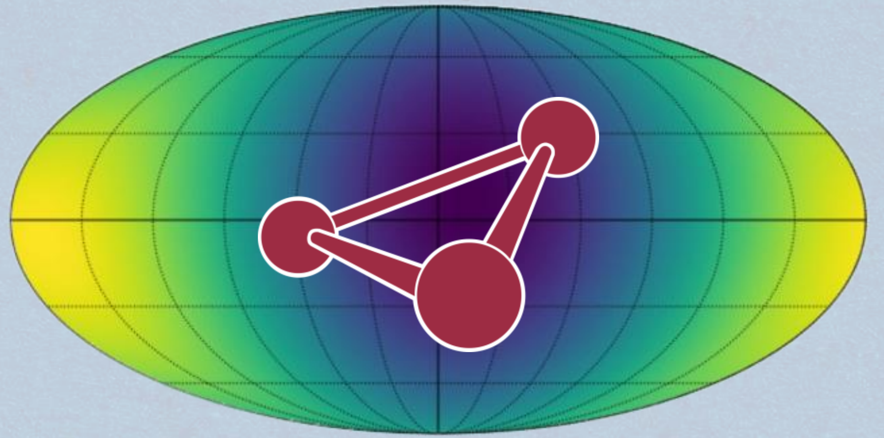

# Observing Kinematic Anisotropies of a Stochastic Gravitational Waves Background with LISA


**Speaker: Dr. Henri Inchauspé**

[http://arxiv.org/abs/2401.14849]

Co-authors: David Maibach, Prof. Lavinia Heisenberg

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**11th LISA CosWG Workshop, Porto 2024**

# Scope and objectives

- Main challenge of the search for SGWB signal with LISA:

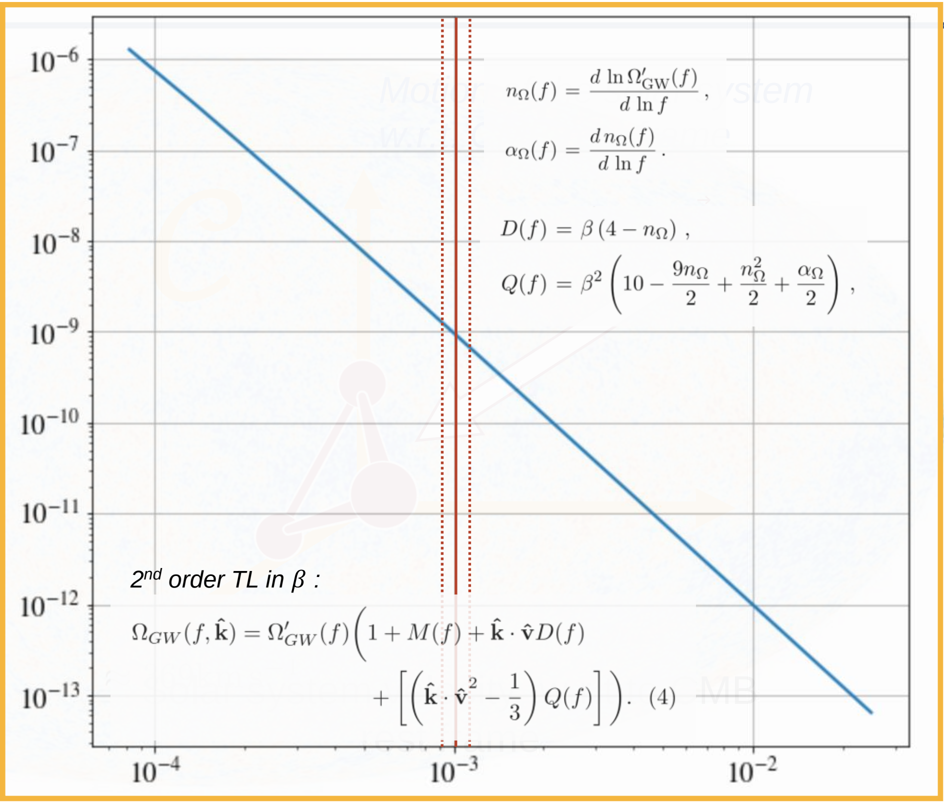
  **How do we distinguish a potential cosmological signal from instrumental noise** (with a single interferometer in space)
  (+ from galactic confusion noise, astrophysical background..)

  **On what kind of evidence can we claim an apparent excess of power is cosmological ?**

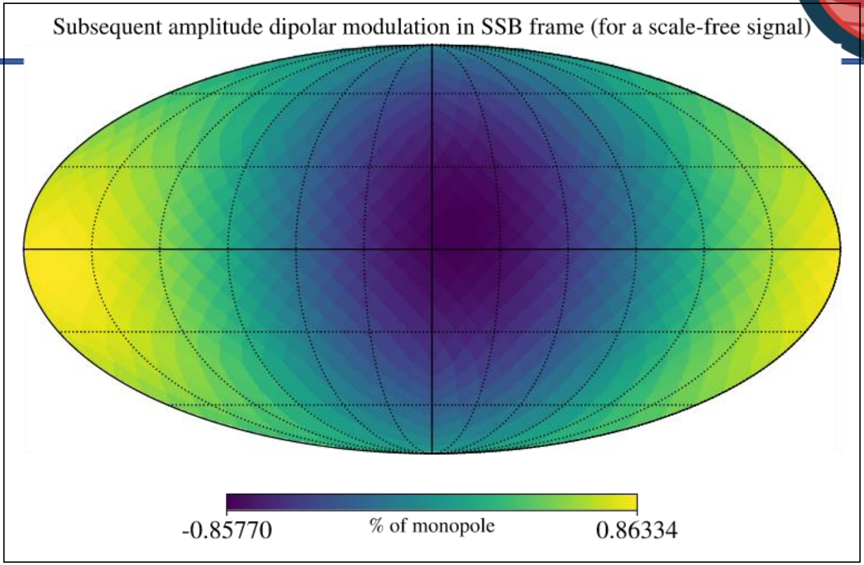
1. ~~The instrument response projects differently noise and signal on data.~~
   The spectrum shapes differ and are distinguishable (relying on assumptions).
   *[Baghi et al. 2023]*
2. The signal has distinctive features (e.g. anisotropy) not shared with the noise.
   *[Heisenberg et al. 2024]*

