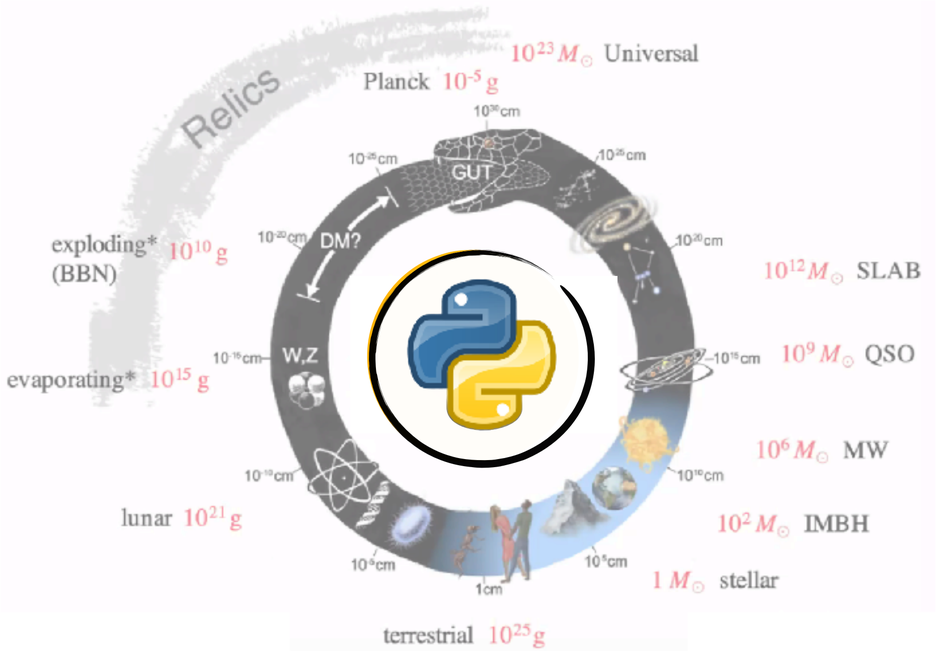
PrimBHoles: Pythonic PBH toolkit



Abundances: mass functions, PBH-DM, non-Gaussianities

Inflation models: Ultra-slow-roll, curvaton, hybrid models.

Thermal history: Phase transitions, formation threshold.

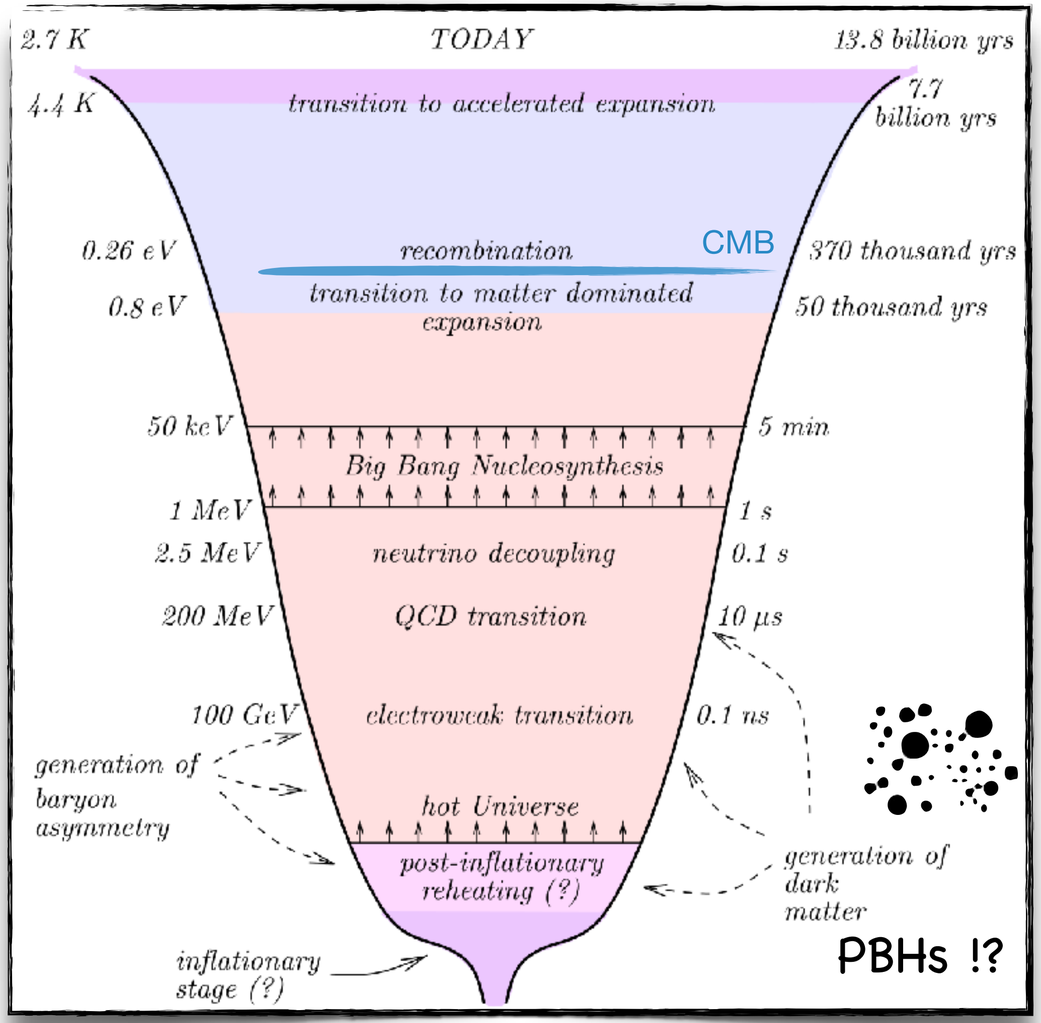
Merger rates: clustering, event rates

GWs: Binaries, SGWB, IGWs.

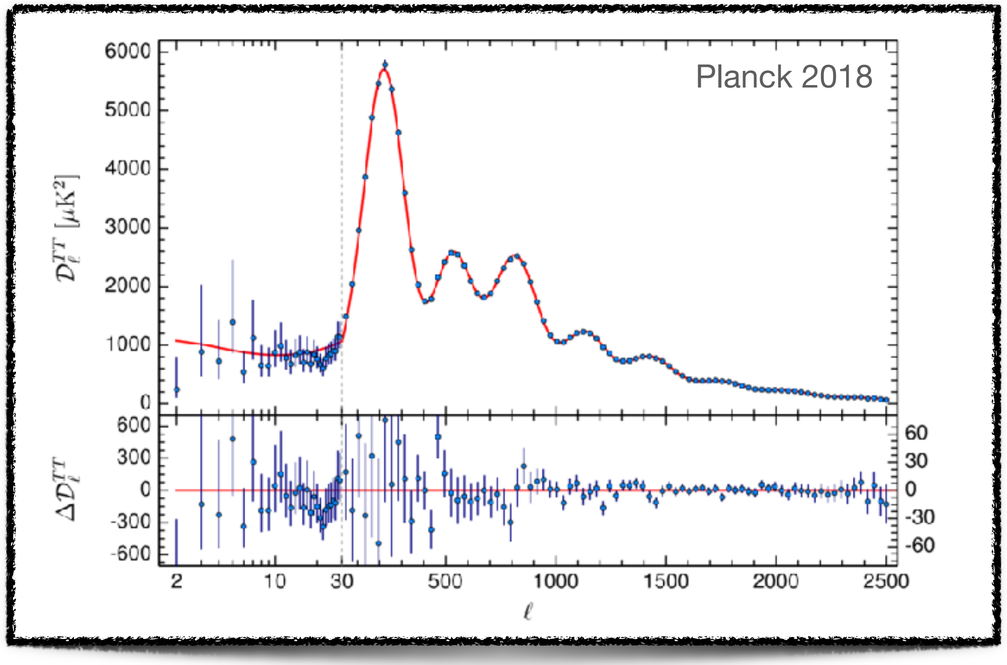
Others: Constraints, Exotic PBH formation channels.

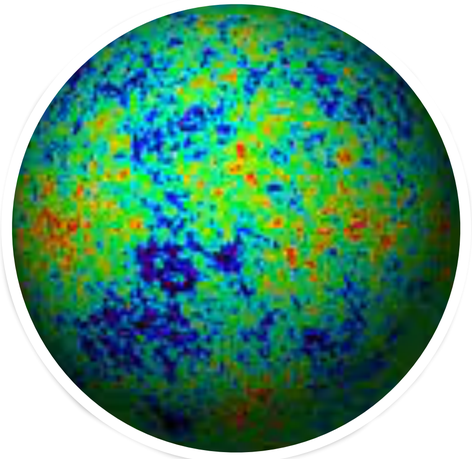
Cristian Joana (ITP-CAS) - LISA CosWG 11th, Porto U.

Inflation + LCDM cosmology ... and Primordial Black Holes!

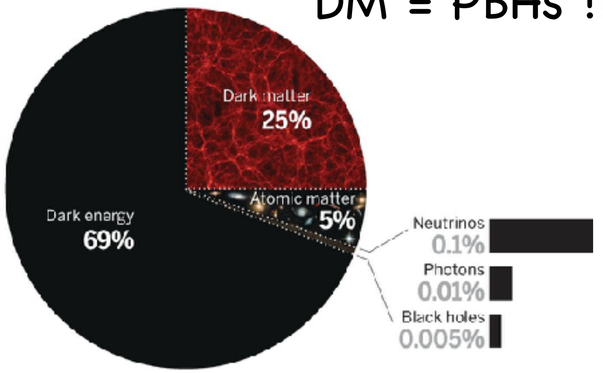


D.S. Gorbunov and V. A. Rubakov's book





DM = PBHs !?



PBH Dark Matter - PBH formation channel

Dark matter = Primordial BHs (?!)

> No new physics (BSM) besides gravity

> Distinguishable obs. from Astro-BHs

> Testable near future!
(LVK, LISA, Taiji, TianQin, PTAs)

> even if not all DM, their existence will tell us about early Cosmology!

Several formation channels:

> Collapse of curvature pert. from inflation

> Phase transitions, bubble nucleation

> Grav. collapse of cosmic strings, domain walls, oscillons, boson stars...

> Inflationary vacuum bubbles

> ...

