High Frequency Gravitational Waves: How and Where to Find Them

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University of Geneva

w/ R. T. D'Agnolo (to appear) and w/ Berlin, Blas, D'Agnolo, Harnik, Kahn, Schütte-Engel (PRD 2022) + Wentzel (PRD 2023)

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UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES
Swiss National Science Foundation

MOTIVATIONS

Why Radio Gravity?

High-Frequency Gravitational Waves

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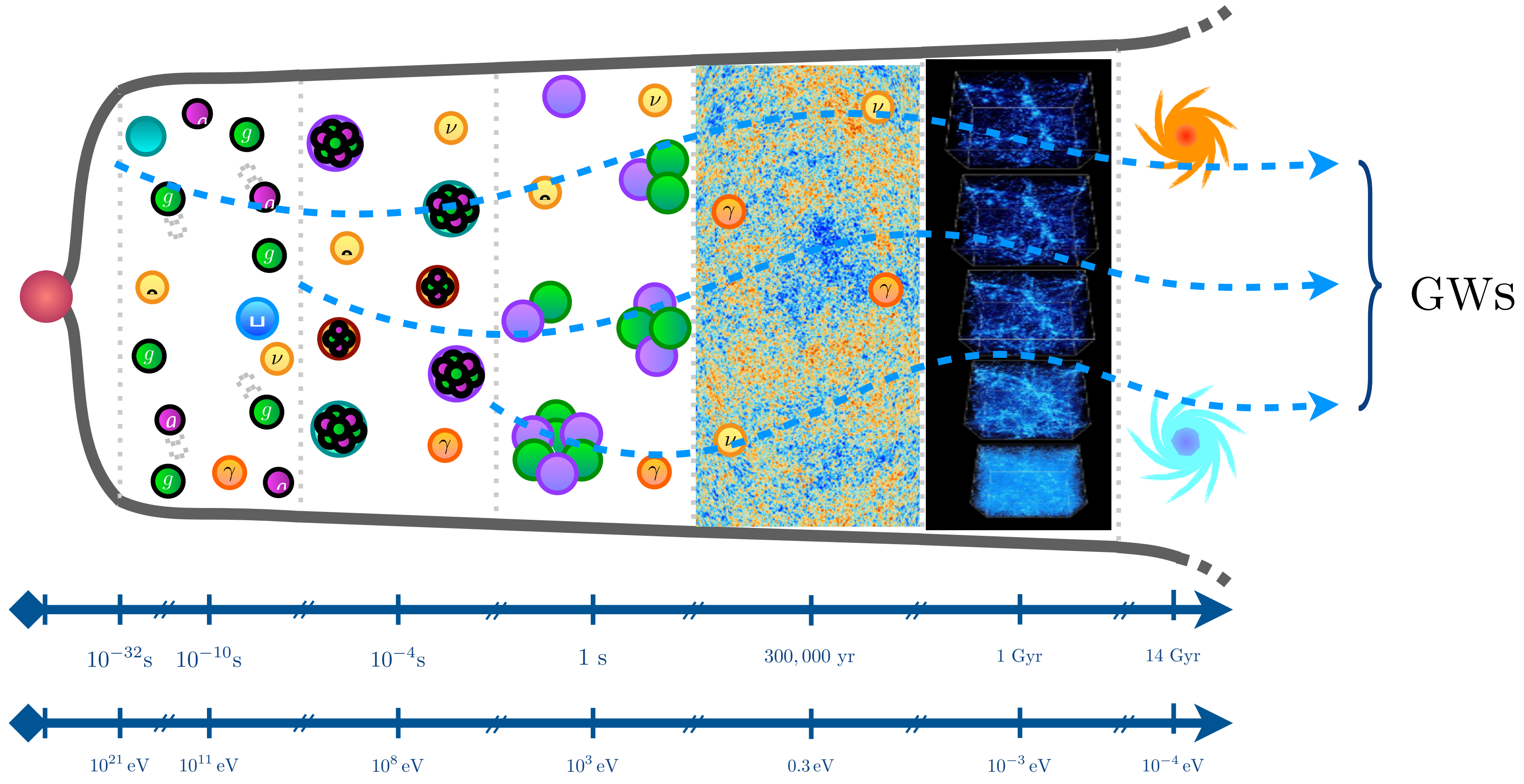
Cosmological GWs

$$\omega_g \Leftrightarrow T_{\text{origin}}$$



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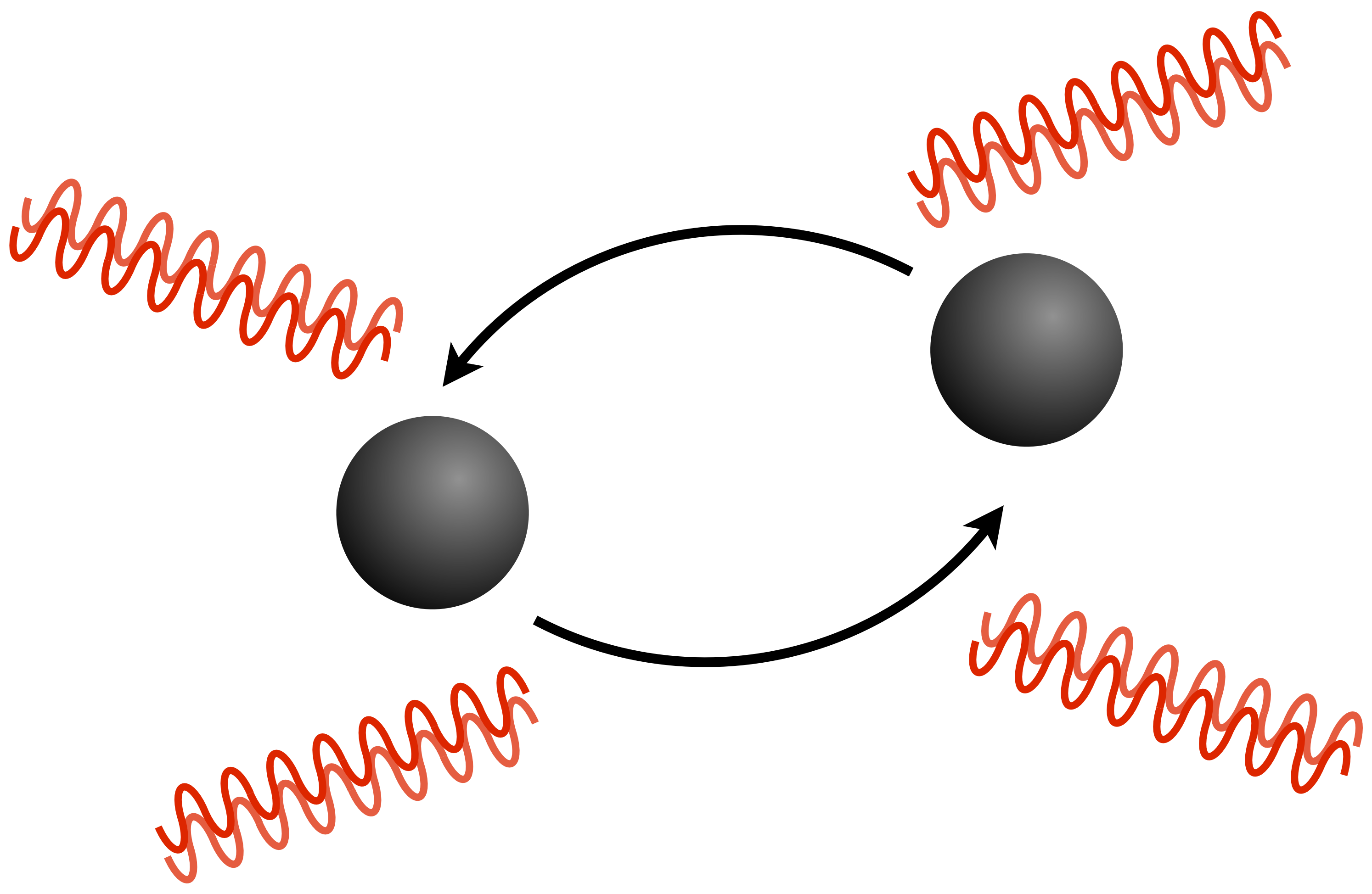
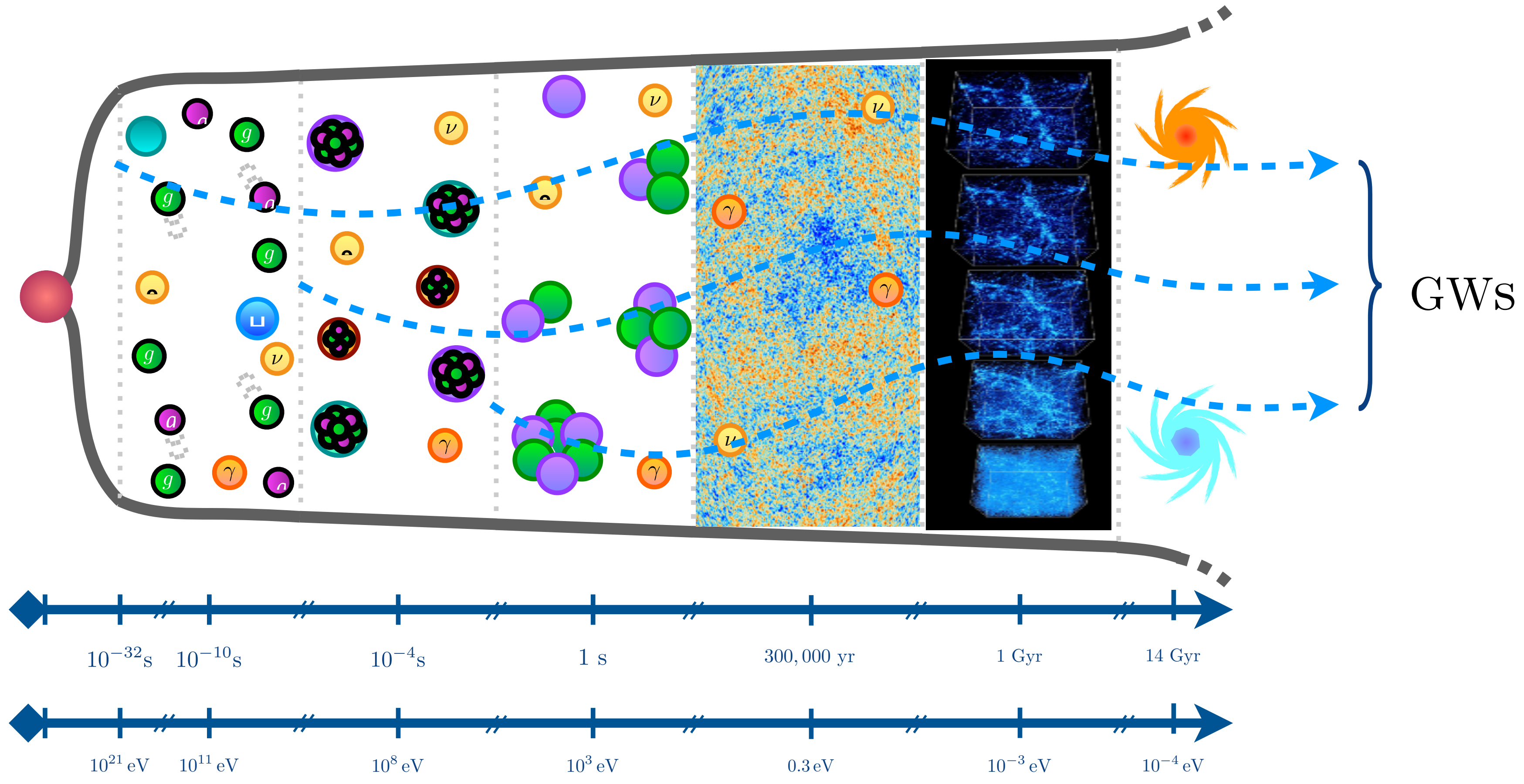


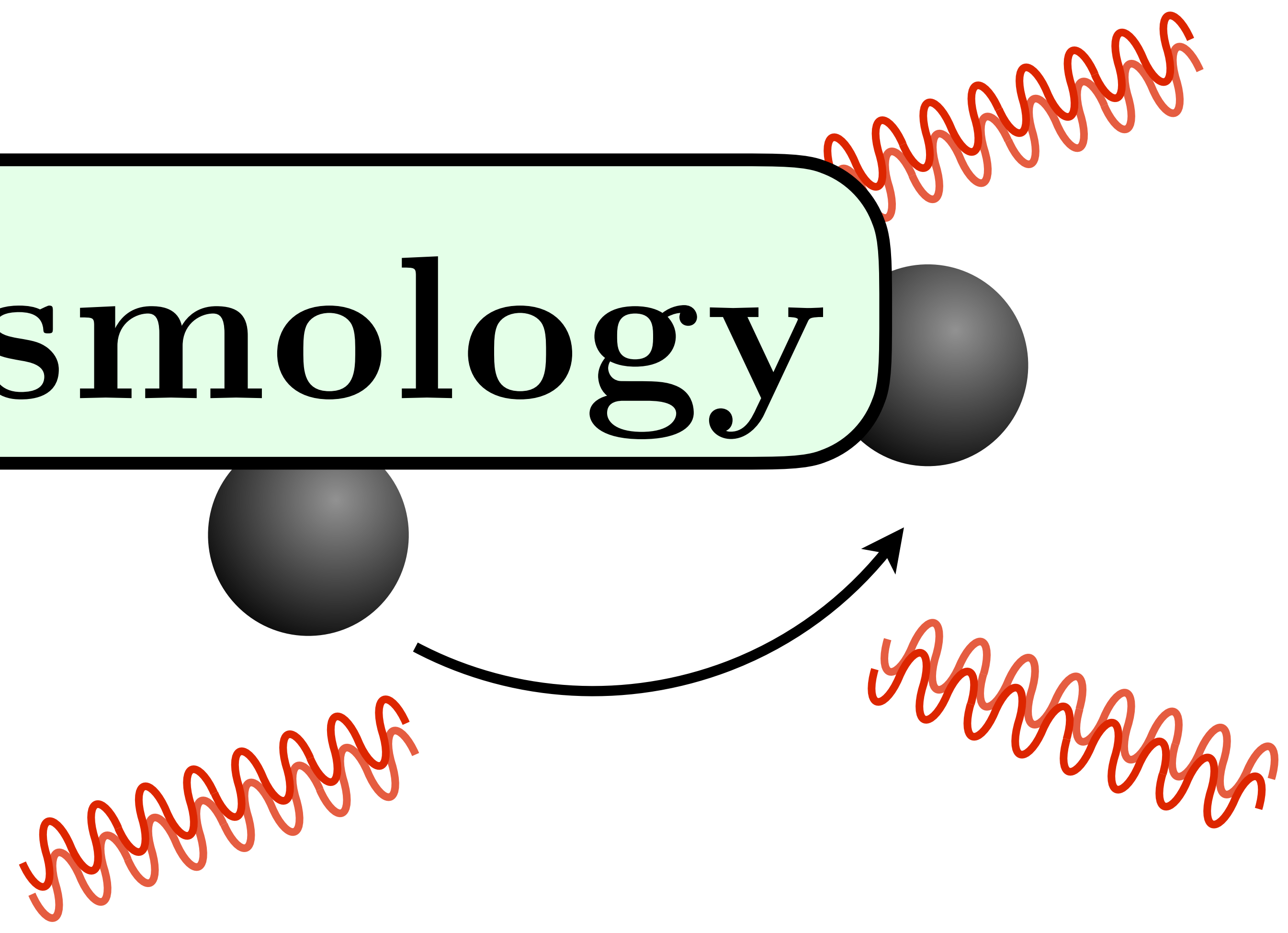


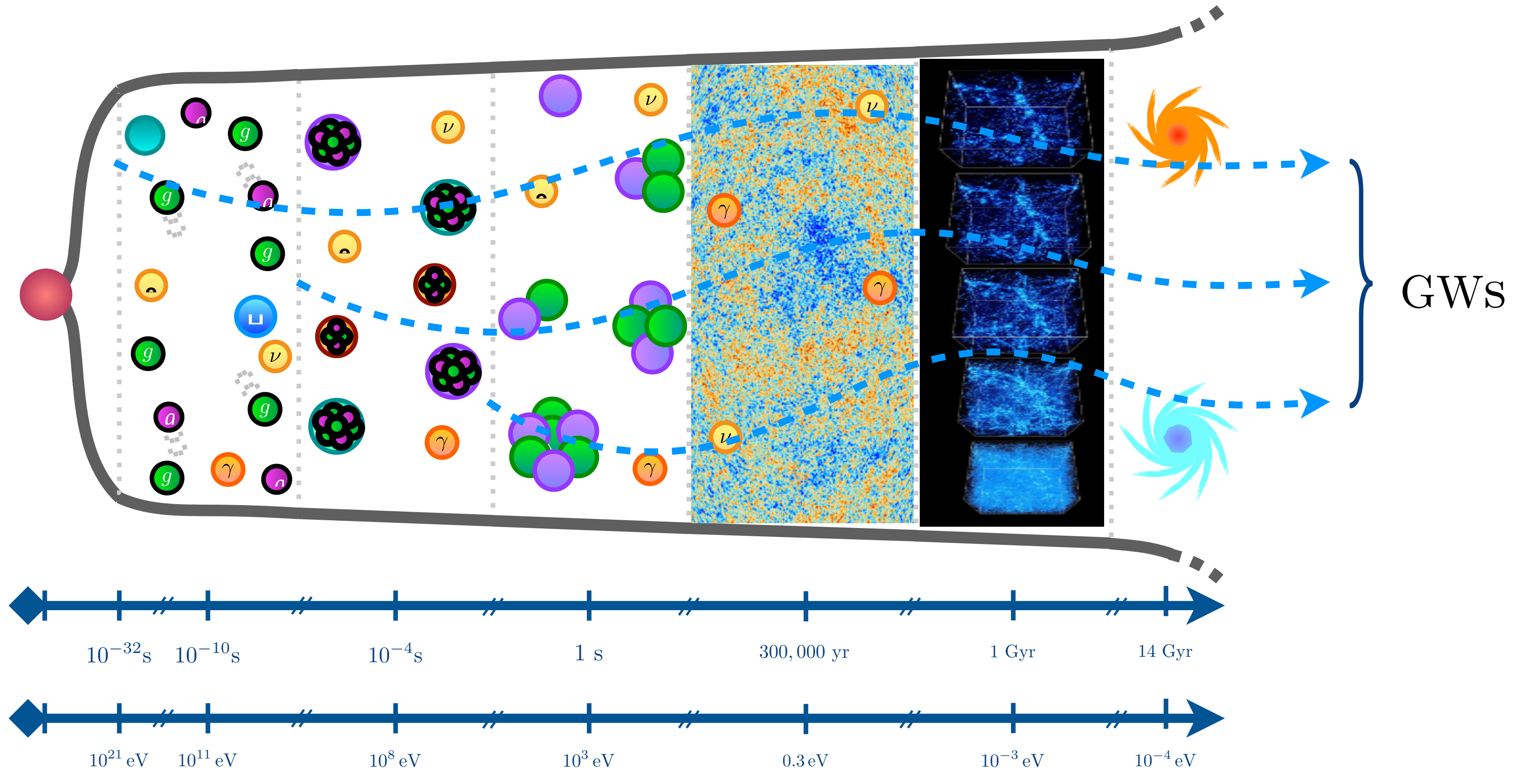
Astrophysical GWs

$$\omega_g \Leftrightarrow \Lambda_{\rm origin}$$

High-Frequency Gravitational Waves

HFGWs to probe BSM Cosmology





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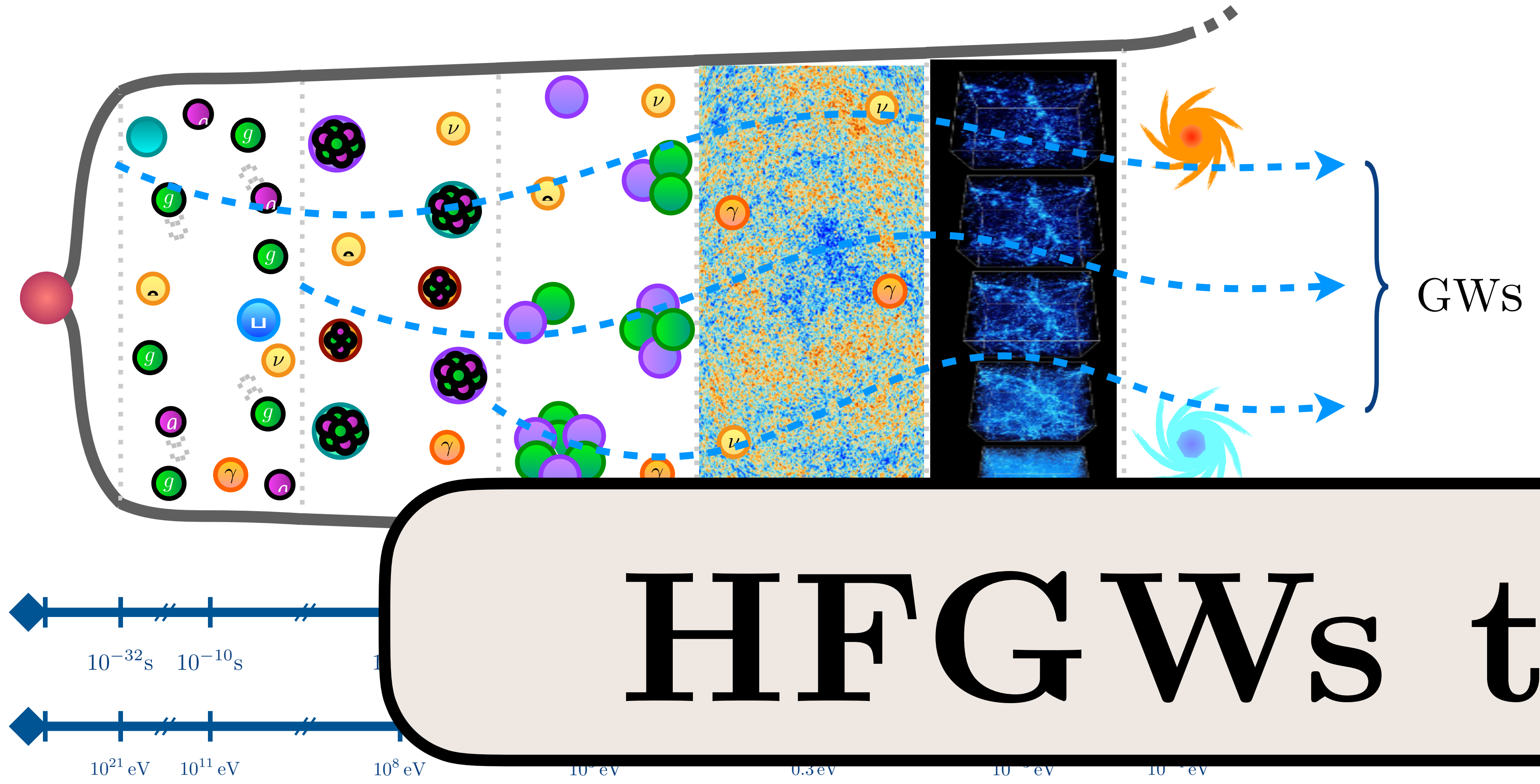

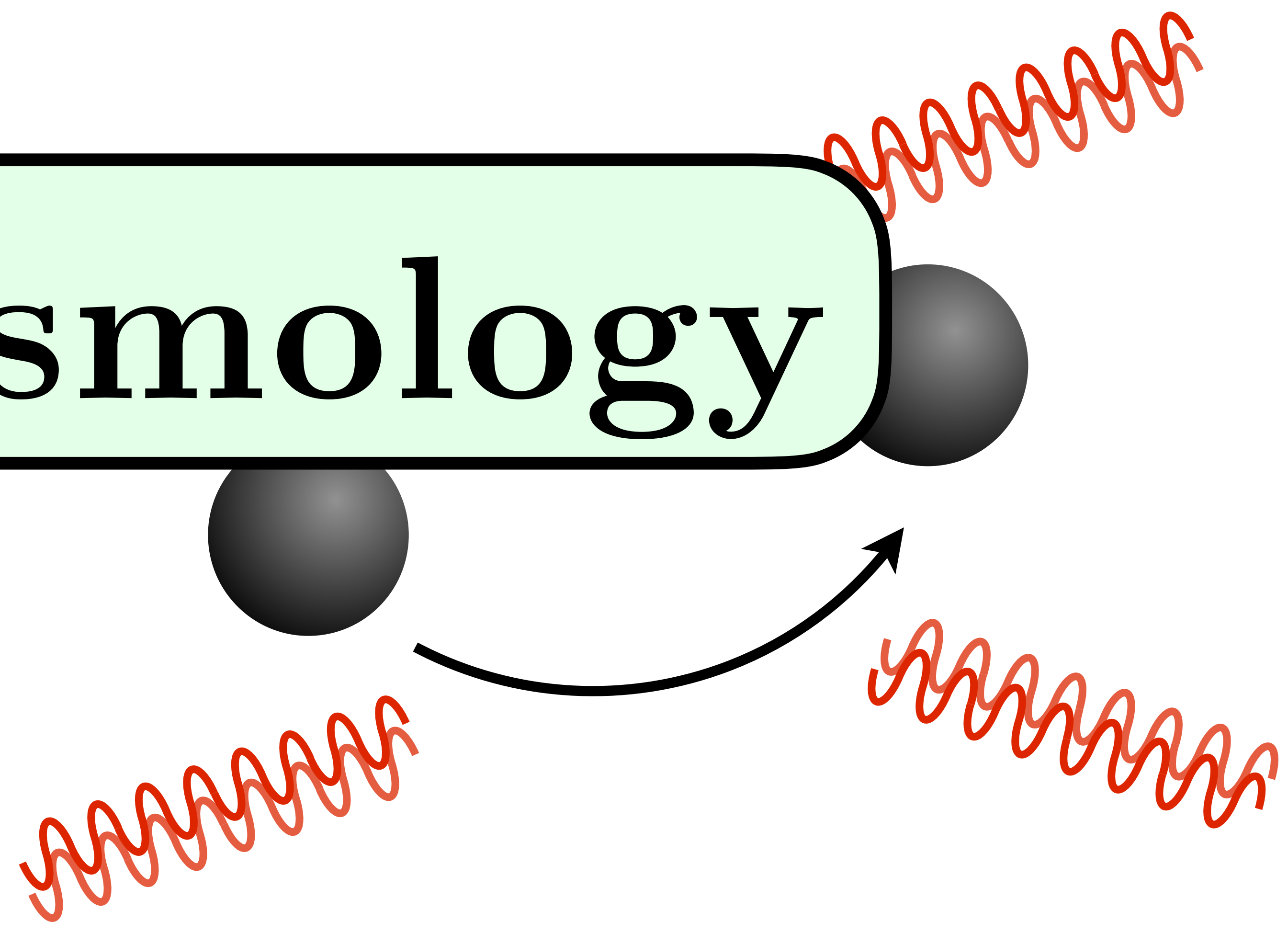

HFGWs to probe Dark Matter

DETECTION HEURISTICS

How do we measure GWs?

Detector Energy

Detector stores EM energy: $U_{\rm in} \sim \langle E_0(t) E_0^*(t) \rangle V_{\rm det}$

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Clearly better, right?

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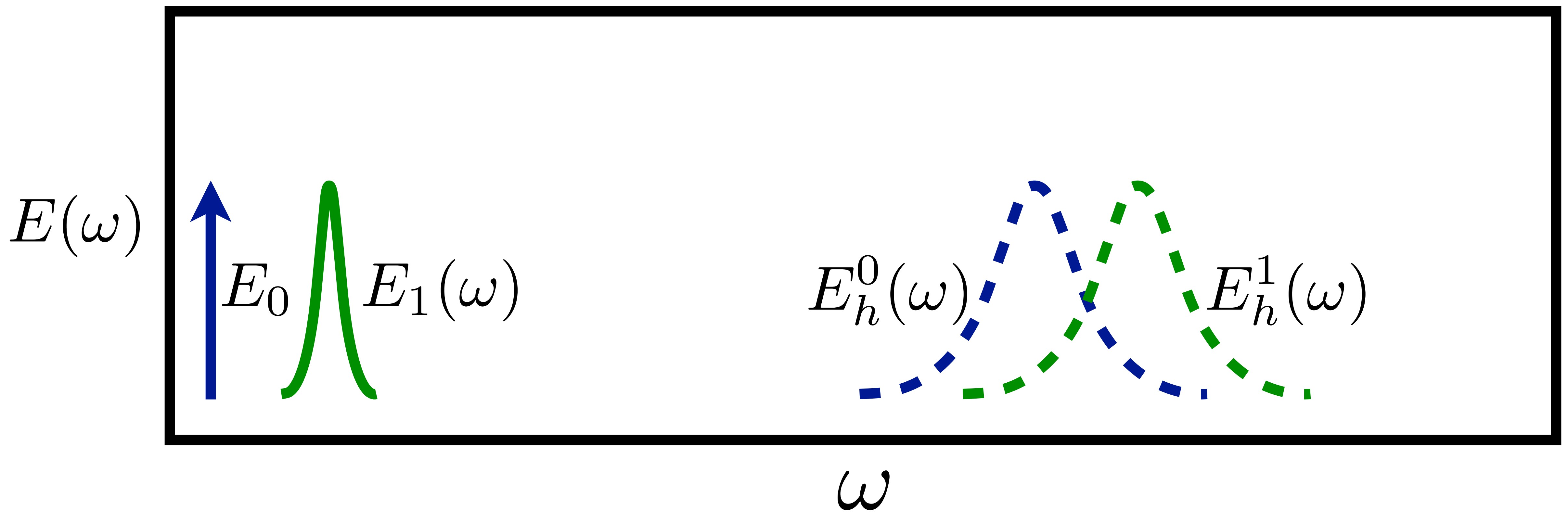

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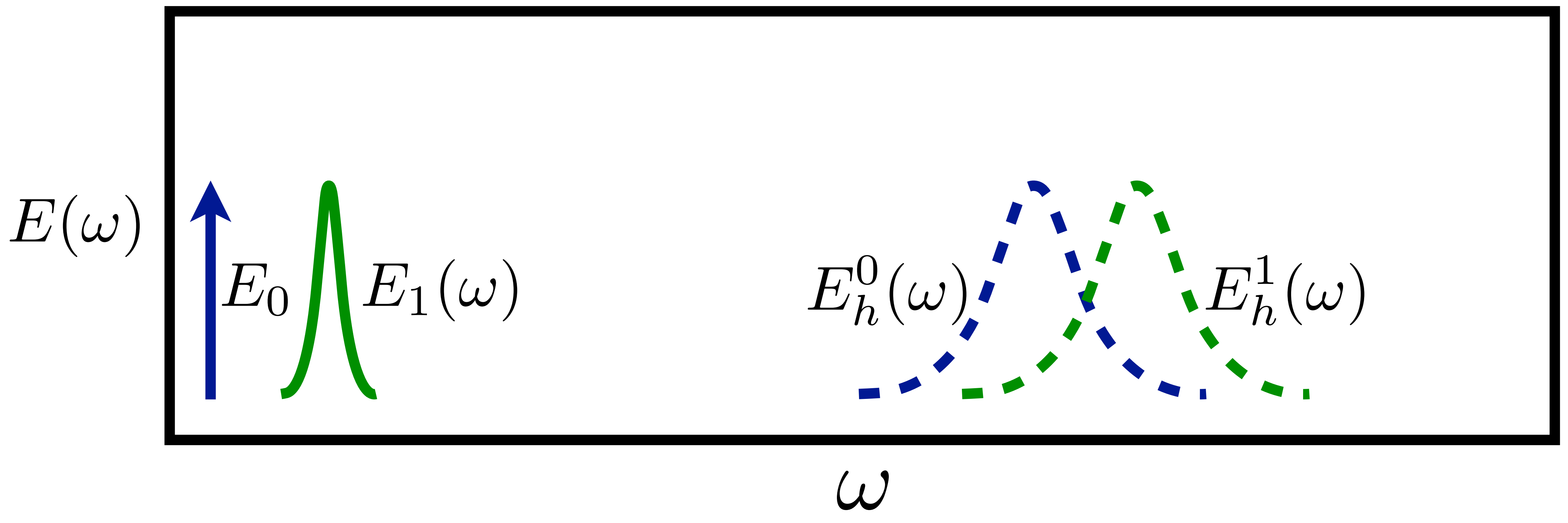

Experiment can be performed such that background energy at detection frequency $\sim$ zero

$$N_{\rm bg}^{\gamma} = \left(\frac{1}{2}\right)_{\rm ba} + \left(\frac{1}{2}\right)_{\rm 0pt}$$

Detector Energy: Quadratic Detector

Minimum detectable power seen by detector

$$P_{\rm min} \sim \frac{(N^{\gamma}_{\rm bg})^{1/2}\,\omega}{t_{\rm int}}$$

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Best possible sensitivity: $h_{\text{min}}^{q} \sim \frac{1}{\sqrt{U_{\text{in}} t_{\text{int}}}} \frac{1}{\mathcal{T}(\omega)}$

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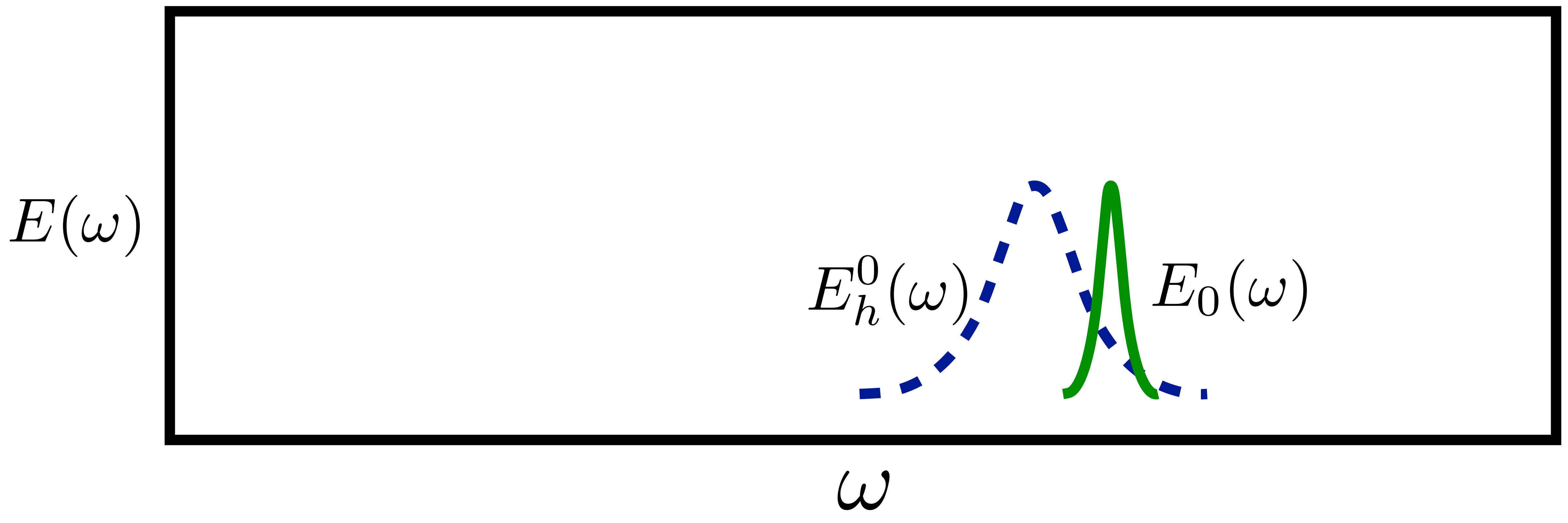

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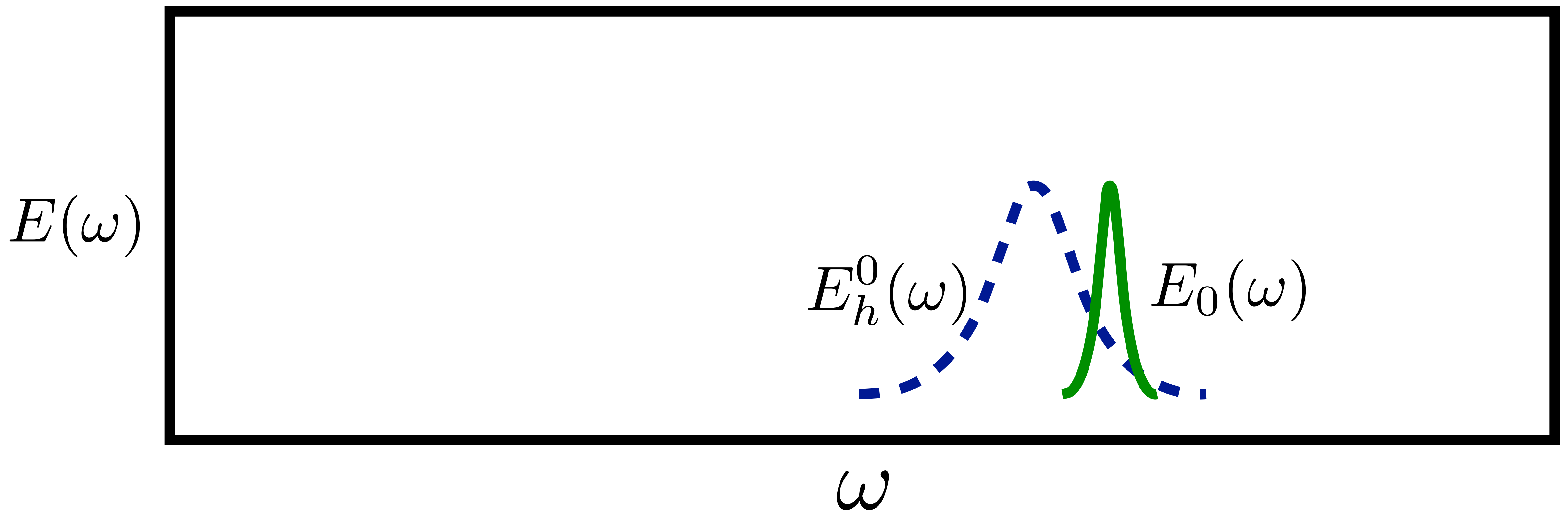

Experiment is performed such that $\langle E_0(\omega) E_0^*(\omega) \rangle \neq 0$

$$N_{\rm bg}^\gamma \sim U_{\rm in}\, t_{\rm int}$$

Detector Energy: Linear Detector

Noise power seen by detector

$$P_{\rm min} \sim \omega \left(\frac{U_{\rm in}}{t_{\rm int}} \right)^{1/2}$$

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🤔

Caveats and Takeaway Messages

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Stochastic backgrounds (almost) always imply quadratic measurements

Caveats and Takeaway Messages

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How about that $\mathcal{T}$ransfer function?

