WEIZMANN INSTITUTE OF SCIENCE

Faculty of Physics
הפקולטה לפיסיקה

GWs from first-order PTs in LISA

reconstruction pipeline and physics interpretation

arXiv:2403.03723 [astro-ph.CO]

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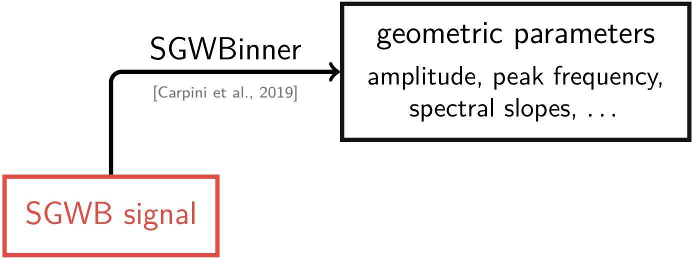
M. Merchand, G. Nardini, M. Pieroni,
A. Roper Pol, and V. Vaskonen

LISA
CosWG-24-01

Parameter reconstruction for cosmological phase transitions

SGWB signal

Parameter reconstruction for cosmological phase transitions



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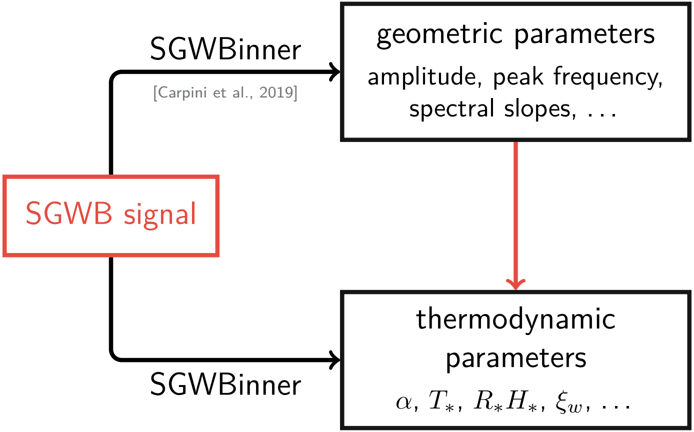

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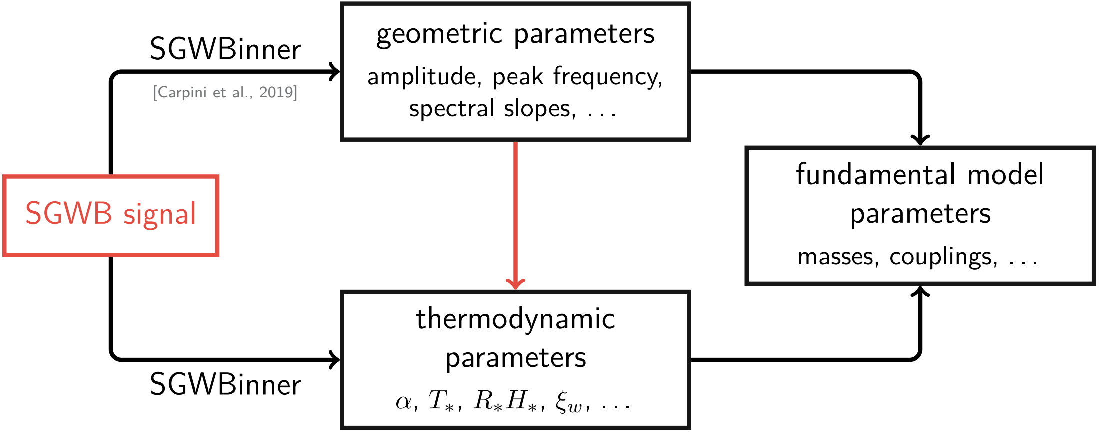
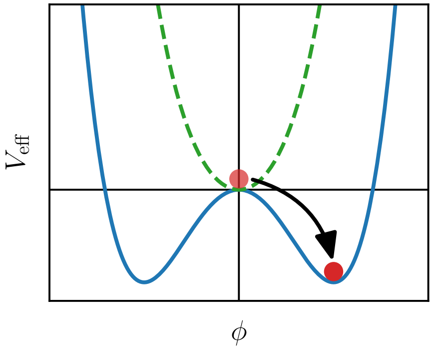


Cosmological phase transitions

- thermal corrections typically restore spontaneously broken symmetries at high temperatures
  $\Longrightarrow$ symmetry breaking phase transition
- can be crossover or first-order