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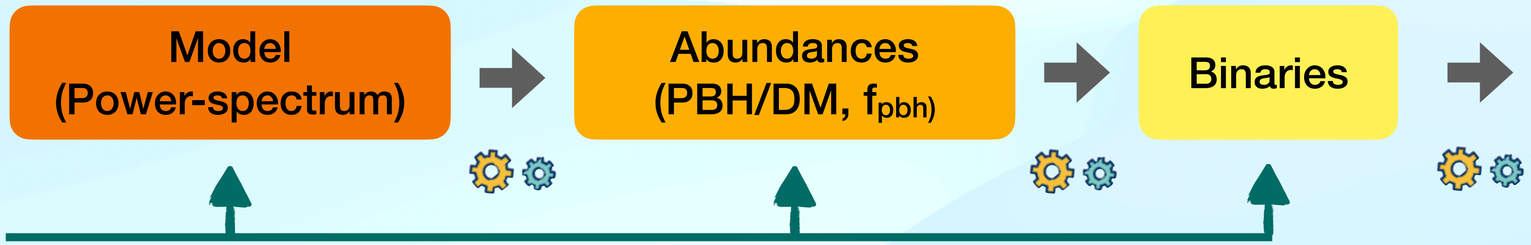
What is PrimBHoles ?

- ✓ Python code to compute PBH signatures
- ✓ Based on LISA PBH living review arXiv: 2310.1985
- ✓ Modular structure :



Merger rates (early/late binaries)

GWB (SIGW,early/ late binaries, close encounters)

Constraints (abundances, GWs)

- ✓ Public, Documented
- ✓ Easy to install, modify and use.

Authors:

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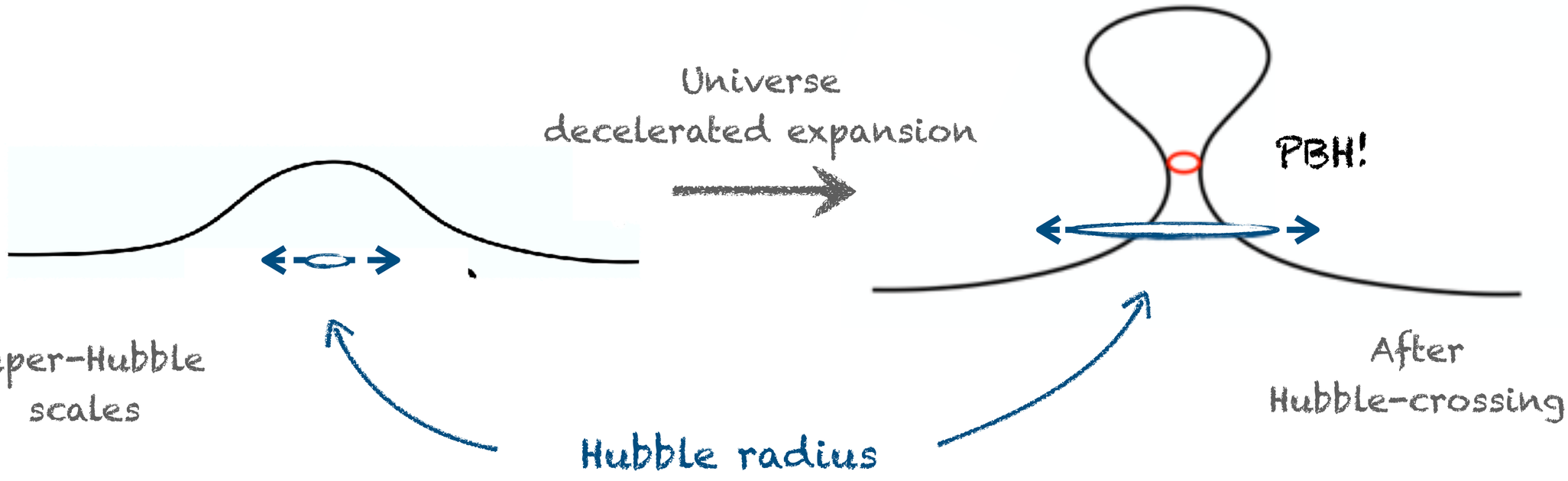
Juan Garcia-Bellido
Cristian Joana
Valerio de Luca
Gabriele Franciolini
Ilia Musco
Rajeev Kumar Jain
Theodoros Papanikolaou
Alvise Reccanelli
Sebastien Renaoux-Petel
Antonio Riotto
Ester Ruiz Morales
Marco Scalisi
Olga Sergijenko
Caner Unal
Vincent Vennin
David Wands

PBH formation: collapse of curvature perturbation

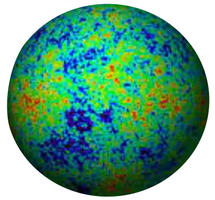
Curvature perturbation re-entry:

$$M_{\rm PBH} \sim \gamma\, M_H \sim k_\star$$



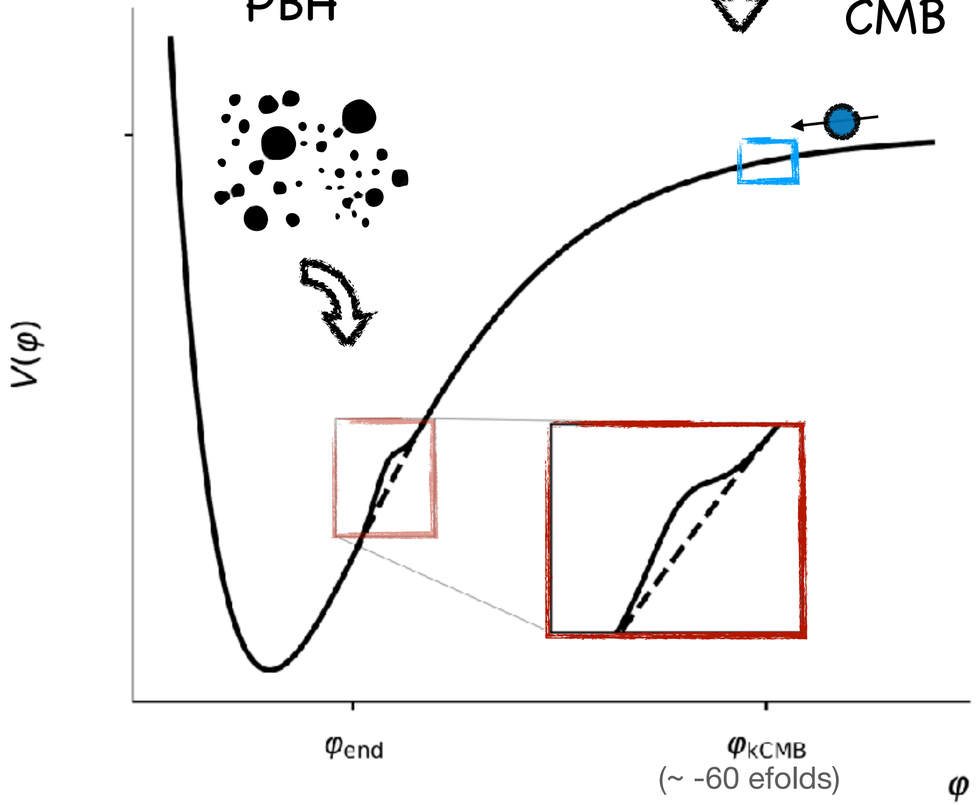
Inflationary - PBH cosmology

Quantum perturbations seed CMB/LSS and PBHs



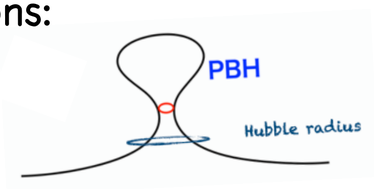
PBH

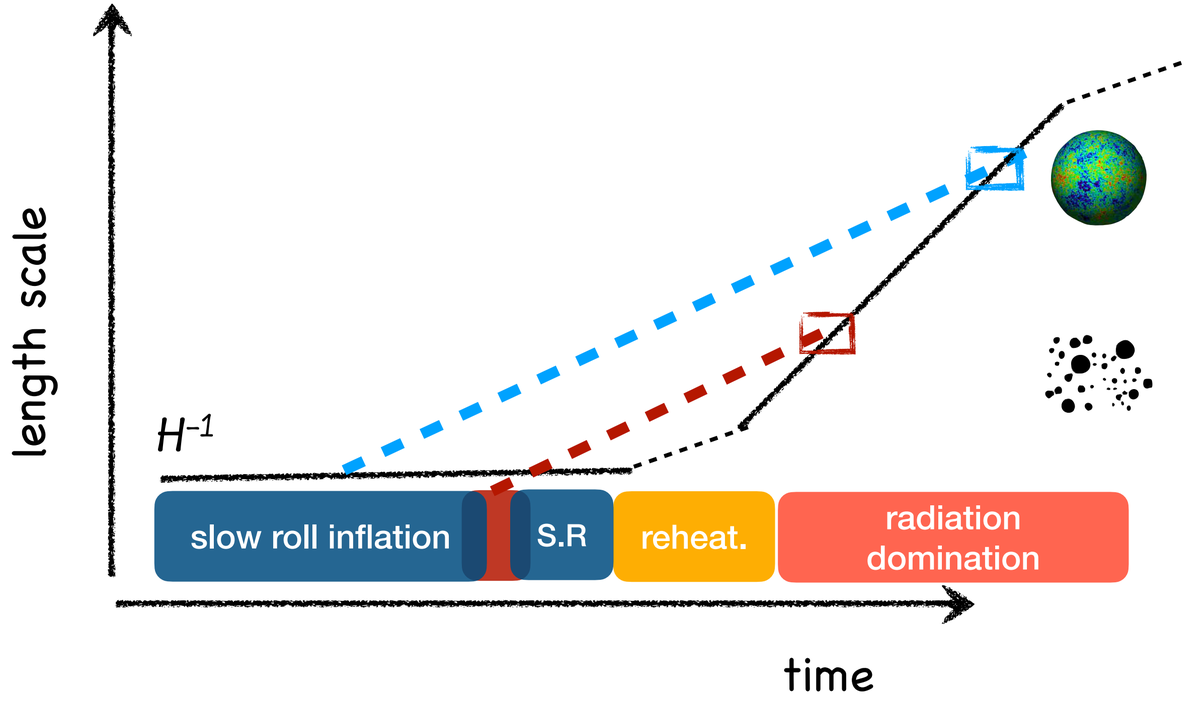
CMB



PBH forms after Hubble reentry of (rare) high-amplitude curvature perturbations:

$$M_{\rm PBH} \sim \gamma\, M_H \sim k_\star$$





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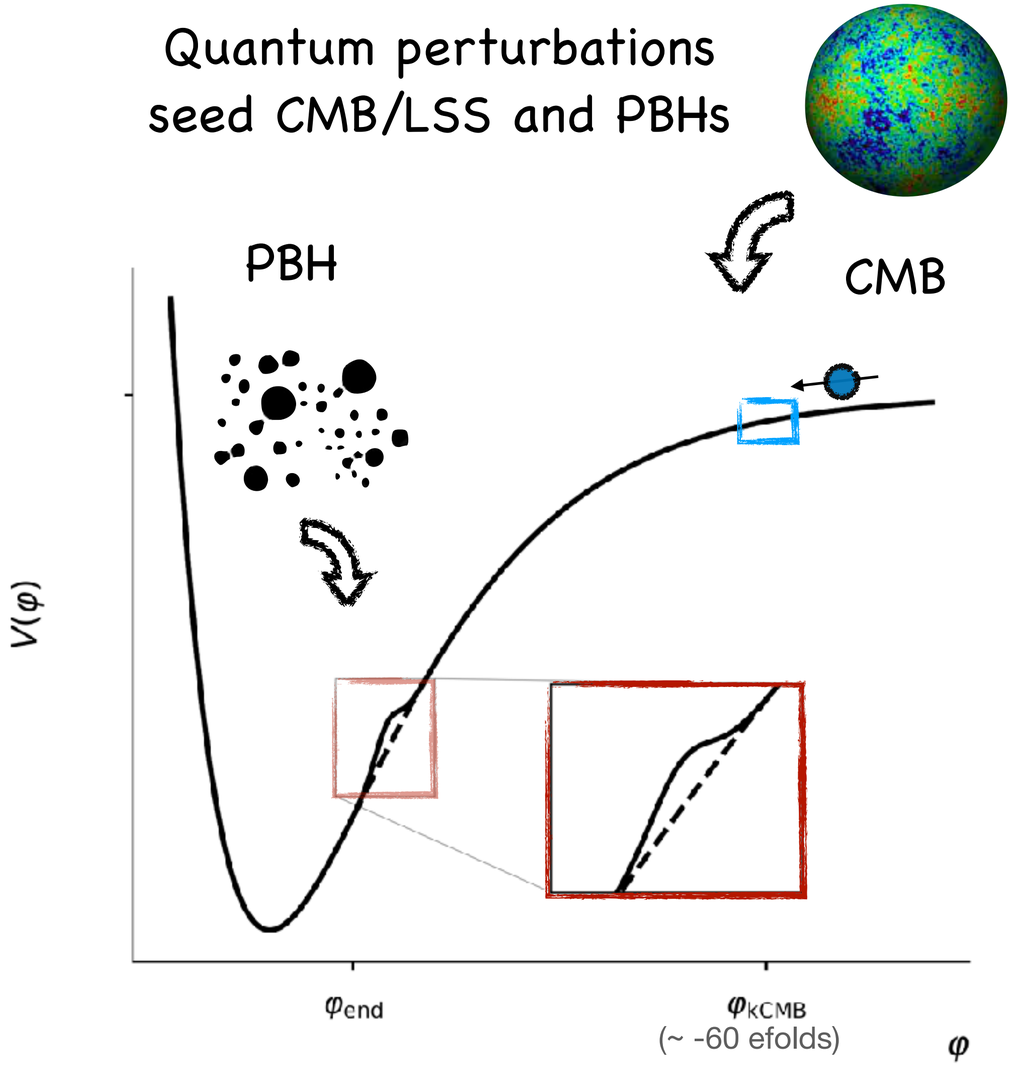
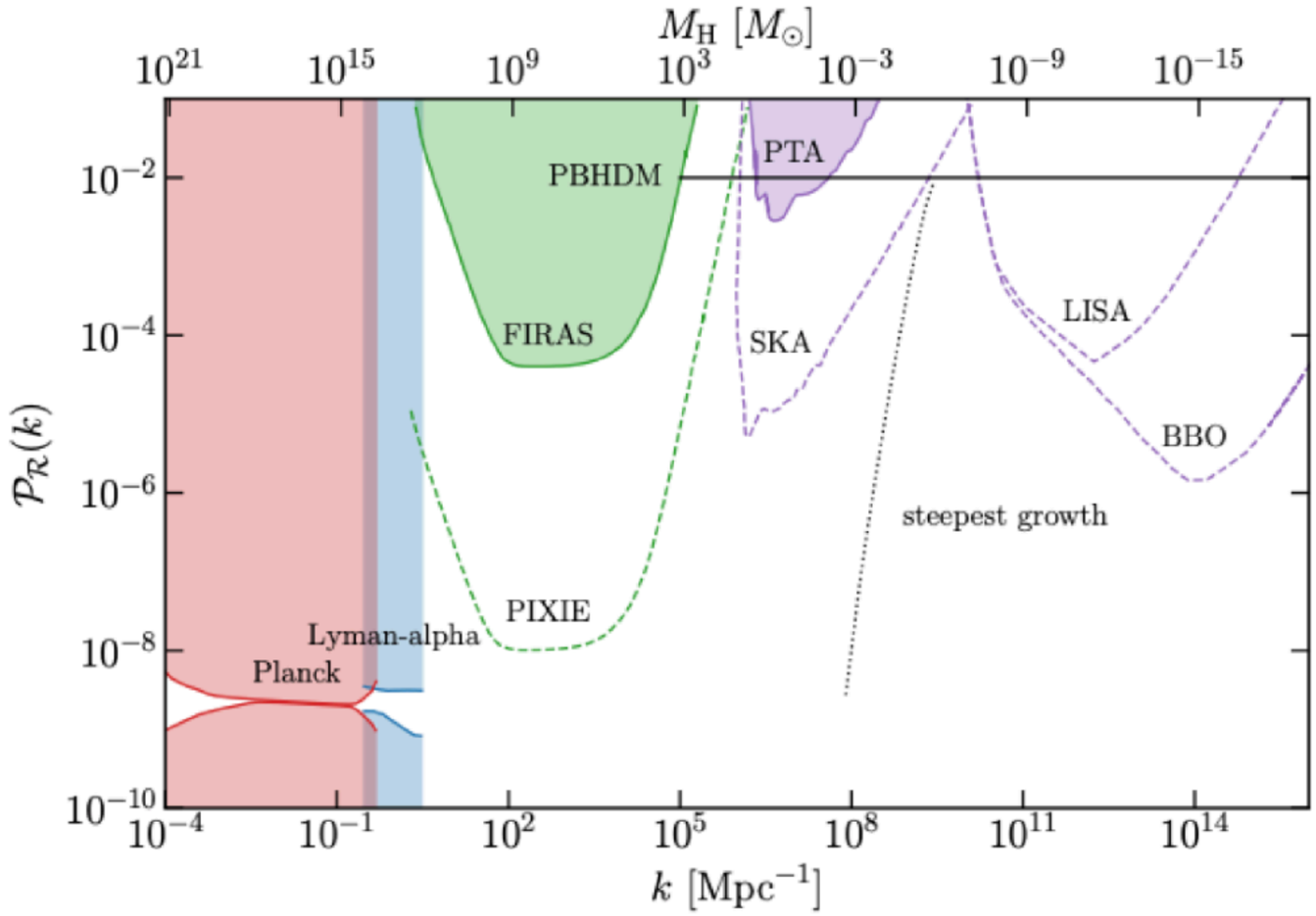
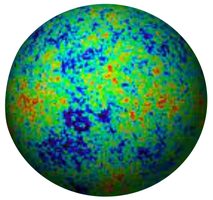