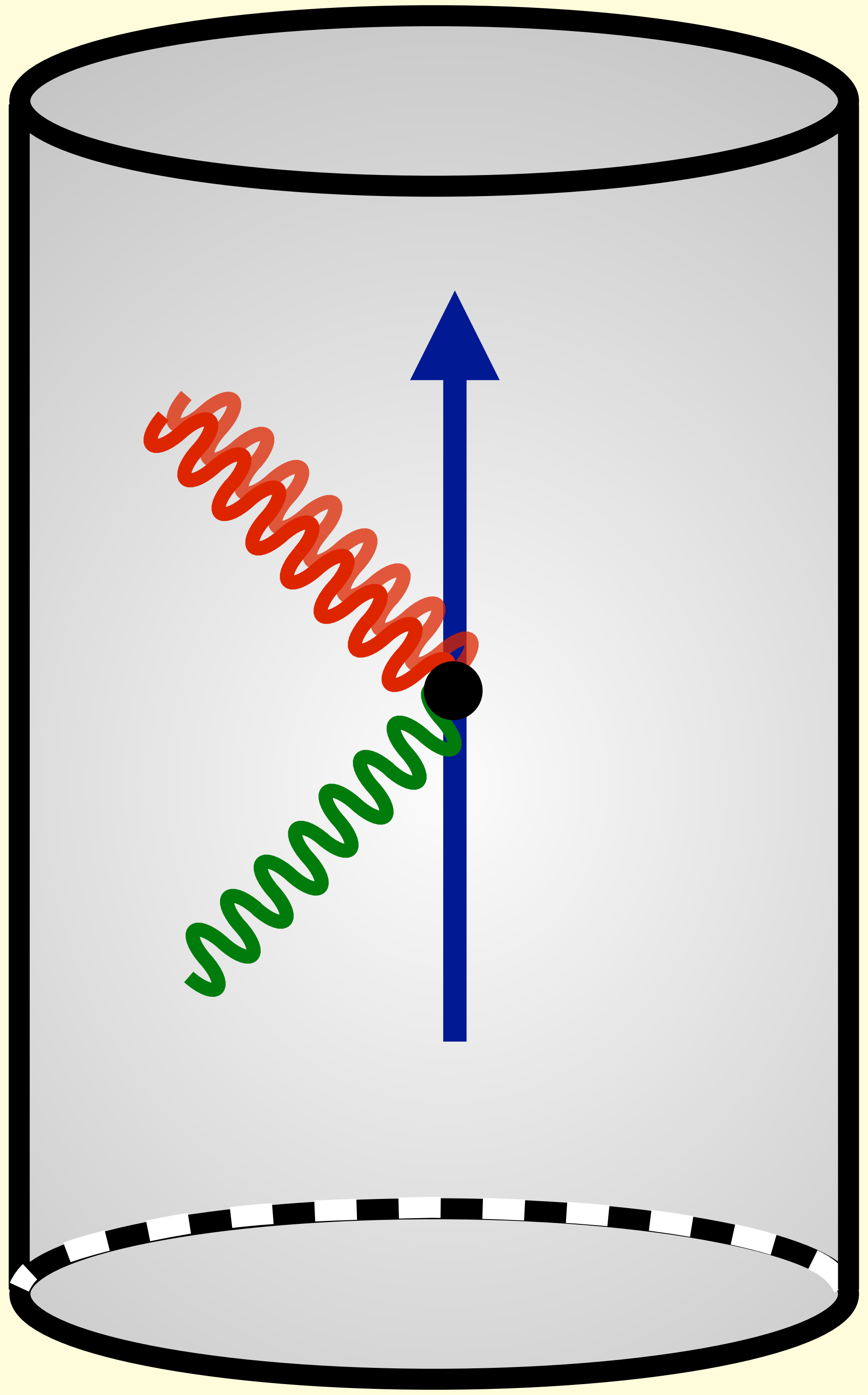
Bandwidth and other detector details shoved under $\mathcal{T}$he rug

Stochastic backgrounds (almost) always imply quadratic measurements

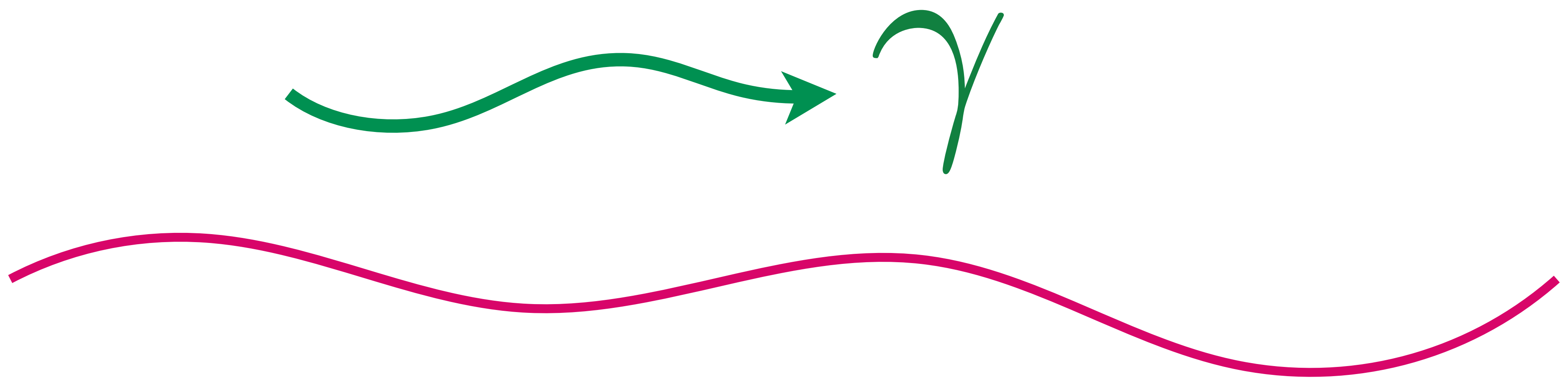
ACTE I

Static B-field

Cavities



Interactions of Gravitational Waves *with light*



$$S_{\rm EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}J_\mu A_\nu\right)$$

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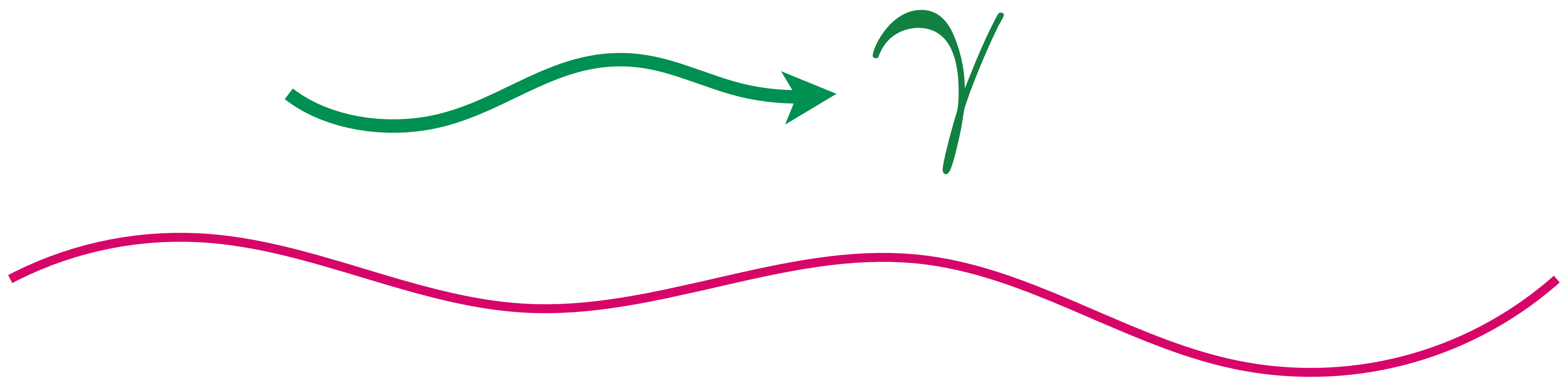

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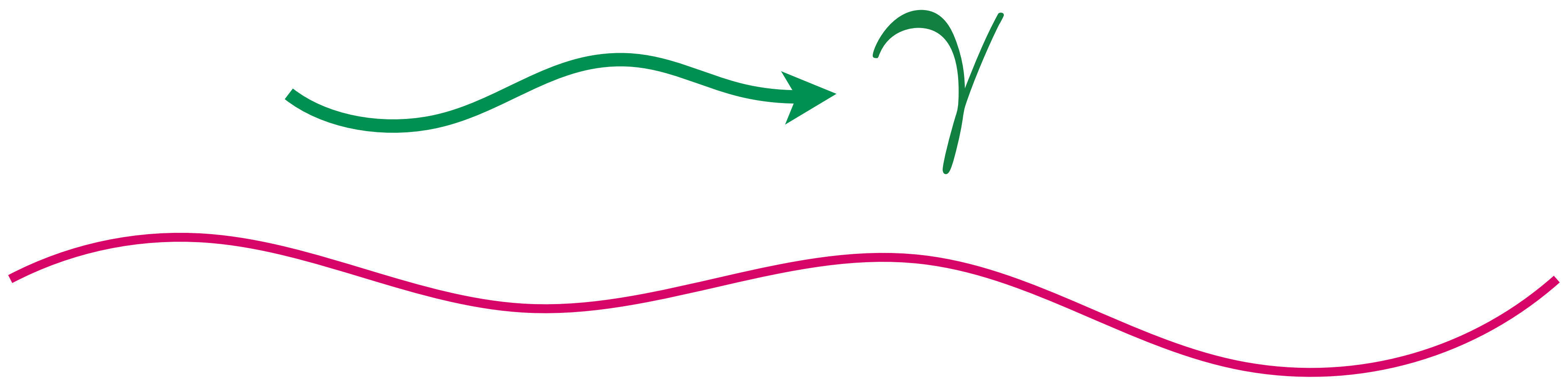

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Equation of motion: $$\partial F \sim -\partial\,(h\,F)$$

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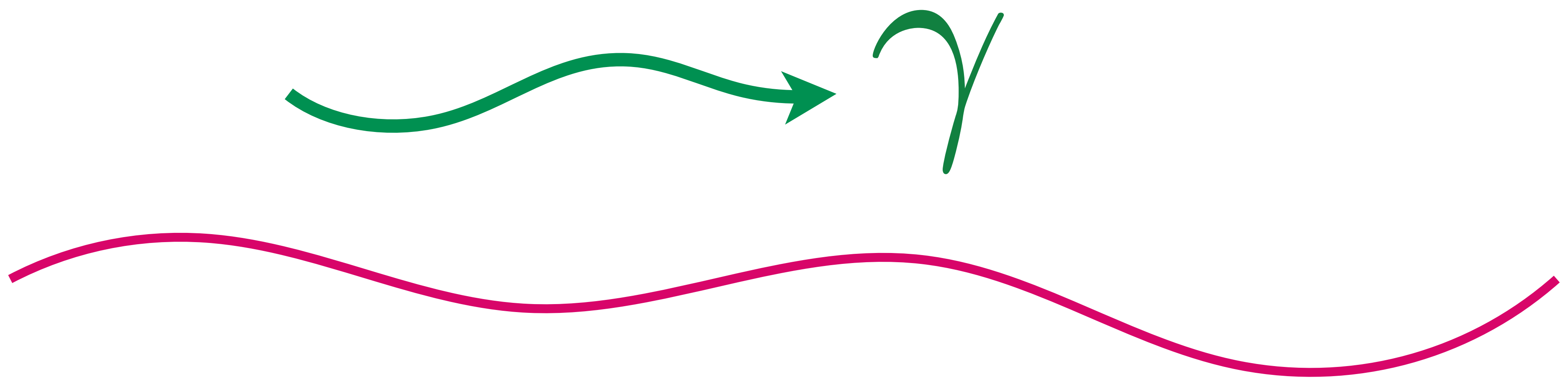

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Effective current from spatial or temporal variations of h or F

$$j^\mu_{\rm eff} \equiv \partial_\nu\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^\nu{}_\alpha\,F^{\alpha\mu} - h^\mu{}_\alpha\,F^{\alpha\nu}\right)$$

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Intuition for EM signal

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Estimate sensitivity to GWs by comparing sizes of currents

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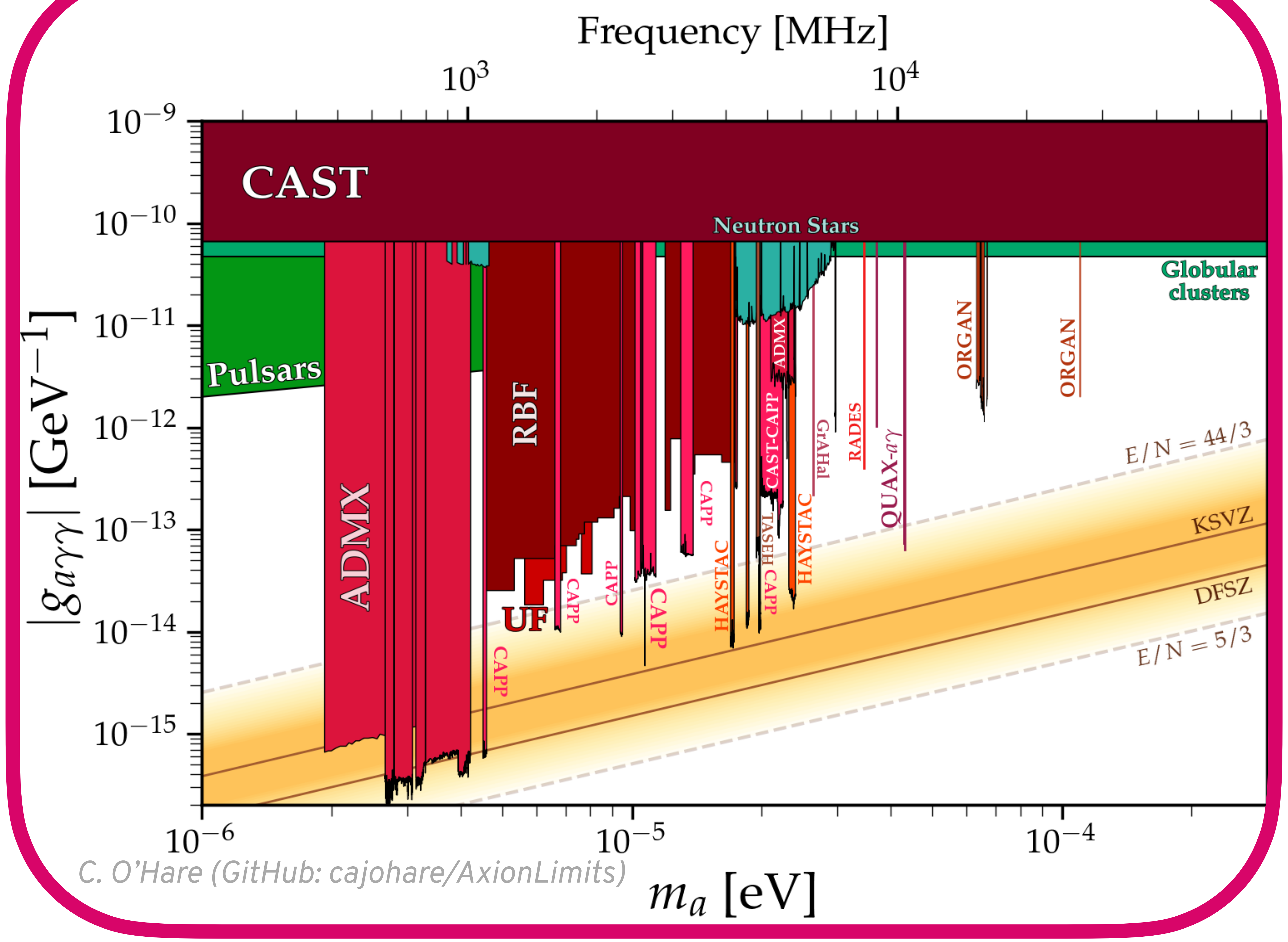

C. O'Hare (GitHub: cajohare/AxionLimits)

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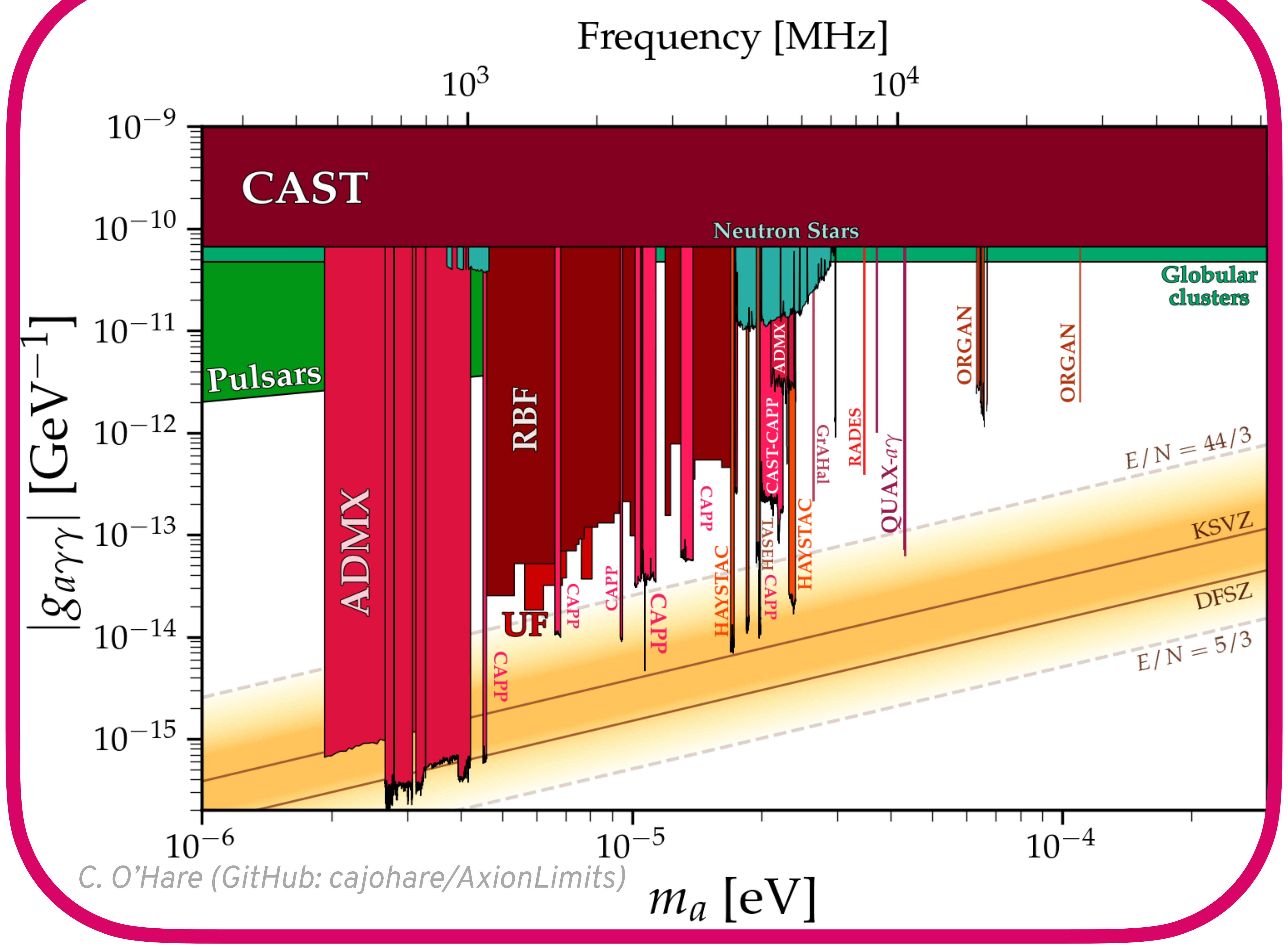

Estimate sensitivity to GWs by comparing sizes of currents

$$j_{\rm eff}^{\rm axion} \sim g_{a\gamma\gamma}\partial_t(a\mathbf{B}) + \mathcal{O}(v)$$

$$j_{\rm eff}^{\rm axion} \lesssim 10^{-19}\,\mathrm{T/m}$$

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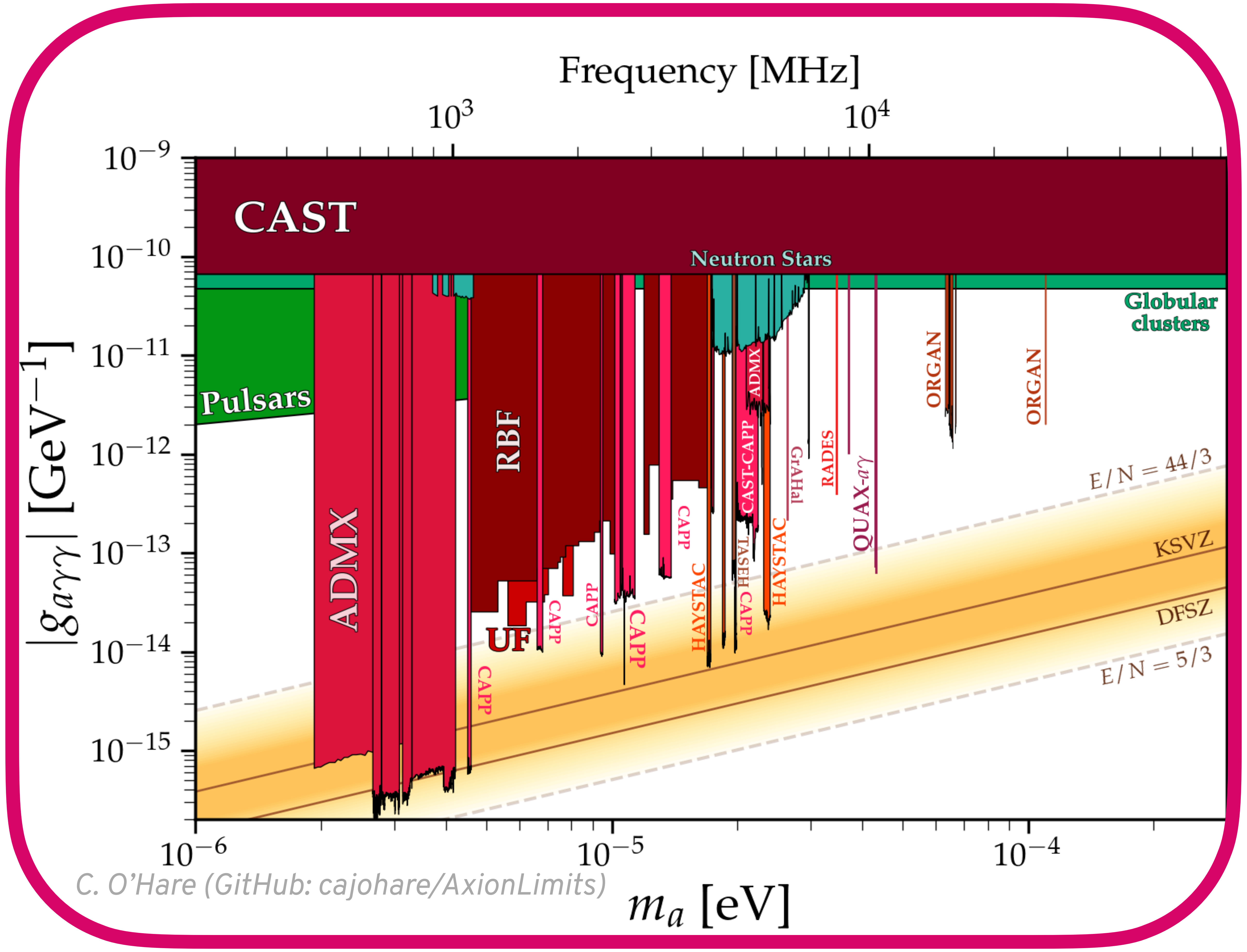

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$$\Downarrow$$

$$j_{\rm eff}^{\rm GW} \sim \partial_t(h\mathbf{B}) + \ldots$$

$$h \lesssim 10^{-21}$$

Framing the Question

Proper Detector Frame — complication

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Textbooks give long-wavelength approximation $\omega_g \, R_{\rm cav} \ll 1$

$$ds^2 \simeq -dt^2(1 + R_{0i0j}x^i x^j) - \frac{4}{3}\, dt\, dx^i \left(R_{0ijk}x^j x^k\right) + dx^i\, dx^j \left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$

e.g. Maggiore (2007)

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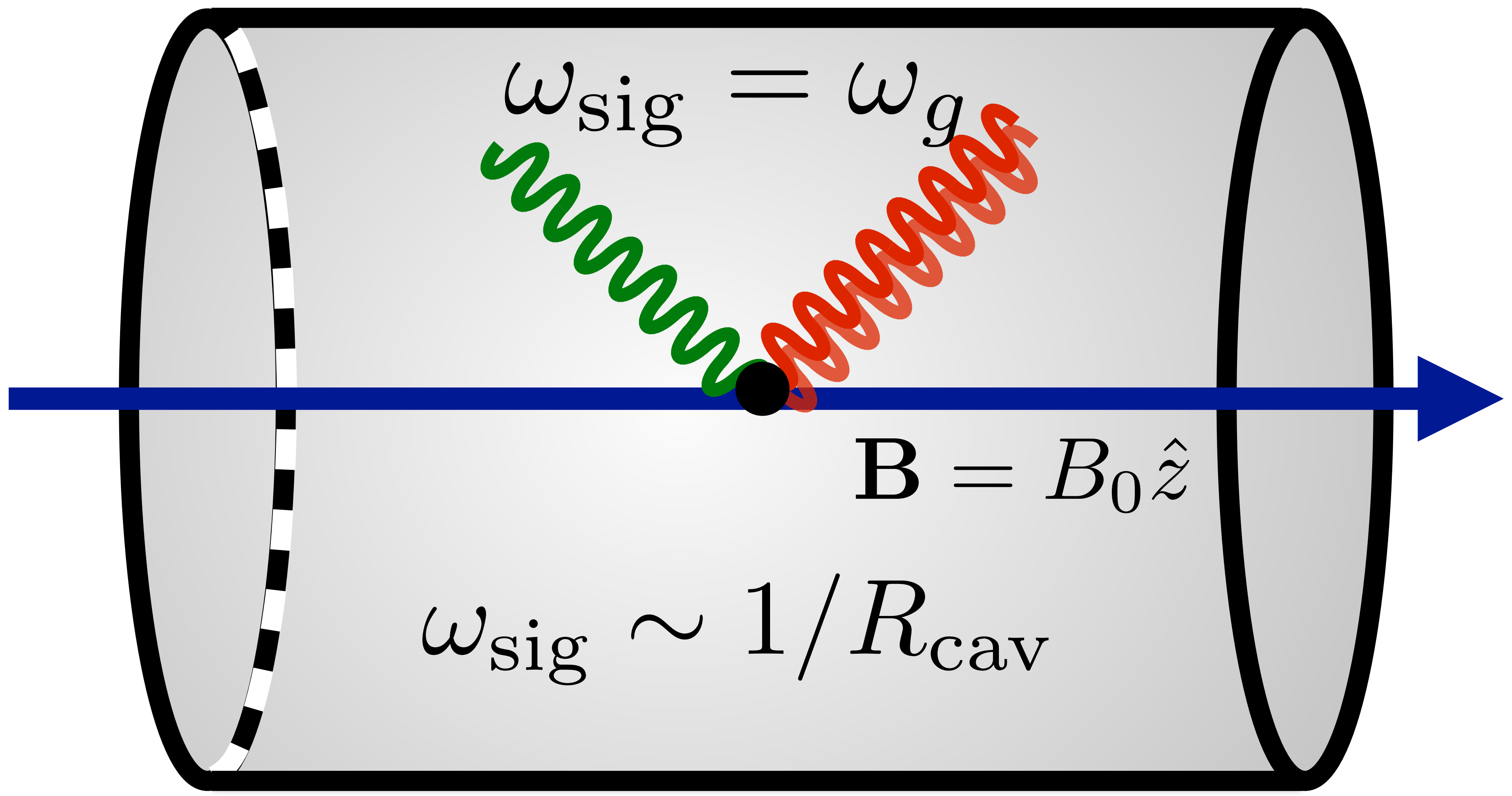
Resonant Cavity:



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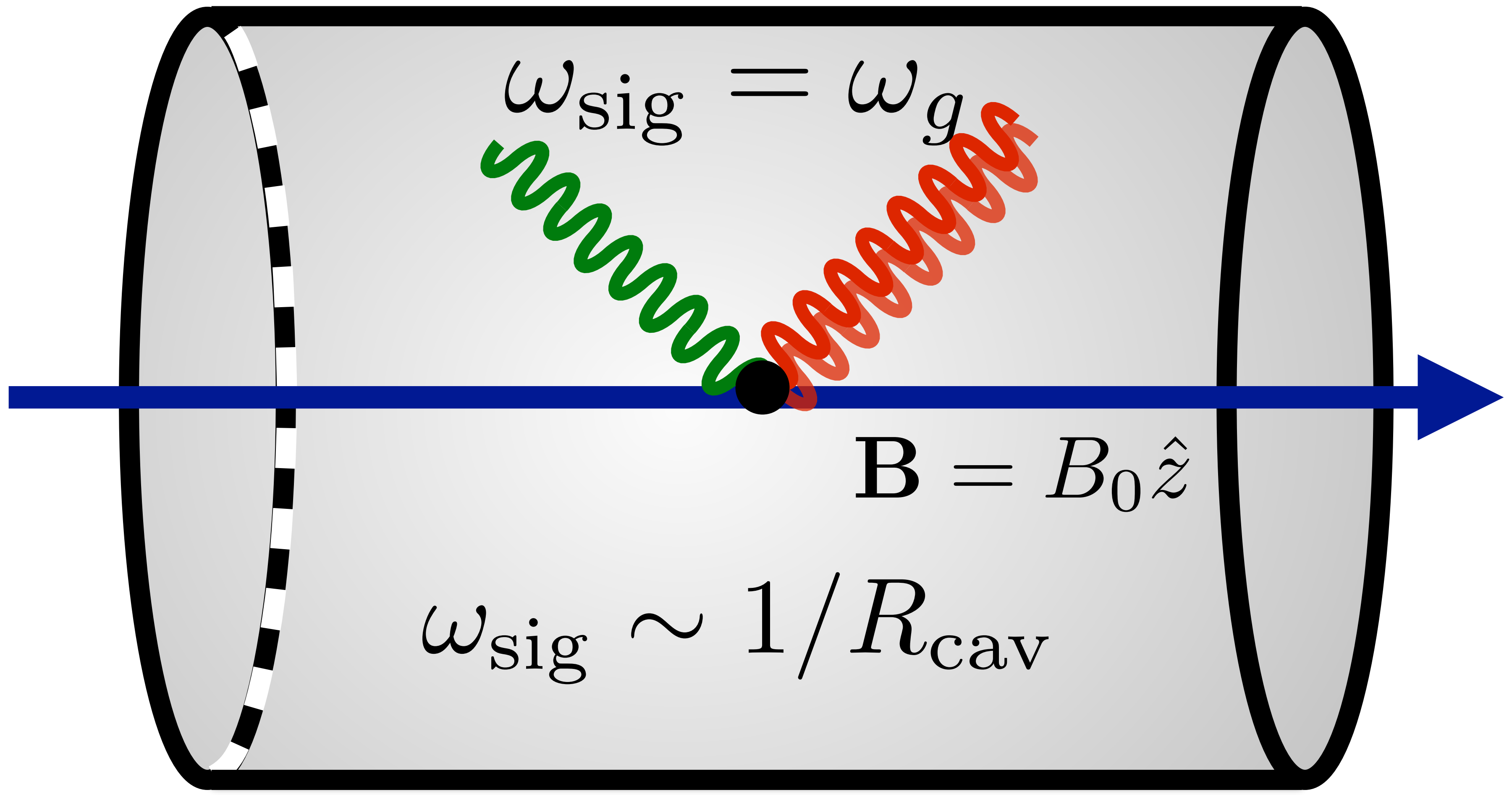

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Long-wavelength limit invalid!

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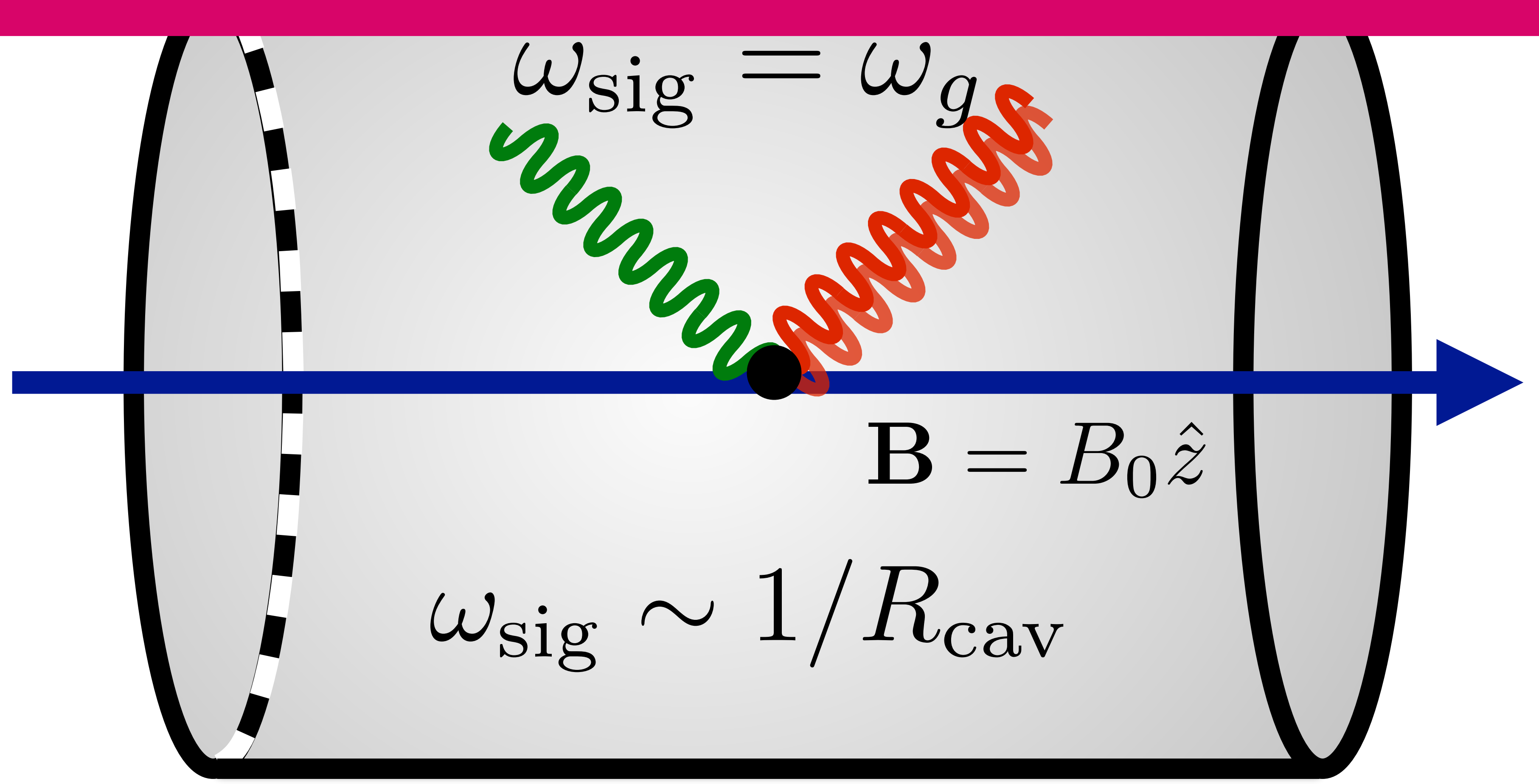
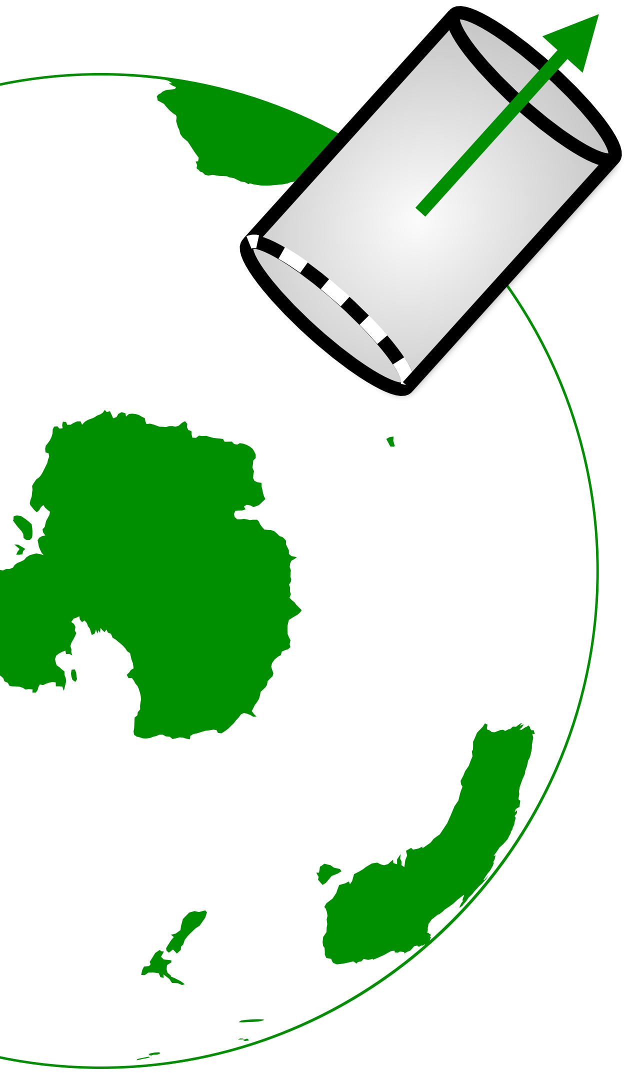


Framing the Question

Solution — GW as sum of plane waves

$$h \propto e^{i\omega_g(t-z)} \longrightarrow \partial_i h_{jk}^{\mathrm{TT}} \sim -\delta_{iz}\partial_t h_{jk}^{\mathrm{TT}}$$

$$x^{k_1}\dots x^{k_r} R_{\mu\nu\rho\sigma,k_1\dots k_r} = (-i\omega_g z)^r R_{\mu\nu\rho\sigma}$$



$$h_{00} = -2\sum_{r=0}^{\infty}\frac{r+3}{(r+3)!}R_{0n0n,k_1,\cdots k_r}x^m x^n x^{k_1}\cdots x^{k_r}$$

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Märzlin (1994)
Rakhmanov (2014)

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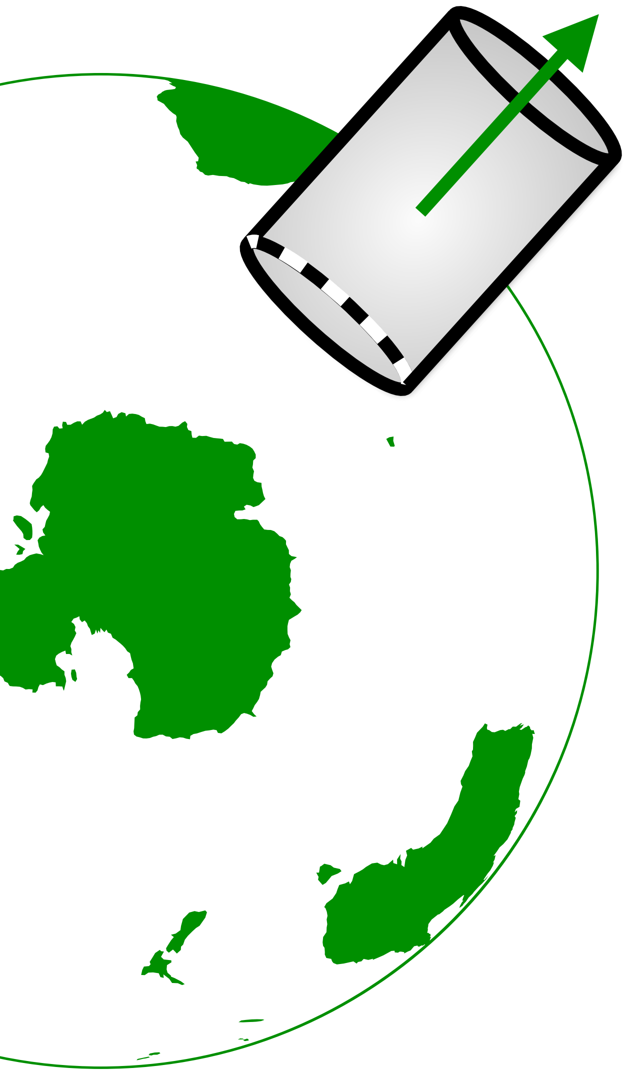

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$$h_{0i} = -2R_{0min}x^m x^n \left(-\frac{i}{2\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

$$h_{ij} = -2R_{imjn}x^m x^n \left(-\frac{1+e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

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Axion Cavity Modes Couple to GWs

$$\eta \propto \int_V \mathbf{E}^*_{\text{cav}} \cdot \mathbf{J}_{\text{eff}}$$

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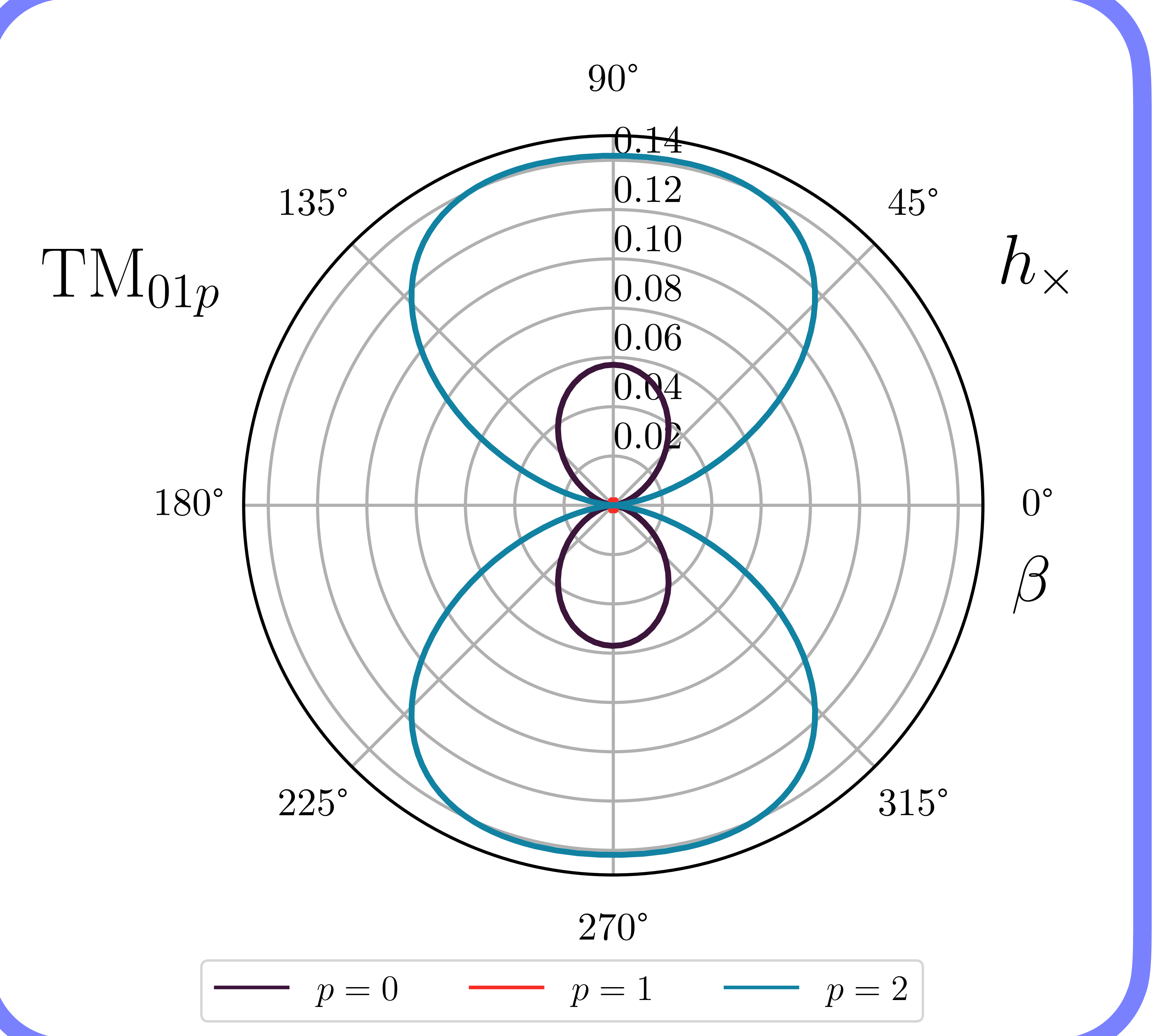

