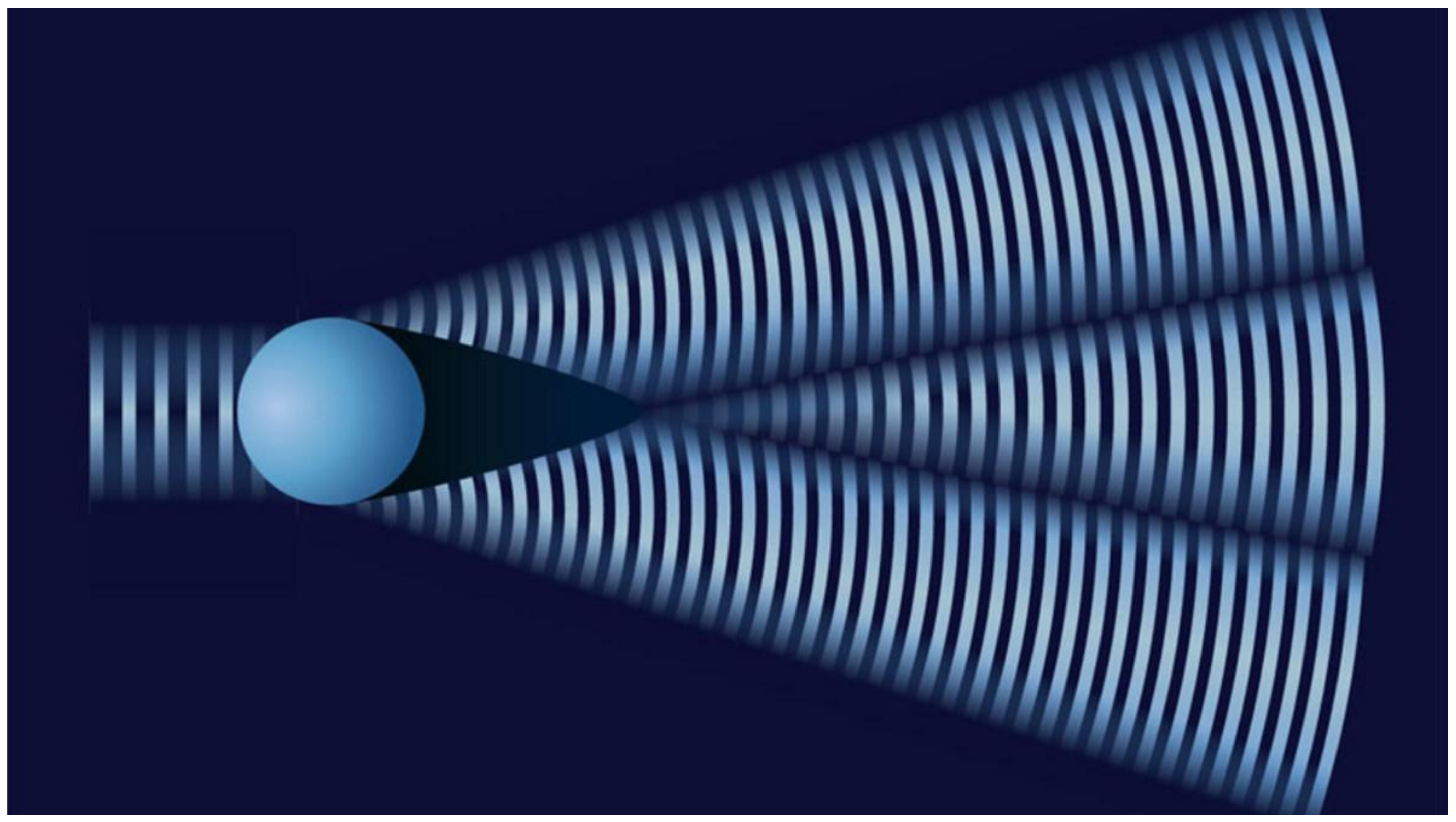
Proper time path integrals for GWs

An improved wave optics framework