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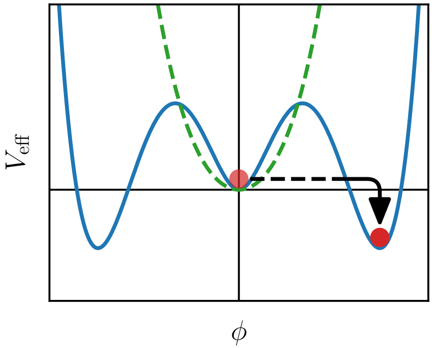

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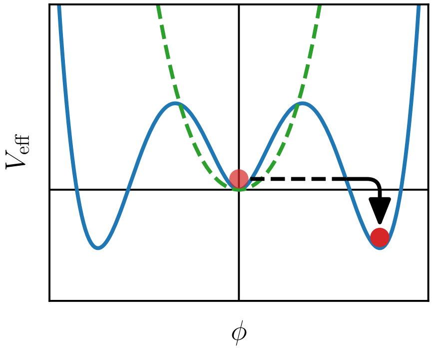

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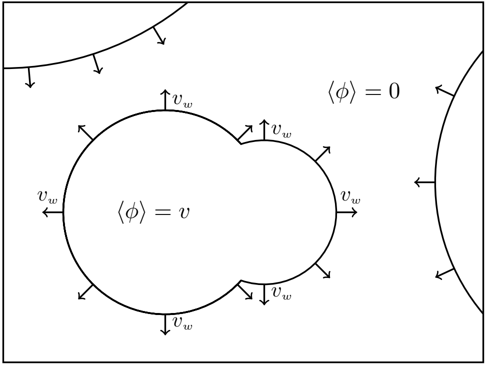

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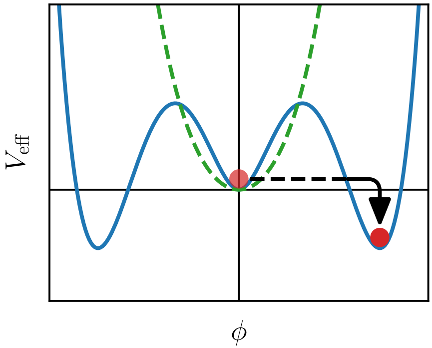

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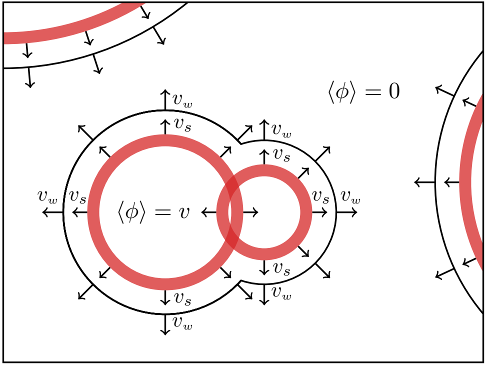

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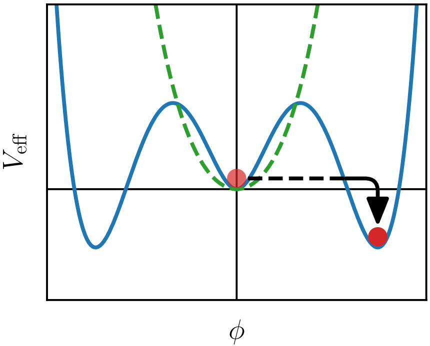
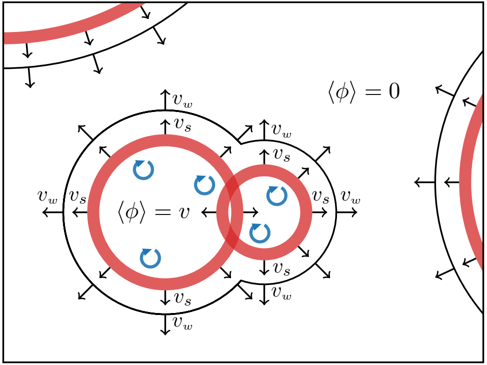


GW production:

1. vacuum bubble collisions
2. sound waves collisions
3. turbulence and vortical motion



Phase transition parameters (thermodynamic parameters)

- nucleation/percolation temperature T_*

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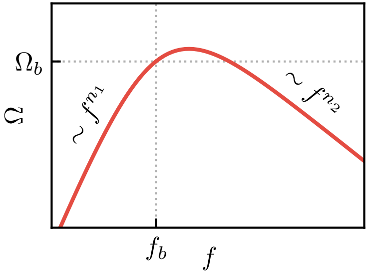
- bubble wall velocity ξ_w

here: $\xi_w \sim 1$

Templates

broken power-law

$$\Omega_b \mathcal{N} \left(\frac{f}{f_b}\right)^{n_1} \left[1 + \left(\frac{f}{f_b}\right)^{a_1}\right]^{\frac{n_2 - n_1}{a_1}}$$



