Reconstructing Scalar-induced Gravitational Waves with LISA

An ongoing project in the Cosmology Working Group

Project coordinators: Gabriele Franciolini, Robert Rosati

Project Logistics

- Proposed in Stavanger, June 2023
- Blue form circulated to group, December 2023
- First call: February 7th, 2024
- Currently 27 members, over 300 person-hours of logged work

Project members

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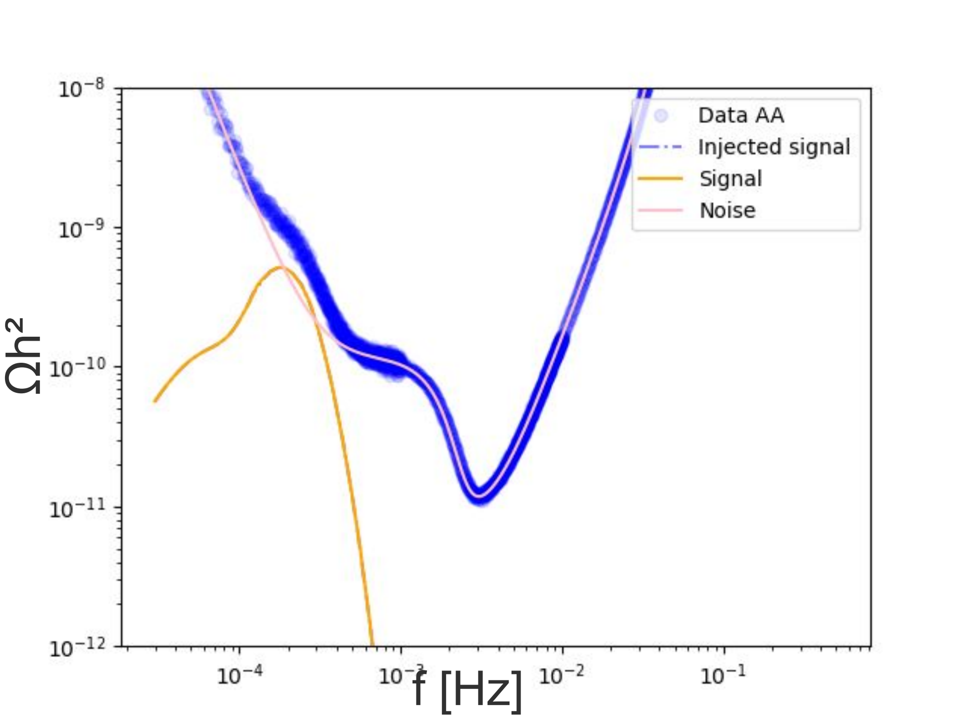
What do we do if we get a detection?

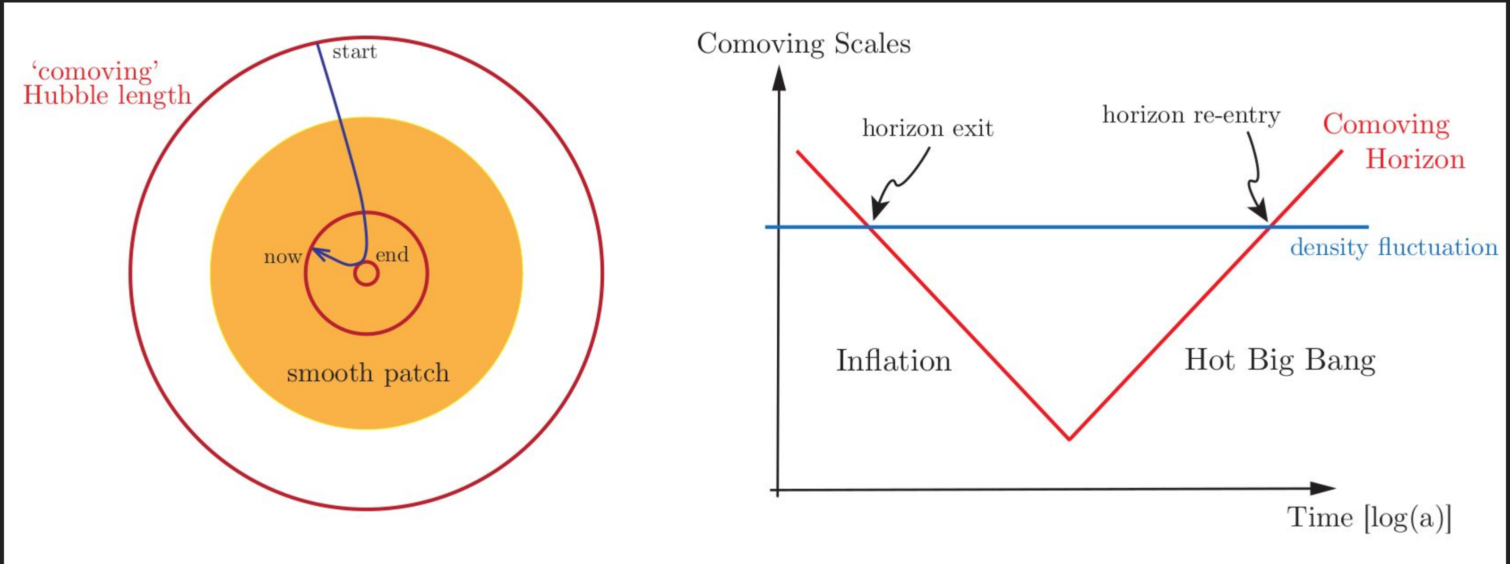
- This signal could be anything!
- Many models predict *scalar-induced SGWBs*
  - Inflation, early PBH production, bouncing cosmologies, etc
- Constrain the scalar perturbations!



Scalar-induced Gravitational Waves

Large scalar perturbations are produced but are causally disconnected

(Re)enter during radiation era, begin collapse



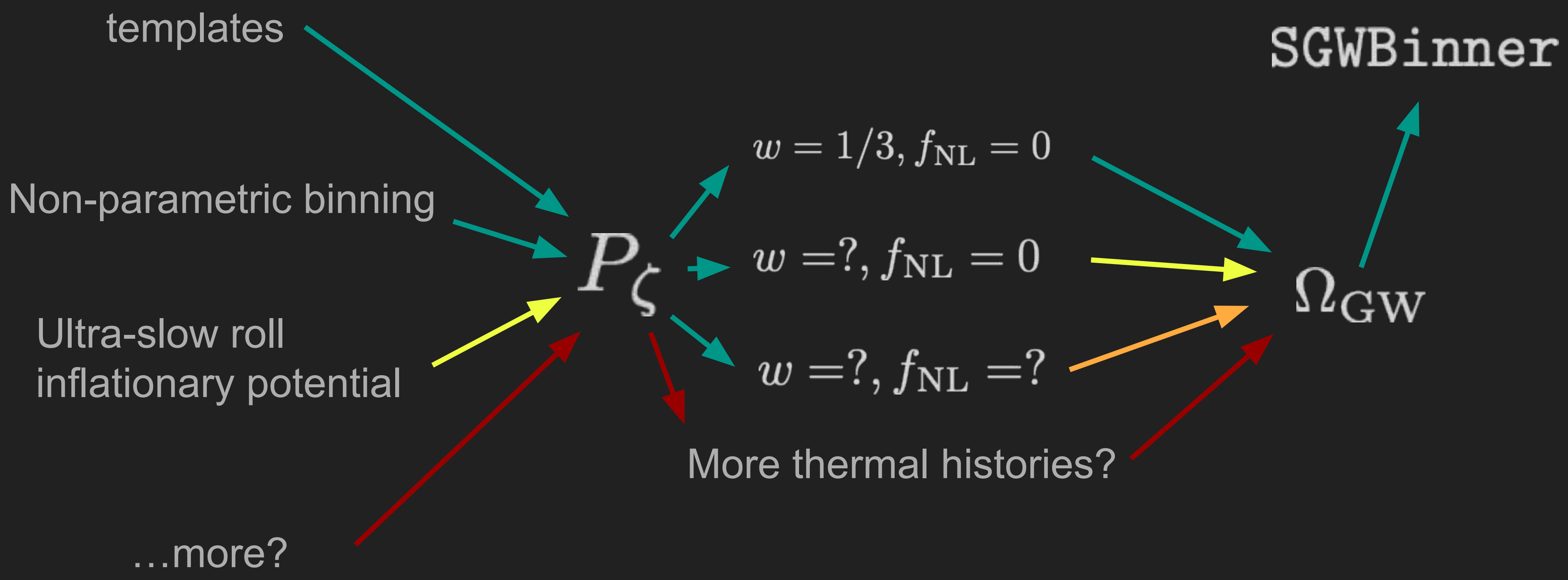
Why constrain scalars?