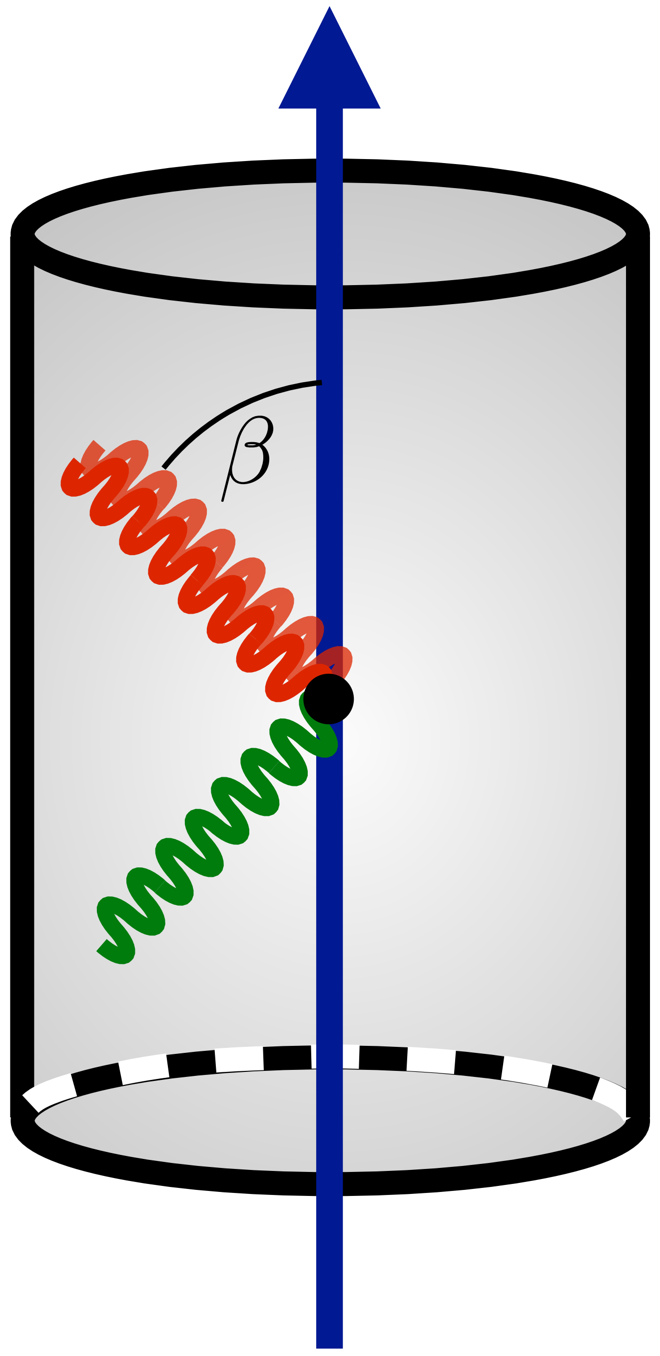

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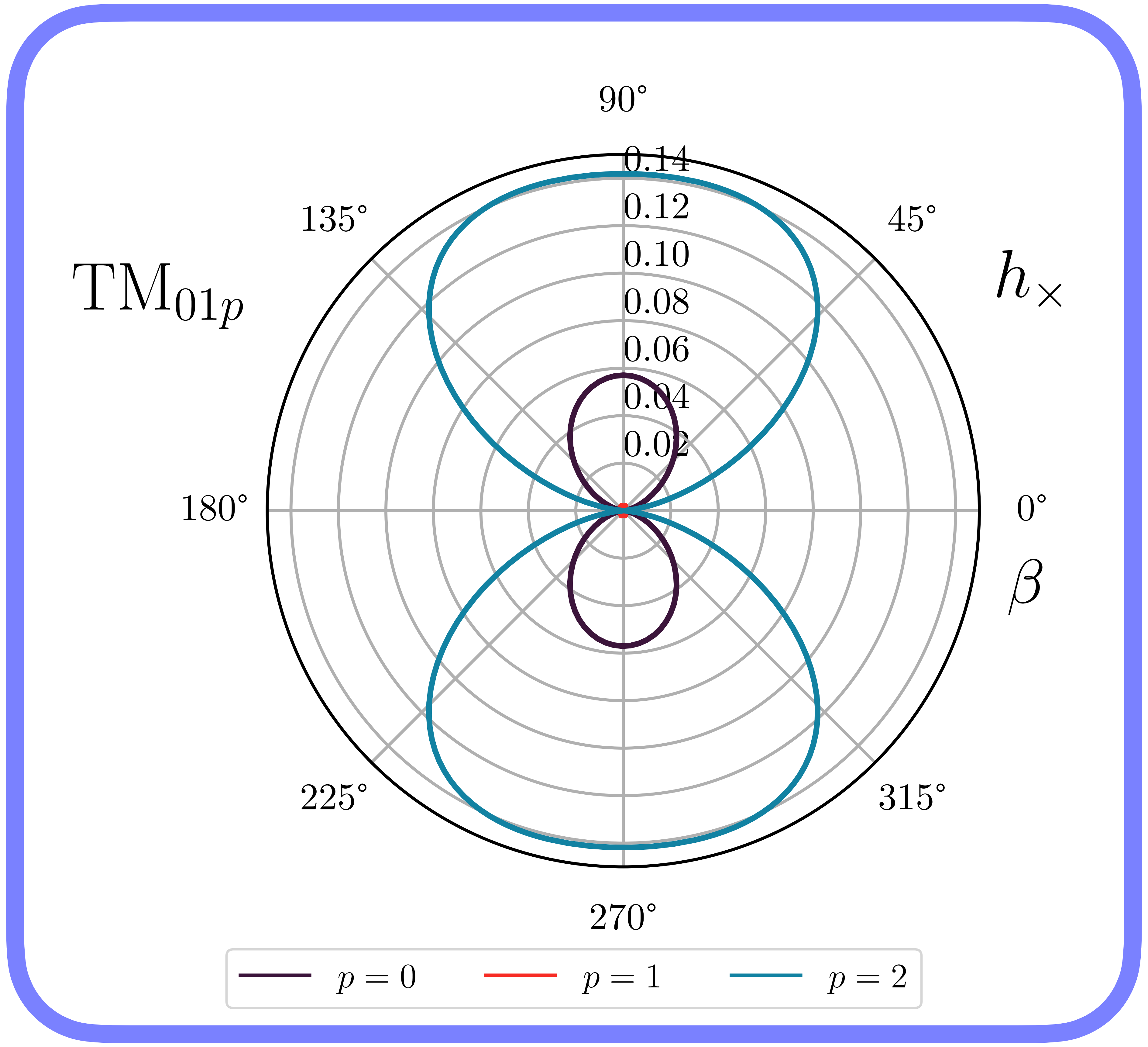
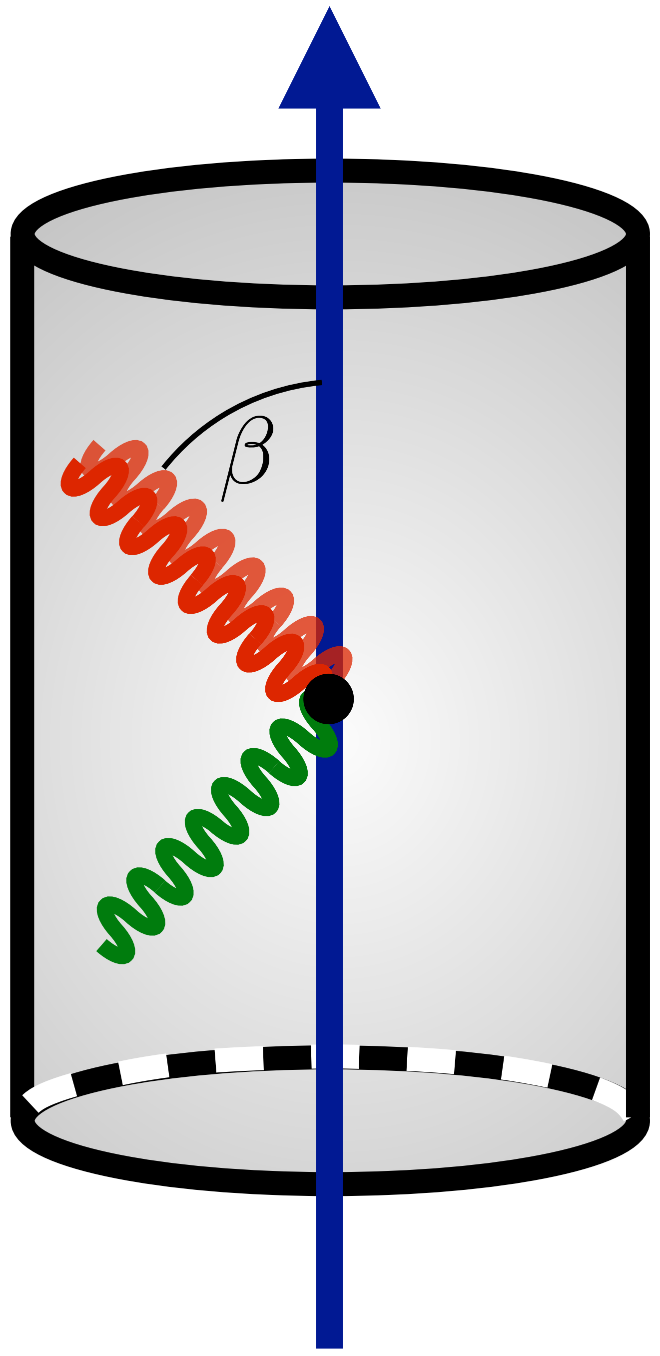
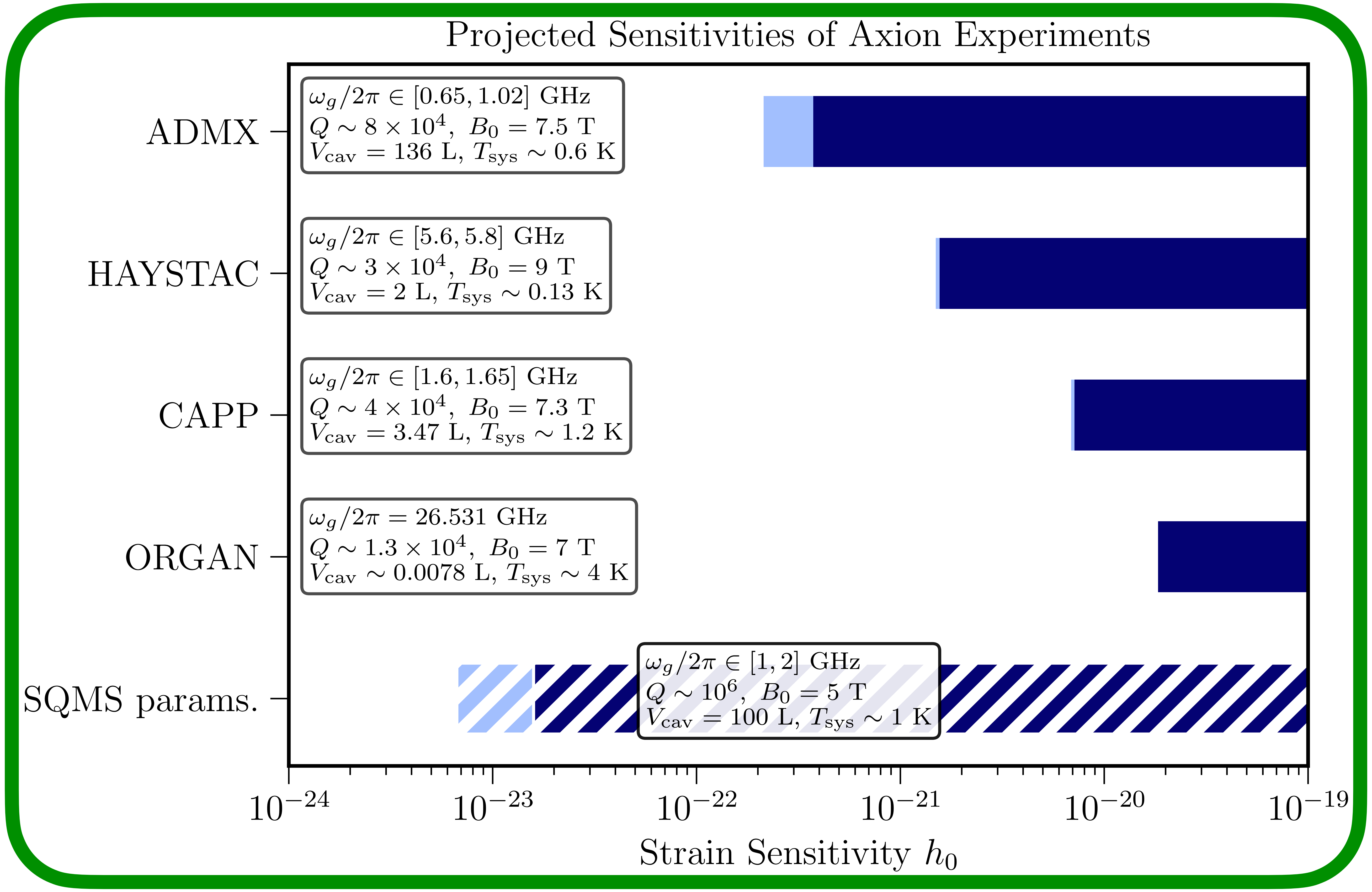


Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

But TM modes not optimal...



Sensitivity of Resonant Cavities



Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Coherent GW

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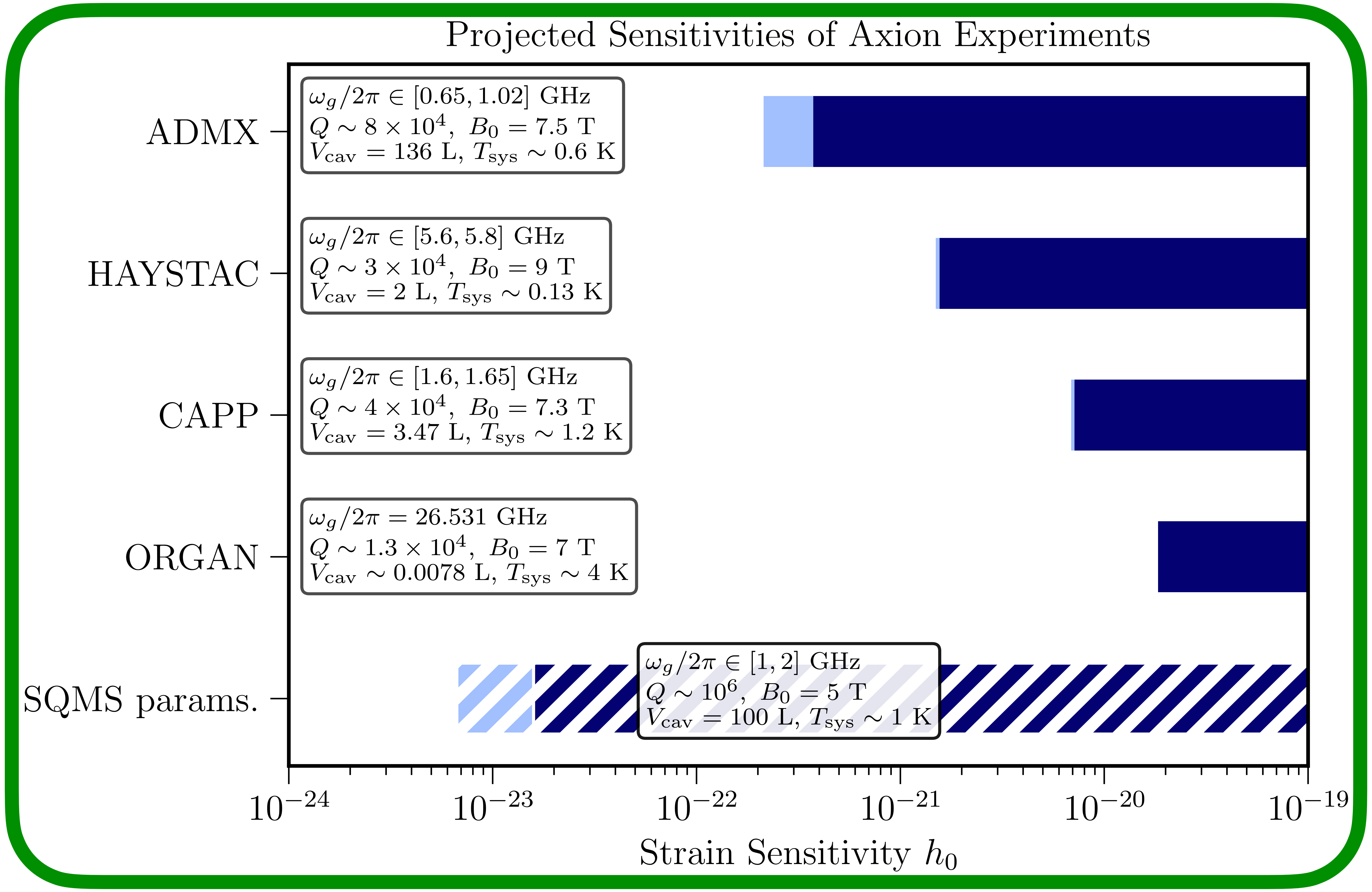

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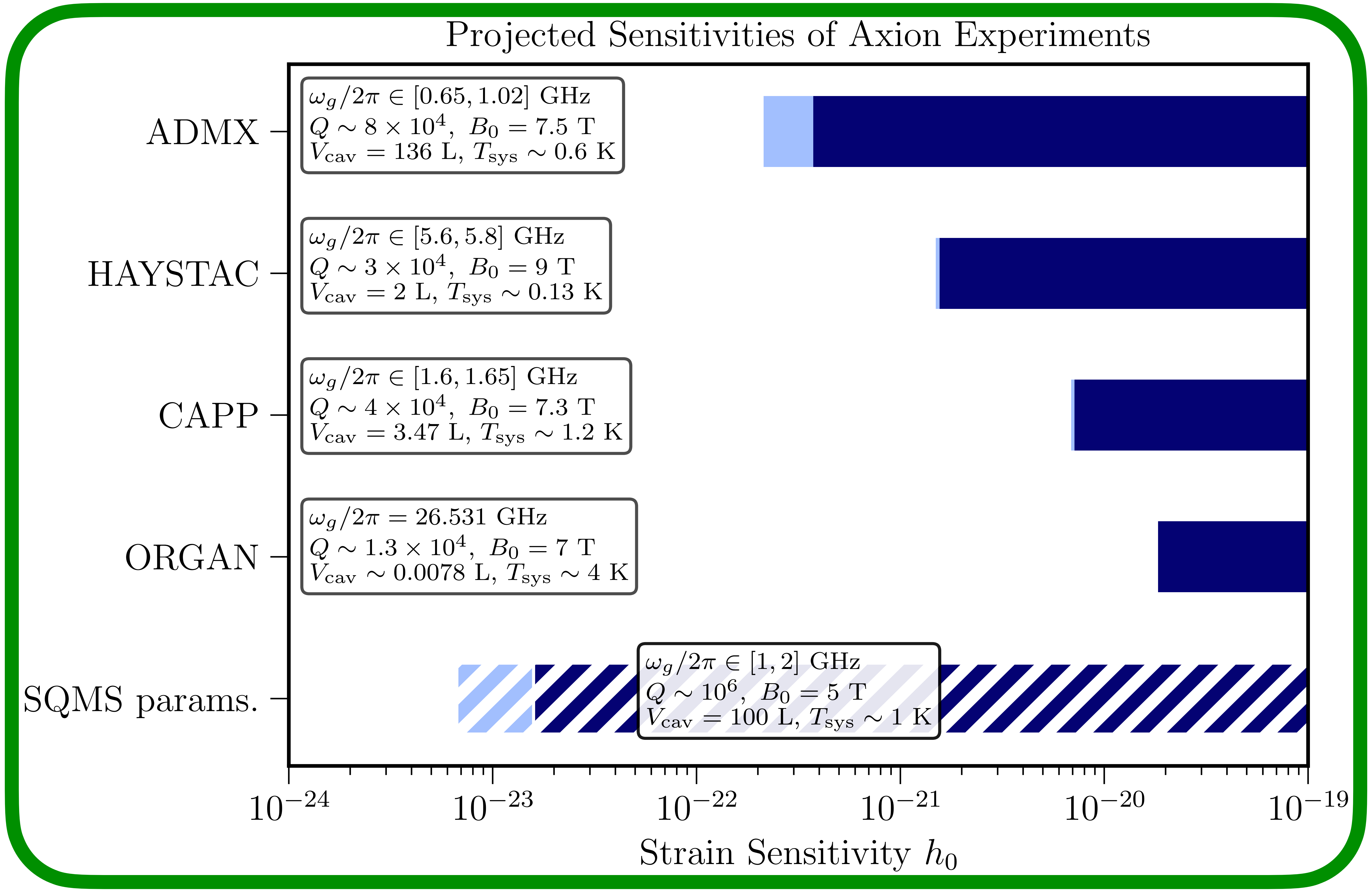
Stochastic GWs

$$\mathrm{SNR} \sim Q\, \omega_g\, \eta_{\rm stoch}^2\, B_0^2\, V_{\rm cav}\, S_h(\omega_g)/T_{\rm sys}$$

$$\Omega_g(\omega_g) \sim \omega_g^3\, S_h(\omega_g)/H_0^2$$

Sensitivity of Resonant Cavities



Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Coherent GW

$$P_{sig} = \frac{1}{2} Q\,\omega_g^3\,V_{cav}^{5/3}\,(\eta_n\,h_0\,B_0)^2$$

Stochastic GWs

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Not beyond BBN bound...

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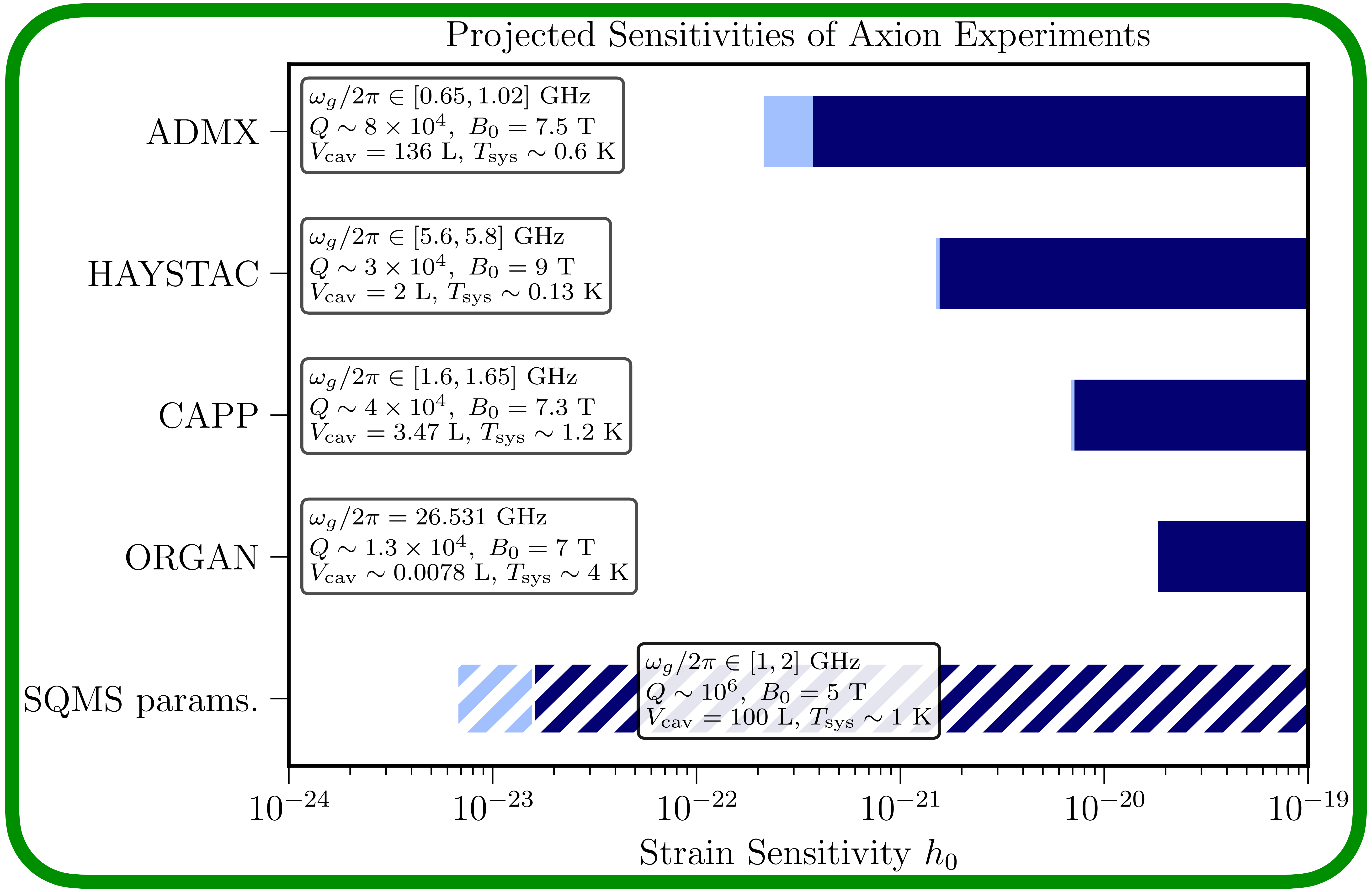

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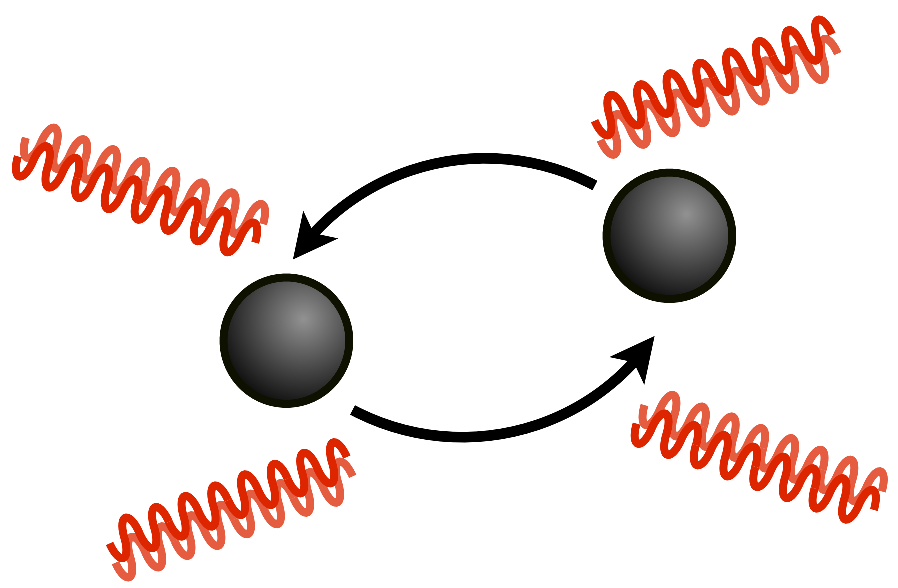
Not beyond BBN bound...

$$\mathcal{T} \sim Q\eta_0(\omega_g V_{\rm cav}^{1/3}) \sim 10^5$$

Axion Cavity Sensitivity to PBH binaries



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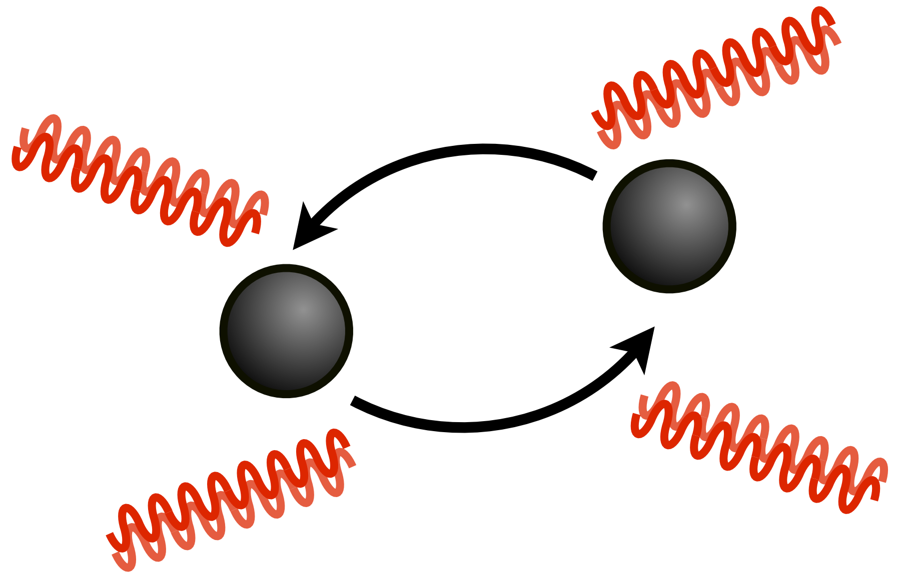
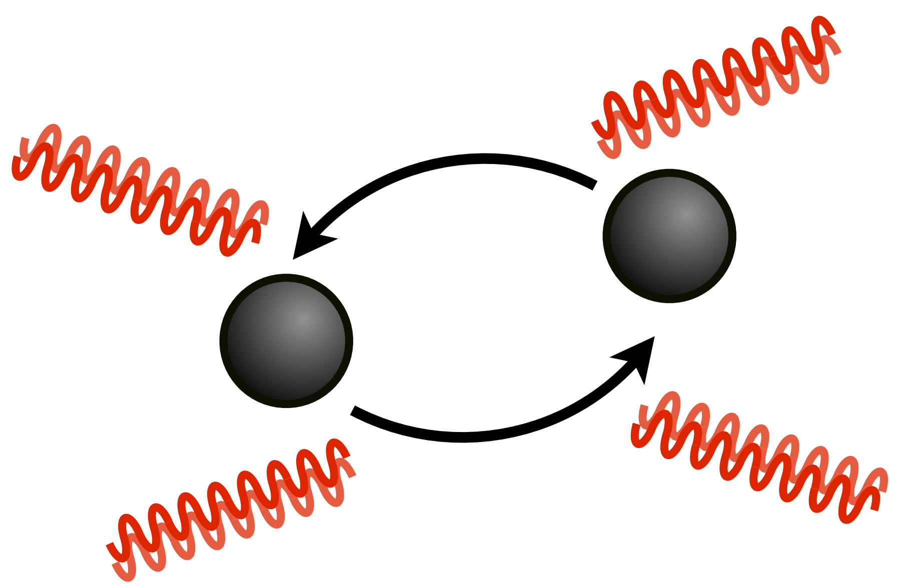


$\omega_g \simeq 14\ \text{GHz} \times (10^{-6}\, M_\odot / M_b)\, (r_{\text{ISCO}}/r_b)^{3/2}$

$d\omega_g/dt \propto (M_b/r_b)^{11/6}$

Axion Cavity Sensitivity to PBH binaries



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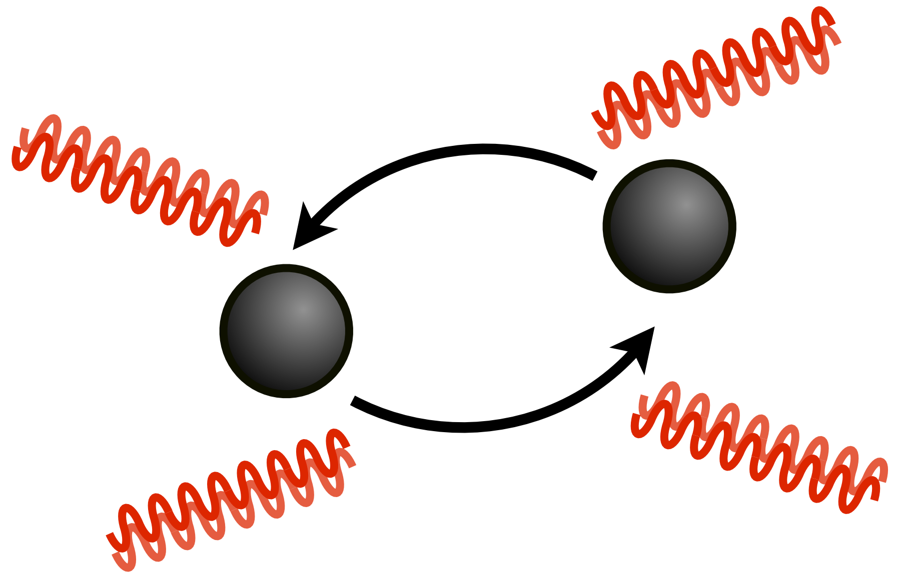
Axion Cavity Sensitivity to PBH binaries



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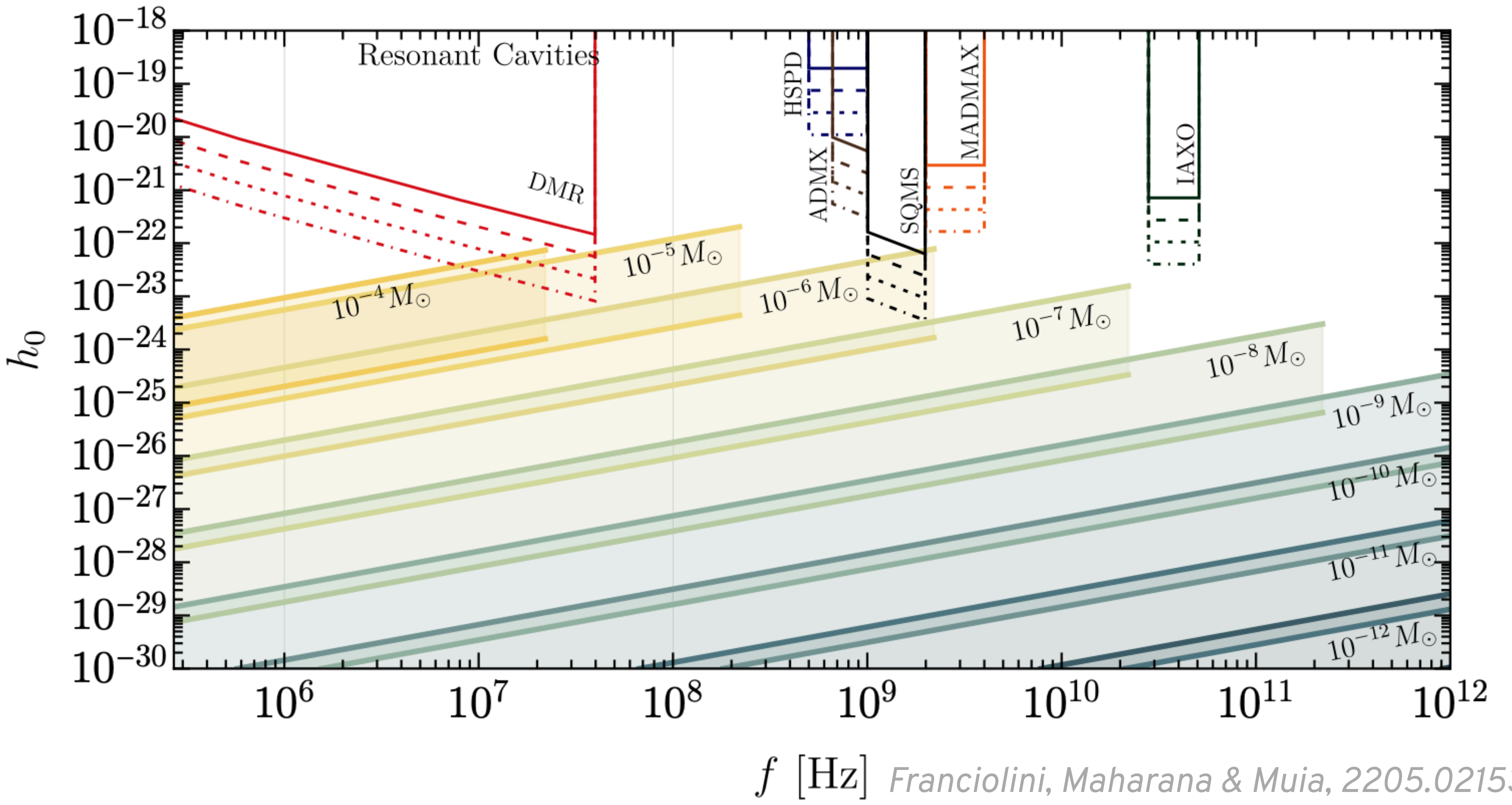
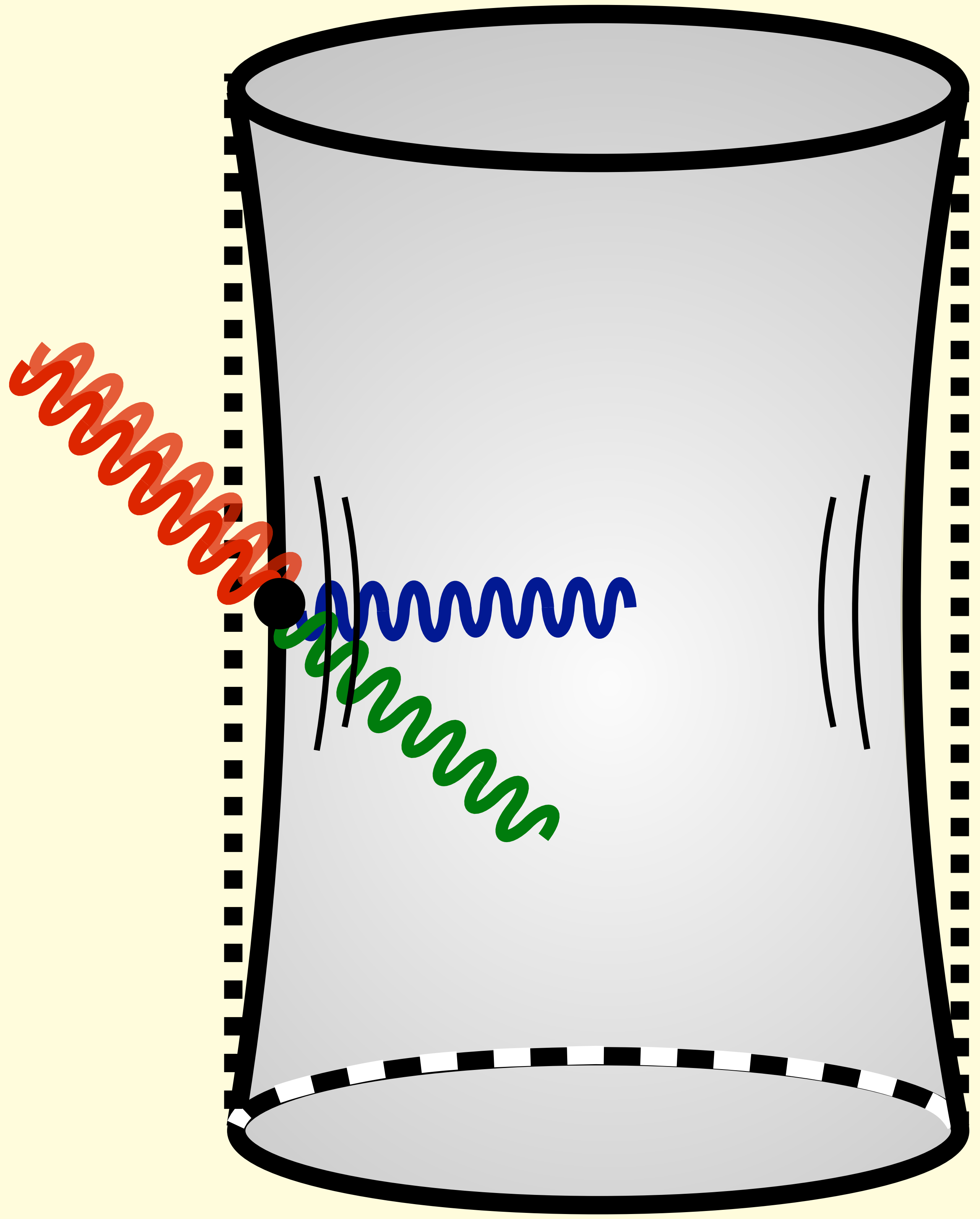


Franciolini, Maharana & Muia, 2205.02153

ACTE II

Loaded Cavities as Weber Bars,

a.k.a. LIGO w/ RF



MAGO 2.0: Mechanical and EM Signals

* “Why Cavities?” in Latin

Berlin, Blas, D’Agnolo, SARE, Harnik, Kahn, Schutte-Engel & Wentzel (PRD 2023)

MAGO 2.0: Mechanical and EM Signals

On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro†, E Picasso‡ and L A Radicati‡§

†Scuola Normale Superiore, Pisa, Italy
‡CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978

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Microwave Apparatus for Gravitational Waves Observation

R. Ballantini, A. Chincarini, S. Cuneo, G. Gemme,* R. Parodi, A. Podestà, and R. Vaccarone
INFN and Università degli Studi di Genova, Genova, Italy

Ph. Bernard, S. Calatroni, E. Chiaveri, and R. Losito
CERN, Geneva, Switzerland

R.P. Croce, V. Galdi, V. Pierro, and I.M. Pinto
INFN, Napoli, and Università degli Studi del Sannio, Benevento, Italy

E. Picasso
INFN and Scuola Normale Superiore, Pisa, Italy and CERN, Geneva, Switzerland

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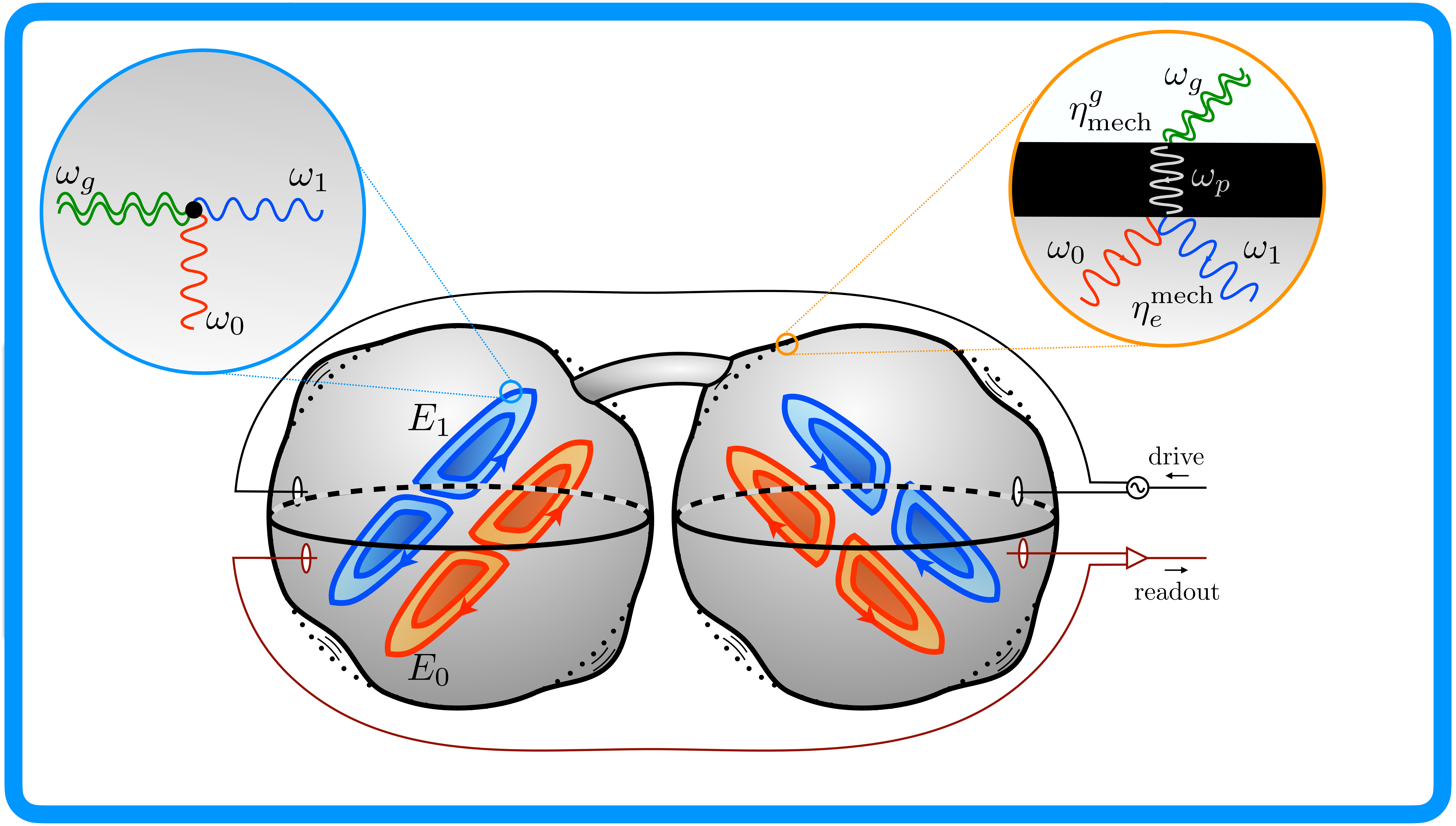

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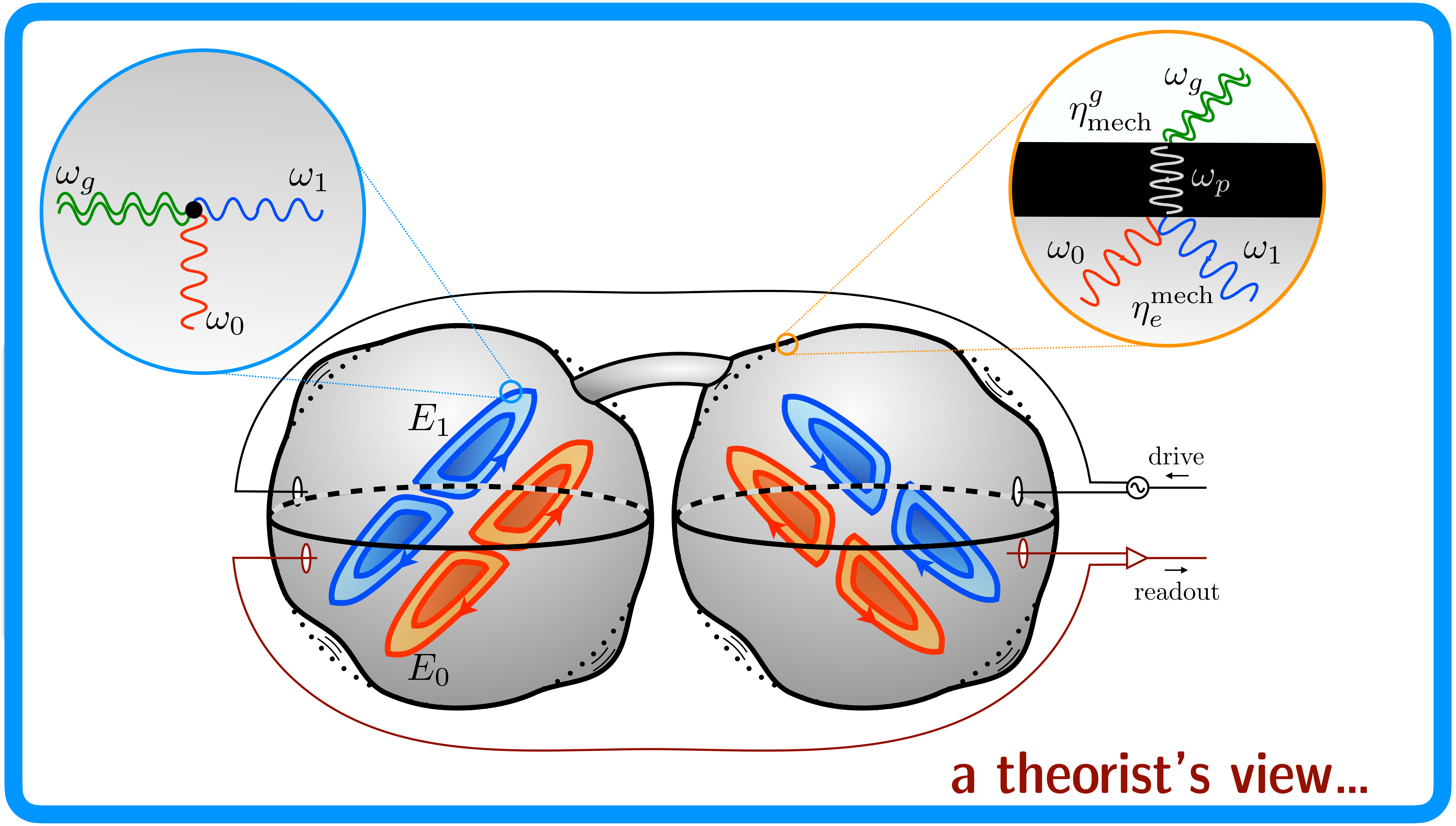