- bubble collisions: [Lewicki & Vaskonen, 2023]
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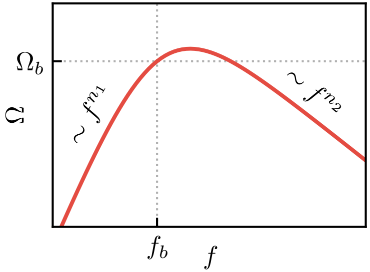

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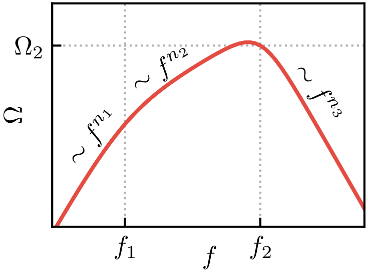
double broken power-law

$$\Omega_2 \mathcal{N}\left(\frac{f}{f_1}\right)^{n_1}\left[1+\left(\frac{f}{f_1}\right)^{a_1}\right]^{\frac{n_2-n_1}{a_1}}\left[1+\left(\frac{f}{f_2}\right)^{a_2}\right]^{\frac{n_3-n_2}{a_2}}$$



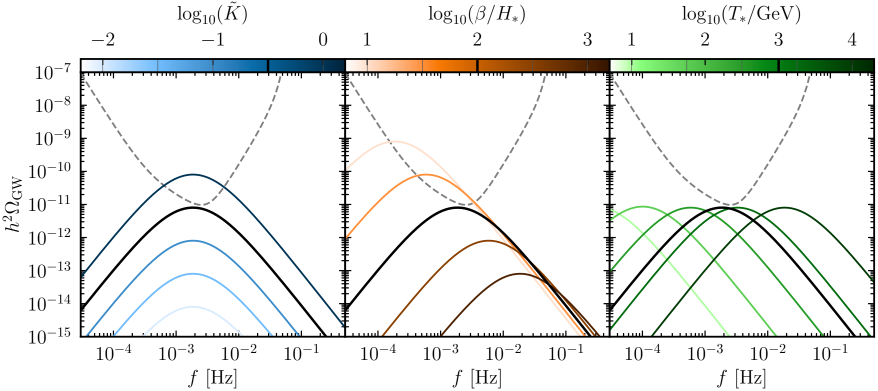
- sound waves: [Jinno et al., 2023]
  $(n_1, n_2, n_2, a_1, a_2) = (3, 1, -3, 2, 4)$
- MHD turbulence: [Roper Pol et al., 2022]
  $(n_1, n_2, n_2, a_1, a_2) = (3, 1, -\frac{8}{3}, 4, 2.15)$

Very strong transitions ($\alpha \gg 1$)

[Koskwsky, Turner & Watkins, 1992; Kosowsky & Turner, 1993; Huber & Konstandin, 2008; Bodeker & Moore, 2009, 2017; Weir, 2016; Jinno & Takimoto, 2017, 2019; Konstandin, 2018; Lewicki & Vaskonen, 2020, 2023; ...]

GWs from vacuum bubble collisions or highly relativistic fluid shells

$$h^2\Omega_b \propto \underbrace{\tilde{K}^2}_{\tilde{K}=\frac{\alpha}{1+\alpha}} \left(\frac{H_*}{\beta}\right)^2 \qquad f_b \propto T_* \frac{\beta}{H_*}$$

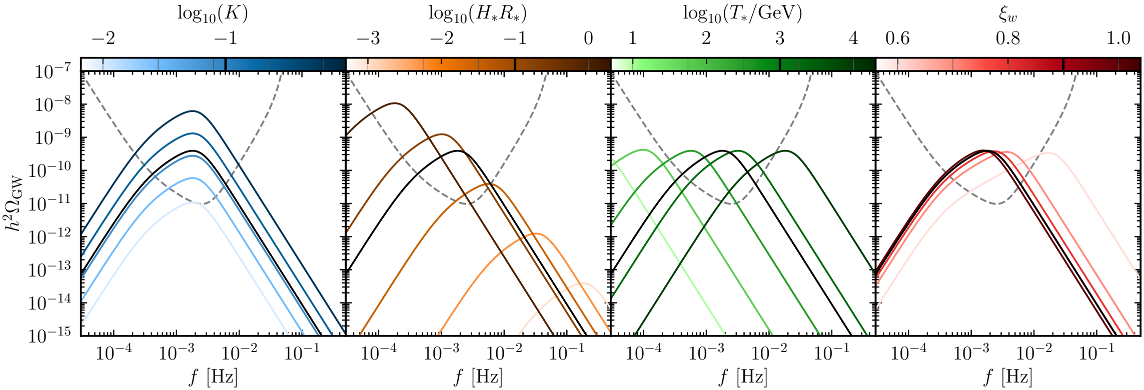


Sound waves

[Hindmarsh et al., 2013, 2015, 2017; Cutting, Hindmarsh & Weir, 2020; Hindmarsh & Hijazi, 2019; Jinno, Konstandin & Rubira, 2019; Jinno et al., 2023; ...]

