The gravitational wave power spectrum from sound waves: Speed of sound and General Relativity beyond the leading order

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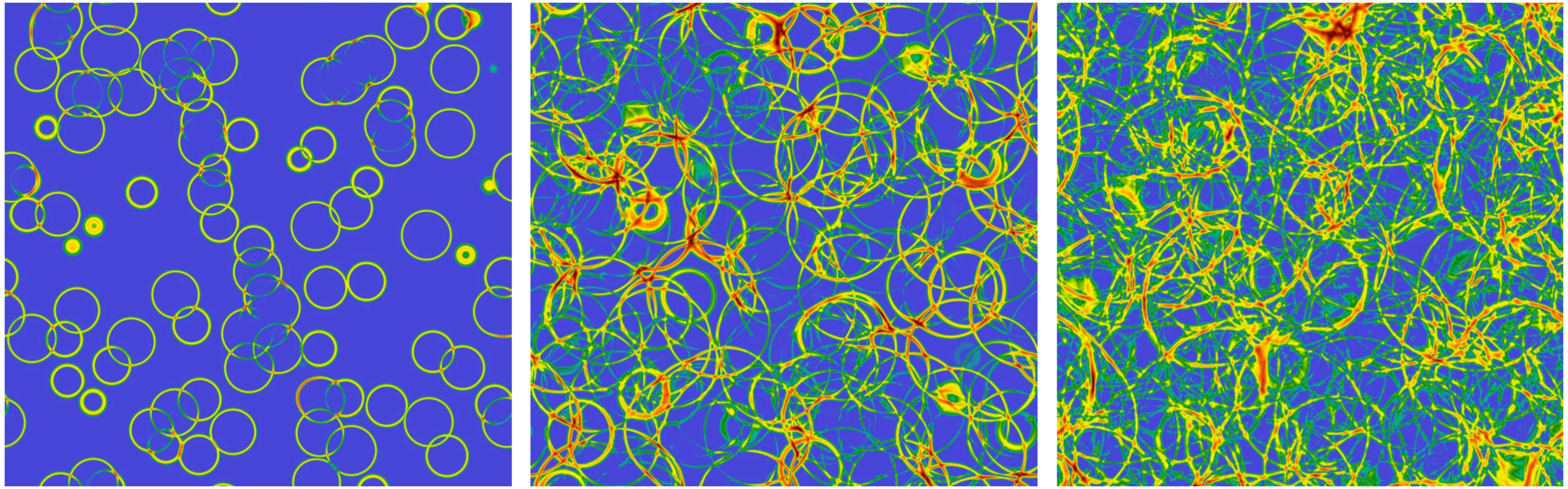
What? First order phase transitions (FOPT)

First order phase transition ⟶ formation of bubbles containing the new phase



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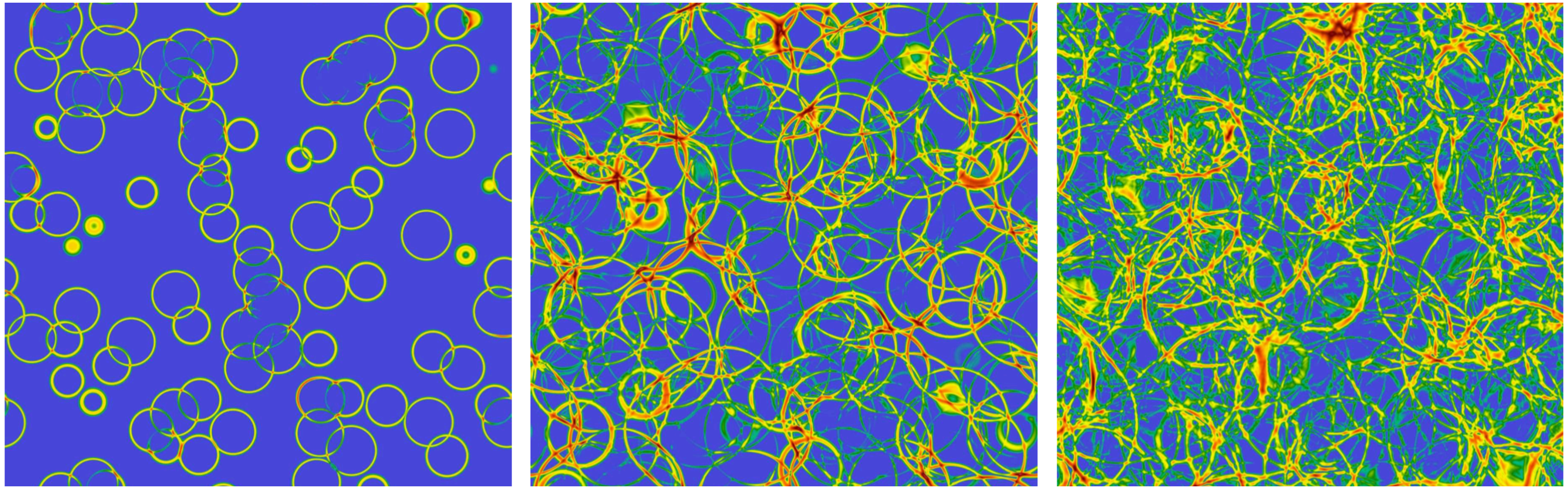

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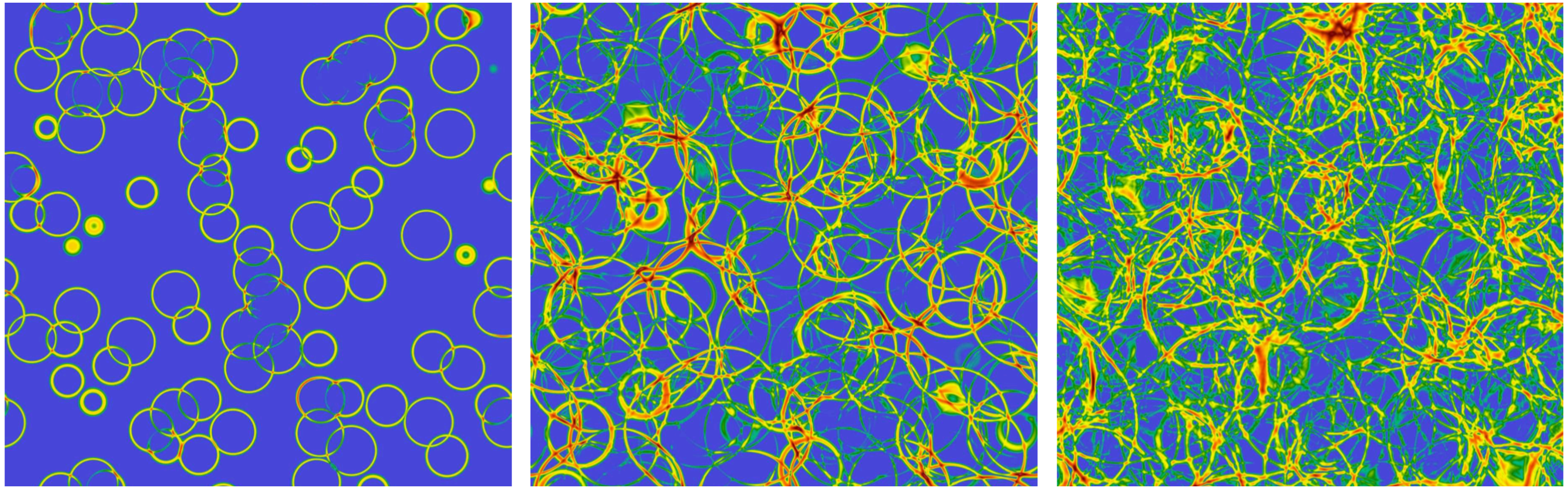

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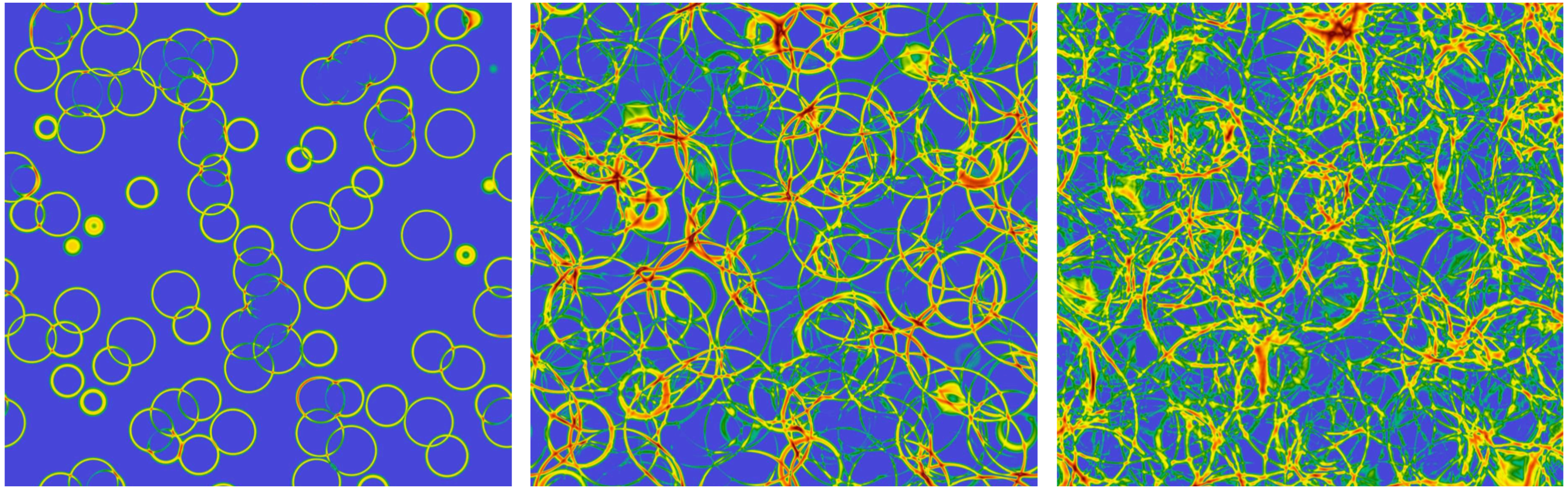

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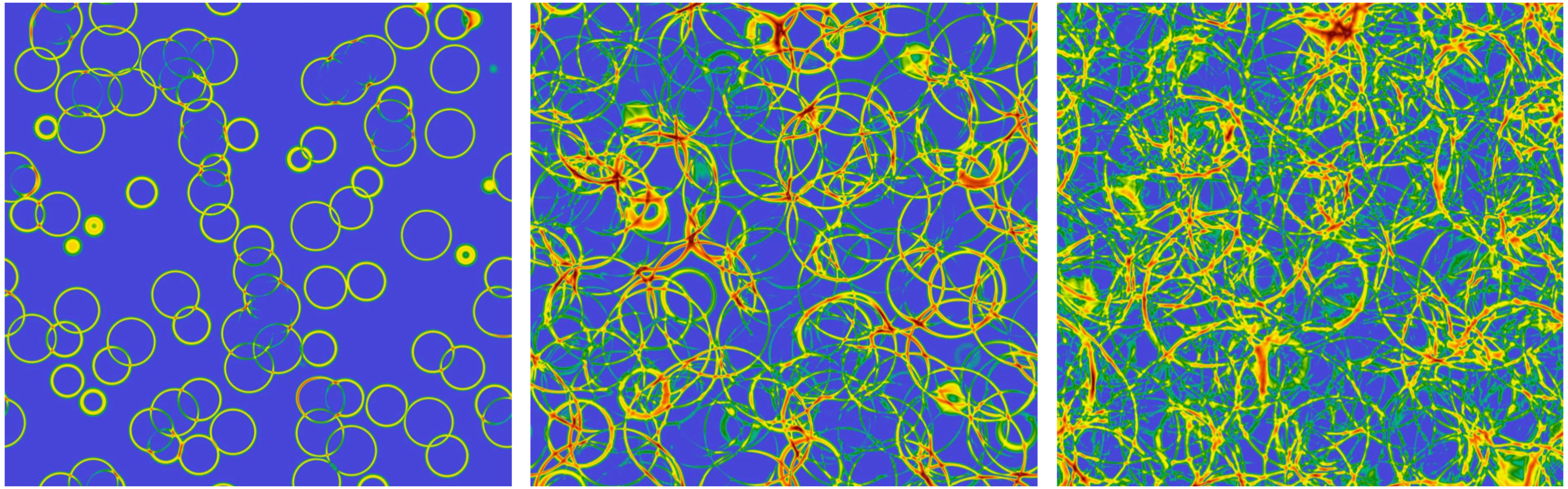

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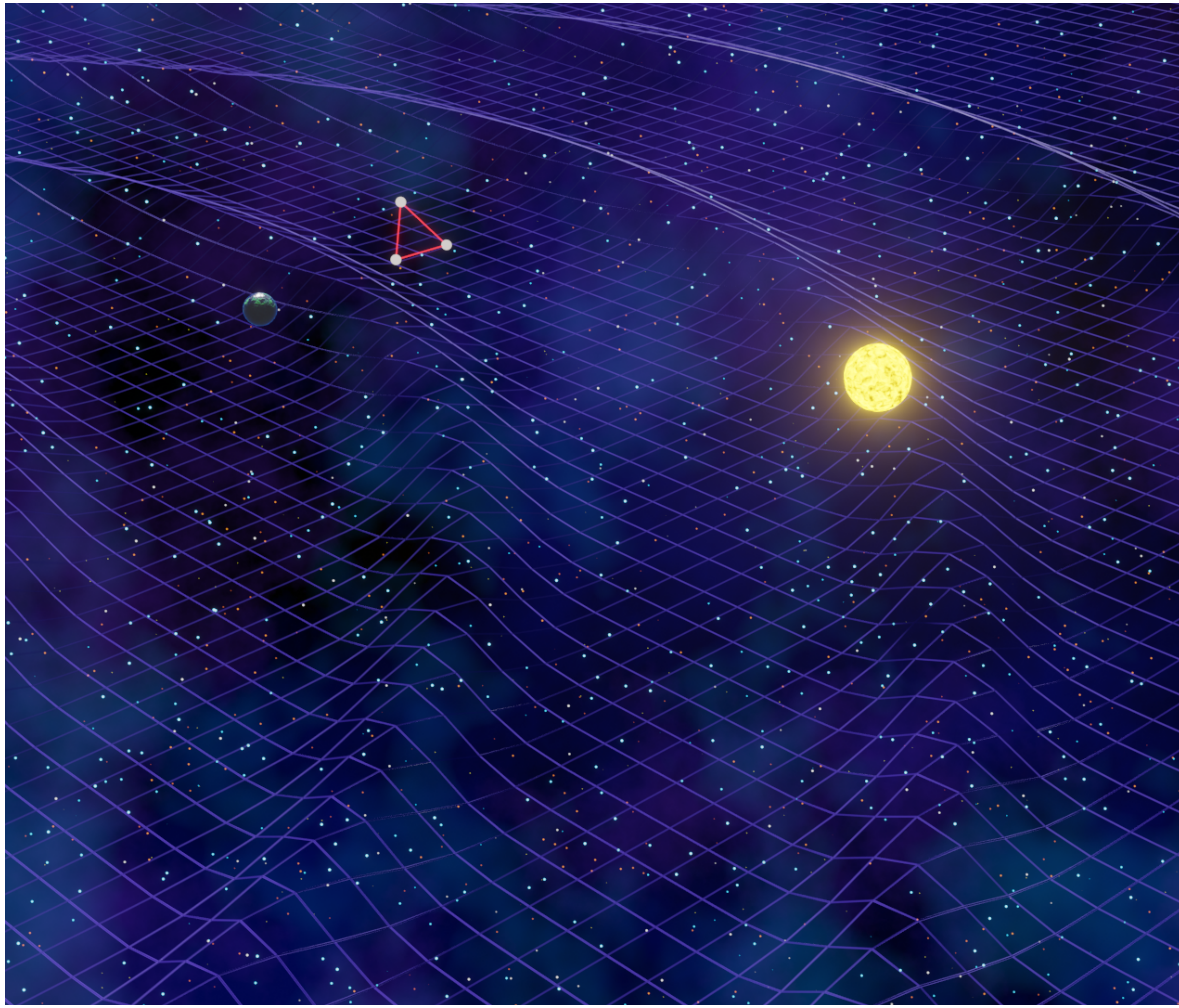
Why? LISA & large bubbles

FOPT at the EW scale (~ 100 GeV) produce gravitational waves (GW) within the LISA frequency band ~ 0.1 mHz - 10 Hz



Credit: Anna Kormu

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Large bubbles is a compelling limit for LISA observations