- Source-model independent constraints
- Testable with astronomy!
  - E.g. [Brito+ 2203.15954]
- Learn about cosmology?
- Some SGWB shapes cannot be scalar-induced

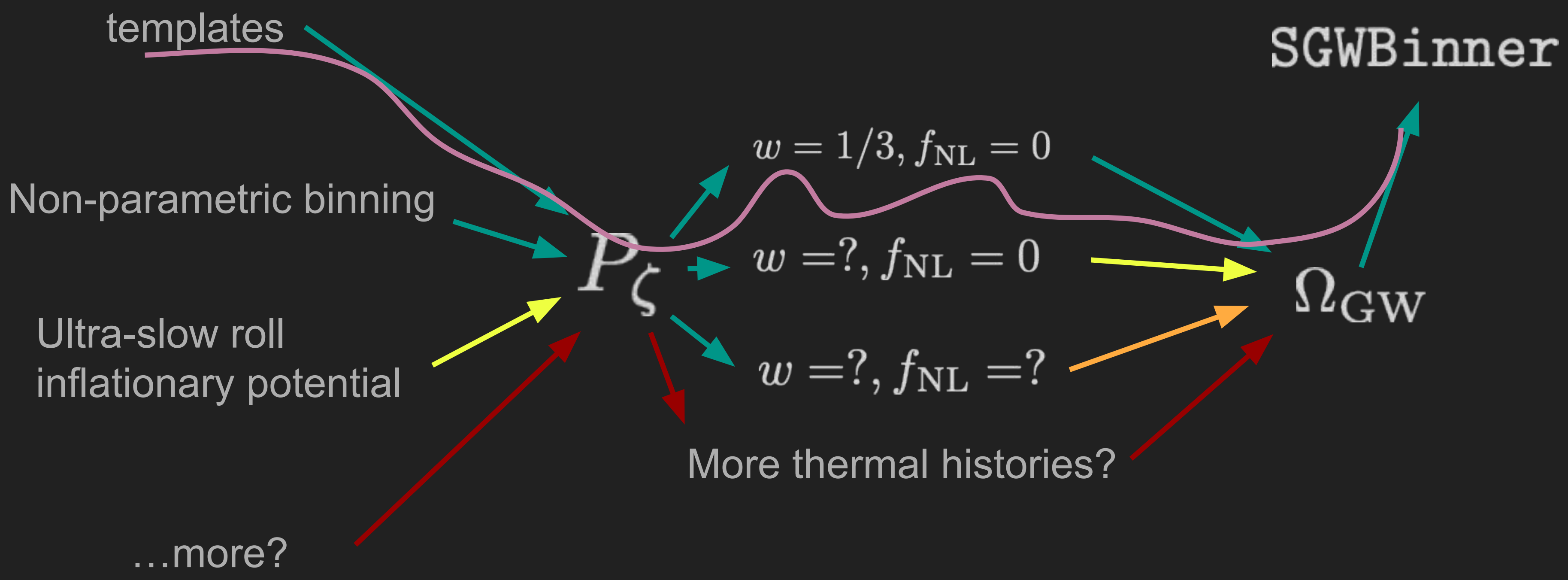
DALLE prompt: collapsing gas cloud

Pipeline

Legend:
- working
- almost working 🤞
- work in progress
- no ongoing work

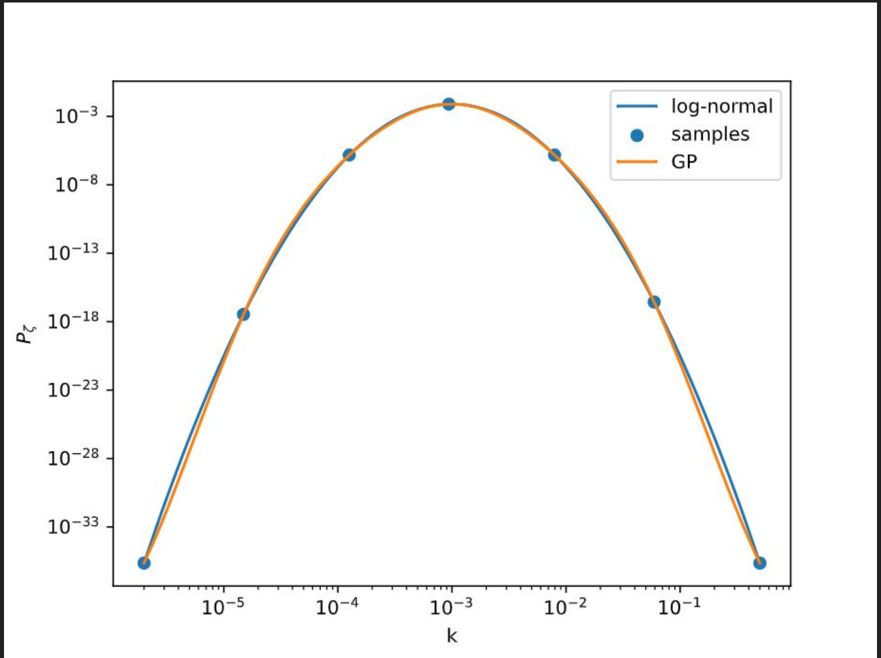


Let's look at a templated search



Templates

- Any parametrized model for $P_\zeta(k)$
- Pros: fast, Fisher posterior estimates available
- Cons: assumes model!
- Interesting cases: log-normal, broken power-law, inflationary sharp features, bouncing cosmologies



Induced Gravitational Waves in Radiation Domination

Large (gaussian) scalar perturbations enter the horizon and collapse!