$$h^2\Omega_2 \propto K^2\,(H_*\tau_{\rm sw})\,(H_*R_*) \qquad f_1 \propto T_*\,(H_*R_*)^{-1} \qquad f_2 \propto T_*\,(H_*R_*)^{-1}\,\frac{\xi_w}{\xi_w - c_s}$$

soundwave lifetime $H_*\tau_{\rm sw} = \max\left[1,\ H_*R_*/\sqrt{\tfrac{4}{3}K}\right]$

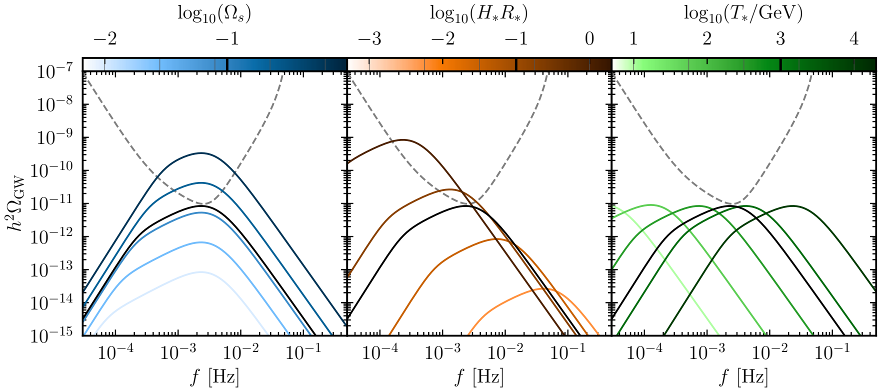


Magnetohydrodynamic turbulence

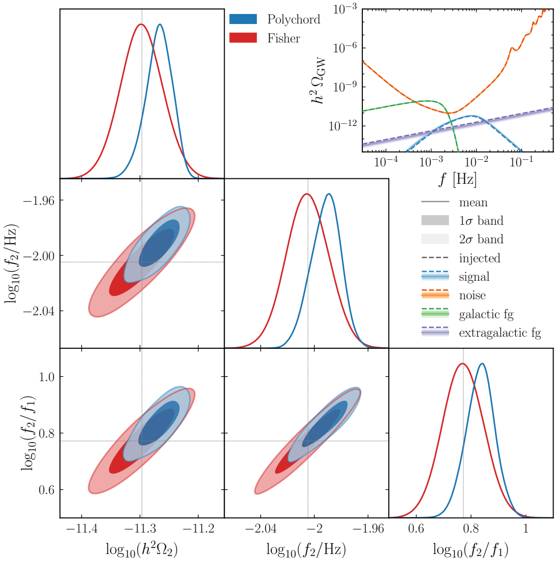
[Caprini, Durrer & Servant, 2009; Roper Pol et al., 2022; Auclari et al, 2022; Roper Pol et al., 2023]

$$h^2\Omega_2 \propto \Omega_s^2 \, (H_* R_*)^2 \qquad f_1 \propto \Omega_s^{\frac{1}{2}} \, T_* \, (H_* R_*)^{-1} \qquad f_2 \propto T_* \, (H_* R_*)^{-1}$$

total energy in turbulent motion $\Omega_s = \varepsilon \, K$



Polychord vs. Fisher analysis



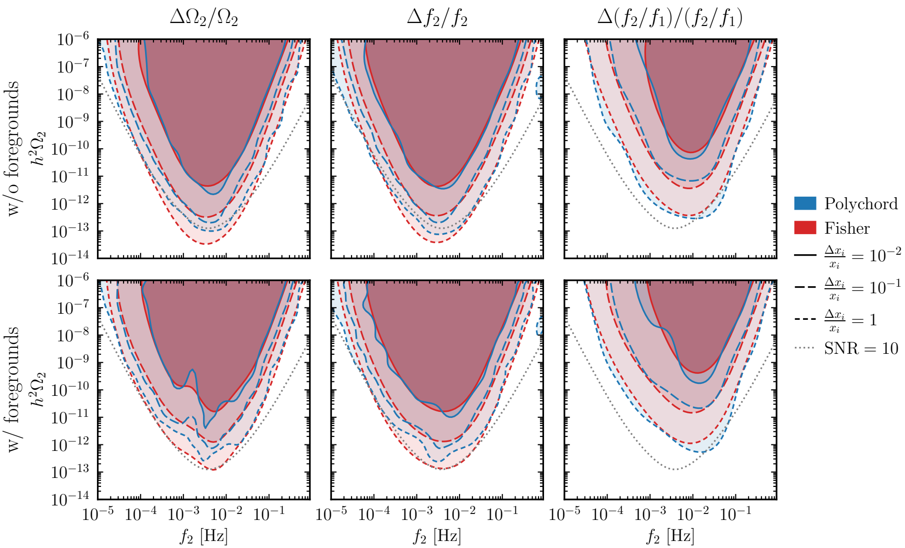
$h^2\Omega_2 = 5\times10^{-12},\; f_2 = 10\,\text{mHz},\; \frac{f_2}{f_1}\approx 6$