Green and Kavanagh arXiv:2007.10722



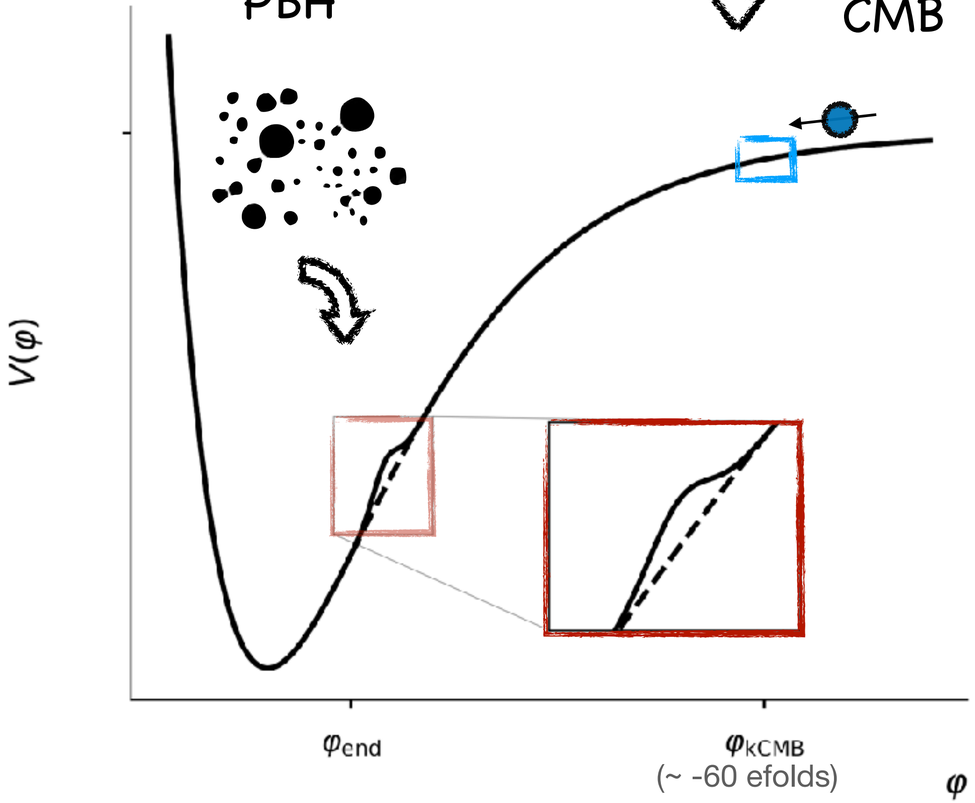
Inflationary - PBH cosmology

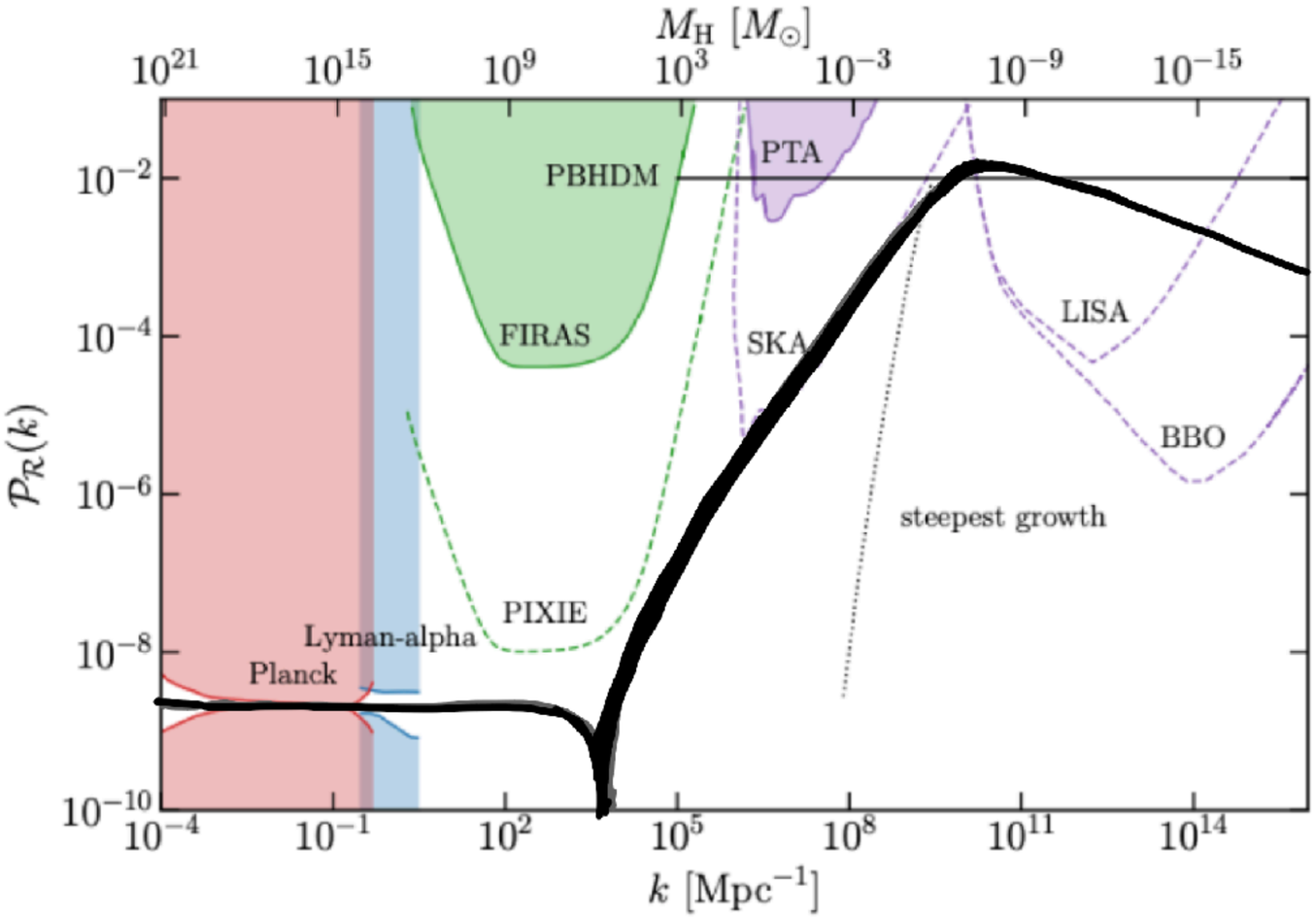
Quantum perturbations
seed CMB/LSS and PBHs



PBH

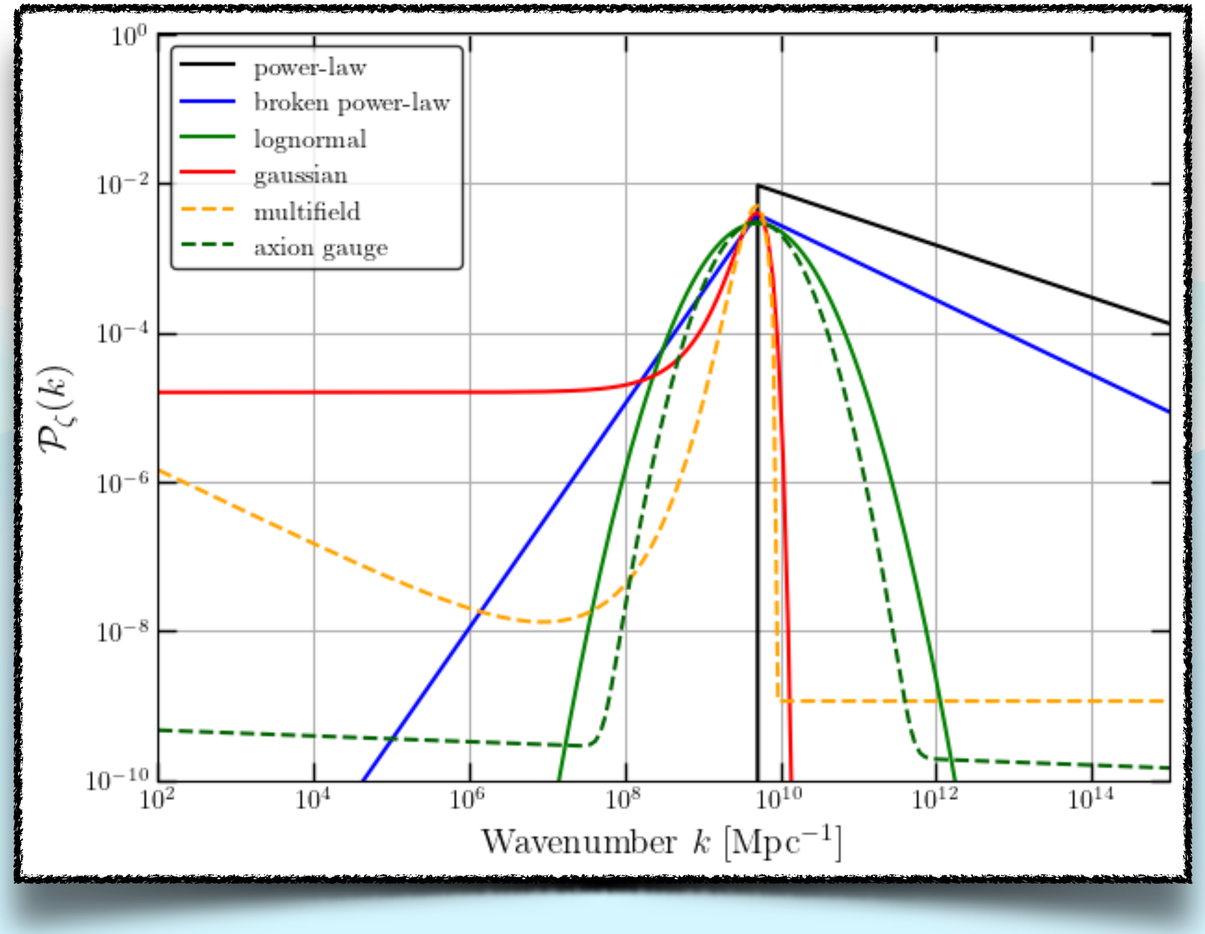
CMB





PrimBHoles — Power-spectrum

```python
import numpy as np
import primbholes
from power_spectrum import PowerSpectrum

# default Gaussian PS   (see param file)
my_PS = PowerSpectrum.gaussian()

# Model A: Gaussian
sigma =  0.3
As = 0.01*sigma
kp = 5e9
my_PS = PowerSpectrum.gaussian(As=As, sigma=sigma, kp=kp)

# own definition (lognormal)
def my_PS(k):
    sigma =  0.25
    As = 0.01#*sigma
    kp = 1e7
    return As * np.exp(- np.log(k / kp) ** 2 / (2 * sigma ** 2))

# call primbholes
pb =  primbholes(ps_function=my_PS)

# .... compute signatures (see next)
```



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Abundances: PBH mass function

Abundances using Press-Shechter for **Gaussian PDF**

$$\beta \equiv \left.\frac{\rho_{\rm PBH}}{\rho_{\rm total}}\right| = \gamma \int_{\delta_c} P(\delta)\mathrm{d}\delta = \gamma \int_{\delta_c} \frac{\mathrm{d}\delta}{\sqrt{2\pi}\sigma_{\rm PBH}} \exp\left[-\frac{\delta^2}{2\sigma^2_{\rm PBH}}\right]$$

where the variance is given by

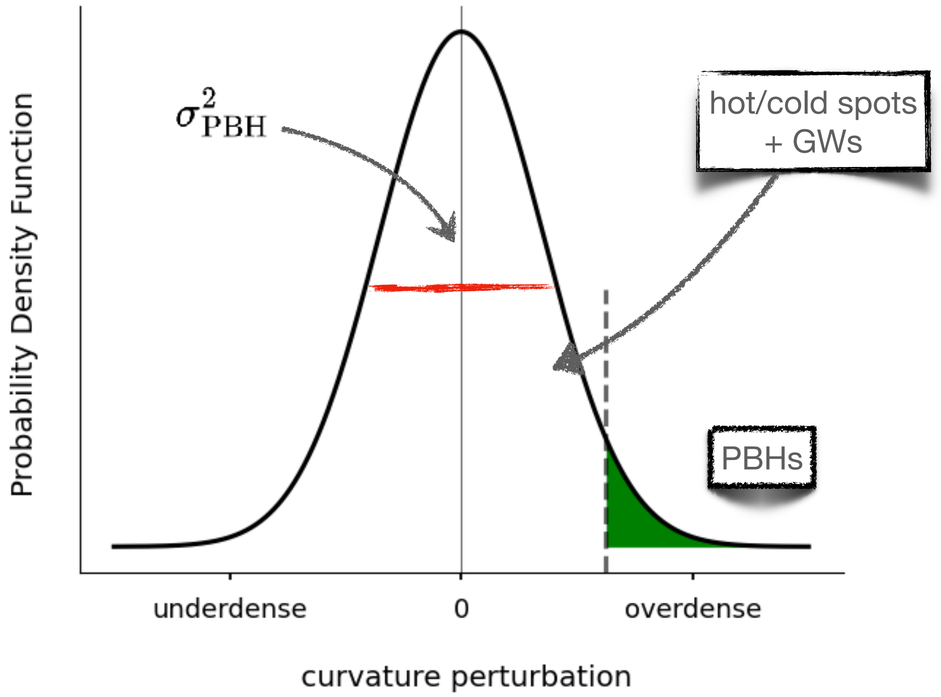
$$\sigma^2_{\rm PBH} = \int_0^\infty \frac{\mathrm{d}k}{k} \mathcal{P}_\delta(k,r) = \frac{16}{81} \int_0^\infty \frac{\mathrm{d}k}{k} (kr)^4 W^2 \mathcal{P}_\zeta(k)$$

the PBH/DM ratio then is $$f_{\rm PBH} = \frac{1}{\Omega_{\rm CDM}} \frac{\mathrm{d}\Omega_{\rm PBH}}{\mathrm{d}\ln M_{\rm PBH}}$$

where $$\Omega_{\rm PBH} \sim \int_{M_{\rm min}}^{M_{\rm max}} \beta(M_{\rm PBH})\, \mathrm{d}\ln M_{\rm PBH}$$