- Maximise the energy density of acoustic GWs

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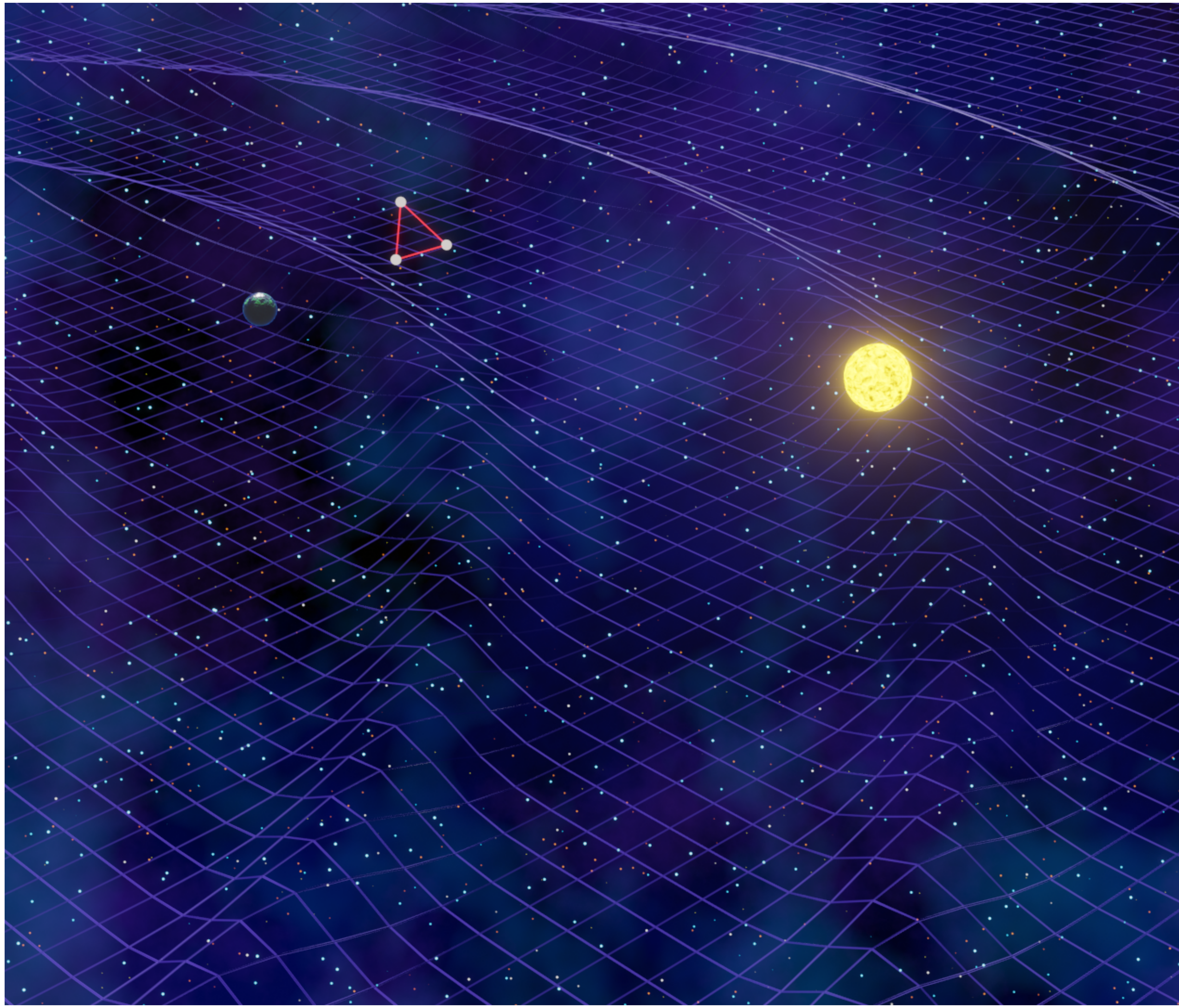

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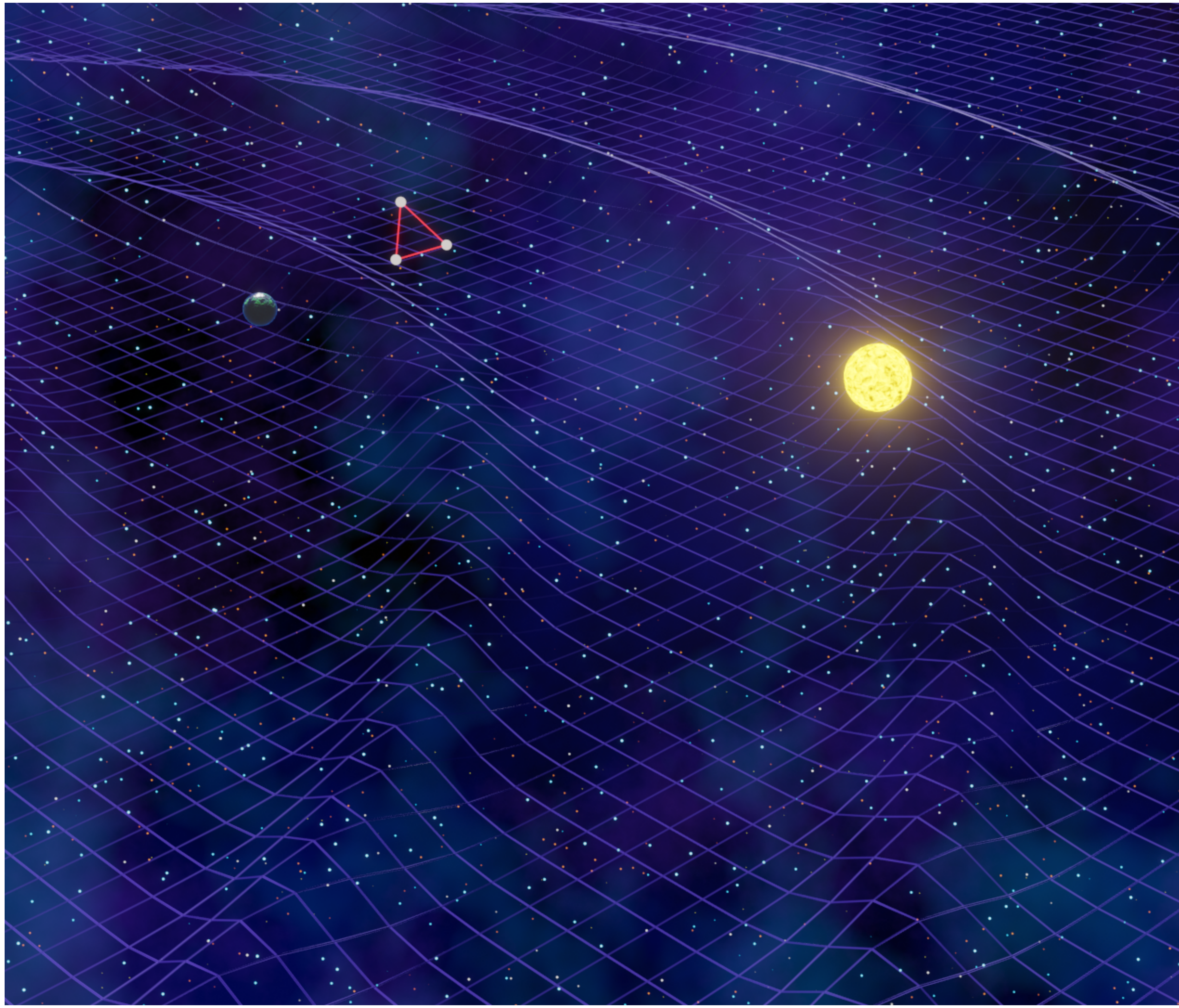
- Enhance secondary gravitational waves from curvature perturbations

  LG and M. Hindmarsh (2024), arXiv:2307.12080v2



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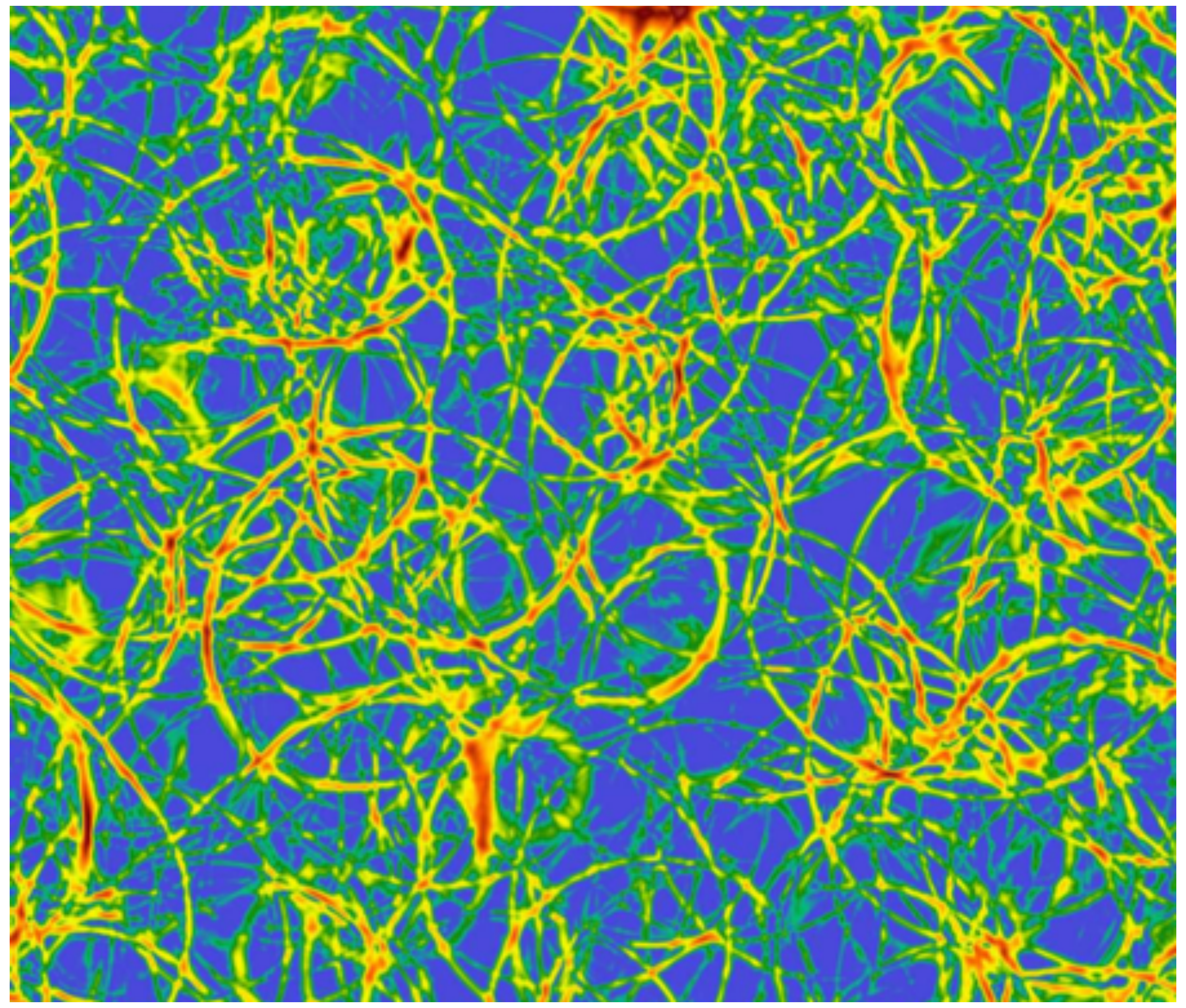
How? Semi-analytic model of gravitational wave production



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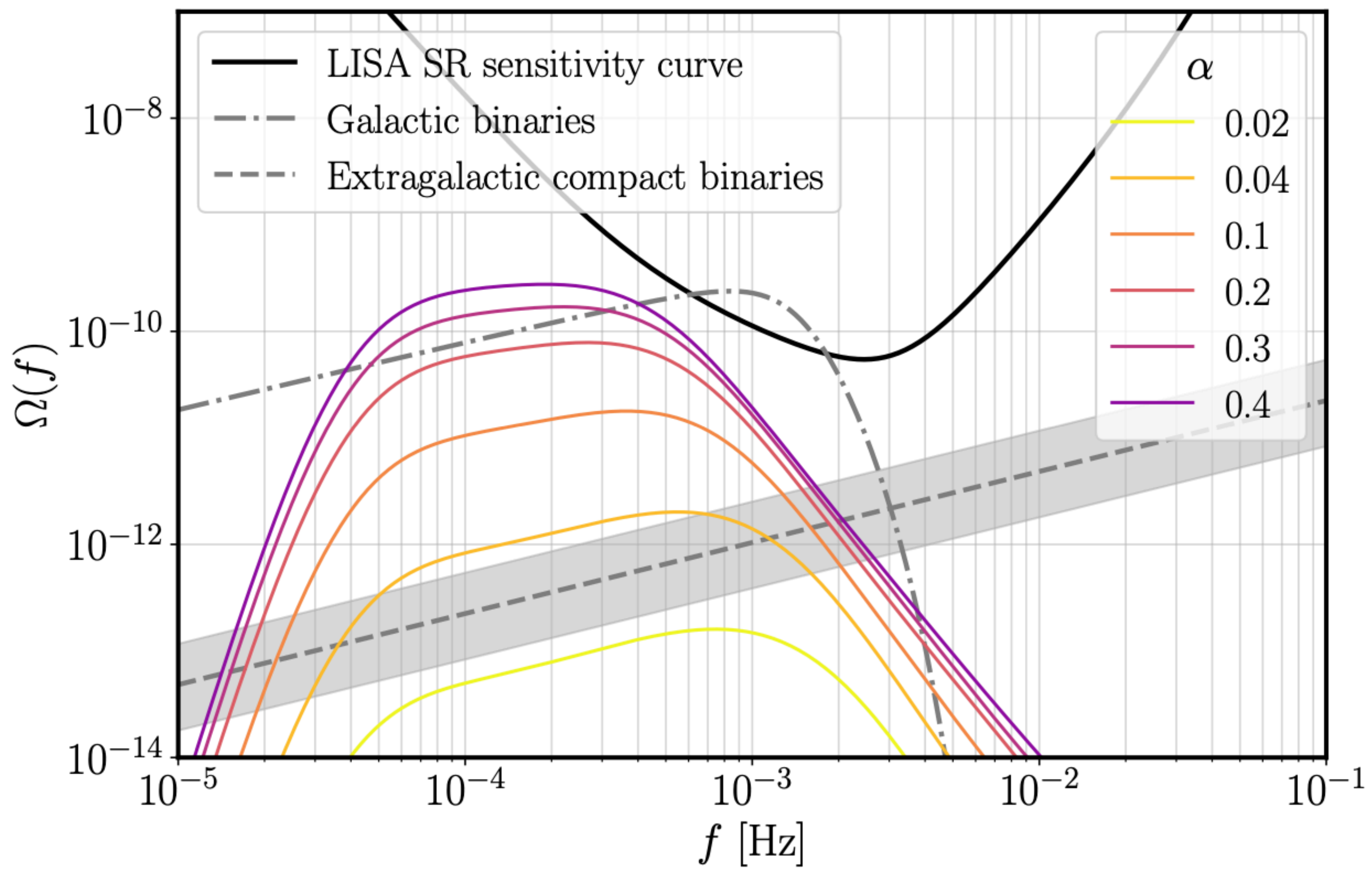

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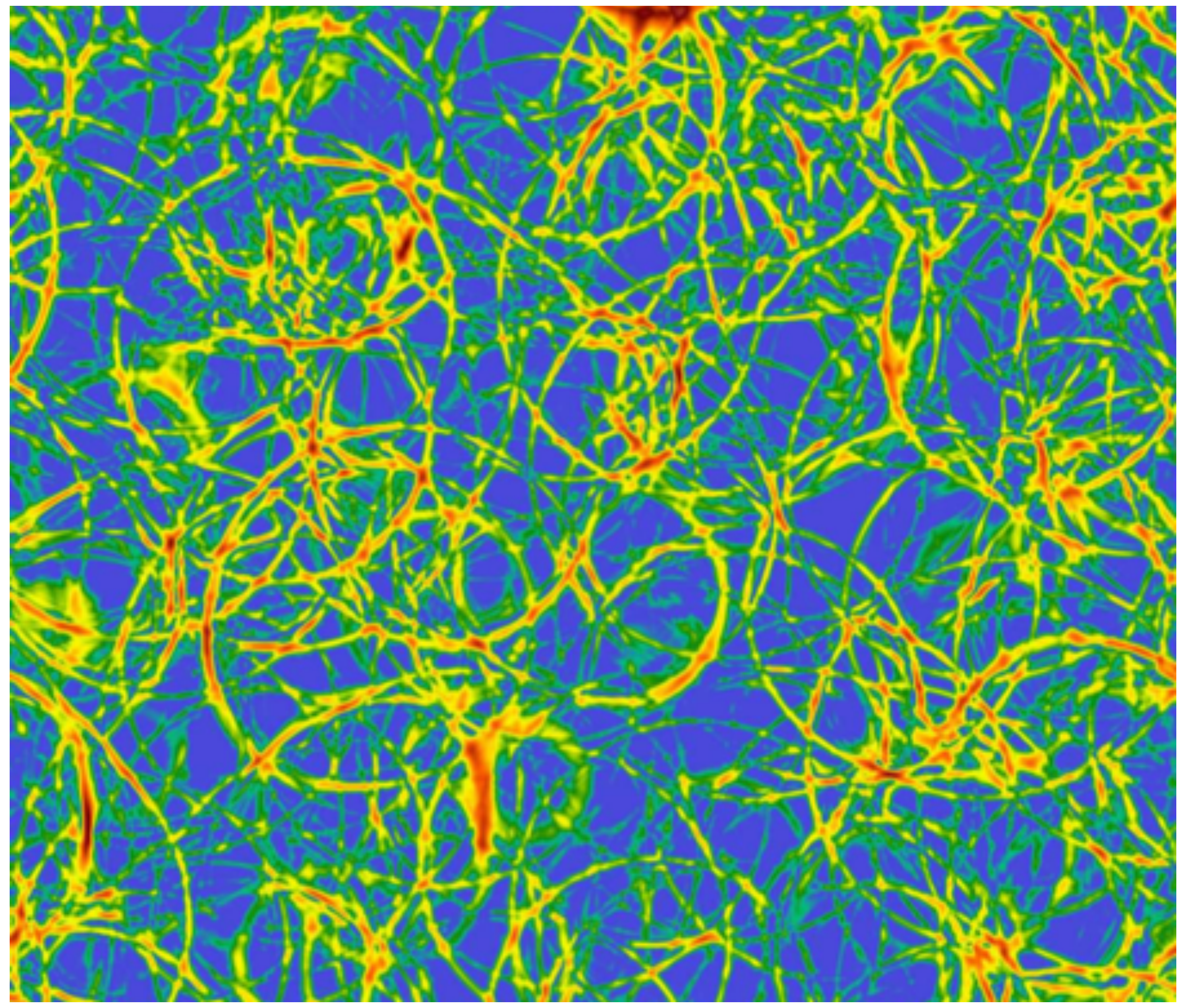