- estimation of parameter reconstruction reach based on generated data + Polychord takes some time
- alternative: Fisher analysis

$$\mathcal{C}_{ij}^{-1} = \mathcal{F}_{ij} = -\frac{\partial^2 \log\mathcal{L}}{\partial\theta_i\partial\theta_j}$$

works well if posteriors are approximately Gaussian

Geometric parameter reconstruction



$$(n_1,\, n_2,\, n_3,\, a_1,\, a_2) = (3,\, 1,\, -3,\, 2,\, 4)$$

Reconstructing thermodynamics parameters

- 3 geom. params.: $\Omega_2,\ f_2,\ f_1$
  4 therm. params.: $K,\ H_*R_*,\ \xi_w,\ T_*$
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- consider fixed T_*:

$$\Omega_2 \propto \frac{\xi_{\text{shell}}}{\xi_w} \begin{cases} K^2 \, H_* R_* & \text{if } H_* \tau > 1 \\ K^{3/2} (H_* R_*)^2 & \text{if } H_* \tau < 1 \end{cases}$$

$$f_2 \propto T_* / (H_* R_*)$$

$$\frac{f_2}{f_1} \propto \xi_w / \xi_{\text{shell}}$$

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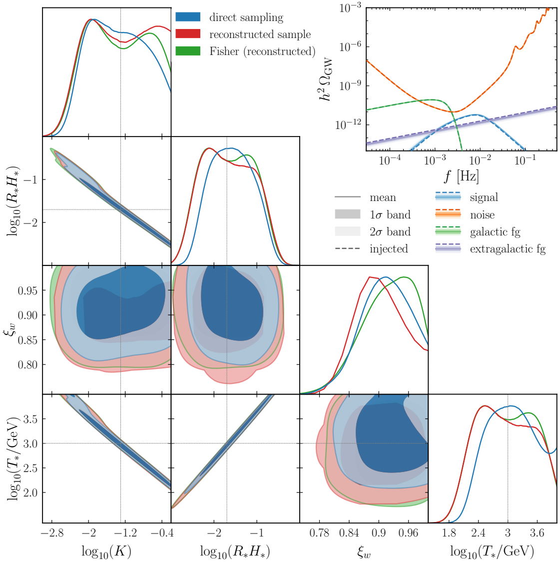

$K = 0.05,\ R_*H_* = 0.02,\ \xi_w = 1,\ T_* = 1$ TeV

Soundwaves + turbulence

- sound waves + turbulence
  $\Longrightarrow$ additional parameter: ϵ
  $\Longrightarrow$ degeneracy broken?

- analytically:

$$
\begin{aligned}
(f_1^{\text{turb}}/f_2^{\text{turb}})^2 &\propto \epsilon\, K \\
\Omega_2^{\text{turb}} &\propto \epsilon^2 K^2 (H_* R_*)^2
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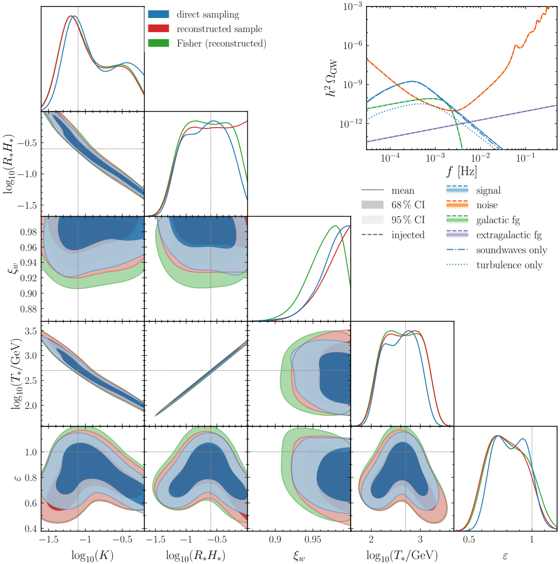