Alice Garoffolo
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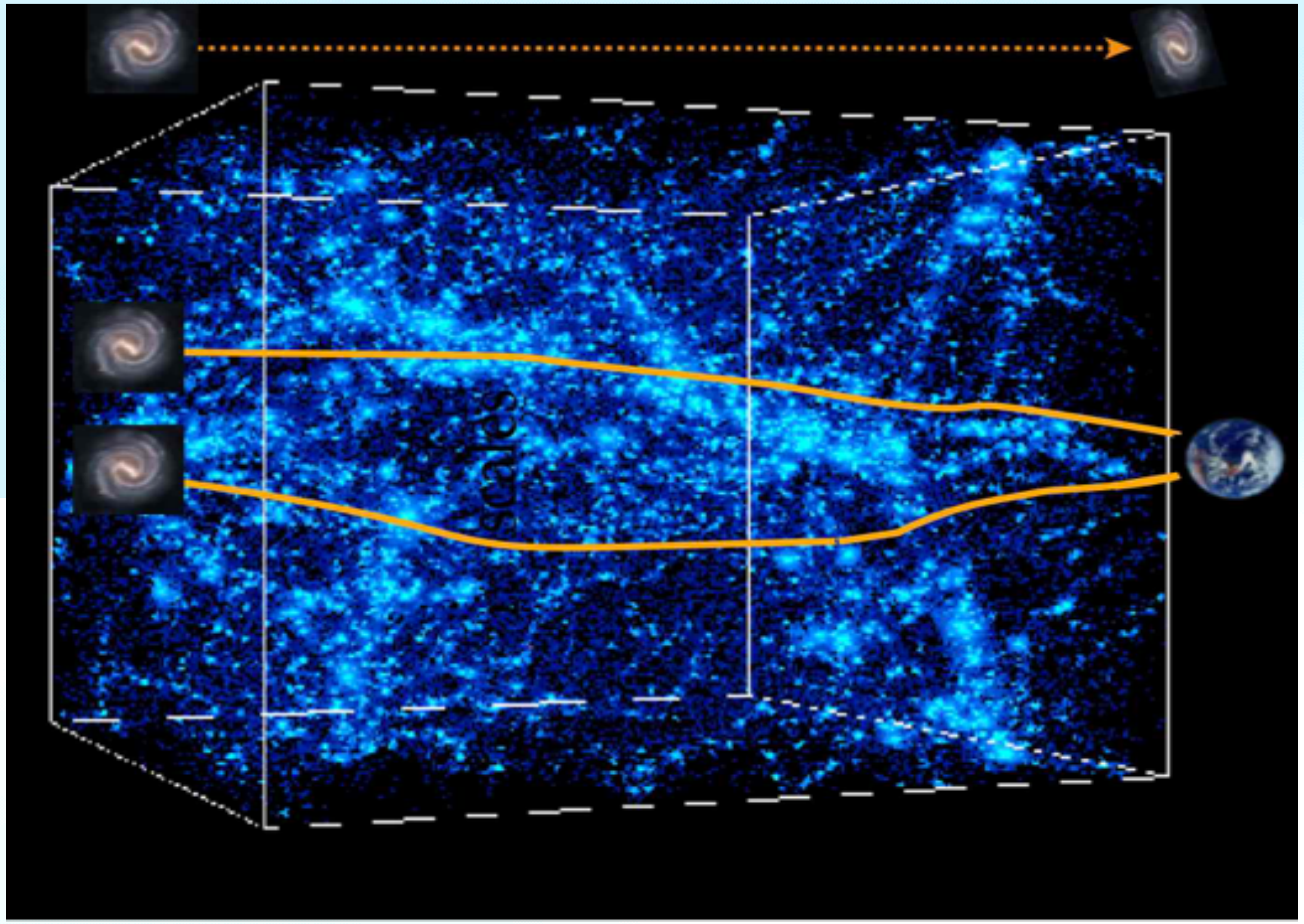