[Aquavaria+ astro-ph/0209156; Baumann+ hep-th/0703290; Espinosa+ 1804.07732; Kohri+ 1804.08577]

$$\overline{\mathcal{P}_h(\eta,k)} = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left[\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right]^2 \overline{I^2(u,v,x)}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

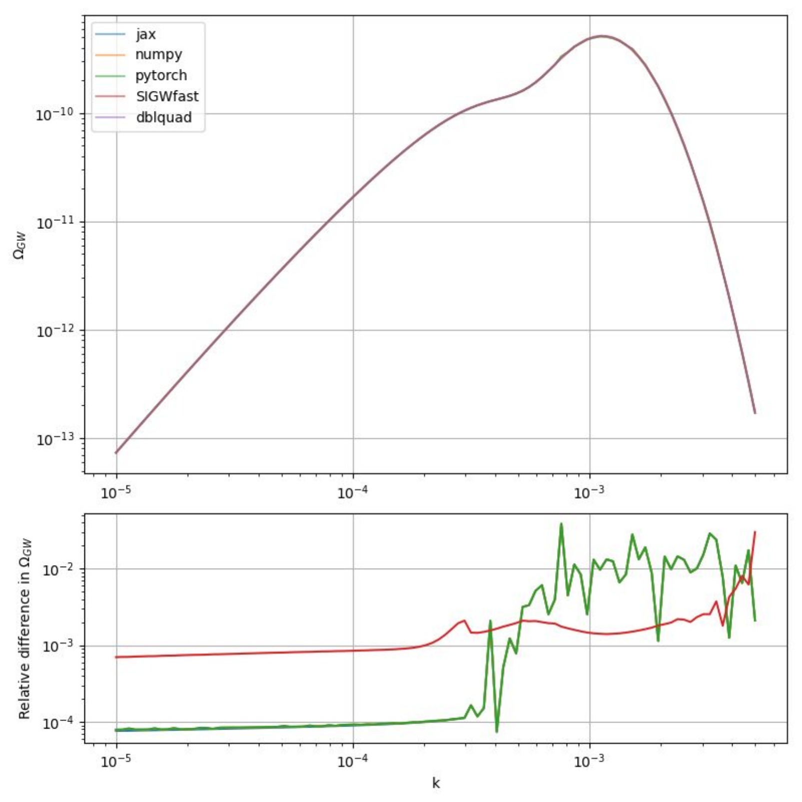
$$\overline{I^2_{\rm RD}(u,v,x\to\infty)} = \frac{1}{2}\left(\frac{3(u^2+v^2-3)}{4u^3v^3x}\right)^2 \left[\pi^2(u^2+v^2-3)^2\Theta(u+v-\sqrt{3}) + \left(-4uv+(u^2+v^2-3)\log\left|\frac{3-(u+v)^2}{3+(u-v)^2}\right|\right)^2\right]$$

Kernels are also known for arbitrary constant *w* [Domènech 2109.01398] or a matter-radiation transition [Domènech+ 2012.08151]

Computing SIGW

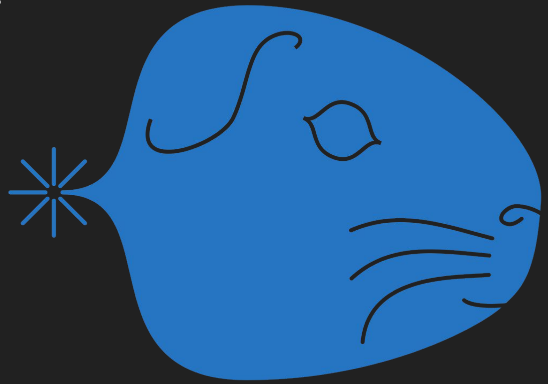
- We developed a **`jax`**-based integrator, needs ~5ms
- Packages exist in the literature, but are too slow for MCMC
  - E.g. **`SIGWFast`** [Witkowski 2209.05296]

method	nevals of integrand	time	rel. precision
scipy dblquad	10^6	5s	"exact"
SIGWfast	400 (interpolates P)	1s	$\sim 10^{-3}$
scipy simps	10^5	50 ms	$\sim 10^{-2}$
PyTorch simps	10^5	50 ms	$\sim 10^{-2}$
jax simps	10^5	5 ms	$\sim 10^{-2}$



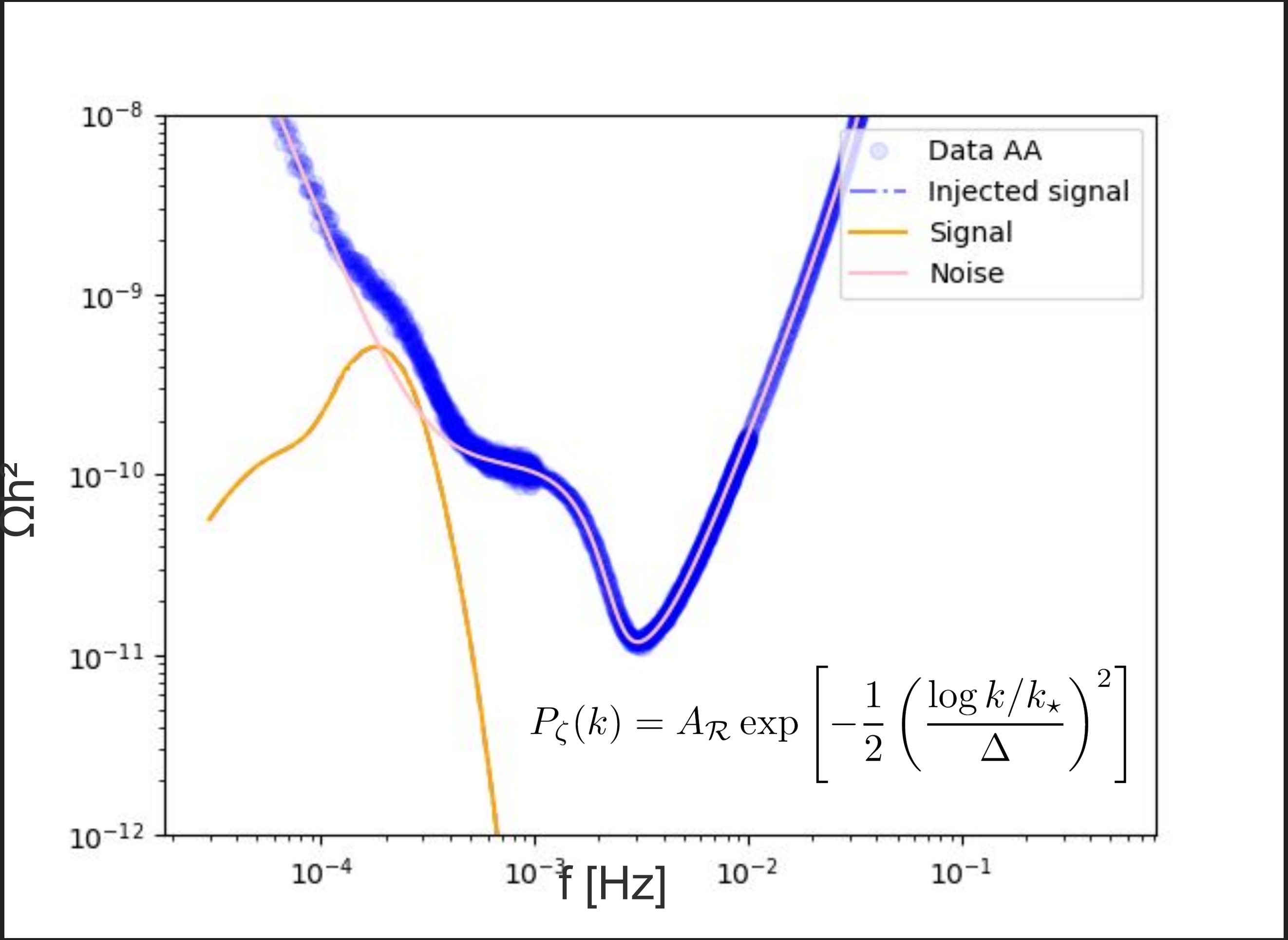
Inference with **Cobaya** and **SGWBinner**

