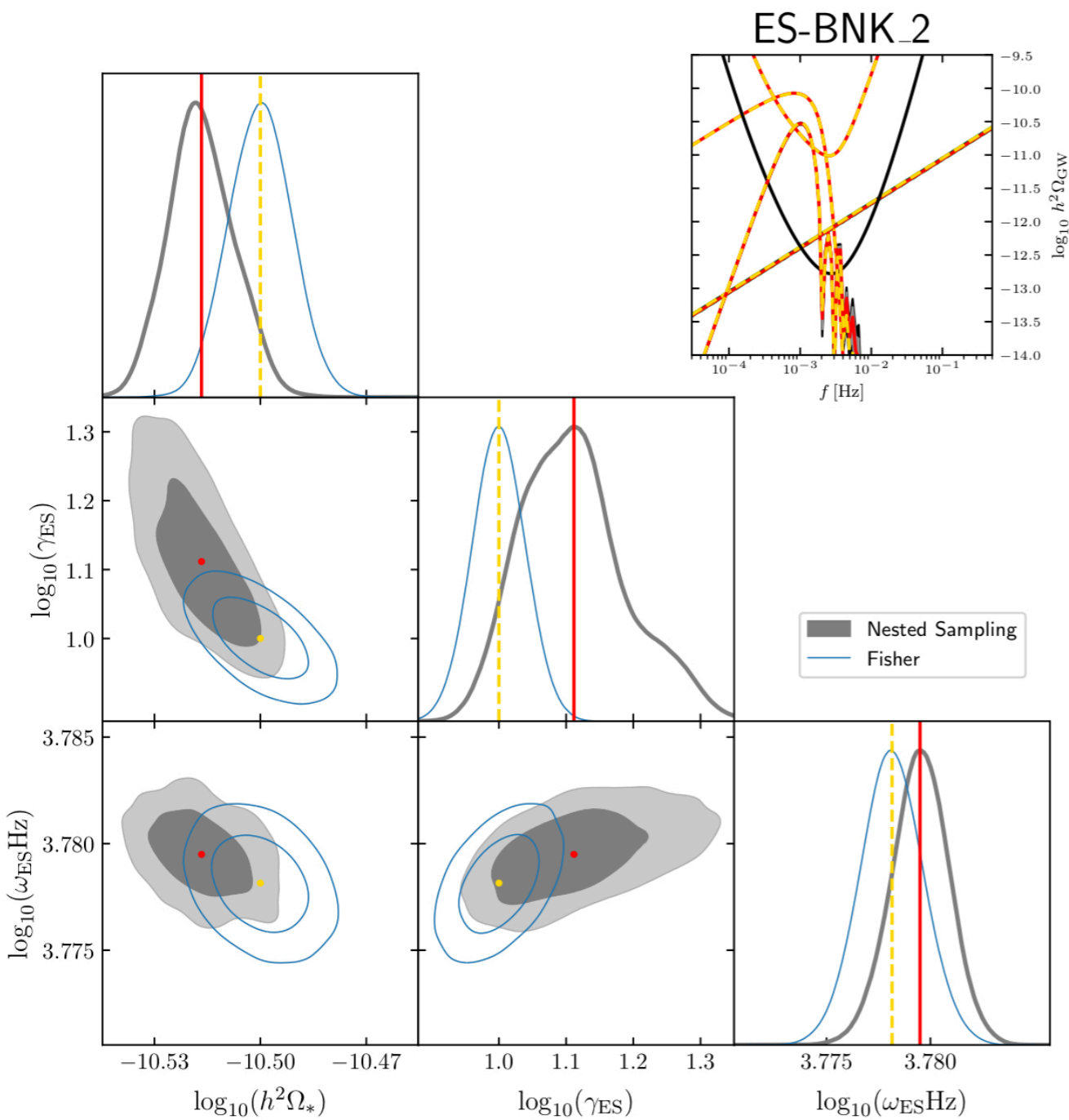
Inomata 2109.03972, Fumagalli et al 2111.14664

A sharp feature induces a temporary large amplification of sub horizon scalar modes.

GWs are amplified by loop effects.



# FORECASTS



Only the amplitude and frequency can be constrained by LISA.