→ **Kinematic anisotropy is a signature of an extragalactic origin**

# Principle: Doppler boosting of the SGWB




$$\rightarrow \beta = 1.23 \times 10^{-3}$$




$$\mathcal{D} = \frac{1 - \beta^2}{1 - \beta \hat{n} \cdot \hat{v}}$$

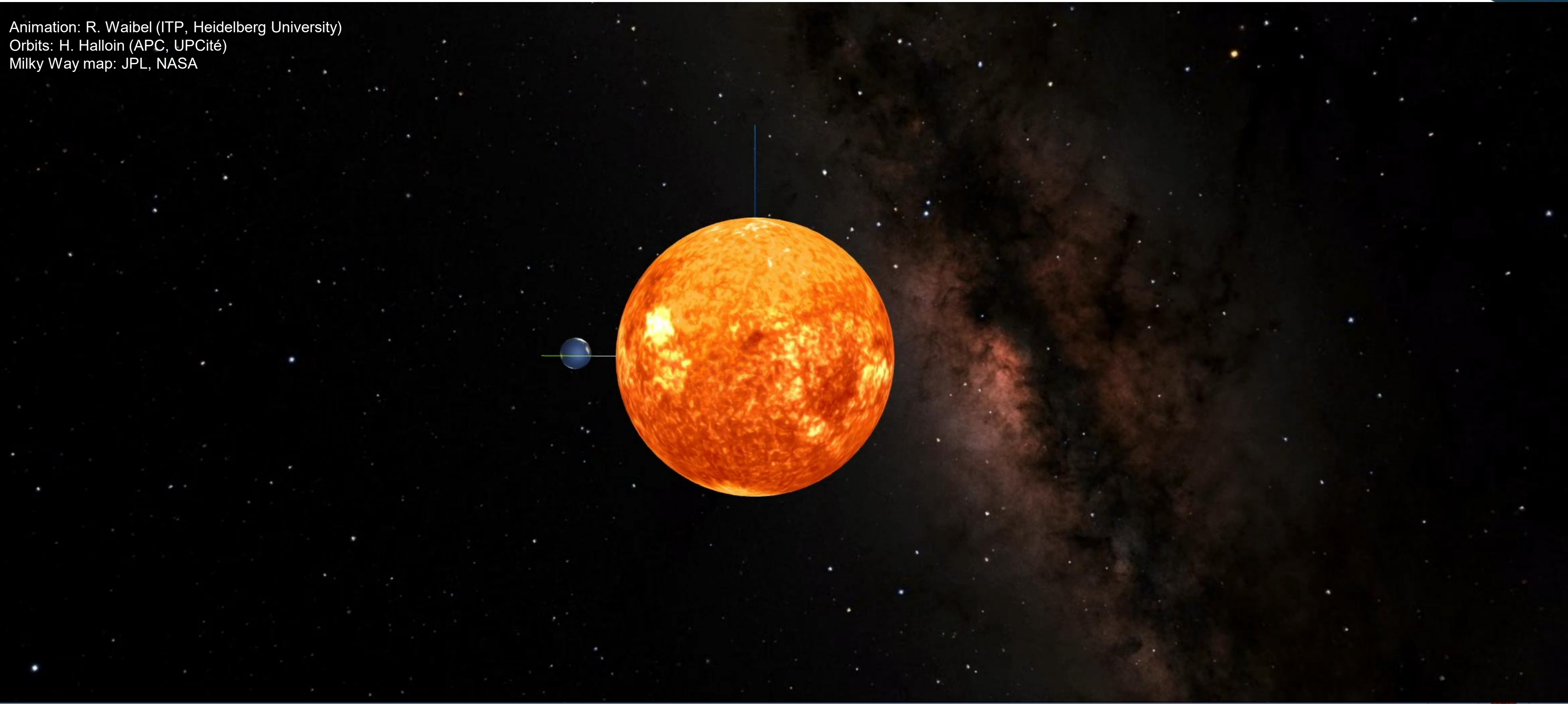
$$\Omega_{GW}(f, \hat{\mathbf{n}}) = \mathcal{D}^4 \Omega'_{GW}(f)(\mathcal{D}^{-1} f)$$

*1. Cusin et al. 2022, "Doppler boosting the stochastic gravitational wave background"*

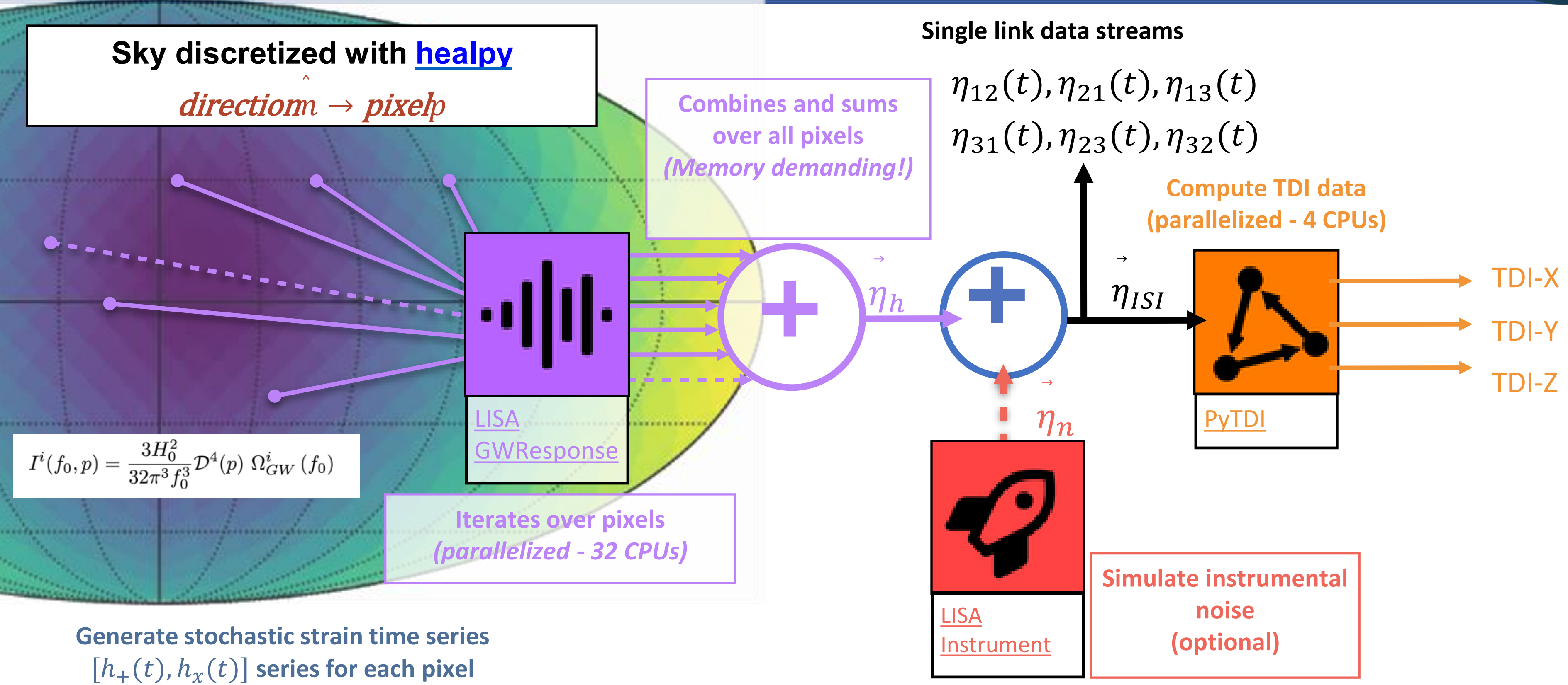
**2. Bartolo et al. 2022, « Probing anisotropies of the Stochastic Gravitational Wave Background with LISA »**