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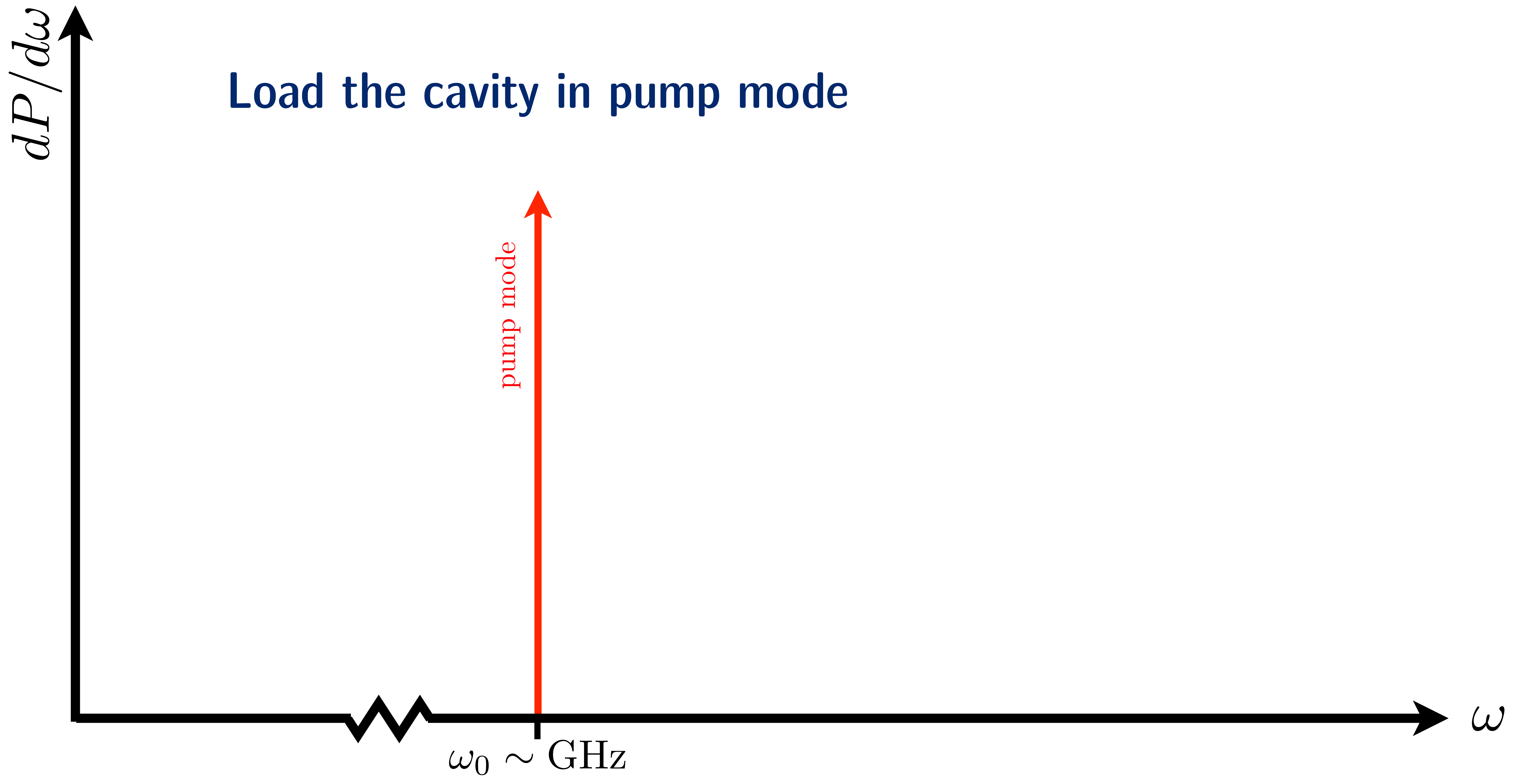
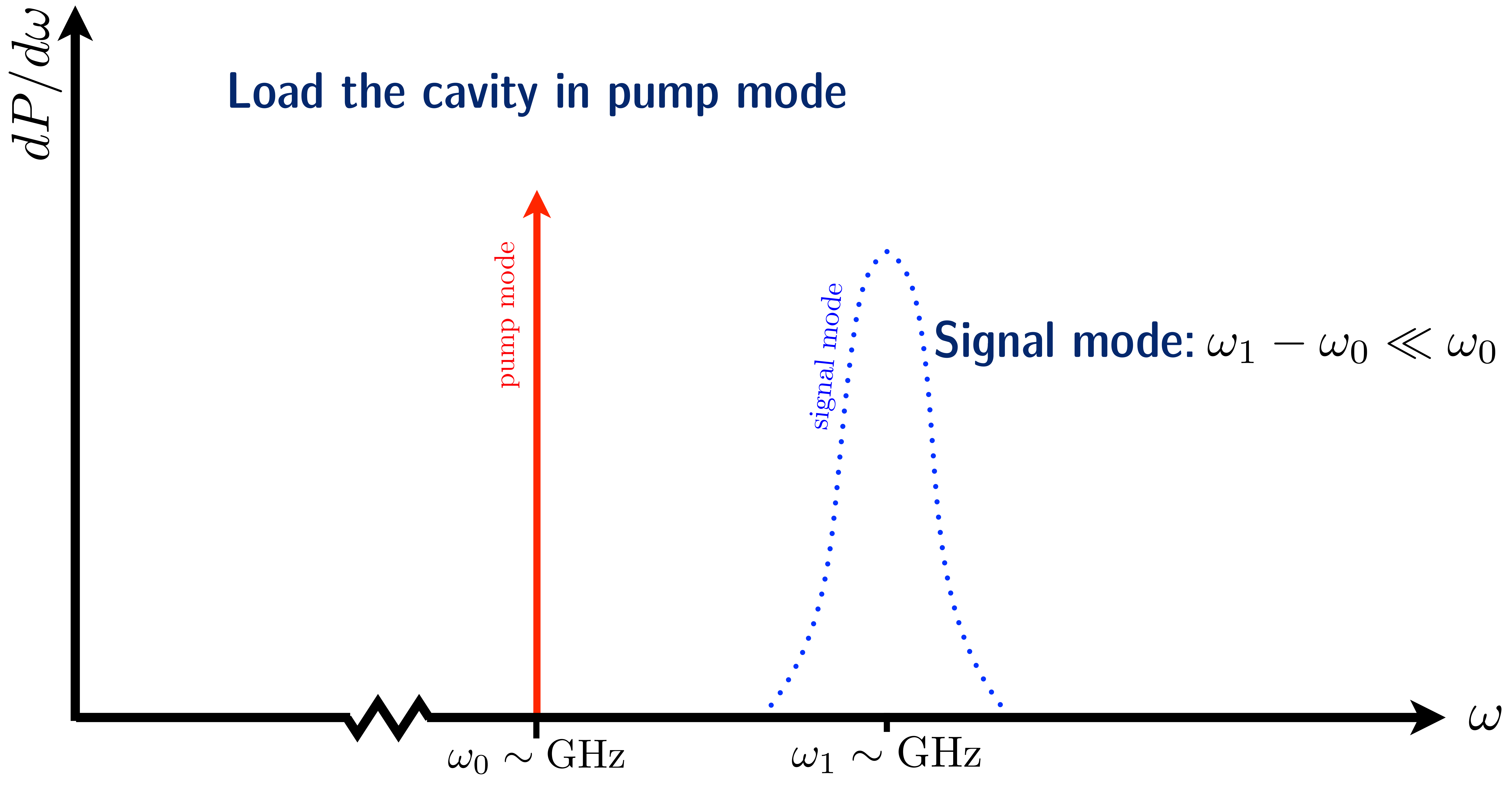
MAGO 2.0



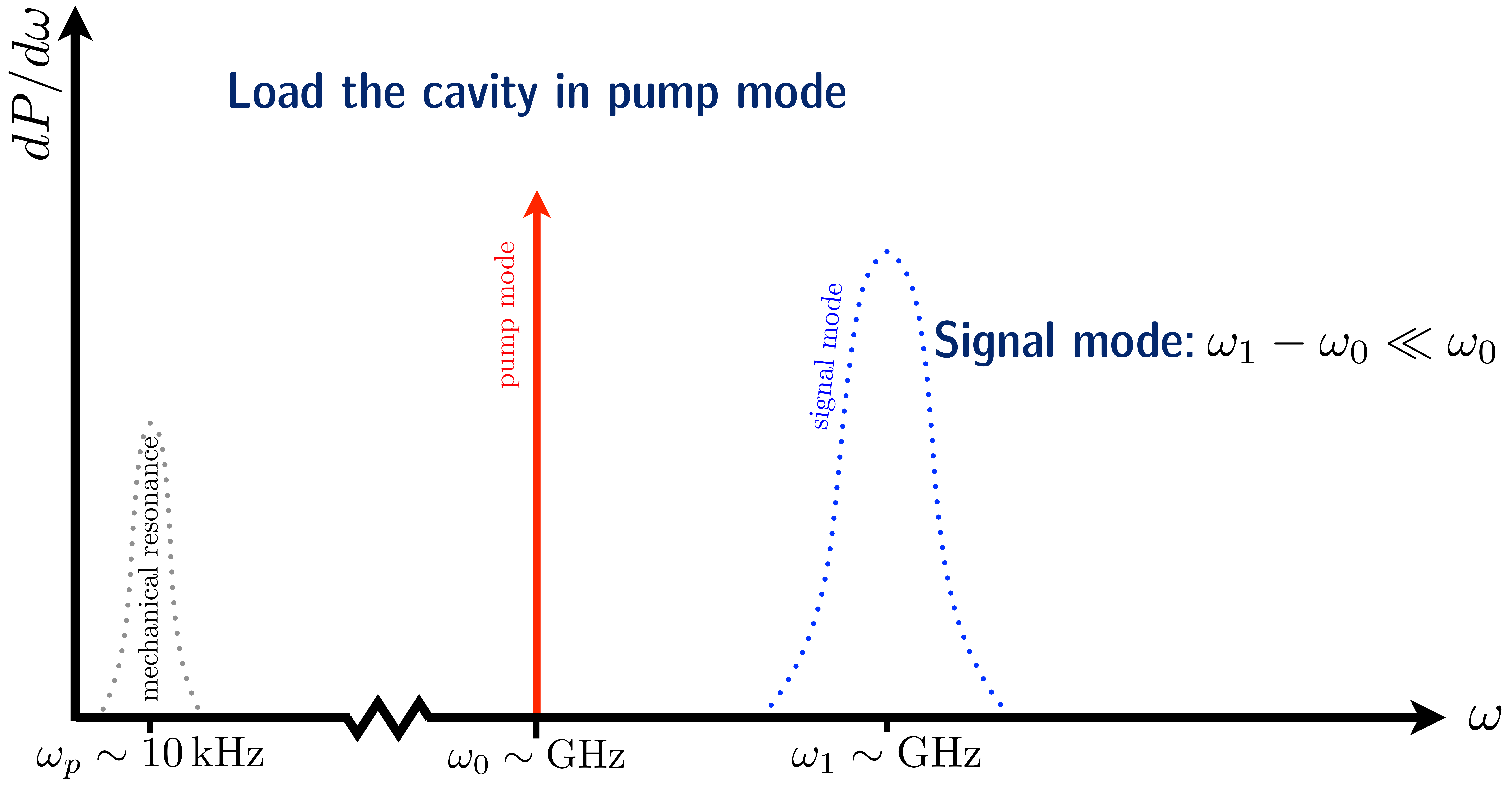
$dP/d\omega$
Load the cavity in pump mode
pump mode
$\omega_0 \sim \mathrm{GHz}$
ω

MAGO 2.0



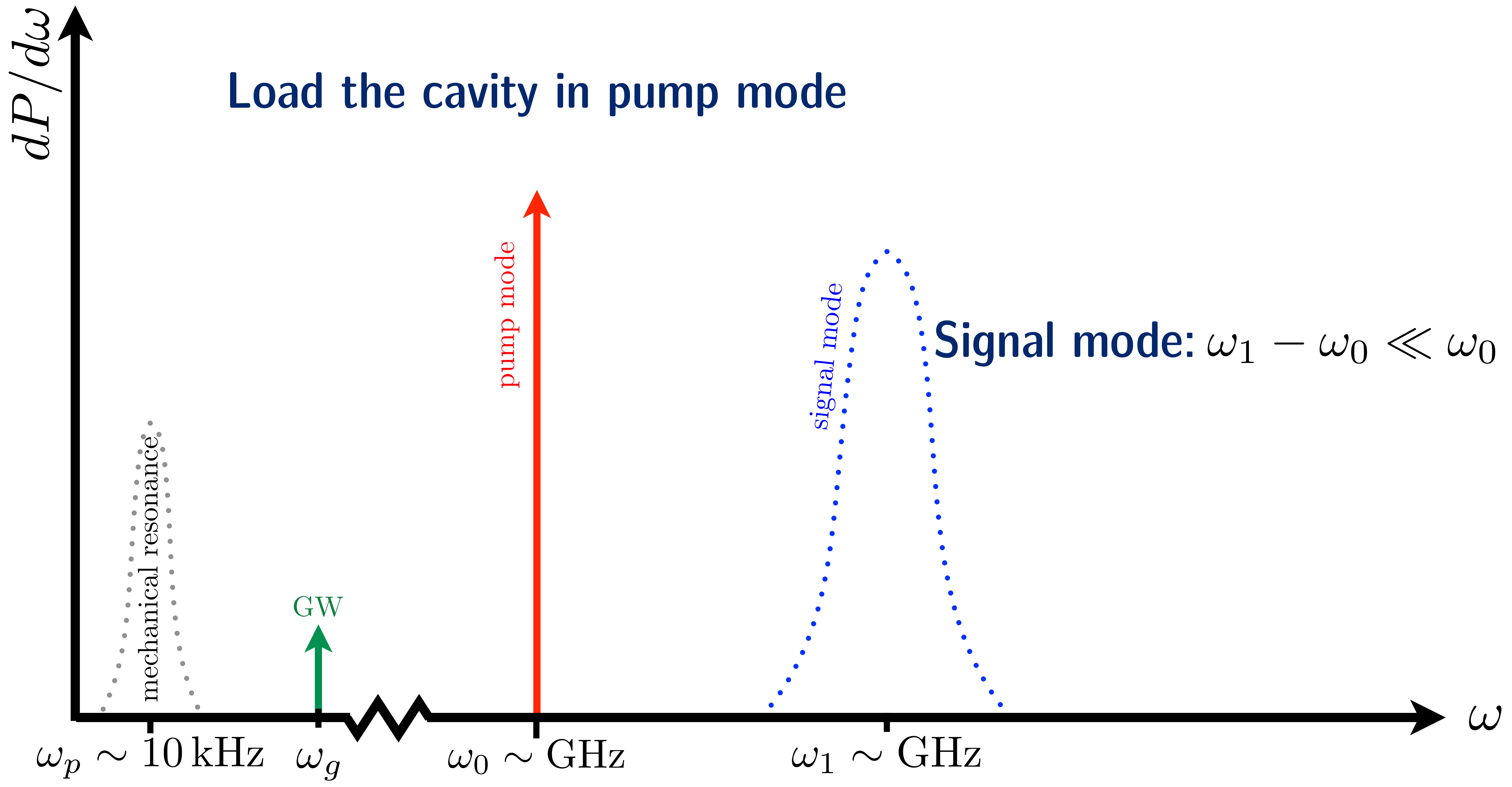
$dP/d\omega$
Load the cavity in pump mode
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
ω
$\omega_0 \sim \text{GHz}$
$\omega_1 \sim \text{GHz}$

MAGO 2.0



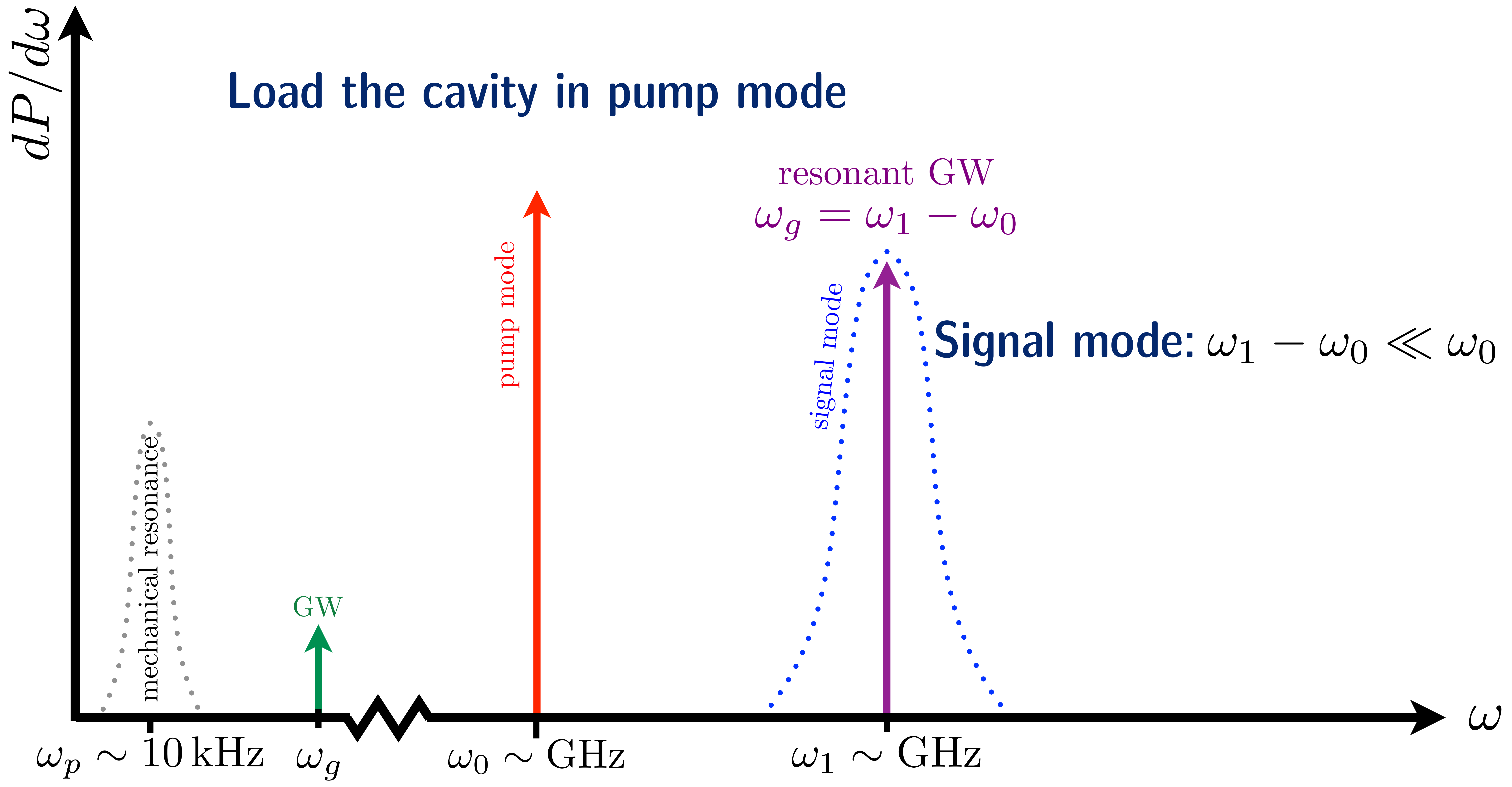
$dP/d\omega$
Load the cavity in pump mode
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
ω
$\omega_p \sim 10\,\mathrm{kHz}$
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$

MAGO 2.0



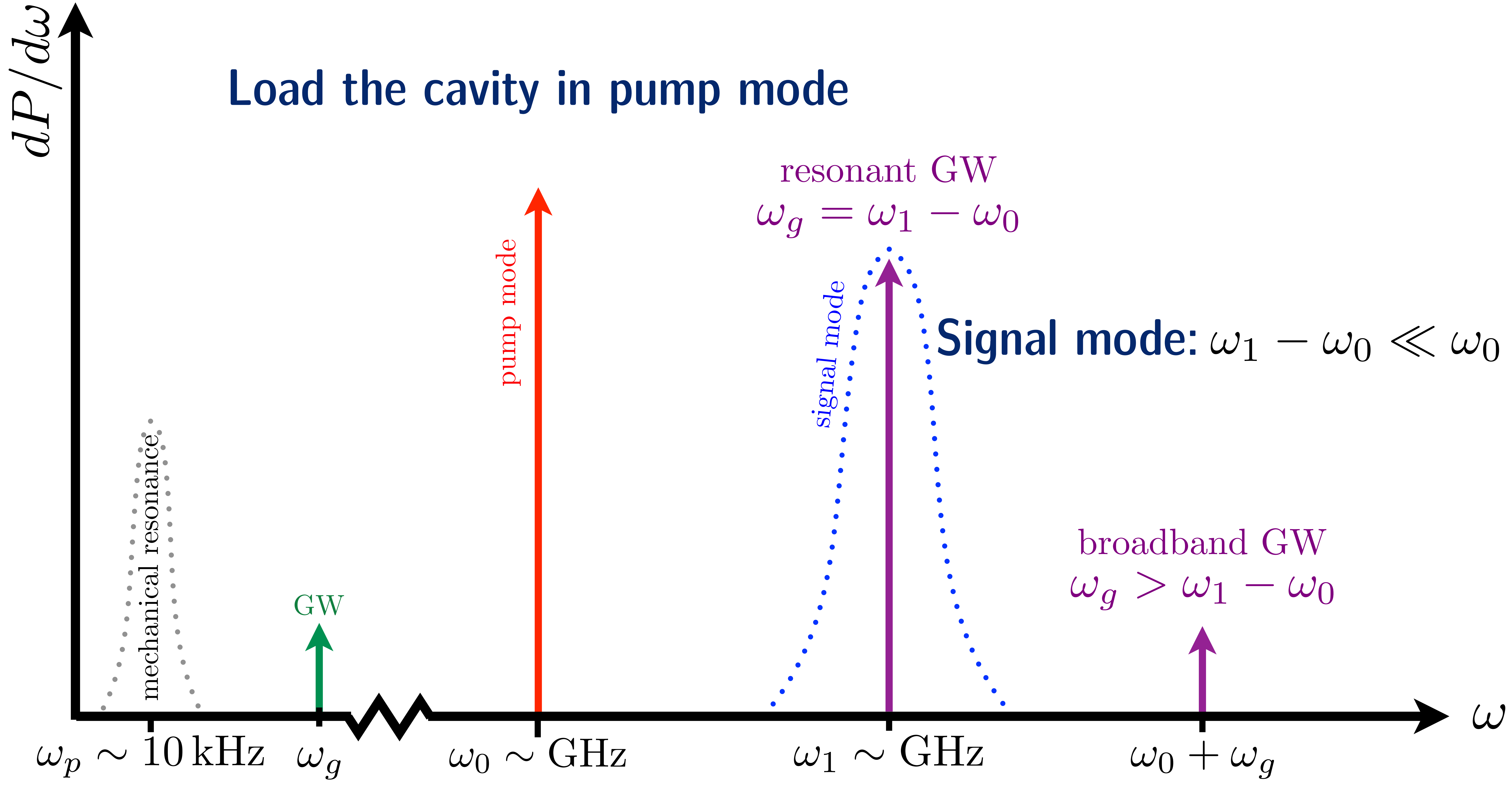
$dP/d\omega$
Load the cavity in pump mode
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
GW
ω
$\omega_p \sim 10\,\text{kHz}$
ω_g
$\omega_0 \sim \text{GHz}$
$\omega_1 \sim \text{GHz}$

MAGO 2.0



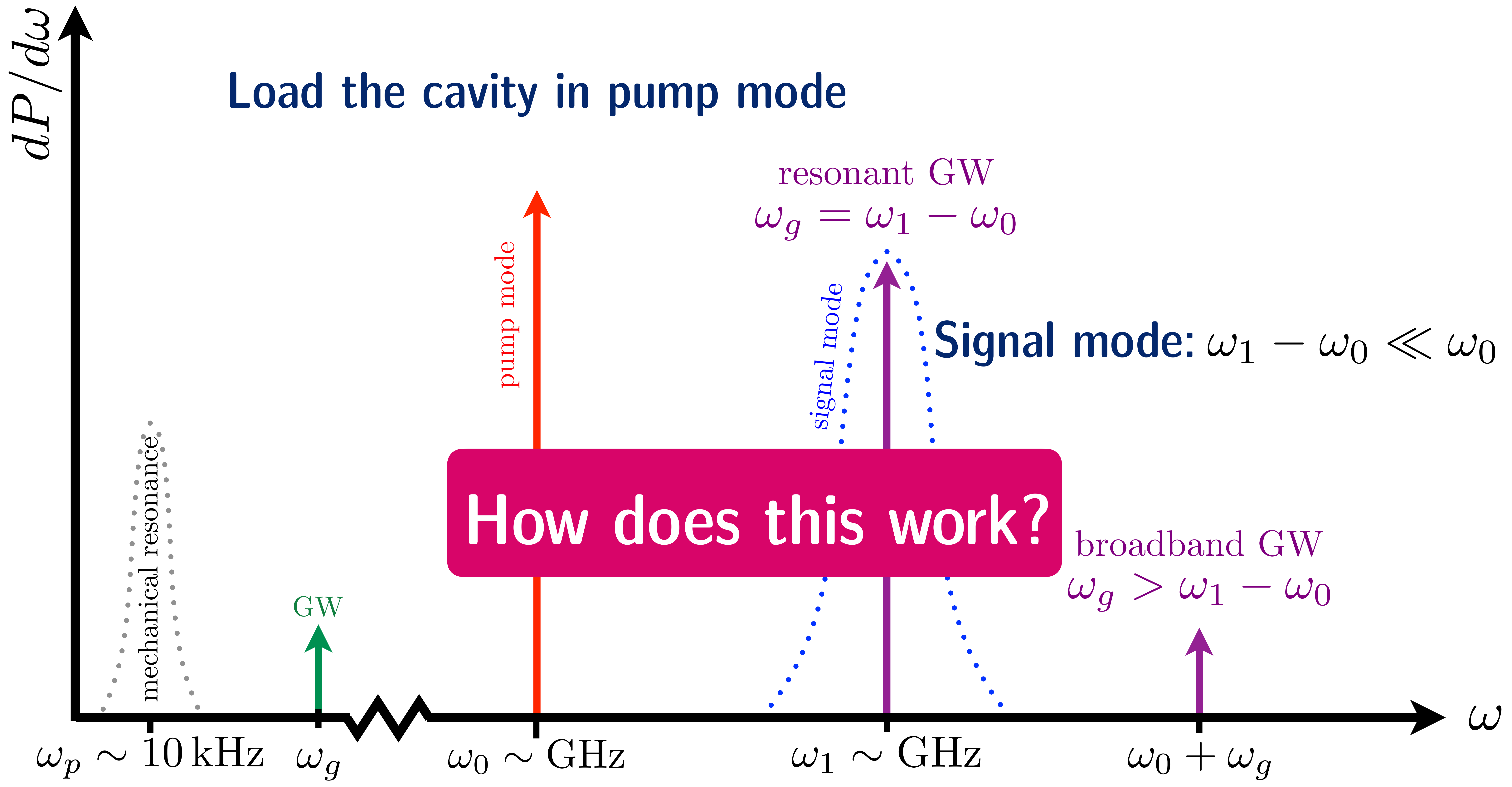
$dP/d\omega$
Load the cavity in pump mode
resonant GW
$\omega_g = \omega_1 - \omega_0$
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
GW
ω
$\omega_p \sim 10\,\text{kHz}$
ω_g
$\omega_0 \sim \text{GHz}$
$\omega_1 \sim \text{GHz}$

MAGO 2.0



$dP/d\omega$
Load the cavity in pump mode
resonant GW
$\omega_g = \omega_1 - \omega_0$
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
GW
broadband GW
$\omega_g > \omega_1 - \omega_0$
ω
$\omega_p \sim 10\,\text{kHz}$
ω_g
$\omega_0 \sim \text{GHz}$
$\omega_1 \sim \text{GHz}$
$\omega_0 + \omega_g$

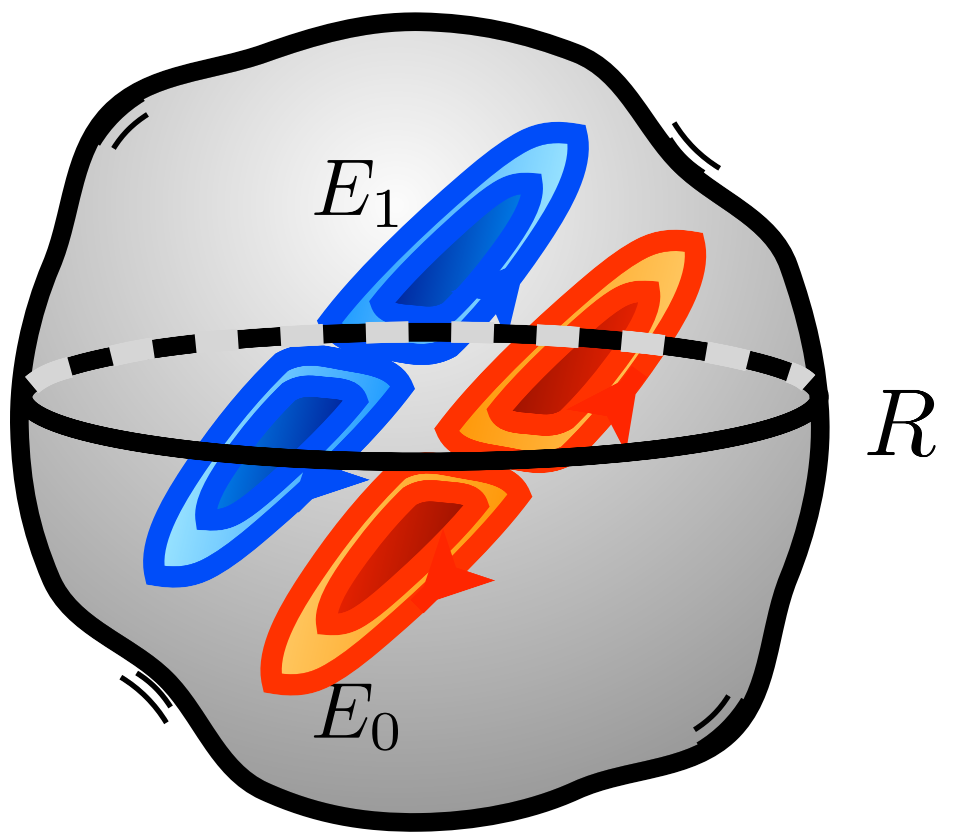
MAGO 2.0



$dP/d\omega$
Load the cavity in pump mode
resonant GW
$\omega_g = \omega_1 - \omega_0$
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
GW
How does this work?
broadband GW
$\omega_g > \omega_1 - \omega_0$
ω
$\omega_p \sim 10\,\mathrm{kHz}$
ω_g
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$
$\omega_0 + \omega_g$

MAGO 2.0

$\omega \sim \text{GHz}$



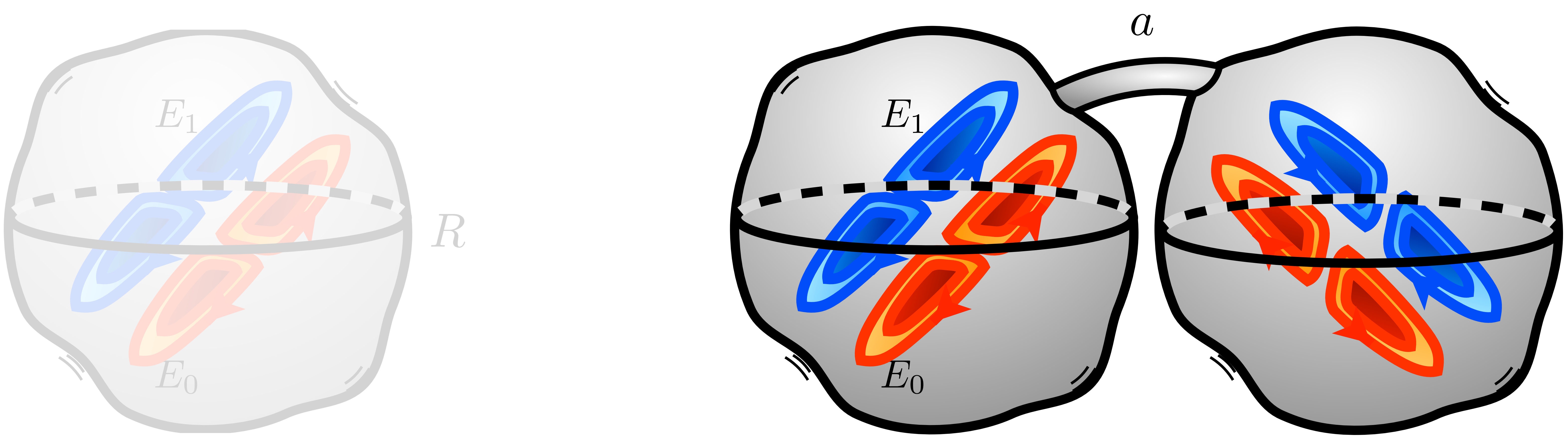
MAGO 2.0

$\omega_a \sim \text{GHz} + \Delta\omega$

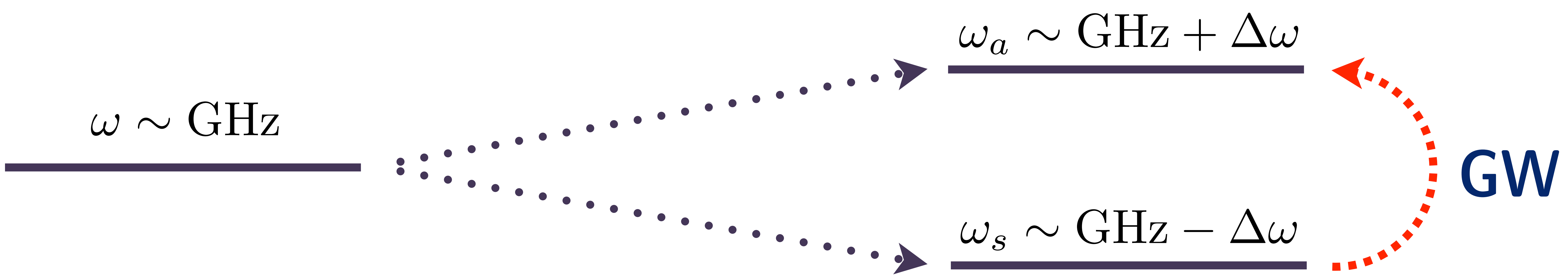
$\omega \sim \text{GHz}$

$\omega_s \sim \text{GHz} - \Delta\omega$

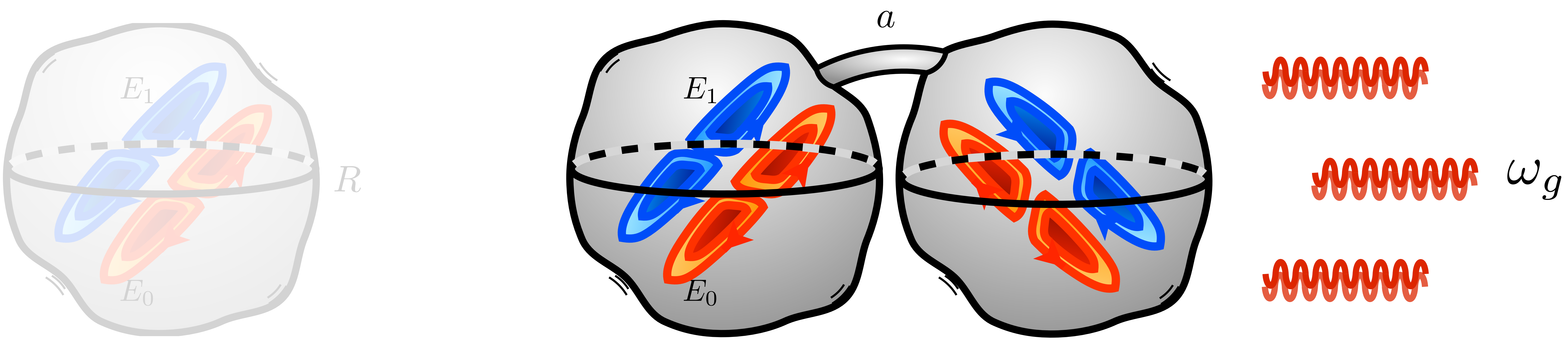
$$\Delta\omega \sim (a/R)^3$$



MAGO 2.0

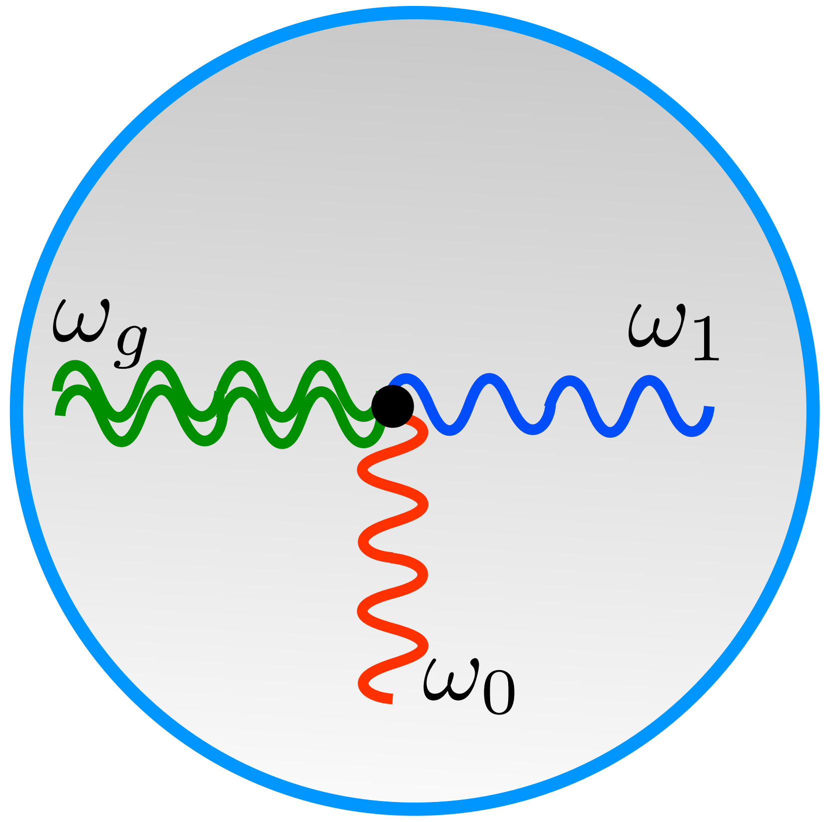


$$\Delta\omega \sim (a/R)^3$$



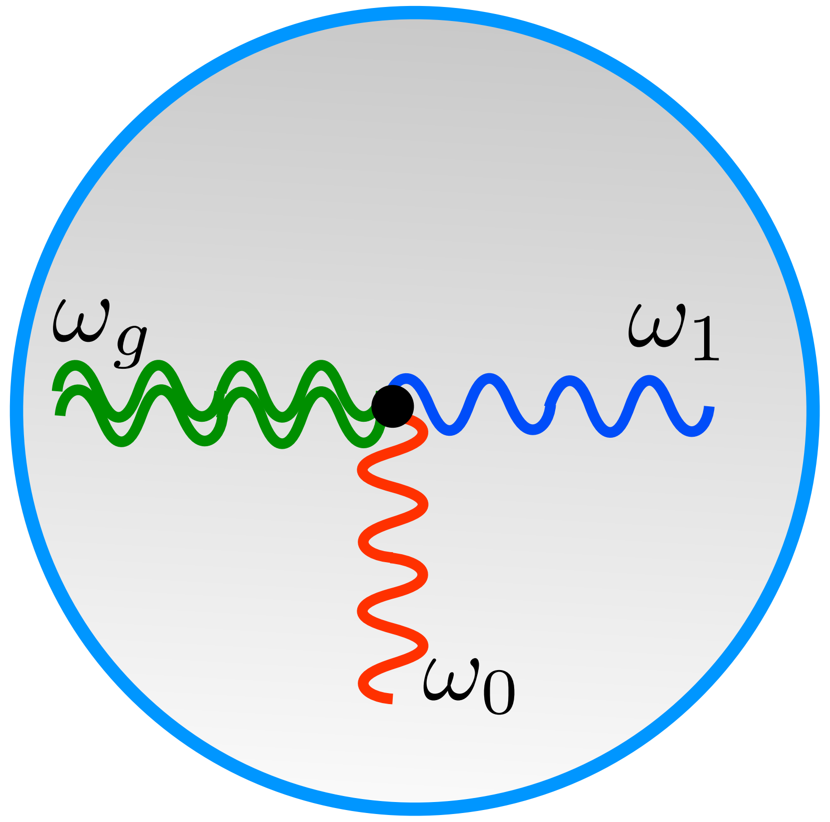
EM and Mechanical signals

EM and Mechanical signals



Parametrics of the EM signal: $E_{\rm sig}^{(\rm EM)} \sim Q_{\rm em}\,(\omega_g\,L_{\rm cav})^2\,h^{\rm TT}\,E_0$

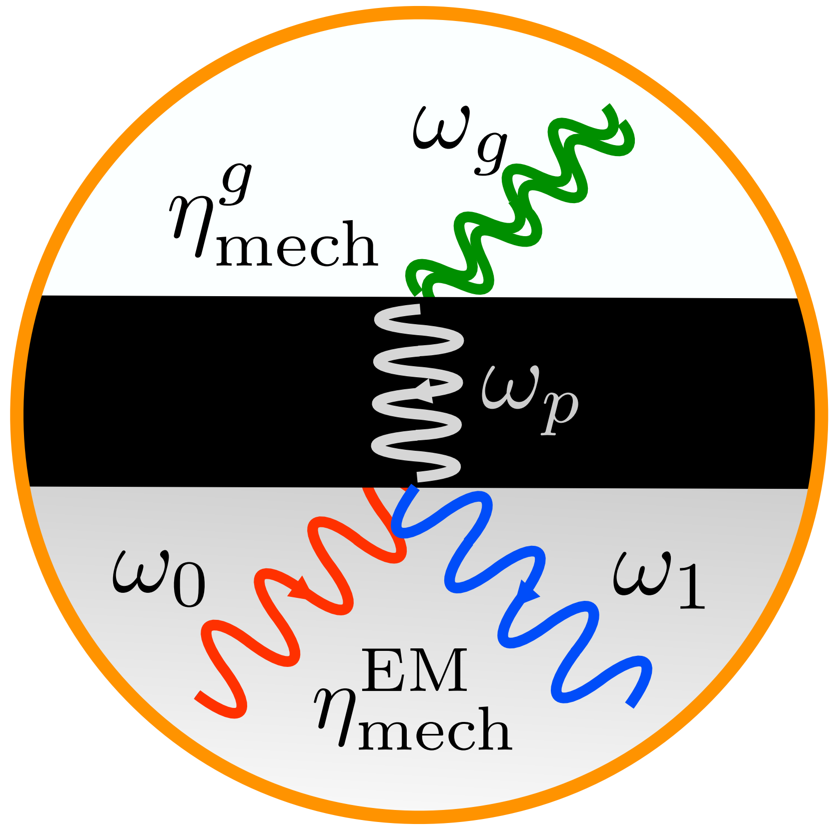
EM and Mechanical signals



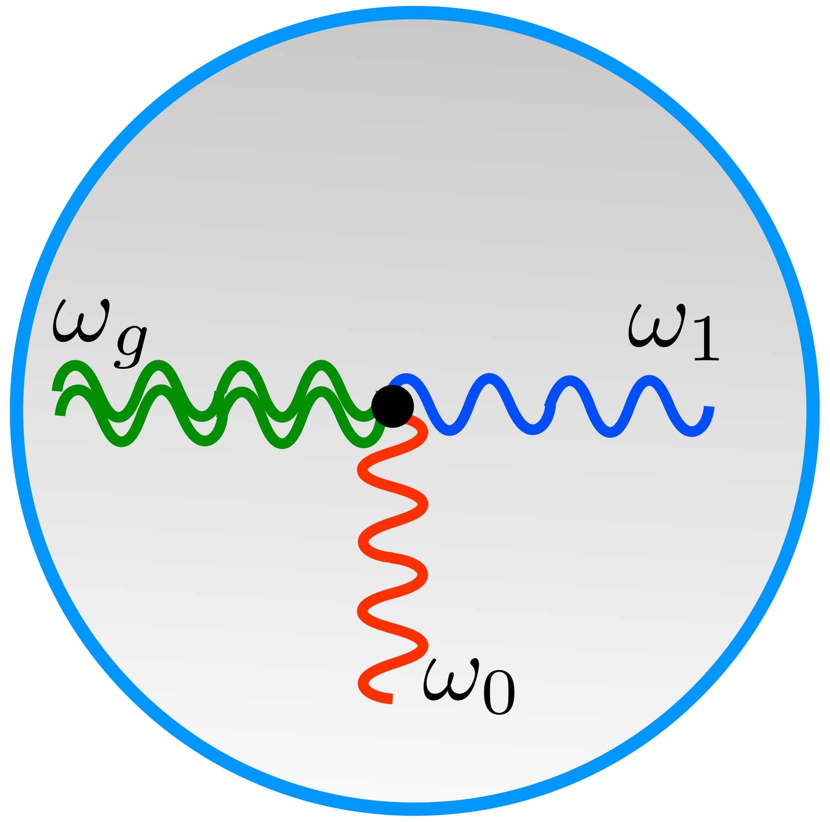
Parametrics of the EM signal: $E_{\rm sig}^{\rm (EM)} \sim Q_{\rm em}\,(\omega_g\,L_{\rm cav})^2\,h^{\rm TT}\,E_0$

Mechanical signal:

$$E_{\rm sig}^{\rm (mech)} \sim Q_{\rm em}\,h^{\rm TT}\,E_0\,\min\left(1\,,\frac{\omega_g\,L_{\rm cav}}{c_s}\right)^2$$