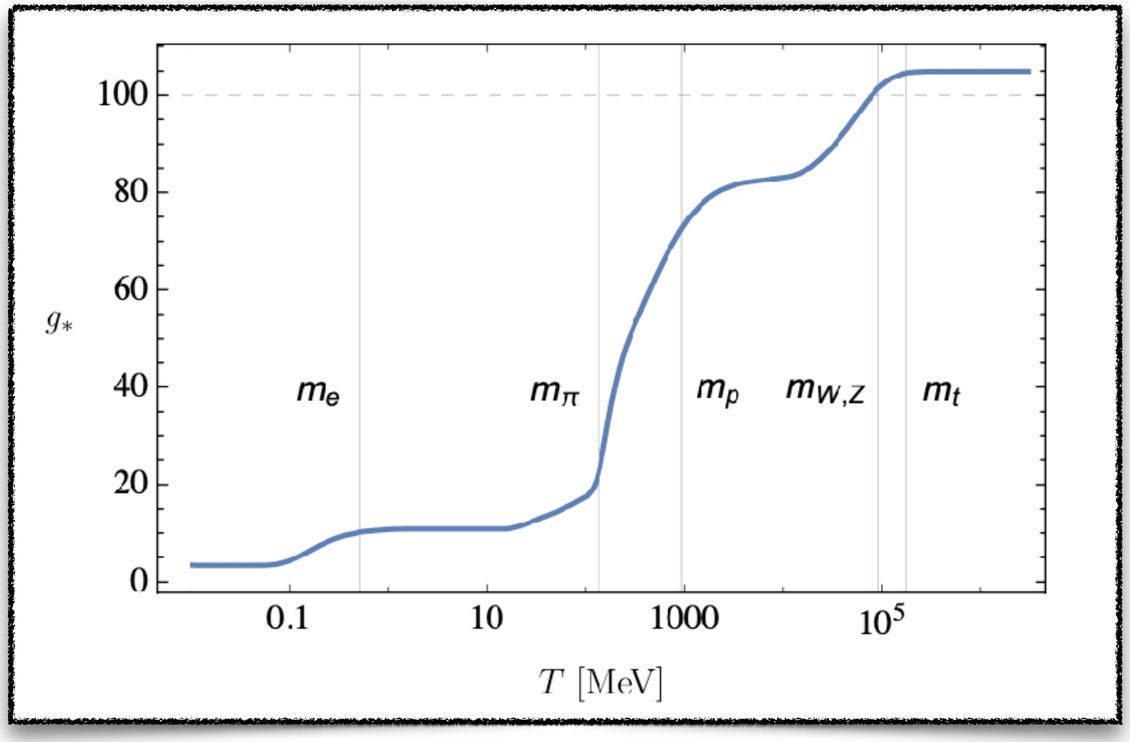
inflation model :

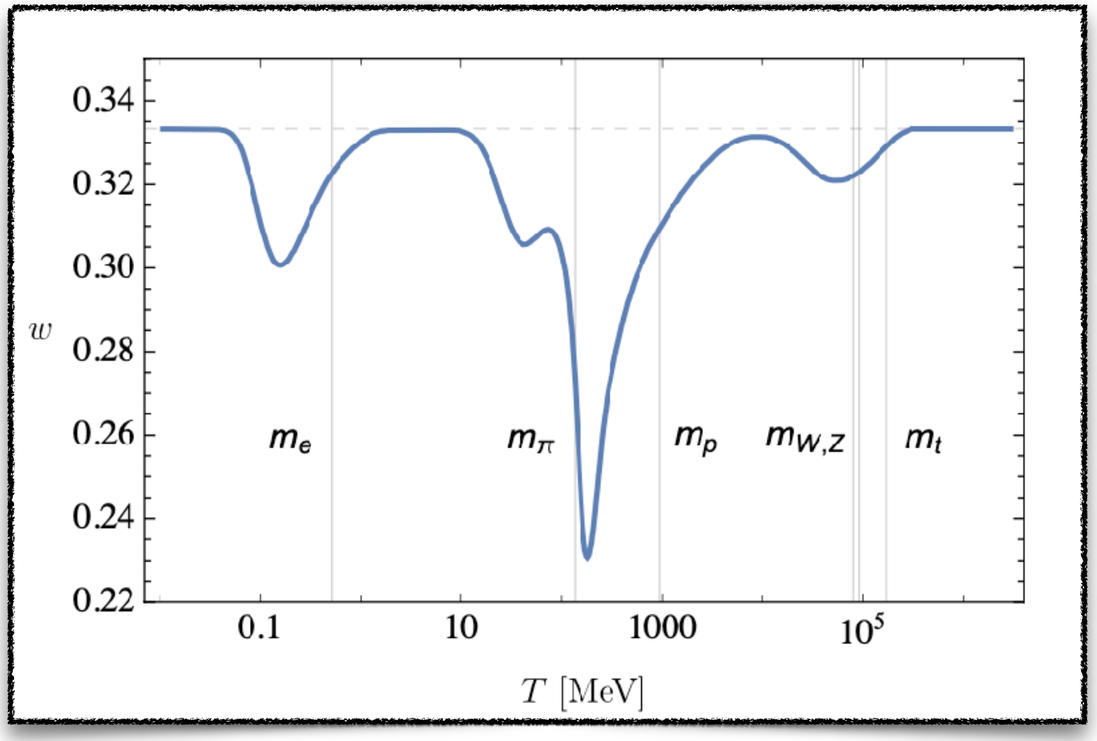
$$P(k_{\rm form}, \delta) \longrightarrow P(M_{\rm PBH}, \delta) \longrightarrow \beta(M_{\rm PBH})$$



Collapse Threshold : Thermal History

Fluctuation in the equation of state, reduce the fluid pressure → enhancement of PBHs!

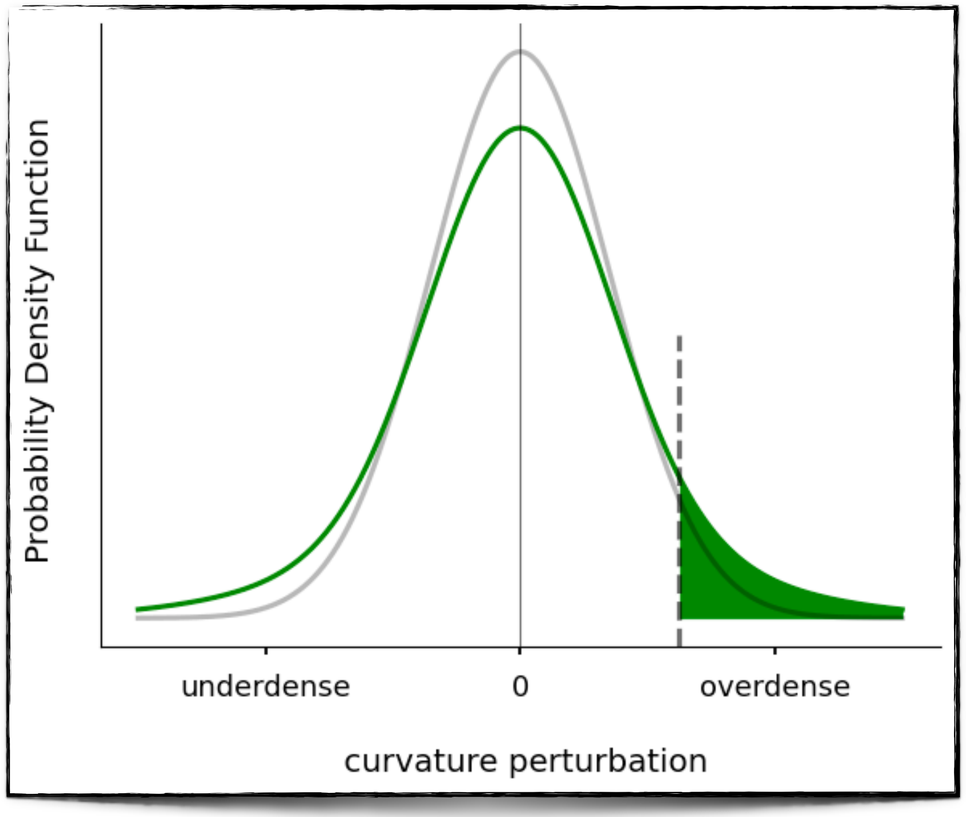


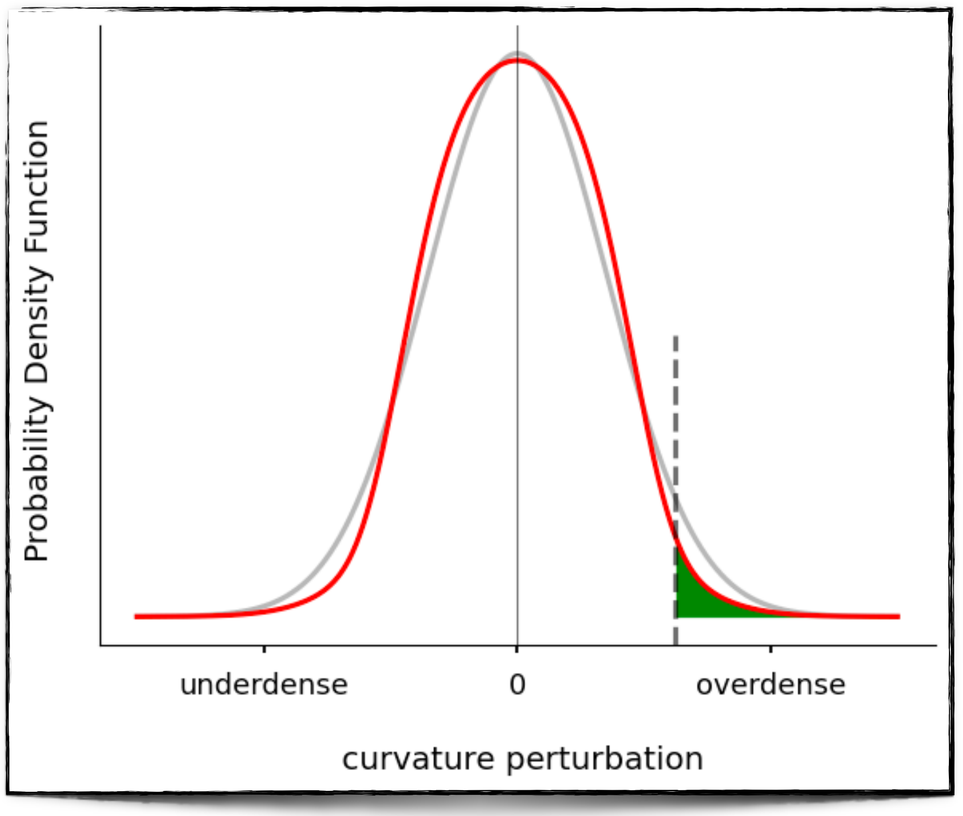


...tabulated input from Numerical Relativity.