# LISA orbital motion —> angular scanning

https://www.thphys.uni-heidelberg.de/~waibel/

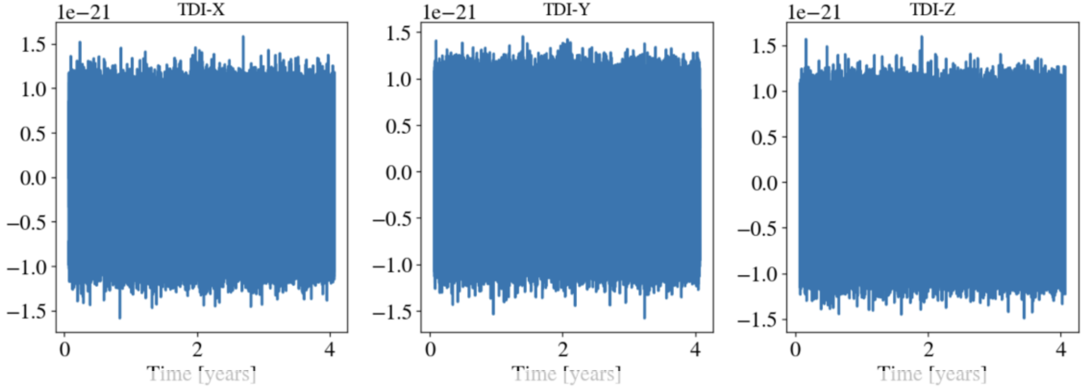


# **DATA** generation: full **time-domain** simulation of GW anisotropic sky, via LISA Simulation Suite

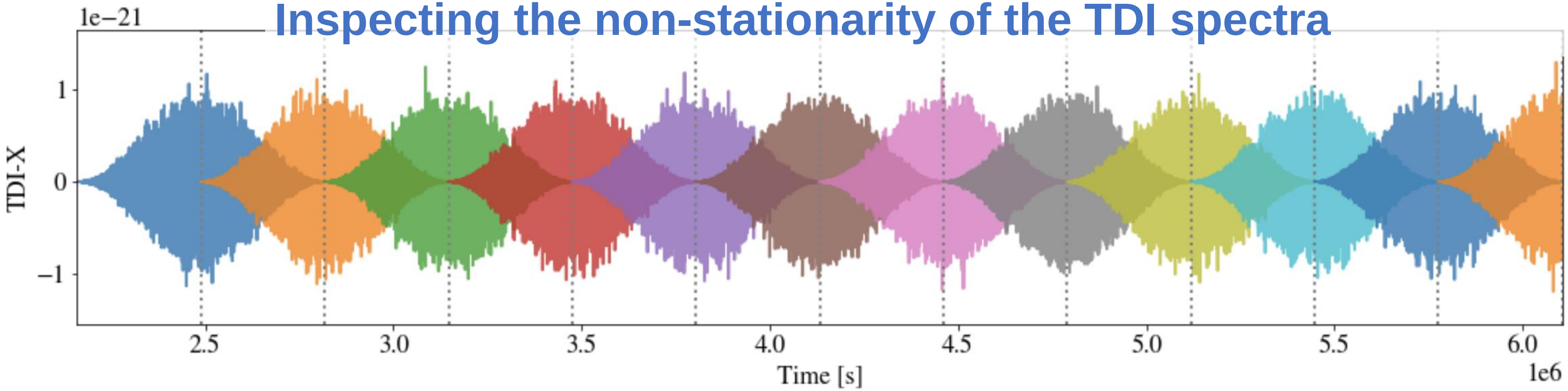


# Map-making strategy: time-frequency analysis with SFT

Output of the simulation:
**3 interferometer channels [X, Y, Z] 4-years long time series**



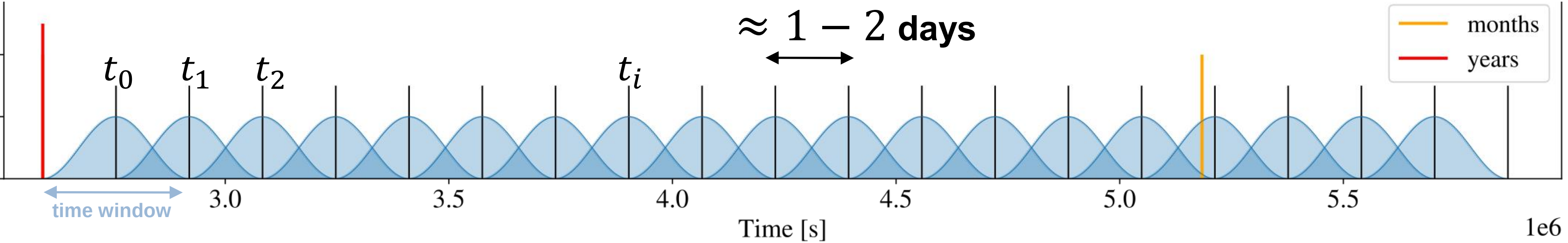

**Inspecting the non-stationarity of the TDI spectra**



# Map-making strategy: pre-processing the **DATA**

[*Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"*]

- Time-splitting the 3 years long TDI data streams
  *Hanning window, 50% overlapping segments*




- Short-time Fourier transforms + Frequency averaging (data compression) :

$$\overline{\mathbf{D}}(t_i, f_j) \equiv \frac{1}{n_j} \sum_{k=j-\frac{n_j}{2}}^{j+\frac{n_j}{2}} \tilde{\mathbf{d}}(t_i, f_k) \otimes \tilde{\mathbf{d}}(t_i, f_k)^{\dagger}.$$

[*Baghi et al. 2023, "Uncovering gravitational-wave backgrounds from noises of unknown shape with LISA"*]

**TDI X, Y, Z data streams**

- DA problem: we're solving for the covariance $C_d$ of the signal $\tilde{\mathbf{d}}$, that is, the expectation of $\overline{\mathbf{D}}(t_i, f_j)$

# Covariance **MODEL** and max likelihood map-making strategy

[Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"]

- Covariance model:

Ultimate parameters to solve for

$$\mathbf{C}_d(t_i, f_j) = \mathbf{A}(t_i, f_j, p) I(f_j, p) + \mathbf{N}(t_i, f_j)$$

$$I(f, p) = \mathcal{D}(\vec{\beta}, p) I'\left(\mathcal{D}^{-1}(\vec{\beta}, p) f\right)$$

