

11th LISA CosWG Workshop

A tool for Cosmological Phase Transitions and GWs

Physics Department – University of Porto

2024-06-18

Marco Finetti

Project ID

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Bolsas de Investigação para
Doutoramento FCT-ECIU

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Marco Matteini (Jožef Stefan Institute)
António Morais (U. of Aveiro)
Miha Nemevšek (Jožef Stefan
Institute)



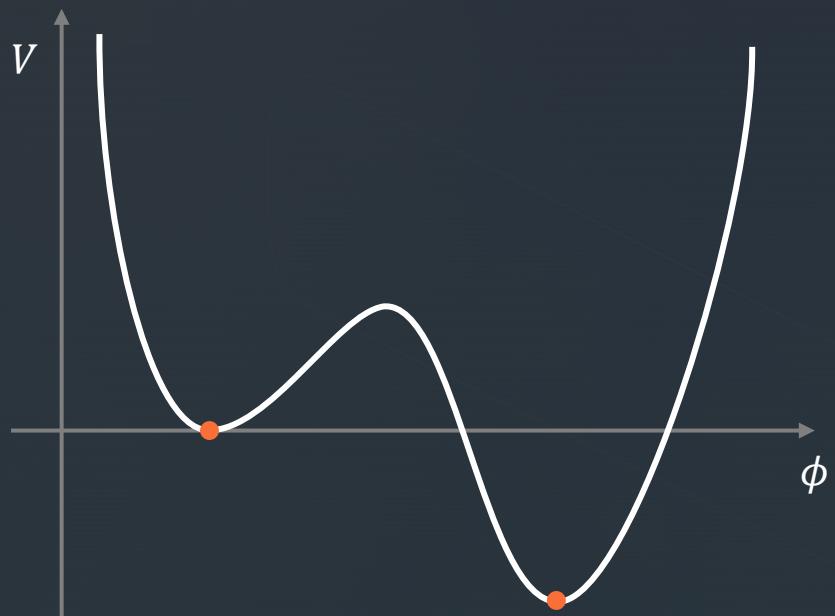
Universidade de Aveiro
theoria poesis praxis



Cosmological Phase Transitions & Single-Field Models

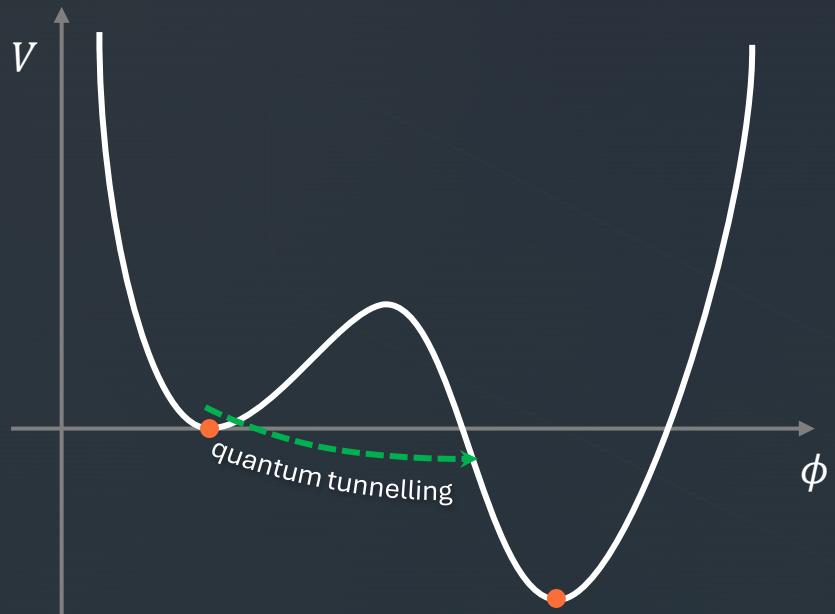
Cosmological Phase Transitions & Single-Field Models

I order phase transitions (FOPTs)



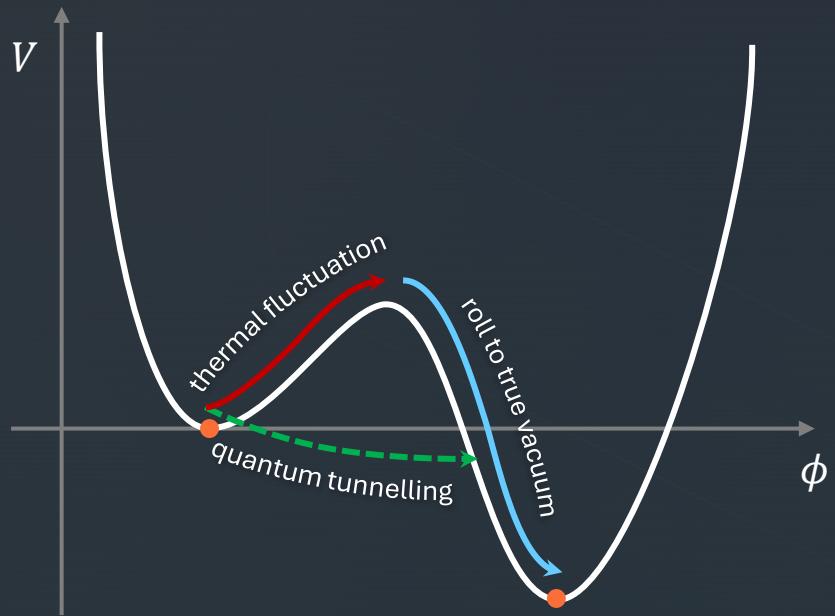
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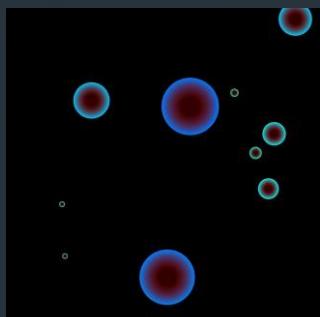
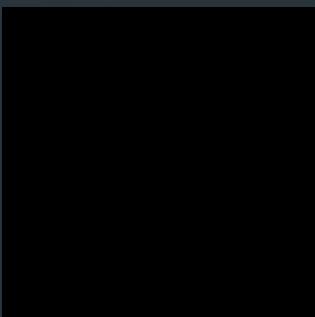
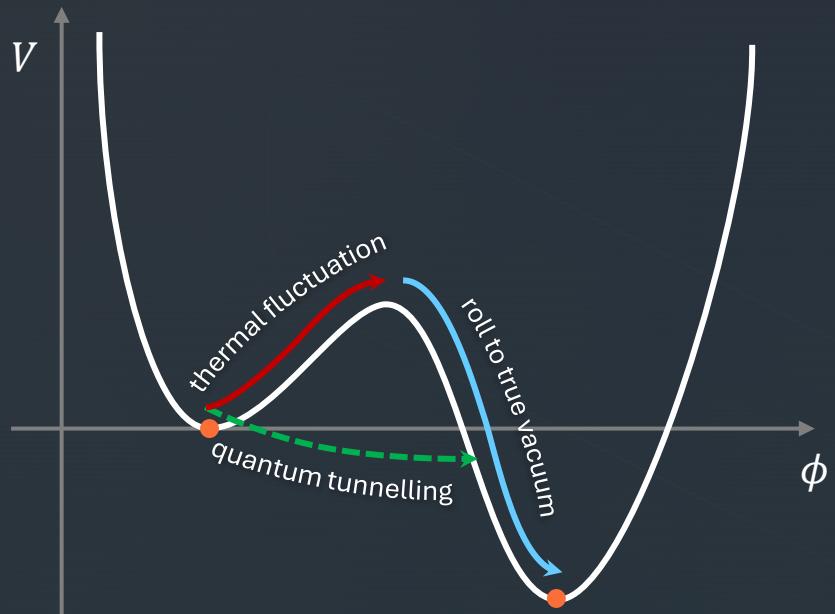
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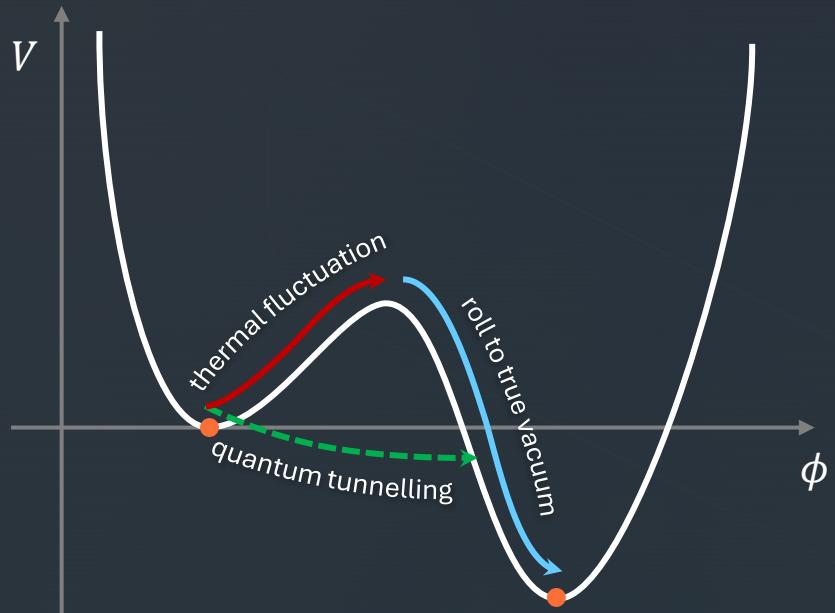
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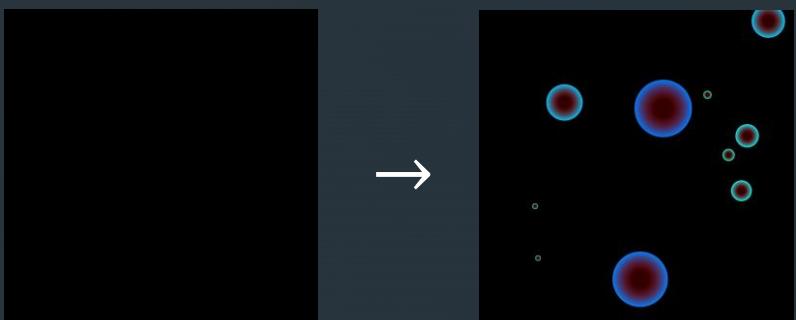


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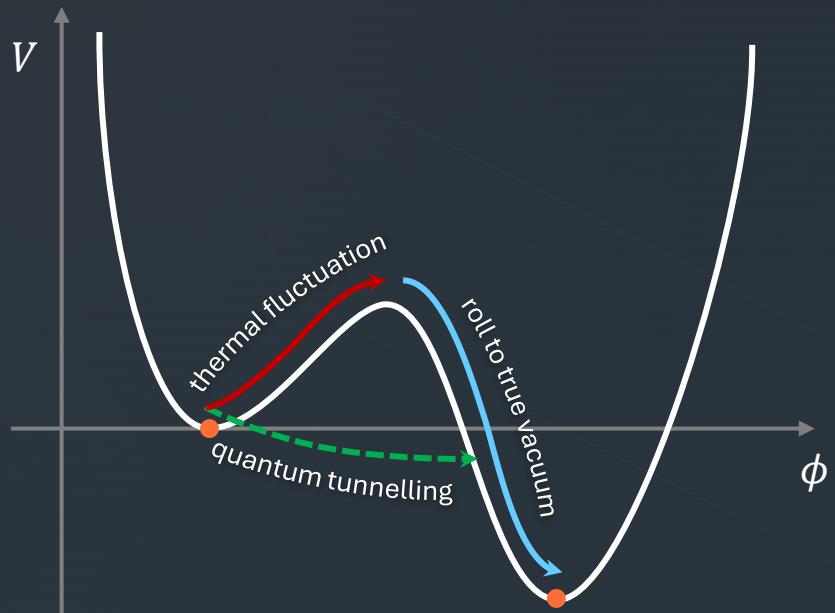