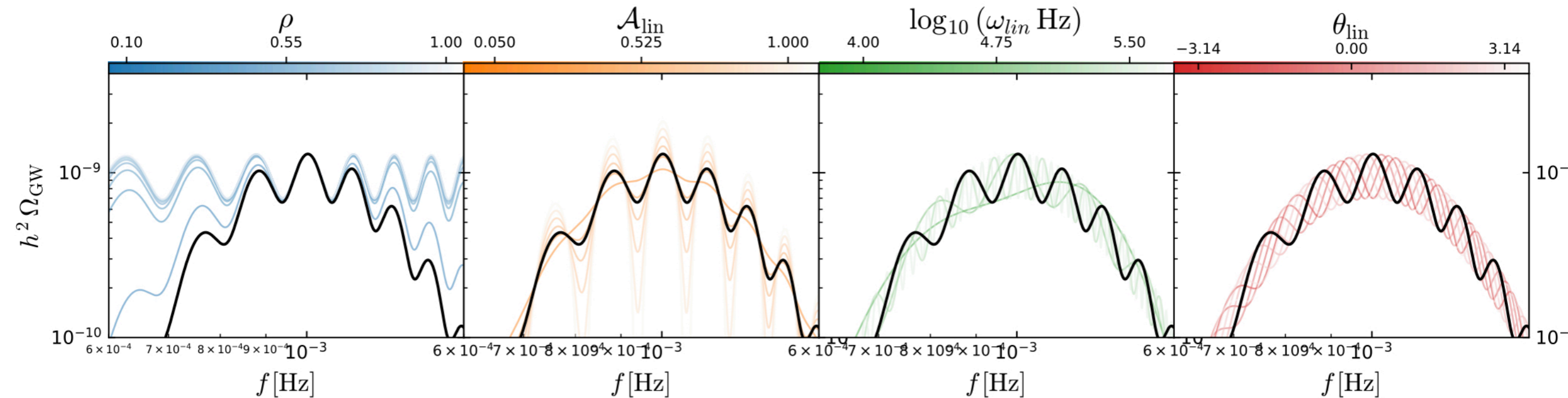


# LINEAR OSCILLATIONS

# TEMPLATE DEFINITION

$$h^2 \Omega_{\text{GW}}^{\text{LO}}(f, \vec{p}) = \left[ 1 + \mathcal{A}_{\text{lin}} \cos(\omega_{\text{lin}} f + \theta_{\text{lin}}) \right] h^2 \Omega_{\text{GW}}^{\text{env}}(f, \vec{p}_{\text{env}})$$

$$\vec{p} = \{ \vec{p}_{\text{env}}, \mathcal{A}_{\text{lin}}, \omega_{\text{lin}}, \theta_{\text{lin}} \}$$