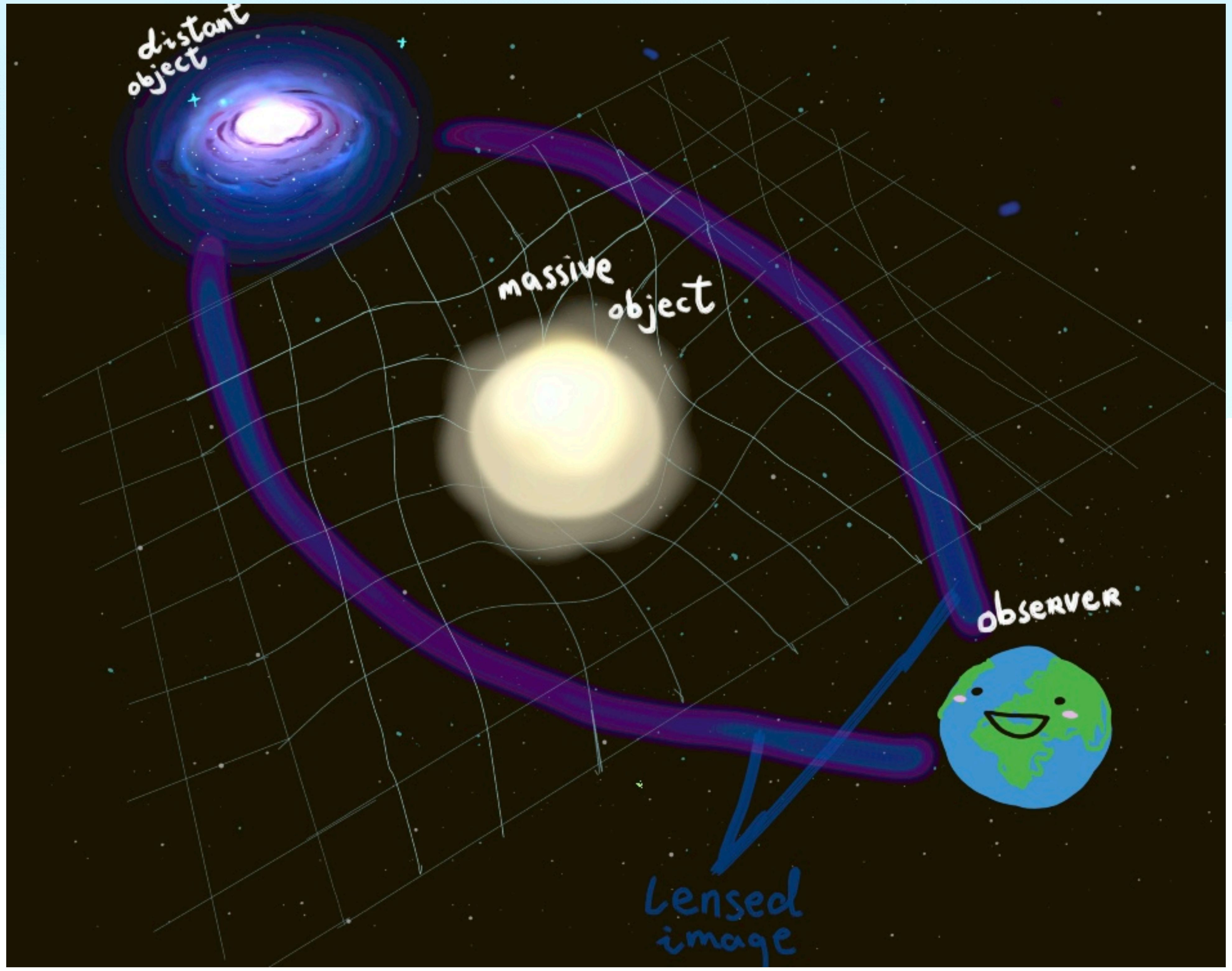
Based on: 2405.20208
G. Braga, AG, A. Ricciardone, N. Bartolo, S. Matarrese