BSM physics



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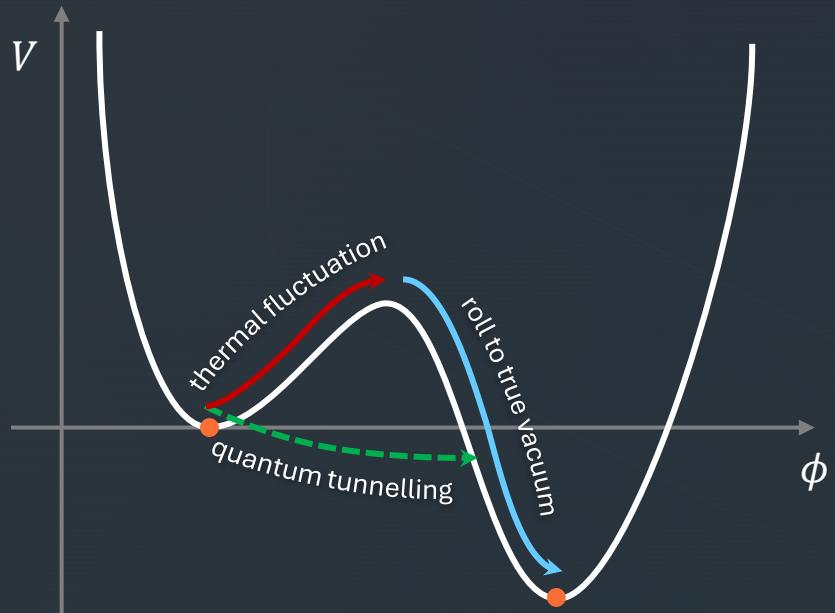
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- EW baryogenesis



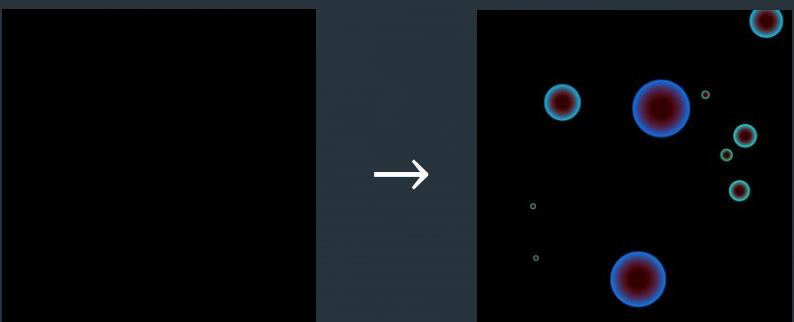
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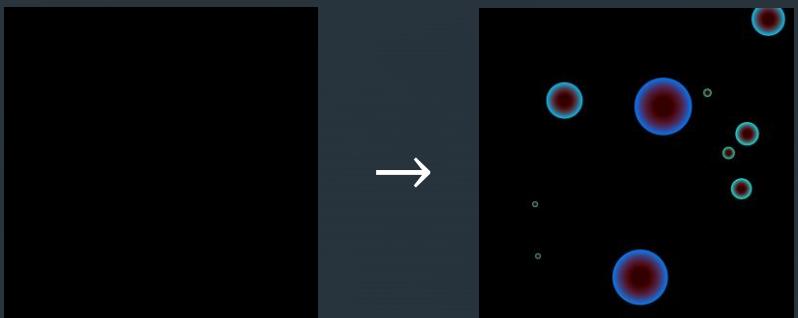
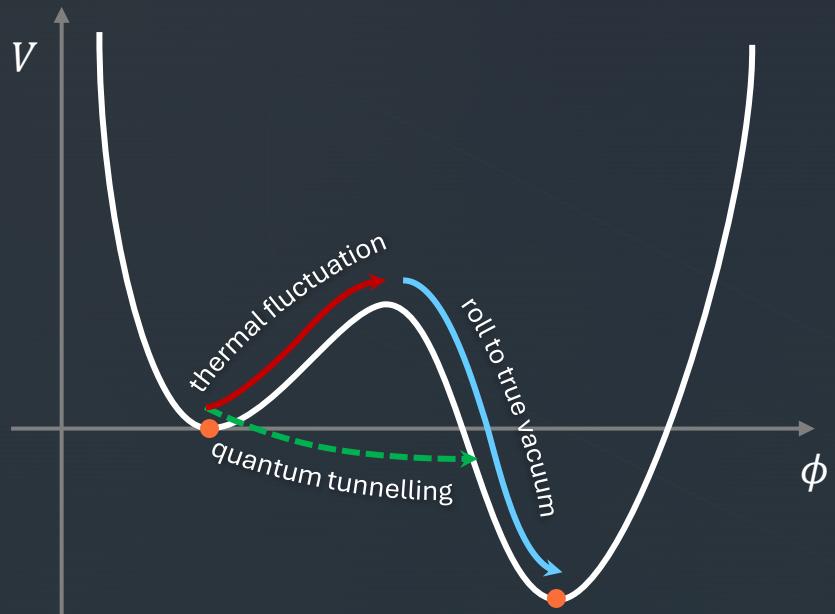
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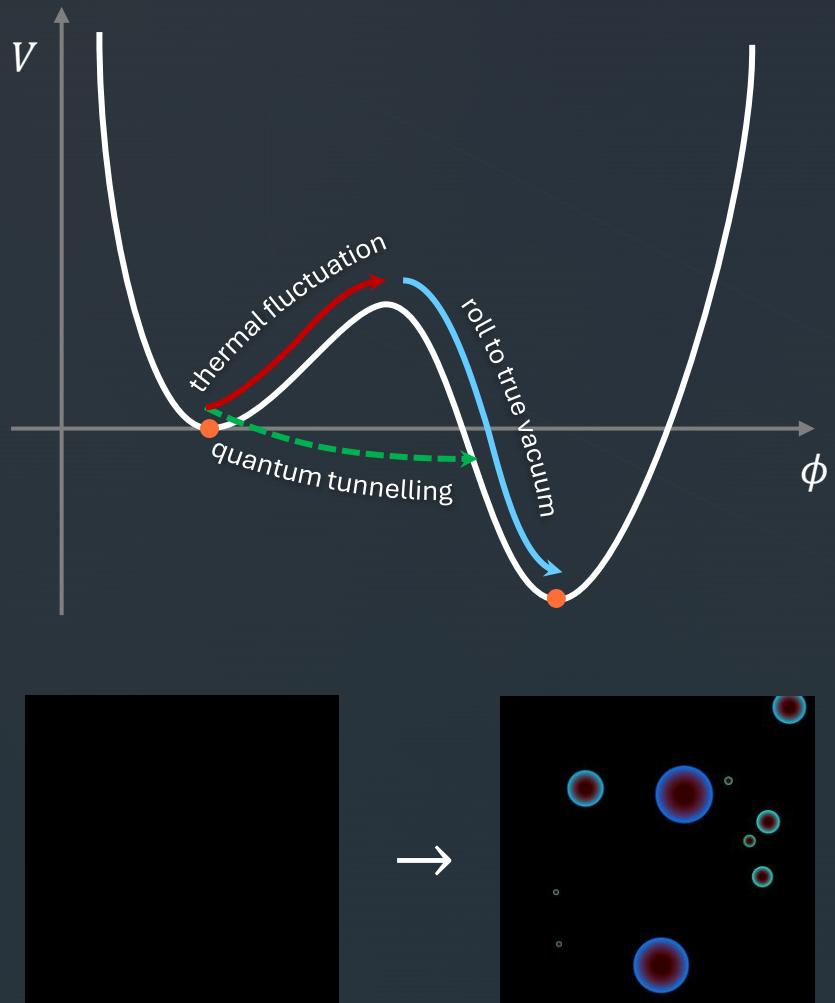


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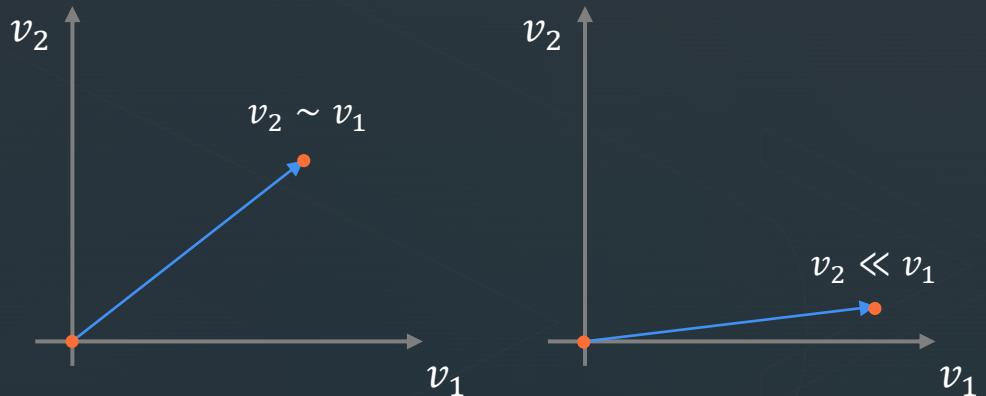
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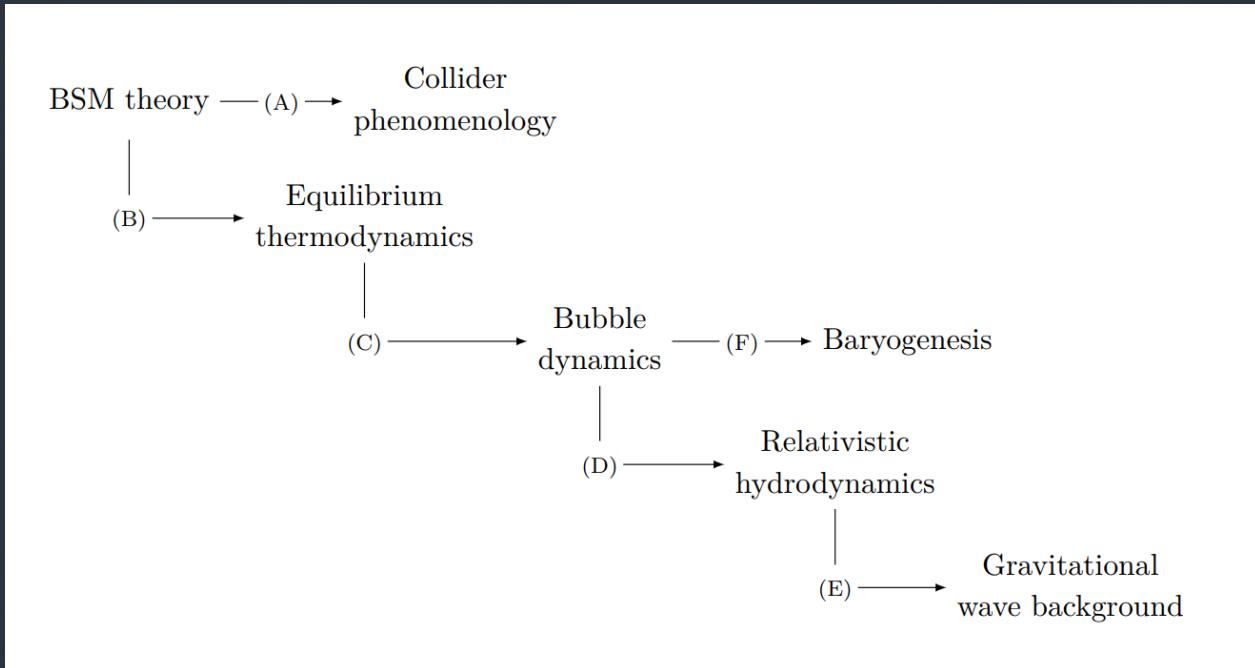
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- Focus on single-field
- strong vev hierarchy
⇒ single-field approximation ✓



BSM → GW
A pipeline

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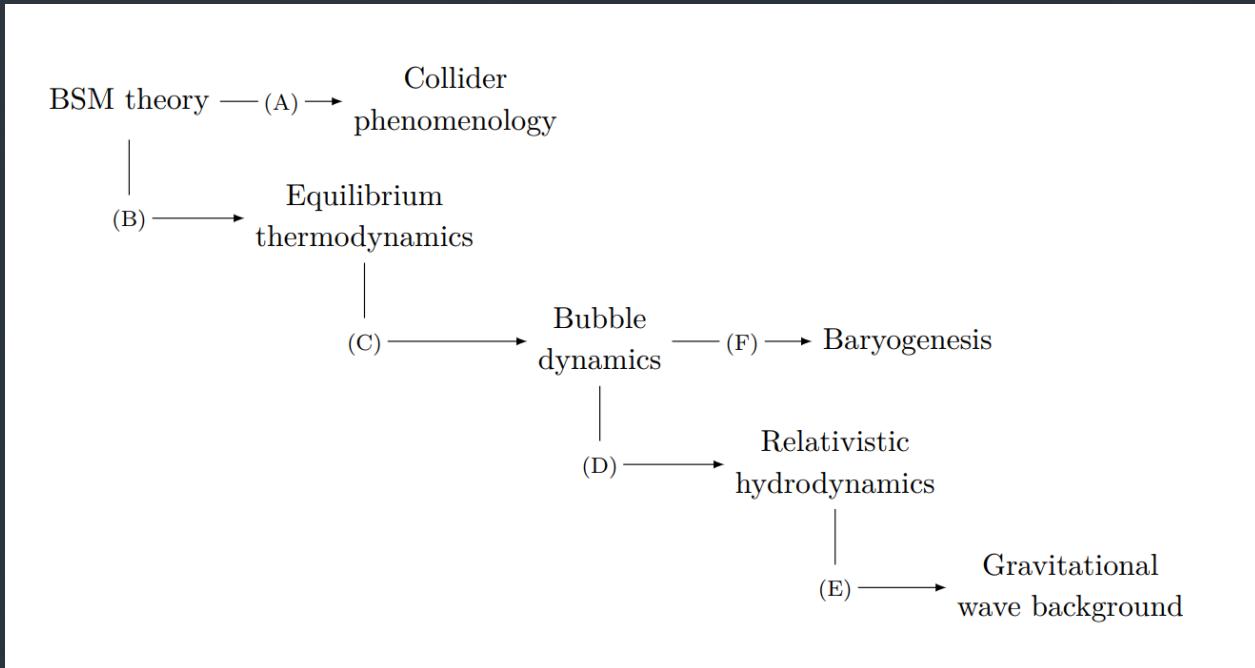