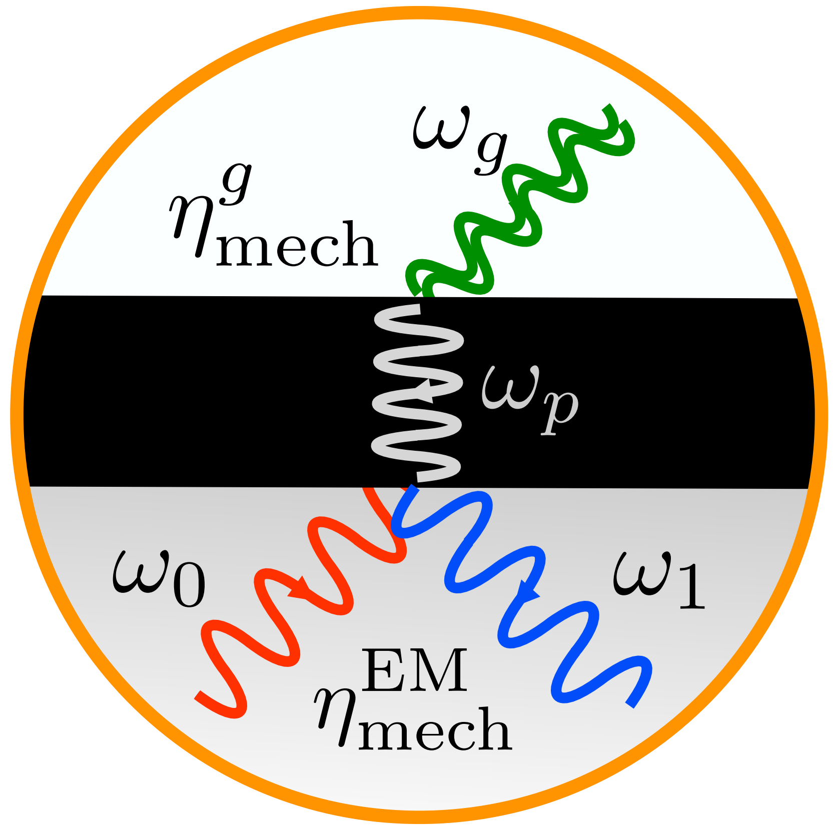
EM and Mechanical signals



Parametrics of the EM signal: $E_{\text{sig}}^{(\text{EM})} \sim Q_{\text{em}}\,(\omega_g\,L_{\text{cav}})^2\,h^{\text{TT}}\,E_0$

Enhanced by $1/c_s^2 \gg 1$ *(!)*

Mechanical signal: $E_{\text{sig}}^{(\text{mech})} \sim Q_{\text{em}}\,h^{\text{TT}}\,E_0\,\min\left(1\,,\,\frac{\omega_g\,L_{\text{cav}}}{c_s}\right)^2$



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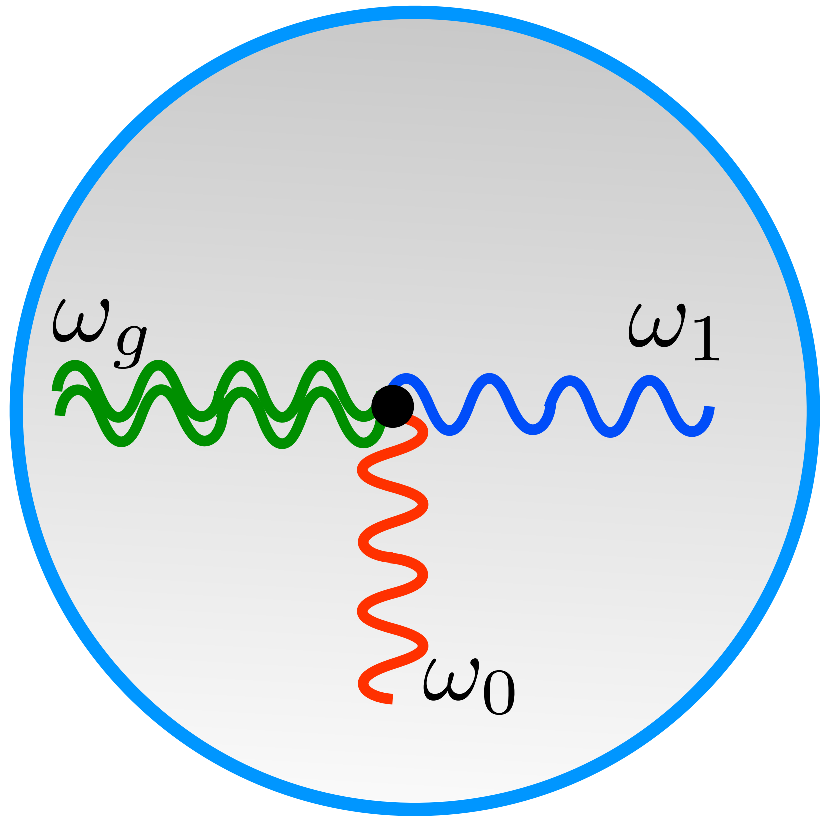

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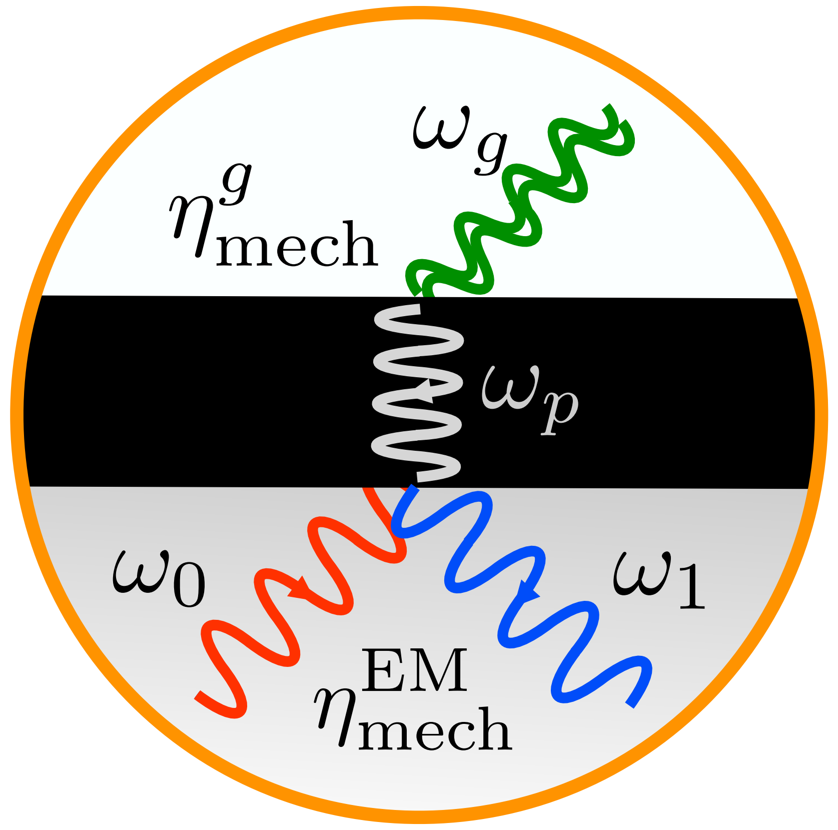
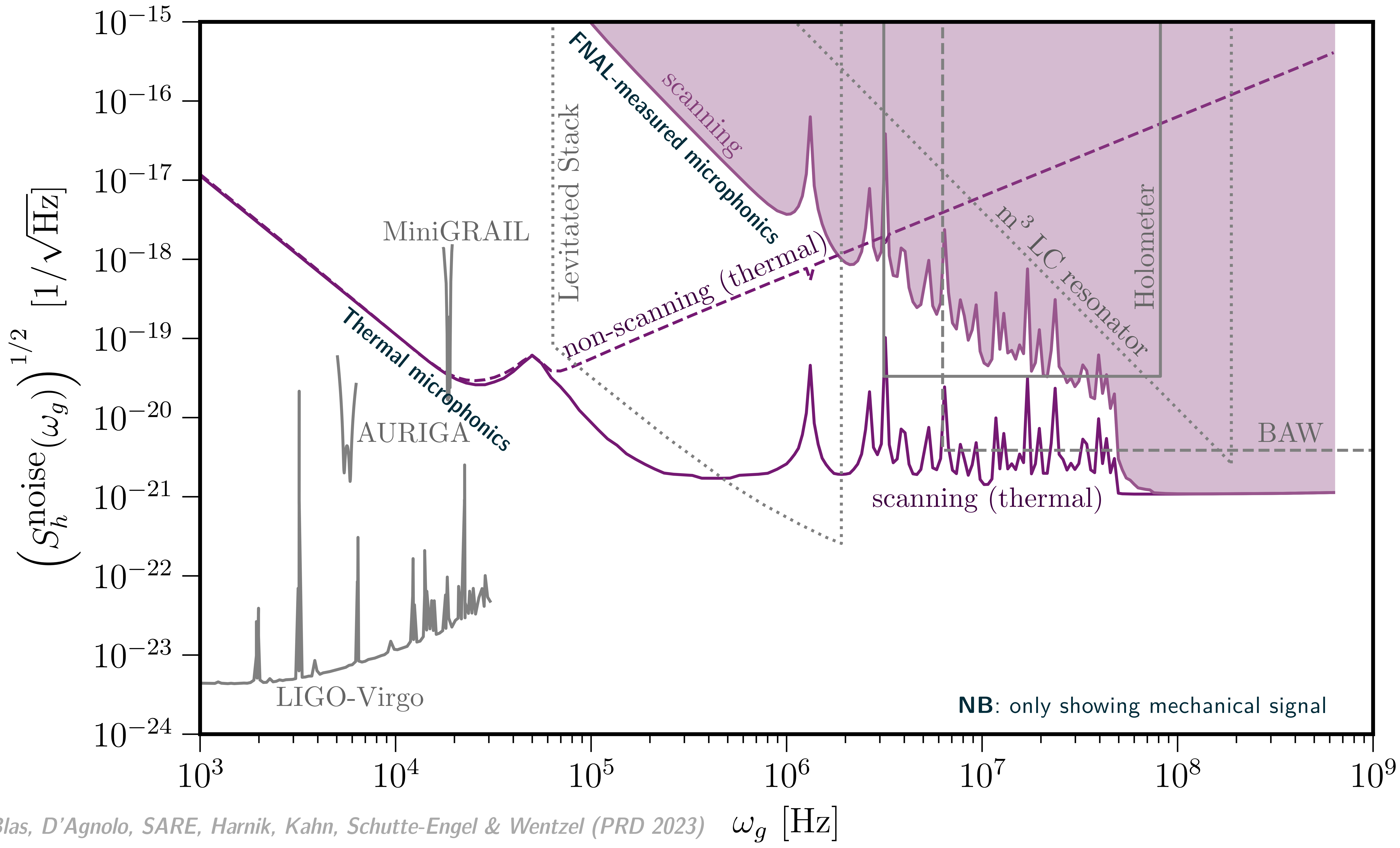


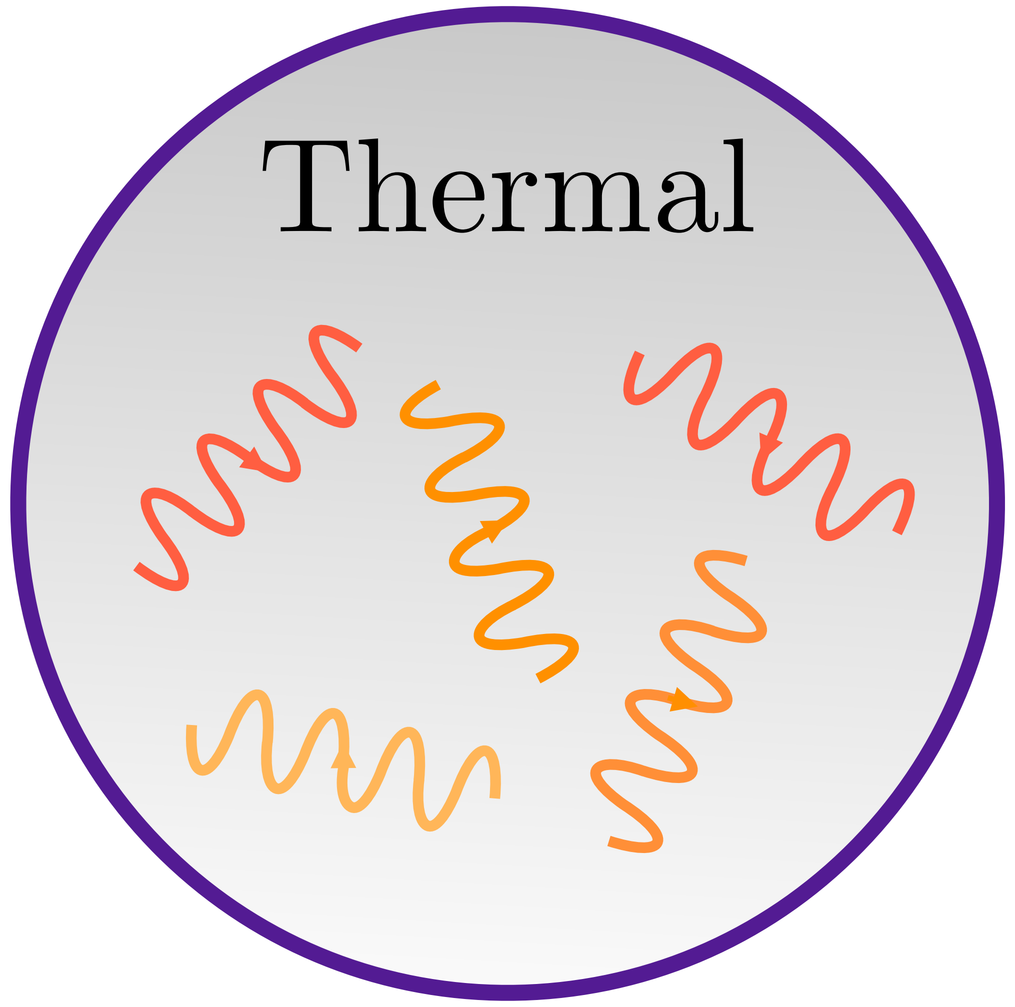
Mechanical modes less "rigid" than EM modes

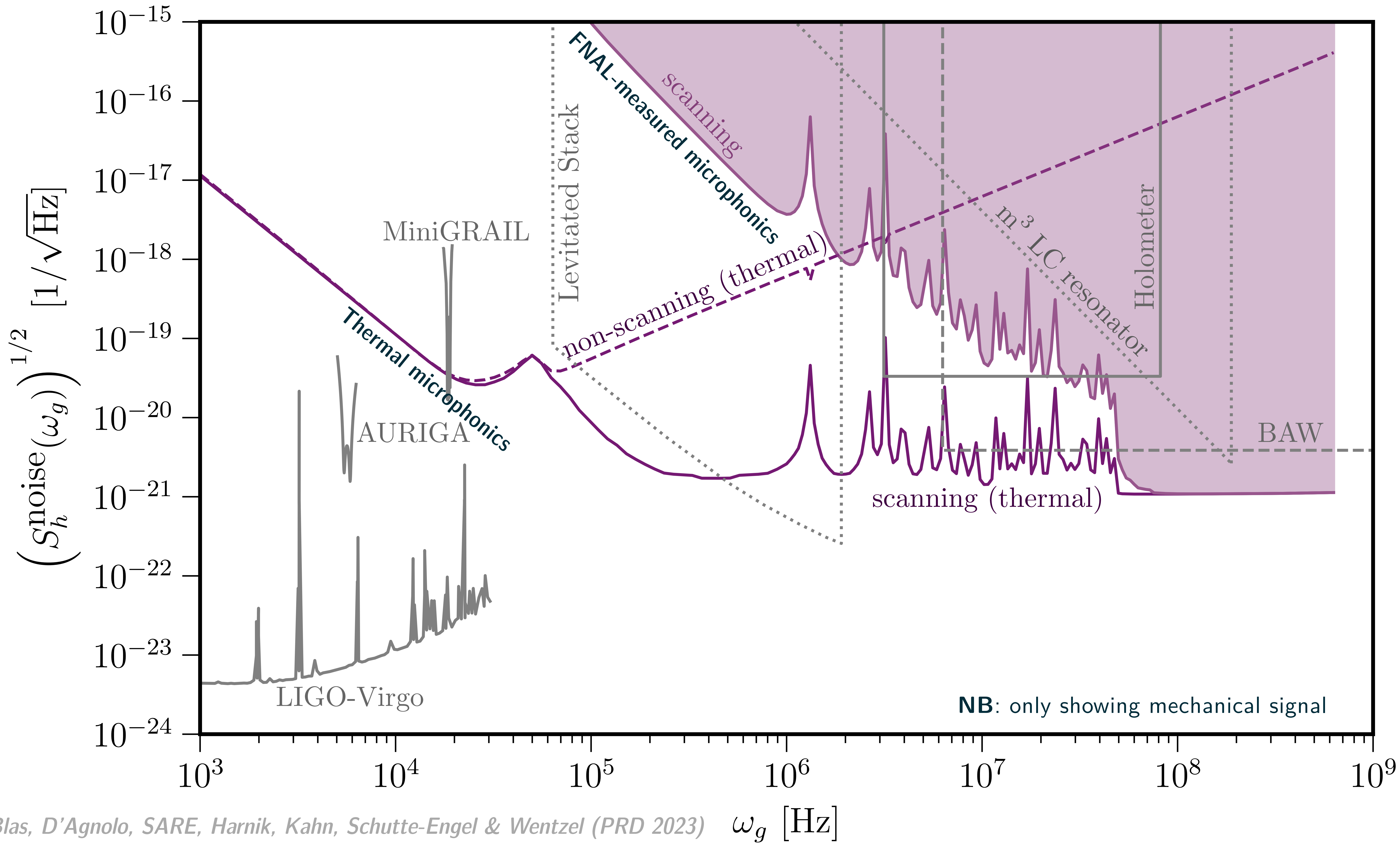
Noise in MAGO 2.0



Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel & Wentzel (PRD 2023)

Noise in MAGO 2.0





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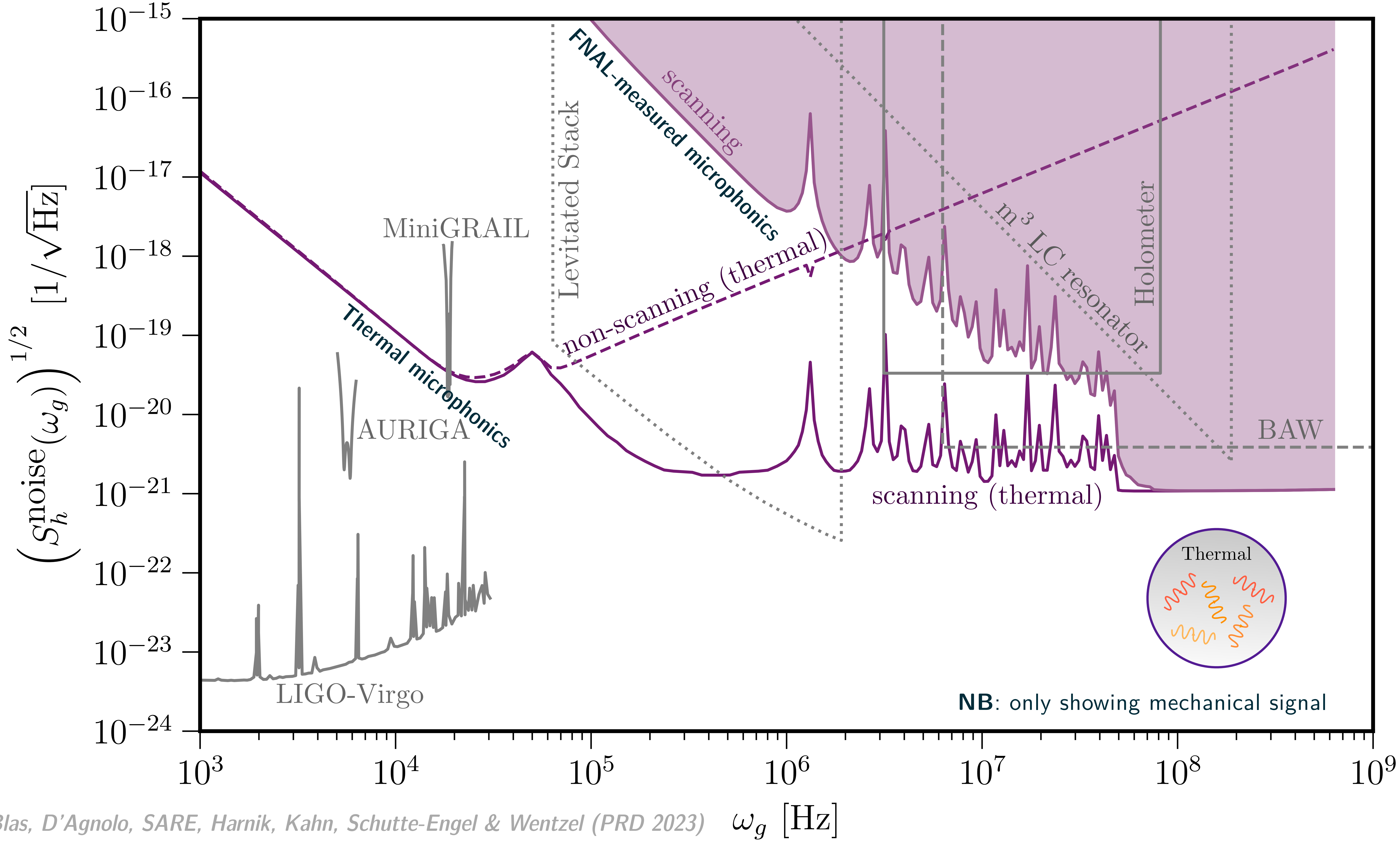

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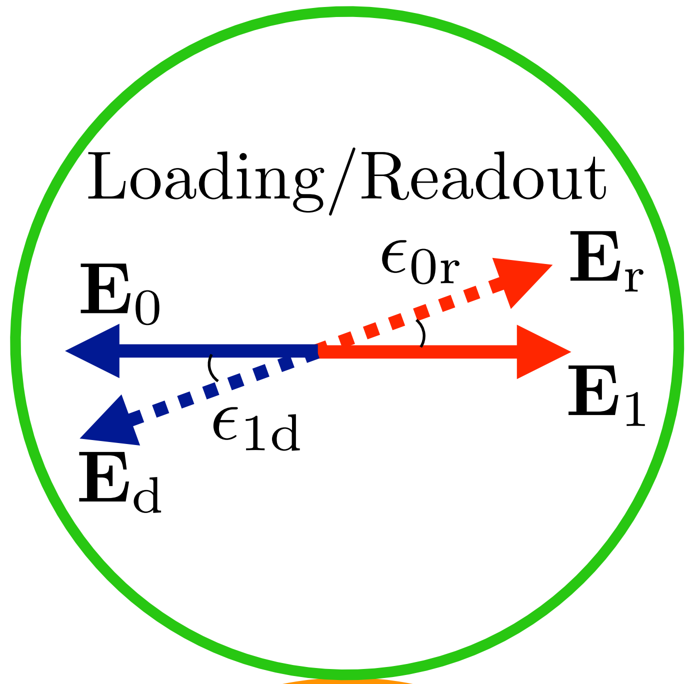

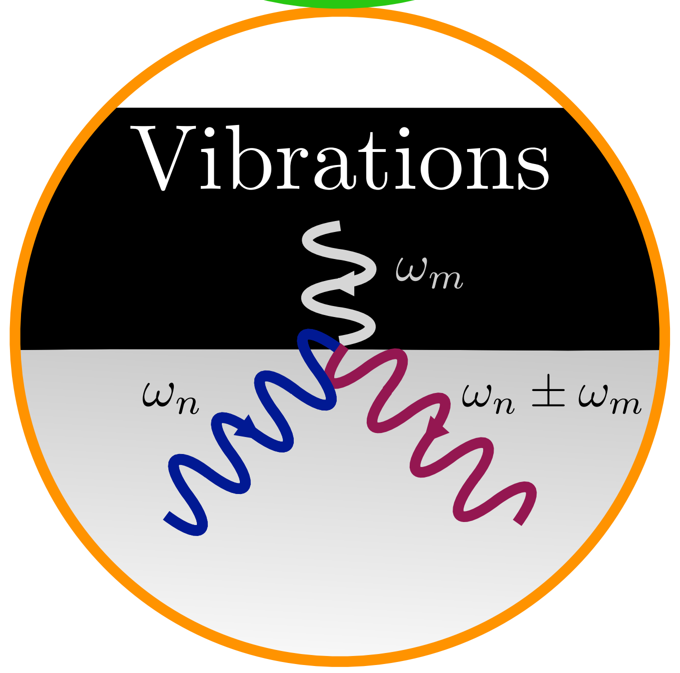

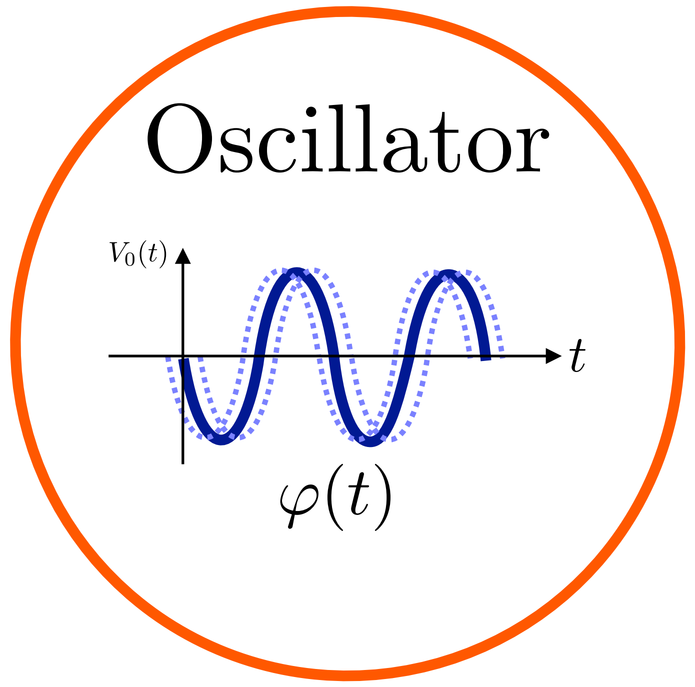

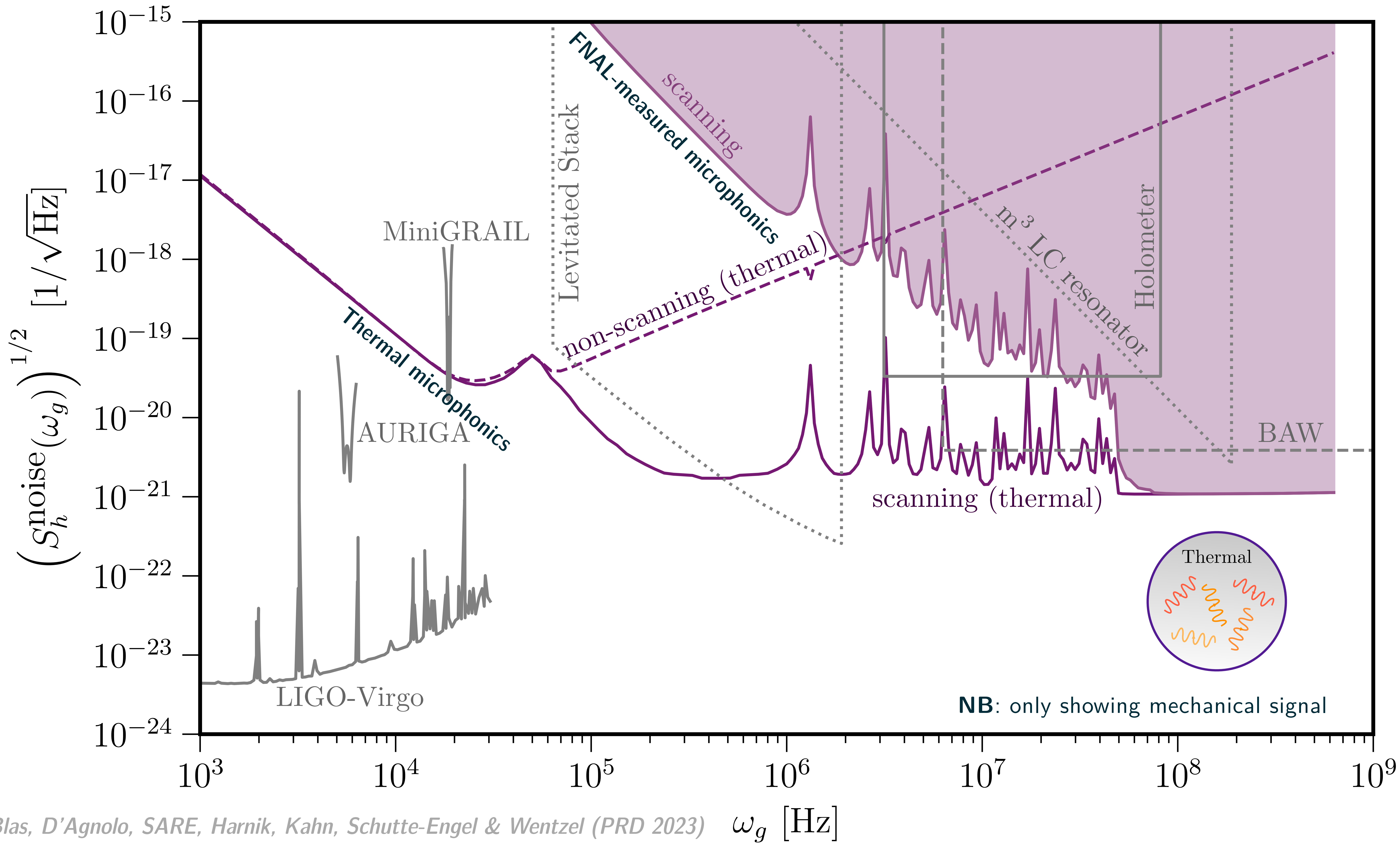

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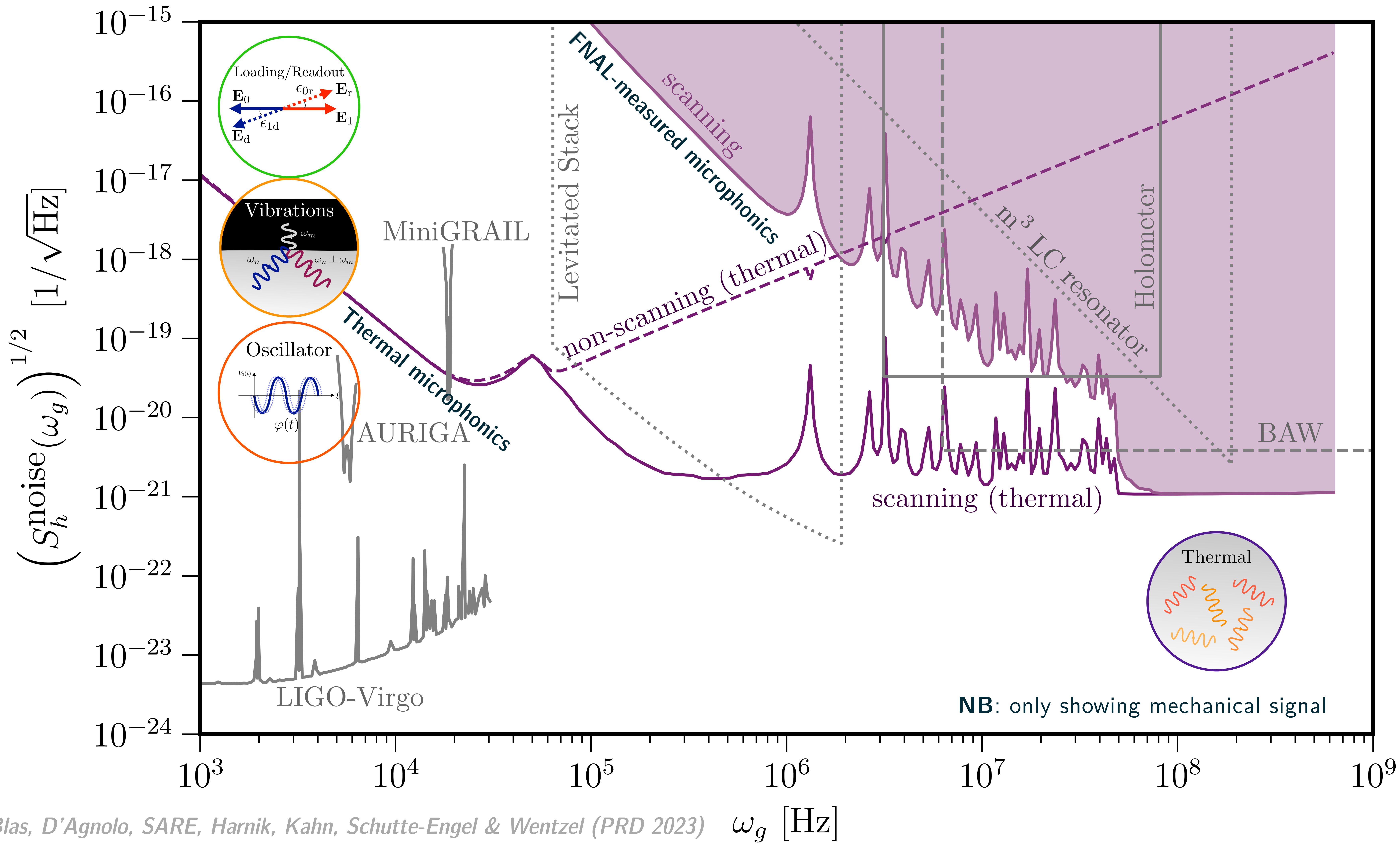

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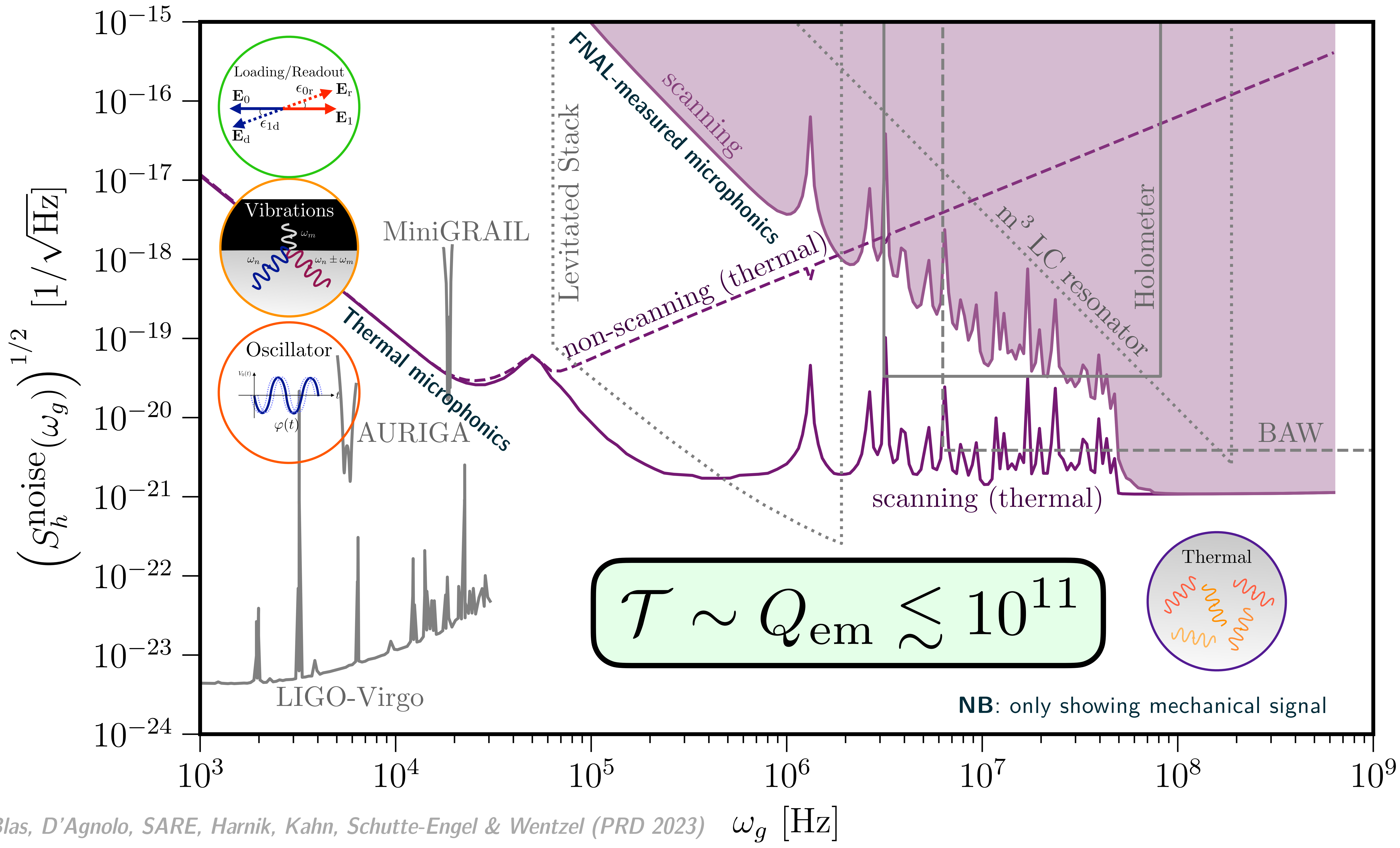
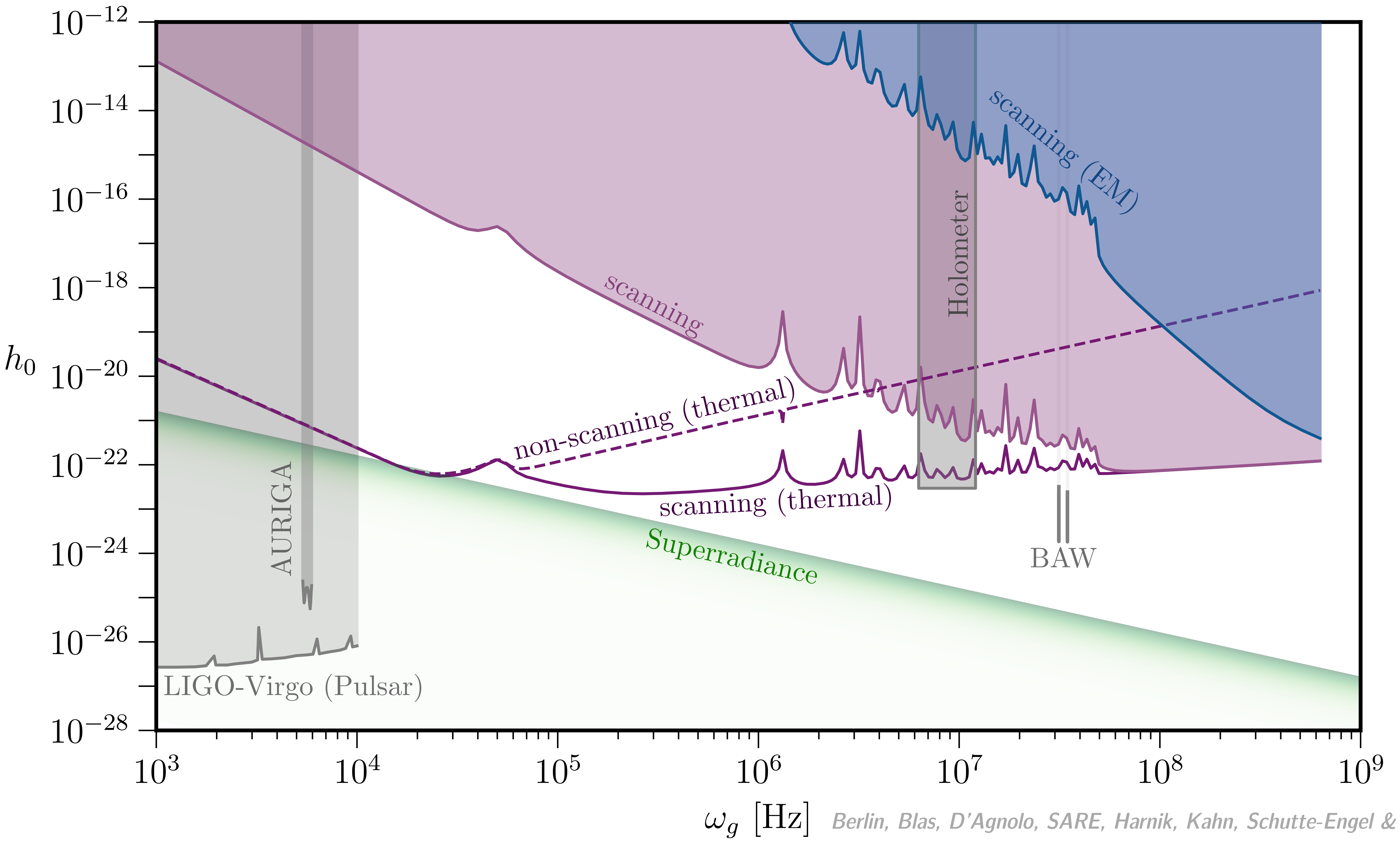


Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel & Wentzel (PRD 2023)

MAGO 2.0 sensitivity to coherent GWs



Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel & Wentzel (PRD 2023)

HEURISTICS II

Prospects for stochastic GWs?