**LISA (+TDI) quadratic response (Freq. domain model)**

**Pixel Map to solve for**

**Instrumental Noise**

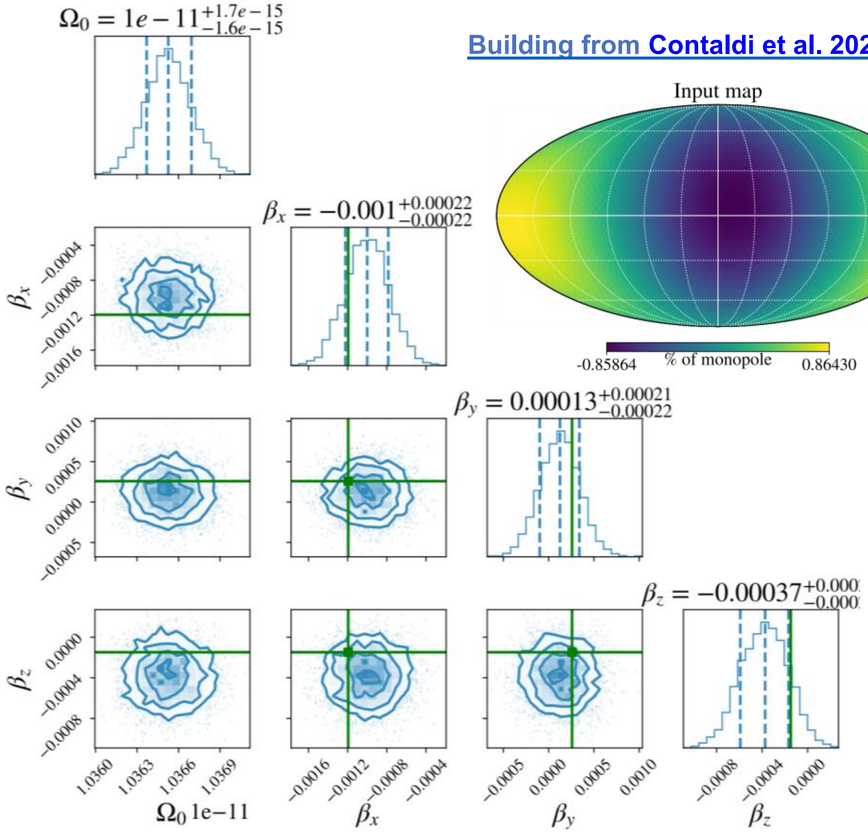
$$\mathcal{D}(\vec{\beta}, p) = \frac{1-\beta^2}{1-\beta\hat{n}(p)\cdot\hat{\beta}}$$

- log-Likelihood, Wishart statistics:

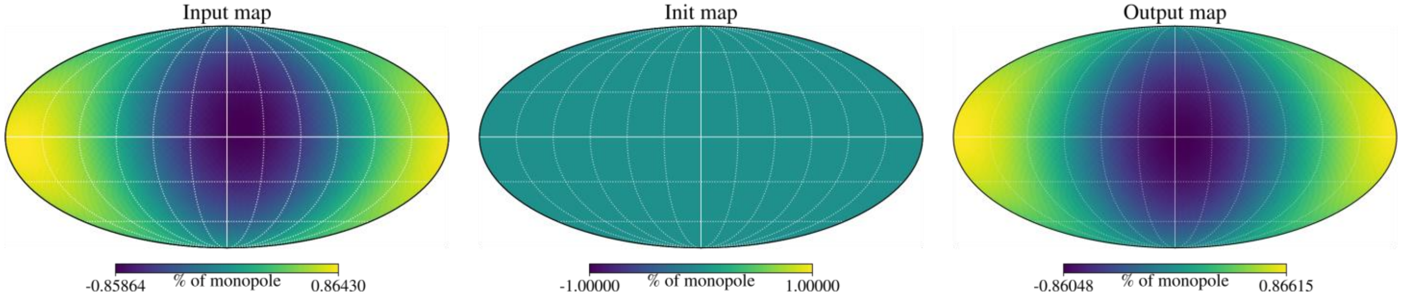
$$\log\mathcal{L} = \sum_{t_i}\sum_{f_j}\left[-\mathrm{tr}\left(\mathbf{C}_\mathbf{d}^{-1}\mathbf{D}(t_i, f_j)\right) - \nu\log\left|\mathbf{C}_\mathbf{d}(\mathbf{t}_\mathbf{i}, \mathbf{f}_\mathbf{j})\right|\right]$$

[Baghi et al. 2023]

# MCMC sampling the velocity: recovered sky maps




[Building from Contaldi et al. 2020, extending to time-domain sim, spectrum averaging, and MCMC sampling]




$$I(f, p) = \mathcal{D}(\vec{\beta}, p) I'\left(\mathcal{D}^{-1}(\vec{\beta}, p) f\right)$$

$$\mathcal{D}(\vec{\beta}, p) = \frac{1 - \beta^2}{1 - \beta \hat{n}(p) \cdot \hat{\beta}}$$

**Fitting for $\vec{\beta}$ and $I'(f_0)$**

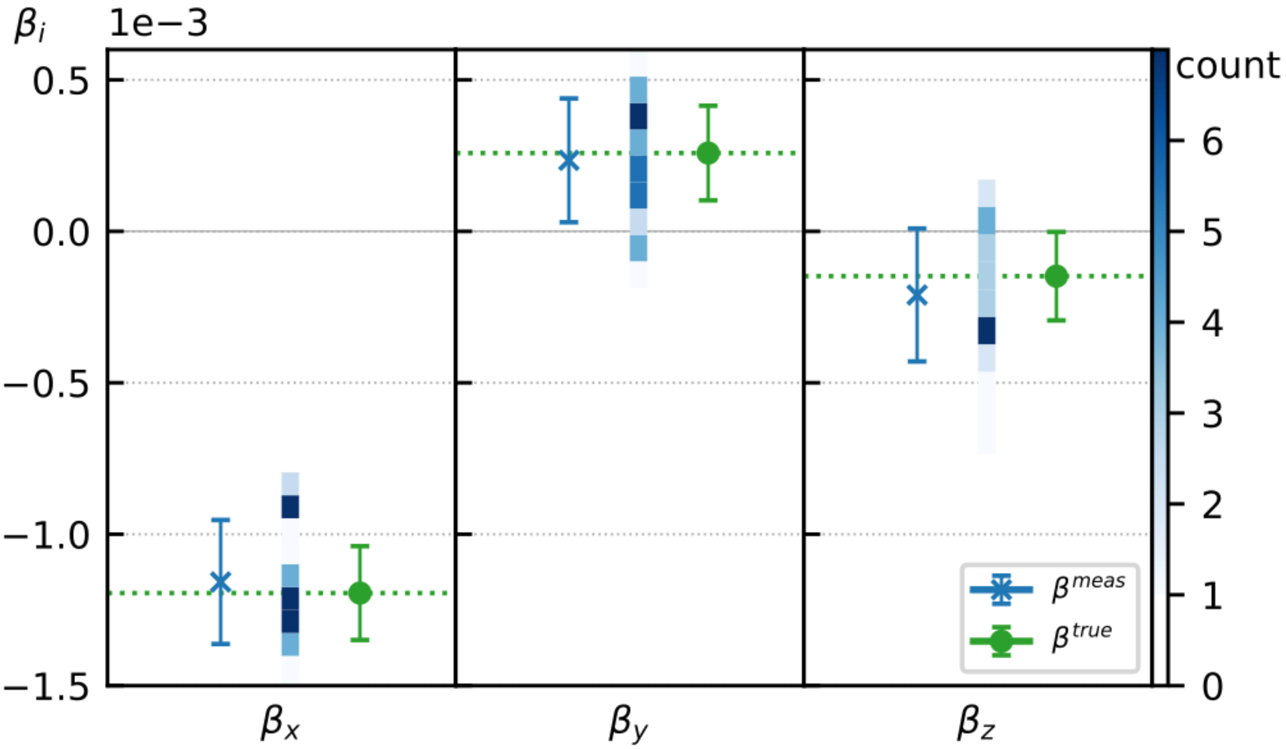
Could also Taylor expand in power of $\beta$:

$$\Omega_{GW}(f, \mathbf{n}) = \Omega'_{GW}(f)\left[1 + M(f) + \mathbf{v} \cdot \mathbf{n} D(f) + ((\mathbf{v} \cdot \mathbf{n}))^2 + \frac{1}{3} Q(f)\right]$$

# 30 sky realization statistical test - velocity $\vec{\beta}$

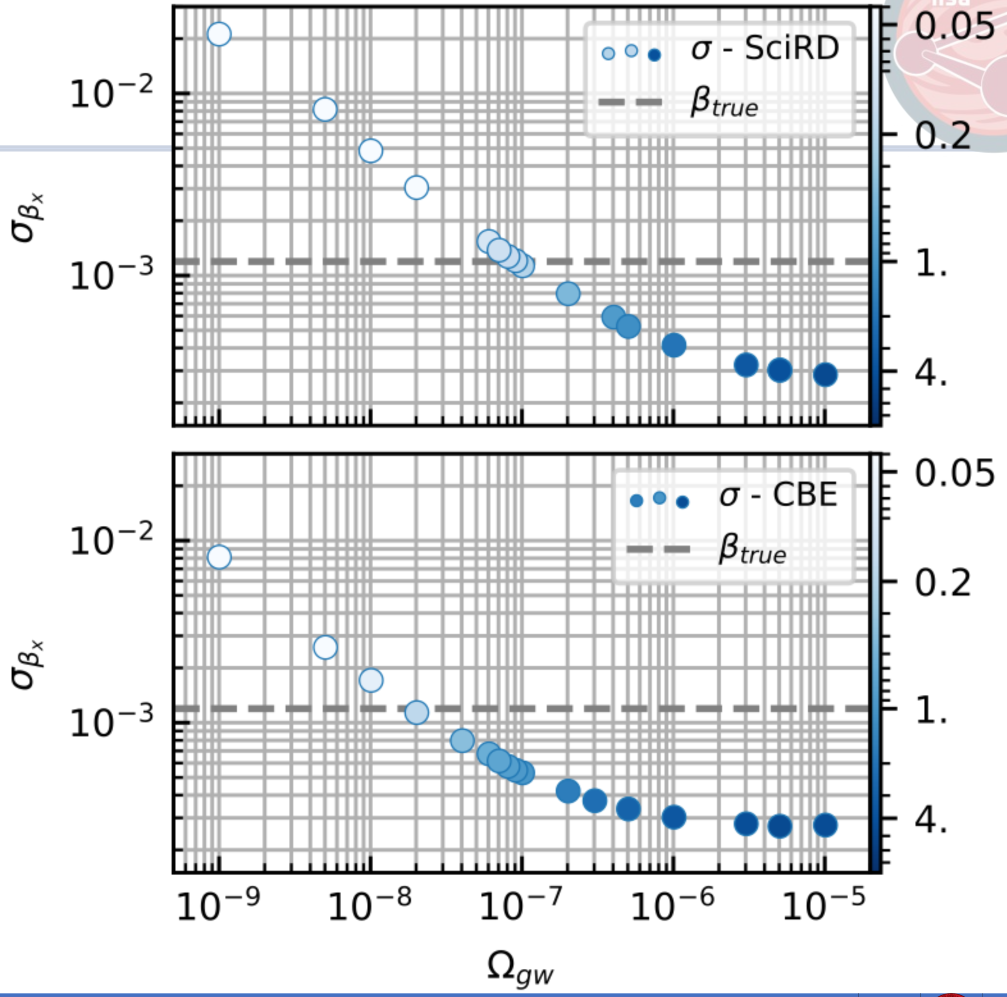
- 30 measured $\vec{\beta}^{meas}$: mean values and standard deviations
- $\vec{\beta}^{true}$ with theoretical error bars from MCMC
- Histogram of $\vec{\beta}^{meas}$ values

$\vec{\beta}_x$ **resolved !**



# Impact of instrumental noise

- At this stage, only source of uncertainty: sample noise —> independent of amplitude of signal $\Omega_{gw}$.
- Precision on $\beta_x$ as a function of the source $\Omega_{gw}$
- 2 noise configuration:
  - Noise specification from SciRD (conservative baseline)
  - Current Best Estimate Models (optimistic baseline)
- Need of high amplitude (scale-invariant) signal to detect anisotropy.



# Conclusion & Perspectives

- End-to-end simulation and analysis of an anisotropic GW sky with LISA.
- With up-to-date and most complete simulation tools of the consortium to date (LISA GWResponse, LISA Instrument, PyTDI)
- Validation of the method to recover kinematic anisotropy, induced on scale-free SGWB signal (spectral index $\alpha = 0$), **for noiseless instrument.**
- What's next ?