P.M. Schicho, T.V.I. Tenkanen and J. Österman (JHEP06(2021)130)

BSM → GW

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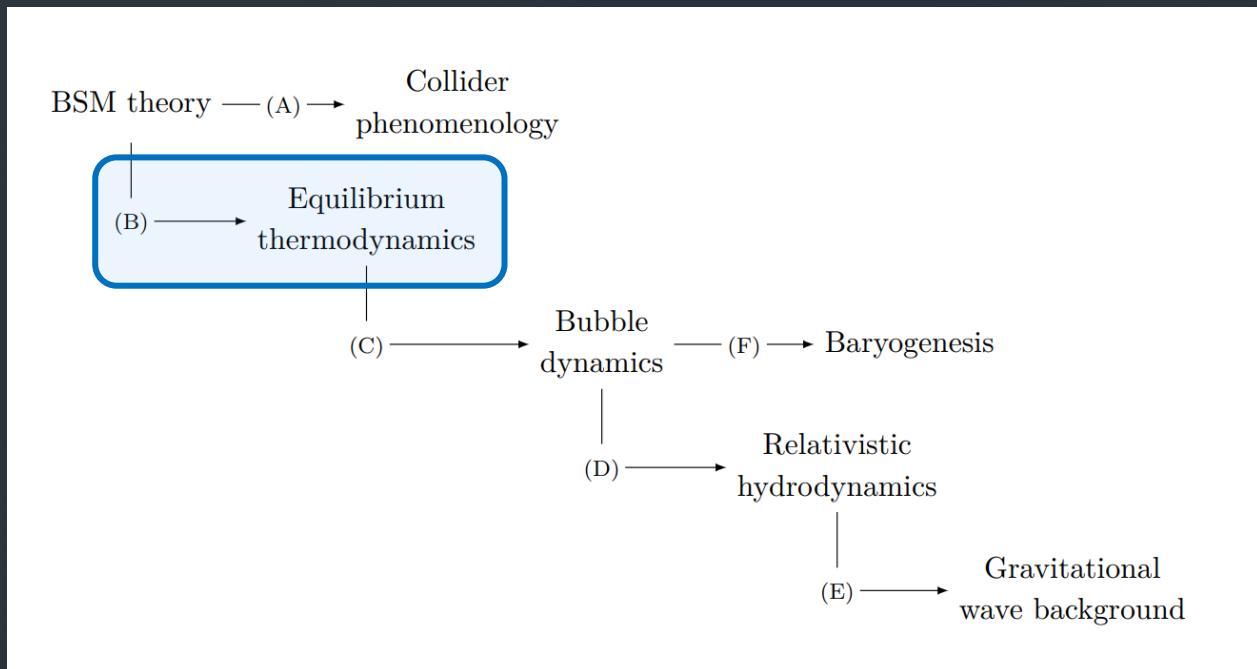
GWCalc Palet



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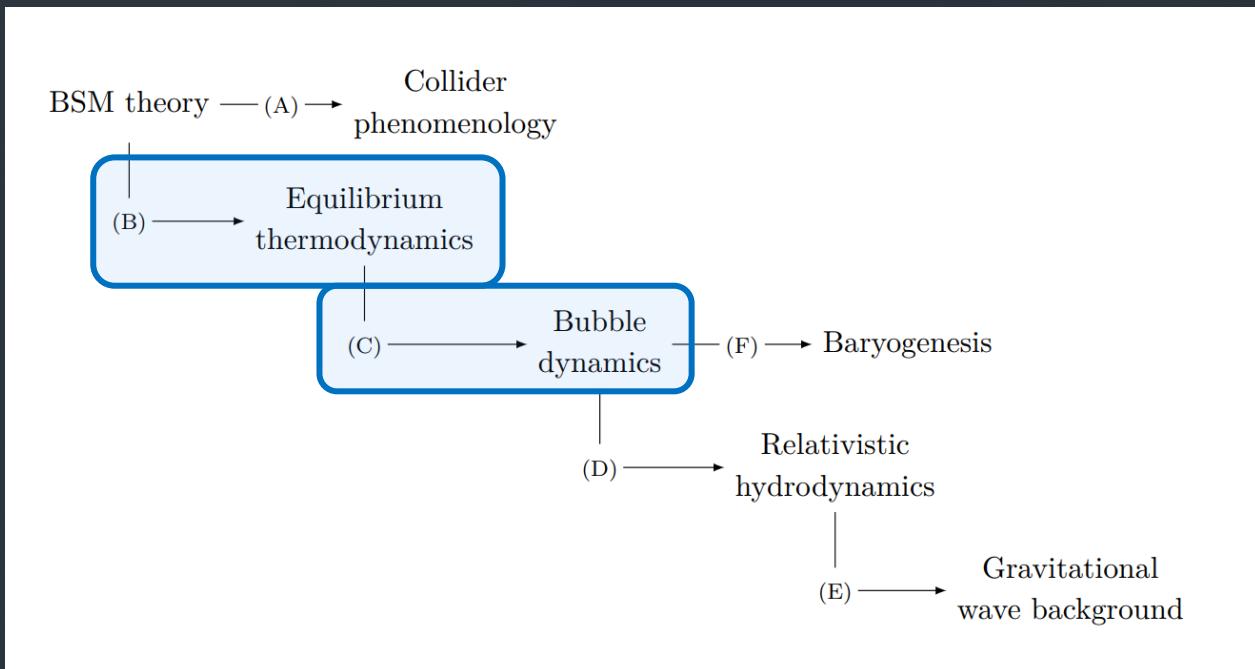
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B. Characterize PTs

- critical temperature
- 1st, 2nd order, cross-over

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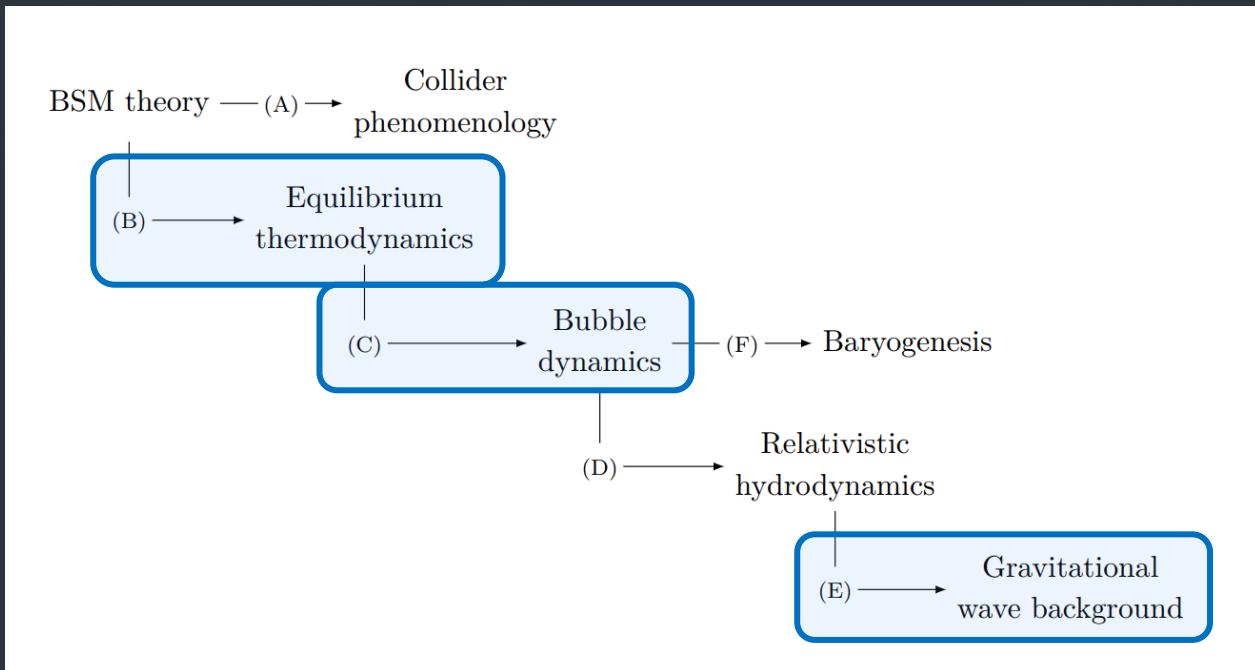
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- T_n, T_p
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FindBounce + action fit

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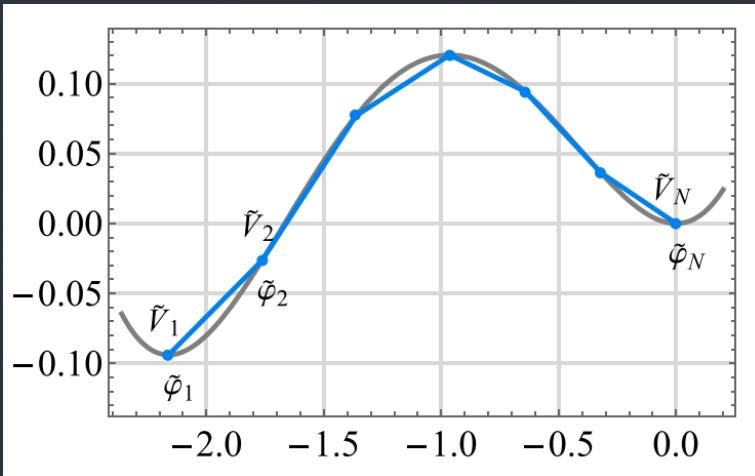
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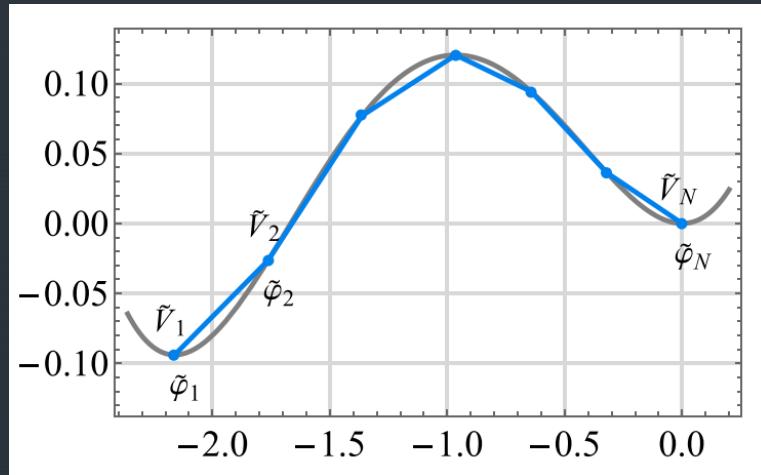
Paclet
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Guada, Nemevšek, Pintar ([CPC 256 \(2020\) 10748](#))