$K = 0.08,\ R_* H_* = 0.25,\ \xi_w = 1,\ T_* = 500$ GeV, $\epsilon = 1$

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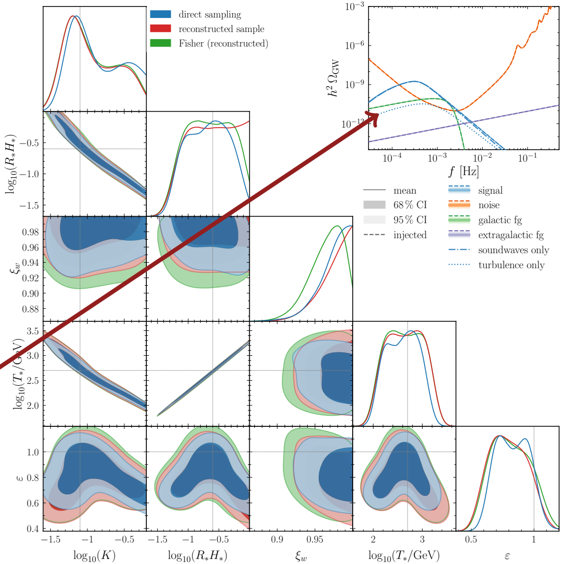

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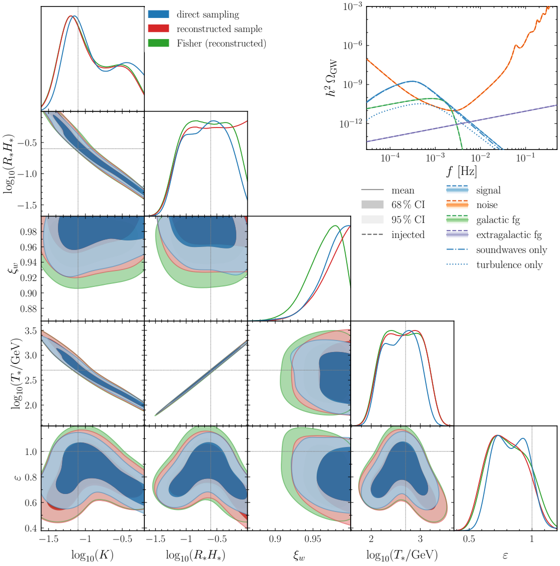

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3. Interpolate on grid to convert geometric parameters to model parameters

Gauge singlet extension with Z_2 symmetry

[see e.g. Lewicki, Merchand, Zych (2022)
Ellis, Lewicki, Merchand, No, Zych (2023)]

$$V(H,s) = -\mu_h^2\, H^\dagger H + \lambda \left(H^\dagger H\right)^2 + \frac{\mu_s^2}{2}\, s^2 + \frac{\lambda_s}{4}\, s^4 + \frac{\lambda_{hs}}{2}\, s^2 H^\dagger H$$

- 2-step transition: $\langle h,\, s\rangle = (0,0) \;\to\; (0, v_s) \;\to\; (v_h, 0)$
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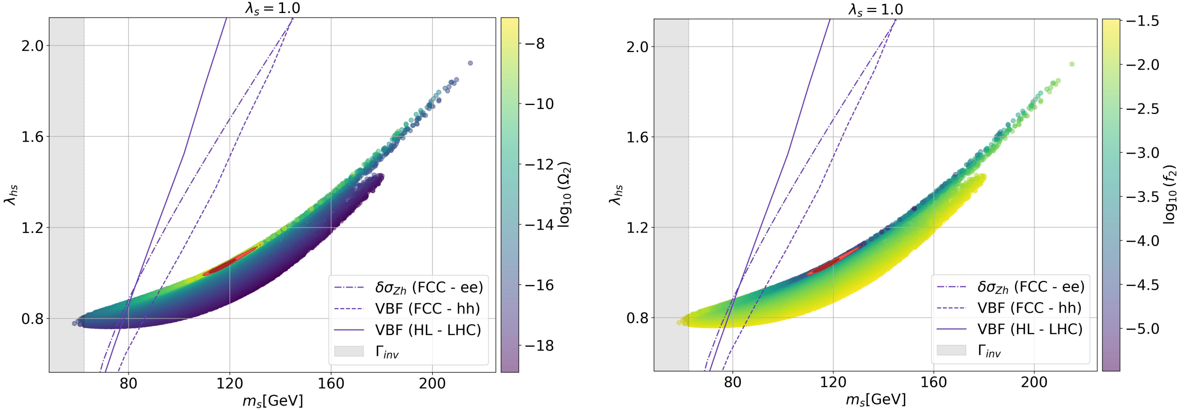