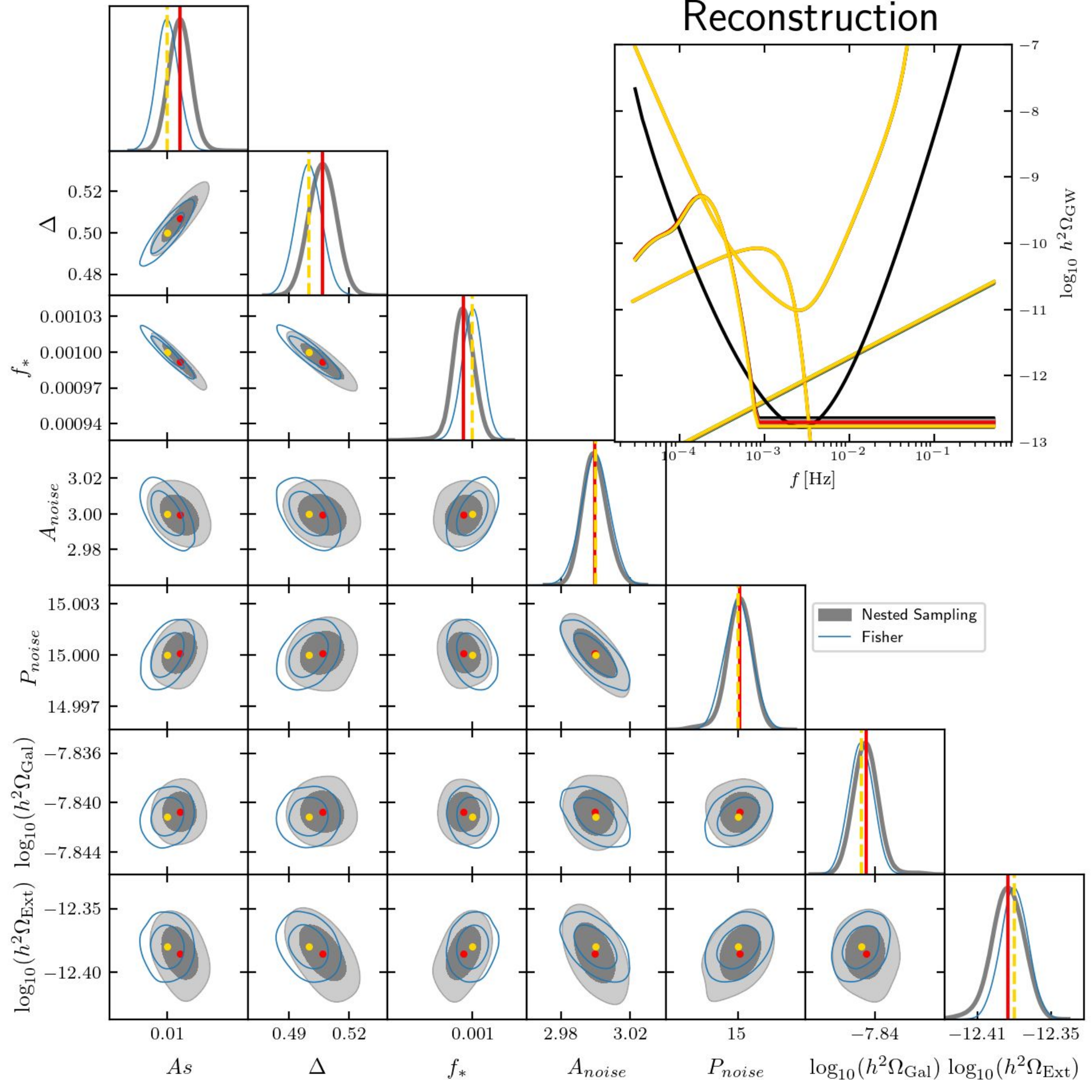
- Nested sampling with **Polychord**



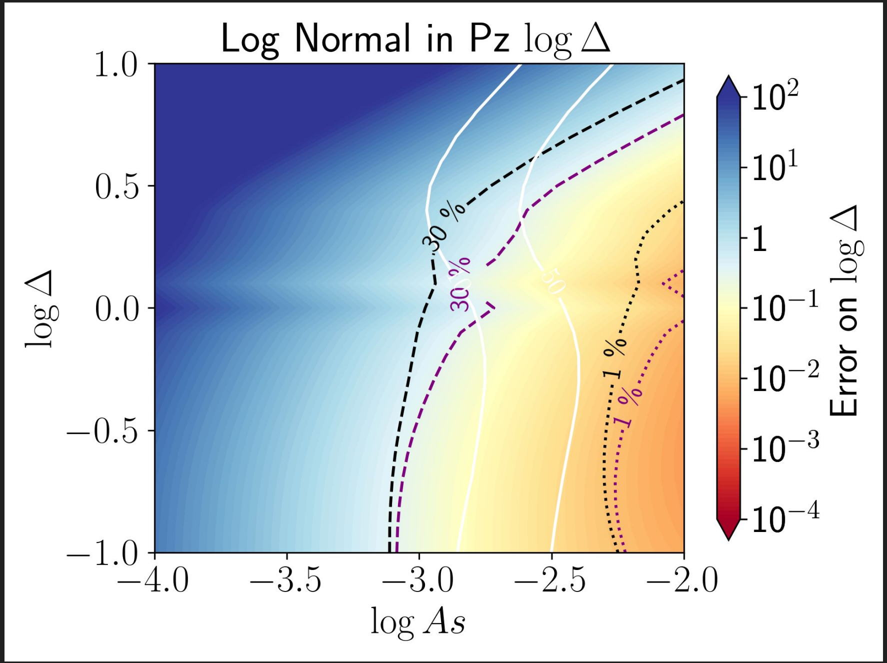
- **SGWBinner** [Flauger+ 2009.11845, Caprini+ 1906.09244]
  - developed in-house, well-tested
  - Parametric or non-parametric searches
  - Fisher estimates readily available