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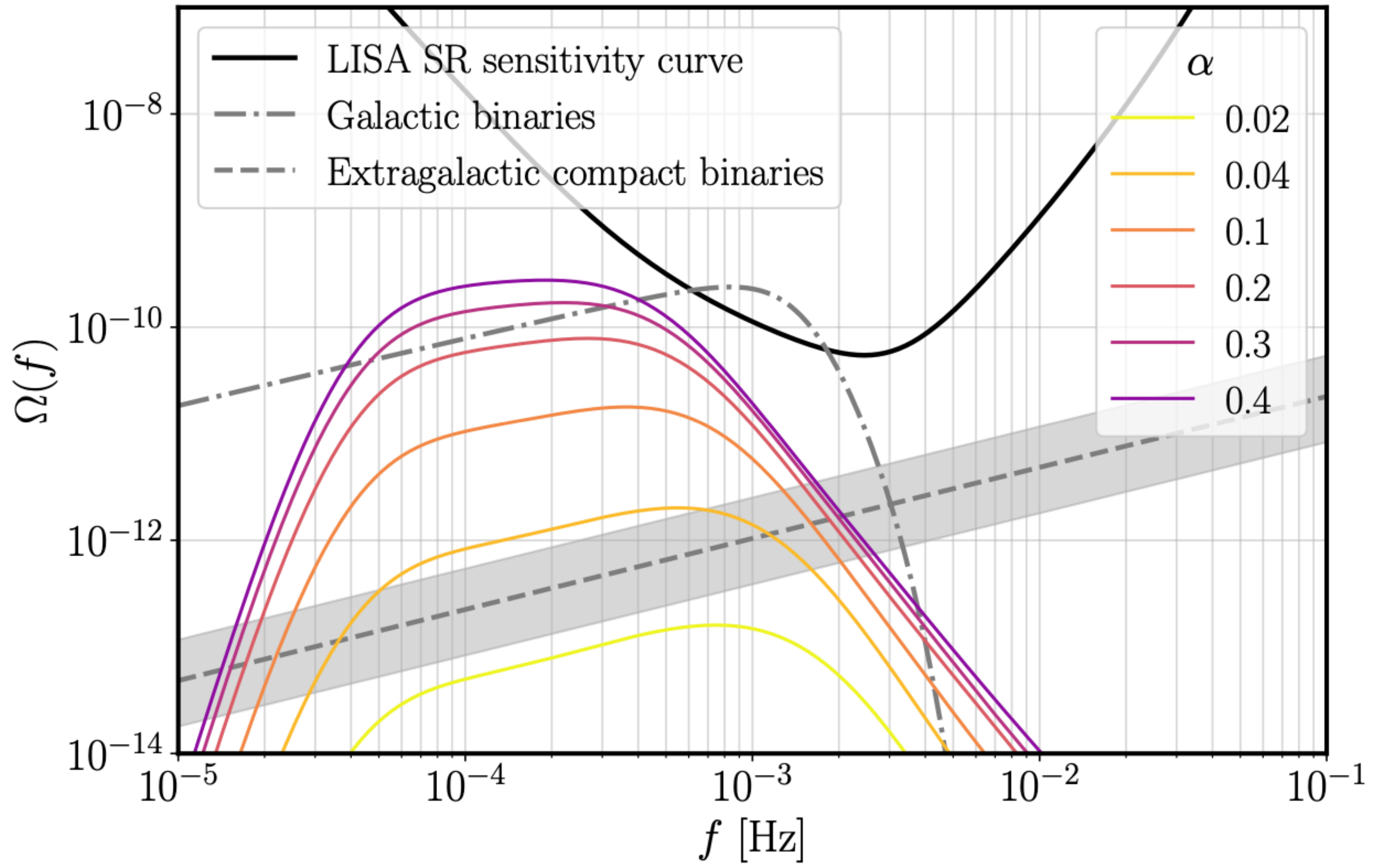
$$h''_{ij} + 2\mathscr{H} h'_{ij} - \nabla^2 h_{ij} = \mathcal{S}_{ij}$$



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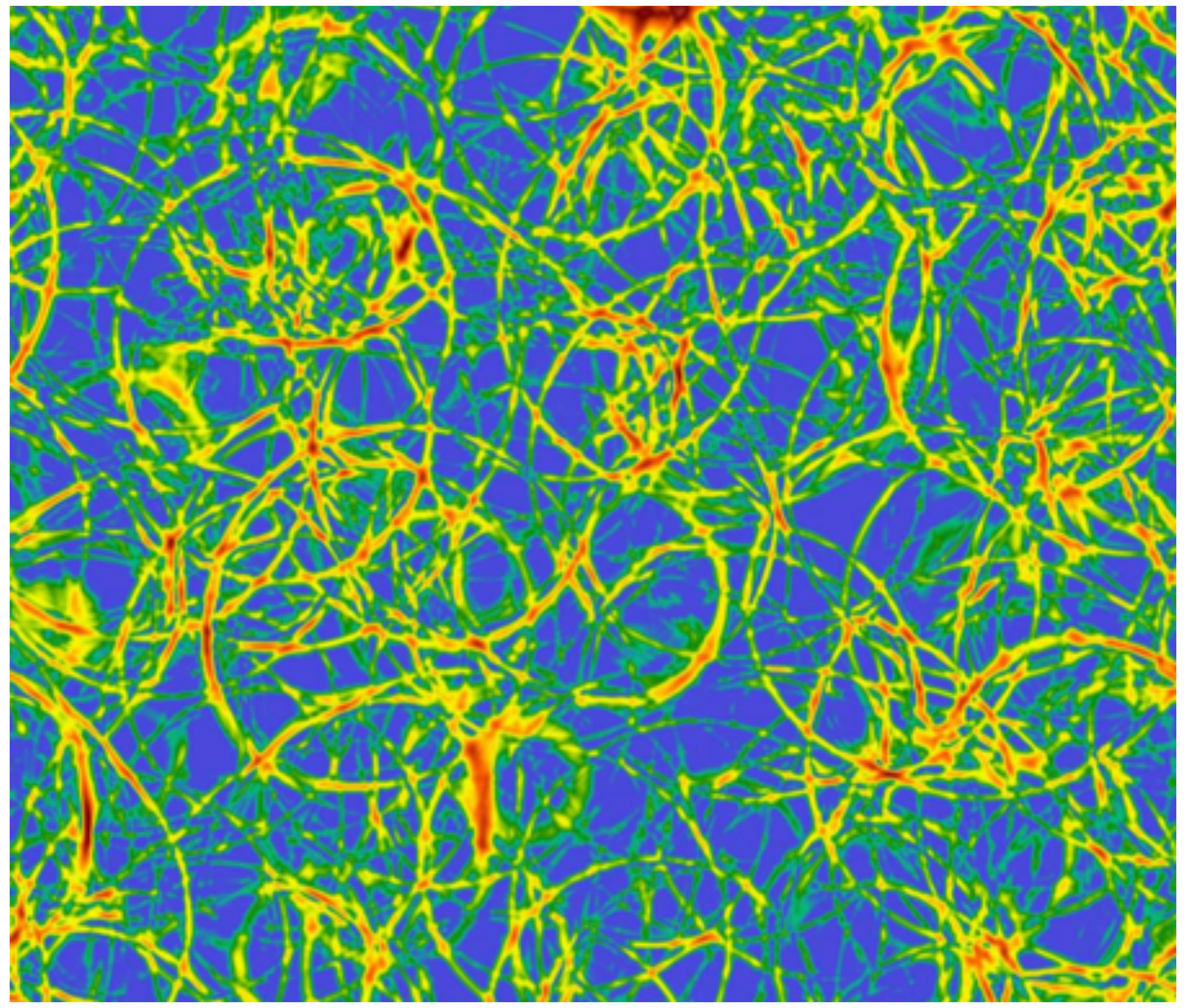

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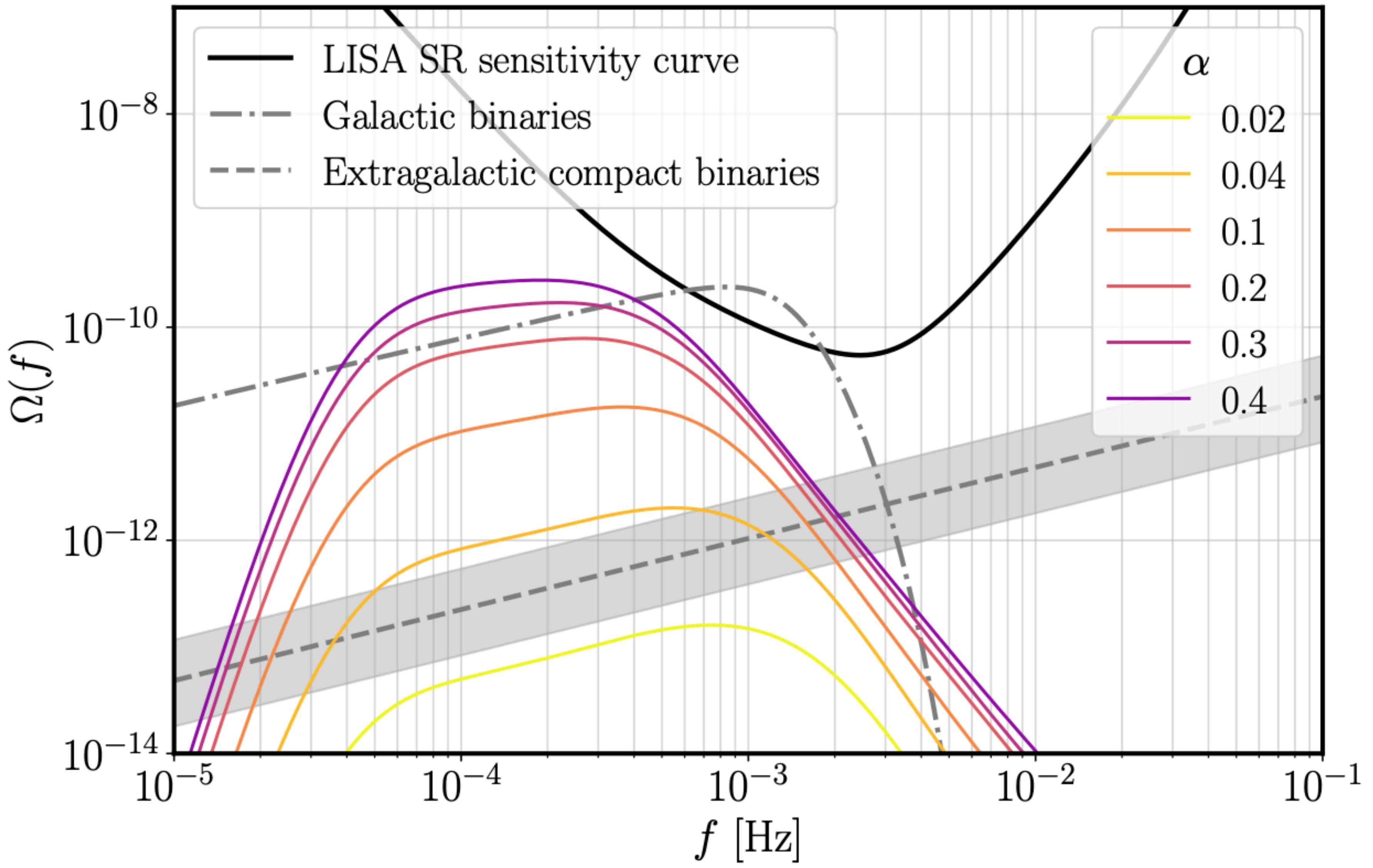

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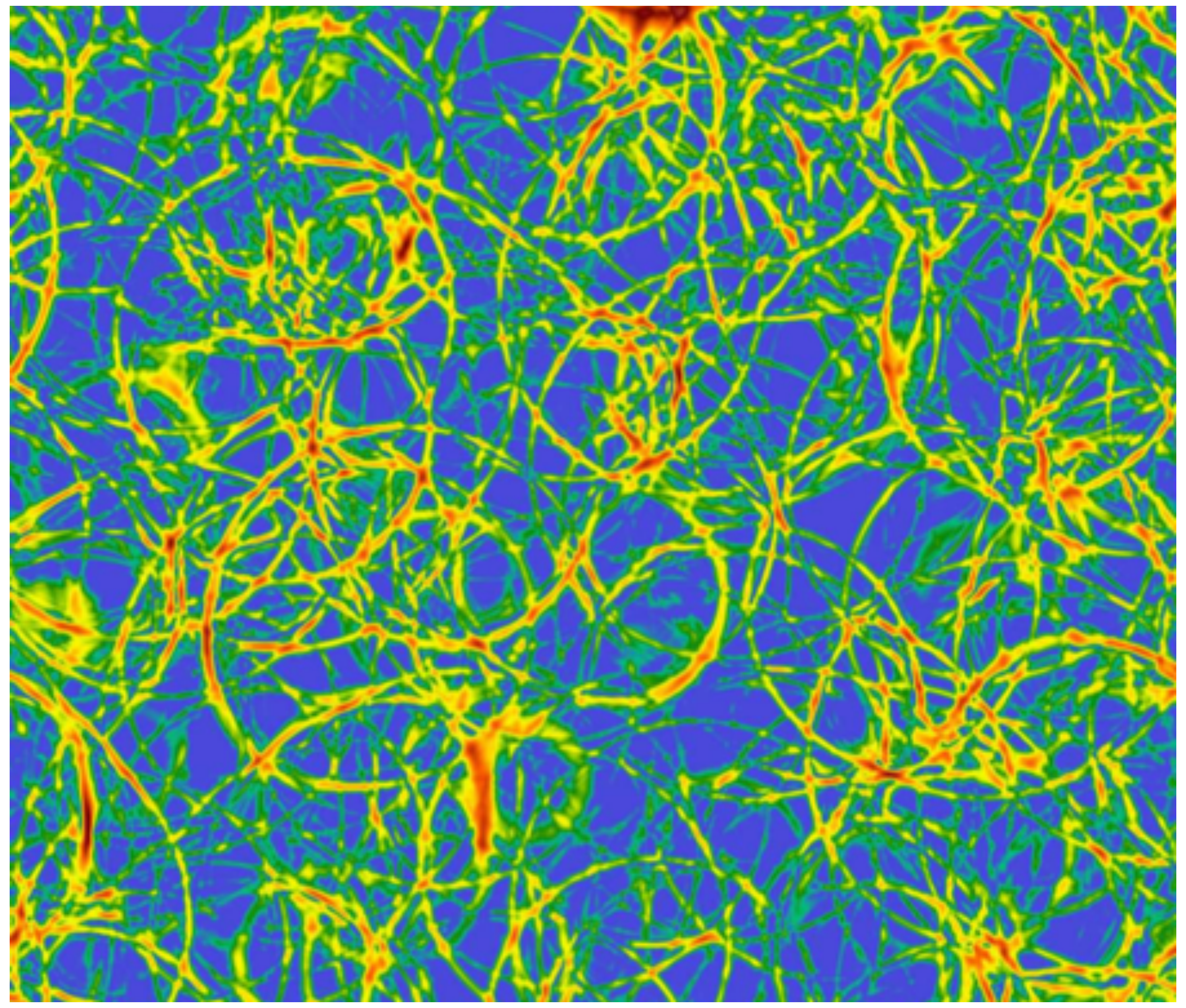