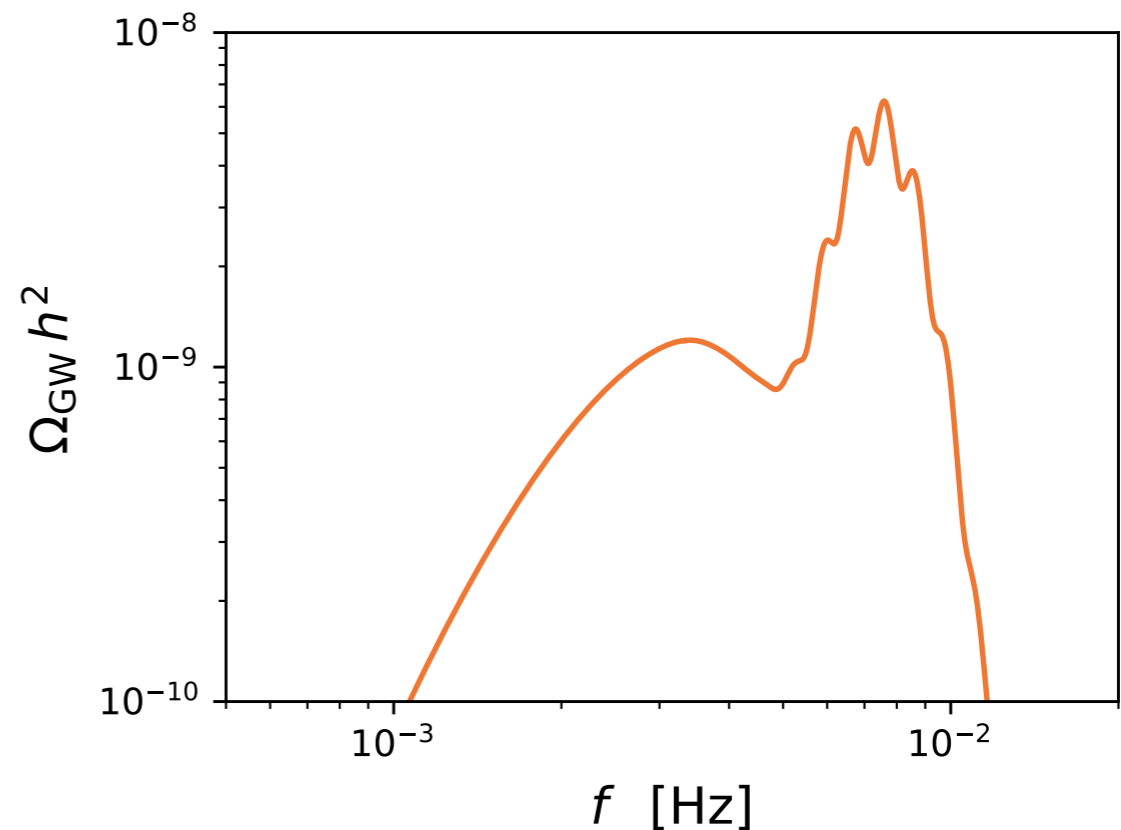
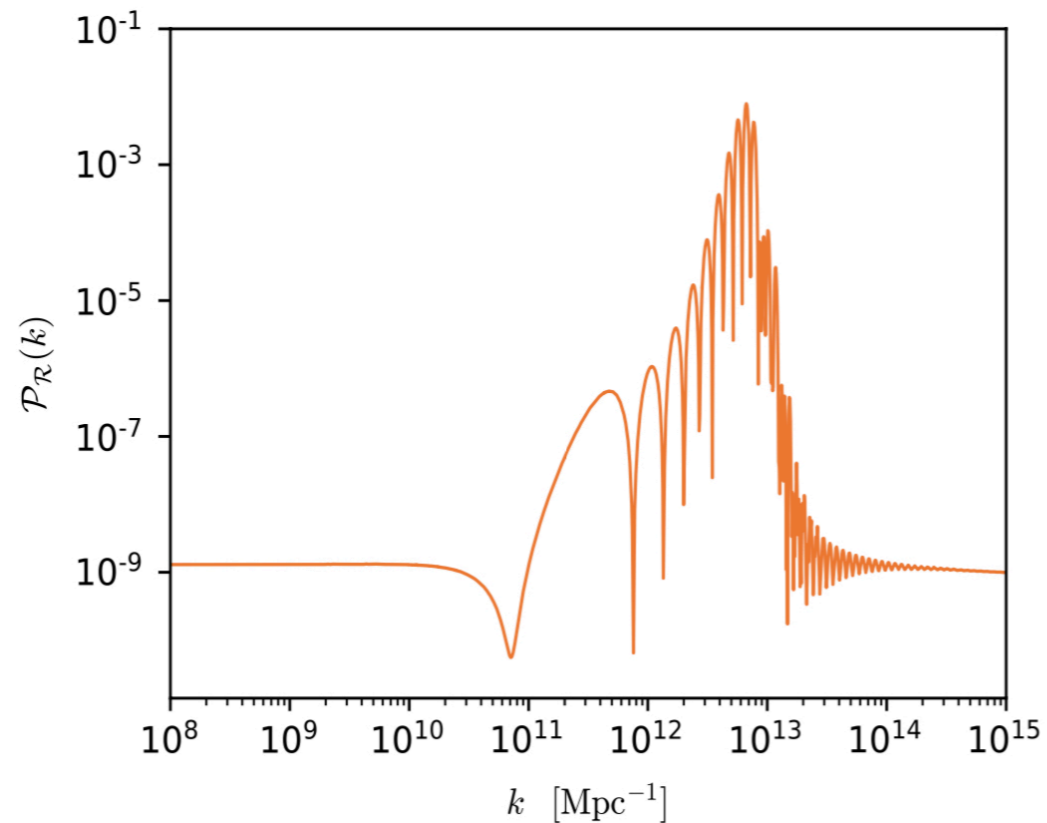
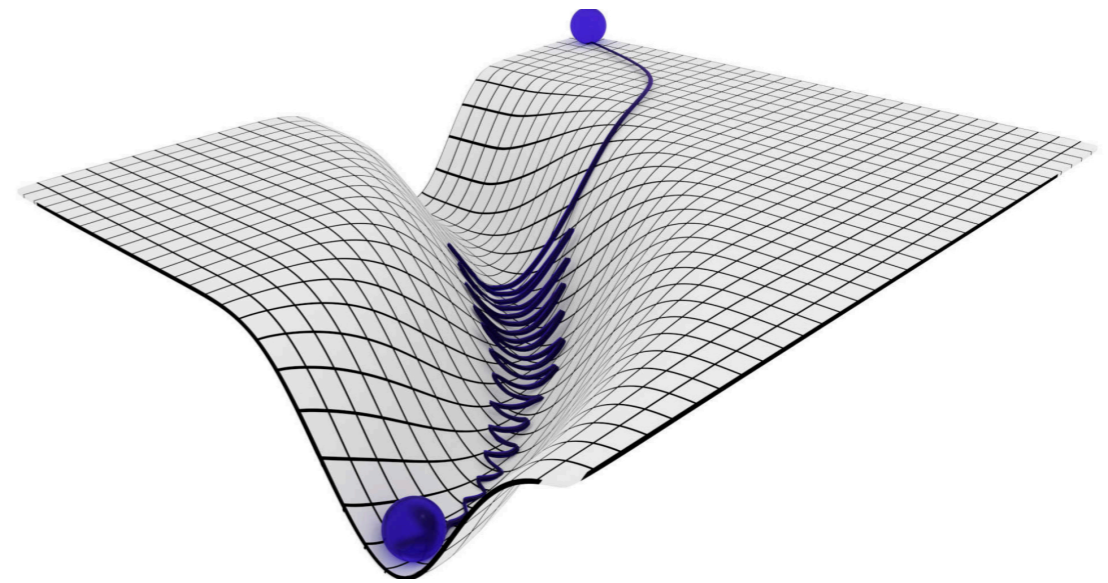