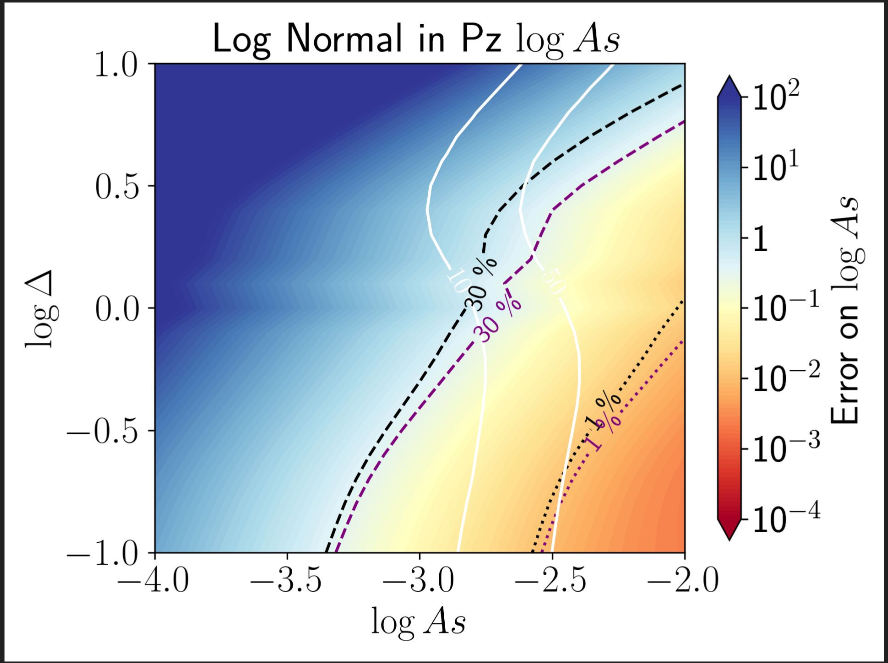
Lognormal results



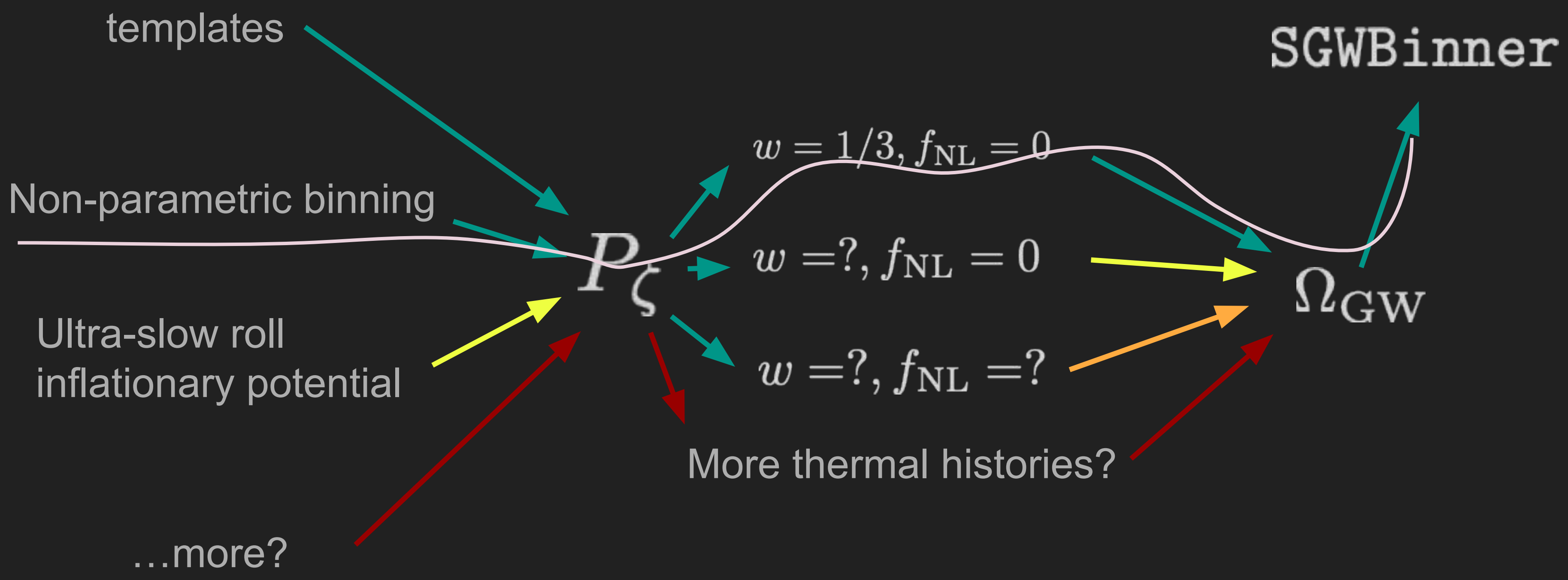

Reconstruction
Nested Sampling
Fisher
f [Hz]
$\log_{10} h^2\Omega_{\rm GW}$
As
Δ
f_*
A_{noise}
P_{noise}
$\log_{10}(h^2\Omega_{\rm Gal})$
$\log_{10}(h^2\Omega_{\rm Ext})$

How accurately can we recover a lognormal?





Let's look at a template-free search



Non-parametric binned powerspectra reconstruction

- Pros: Model-independent, can set constraints, reveal degeneracies, maybe invertible? (no)

- Cons: no analytic form, *many more* parameters

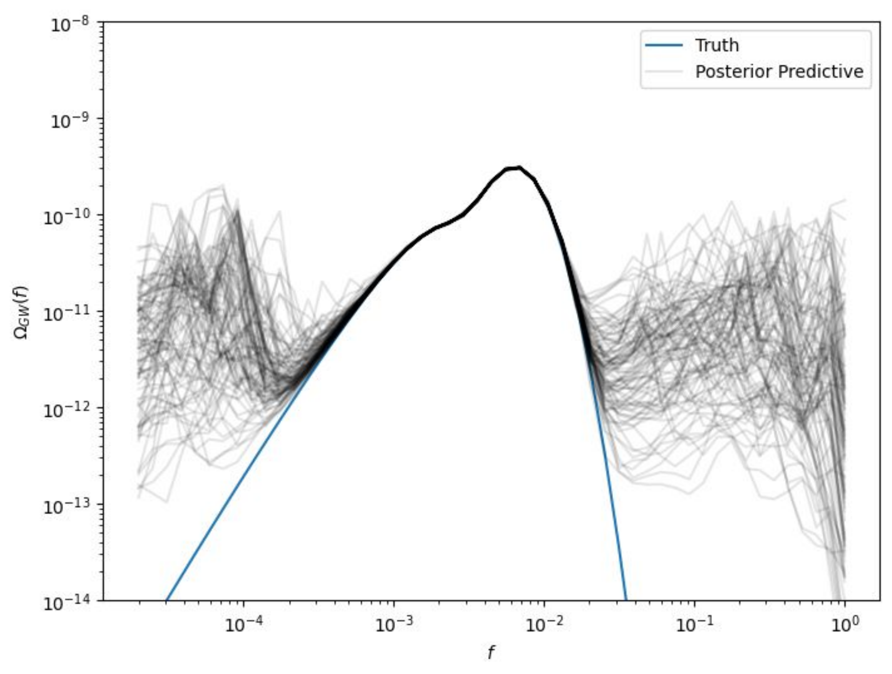
We use a very simple rectangular binning in log(f):

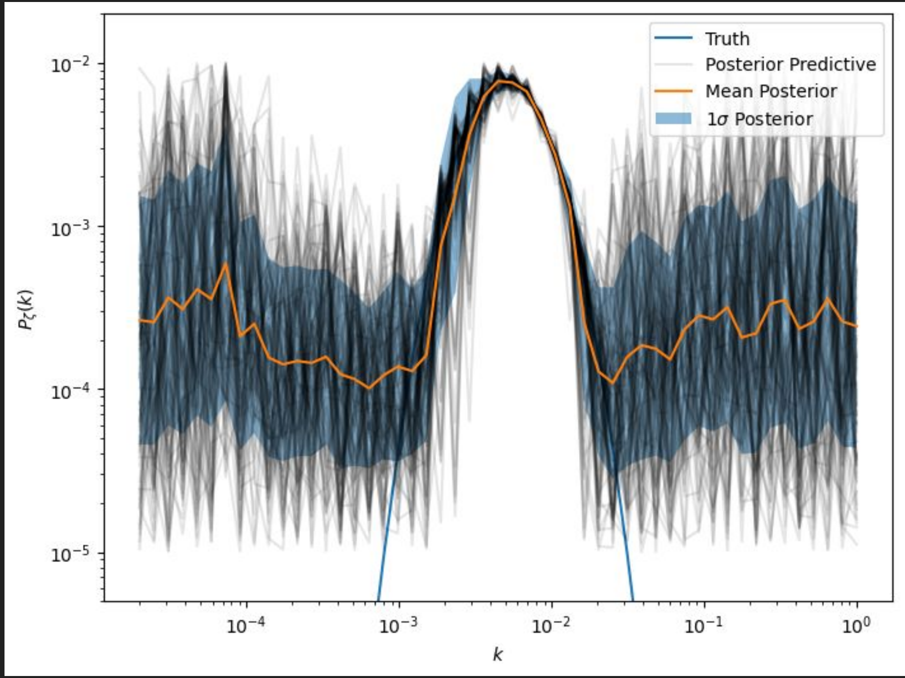
$$P_\zeta(k) = \sum_{i=1}^{N_p-1} A_i \theta(k - k_i)\theta(k_{i+1} - k)$$

$$\Omega_{\mathrm{GW}}(k) = \sum_{i,j=1}^{N_p-1} A_i A_j \Omega_{\mathrm{GW}}^{(i,j)}(k)$$