Collapse Threshold : Non-Gaussianities

Shape of the curvature Prob. Distr. Func. (and Non-Gaussianities) modify the critical threshold





...analytical prescription fitted from Numerical Relativity.

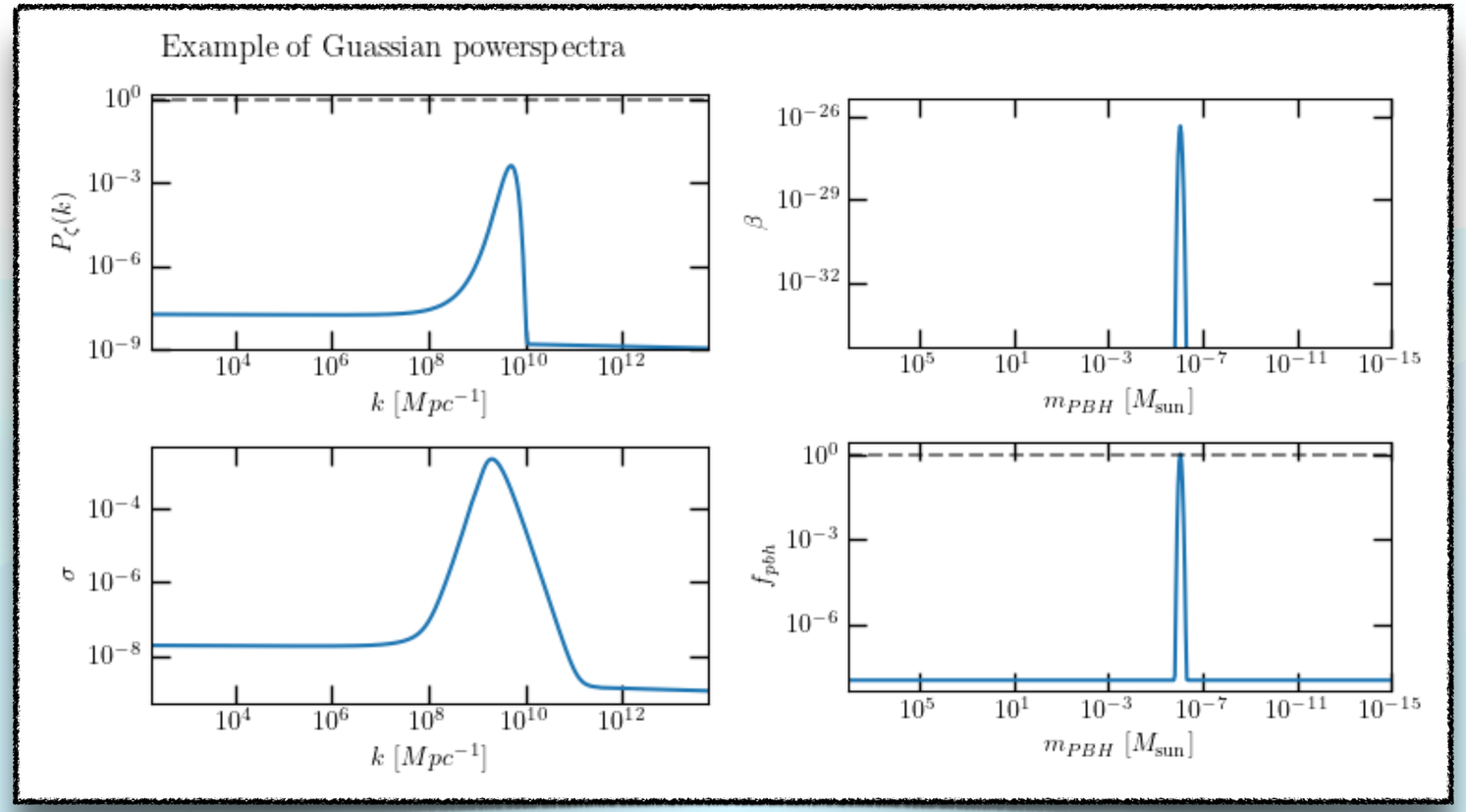
PrimBHoles — Abundances

```python
import numpy as np
import primbholes
from power_spectrum import PowerSpectrum

# own definition
def my_PS(k):
    ...

# call primbholes
pb = primbholes(ps_function = my_PS,
        threshold_method="ShapePrescription",  # numeric
        # threshold_method= "Standard",        # i.e. de
        thermal_history = False, # True
        fpbh_rescaling=False,     # Rescale PS, to enfor
        #...
        )

# compute abundances
fpbh = lambda mass: pb.get_fPBH(mass)
beta = lambda mass: pb.get_beta(mass)
```



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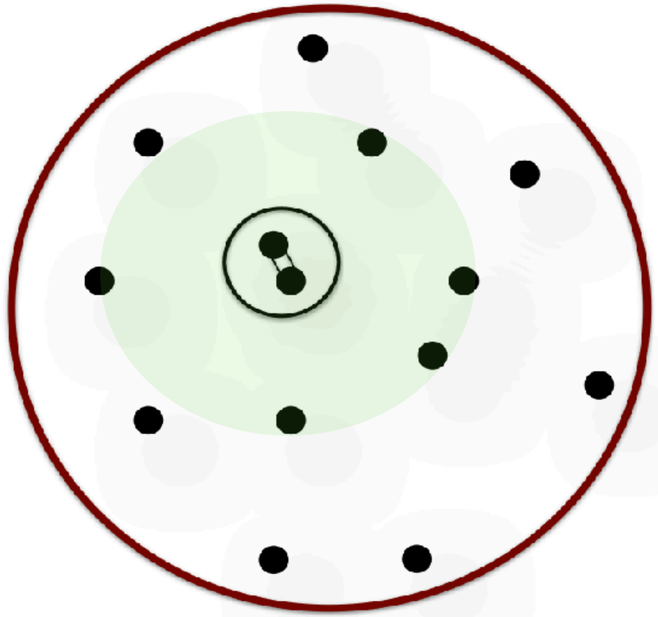
Binary formation, merger rates

- ***Initial spatial Poisson distribution***
- ***Random decoupling of binary systems from the Hubble flow***

↓

- ***Binary formation happening before matter-radiation equality***
- ***The distribution of initial semi-major axis and eccentricity determines the merger rate***



$$\frac{\mathrm{d}R}{\mathrm{d}m_1\mathrm{d}m_2} = \frac{1.6\times10^6}{\mathrm{Gpc^3\,yr}} f_{\rm PBH}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \left(\frac{M_{\rm tot}}{M_\odot}\right)^{-\frac{32}{37}} S\left(M_{\rm tot}, f_{\rm PBH}\right)\ \psi(m_1)\psi(m_2)$$

Suppression factors (enviromental,...)

$$S_{\rm early} \approx 1.42 \left[\frac{\langle m^2\rangle/\langle m\rangle^2}{\bar{N}(y)+C} + \frac{\sigma_{\rm M}^2}{f_{\rm PBH}^2}\right]^{-21/74} \exp\left[-\bar{N}(y)\right]$$

$$\text{with} \qquad \bar{N}(y) \equiv \frac{M}{\langle m\rangle}\left(\frac{f_{\rm PBH}}{f_{\rm PBH}+\sigma_{\rm M}}\right)$$

T. Nakamura, M. Sasaki, T. Tanaka, and K. S. Thorne, Astrophys. J. Lett. 487, L139 (1997), [arXiv:9708060]
K. Ioka, T. Chiba, T. Tanaka, and T. Nakamura, Phys. Rev. D 58, 063003 (1998), [arXiv:9807018]

Y. N. Eroshenko, J. Phys. Conf. Ser. **1051**, no.1, 012010 (2018) [arXiv:1604.04932]
Y. Ali-Haïmoud, E. D. Kovetz, and M. Kamionkowski, Phys. Rev. D **96**, 123523 (2017), [arXiv:1709.06576]
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Initial spatial Clustering

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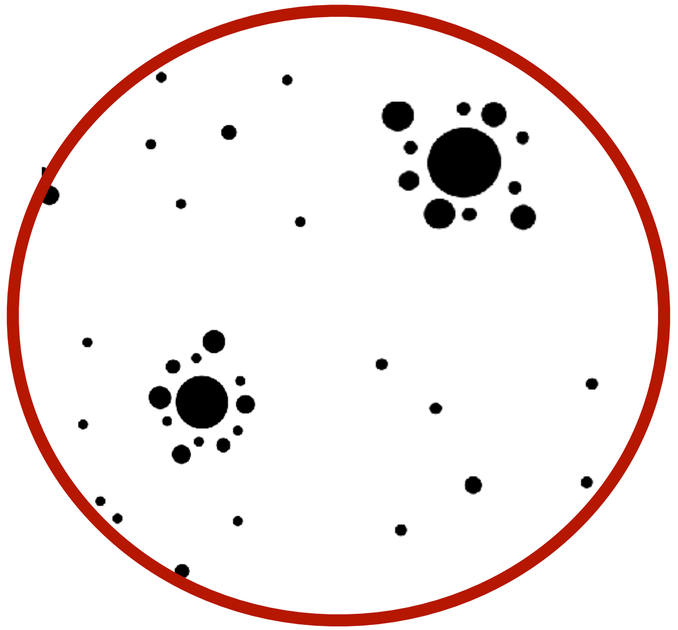
late binaries



$$R_{\rm LB} \approx R_{\rm clust} f_{\rm PBH}^2 f(m_1) f(m_2) \frac{(m_1+m_2)^{10/7}}{(m_1 m_2)^{5/7}} {\rm yr}^{-1}{\rm Gpc}^{-3}$$