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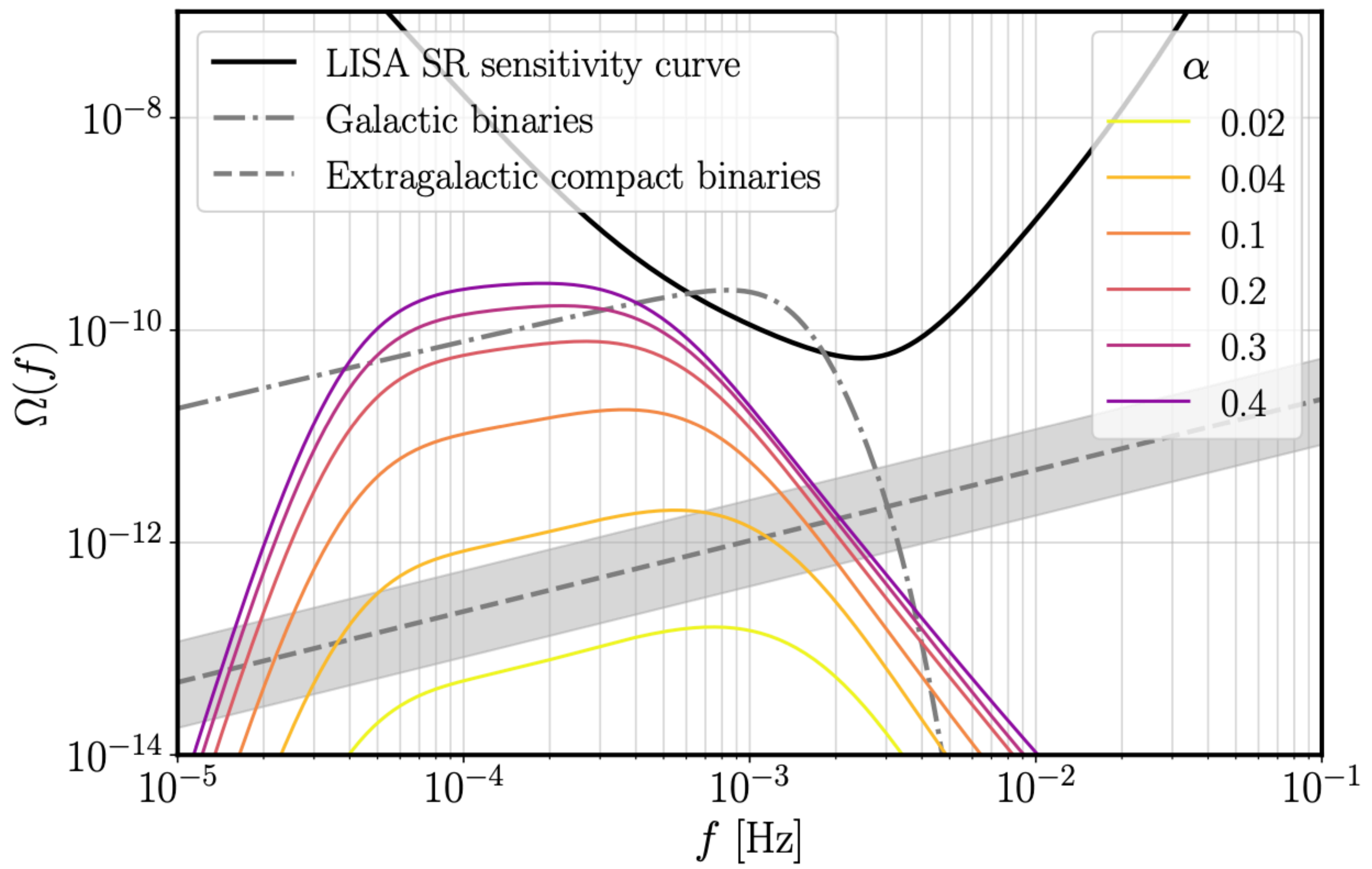

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Energy density GW

R. A. Isaacson (1968)

$$e_{gw} = -\frac{1}{32\pi G}\langle \mathscr{D}_0 h_{ij} \mathscr{D}^0 h^{ij} \rangle$$

Power spectrum

$$\mathscr{P}_{gw} = \frac{1}{e_c}\frac{de_{gw}}{d\ln k}$$

e_c Critical energy density

k Gravitational wave wavenumber

Stationary source - the state of the art

$$\mathcal{P}_{gw} = 3\left(\Gamma \bar{U}_f^2\right)^2 (\mathcal{H}_* R_*)(\mathcal{H}_* \tau_v)\, \mathcal{P}(kR_*)$$

$\bar{U}_f$ root mean square fluid four-velocity

$\Gamma = \dfrac{\bar{w}}{\bar{e}}$ adiabatic index

R_* mean bubble spacing & sw-wavelength

$\mathcal{H}_*$ Hubble constant beginning acoustic phase

τ_v lifetime of the source (sound waves)

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Incoming
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- $dq\, d\tilde{q}$: Incoming sw momenta
- $E_k(q)E_k(\tilde{q})$: Kinetic energy of sw
- $\rho(k,q,\tilde{q})$: Geom. factor
- $\Delta(k,q,\tilde{q})$: Kernel

Vertex
Interference sw - GW

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__Assumptions__:

(1) Equation of state $\bar{p} = \dfrac{\bar{e}}{3}$

But $c_s^2 < 1/3$ is possible

F.R. Ares et al. (2020), arXiv:2011.12878v2

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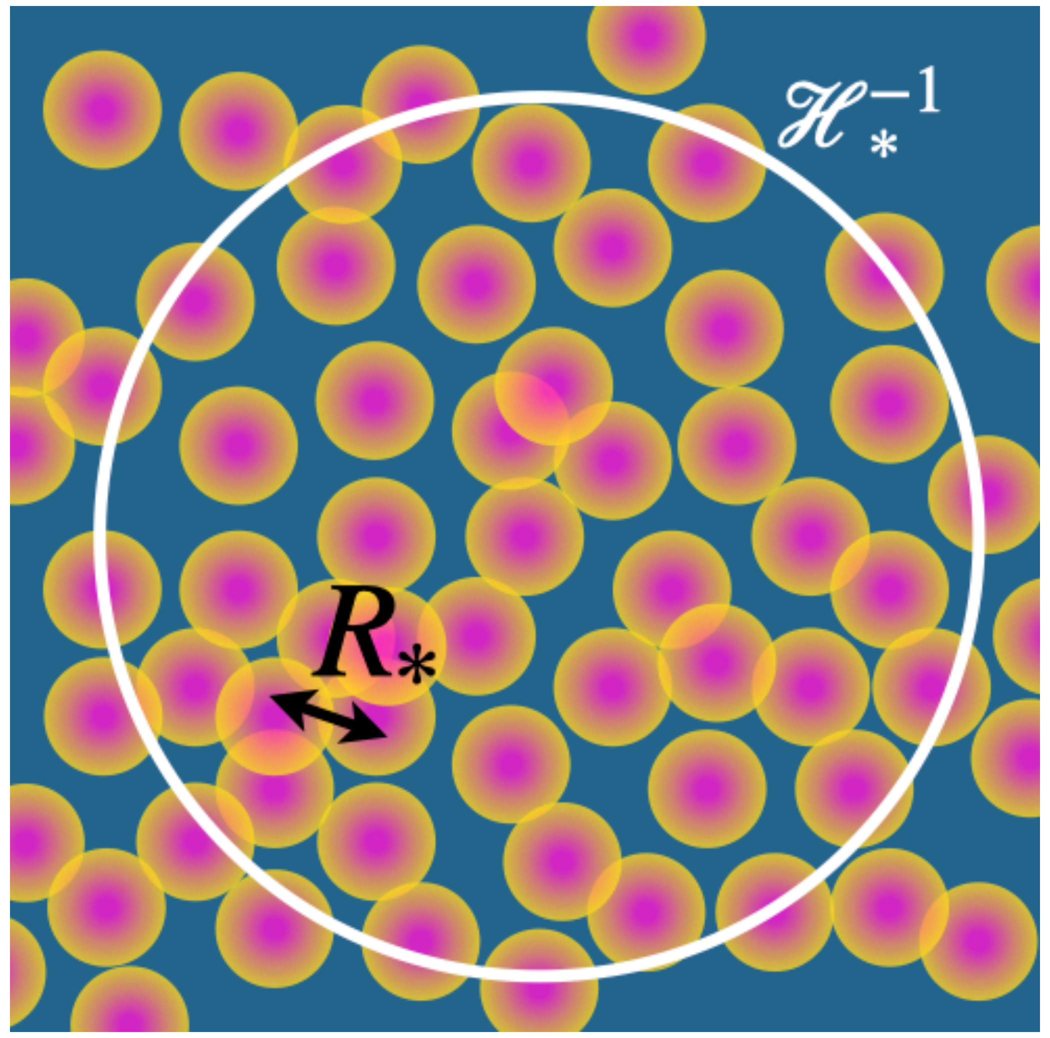

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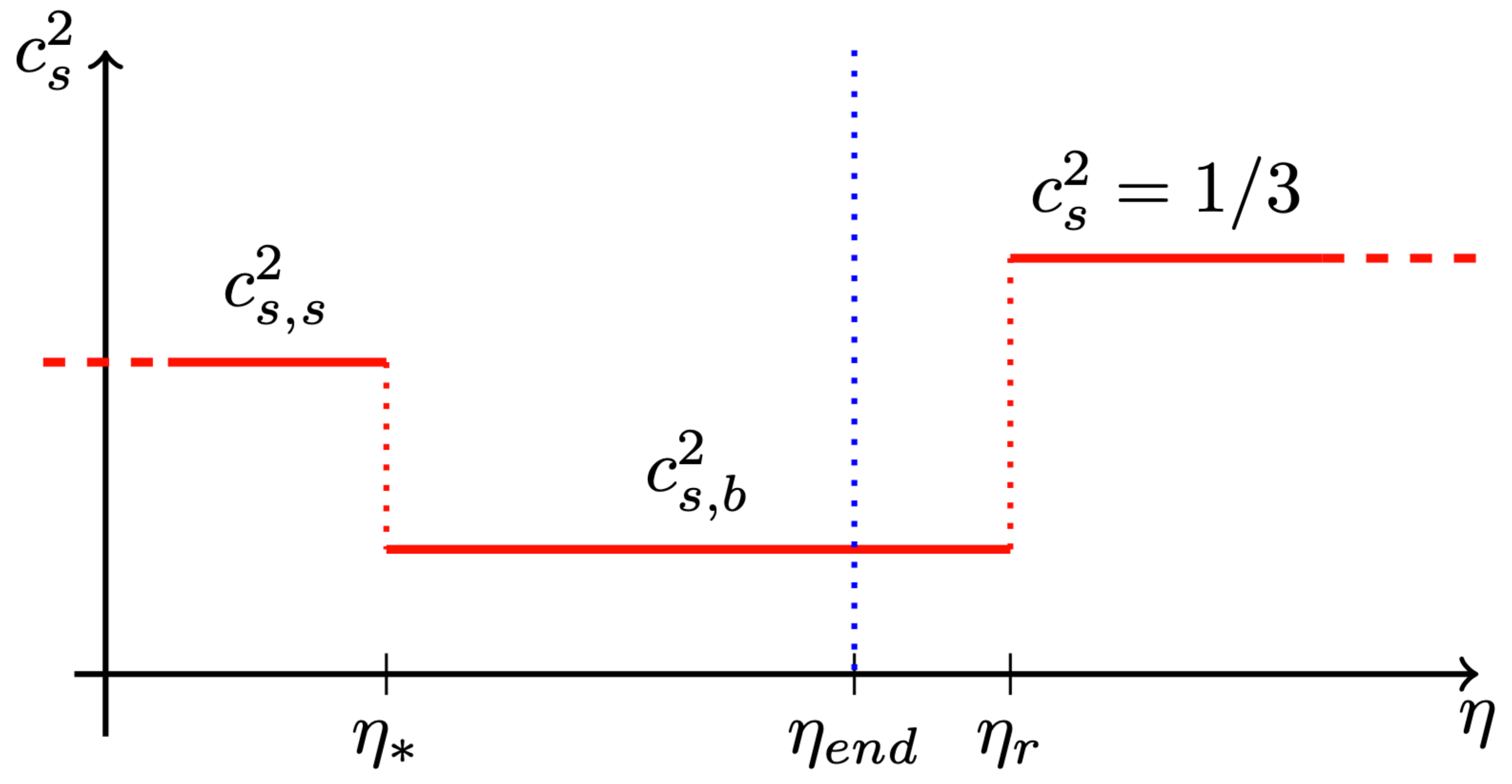
(2) Small bubbles: $R_*\mathscr{H}_* \ll 1$

What about large bubbles?

(1) Softening the Equation of State



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η_* : begin acoustic phase

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η_r : begin radiation domination

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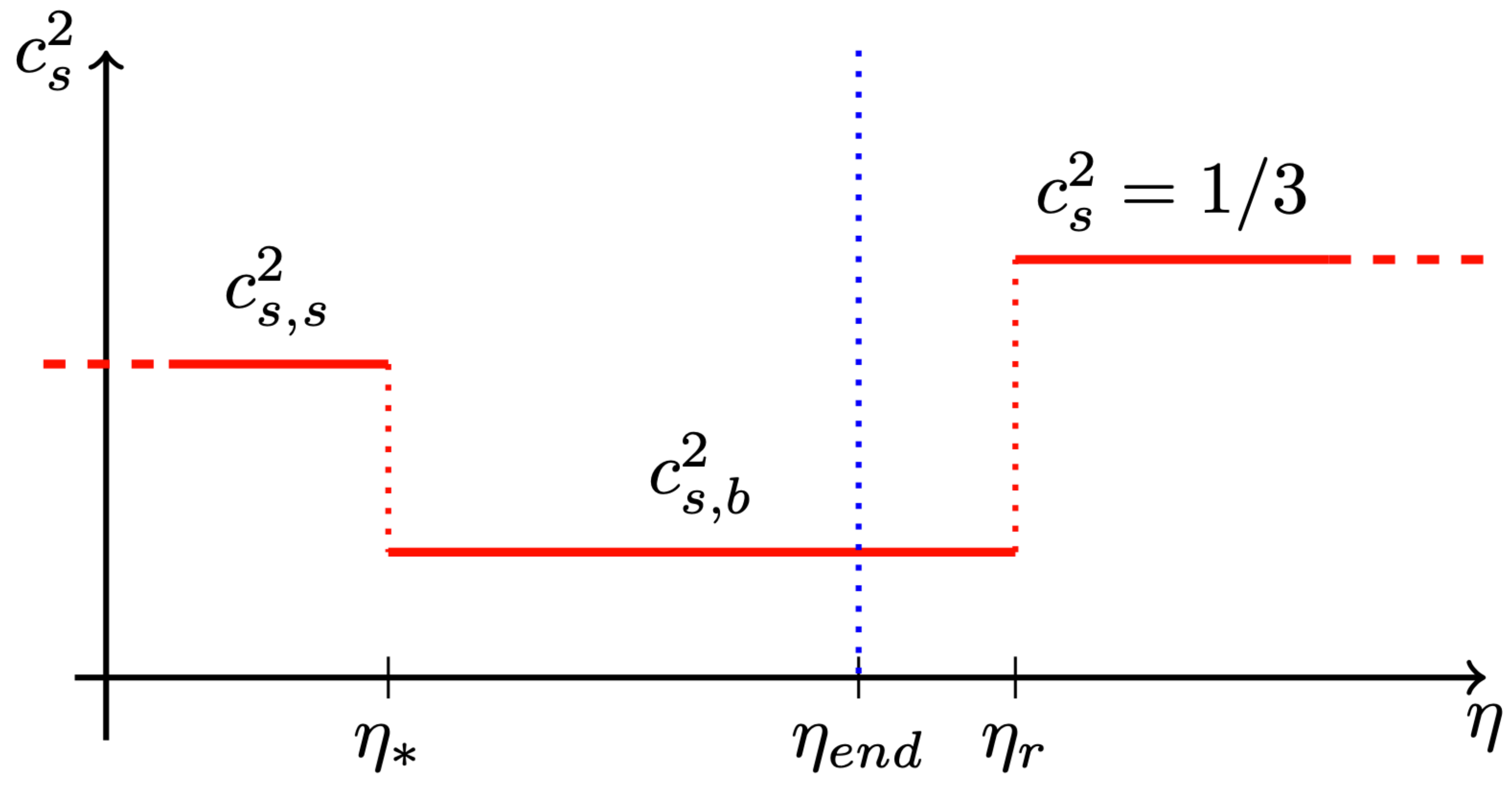

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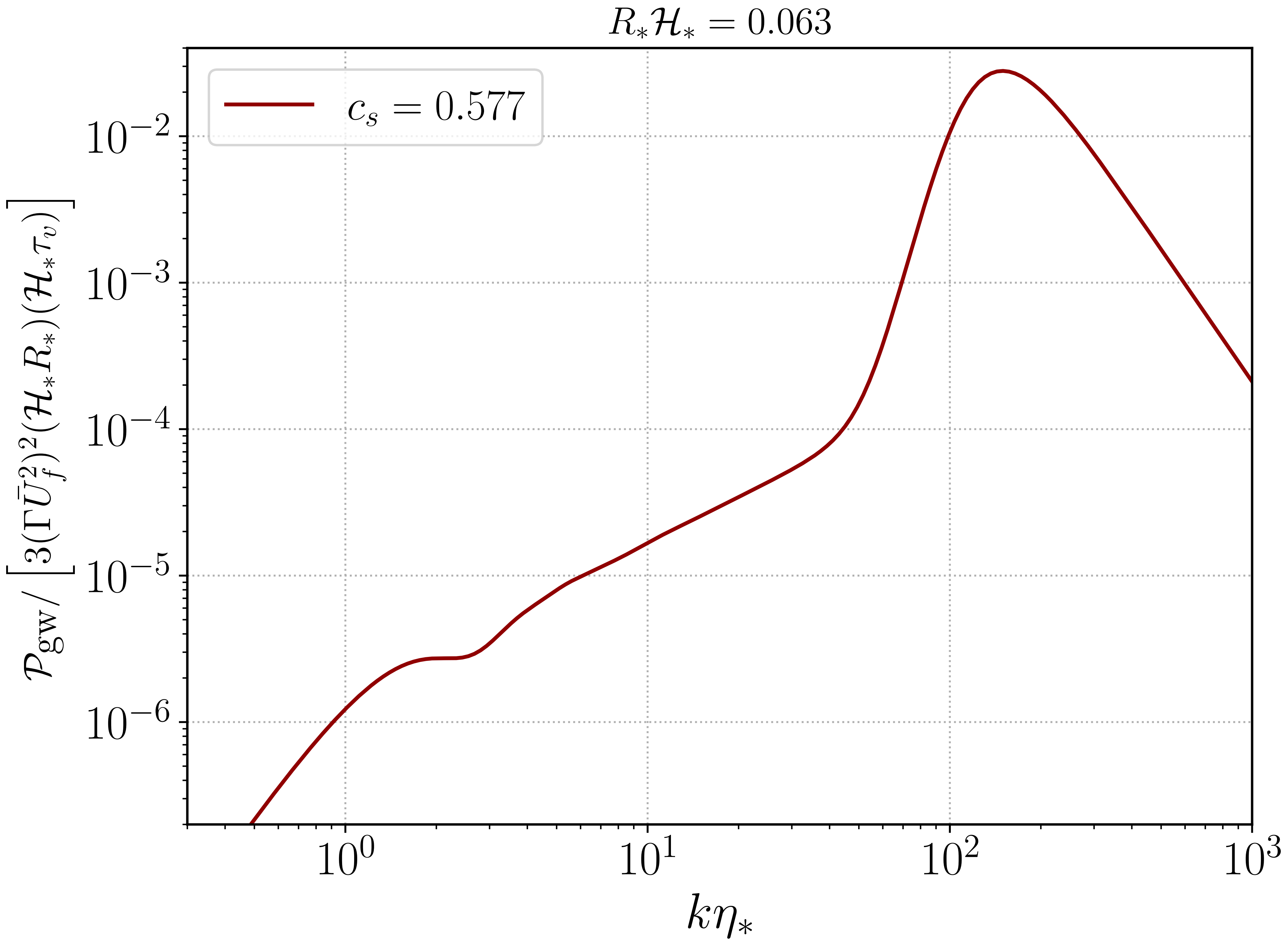