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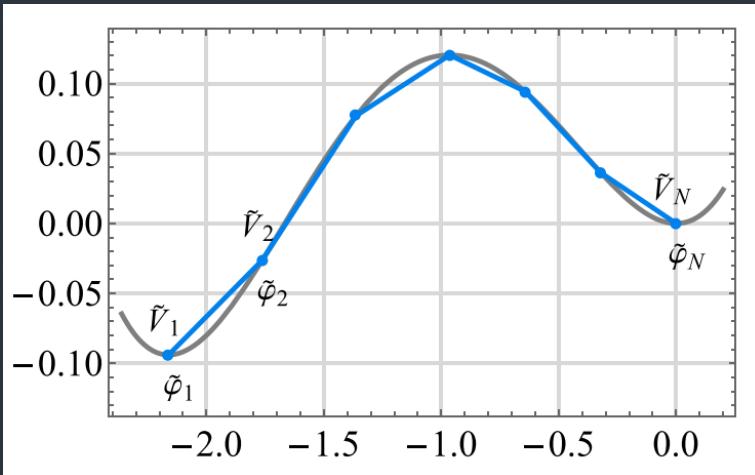
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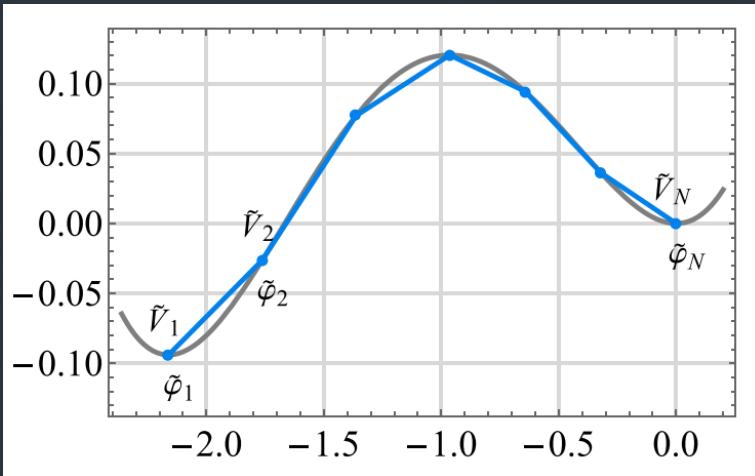
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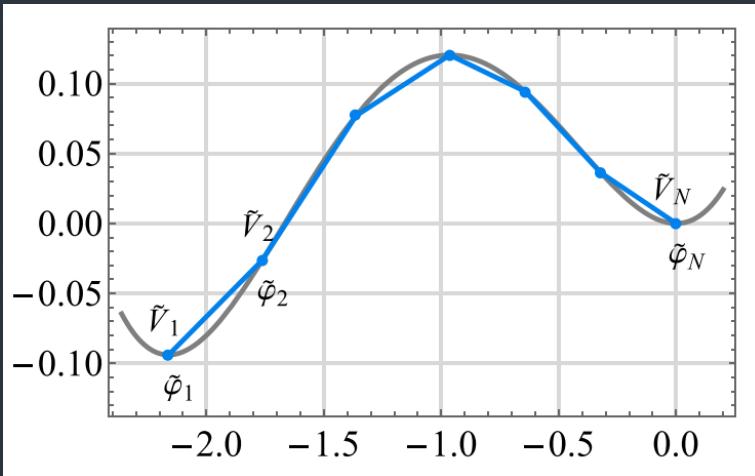
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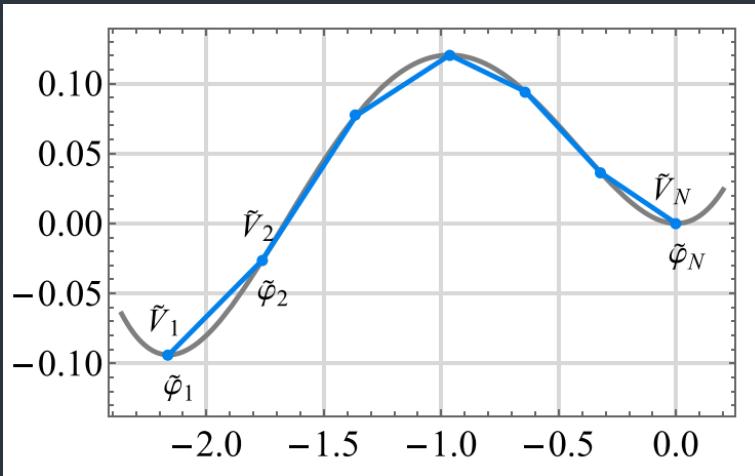
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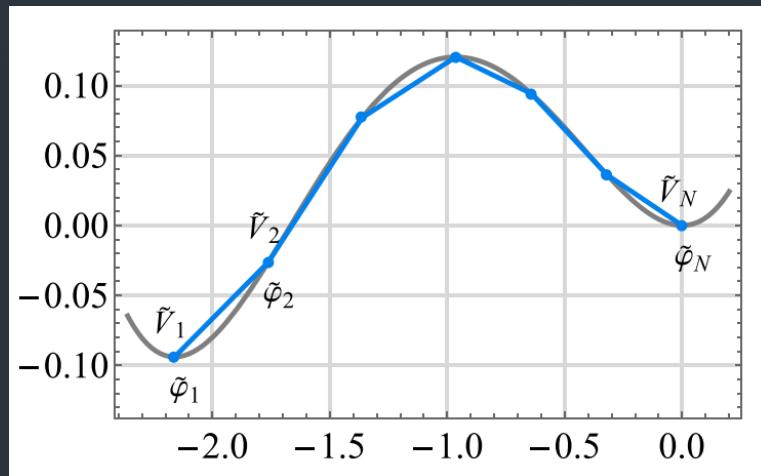
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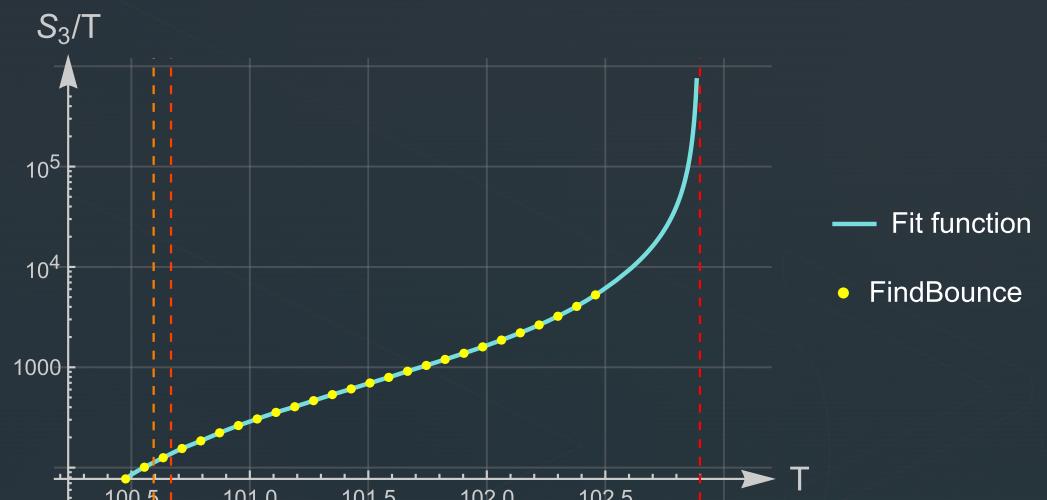
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Paclet
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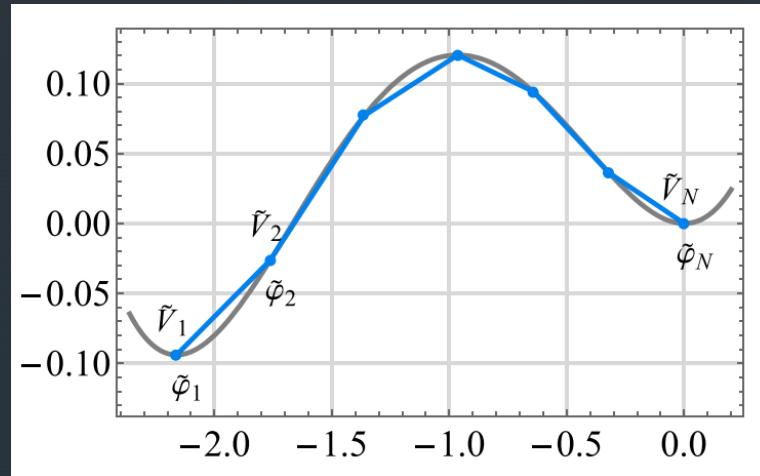


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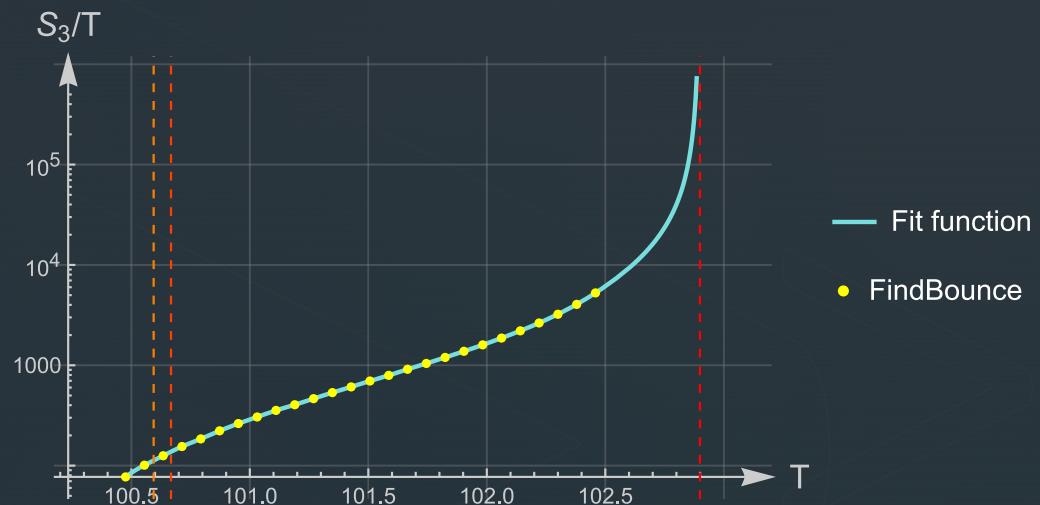


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 3. T_n, T_p via above integrals

Paclet
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Guada, Nemevšek, Pintar (CPC 256 (2020) 10748)



Example I

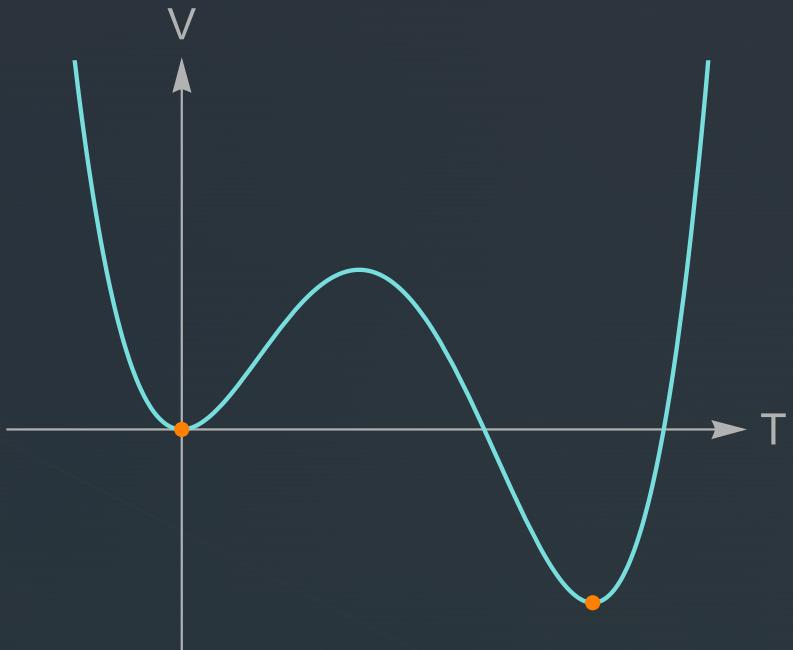
Fluid-field model

Example I

Fluid-field model

- Scalar potential

$$V(\phi, T) = \frac{c_2}{2}(T^2 - T_0^2)\phi^2 - \frac{c_3}{3}T\phi^3 + \frac{c_4}{4}\phi^4$$



Linde (1983, NPB 216. 2)

Hindmarsh, Huber, Rummukainen, Weir (PRD.92.123009)