Stochastic GW SNR

Stochastic backgrounds have zero mean — necessarily quadratic measurement

Stochastic GW SNR

Stochastic backgrounds have zero mean — necessarily quadratic measurement

Single detector: $\mathrm{SNR} \sim \frac{S_{\mathrm{sig}}(\omega)}{S_{\mathrm{noise}}(\omega)}$

Two detectors: $\mathrm{SNR} \sim \left(t_{\mathrm{int}} \int df \, \left(\frac{S_{\mathrm{sig}}(\omega)}{S_{\mathrm{noise}}(\omega)} \right)^2 \right)^{1/2}$

Stochastic GW SNR

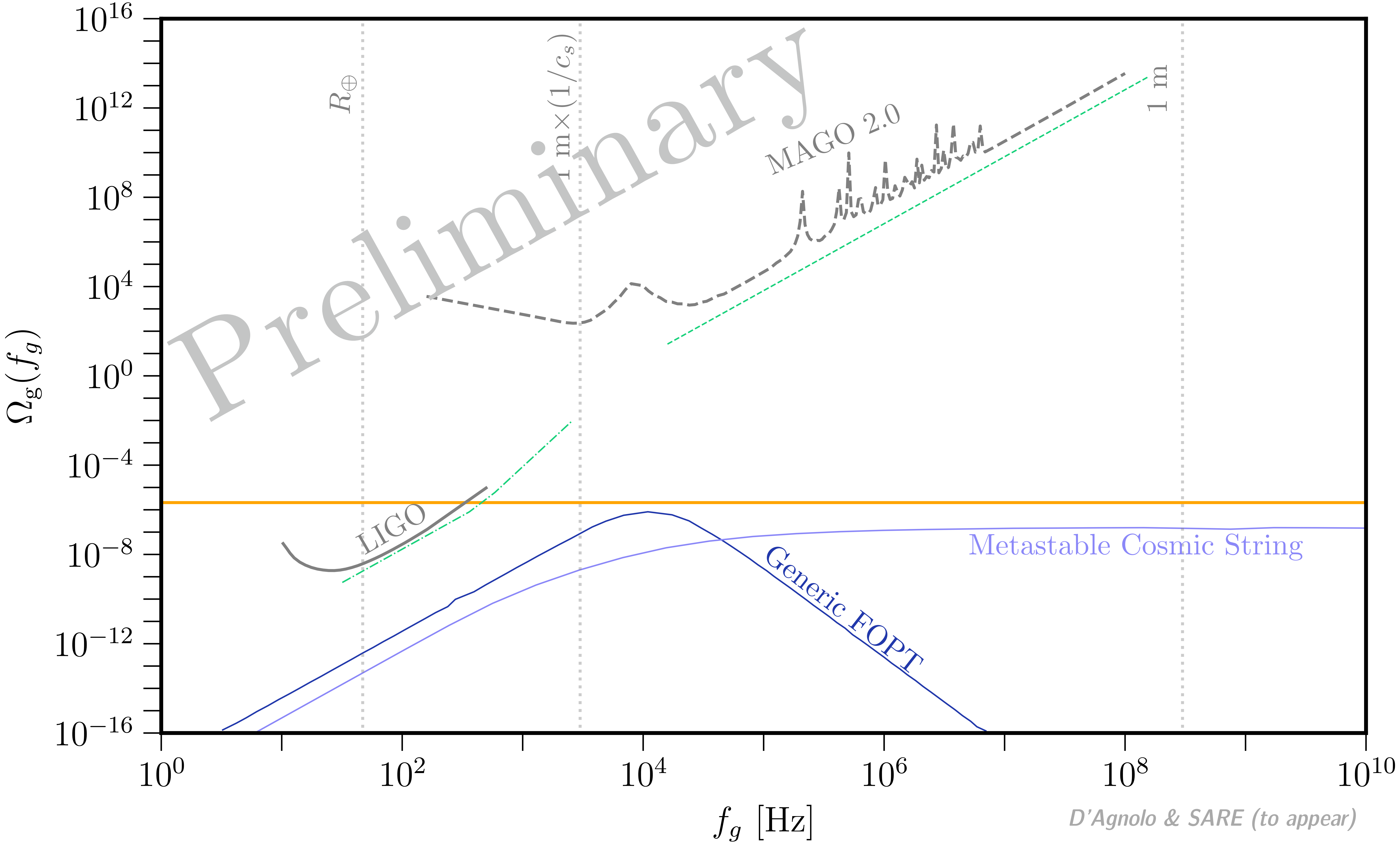
Stochastic backgrounds have zero mean — necessarily quadratic measurement

Single detector: $$\mathrm{SNR} \sim \frac{S_{\mathrm{sig}}(\omega)}{S_{\mathrm{noise}}(\omega)}$$

Two detectors: $$\mathrm{SNR} \sim \left(t_{\mathrm{int}} \int df \, \left(\frac{S_{\mathrm{sig}}(\omega)}{S_{\mathrm{noise}}(\omega)} \right)^2 \right)^{1/2}$$

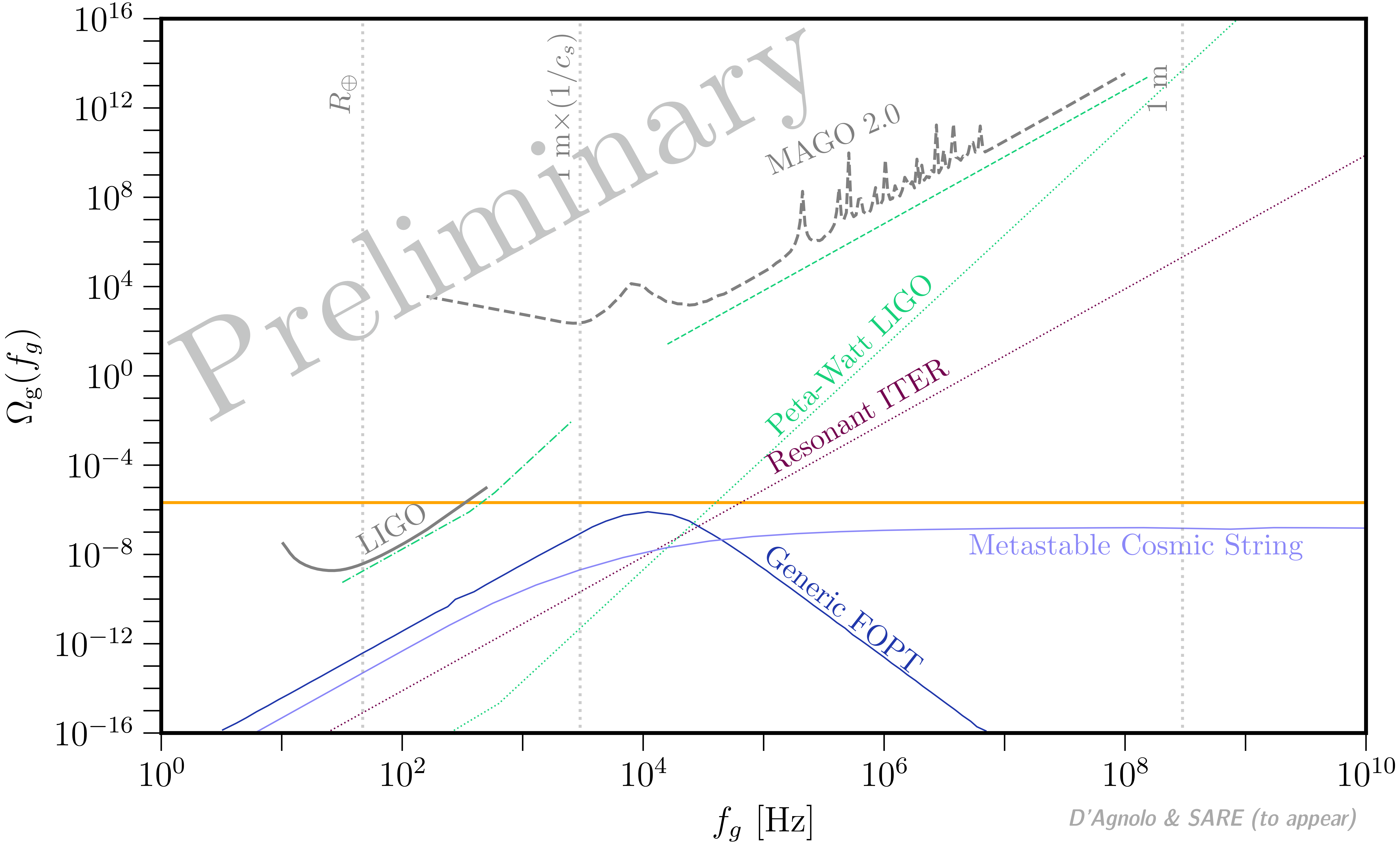
$$\Omega_g(\omega) \propto \frac{\omega^3 S_h(\omega)}{H_0^2} \qquad S_{\mathrm{sig}}(\omega) \sim |\mathcal{T}(\omega)|^2 S_h(\omega)$$

Sensitivity to stochastic HFGWs



D'Agnolo & SARE (to appear)

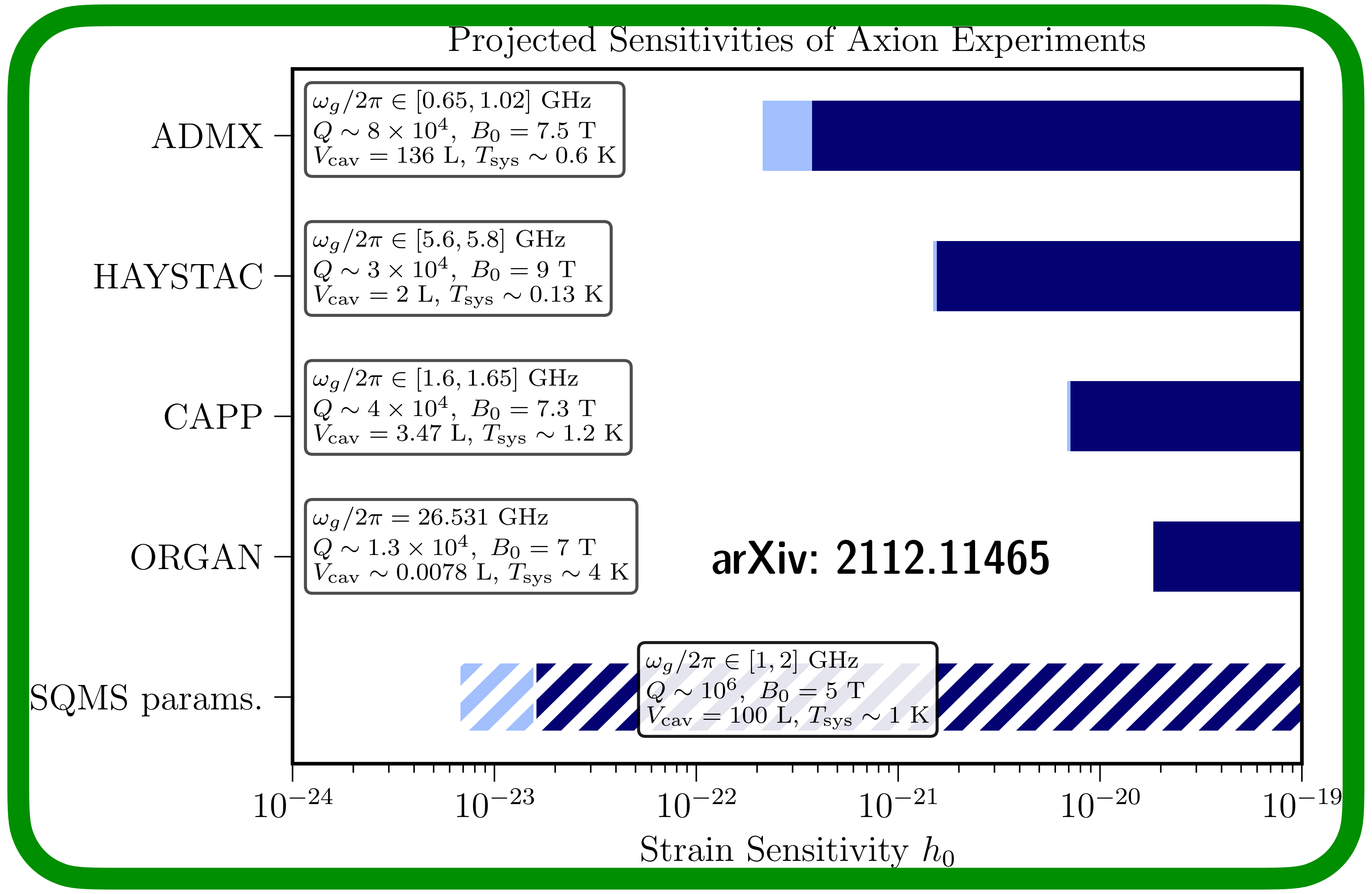
Sensitivity to stochastic HFGWs

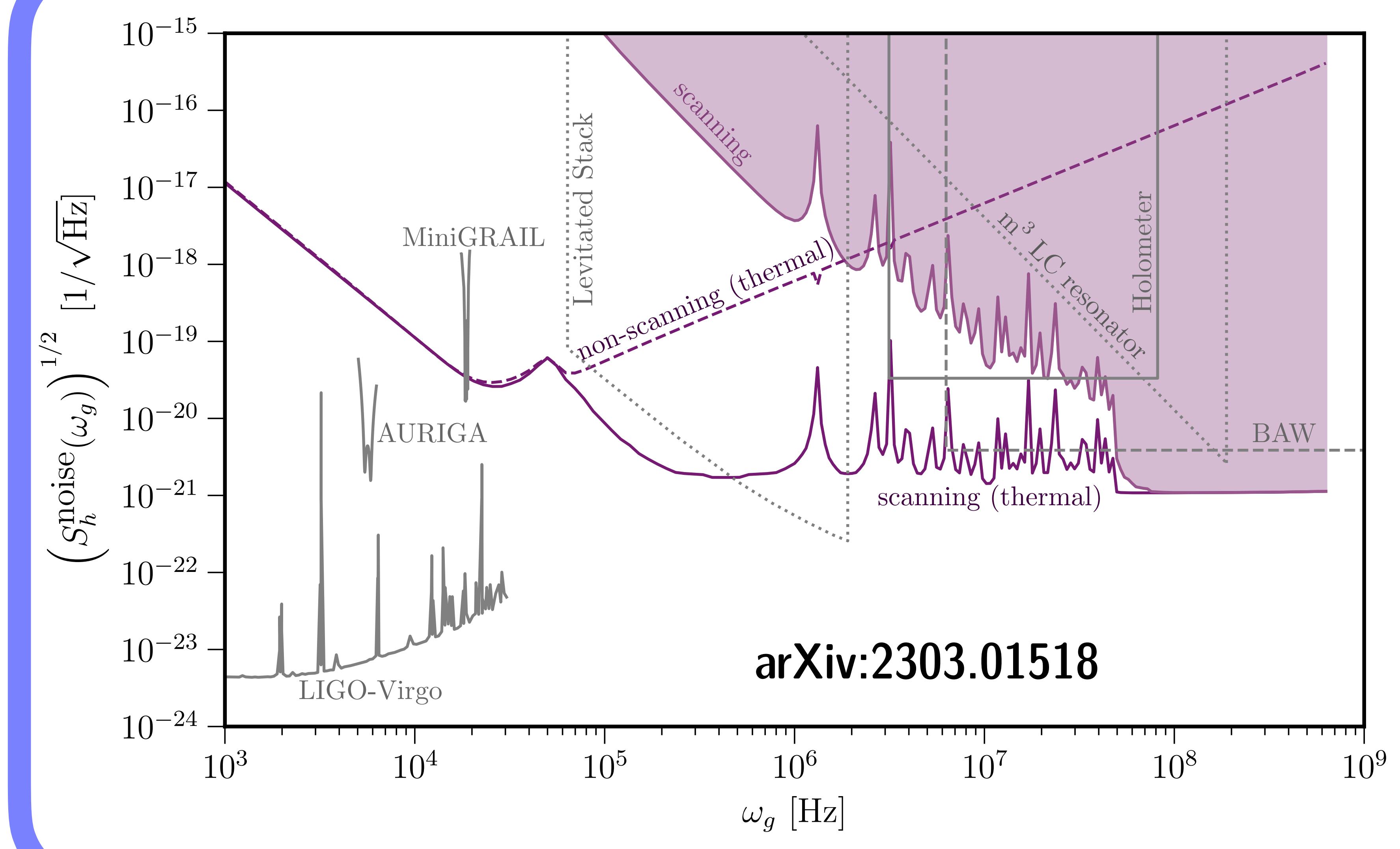


D'Agnolo & SARE (to appear)

Open questions

How to optimise the transfer function?





Open questions

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Advances in readout

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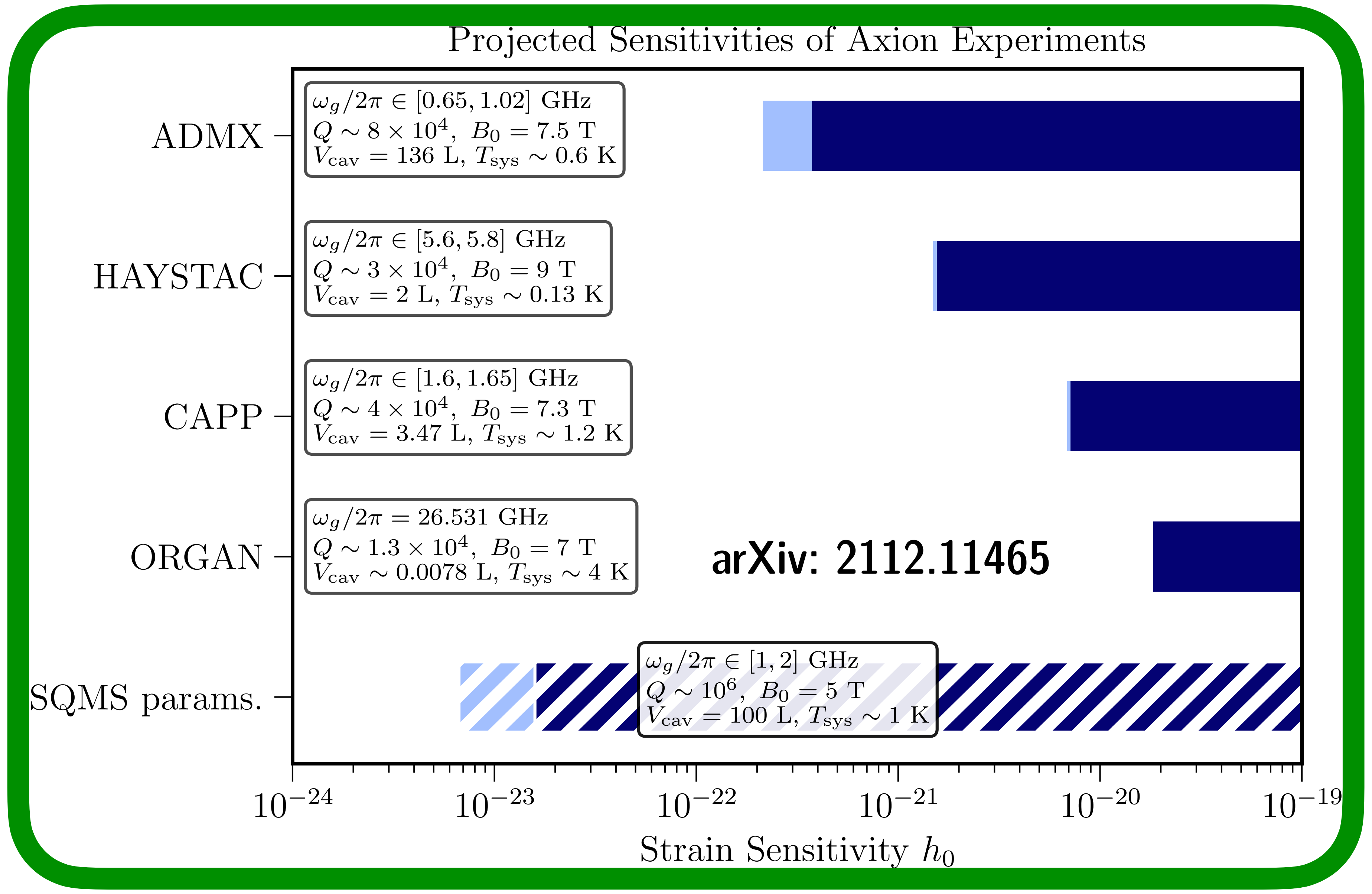
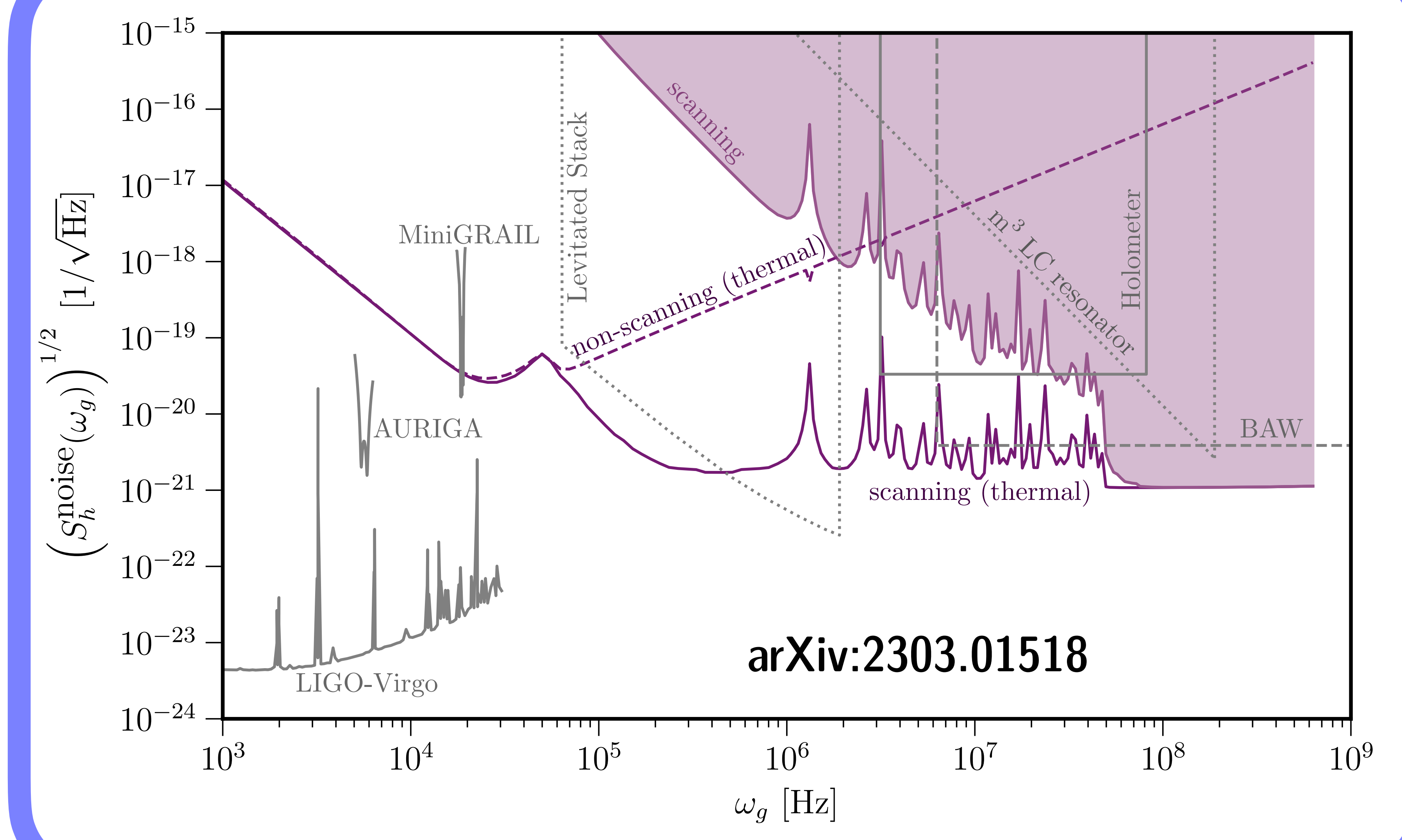
synergies w/ Axion searches, QC(?)

see e.g. arXiv:2308.11497 by Schmieden & Schott





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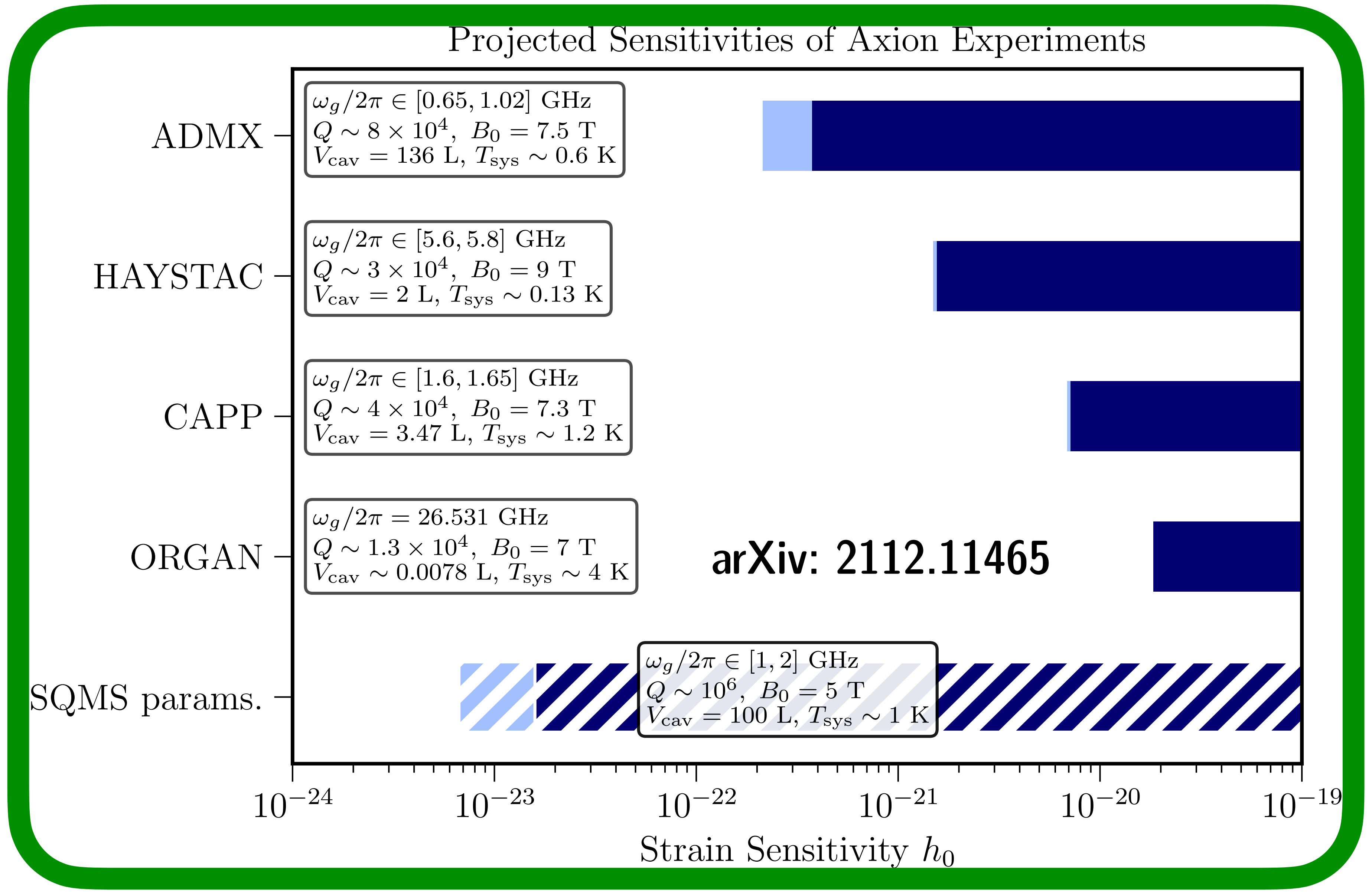

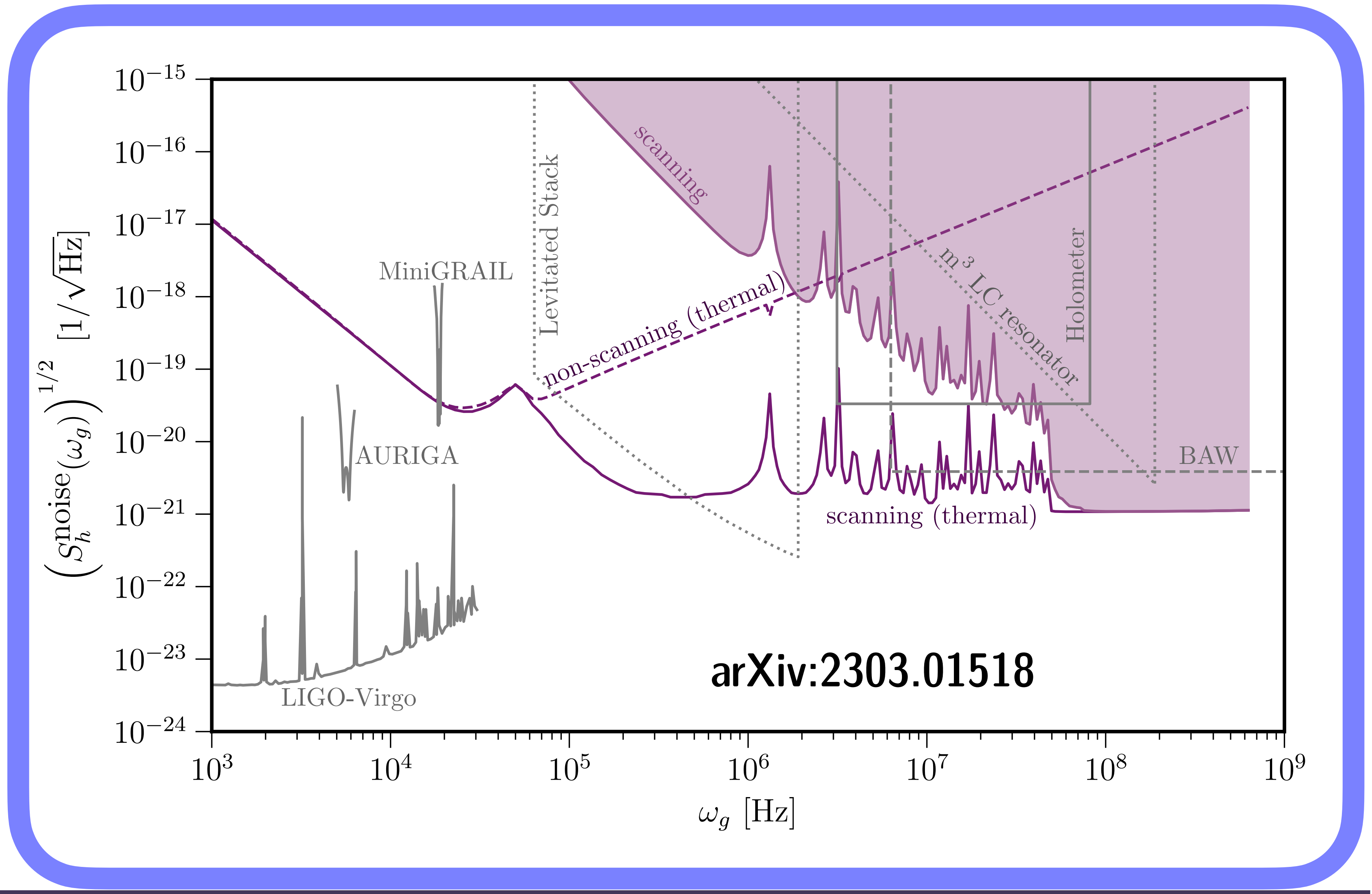

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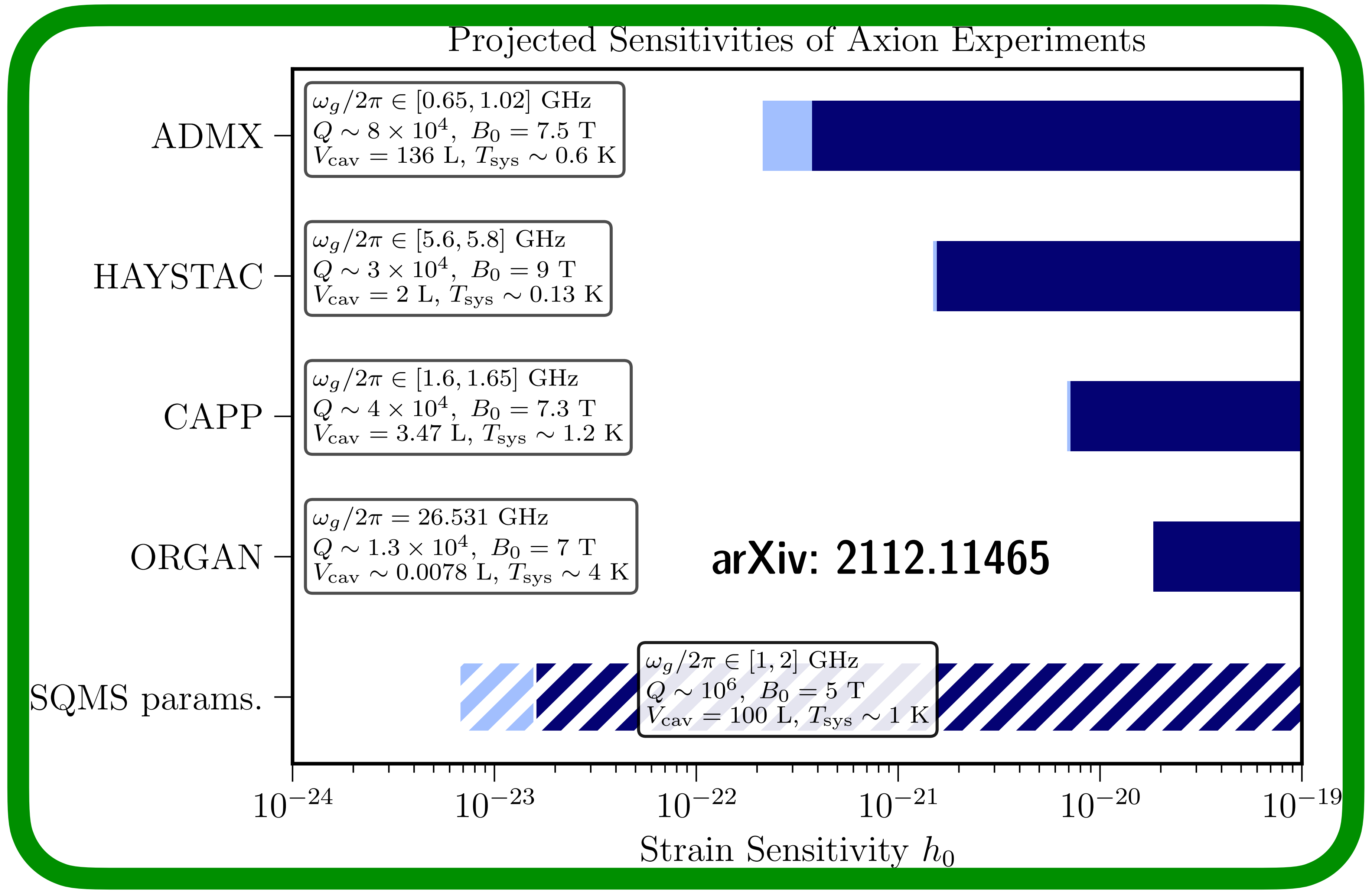

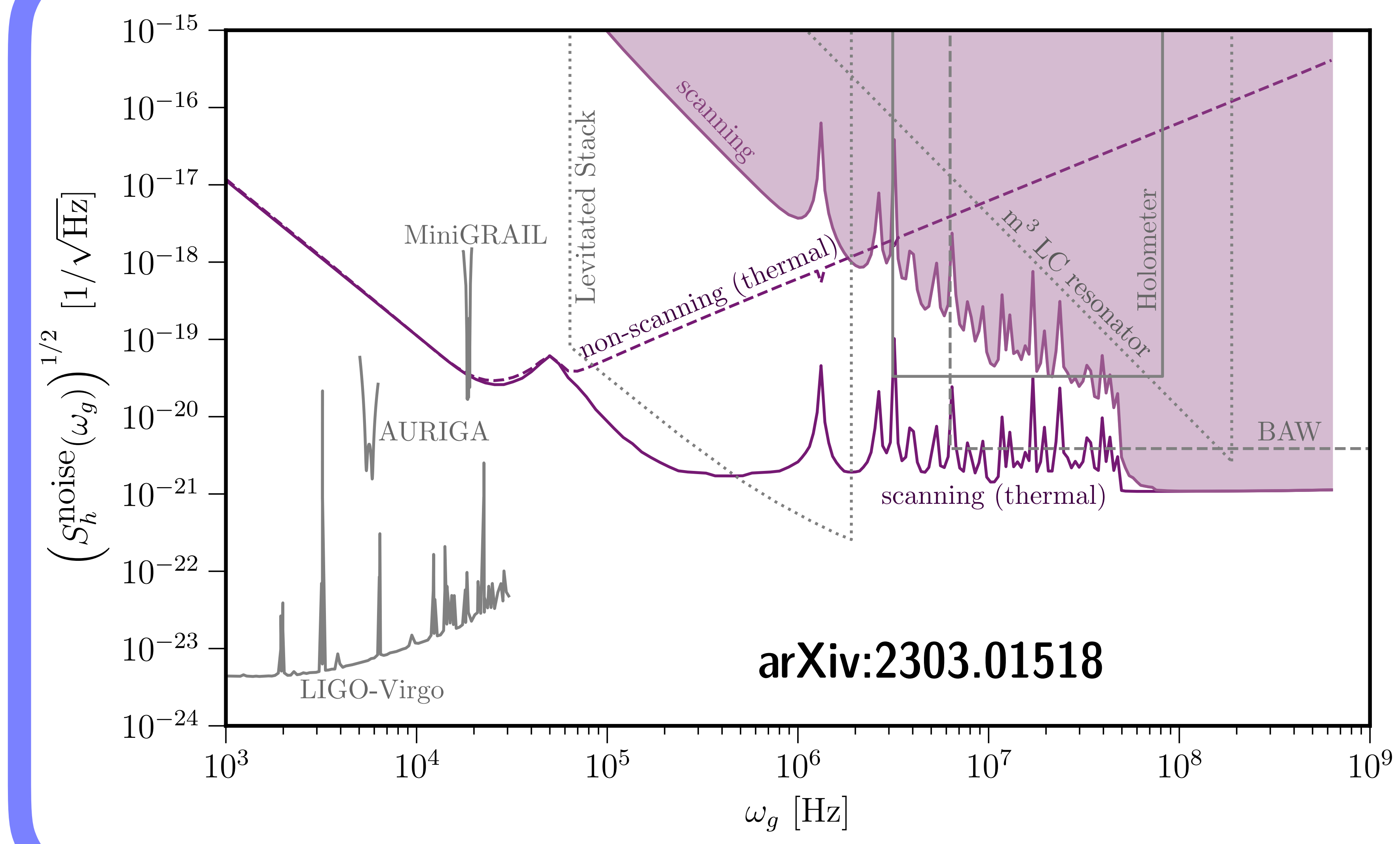

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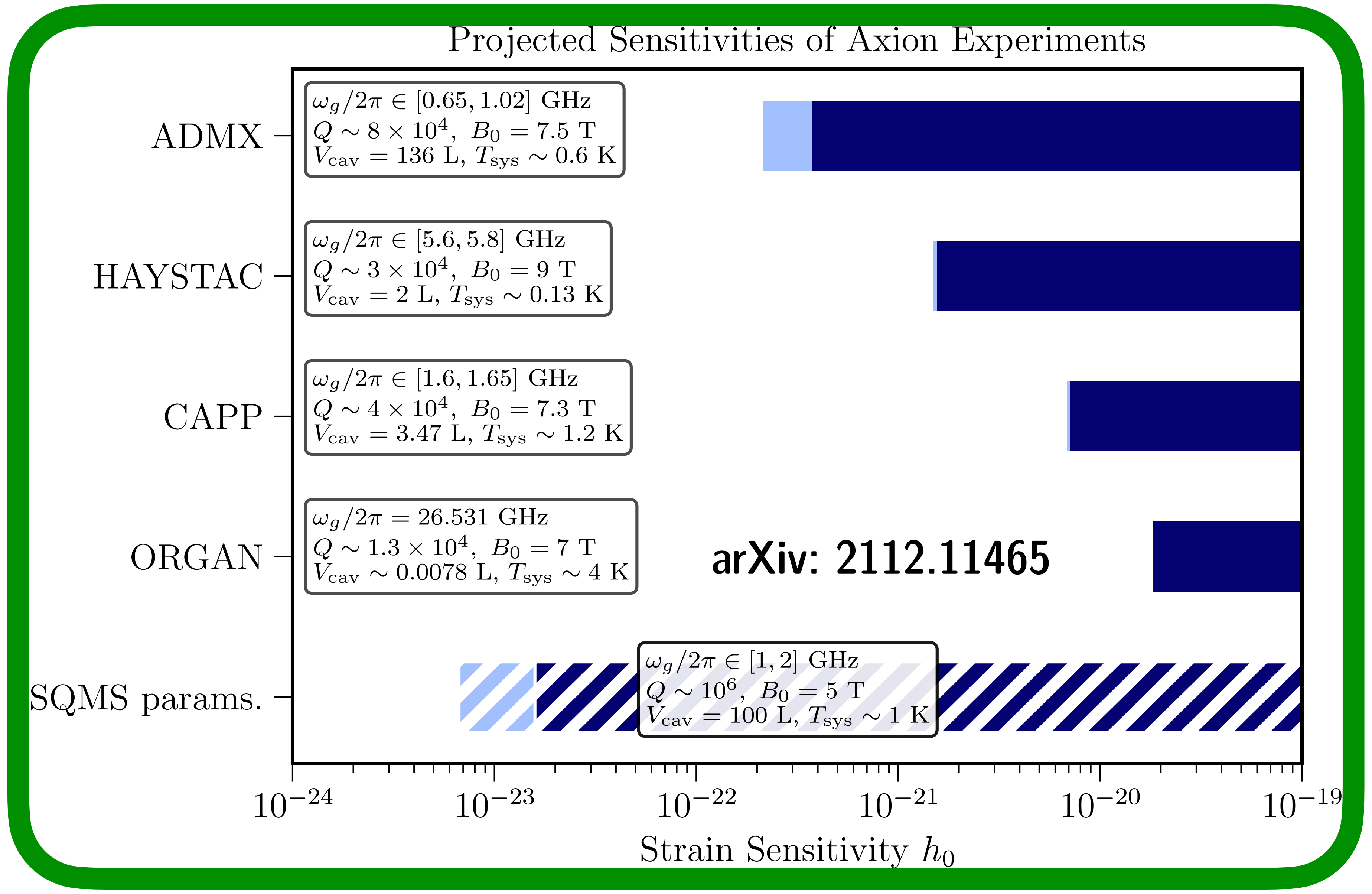