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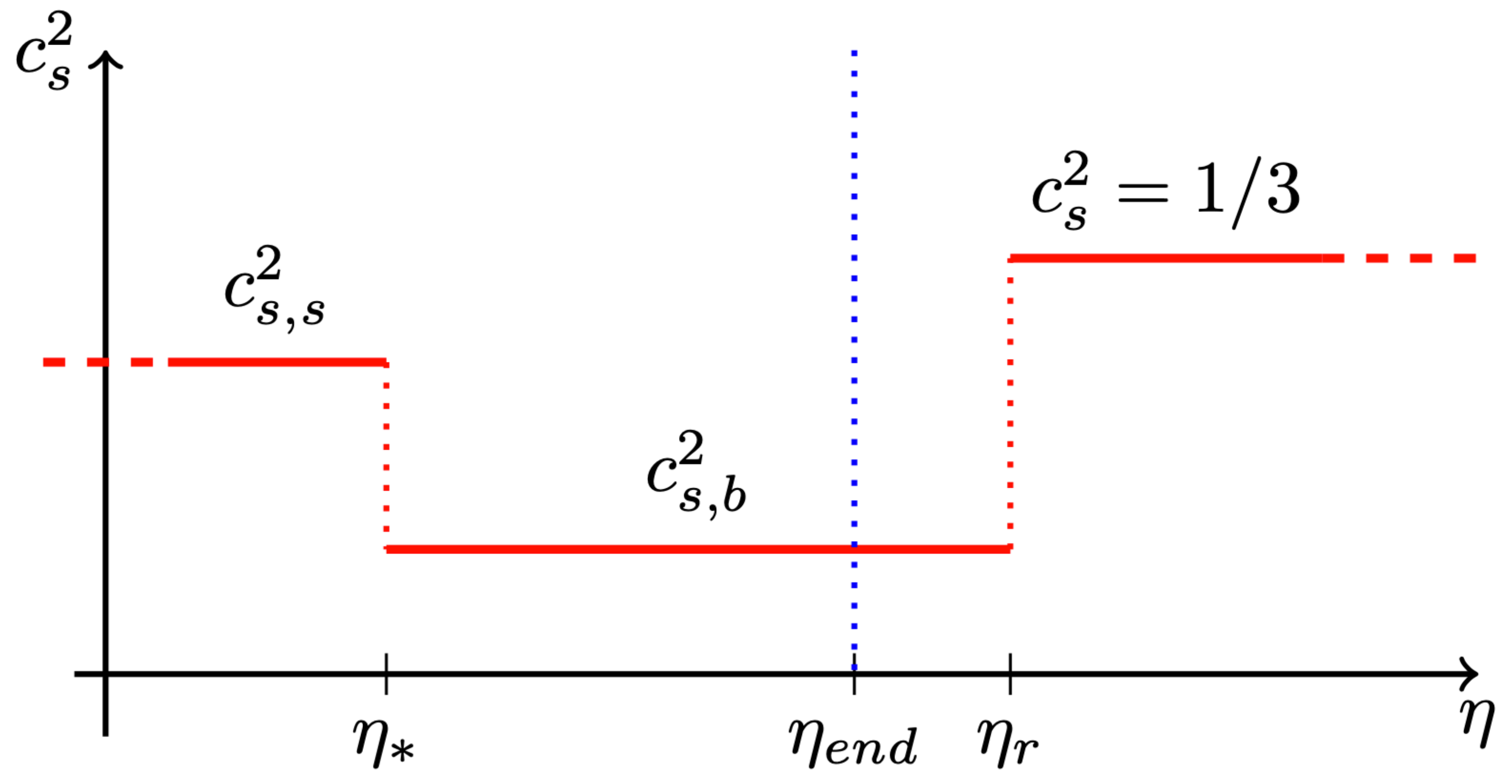

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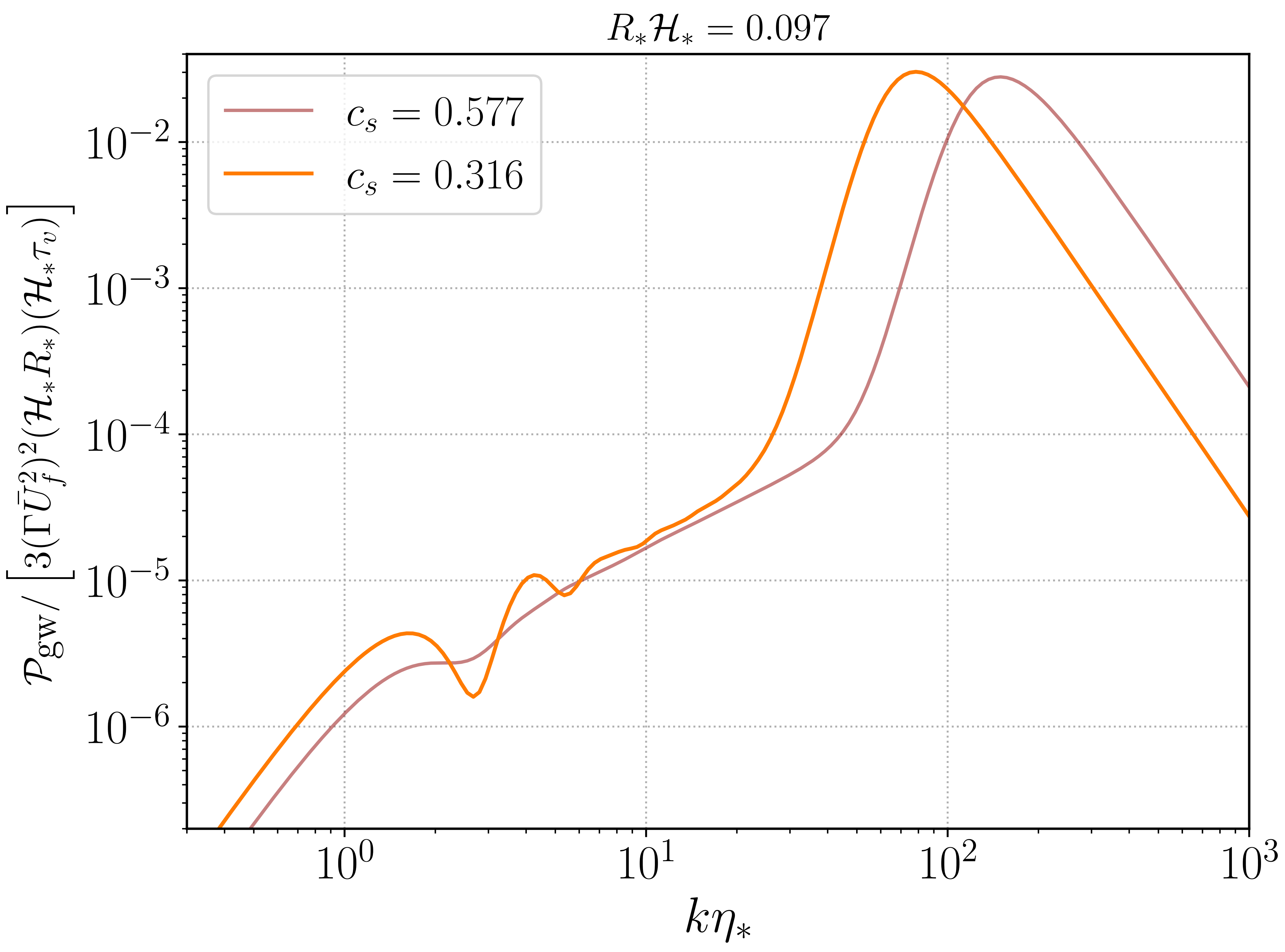

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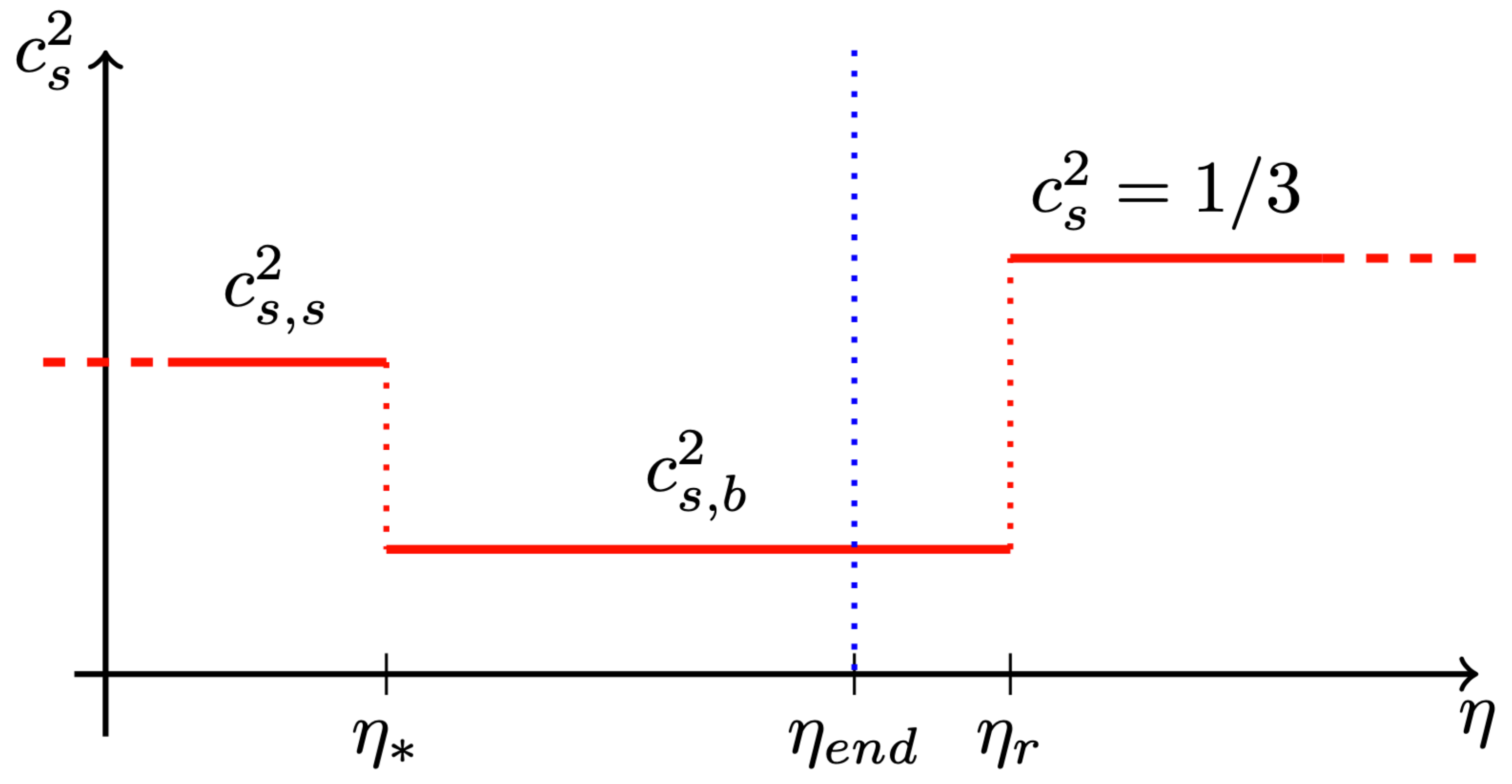

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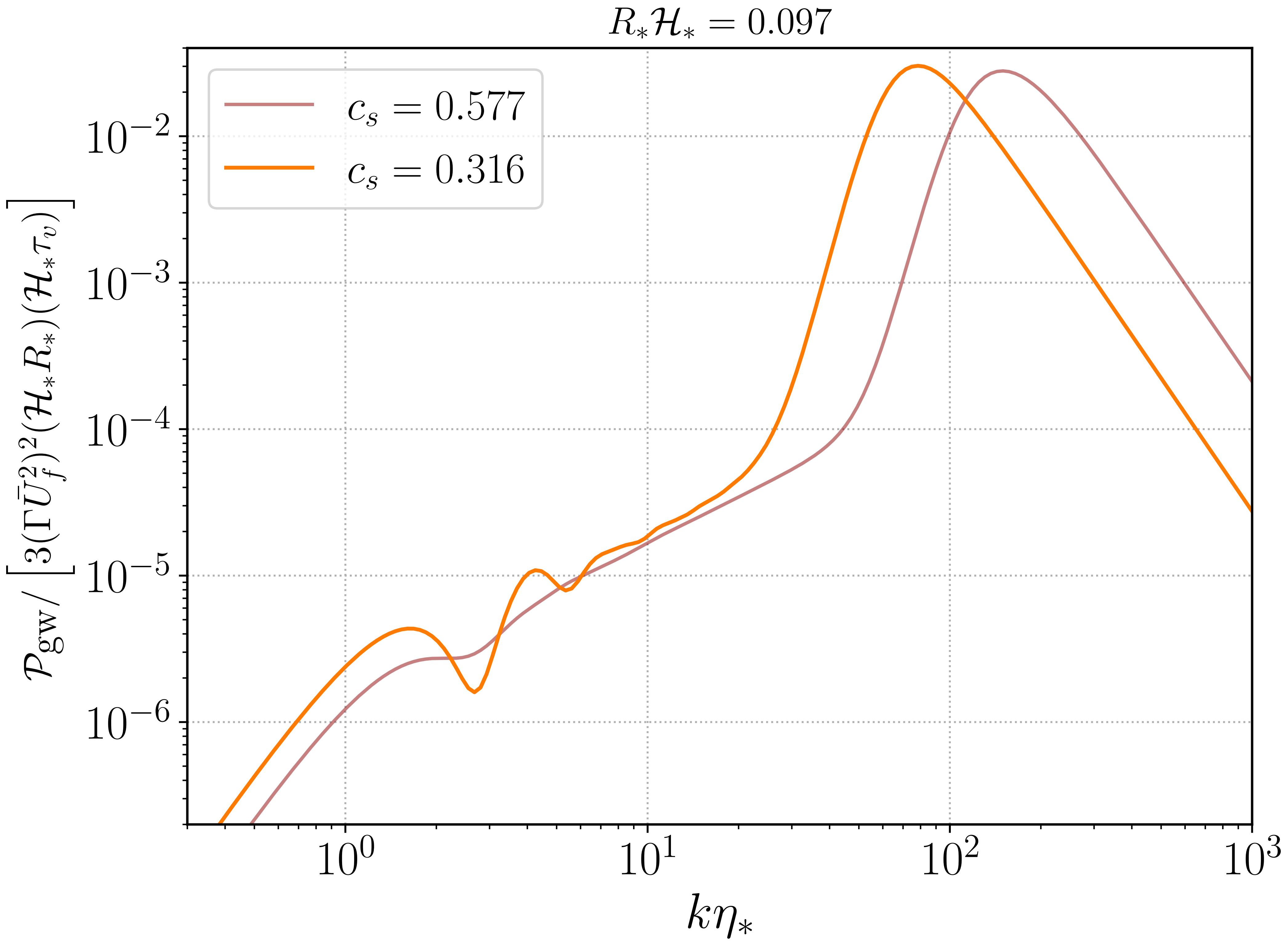