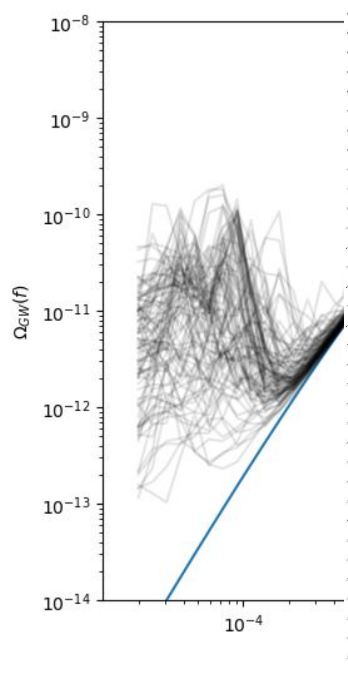
Only need to calculate basis functions once!

Lognormal - binned recovery (50x50x50 bins)

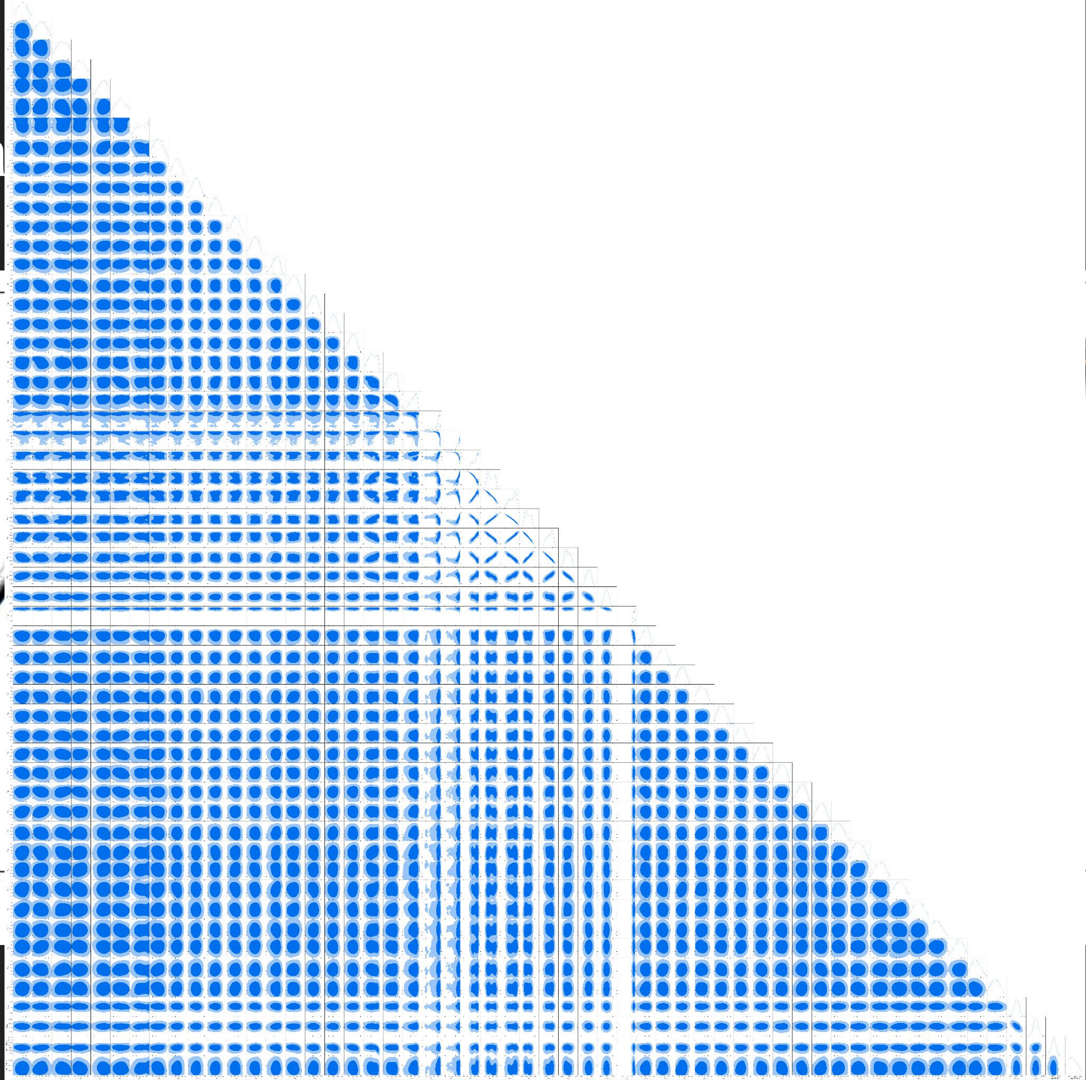


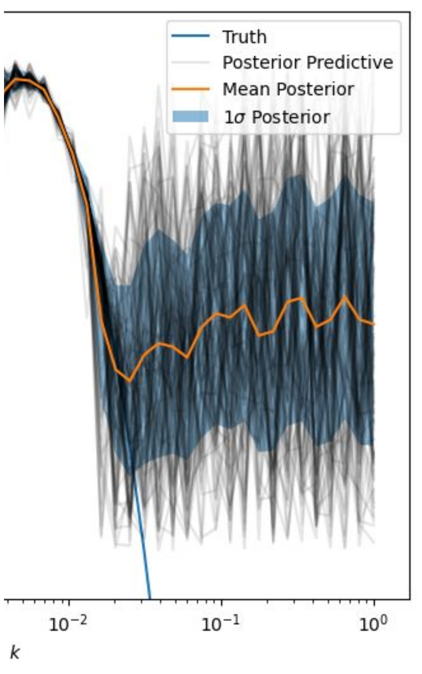


Lognorma



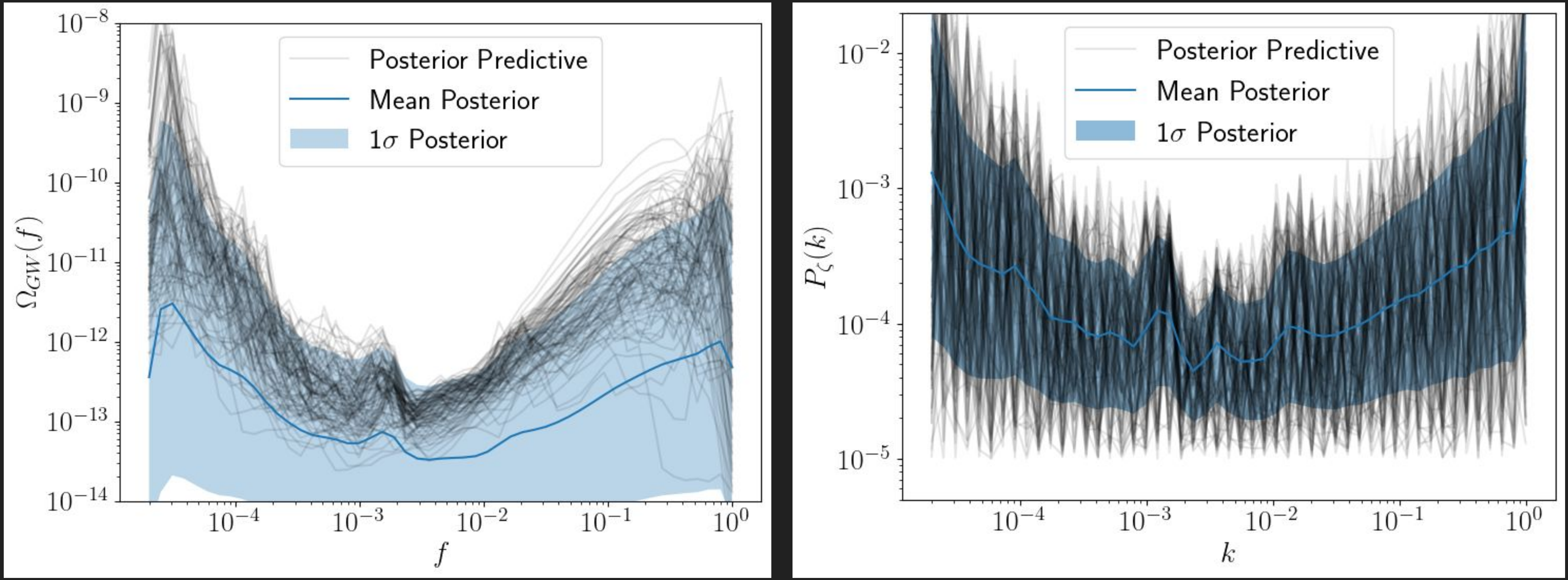
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
$\Omega_{GW}(f)$
10^{-4}