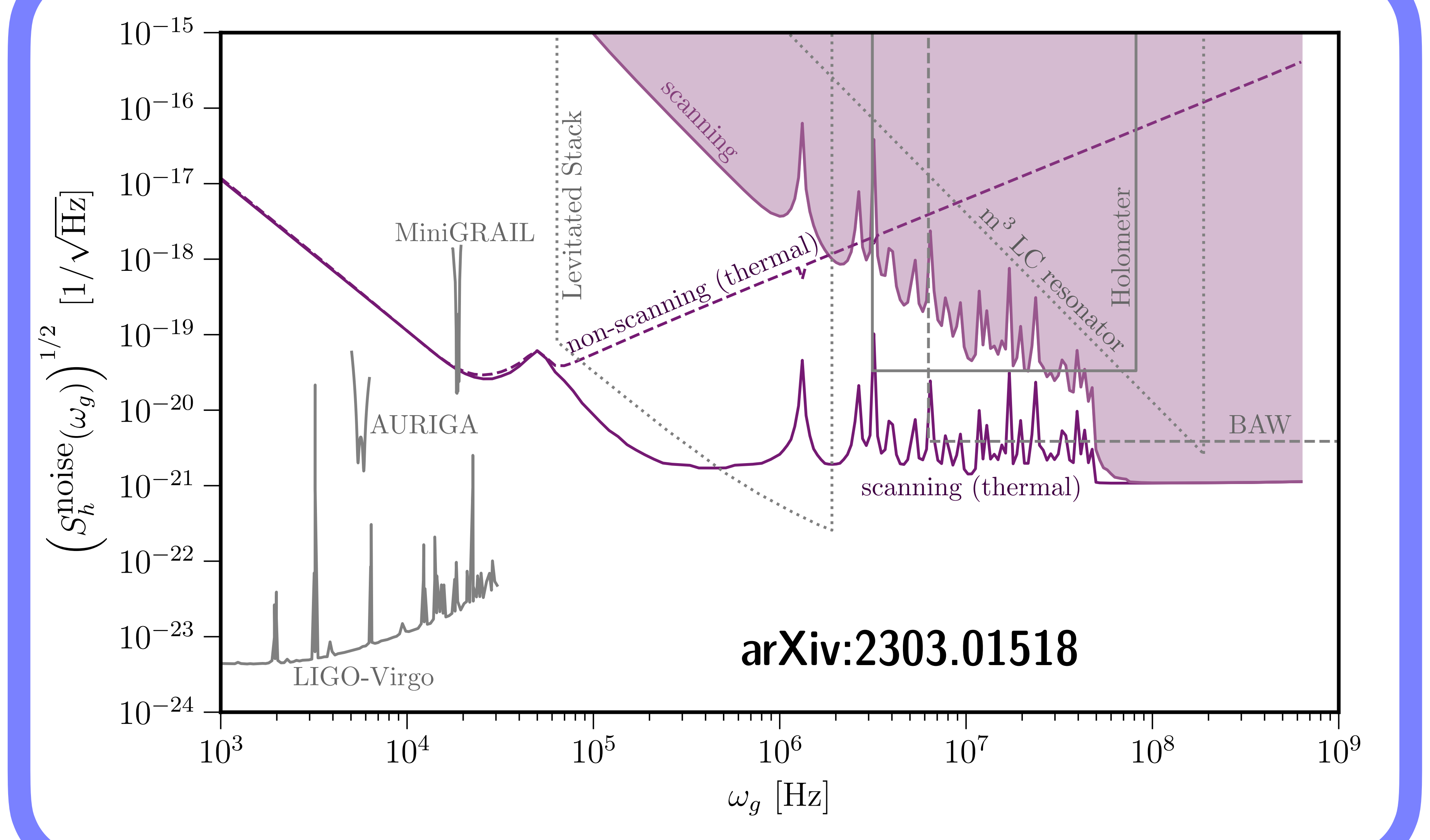

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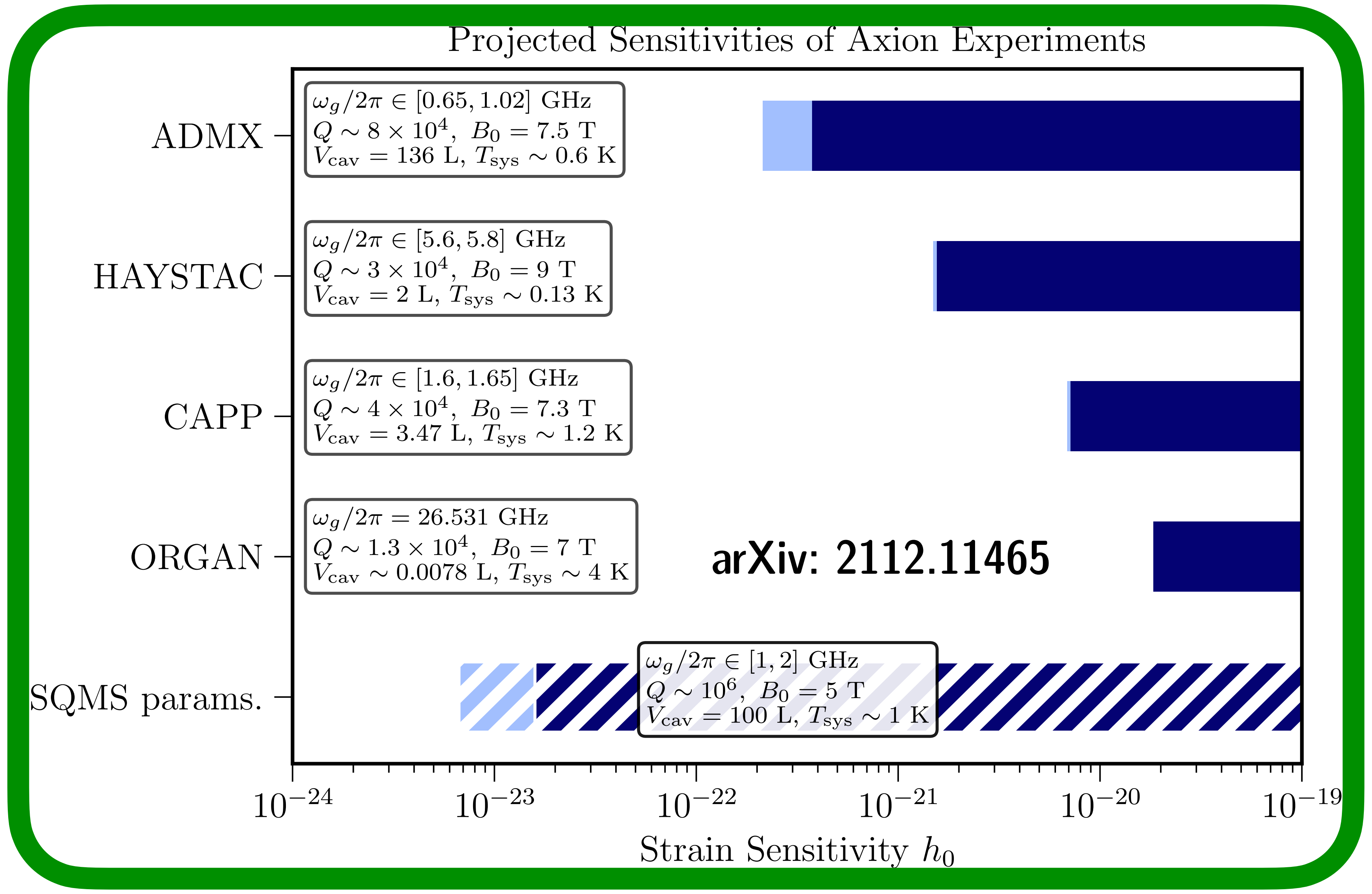
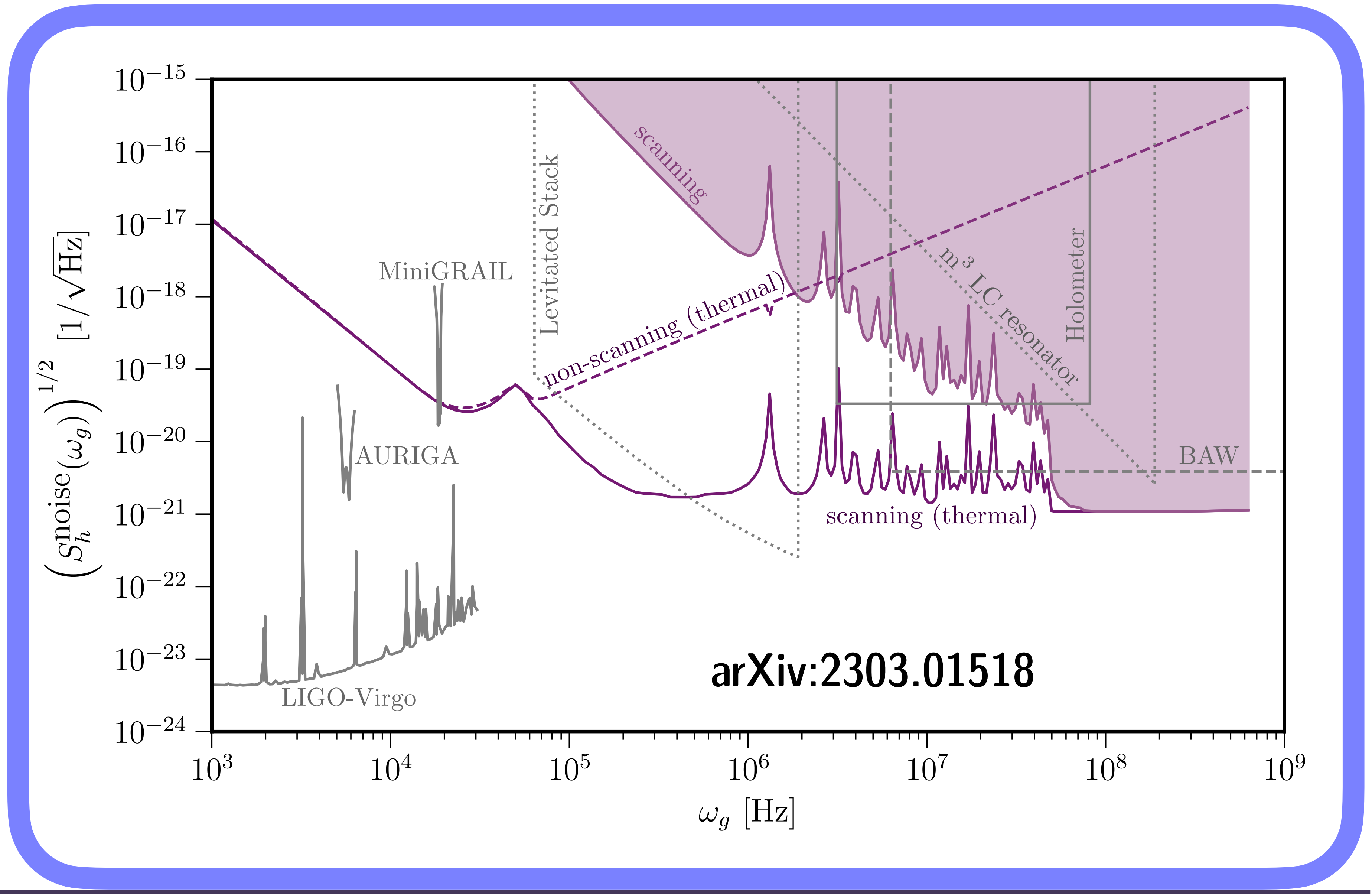
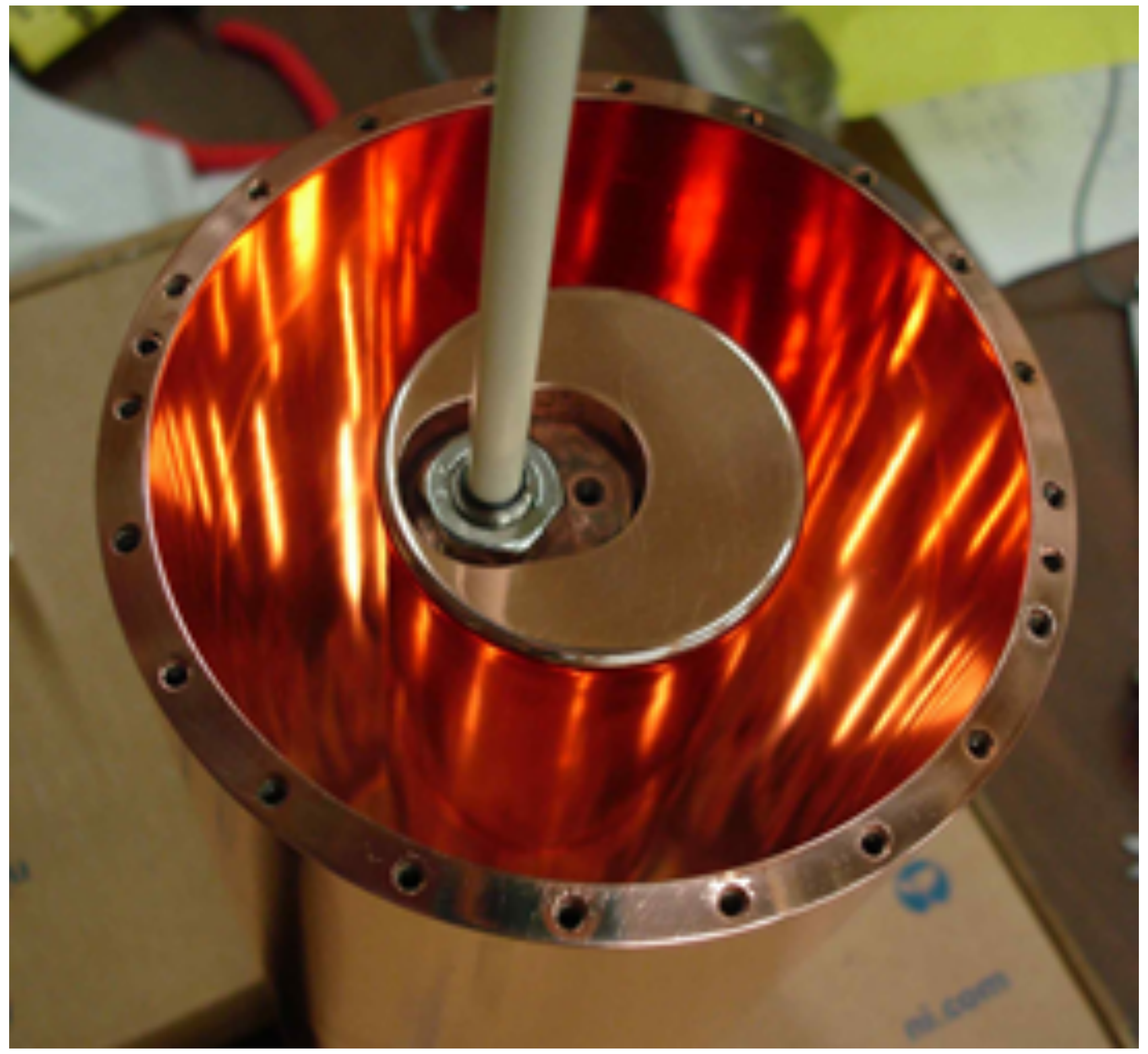




BACKUP

Resonant Cavities

Why?



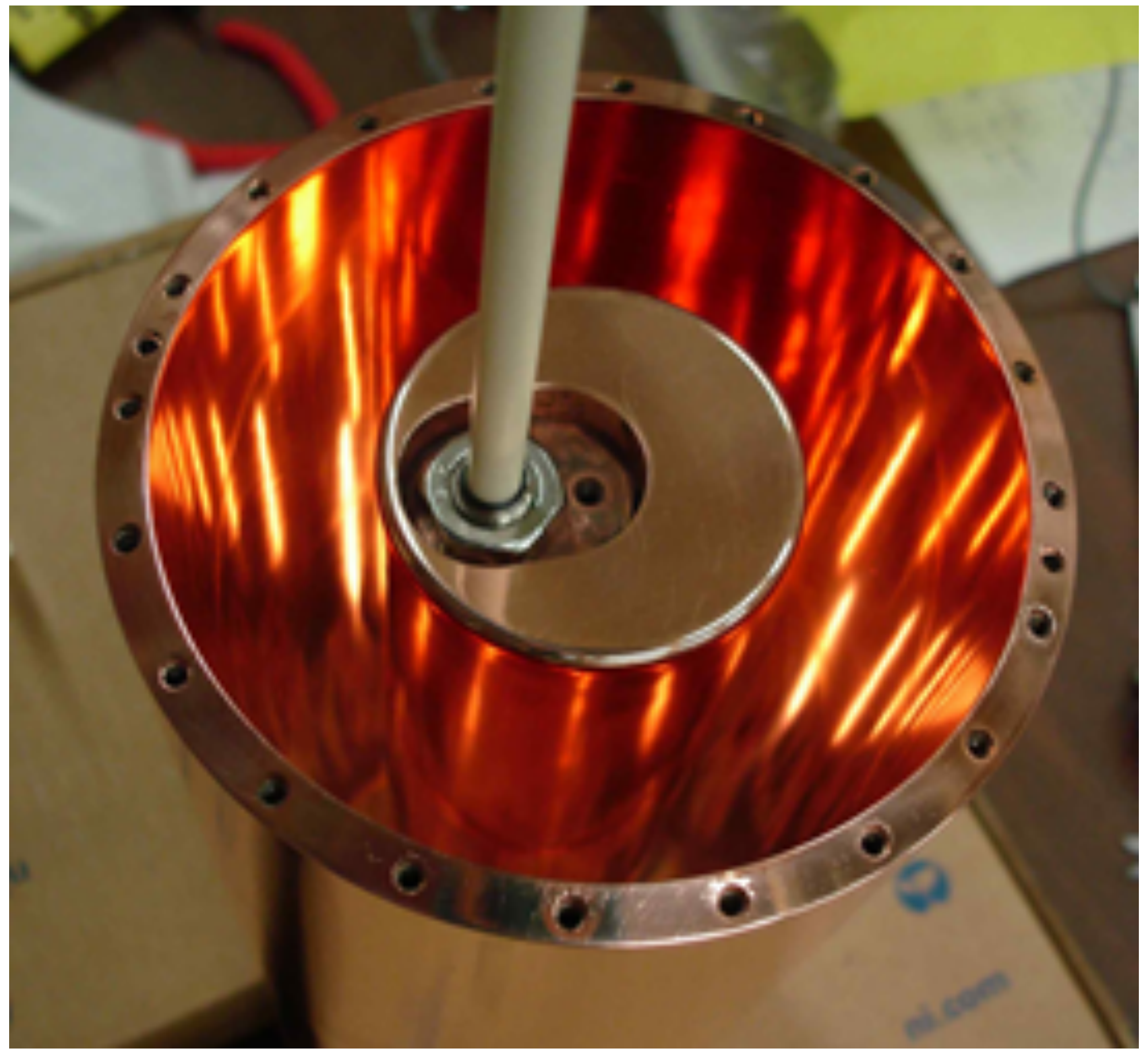
HAYSTAC

Resonant Cavities

Why?

Mature technology & constantly improving

Benefit from decades of development for accelerator use



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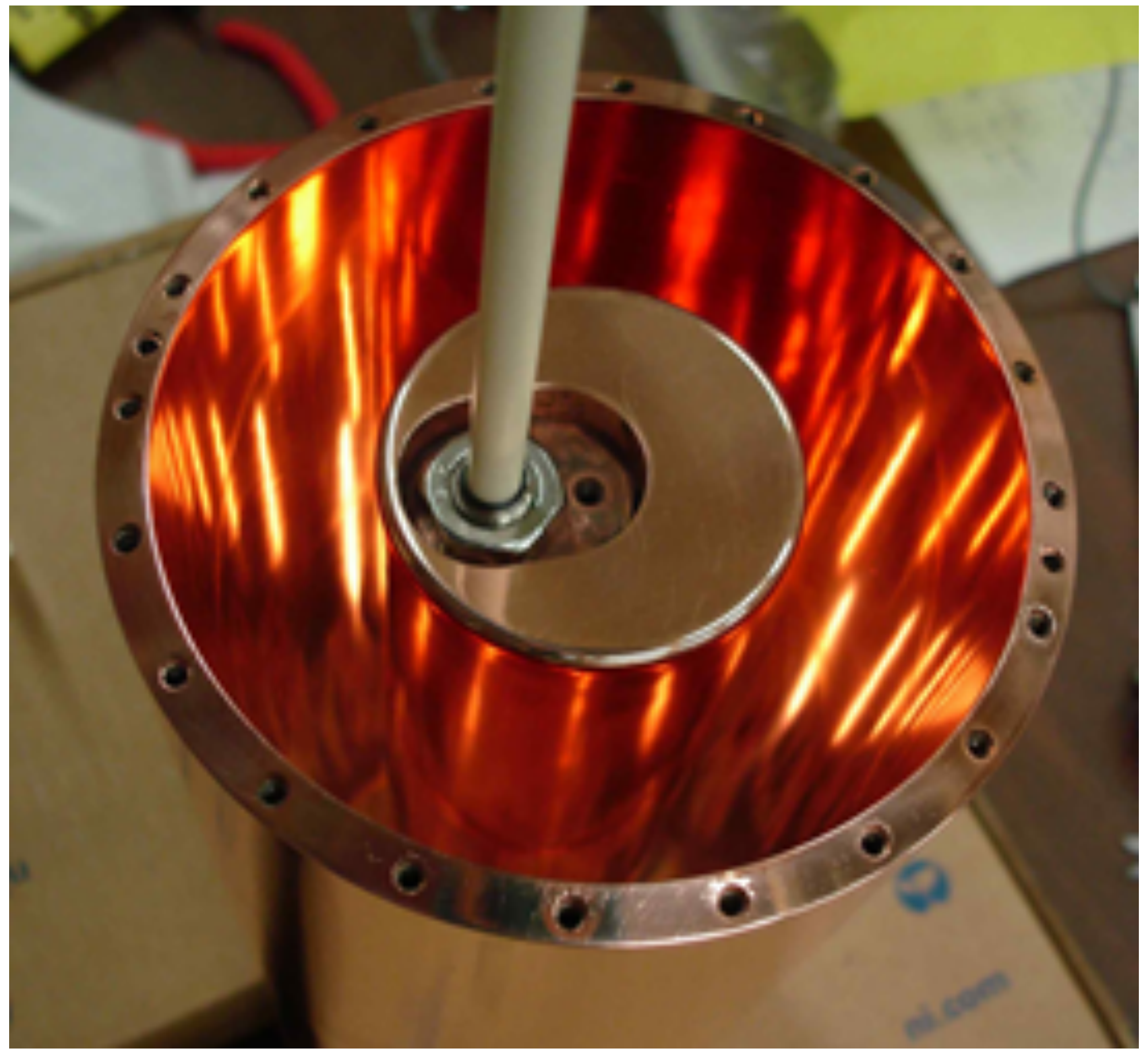

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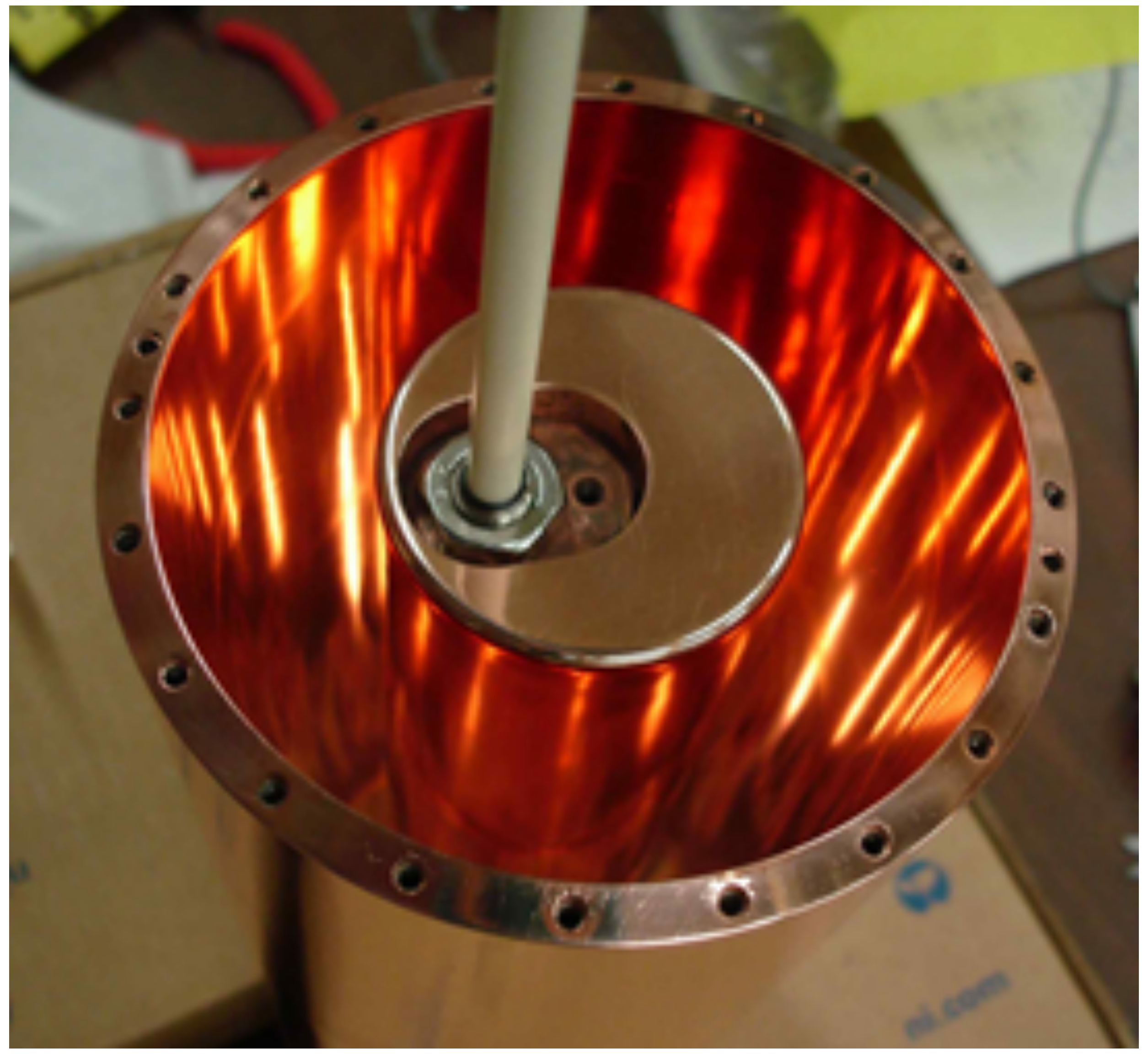

HAYSTAC

Domain where various subtleties arise...

Framing the Question

A more detailed estimate requires some GR

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GW in TT gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$

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GW in TT gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$

Riemann tensor invariant at O(h):

$$R_{0i0j} = -\frac{1}{2}\partial_t^2 h_{ij}^{\mathrm{TT}},$$

$$R_{0ijk} = \frac{1}{2}\partial_t \left(\partial_k h_{ij}^{\mathrm{TT}} - \partial_j h_{ik}^{\mathrm{TT}}\right),$$

$$R_{ikjl} = \frac{1}{2}\left(\partial_k\partial_j h_{il}^{\mathrm{TT}} + \partial_i\partial_l h_{jk}^{\mathrm{TT}} - \partial_i\partial_j h_{kl}^{\mathrm{TT}} - \partial_k\partial_l h_{ij}^{\mathrm{TT}}\right)$$

Gravitational Wave and a Hollow Sphere

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Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

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$$\ddot{u}_p + \frac{\omega_p}{Q_p}\,\dot{u}_p + \omega_p^2\,u_p \simeq -\frac{1}{2}\,\omega_g^2\,V_{\text{cav}}^{1/3}\,\eta^g_{\text{mech}}\,h_0\,e^{i\omega_g t}$$

$$\eta^g_{\text{mech}} = \frac{\hat{h}^{TT}_{ij}}{V_{\text{cav}}^{1/3}\,V_{\text{shell}}}\int_{V_{\text{shell}}} d^3\mathbf{x}\,U_p^{*i}\,x^j$$

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Tiny displacement $\ll$ nm

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Cur Cavis?* pt. 2

MAGO 2.0

* “Why Cavities?” in Latin

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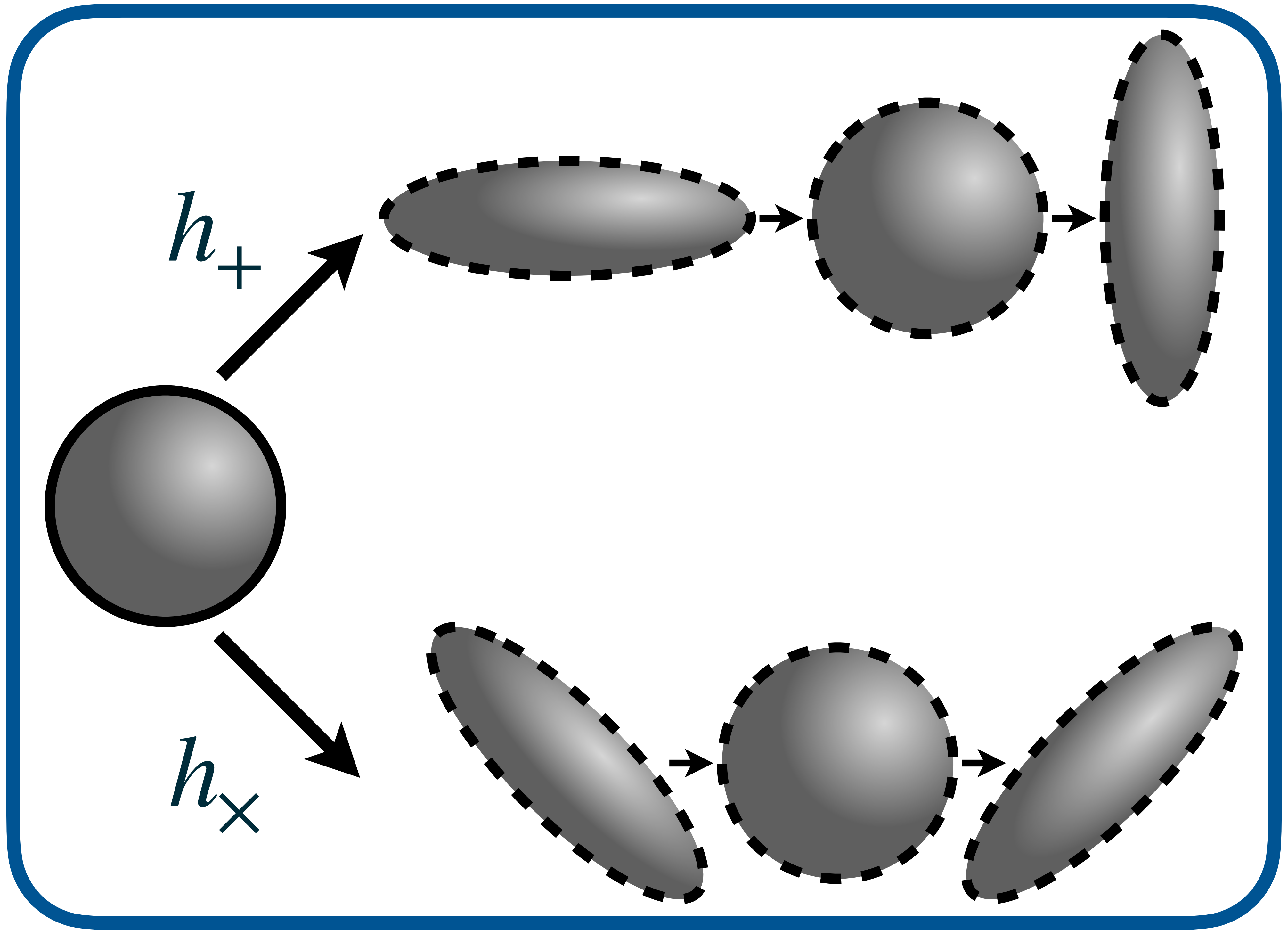

TT frame intuition

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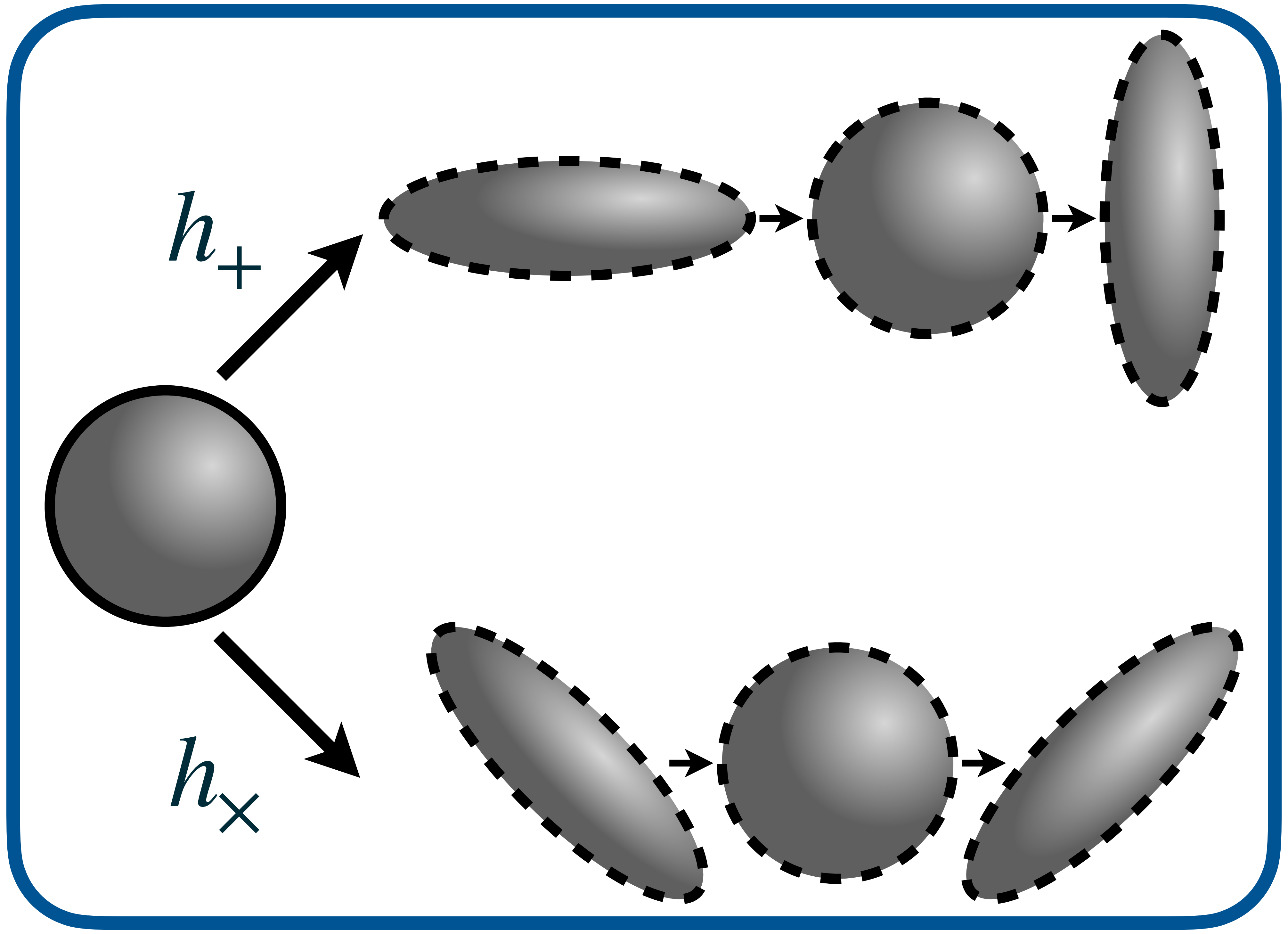

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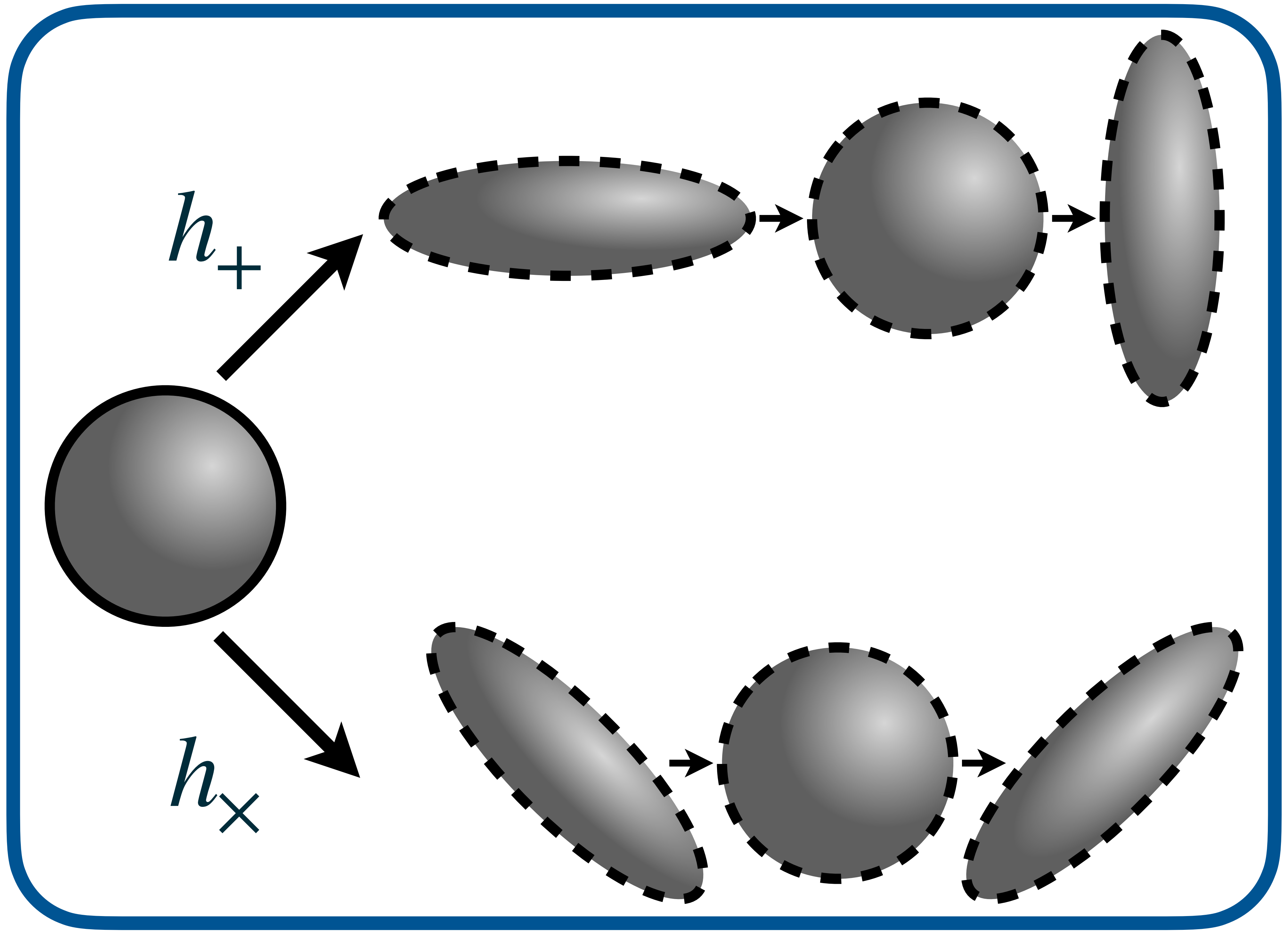

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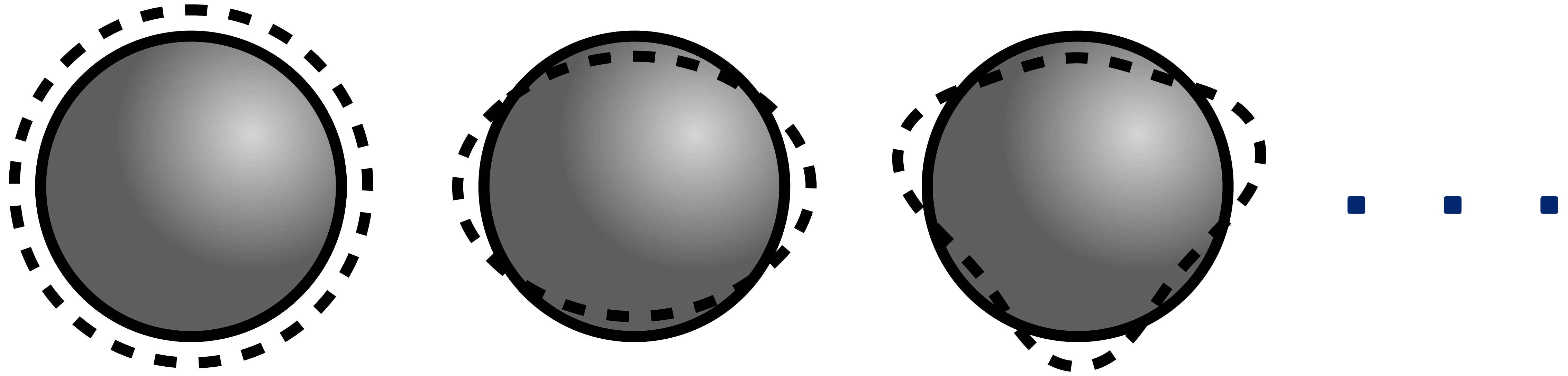
Spheroidal



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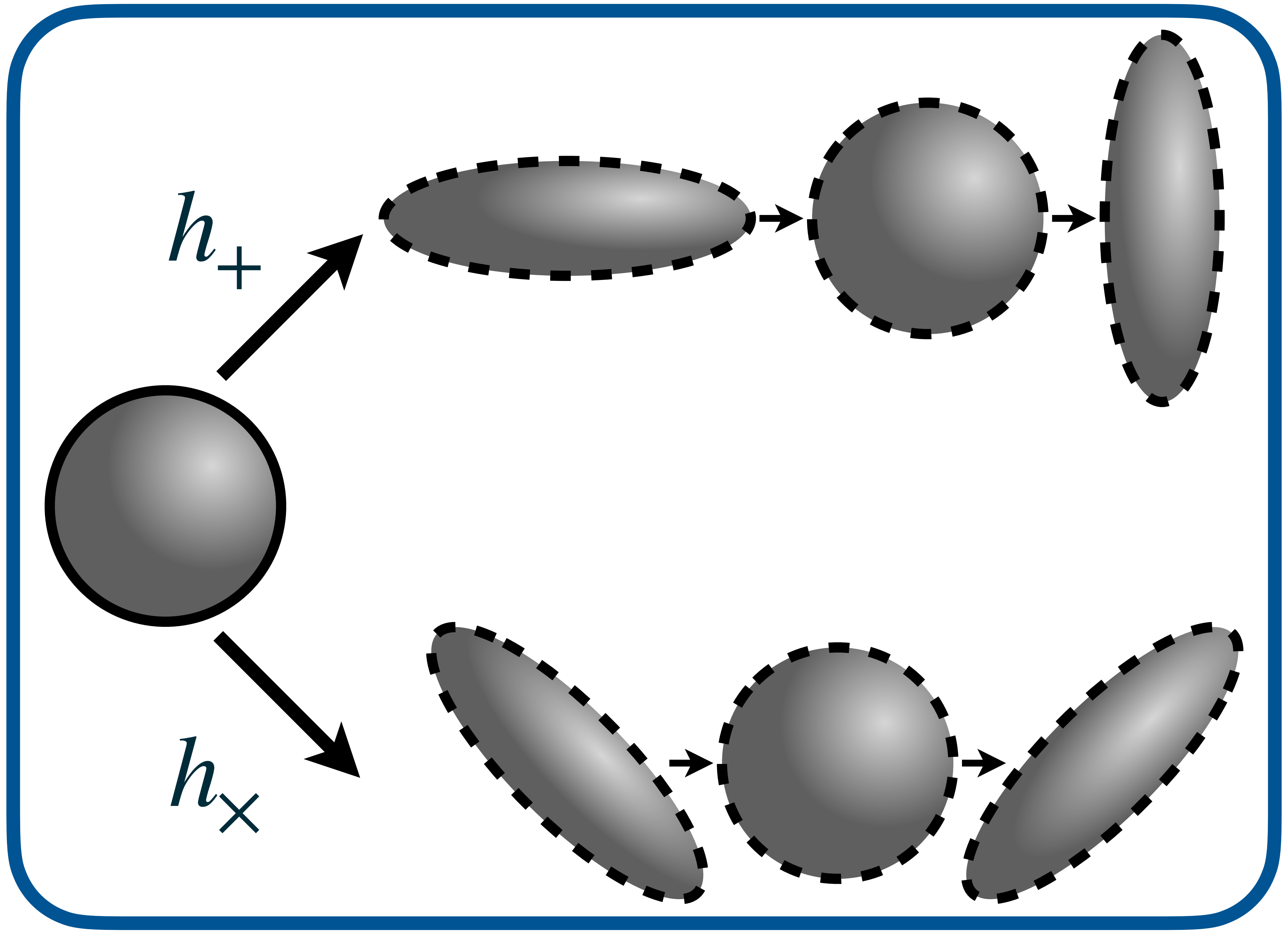

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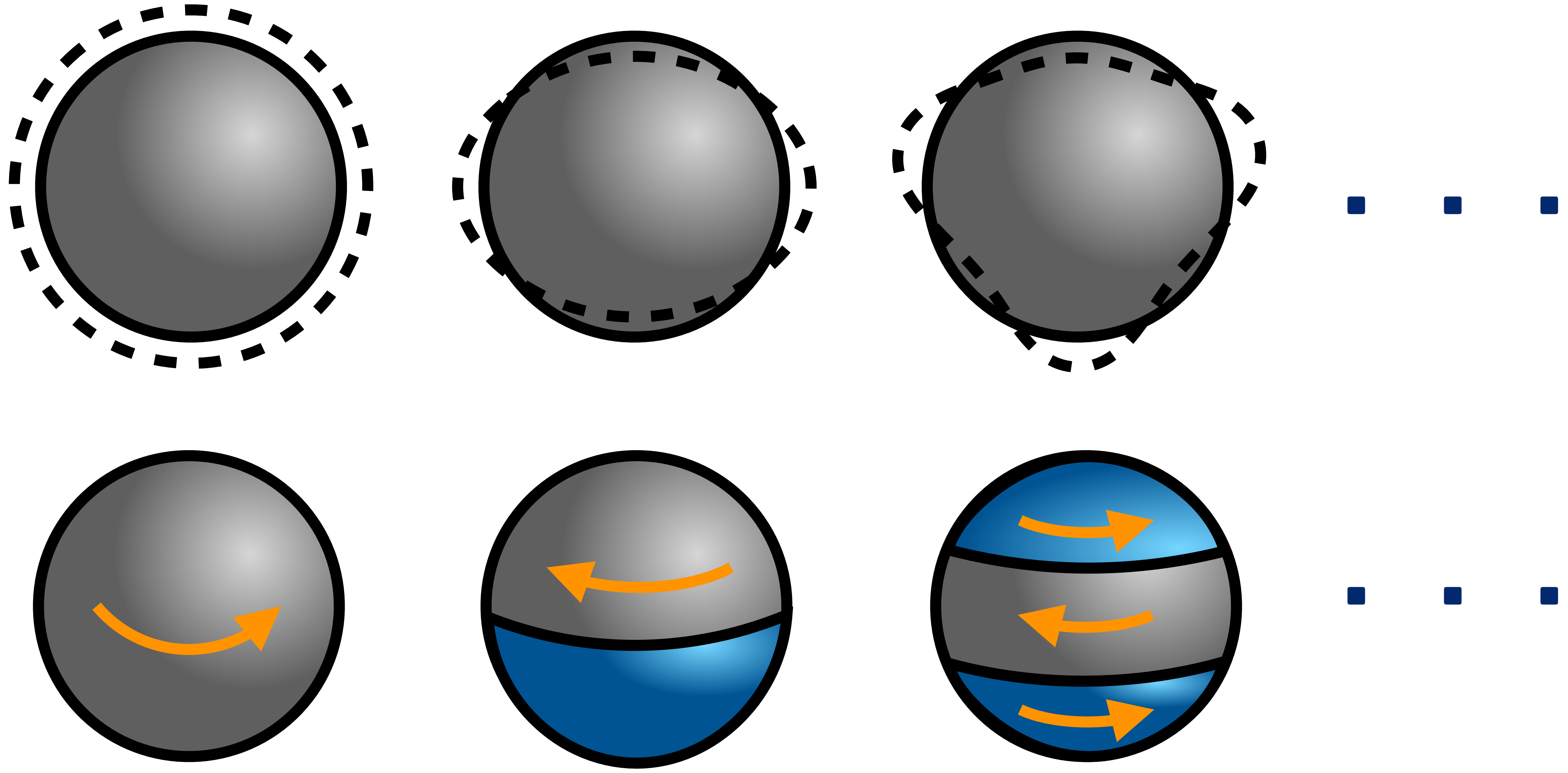
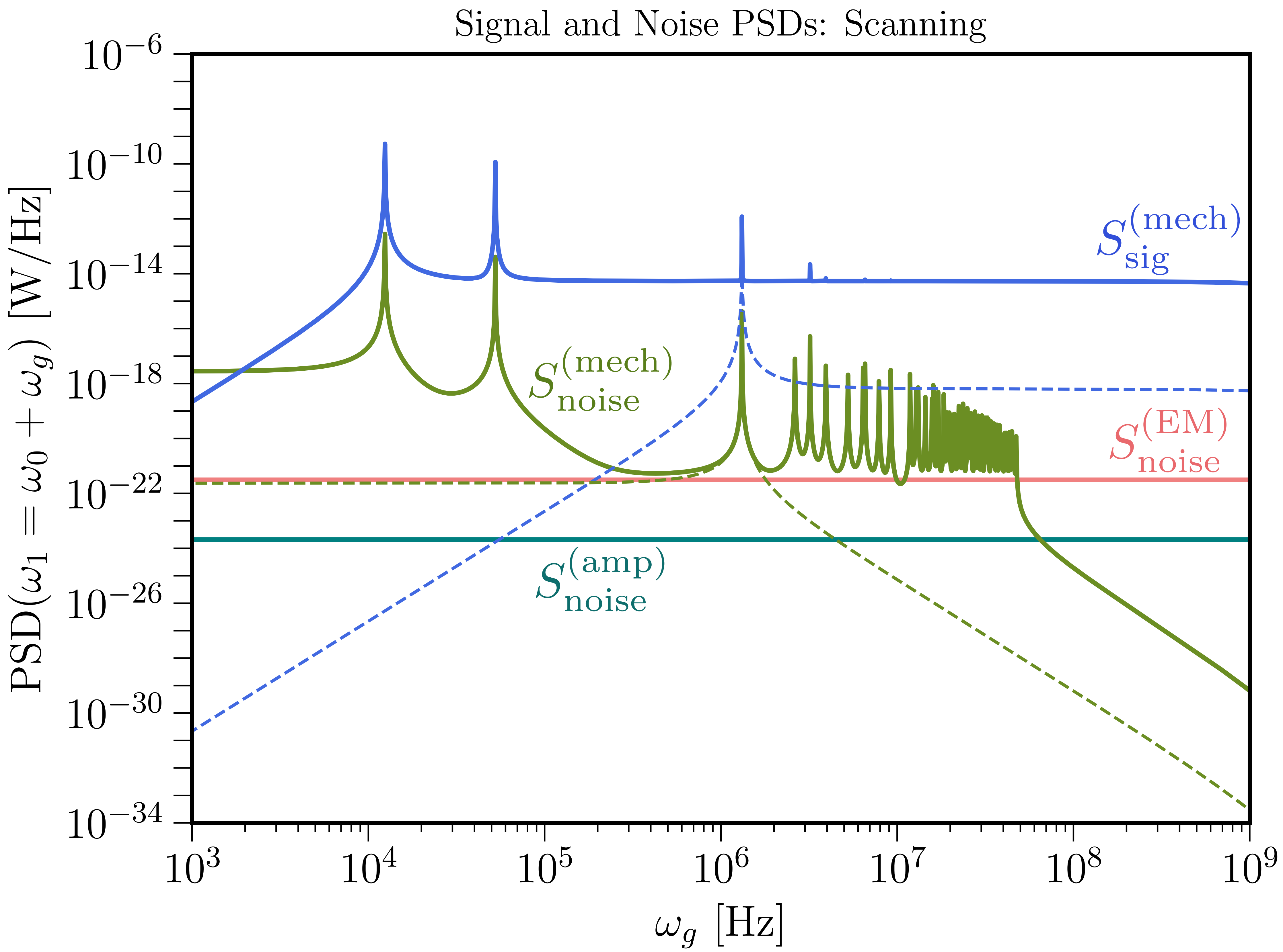
Spheroidal — Toroidal