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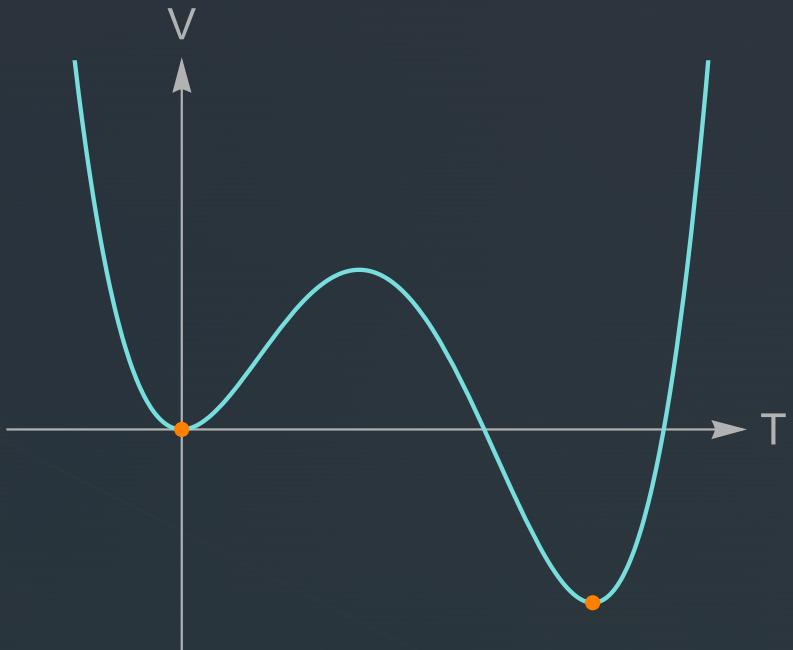
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- Analytic derivation of the action
 - in thin/thick wall regimes
 - intermediate interpolation

Matteini, Nemevšek, Shoji, Ubaldi (2024, [2404.17632](#))



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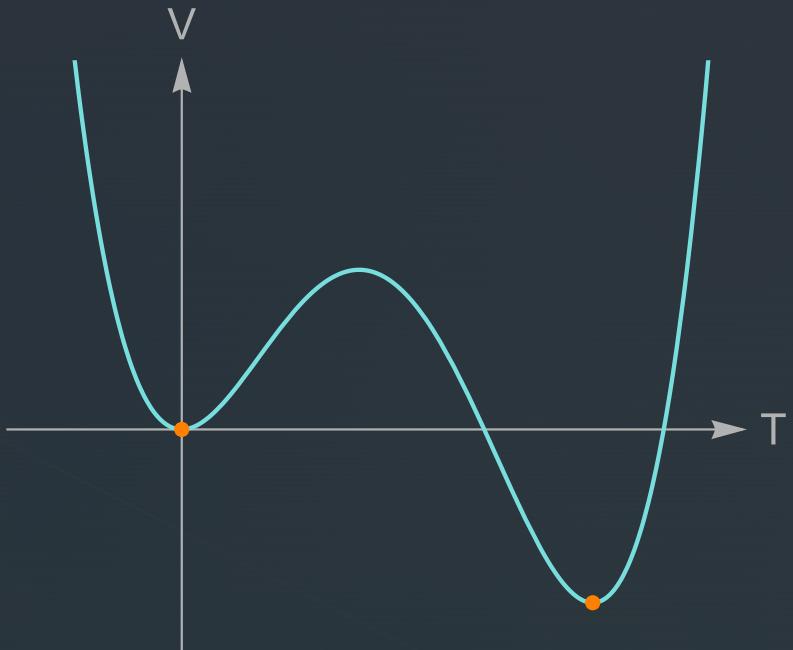
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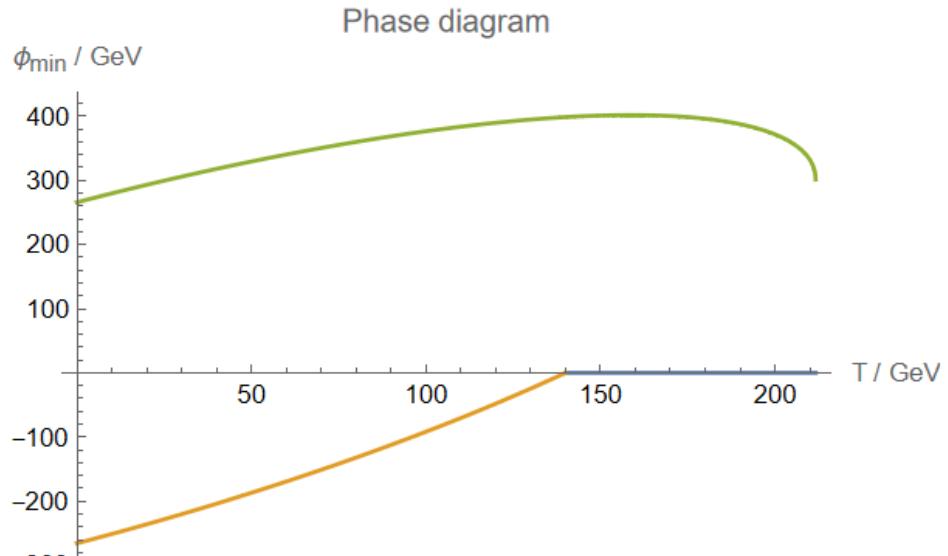
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Example I Fluid-field model

```
In[71]:= Trs = TBounce[V, vw, "TracingMethod" → NSolve,
  "PlotAction" → True, "PlotGWSpectrum" → True]
```

Determining phase structure



Looping over pairs of phases

Found transition at critical temperature

» $T_c \rightarrow 197.99$

Computing nucleation temperature via $\Gamma/H^4 \approx 1$ criterion and bisection method...

» $T_n^{\text{estimate}} \rightarrow 170.703$ $S_3/T = 148.609$ $\Gamma/H^4 = 1.00114$

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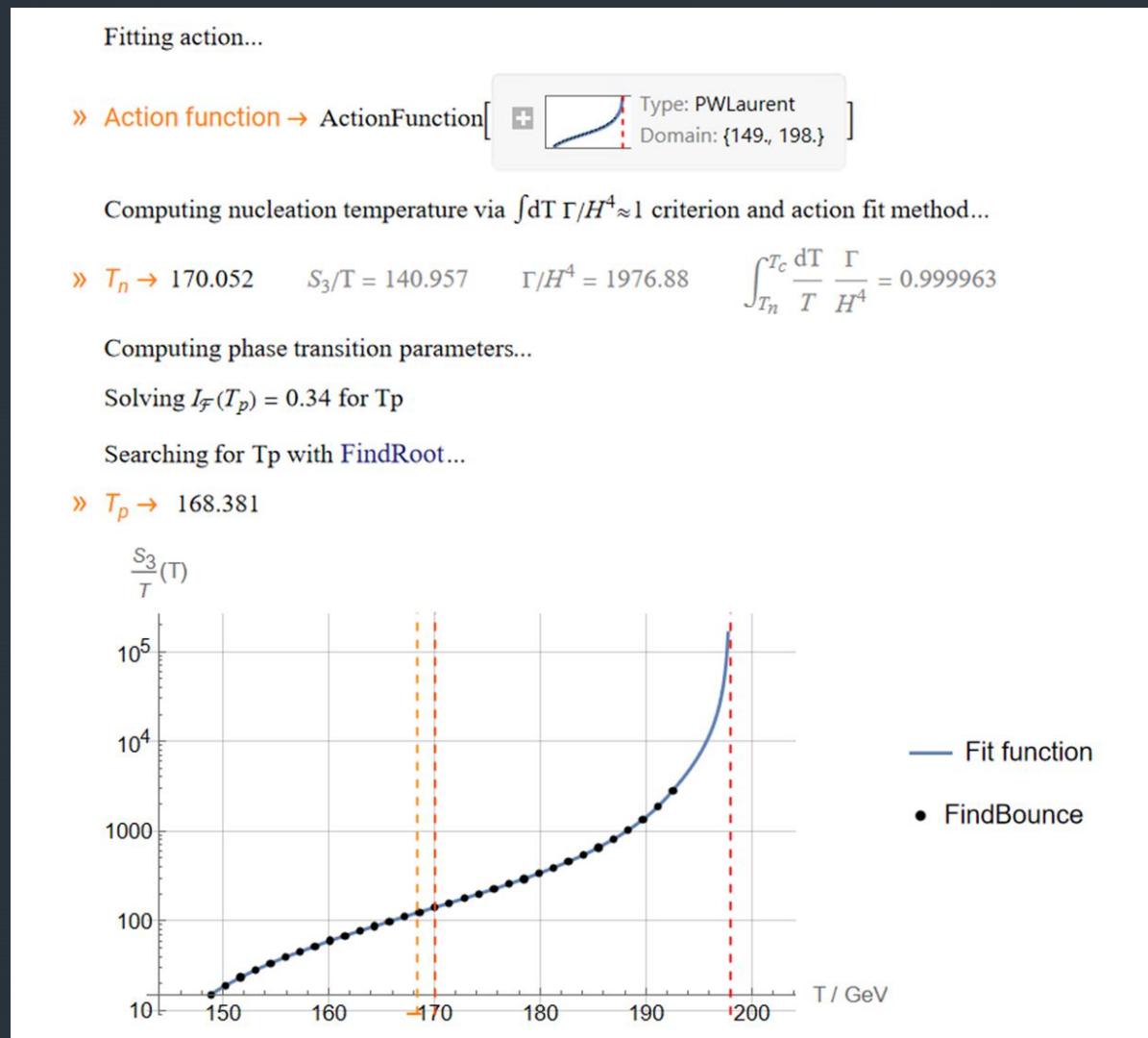
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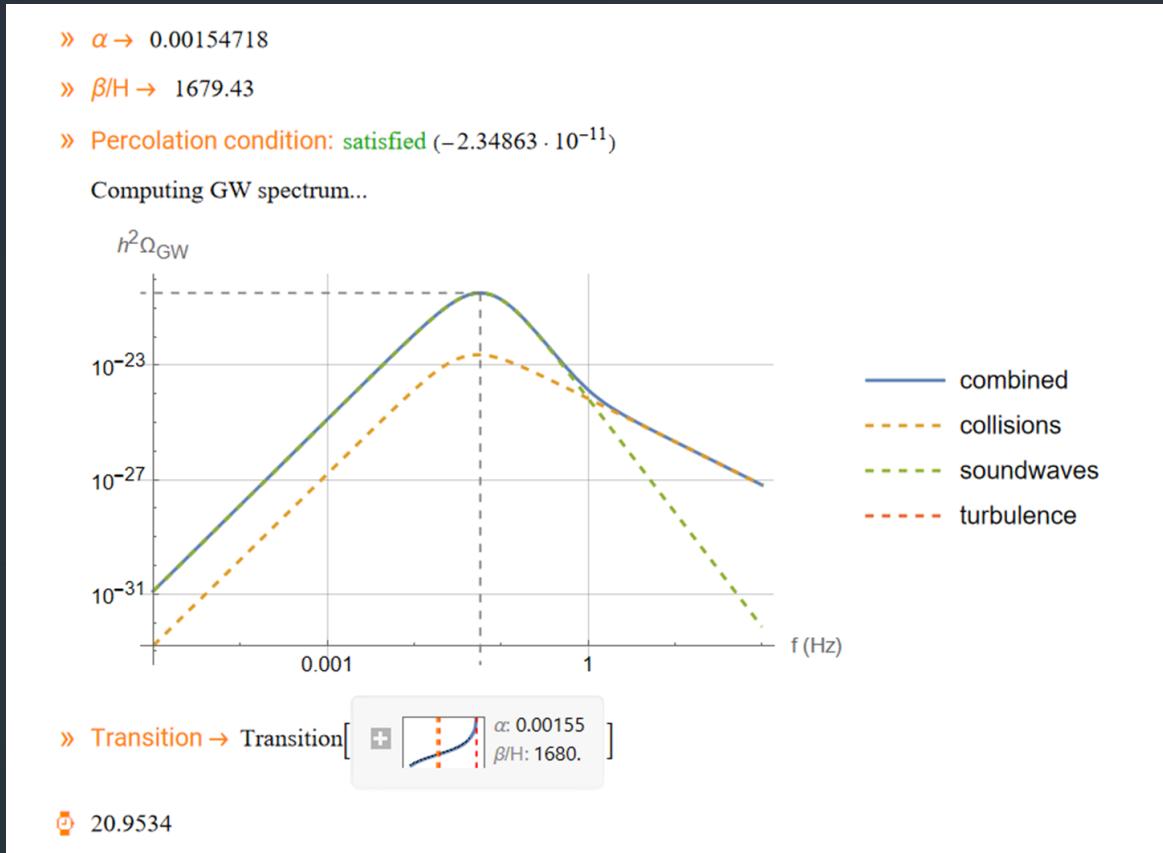
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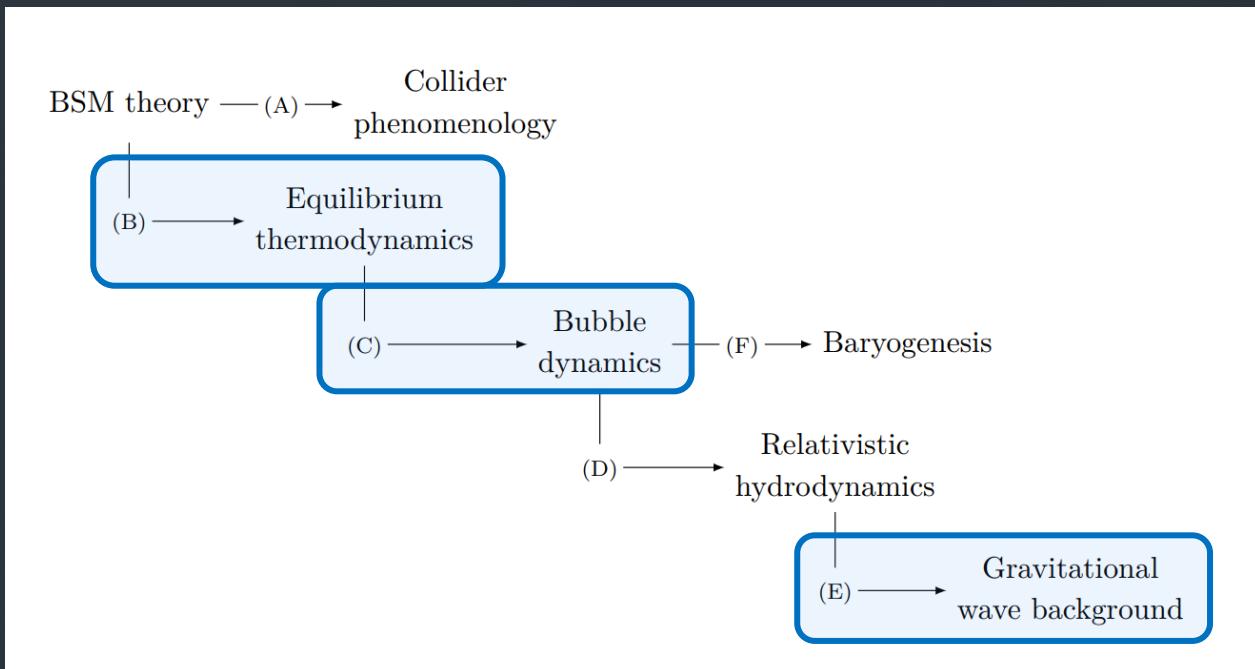
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$\text{BSM} \rightarrow \text{GW}$