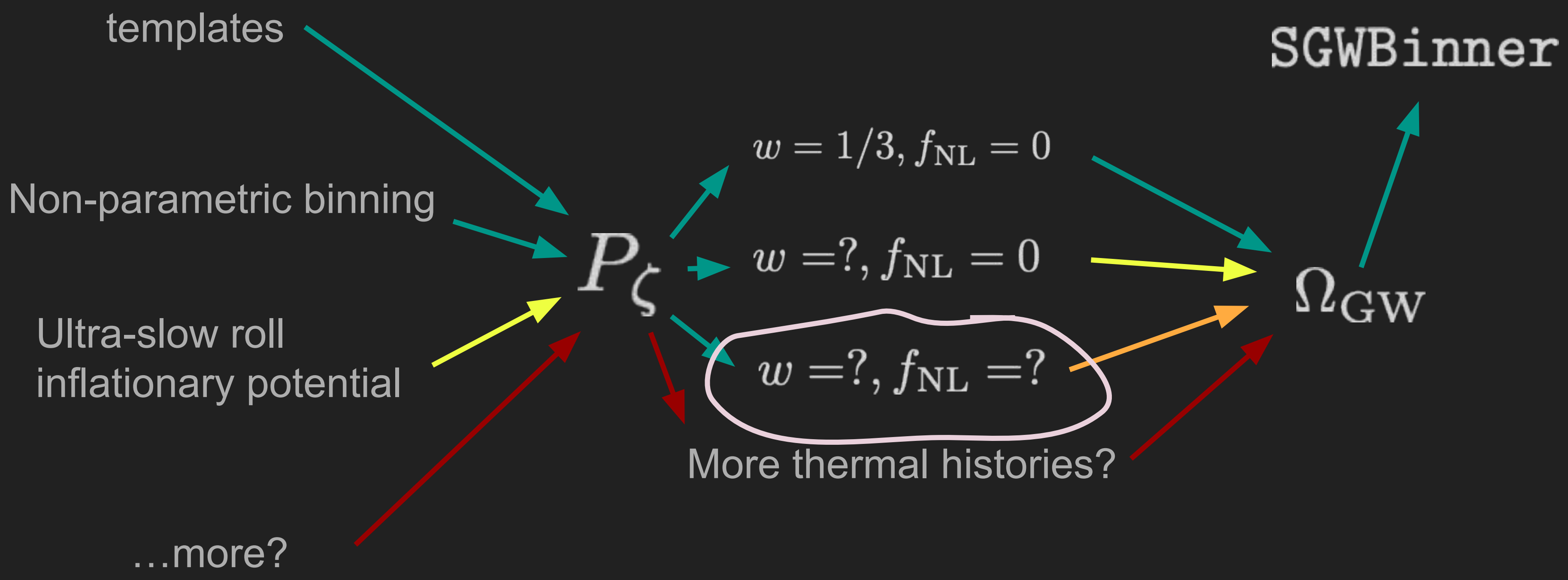


Truth
Posterior Predictive
Mean Posterior
1σ Posterior
10^{-2}
10^{-1}
10^{0}
k

Upper limits! No injection

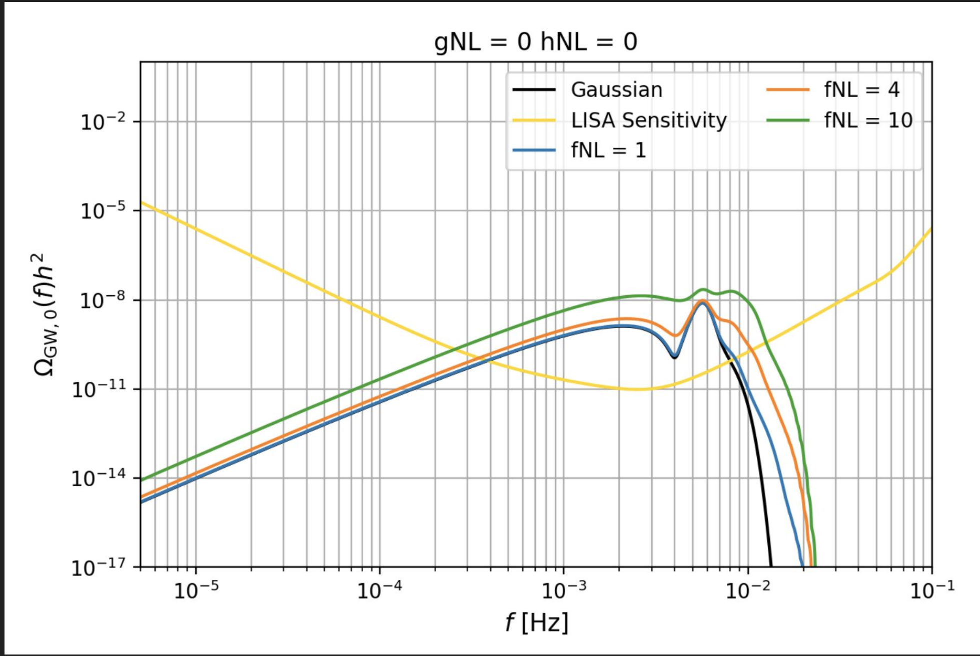


What about non-gaussianities?