❋ Reduces the background energy density

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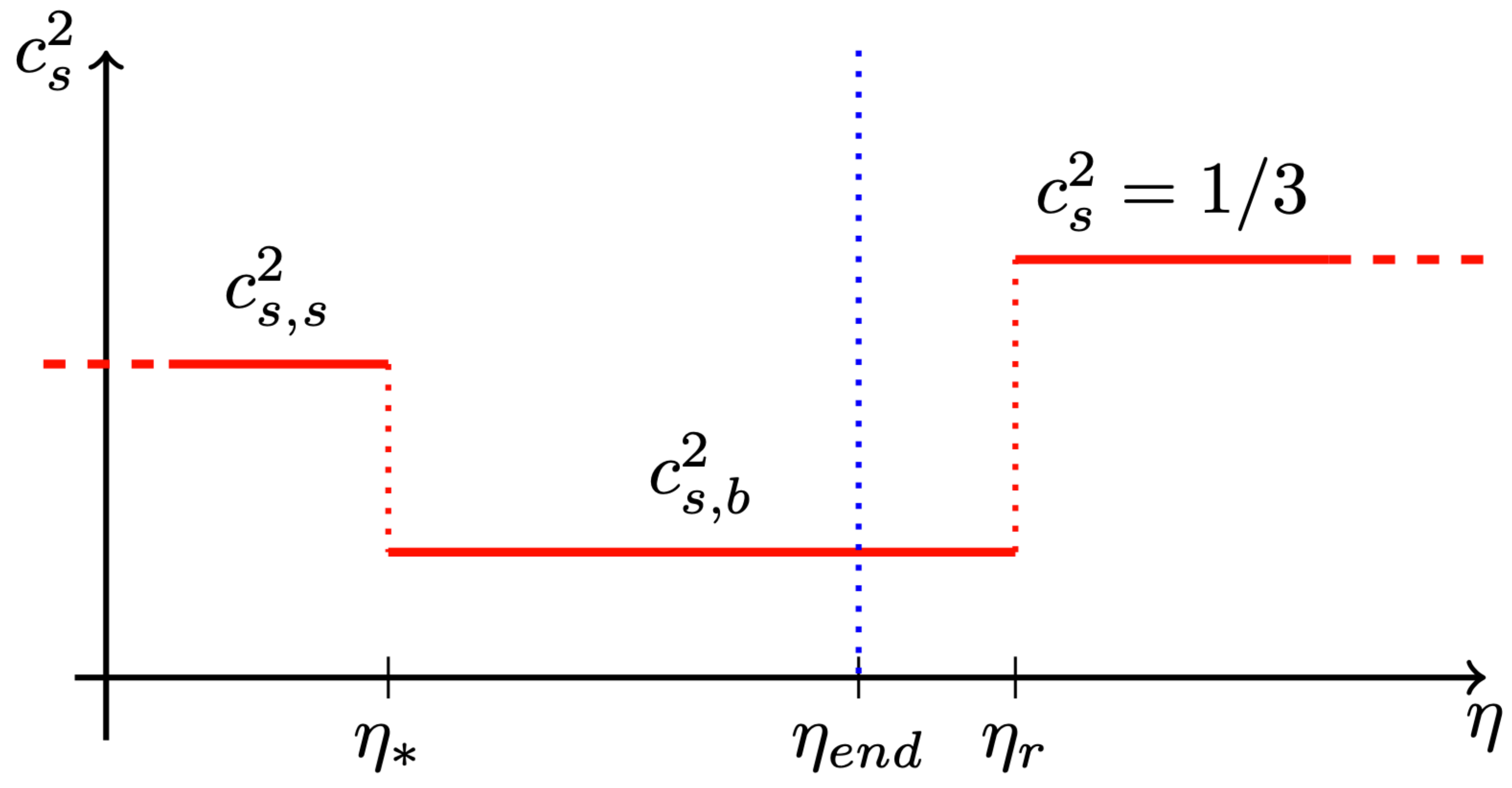

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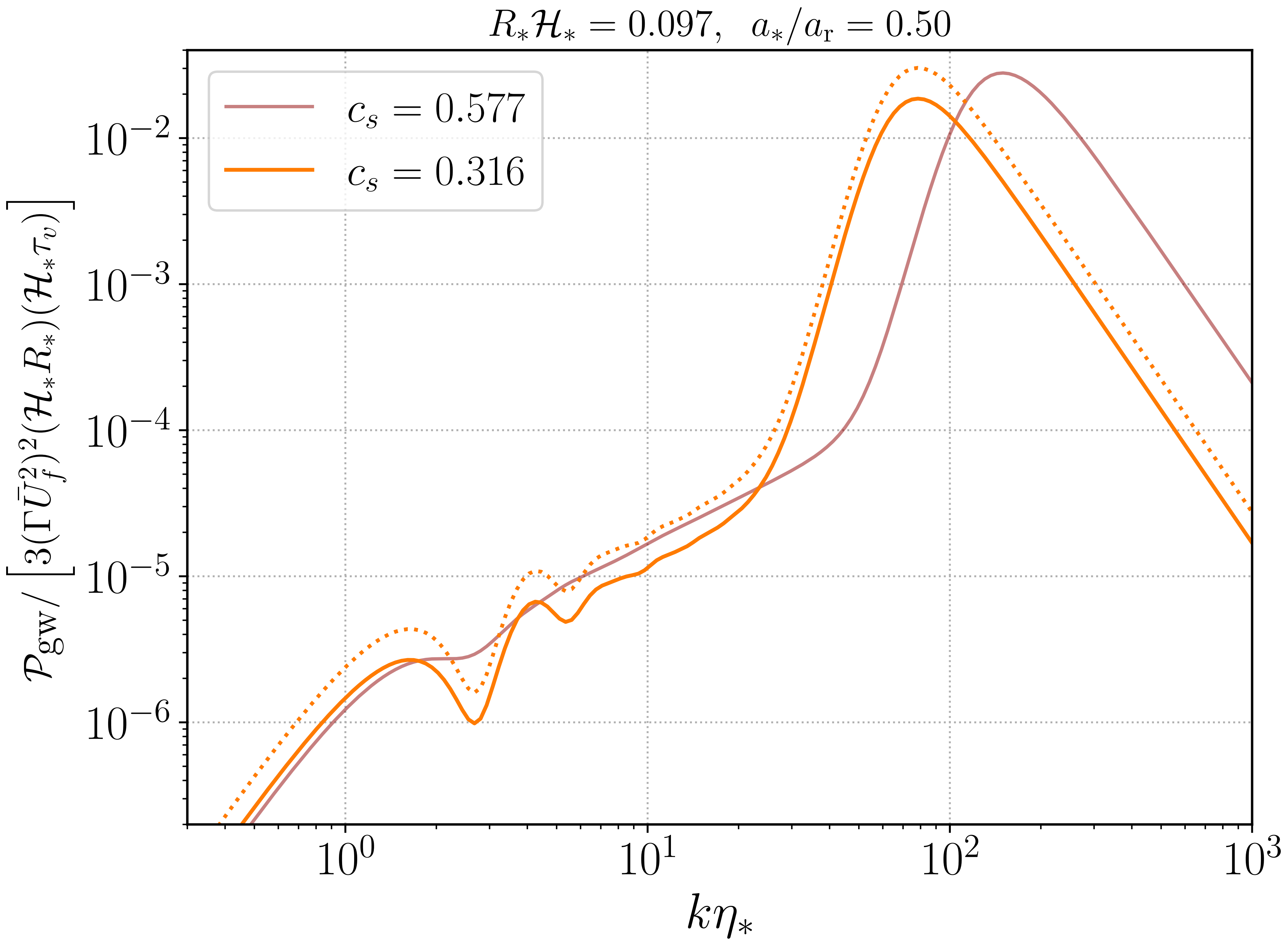

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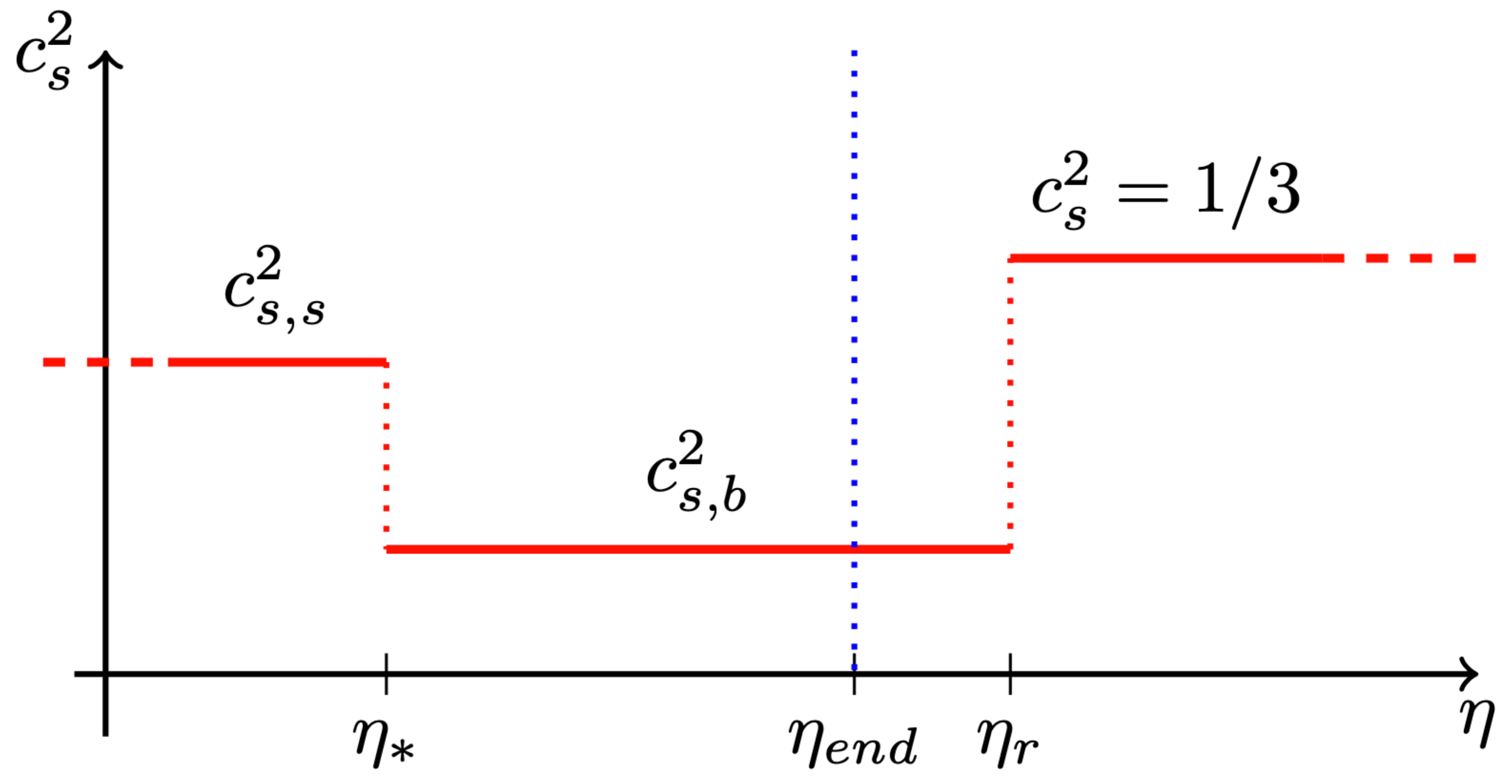

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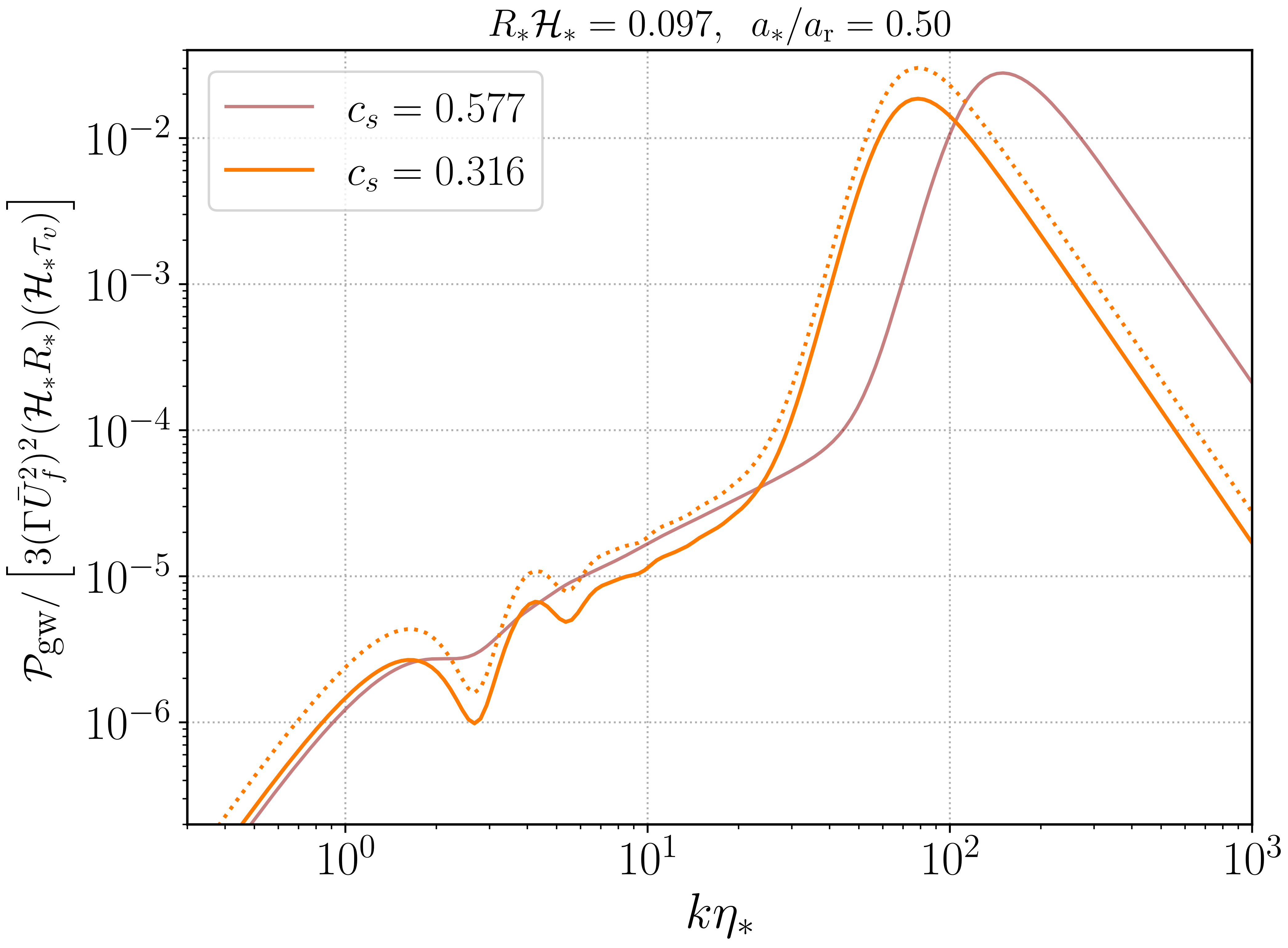

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—> friction on sound wave propagation

$$\tilde{v}_k \sim e^{ic_s k\eta}$$

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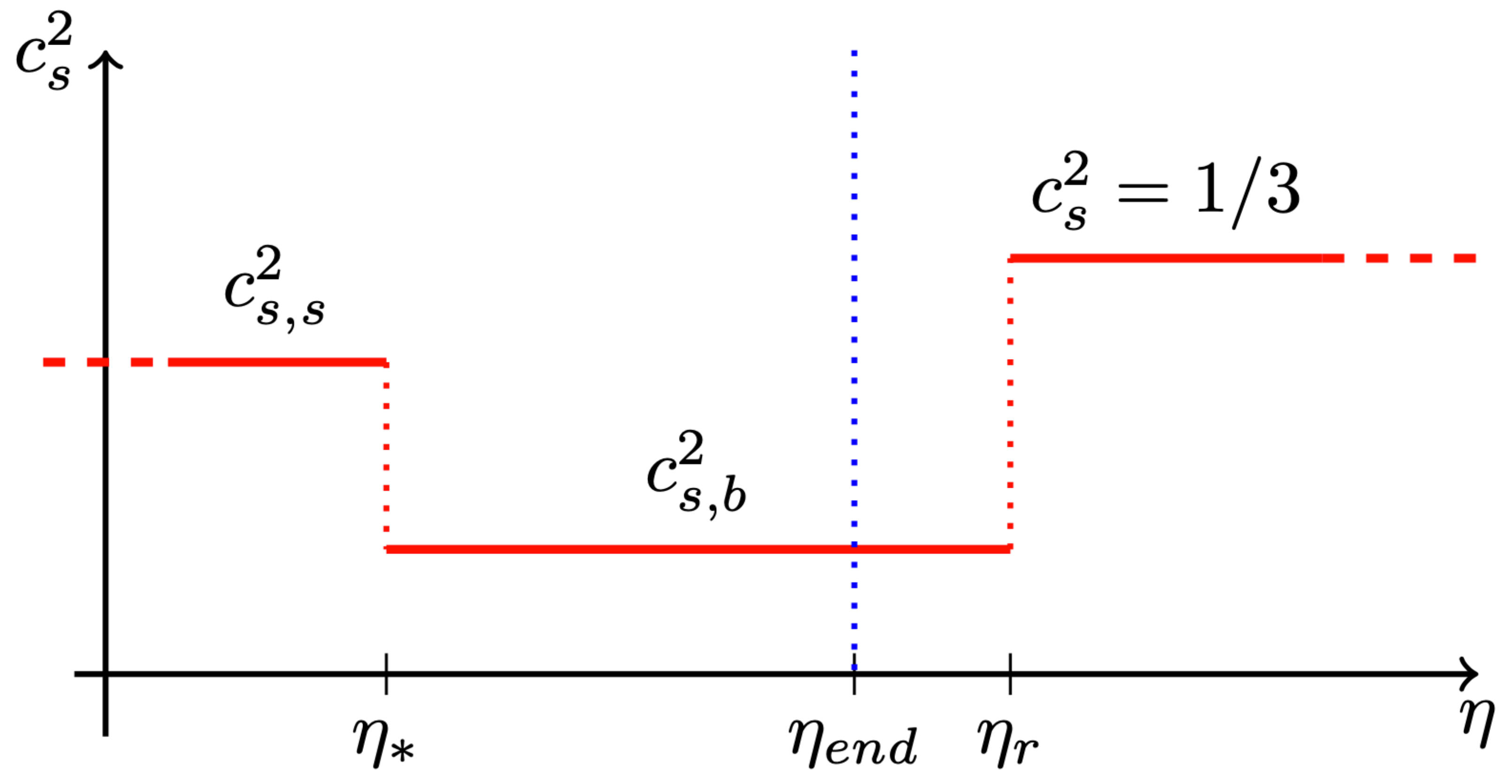