  1. **Investigate time-frequency data representation (wavelets).**
  2. Apply the method to SGWB with **richer spectrum profiles** (broken power laws, peaks)
     **Sharp spectrum transition, breaks, peaks…**
     **$\rightarrow$ can boost the SNR a lot (dipole AND quadrupole)**

$$D(f) = \beta\,(4 - n_\Omega)\ ,$$

$$Q(f) = \beta^2 \left(10 - \frac{9n_\Omega}{2} + \frac{n_\Omega^2}{2} + \frac{\alpha_\Omega}{2}\right)\ ,$$

  3. Apply the method to **the mapping of the galactic foreground** (on LDC data!)

# PBHs

*from* ***[LISA Definition Study Report [arXiv:2402.07571]]***

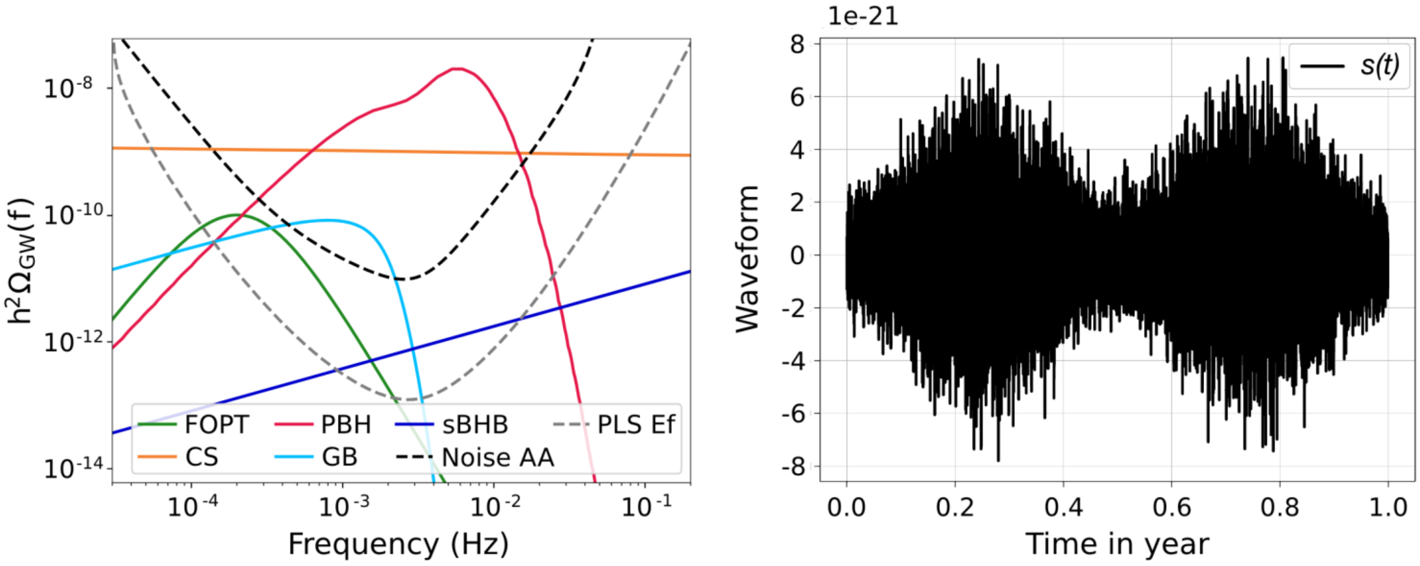



**Figure 3.8: Left panel:** Examples of SGWBs in the LISA band, together with the instrument sensitivity in the *A*-channel (*black, dashed*) and the effective Power Law Sensitivity [399] (*grey, dashed*). The cosmological SGWBs are: in *red*, the SGWB from Primordial Black Holes (PBHs) in a mass range for which they could constitute the totality of the Dark Matter today [77]; in *orange*, the SGWB from Cosmic Strings (CSs) with tension providing a signal that would account for the SGWB detection by PTAs [25]; in *green*, the SGWB from a primordial First-Order Phase Transition (FOPT) at the Electroweak (EW) scale, in the context of a singlet extension of the Standard Model of particle physics, testable at particle colliders. The astrophysical SGWB from unresolved stellar-mass Black Hole binaries (sBHBs), taken from [54] assuming GWTC-3 population constraints [396] is shown in *dark blue*. The Galactic foreground is shown in *light blue*, taken from [251], averaged over time. **Right panel:** The stochastic Galactic foreground in the time domain, where the periodic time variability of the signal amplitude is apparent (figure taken from [104]).

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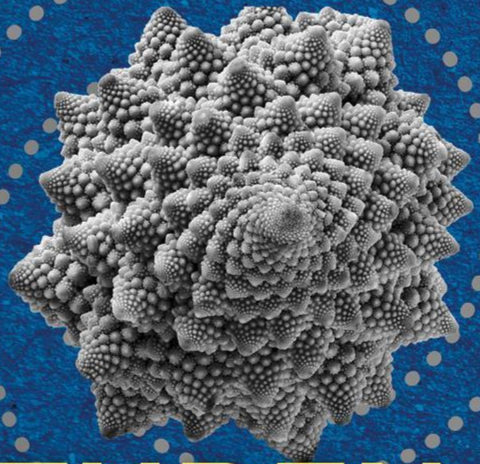


# STRUKTUREN IN DER WELT

# Back slides

# CMB Dipole

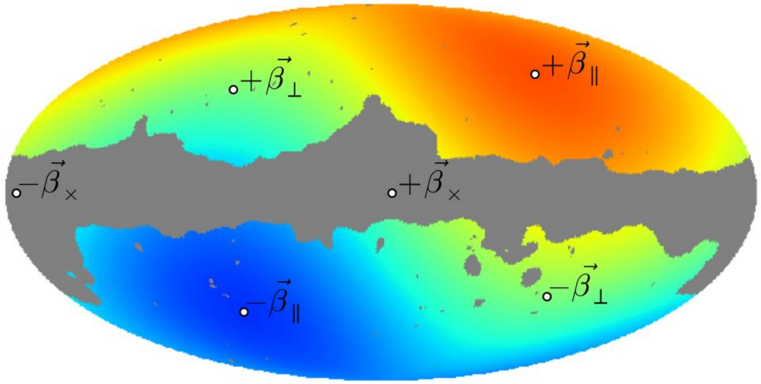



**Fig. 2.** Specific choice for the decomposition of the dipole vector $\boldsymbol{\beta}$ in Galactic coordinates. The CMB dipole direction $(l, b) = (263^\circ.99, 48^\circ.26)$ is given as $\boldsymbol{\beta}_\parallel$, while two directions orthogonal to it (and each other) are denoted as $\boldsymbol{\beta}_\perp$ and $\boldsymbol{\beta}_\times$. The vector $\boldsymbol{\beta}_\times$ lies within the Galactic plane.