# INFLATIONARY MODELS PRODUCING THE SIGNAL. MODEL 1

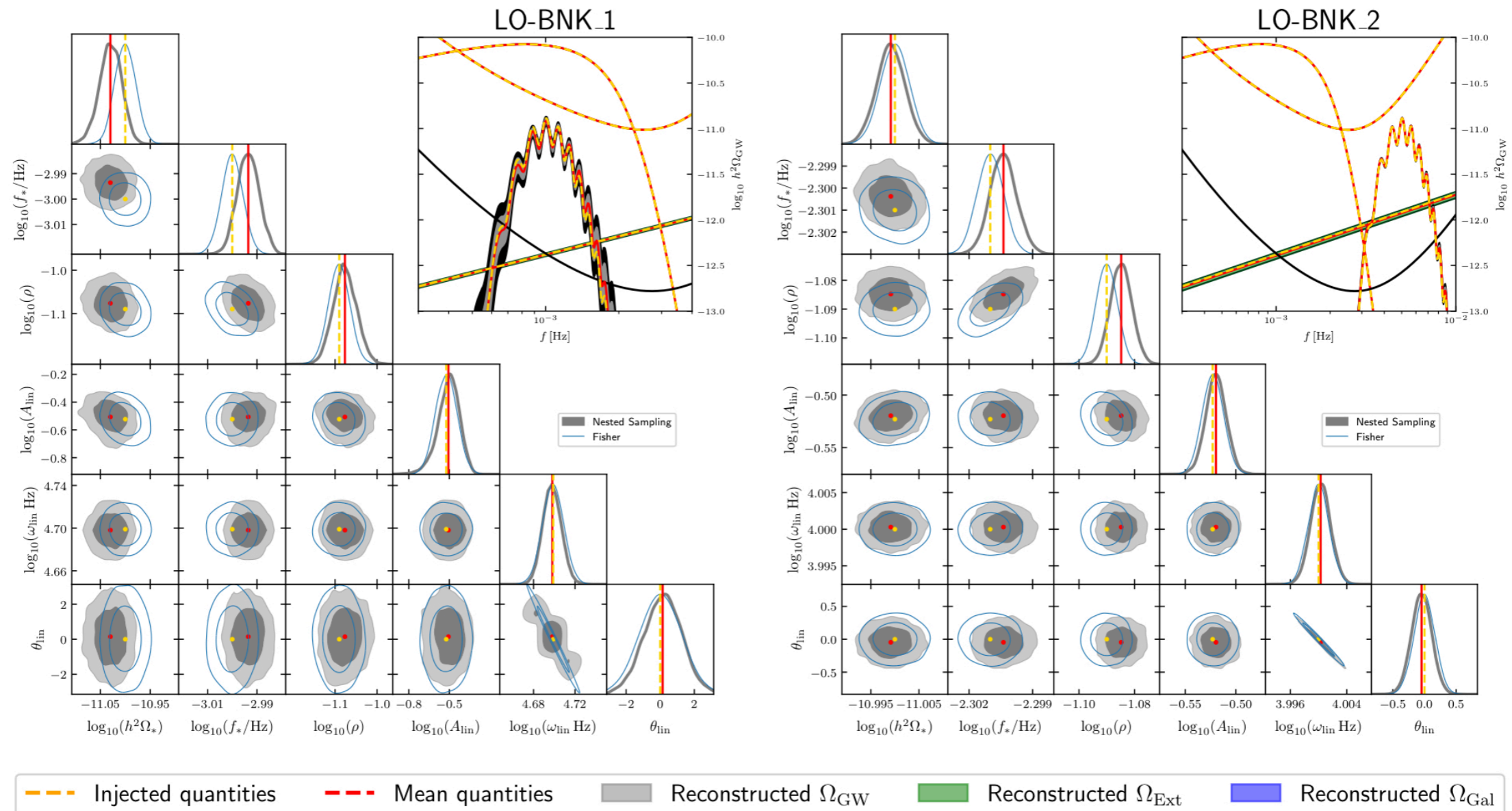
## Sharp features during inflation:

Fumagalli et al 2012.02861, Braglia, Chen, Hazra 2012.05821

Scalar induced signal from sharp features during inflation produced at horizon re-entry.