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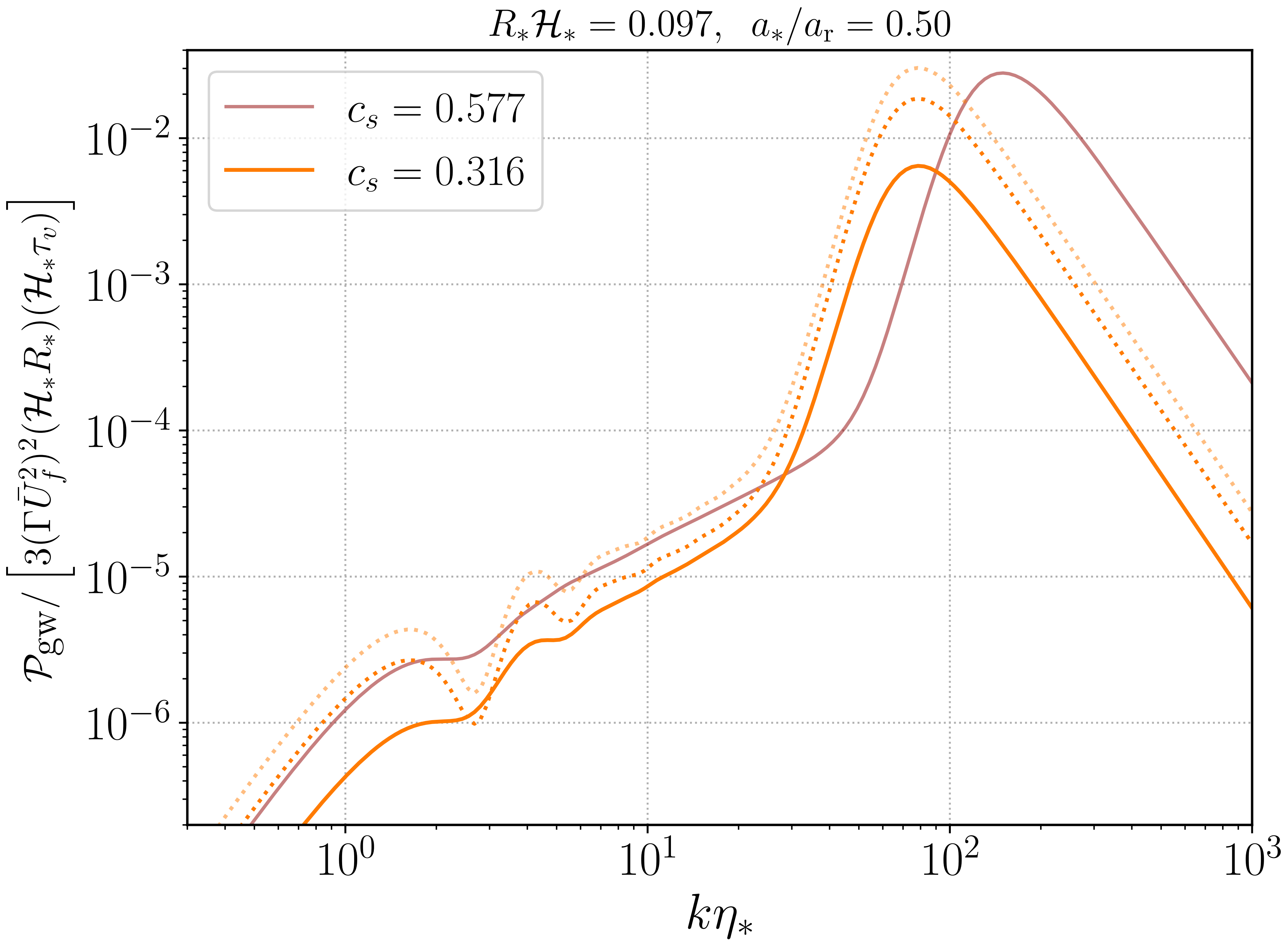

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- ❋ Accelerates the expansion of the Universe
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$$\tilde{v}_k \sim (\eta_*/\eta)^\nu e^{ic_s k\eta} \qquad \nu = \frac{1-c_s^2}{1+c_s^2}$$

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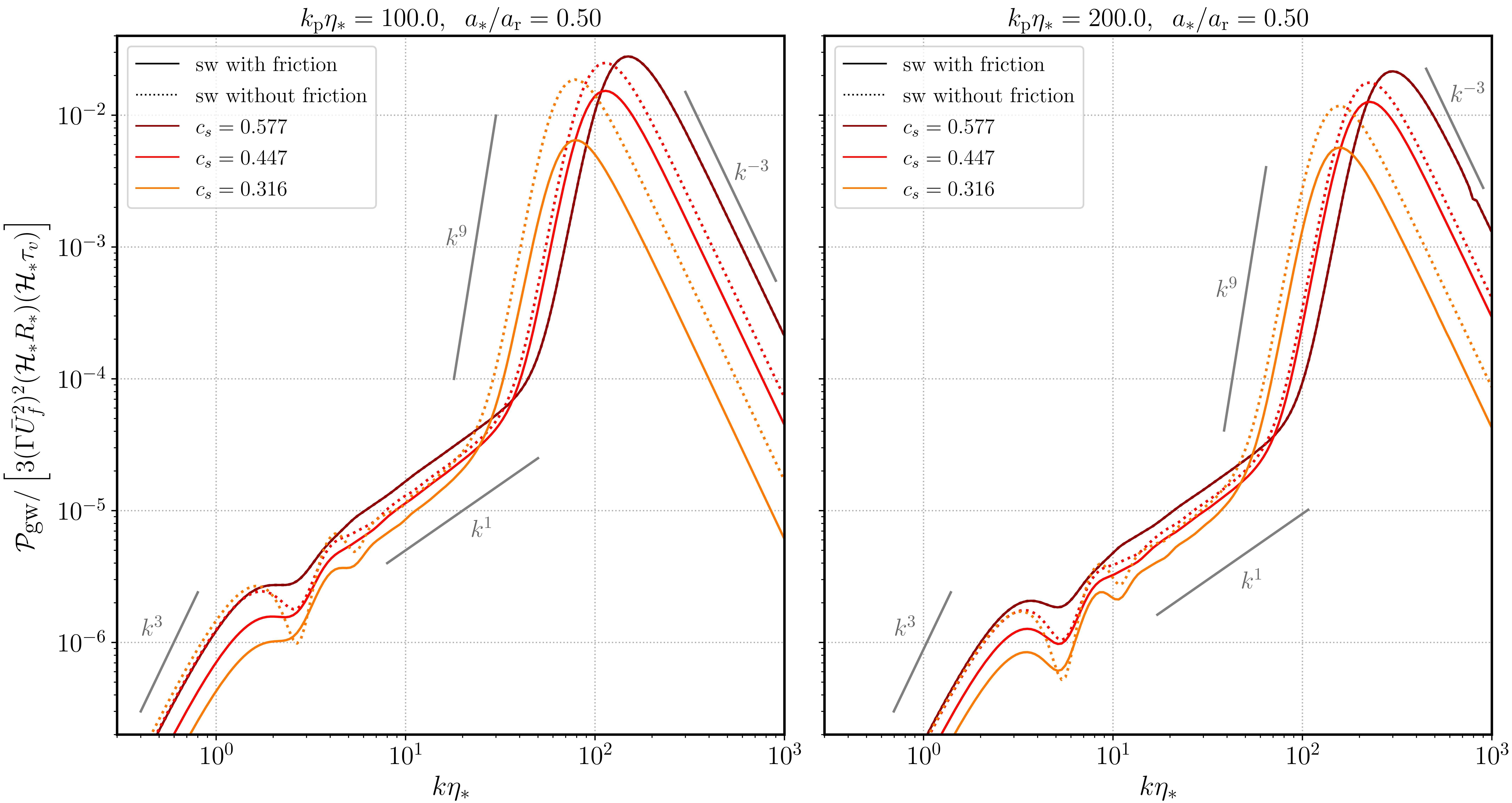
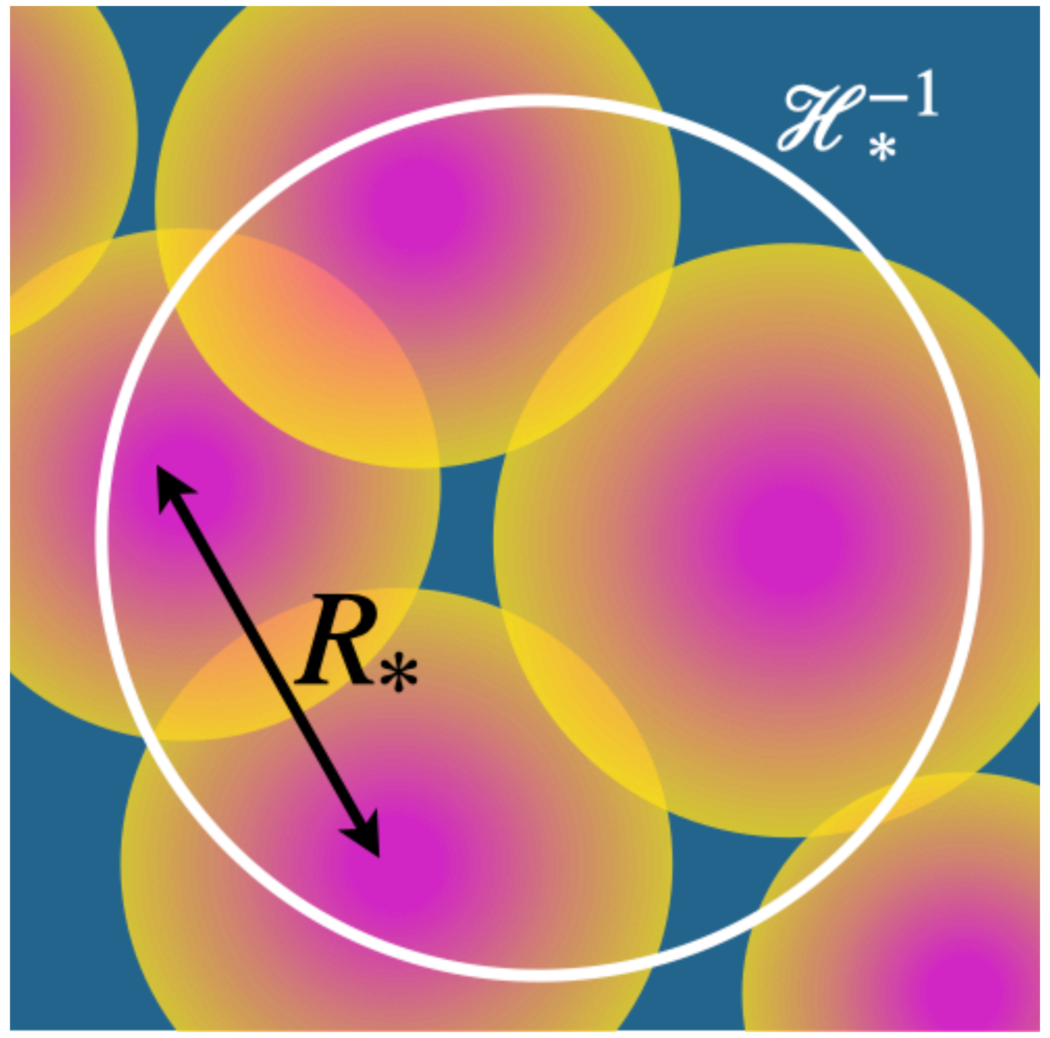


$$k_p = \frac{2\pi}{R_*}$$

(2) General relativity at next to leading order in $R_*\mathscr{H}_*$

Short wavelength expansion $R_*\mathscr{H}_* \lesssim \mathcal{O}(1)$



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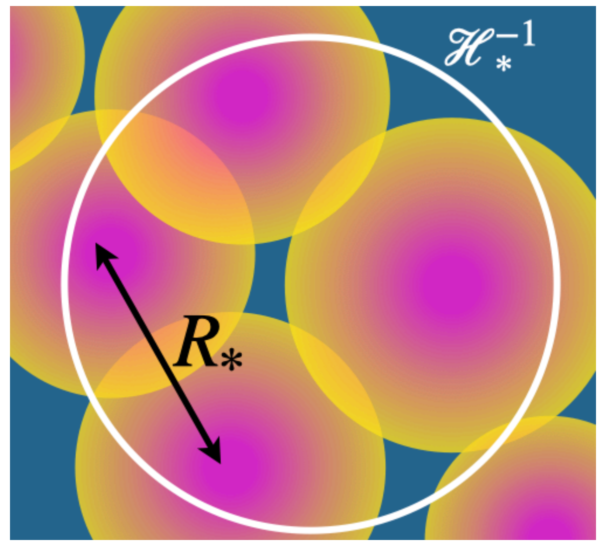

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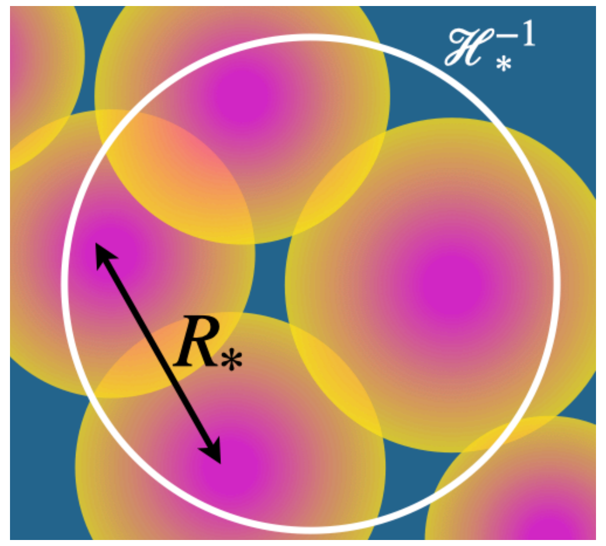

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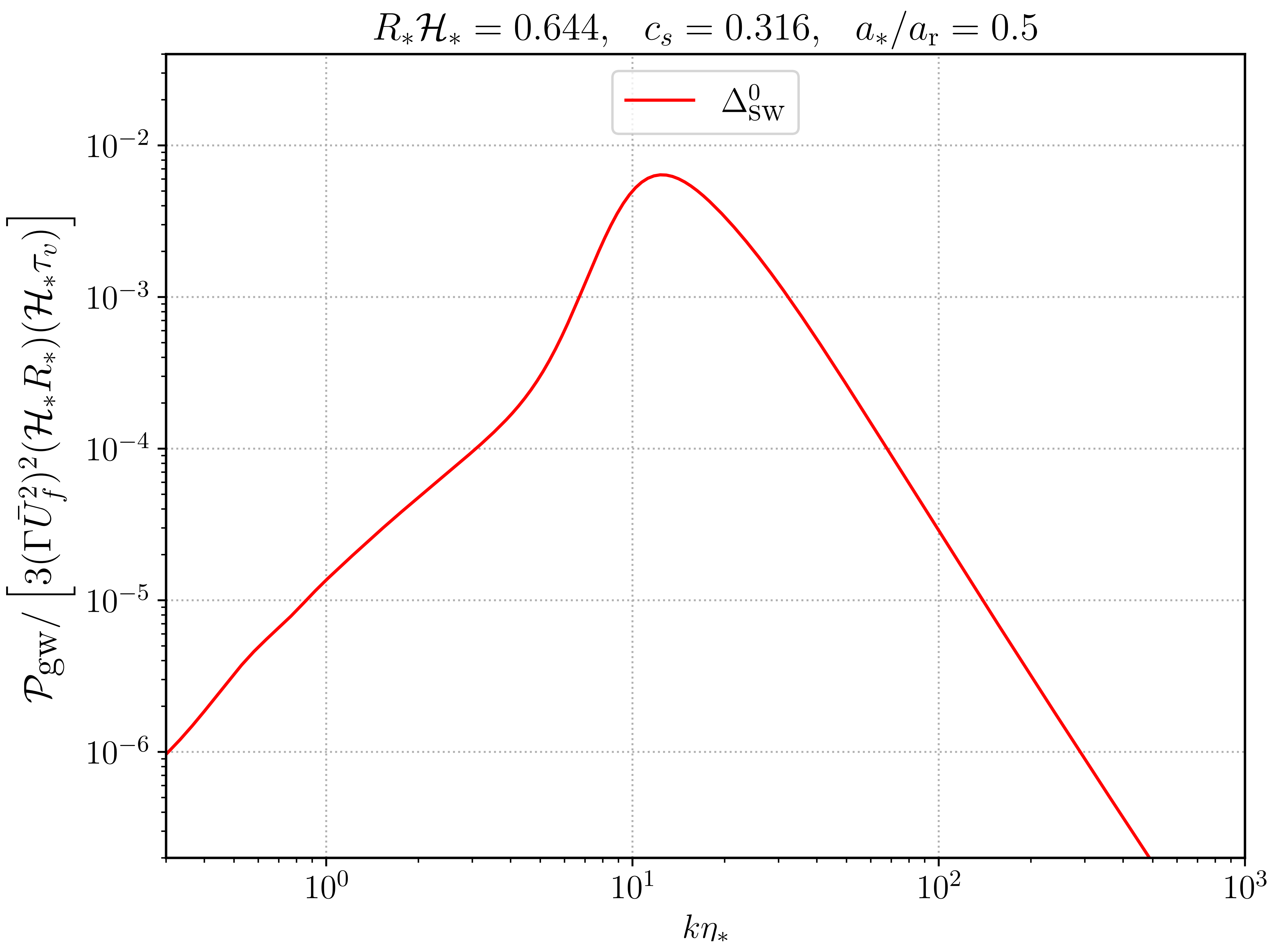
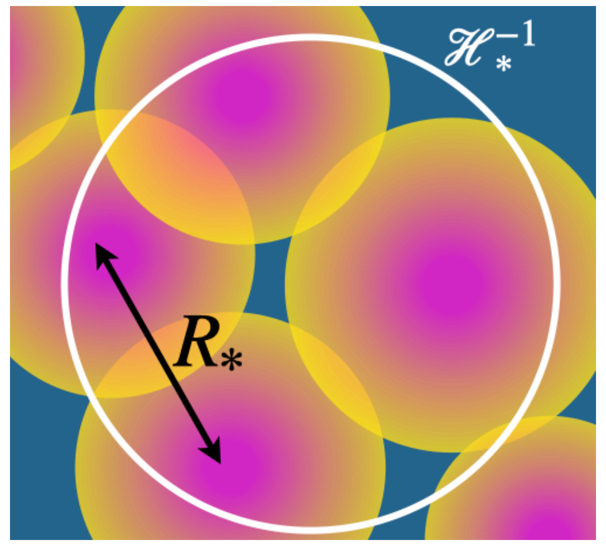
$$\mathscr{P}(kR_*) \sim \iint dq\, d\tilde{q}\, E_k(q) E_k(\tilde{q})\, \rho(k,q,\tilde{q})\, \Delta\left(k,q,\tilde{q}\right)$$