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P.M. Schicho, T.V.I. Tenkanen and J. Österman (JHEP06(2021)130)

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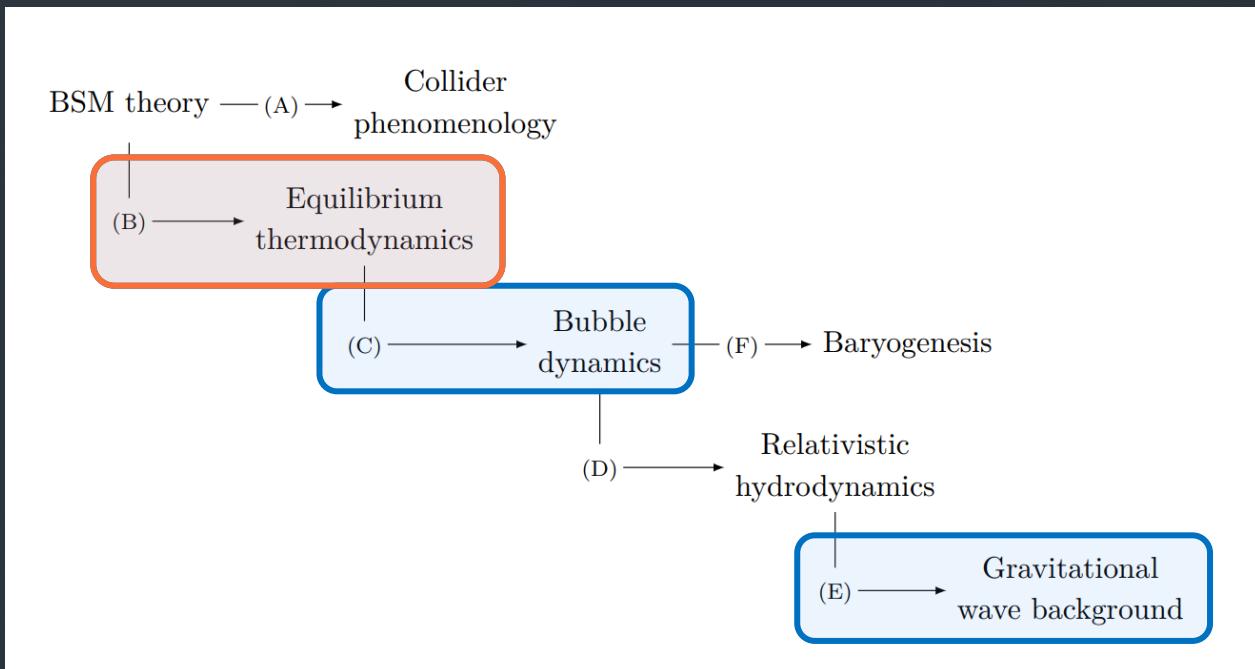
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B. Dimensional reduction

- Interface with DRalgo

Dimensional Reduction

An improved recipe for thermal EFTs

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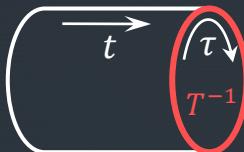
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$$4d \xrightarrow{t \rightarrow t+i/T} 3d \text{ EFT}$$

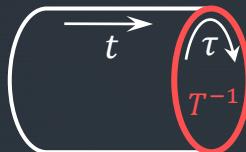


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 - include systematically higher-order resummations

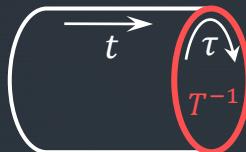
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Dimensional Reduction An improved recipe for thermal EFTs

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- Narrower theoretical uncertainties

$$4d \xrightarrow{t \rightarrow t+i/T} 3d \text{ EFT}$$

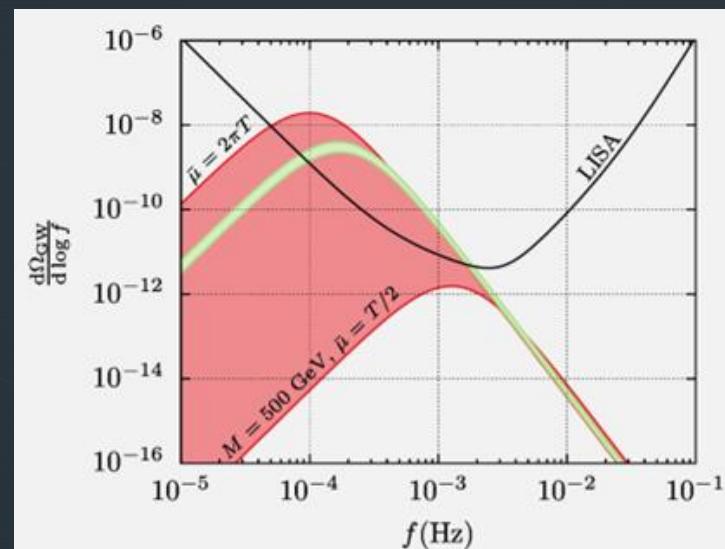
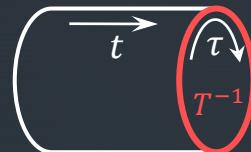


Dimensional Reduction

An improved recipe for thermal EFTs

- Dimensional reduction (DR)
 - time \rightarrow temperature \Rightarrow high- T approach
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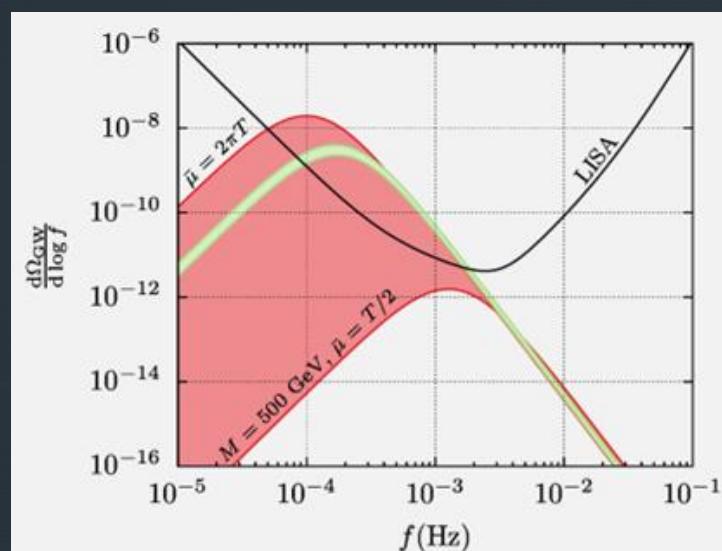
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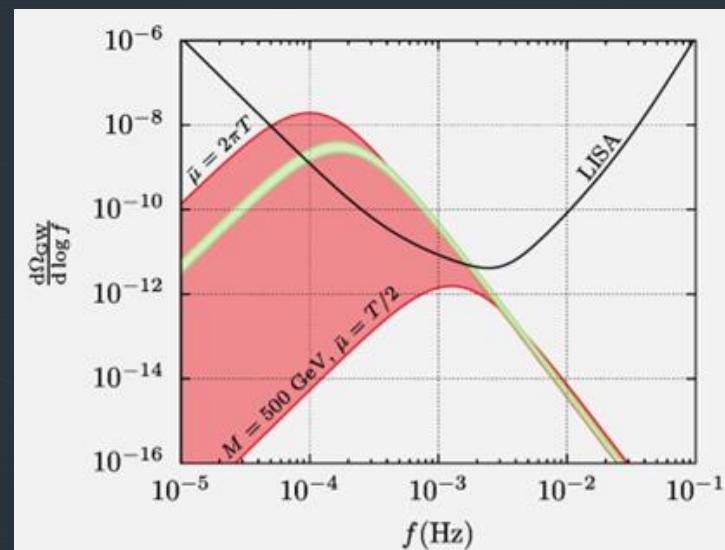
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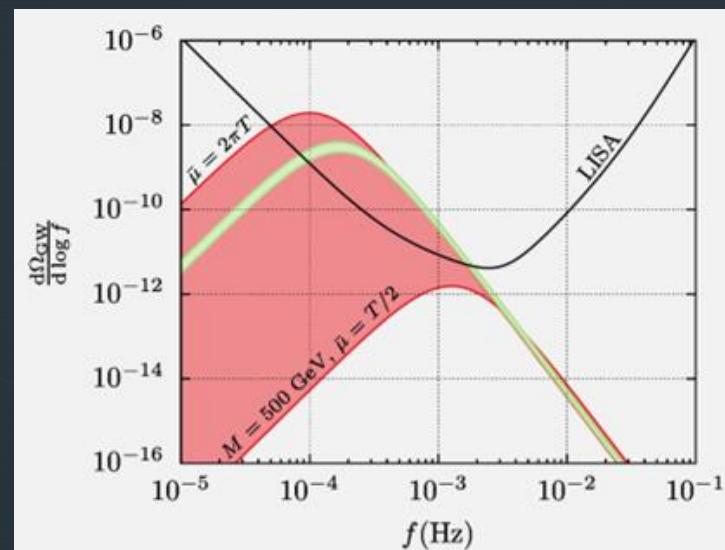
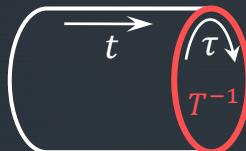
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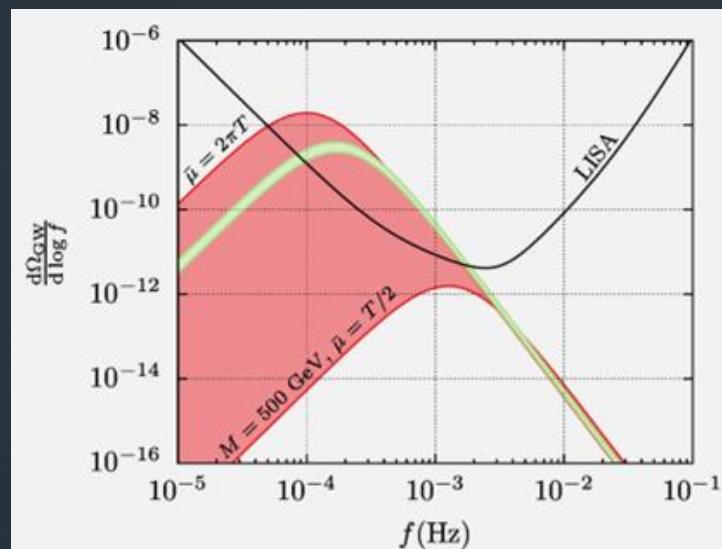
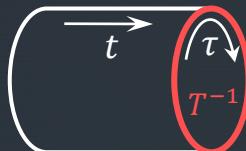
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 - RG resolution
 - closed-form $V_{\text{eff}}(\phi, T)$