From: Planck 2013 results. XXVII. Doppler boosting of the CMB

$$\rightarrow \beta = 1.23 \times 10^{-3}$$

If $T'$ and $\hat{\boldsymbol{n}}'$ are the CMB temperature and direction as viewed in the CMB frame, then the temperature in the observed frame is given by the Lorentz transformation (see, e.g., Challinor & van Leeuwen 2002; Sollom 2010),

$$T(\hat{\boldsymbol{n}}) = \frac{T'(\hat{\boldsymbol{n}}')}{\gamma(1 - \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})}, \quad (1)$$

where the observed direction $\hat{\boldsymbol{n}}$ is given by

$$\hat{\boldsymbol{n}} = \frac{\hat{\boldsymbol{n}}' + [(\gamma - 1)\hat{\boldsymbol{n}}' \cdot \hat{\boldsymbol{v}} + \gamma\beta]\hat{\boldsymbol{v}}}{\gamma(1 + \hat{\boldsymbol{n}}' \cdot \boldsymbol{\beta})}, \quad (2)$$

and $\gamma \equiv (1 - \beta^2)^{-1/2}$. Expanding to linear order in $\beta$ gives

$$T'(\hat{\boldsymbol{n}}') = T'(\hat{\boldsymbol{n}} - \nabla(\hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})) \equiv T_0 + \delta T'(\hat{\boldsymbol{n}} - \nabla(\hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})), \quad (3)$$

so that we can write the observed temperature fluctuations as

$$\delta T(\hat{\boldsymbol{n}}) = T_0 \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta} + \delta T'(\hat{\boldsymbol{n}} - \nabla(\hat{\boldsymbol{n}} \cdot \boldsymbol{\beta}))(1 + \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta}). \quad (4)$$

Doppler Modulation

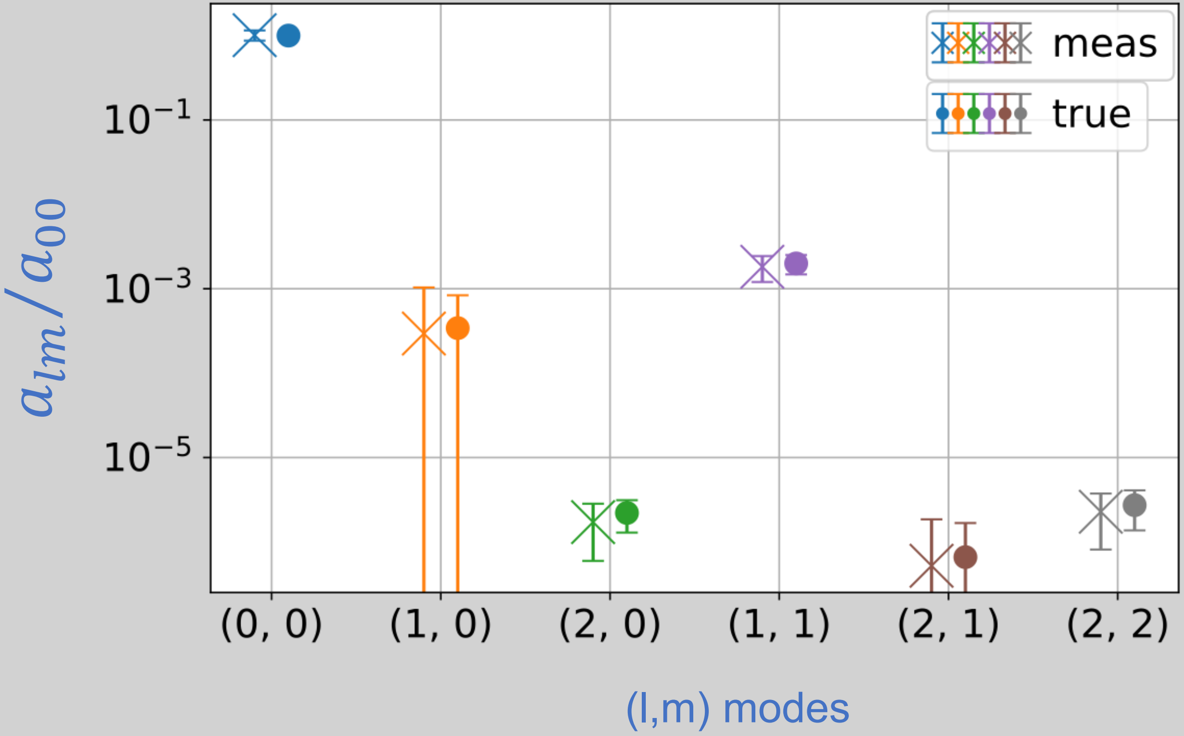
Aberration

# 30 sky realization statistical test - $a_{\ell m}$

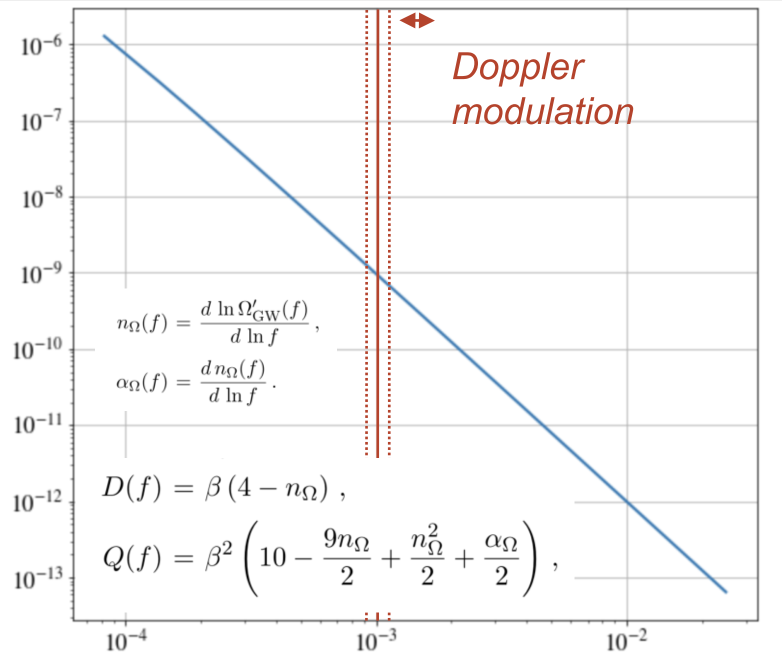
- Conversion of $\vec{\beta}^{meas}$ statistics to $a_{\ell m}$ counterpart
- Main mode resolved (1,1)
- Start to be sensitive to (2,0) and (2,2) !