Noise in MAGO 2.0



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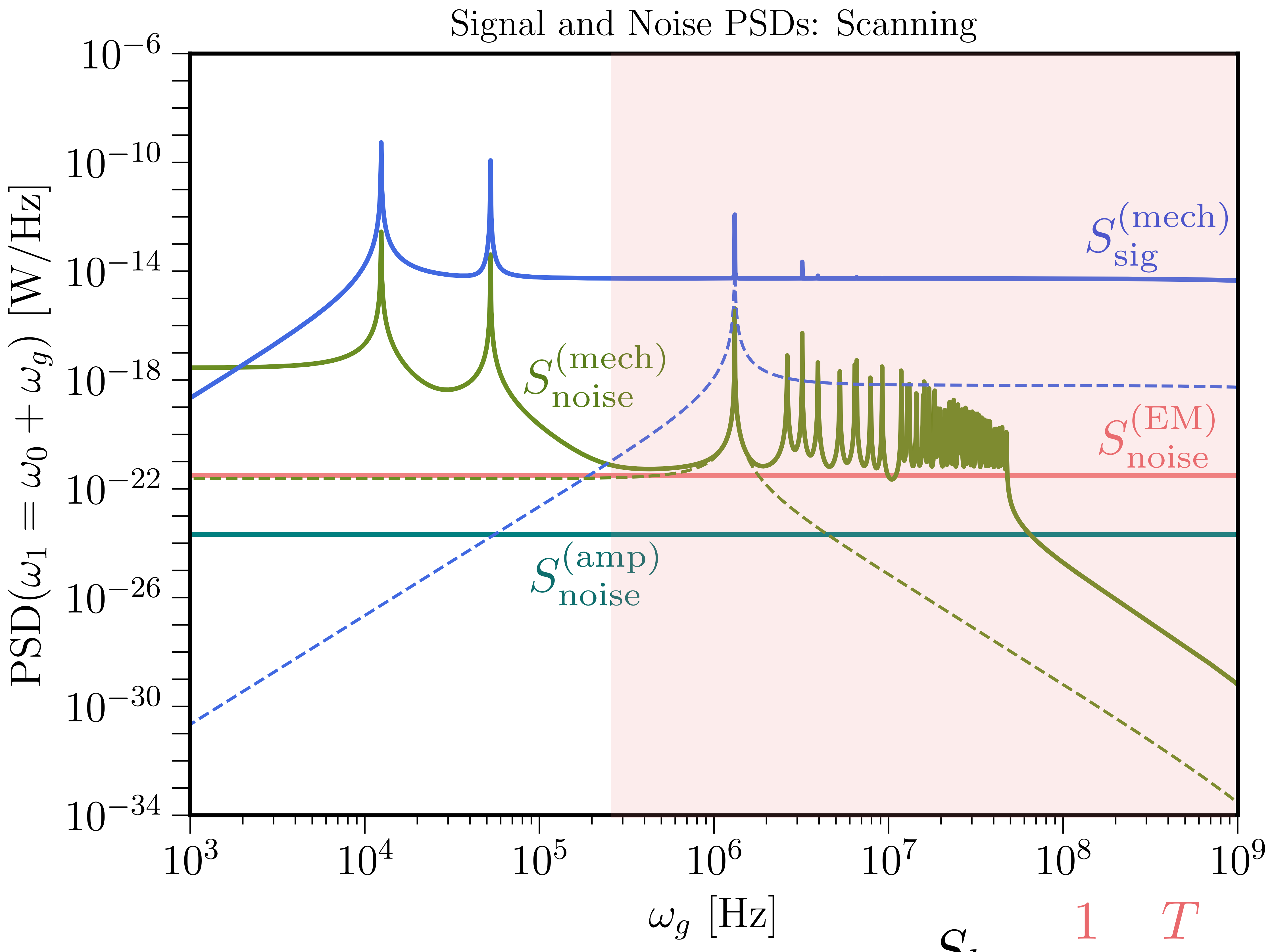

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EM Th. > MHz

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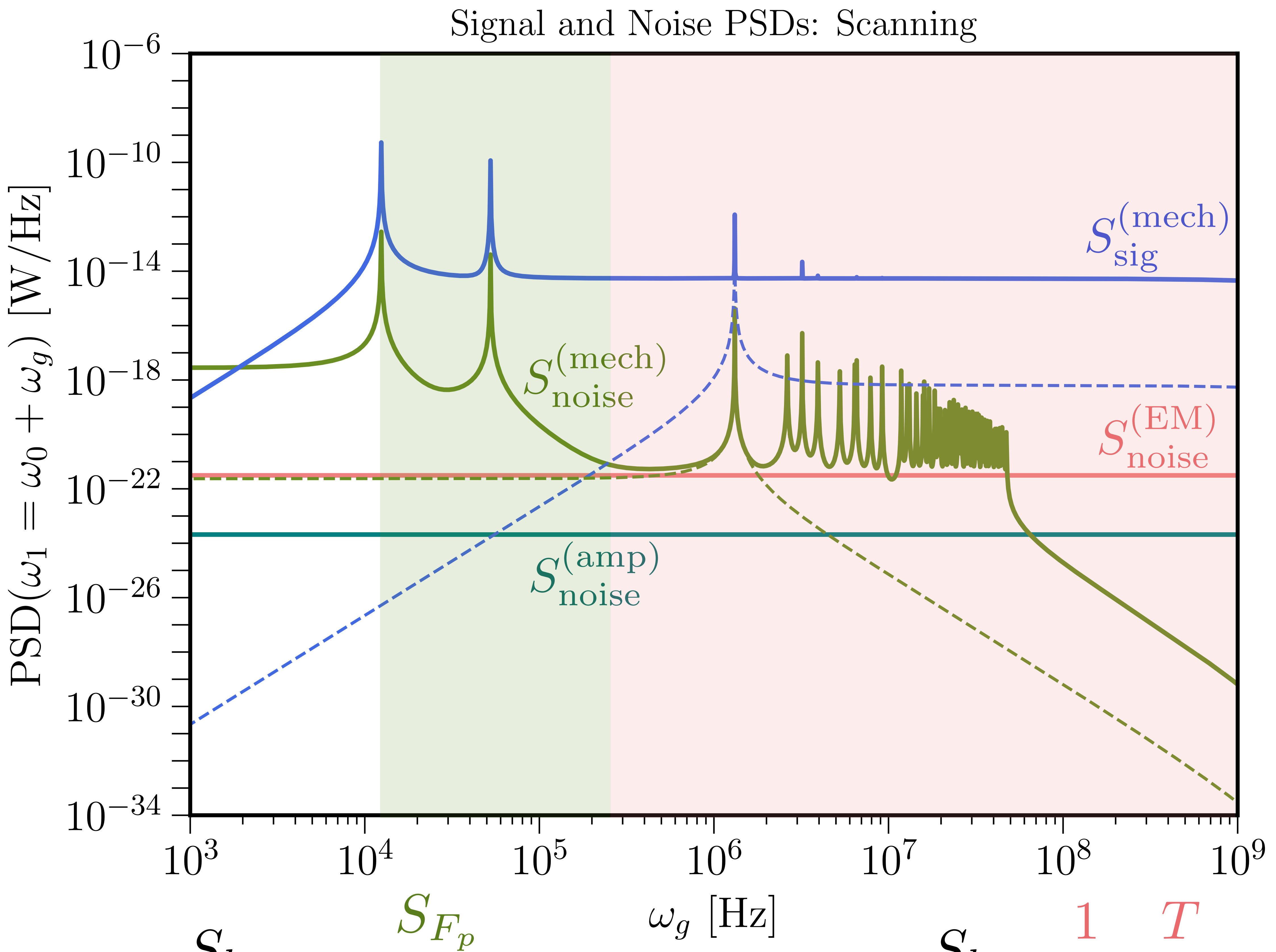

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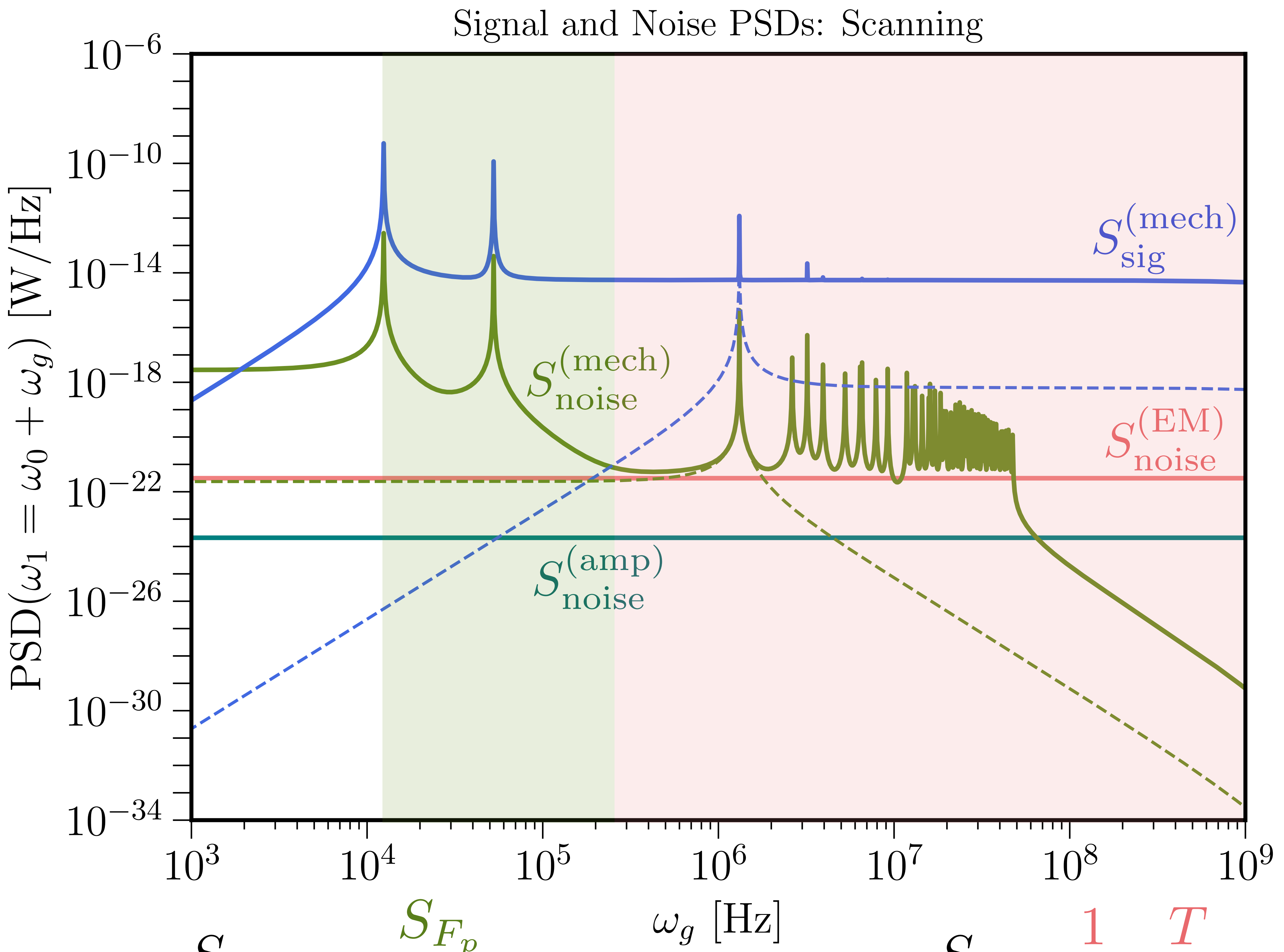

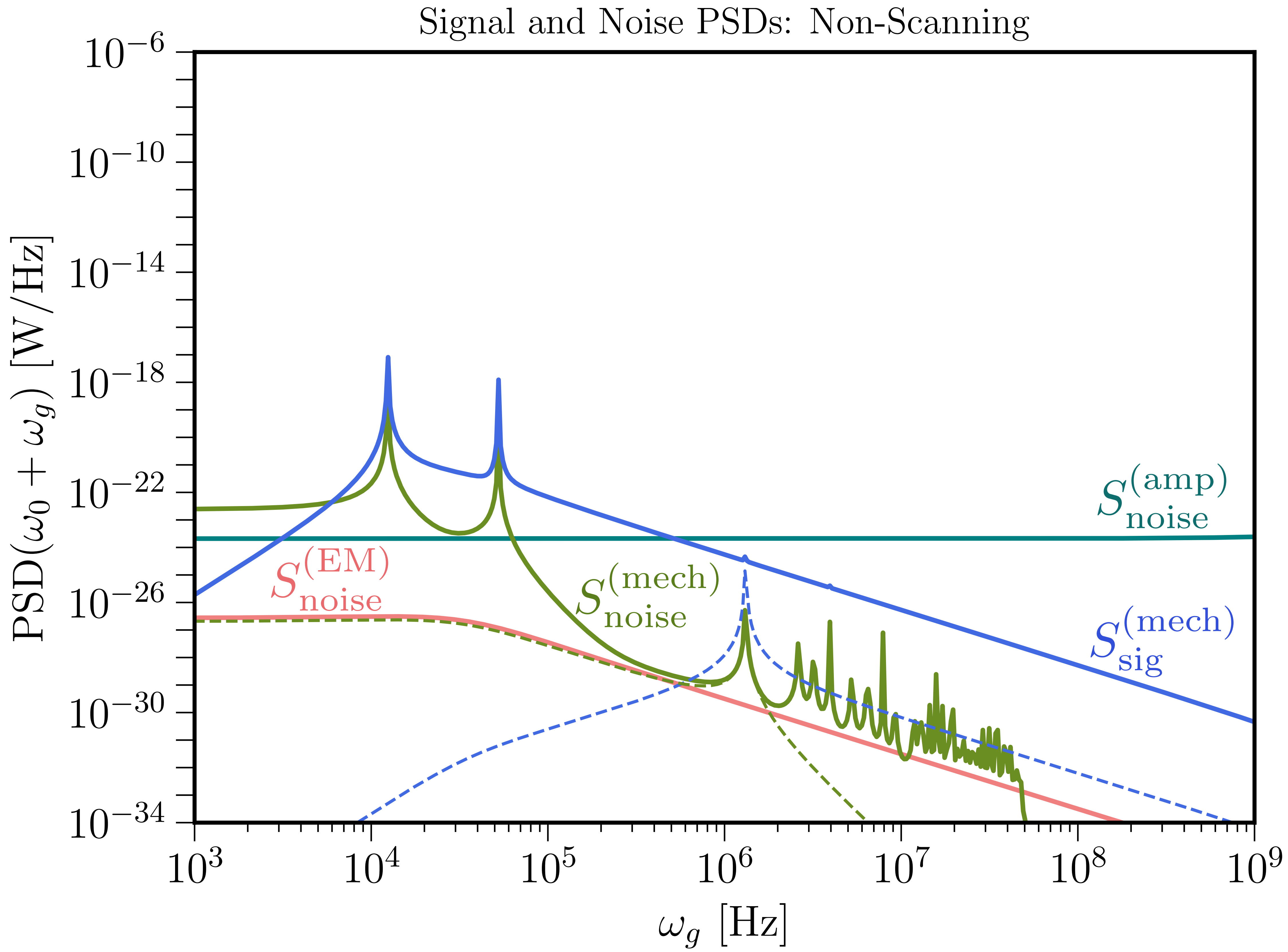

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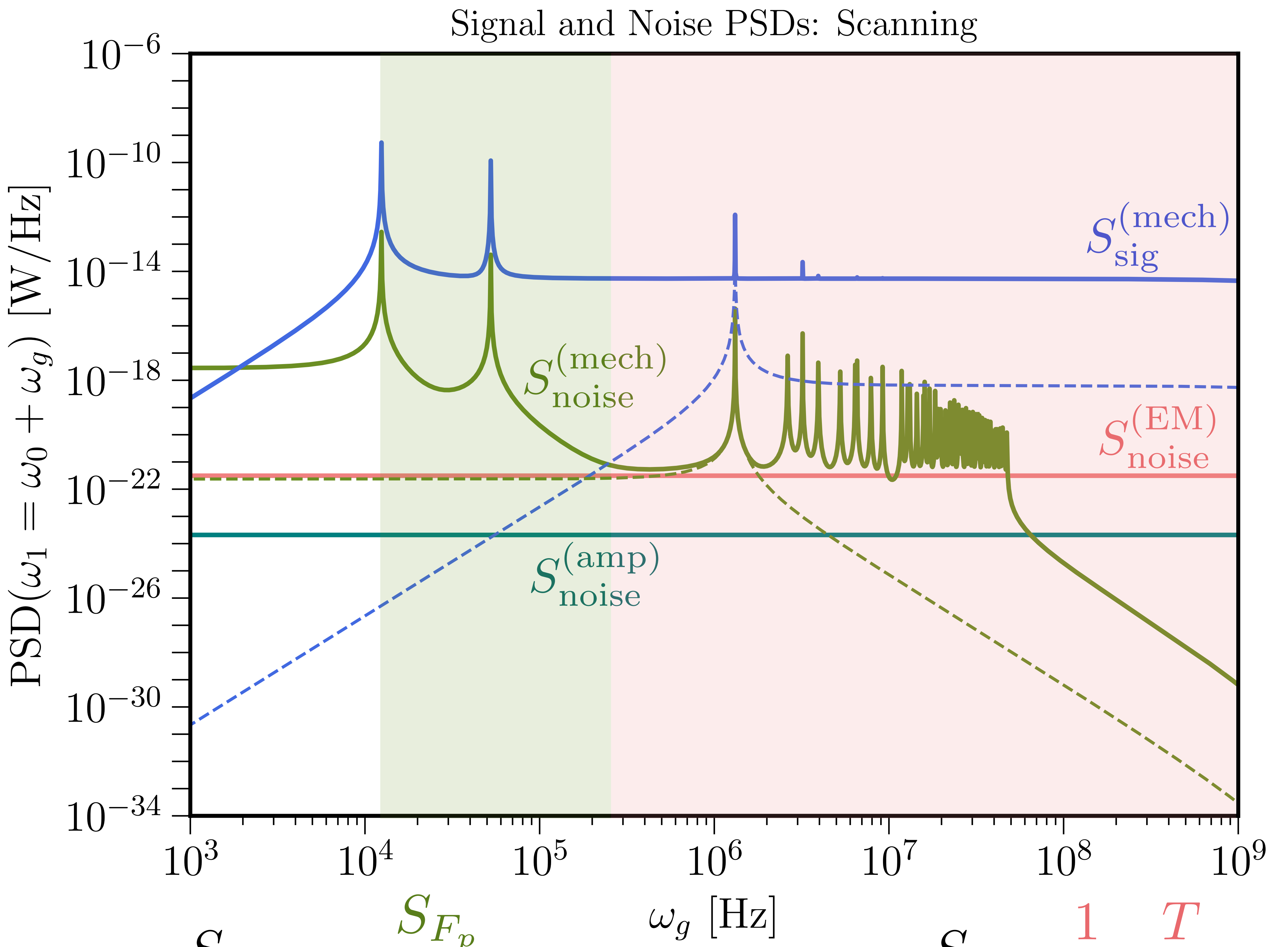

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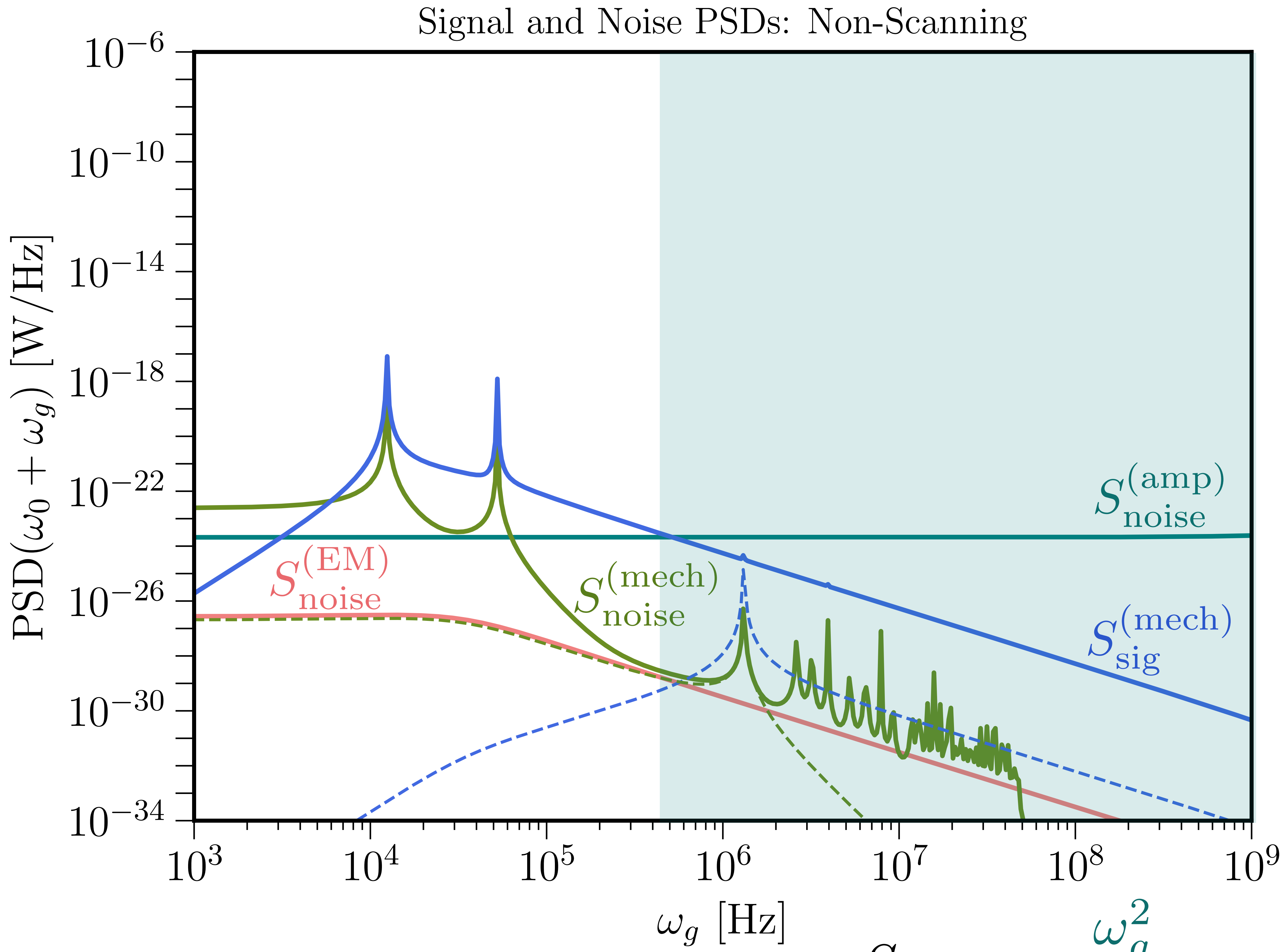

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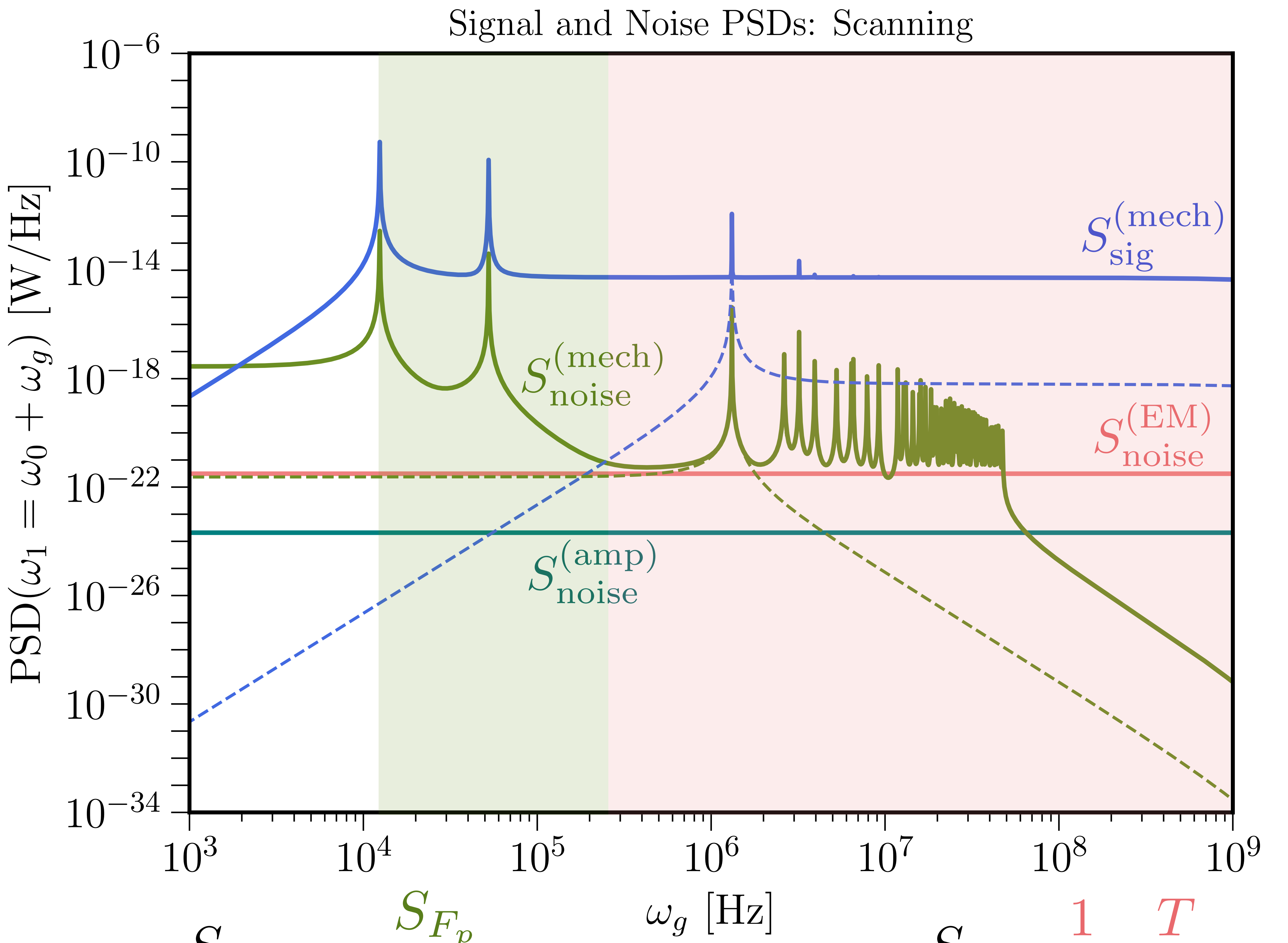

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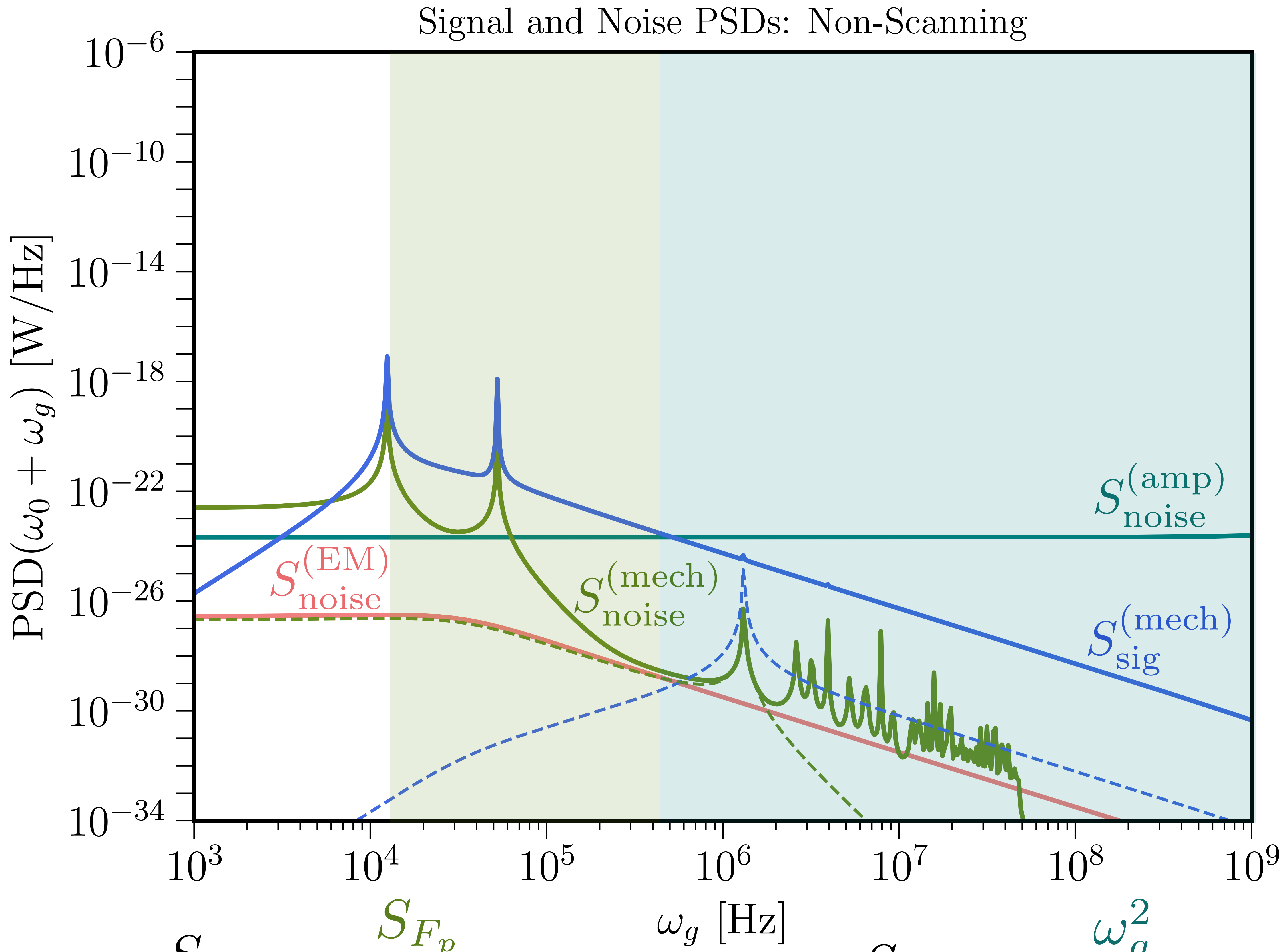

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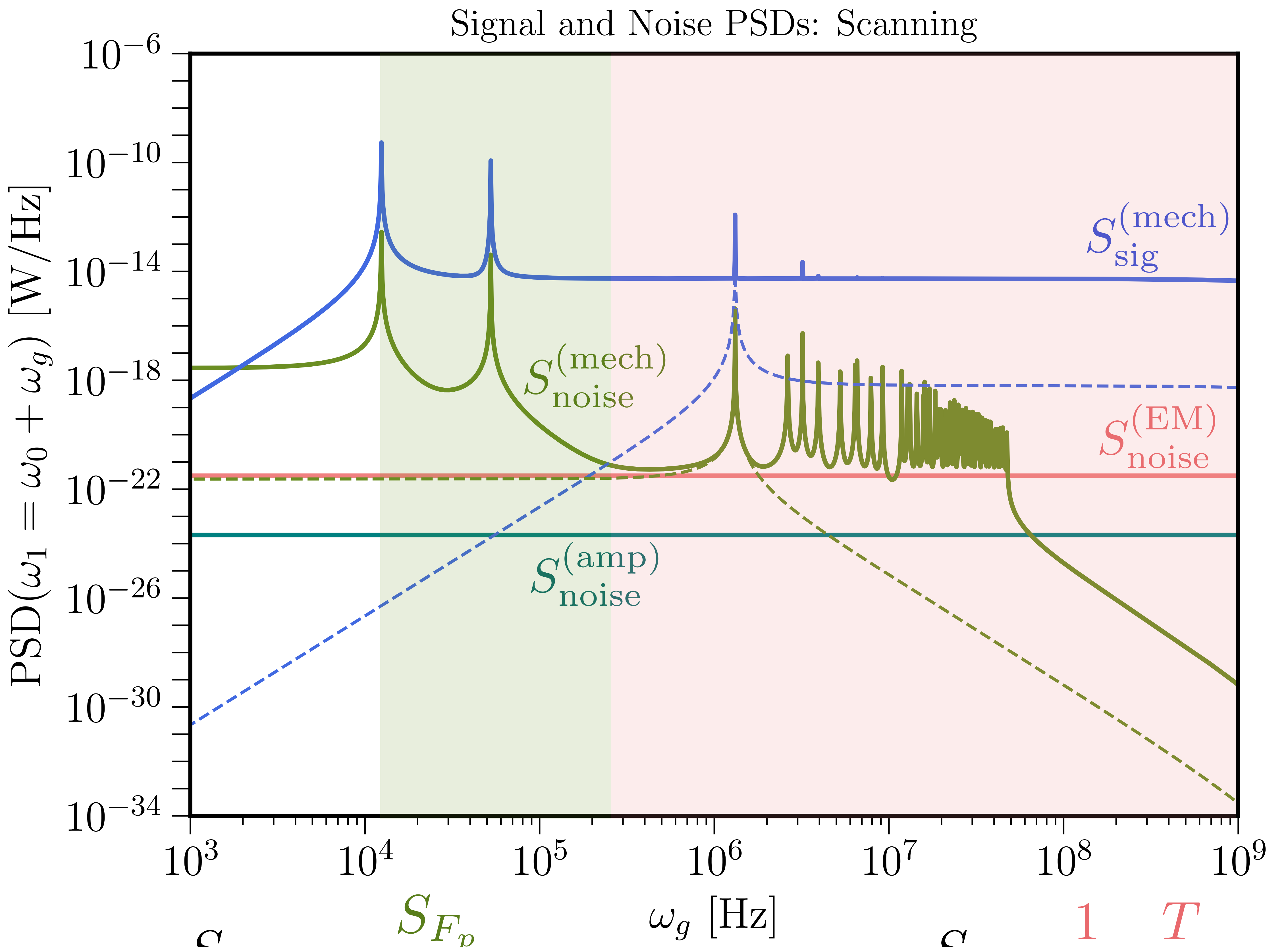

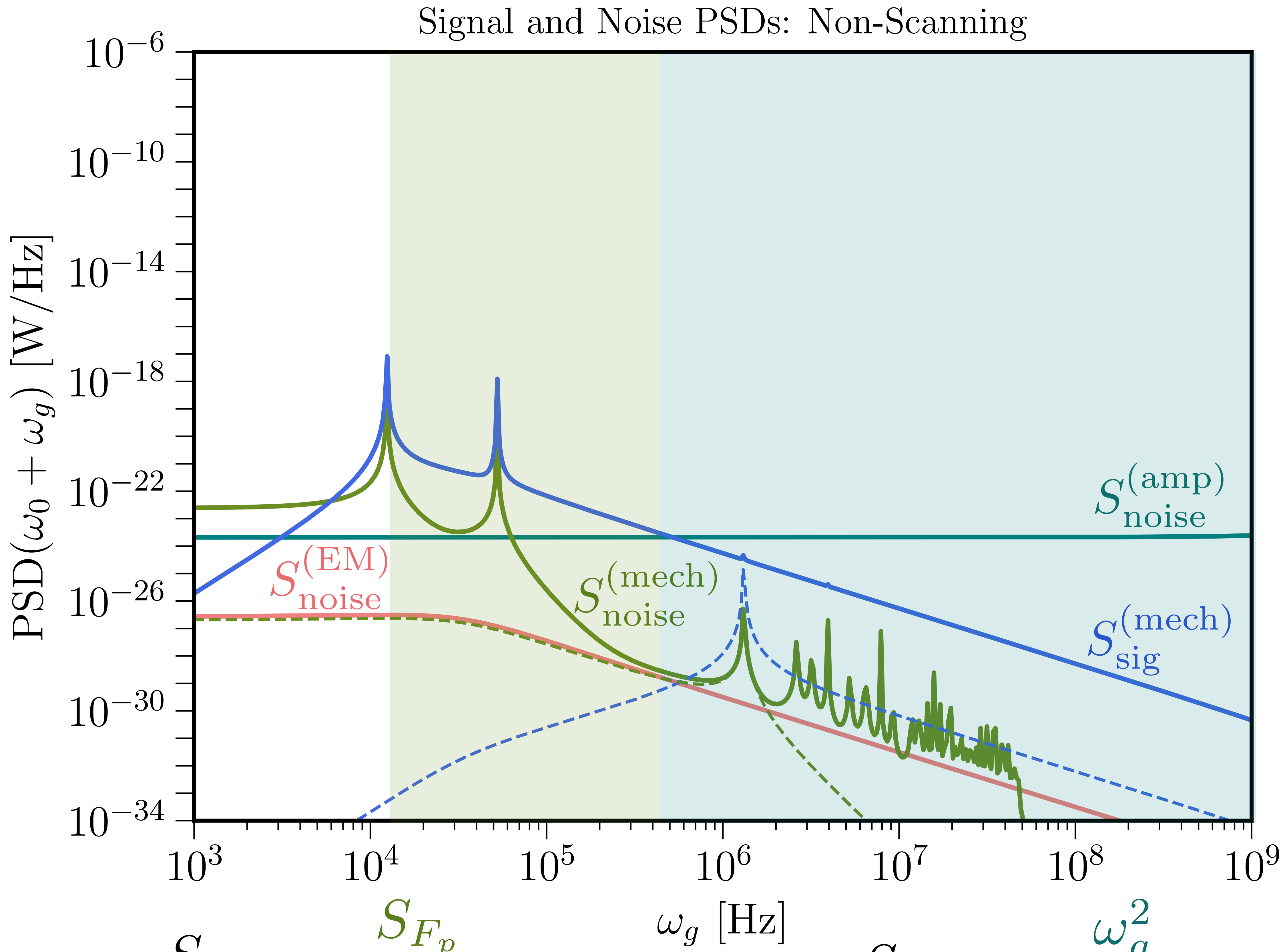

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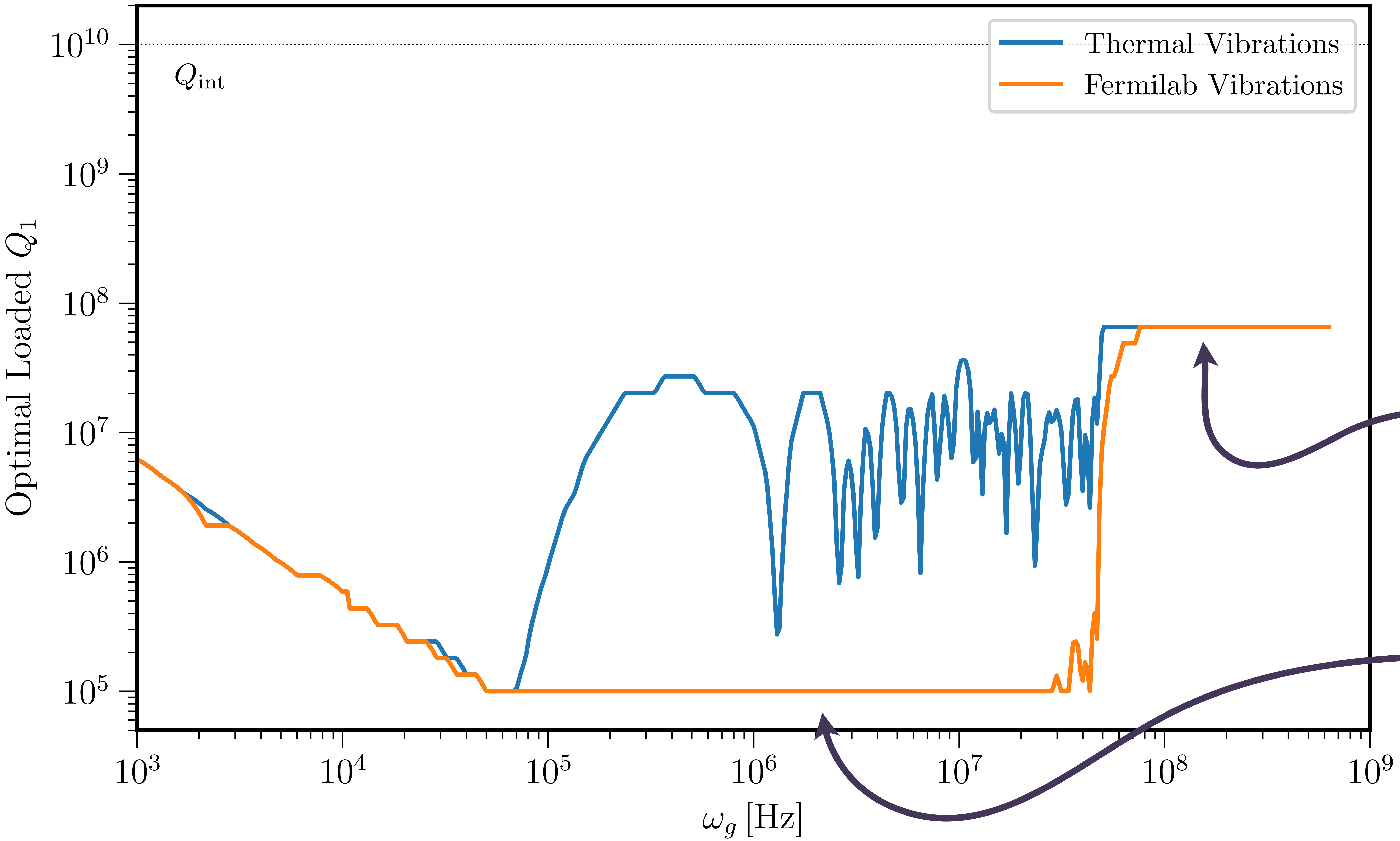
NB: missing radiation damping effect studied in *Löwenberg, Moortgat-Pick: 2307.14379*

Optimal Scanning



Integration time:

$$t_{\rm int} \sim t_e \min\left(\frac{\omega_1}{Q_1\,\omega_g}\,,\,1\right)$$

Thermal:

$$Q_1 \sim Q_{\rm int}(\omega_1/T)$$

Vibrations:

$$Q_1 \sim Q_1^{\rm min}$$