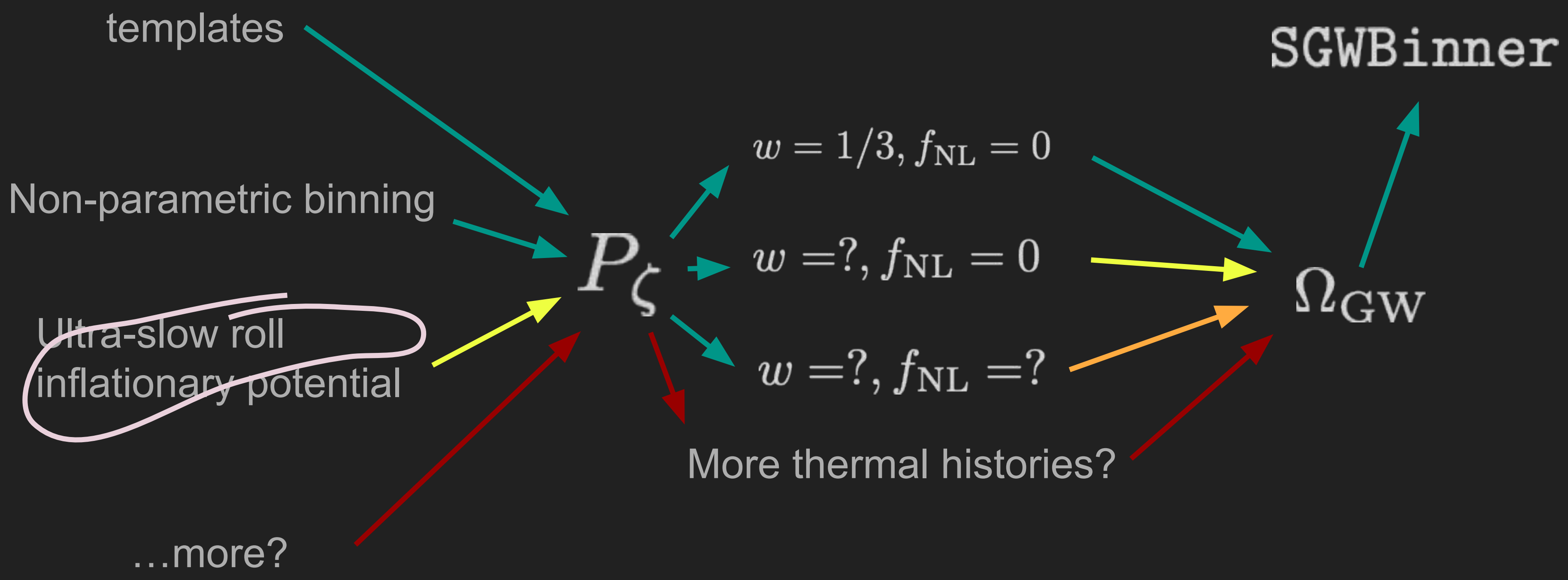


(Local) Non-gaussianities

- Use recent work from [Perna+ 2403.06962]
- High order results, include effects from $f_{\rm NL}, g_{\rm NL}, h_{\rm NL}$
- Needs 5D integrals evaluated, currently very slow
- Speed up with binning technique?



Inflationary reconstruction



Ultra slow-roll inflationary models

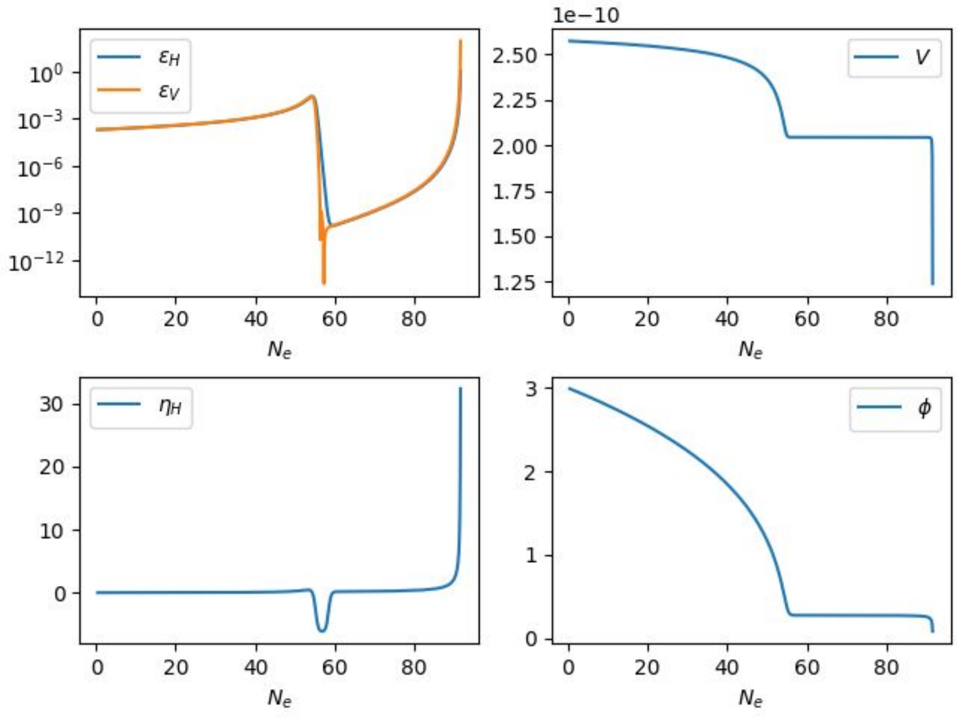
Many packages in literature:

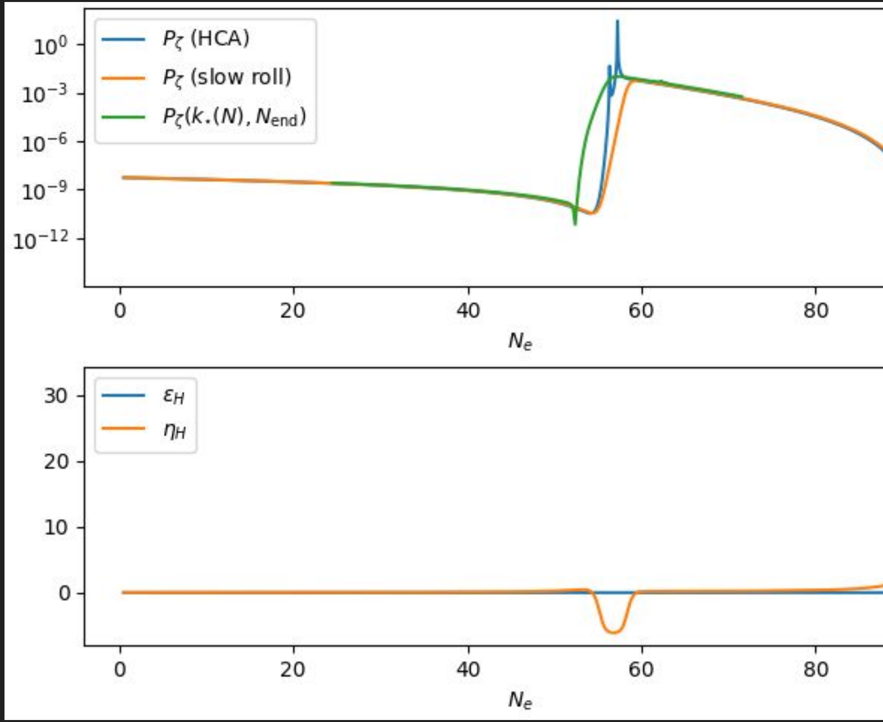
- `PyTransport` [Mulryne+ 1609.00381]
- `BINGO` [Hazra+ 1201.0926]
- `MultiModeCode` [Price+ 1410.0685]
- `Inflation.jl` [RR 2020]
- …

But we have a *need for speed*:

- New **`jax`** and **`diffrax`**-based solver for background and perturbations
- Re-scale the potential so solvers work with O(1) quantities
- 100 k-modes in ~0.3s

Some examples from [Cole+ 2304.01997]





Integration into `SGWBinner` ongoing

Conclusions / future work

- New methods for SIGW reconstruction (NG, EoS)
- Upper limits on scalar sensitivity
- New codes, available to CosmoWG
  - Fast inflationary solver
  - Scalar constraints module for `SGWBinner`
  - Inflationary model constraint module for `SGWBinner`
  - Fast non-gaussian SIGW code?
- Degeneracies?
- Are there shapes that *cannot be* scalar-induced?