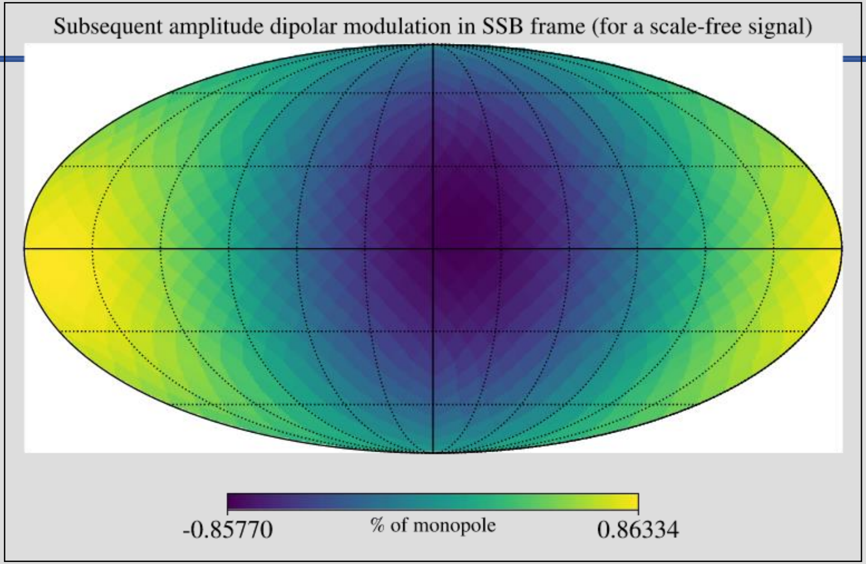
*Dipole: higher signal, but less responsive*

*Quadrupole: reduced signal, but more responsive*



# Principle: Doppler boosting of the SGWB




$$\mathcal{D} = \frac{1-\beta^2}{1-\beta\hat{n}\cdot\hat{v}}$$

$$\Omega_{GW}(f,\hat{\mathbf{n}}) = \mathcal{D}^4\Omega'_{GW}(f)(\mathcal{D}^{-1}f)$$

1. Cusin et al. 2022, "Doppler boosting the stochastic gravitational wave background"

2. Bartolo et al. 2022, « Probing anisotropies of the Stochastic Gravitational Wave Background with LISA »

# **DATA** generation: full **time-domain** simulation of GW anisotropic sky, via LISA Simulation Suite

- Physical assumptions:
  - ✓Pixel stochastic strain time series uncorrelated
  - ✓*Equal arm* or *keplerian* orbits.
  - ✓TDI 2.0
  - ✓**Arm propag. delays in TCB time**
  - ✓Secondary noise only (when noise on)

- Simulation settings
  - 3 years, sampling frequency = [0.05 Hz / 0.2 Hz]
  - Number of pixels: [12288 / 3072]
  - Cosmological signal: $\alpha = 0, \Omega = [10^{-12}, 10^{-7}]$

# LISA response

$$G_{lm,p}(f', t, \hat{\mathbf{k}}) = \frac{\xi_p(\hat{\mathbf{u}}_k, \hat{\mathbf{v}}_k, \hat{\mathbf{n}}_{lm})}{2\left(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_{lm}(t)\right)} \left[ e^{-\frac{2\pi i f'}{c}\left(L_{lm}(t) + \hat{\mathbf{k}}\cdot\mathbf{x}_m(t)\right)} - e^{-\frac{2\pi i f'}{c}\hat{\mathbf{k}}\cdot\mathbf{x}_l(t)} \right]. \qquad \text{(B.5)}$$

From Baghi et al. 2023

$$\begin{aligned} X_2 = X_1 &+ \mathbf{D}_{13121} y_{12} + \mathbf{D}_{131212} y_{21} + \mathbf{D}_{1312121} y_{13} + \mathbf{D}_{13121213} y_{31} \\ &- \left[\mathbf{D}_{12131} y_{13} + \mathbf{D}_{121313} y_{31} + \mathbf{D}_{1213131} y_{12} + \mathbf{D}_{12131312} y_{21}\right], \end{aligned}$$

$$\mathbf{D}_{ij}\tilde{x}(f) \approx \tilde{x}(f) e^{-2\pi i f L_{ij}}.$$

# Covariance **MODEL** and max likelihood map-making strategy

Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"

- Covariance model:

$$\mathbf{C}_d(t_i, f_j) = \mathbf{A}(t_i, f_j, p) I(p) + \mathbf{N}(t_i, f_j)$$

**LISA quadratic response** — **Pixel Map to solve for** — **Instrumental Noise**

$$I(f, \hat{n}) = \Omega_{\mathrm{GW}}(f, \hat{n}) \frac{3H_0^2}{4\pi^2 f^3}$$

**Sky discretized with healpy**

*direction* $\hat{n} \rightarrow$ *pixel* $p$

- LISA quadratic response:

$$A(t_i, f_j, p) = R_+(t_i, f_j, p) \otimes R_+(t_i, f_j, p)^* + R_\times(t_i, f_j, p) \otimes R_\times(t_i, f_j, p)^* \quad R_P(t_i, f_j, p) = M_{TDI}(t_i, f_j) G_P(t_i, f_j, p) M_{TDI}(t_i, f_j)^\dagger$$

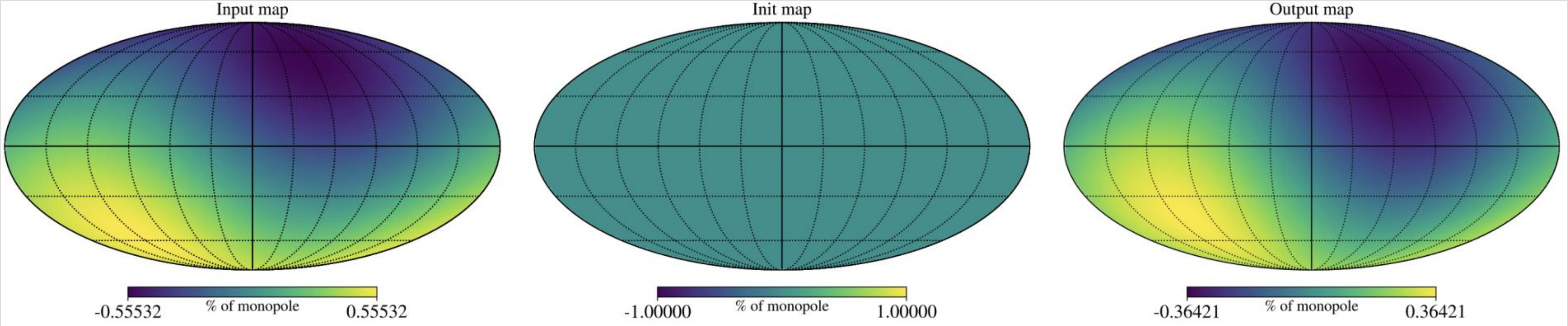
TDI matrix (phasing operators) — single link response (freq. domain, at time $t_i$) — TDI matrix (phasing operators)

Baghi et al. 2023

- log-Likelihood, Wishart statistics:

$$\log\mathcal{L} = \sum_{t_i f_j}\sum \left[-\mathrm{tr}\left(\mathbf{C}_d^{-1}\mathbf{D}(t_i, f_j)\right) - \nu \log\left|\mathbf{C}_d(t_i, f_j)\right|\right]$$

# MCMC sampling the alms: **artificially rotated input** - *Sanity check*




*Work from D. Maibach (Heidelberg Univ.)*