# FORECASTS



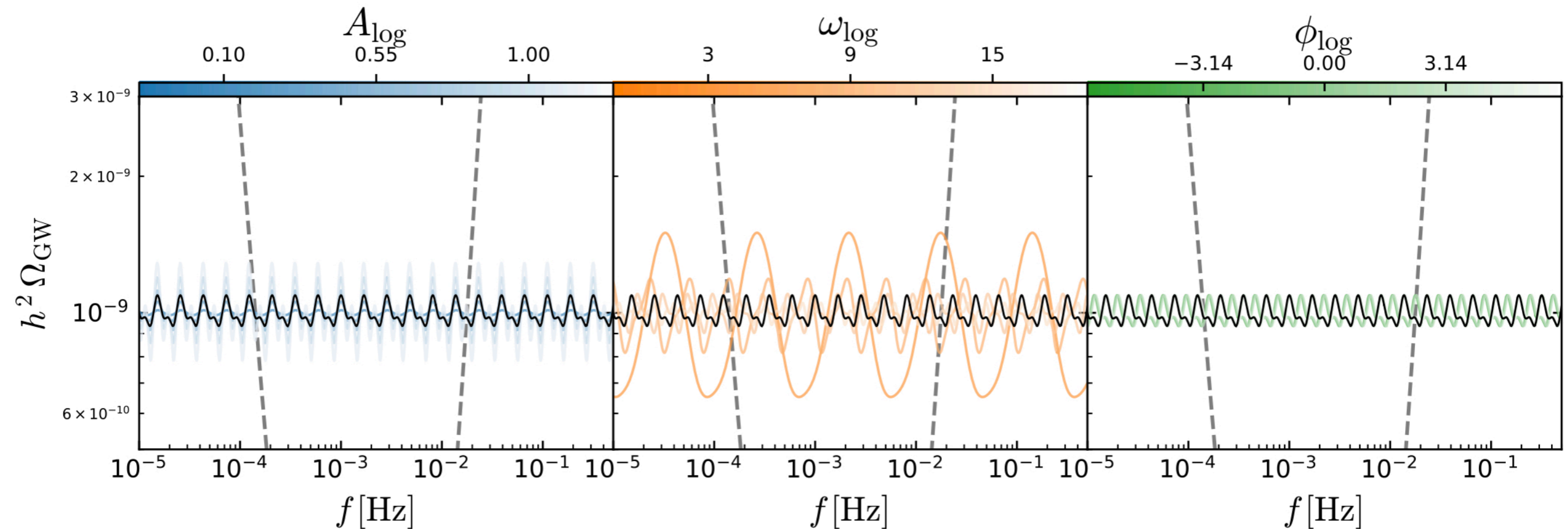
Because of their particular spectral shape oscillations can be well constrained by LISA.

# **RESONANT (LOGARITHMIC) OSCILLATIONS**

# TEMPLATE DEFINITION

$$h^2 \Omega_{\text{GW}}^{\text{RO}}(f, \vec{p}) = \left\{ 1 + \mathcal{A}_1(A_{\log}, \omega_{\log}) \cos [\omega_{\log} \ln(f/\text{Hz}) + \theta_{\log,1}] \right. \\ \left. + \mathcal{A}_2(A_{\log}, \omega_{\log}) \cos [2\omega_{\log} \ln(f/\text{Hz}) + \theta_{\log,2}] \right\} h^2 \Omega_{\text{GW}}^{\text{env}}(f, \vec{p}_{\text{env}})$$