$$\Delta = \Delta^0_{sw} + \Delta^1_{sw} + \Delta^1_{gw} + \Delta_\Phi$$



(2) General relativity at next to leading order in $R_*\mathscr{H}_*$



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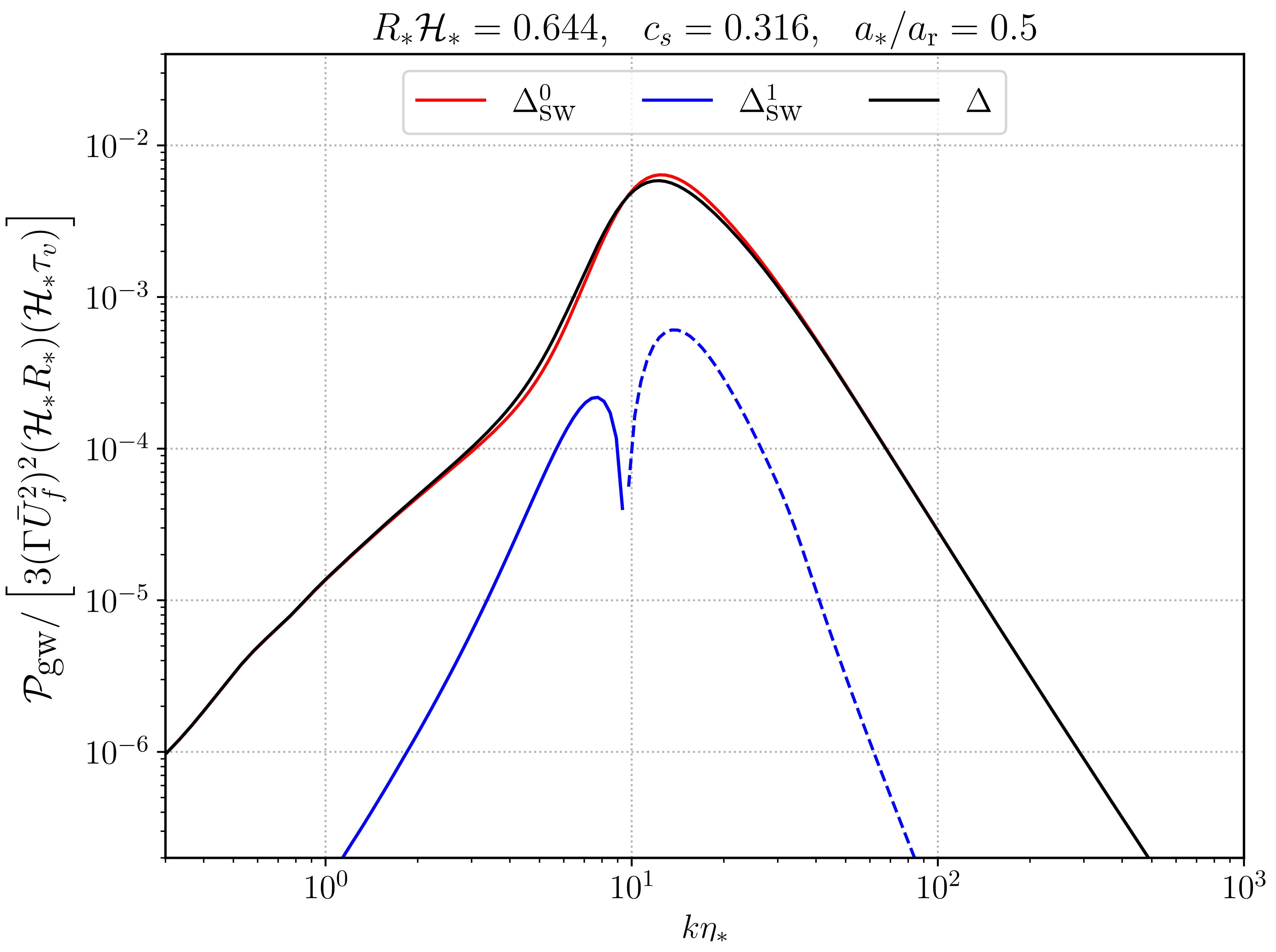

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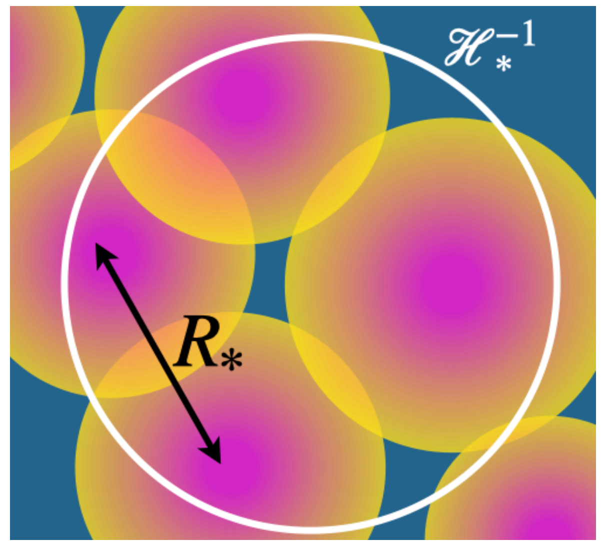

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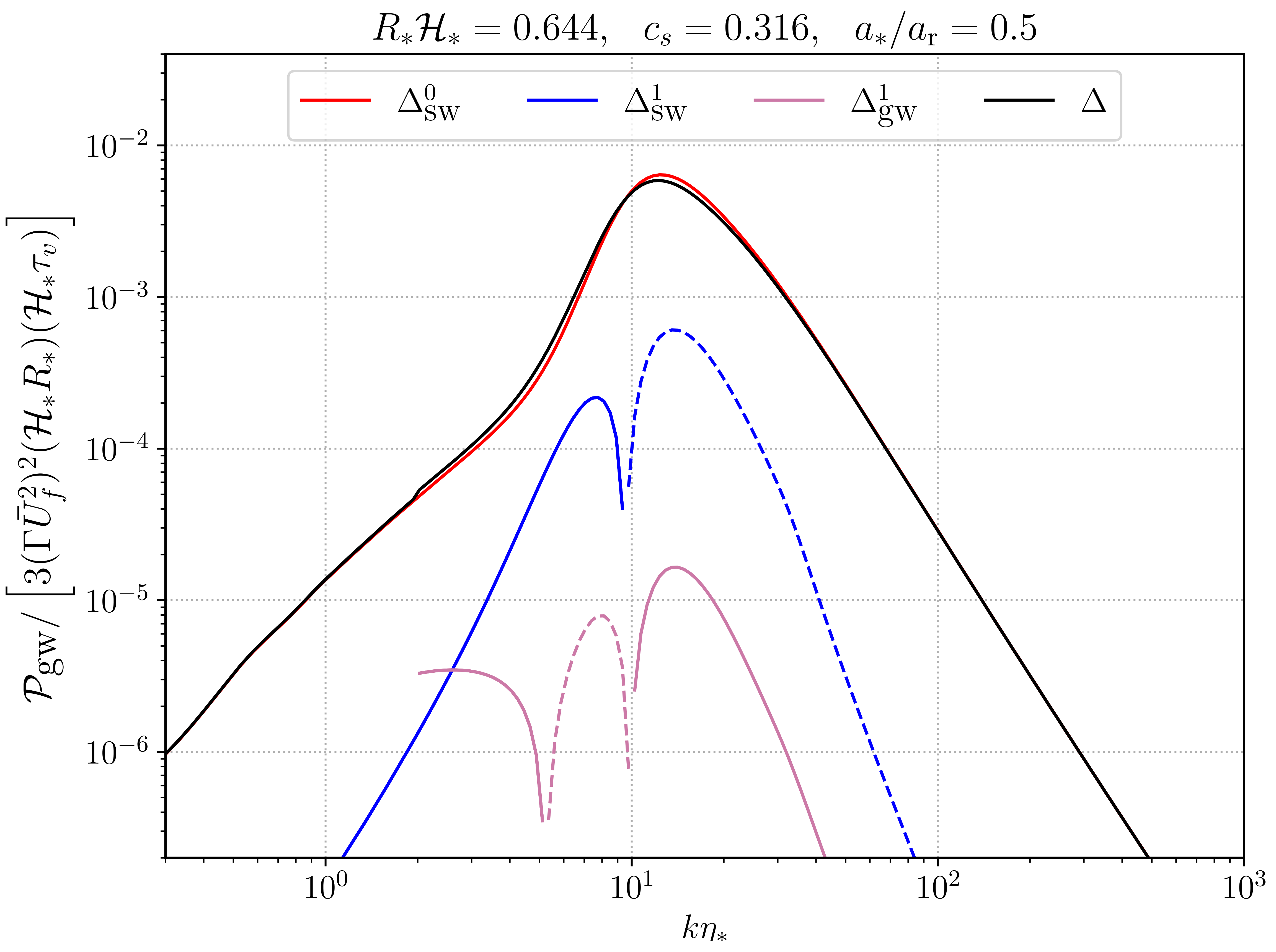

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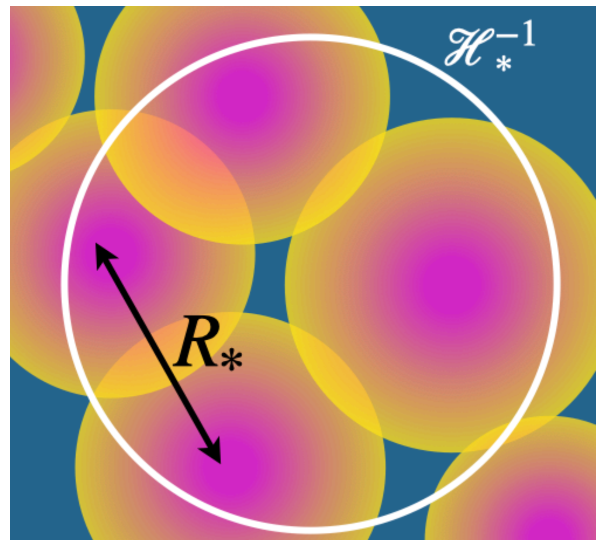

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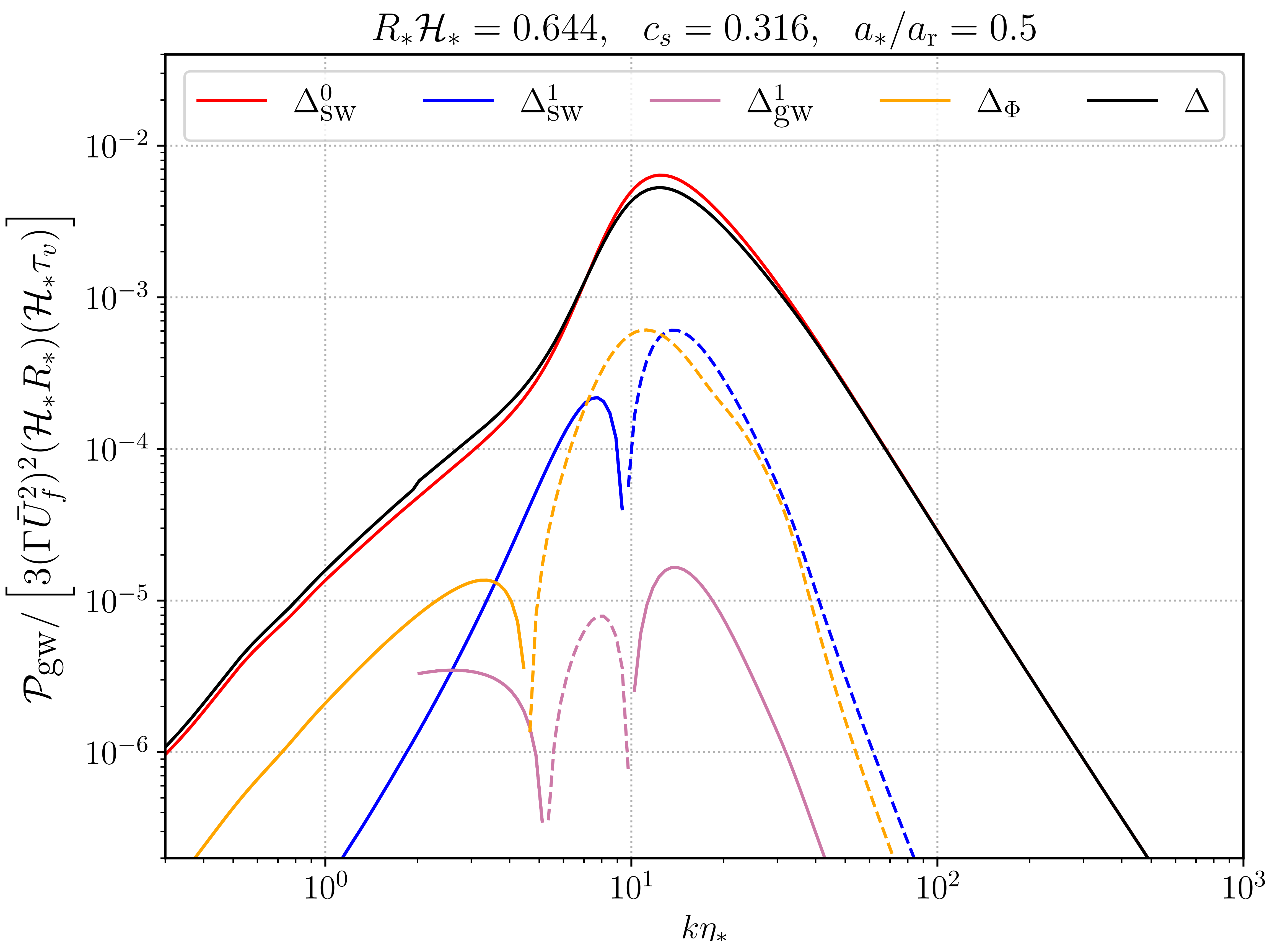
- Δ_Φ : secondary GWs sourced by curvature perturbations Φ

$$\mathcal{S}_{ij} = v_i v_j + \frac{1}{4\pi G \bar{w} a^2} \partial_i \Phi \partial_j \Phi$$



Summary

❋ Softening of the Equation of State:

- Suppression of background energy
- Friction in sound waves

} Homogeneous suppression $\mathcal{O}(1)$ at all frequencies
Independent on the bubble size

❋ General relativistic effects at quadratic order:

- Modify propagation of sw and GW
- Secondary GWs from curvature Φ

} Frequency-dependent corrections $\mathcal{O}(R_* \mathcal{H}_*)^2$
Relevant for large bubbles

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❋ Temperature-dependent equation of state

Backup slides

Parameters used in the integration

Lifetime of the source: $\tau_v = N_{sh}\dfrac{R_*}{\bar{U}_f}$

End of acoustic phase: $\eta_{end} = \eta_* + N_{sh}\eta_{sh}$

Shock formation time: $\eta_{sh} = \dfrac{\xi_*}{\bar{U}_f}$ $\xi_* = \dfrac{1}{\bar{U}_f^2}\displaystyle\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} k^{-1} P_v(k)$

$$P_v(k) = 6\pi \frac{\bar{U}_f^2}{k_p^3} \frac{(k/k_p)^2}{1 + (k/k_p)^6}$$

R. Durrer & C. Caprini (2003), arXiv:astro-ph/0305059
J. Dahl et al. (2022), arXiv:2112.12013