→ Binaries from 3-body interactions

\+

→ Binaries from Hyperbolic encounters

T. Nakamura, M. Sasaki, T. Tanaka, and K. S. Thorne, Astrophys. J. Lett. 487, L139 (1997), [arXiv:9708060]
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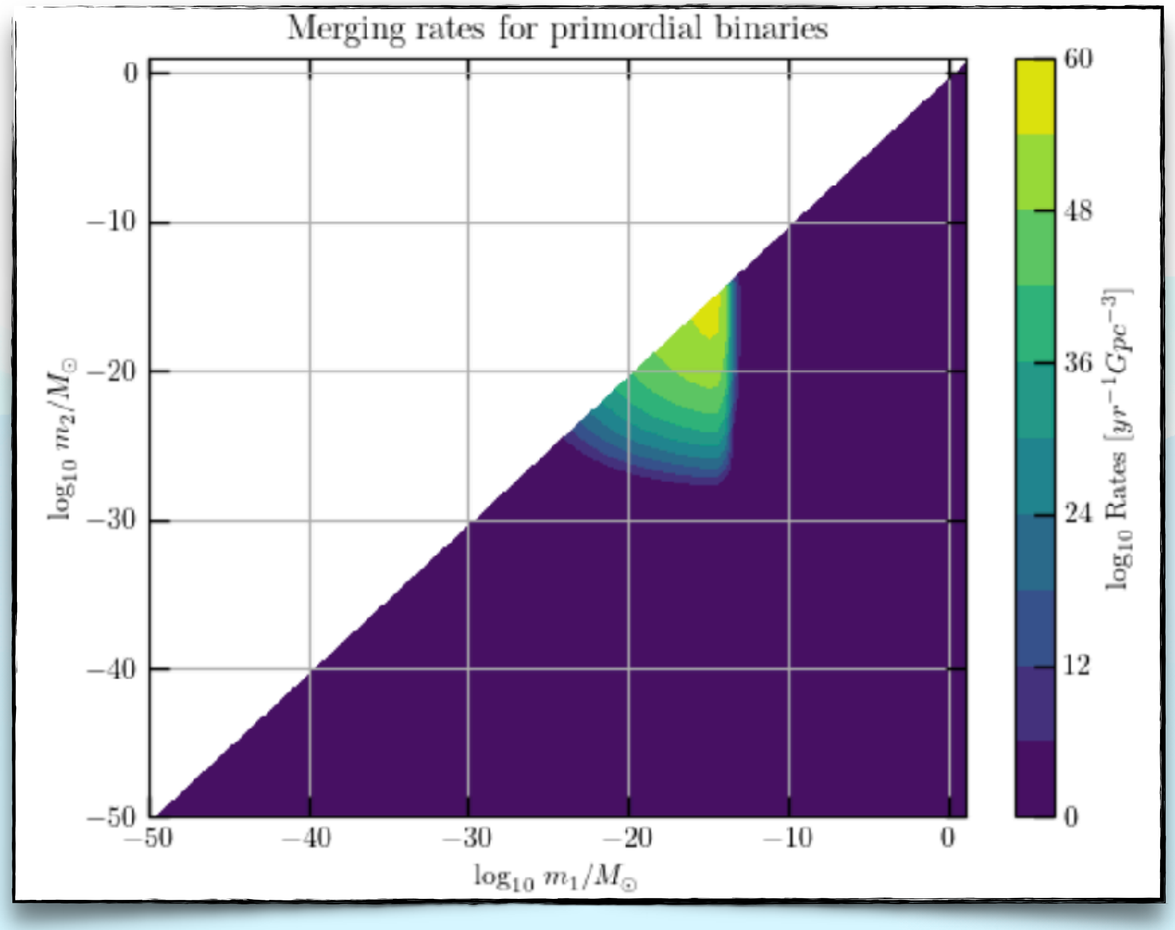
rates_EB = lambda masses: pb.get_rates_primordial(masses)
rates_LB = lambda masses: pb.get_rates_clusters(masses)