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Example II

Dark photon model

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Dark photon model

- Dark $U(1)$ gauge sector

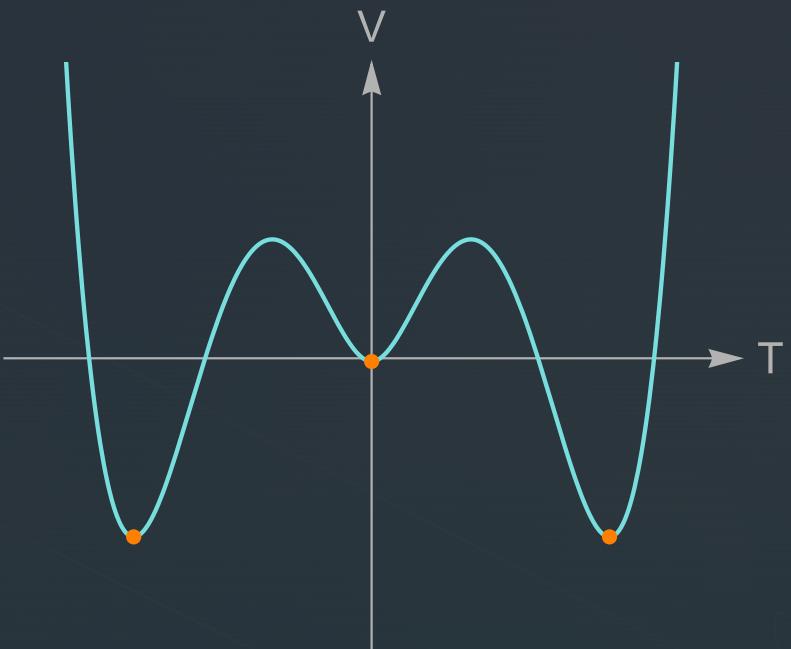
Example II

Dark photon model

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- Scalar content:

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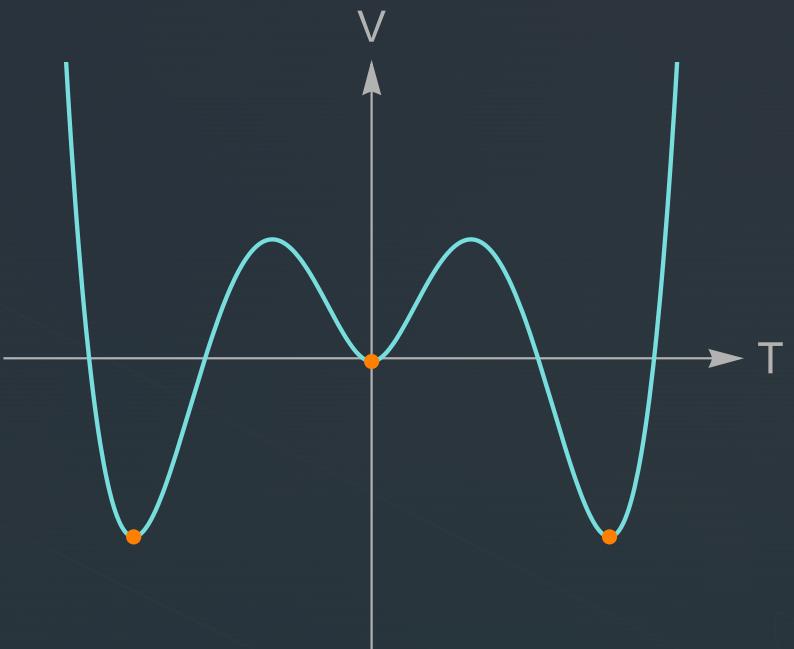
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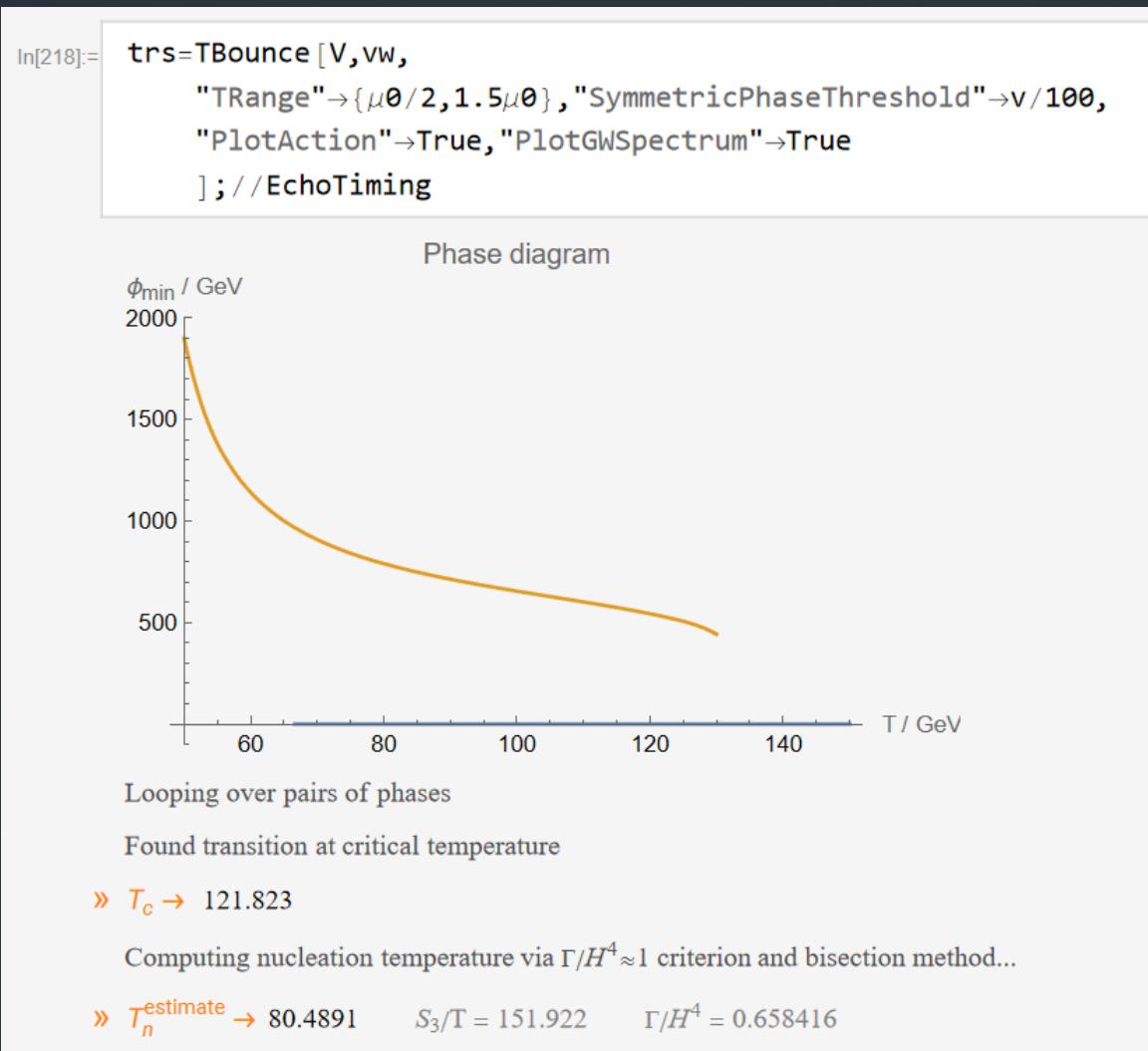
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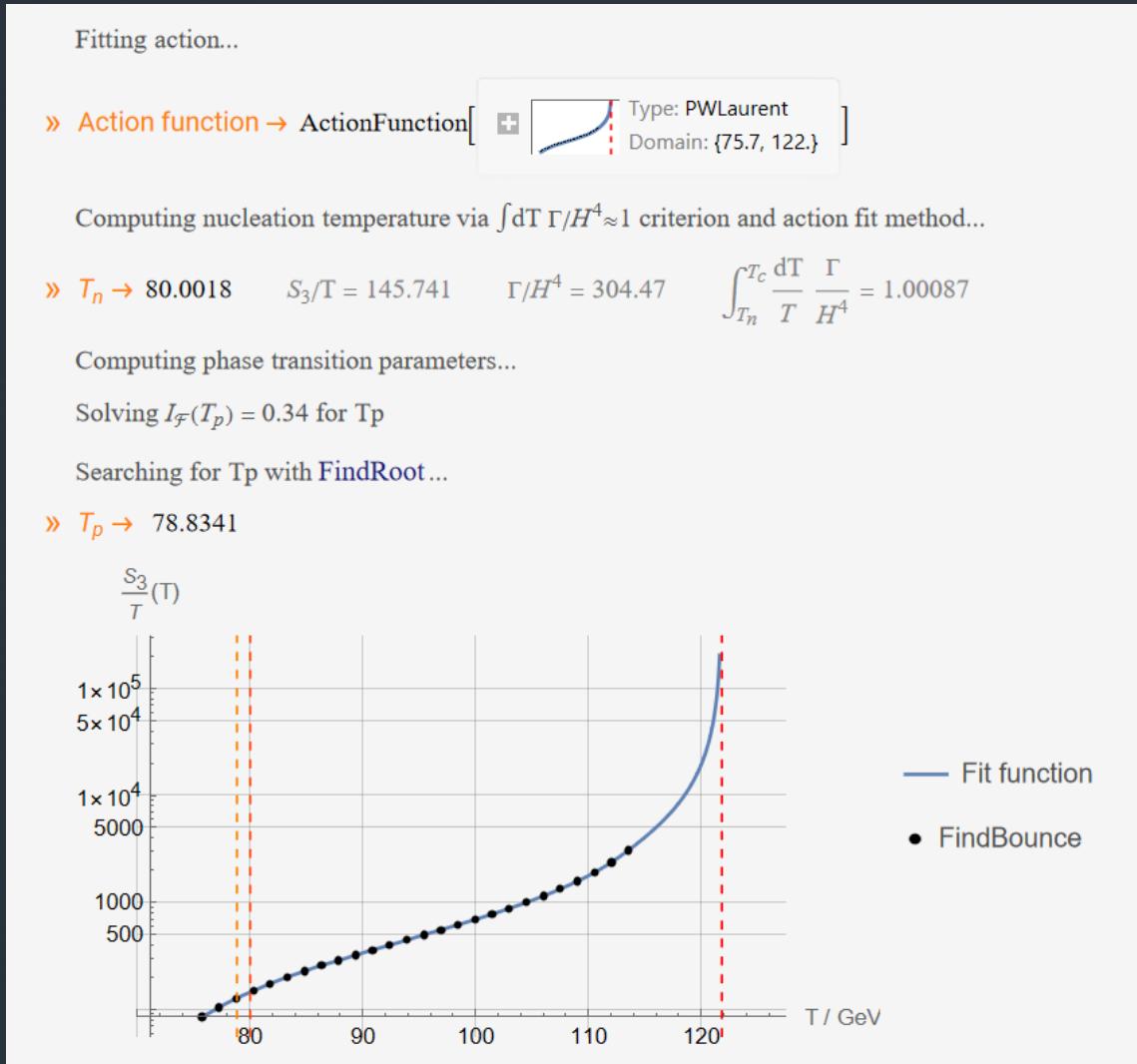