$$\vec{p}_{\text{RO}} = \{ \alpha_*, A_{\log}, \omega_{\log}, \phi_{\log} \}$$



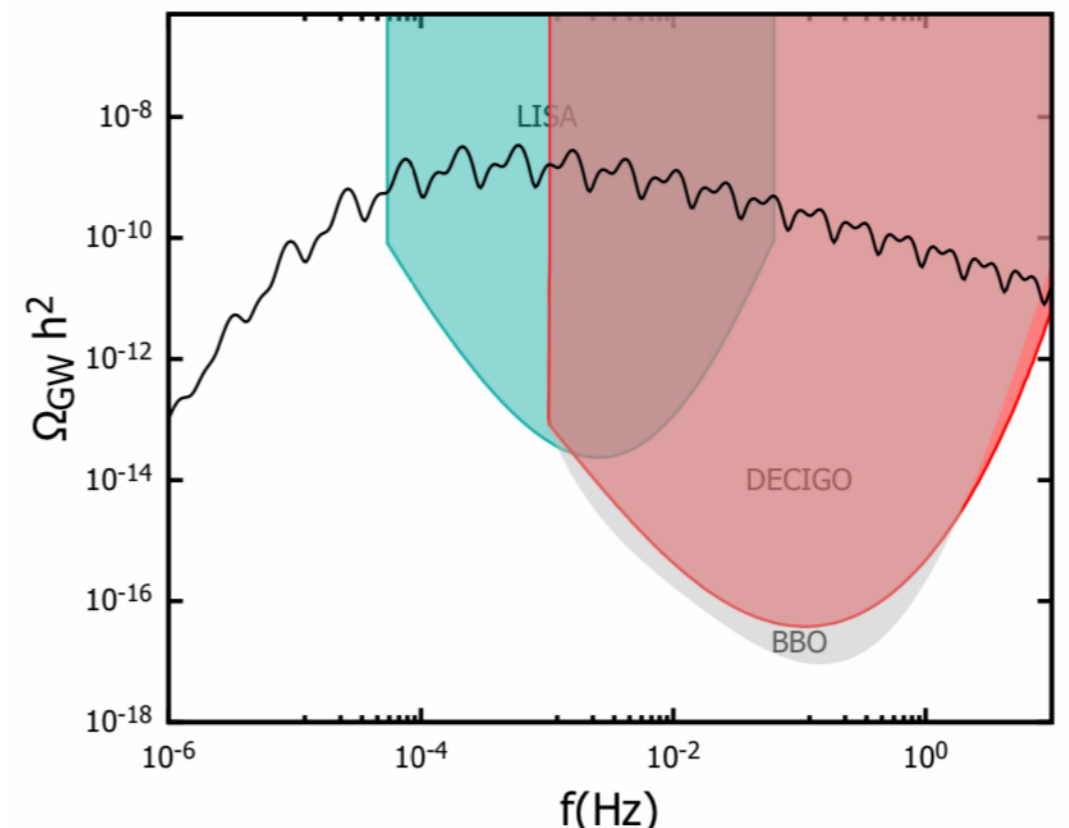
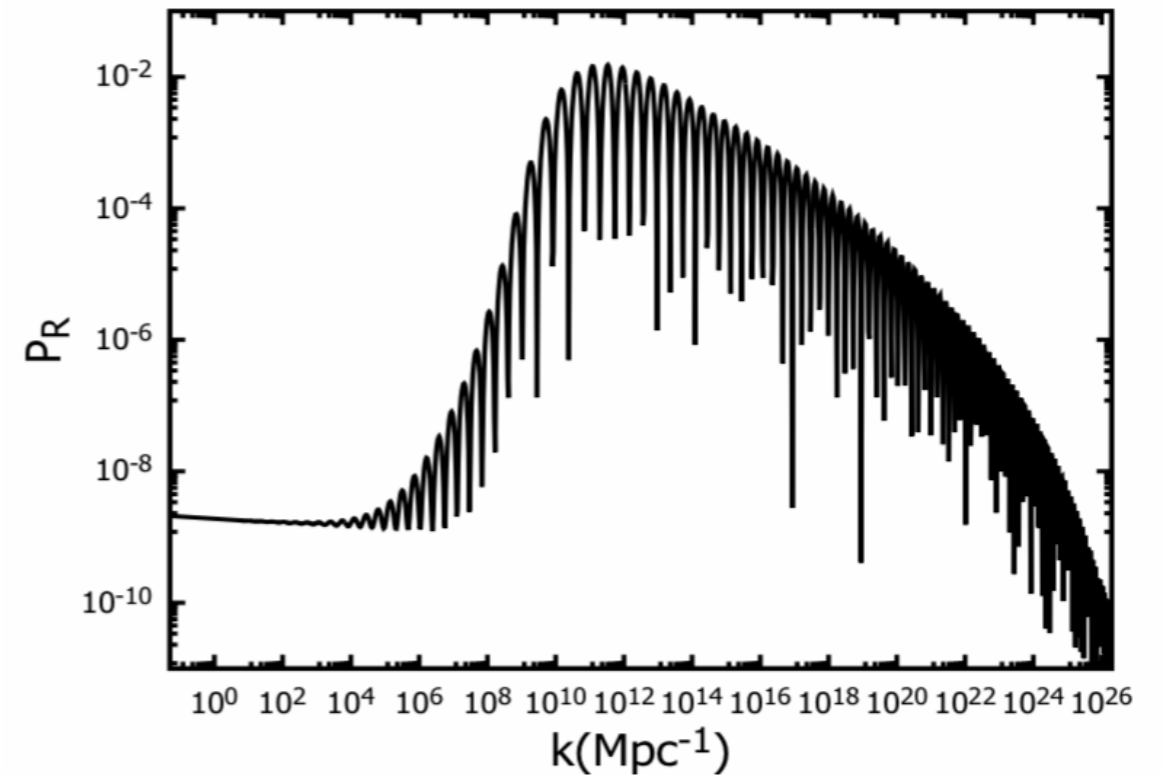
# INFLATIONARY MODELS PRODUCING THE SIGNAL. MODEL 1