GWB_EB = lambda freq: pb.Get_GW_bkg_primordial_binary(freq)
GWB_LB = lambda freq: pb.Get_GW_bkg_cluster_binary(freq)
```



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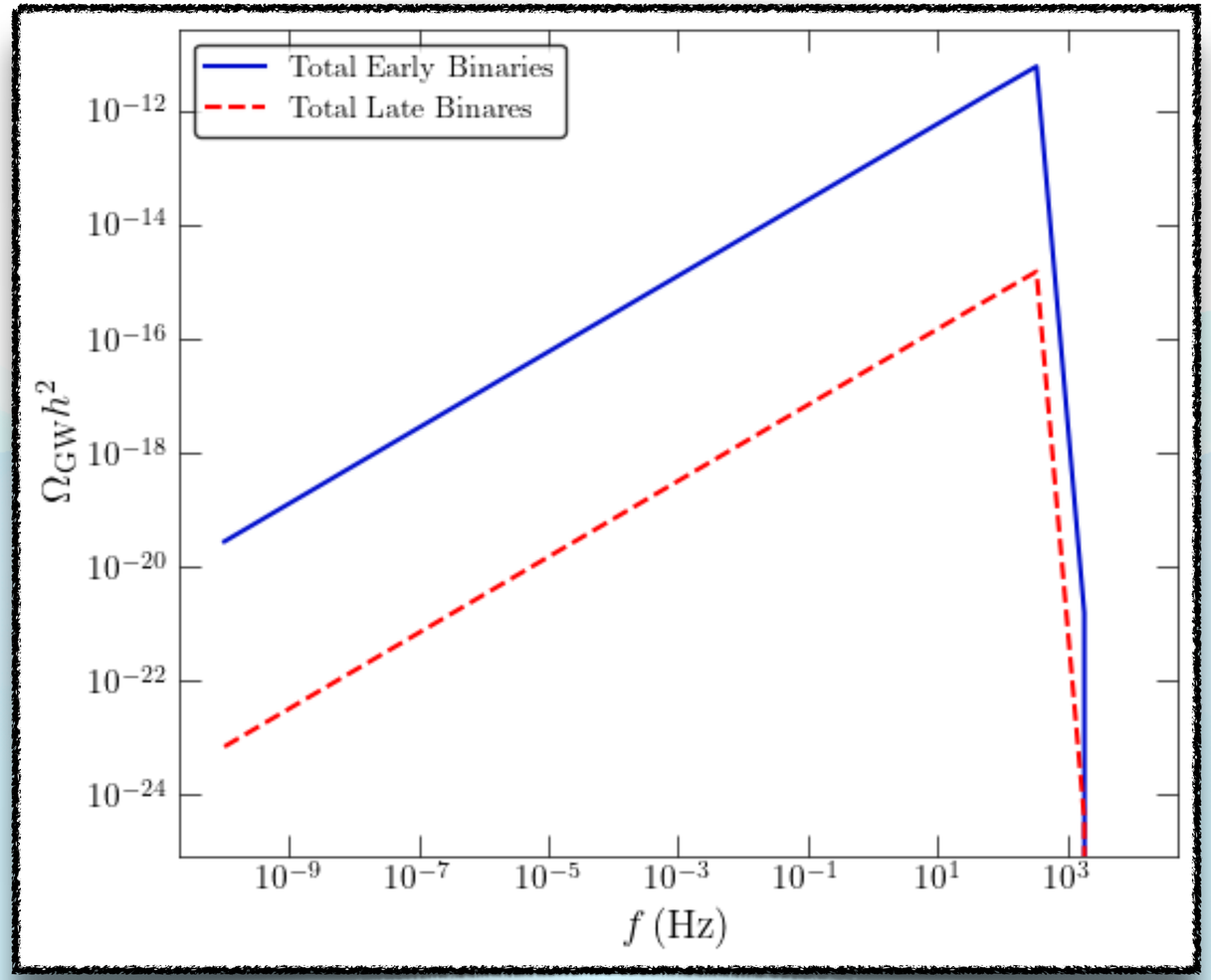

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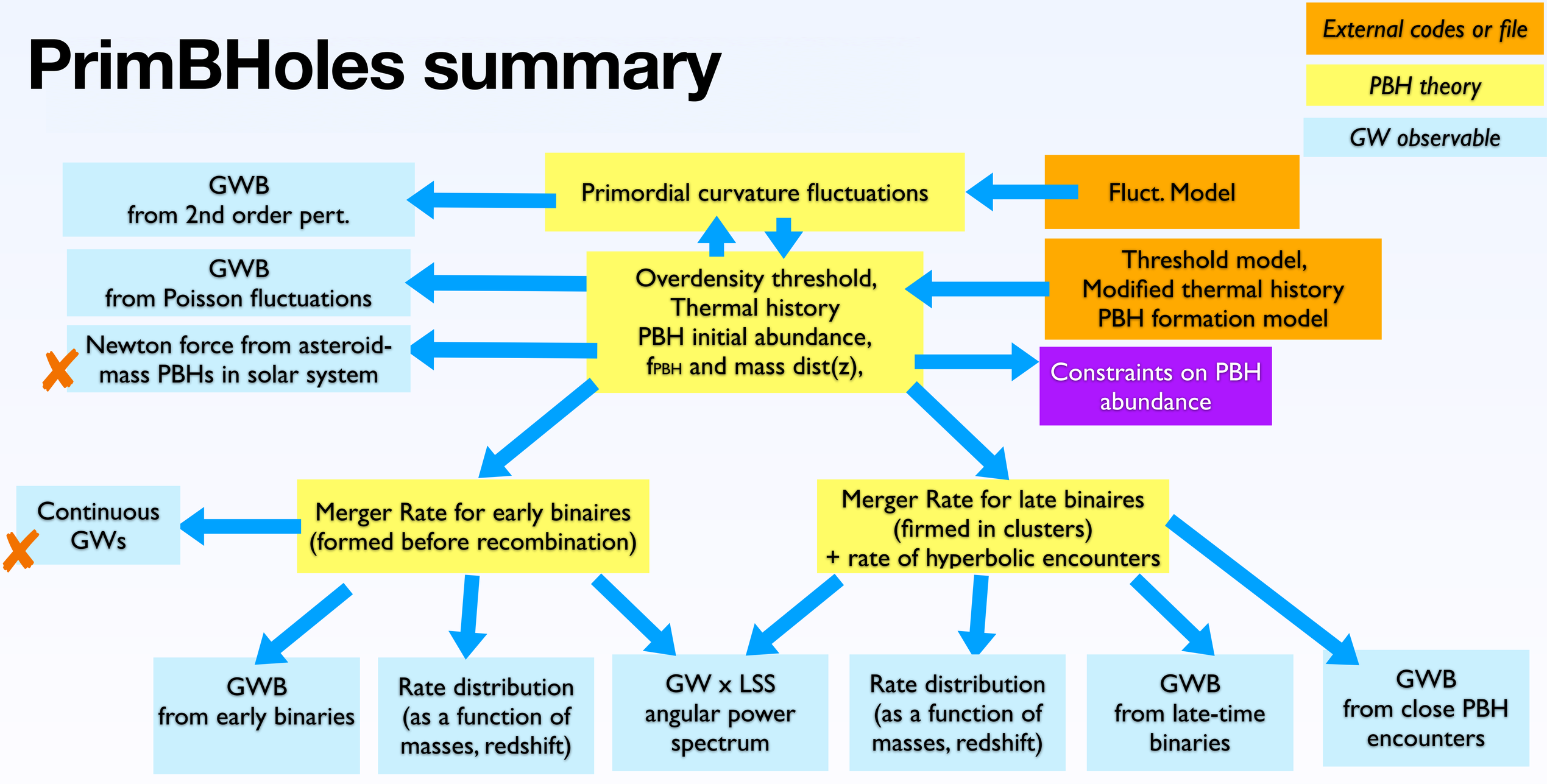
PrimBHoles summary



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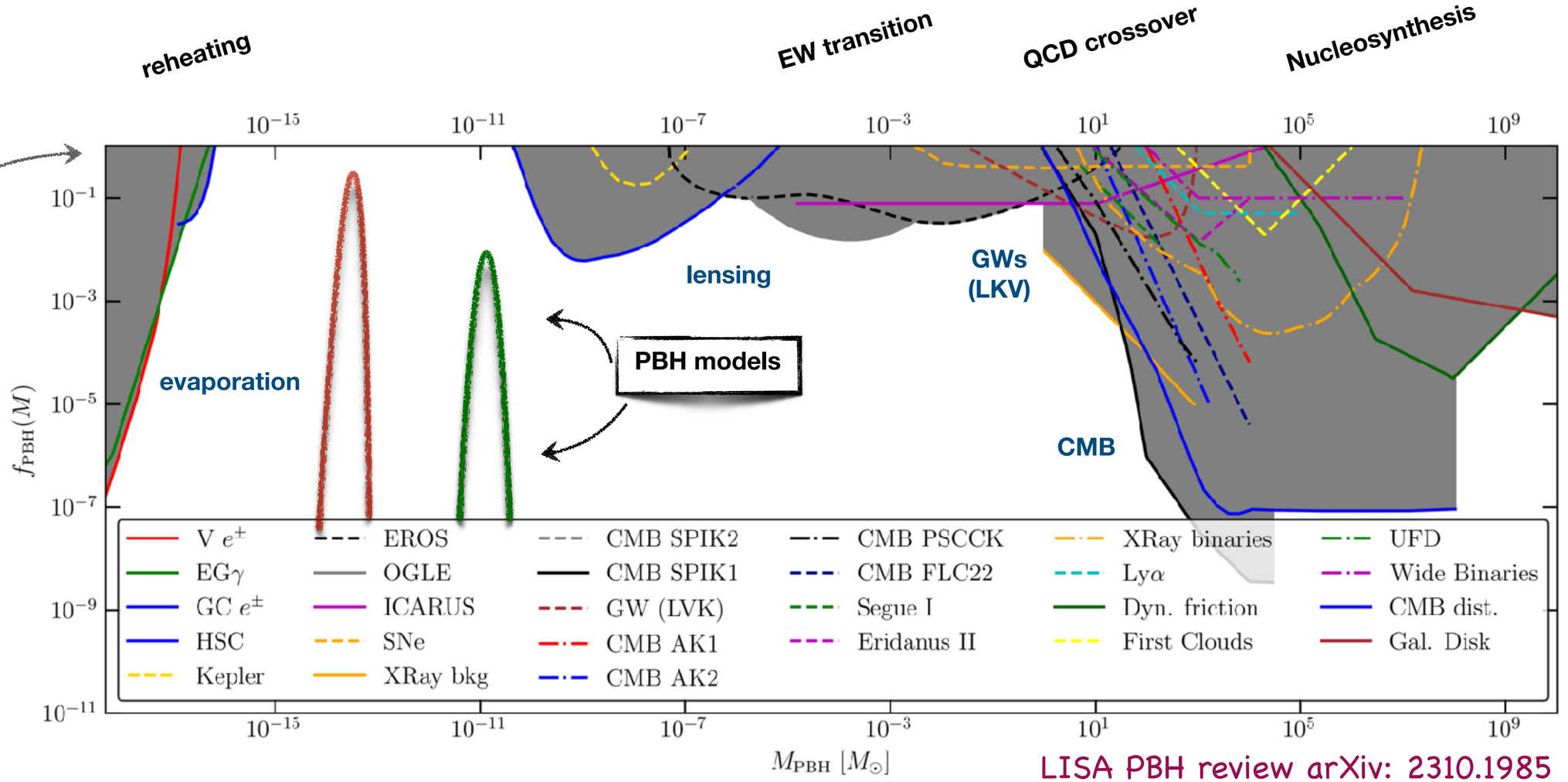
Abundances constraints / Model signatures



LISA PBH review arXiv: 2310.1985

E. Bagui, S. Clesse, J. Garcia-Bellido, **C. Joana**, V. d. Luca, G. Franciolini, I. Musco, R.K. Jain, T. Papanikolaou, A. Reccanelli, S. Renaoux-Petel, A. Riotto, E.R. Morales, M. Scalisi, O. Sergijenko, C. Unal, V. Vennin. D. Wands

… more under development

- **Abundances**:
  - (Non-) Gaussianities (based on peak-theory)
  - Mass functions and accretion
  - PBHs from reheating
  - Models: curvaton, ultra-slowroll, linear Starobinsky…
- **Rates/ GWB:**
  - Rates close encounters
  - GWB from Quantum diffusion, PBH mergers,
  - Continuous GWs from PBH inspirals
- **First Release (v1) … around October 2024 !**

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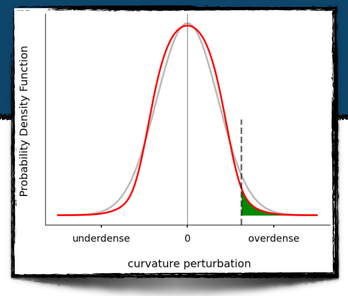
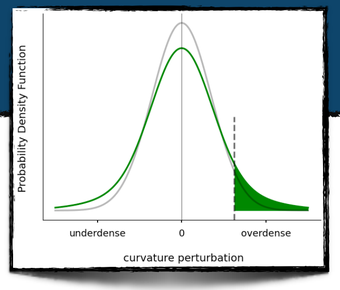
Thank you!

Thank you!

Back-up

PBH formation: thresholds



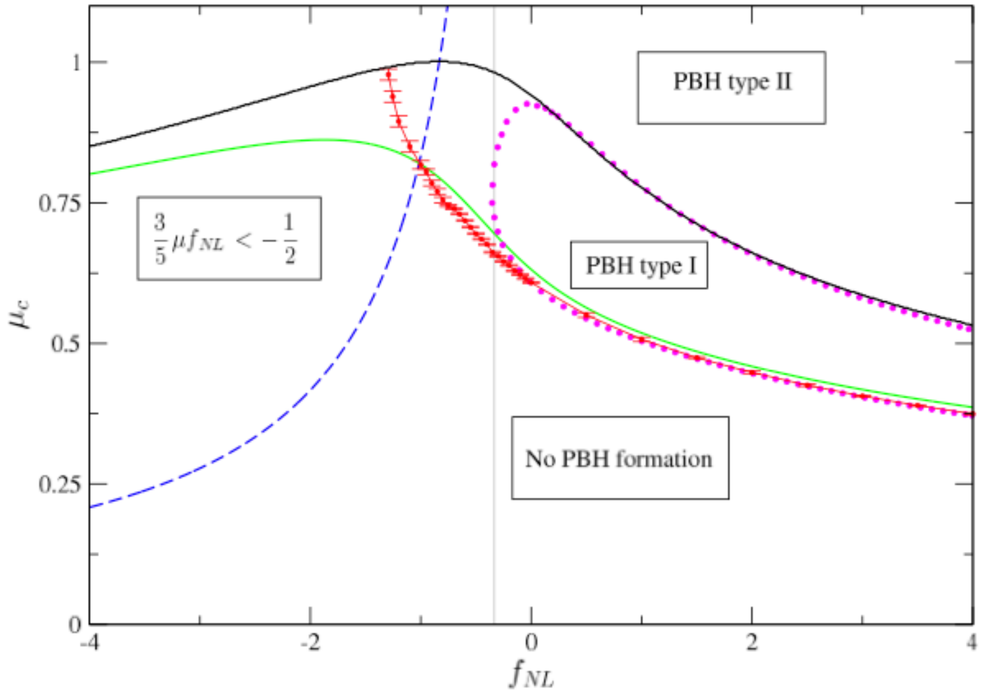


$$\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2 + \frac{9}{25} g_{NL} \zeta_G^3 + \dots$$

Critical amplitudes (thresholds) require Numerical Relativity simulations.

Non-Gaussianities: expansion in f_{NL} , g_{NL} …

- Solve GR : non-linear grav. collapse.
- Dependence on:
  - medium & equation of state (i.e. fluid pressur
  - profile shape of the perturbation (incl. deviations from non-gaussian statistics)
- There are two type of gravitational collapse: Type I, and Type II.



- NR
- semi-analytical ("q approach")
- analytical (i.e. C=2/5)

[Escrivà et al 2022]

PBH formation