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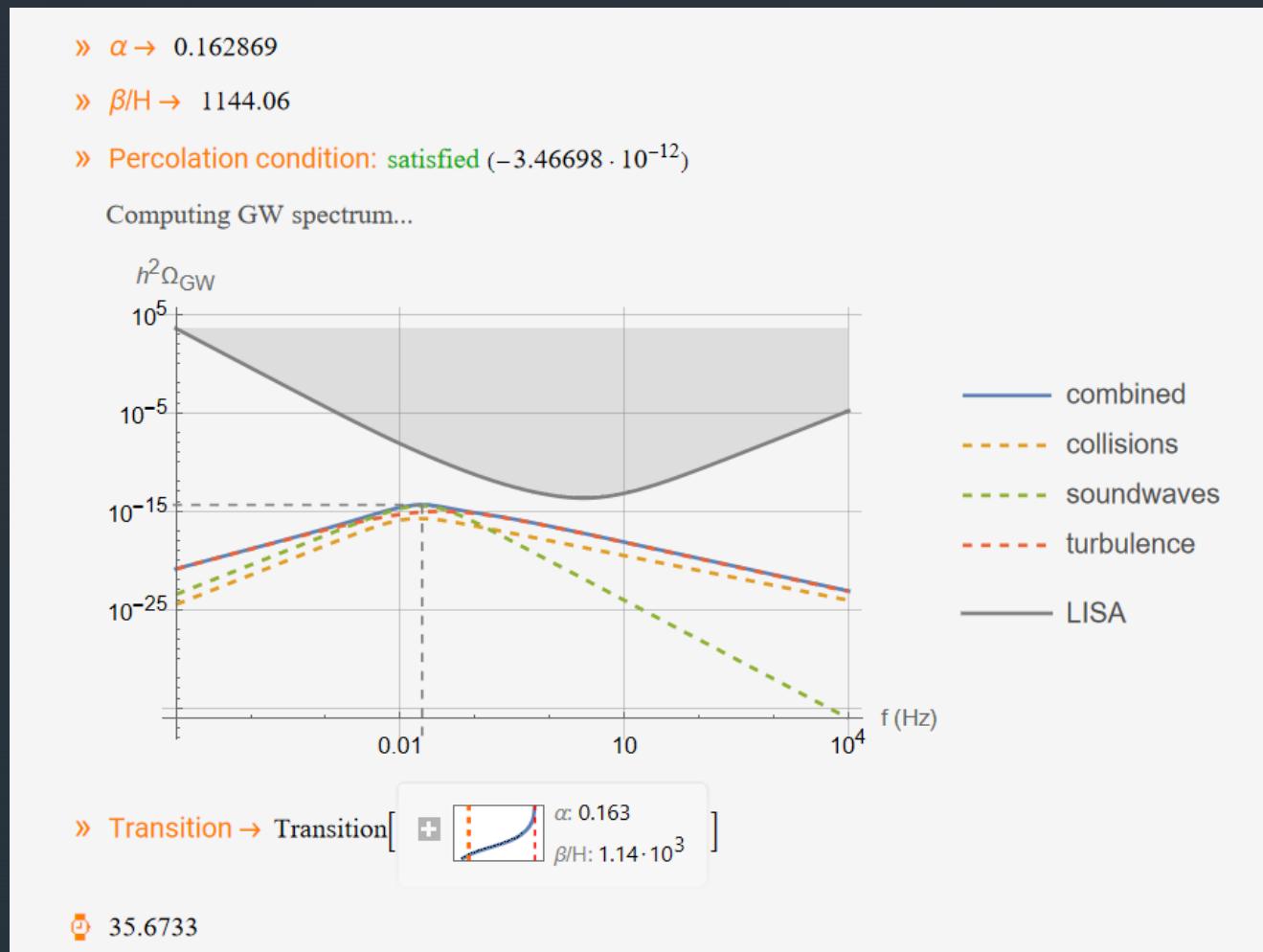
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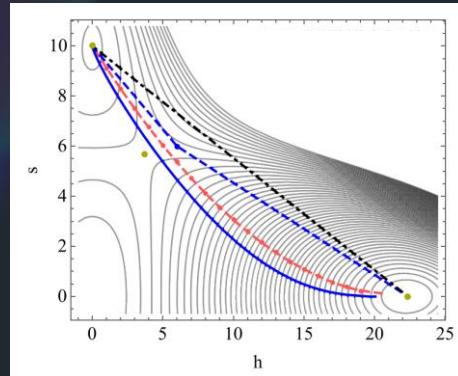
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Outcome & Future Endeavours

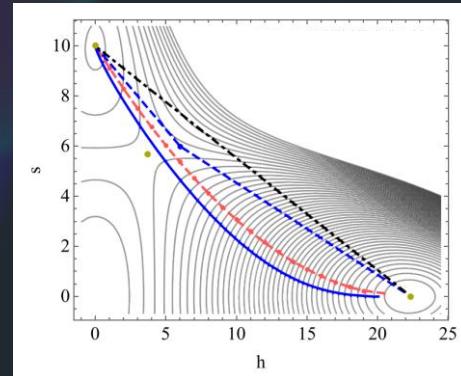
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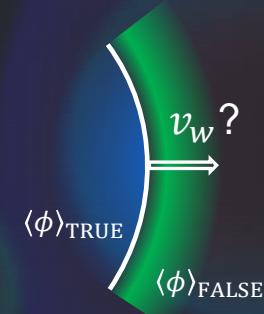


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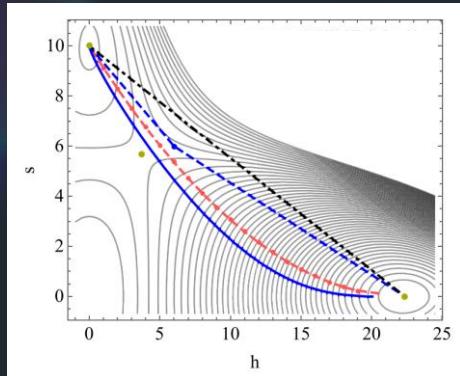


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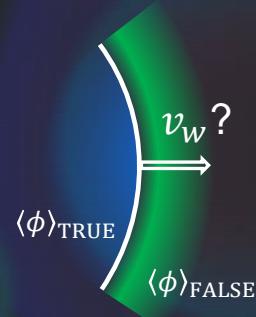


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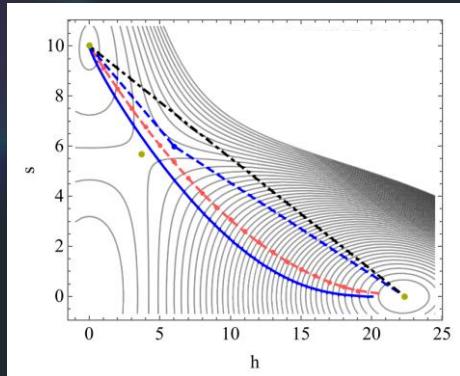


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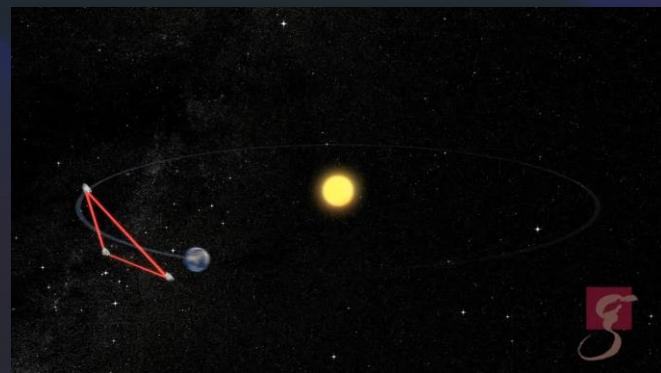


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Suggestions are welcome!

EW Baryogenesis

The matter-antimatter problem

- Fundamental problem: baryon asymmetry

Sakharov conditions (1967)

1. B-number violation

2. C & P violation

3. Departure from
 T -equilibrium

SM

✓ → non-perturbatively

✓ → weakly

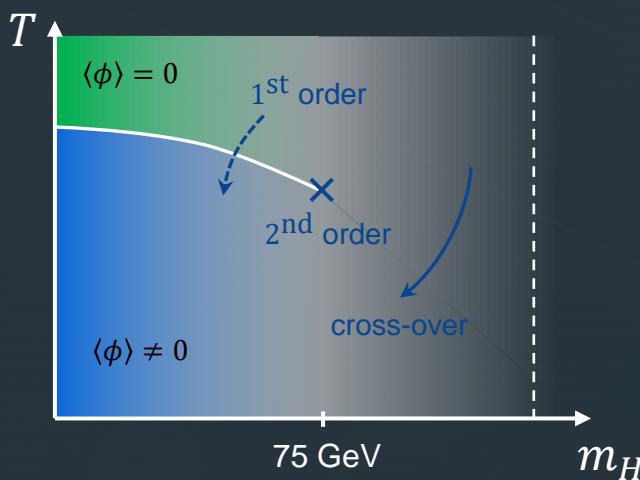
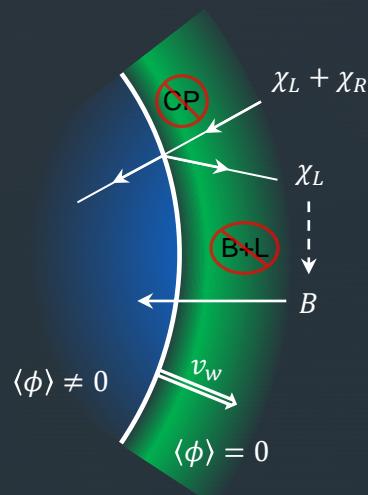
\times → cross-over

LQ Model

✓ → LQs acquire vev

✓ → potential

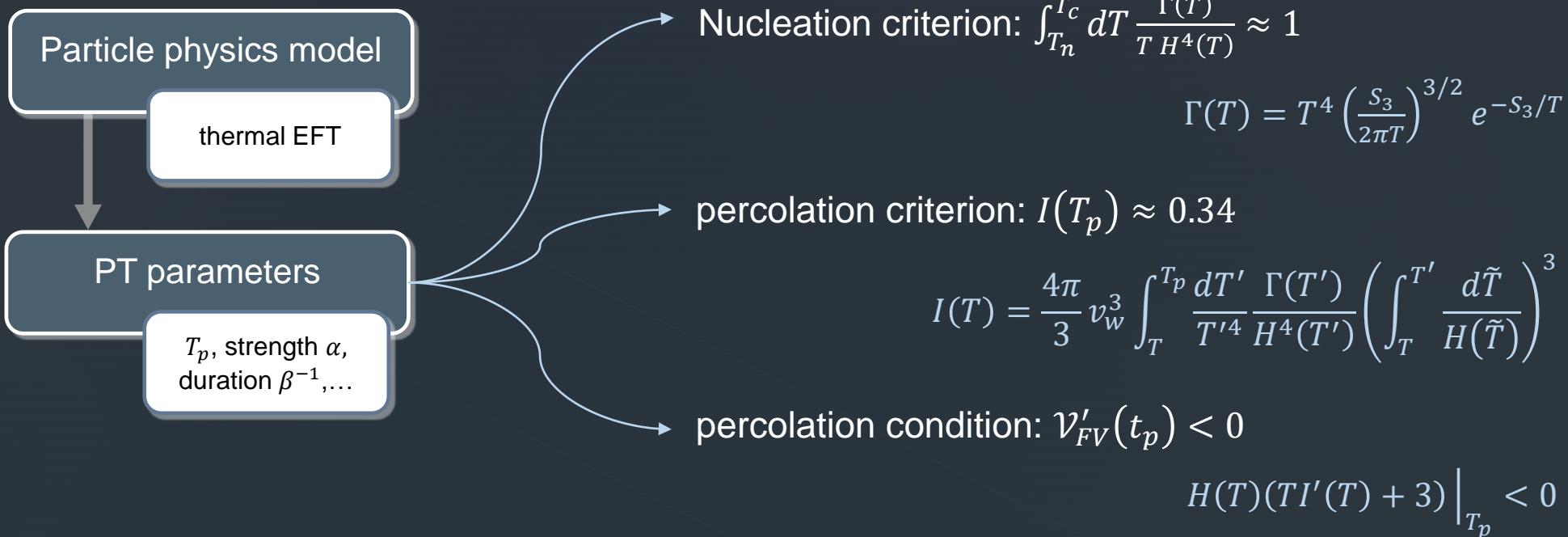
✓ → strong FOPTs



BSM physics
required!

Outcome

From Particle Physics to Cosmology



- **strength** $\alpha = \frac{1}{\rho} \Delta \left[V - \frac{T}{4} \partial_T V \right]$
- **duration⁻¹** $\frac{\beta}{H} = T \frac{d}{dT} \left(\frac{S_3}{T} \right)$

EW Baryogenesis

The matter-antimatter problem

```
In[5]:= SetDirectory[NotebookDirectory[]];
LoadDRExpressions["ahDRExpressions.m"]

ComputeEffectivePotential[{gsq0, λ0, msq0}, {μ0, μ0/10, 100 μ0},
  subRules, "OrderVeff" → "NLO", "LoadDRFrom" → "ahDRExpressions.m"]

gsq → InterpolatingFunction[ +  Domain: (10. 1.00·104) ]
Output: scalar
```

» RG solutions $\lambda \rightarrow$ InterpolatingFunction[+  Domain: (10. 1.00·10⁴)]
Output: scalar

```
msq → InterpolatingFunction[ +  Domain: (10. 1.00·104) ]
Output: scalar
```

```
In[18]:= V[ϕ, μ0]

Out[18]= -1.06103 ((53.3507 - 0.00338267 ϕ2)3/2 + (53.3507 - 0.00112756 ϕ2)3/2 + 0.000265675 ϕ4 - 25.1409 ϕ2 + 0.118862 (ϕ2)3/2)
```

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