## Resonant features during inflation:

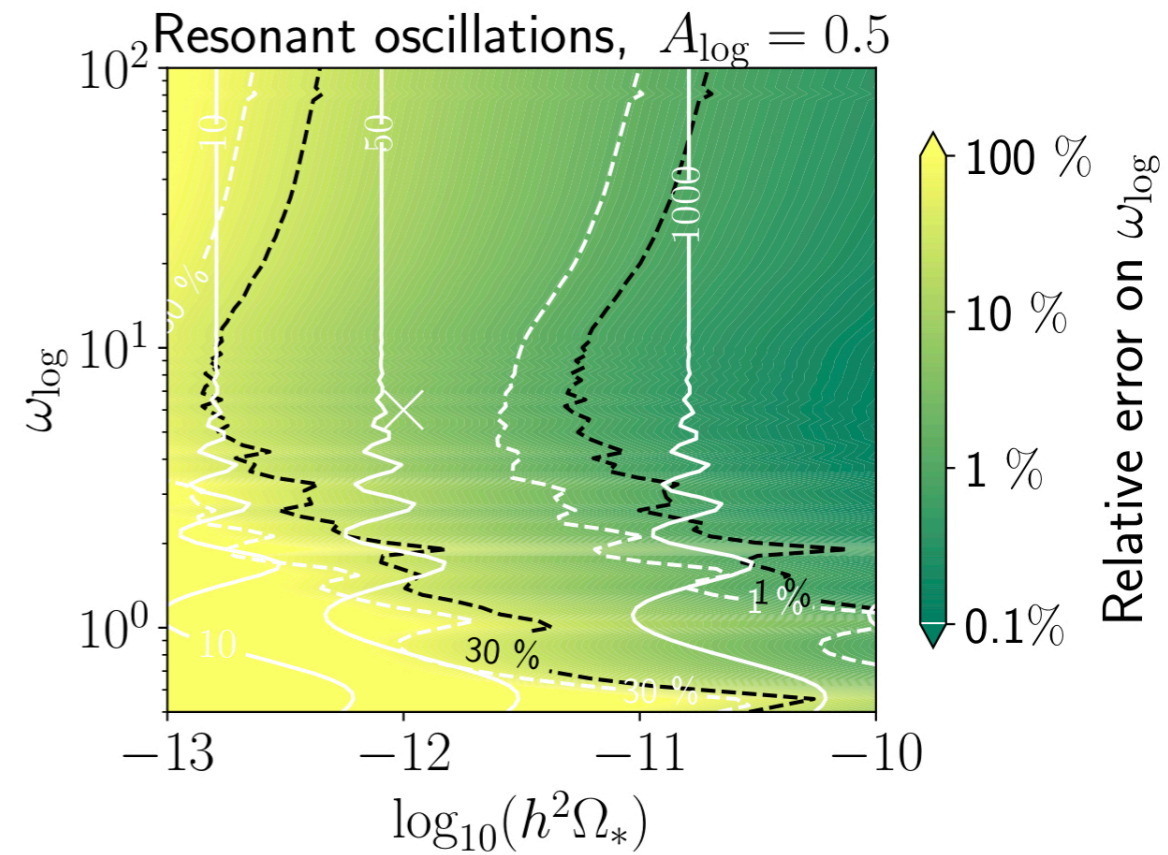
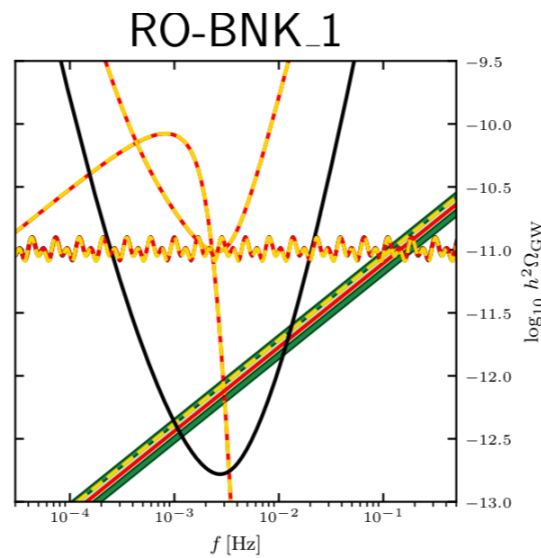
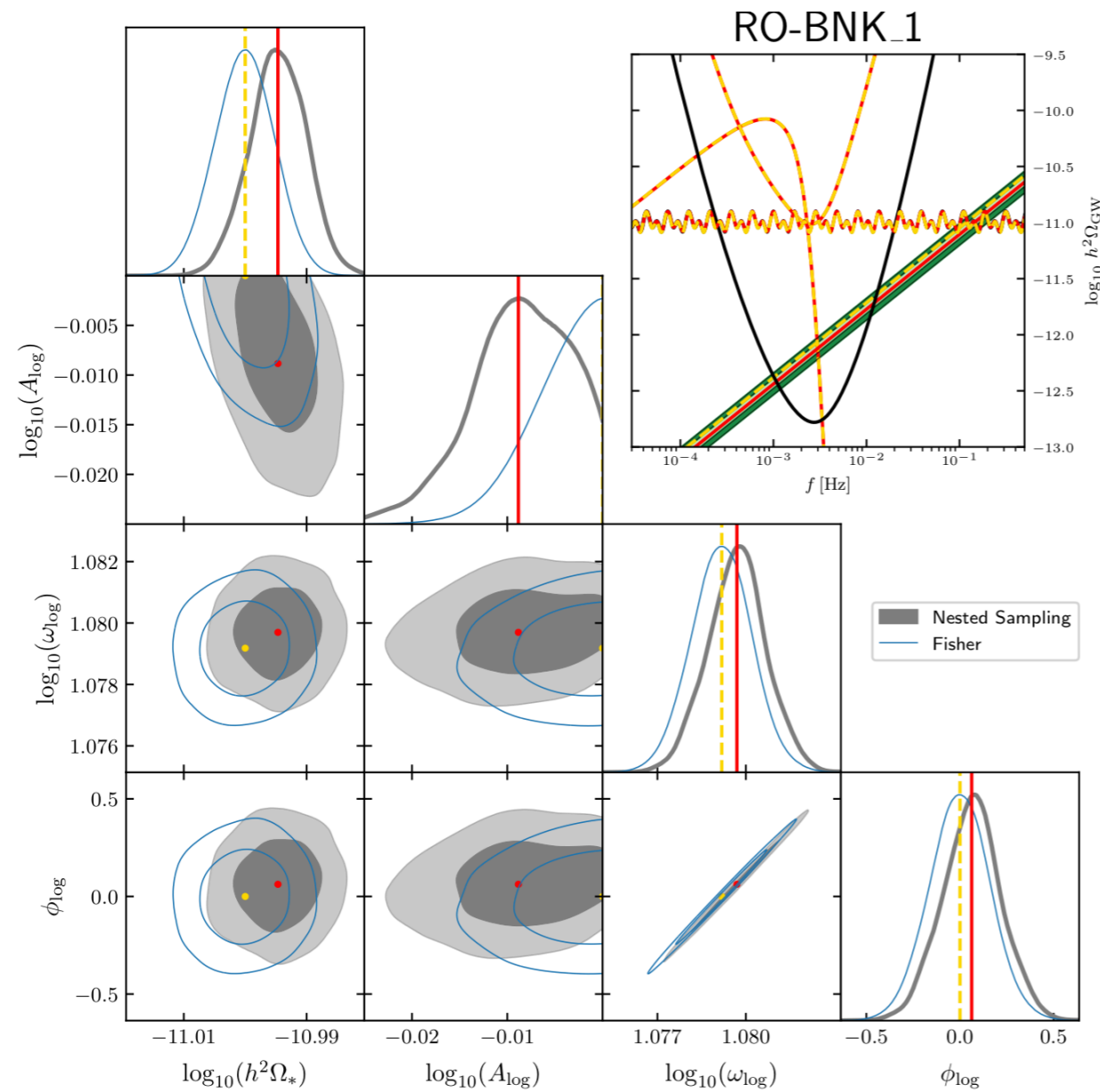
Fumagalli et al 2105.06481

Currently a theoretical proposal based on analogies with resonant mechanisms in axion monodromy or primordial standard clock models. Models going in this direction include:

Battacharya & Zavala 2205.06065, Mavromatos et al 2206.07963,  
Calcagni & Kuroyanagi 2308.05904



# FORECASTS



Constraints are dependent on the overall amplitude and the frequency of the signal.



# SUMMARY

- We initiated the collection of a template bank for  $\Omega_{\text{GW}}(f)$  based on motivated inflationary scenarios.
- Within the LISA CosWG ongoing efforts for Cosmic strings and phase transitions
- Our results showcase the potential of LISA to constrain Inflation based on the reconstruction of the spectral shape of the GW background
- Other observables or correlation with other experiments may be needed to underpin the inflationary origin of the background if the spectral shape is degenerate with other models