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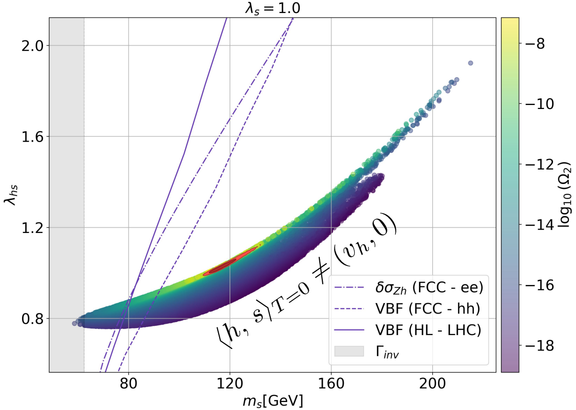

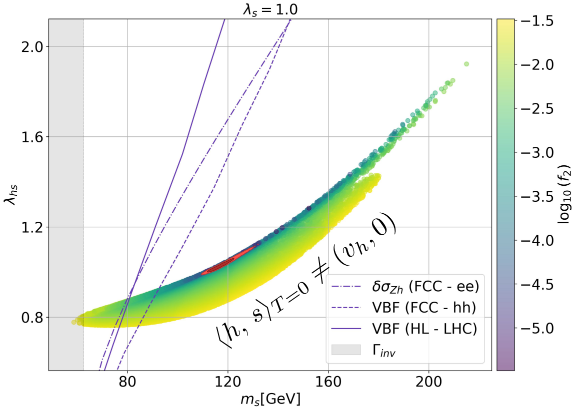

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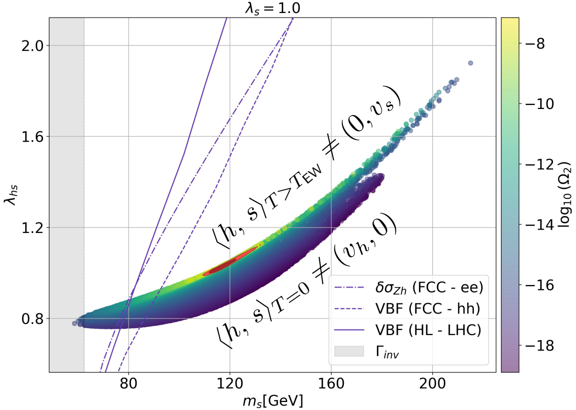
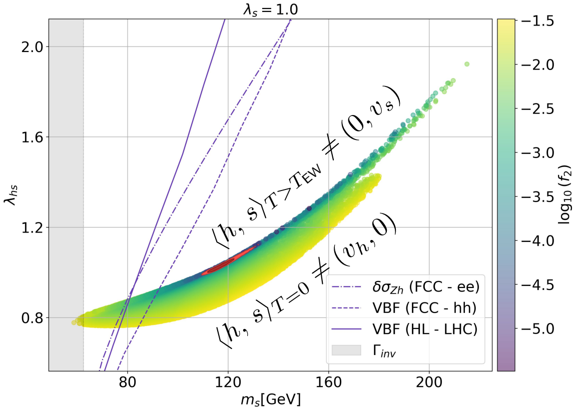
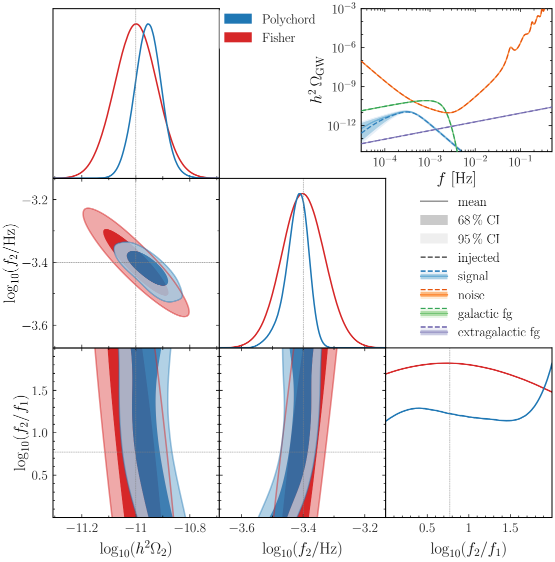
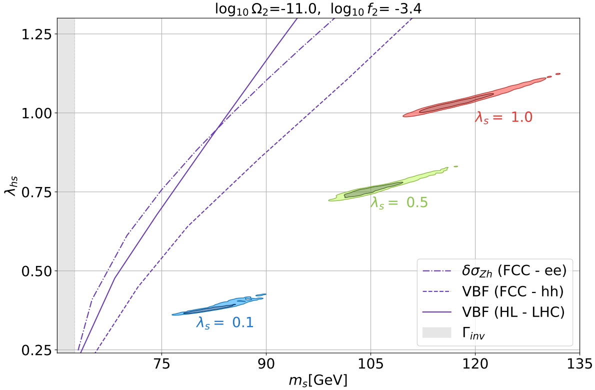




Gauge singlet parameter reconstruction





Classically conformal $U(1)_{B\text{-}L}$ model

[see e.g. Jinno, Takimoto (2009)
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$$V(H,\phi) = \lambda_H \left(H^\dagger H\right)^2 + \lambda_\phi \left(\phi^\dagger \phi\right)^2 - \lambda_p \left(H^\dagger H\right)\left(\phi^\dagger \phi\right)$$

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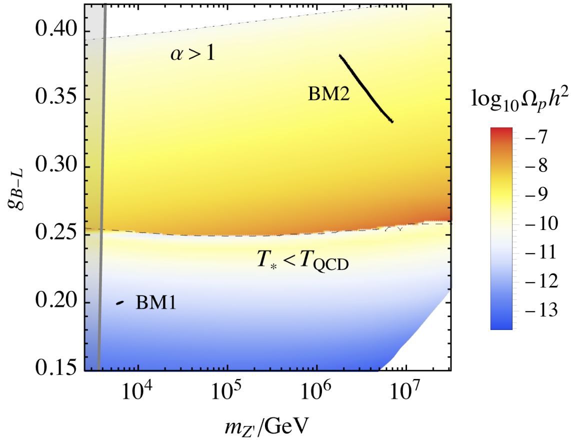
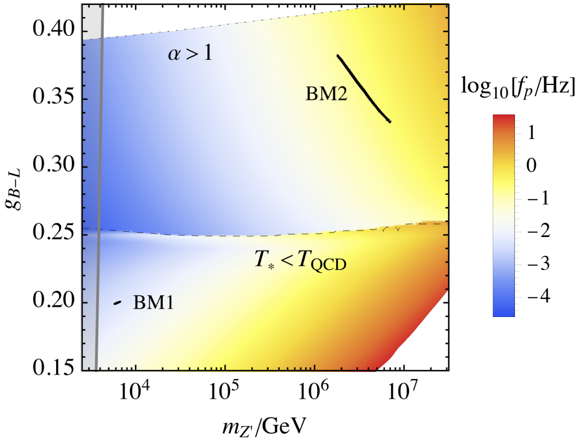
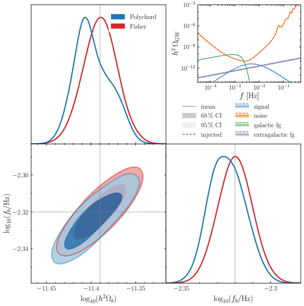
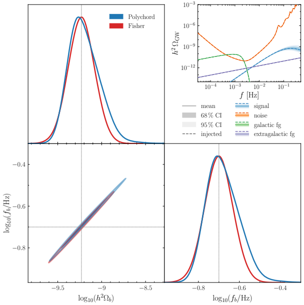
supercooled PT $\Longrightarrow$ bubble collision / highly relativistic fluid shells

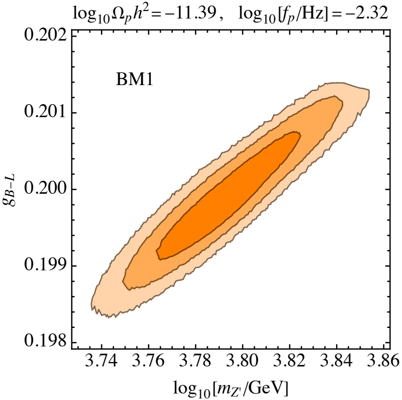


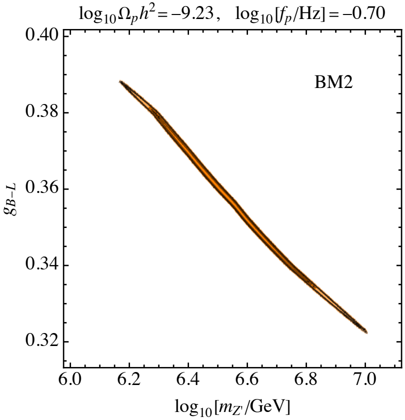


$U(1)_{B\text{-}L}$ parameter reconstruction









Conclusions

- for the geometric parameters (amplitude, peak/break frequencies, ...), we can estimate the reach using Fisher analysis
- the reconstruction of thermodynamic parameters of cosmological phase transitions (α, $H_* R_*$, T_*, $\ldots$) suffers from degeneracies
- a potential observed SGWB signal can determine/constrain fundamental model parameters

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Thank you for your attention!