UNIVERSITY of PENNSYLVANIA

GWs through the perturbed Universe

Probe of large scale structures and compact objects



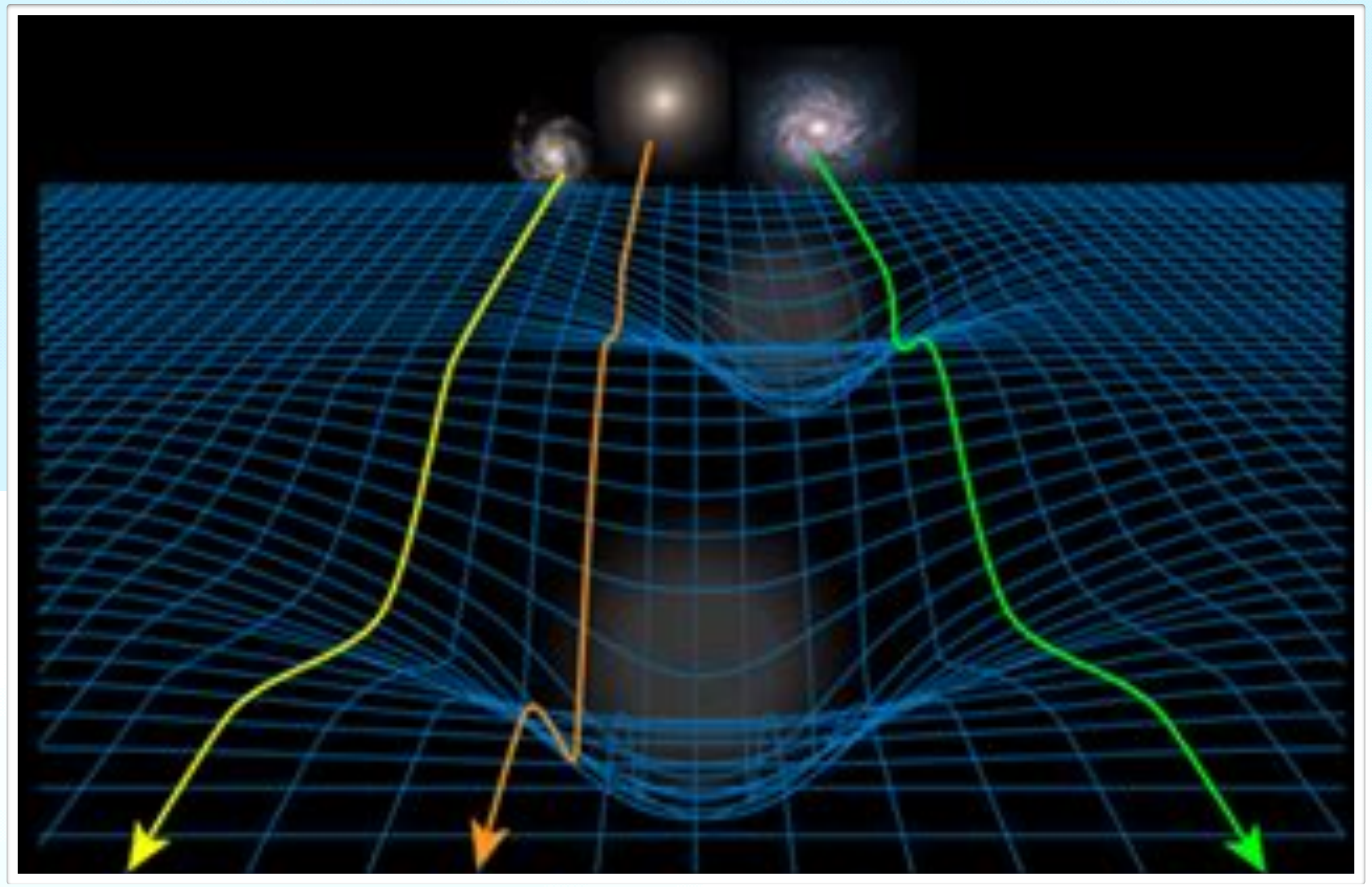


Propagation effects carry cosmological and astrophysical information

Optical regimes

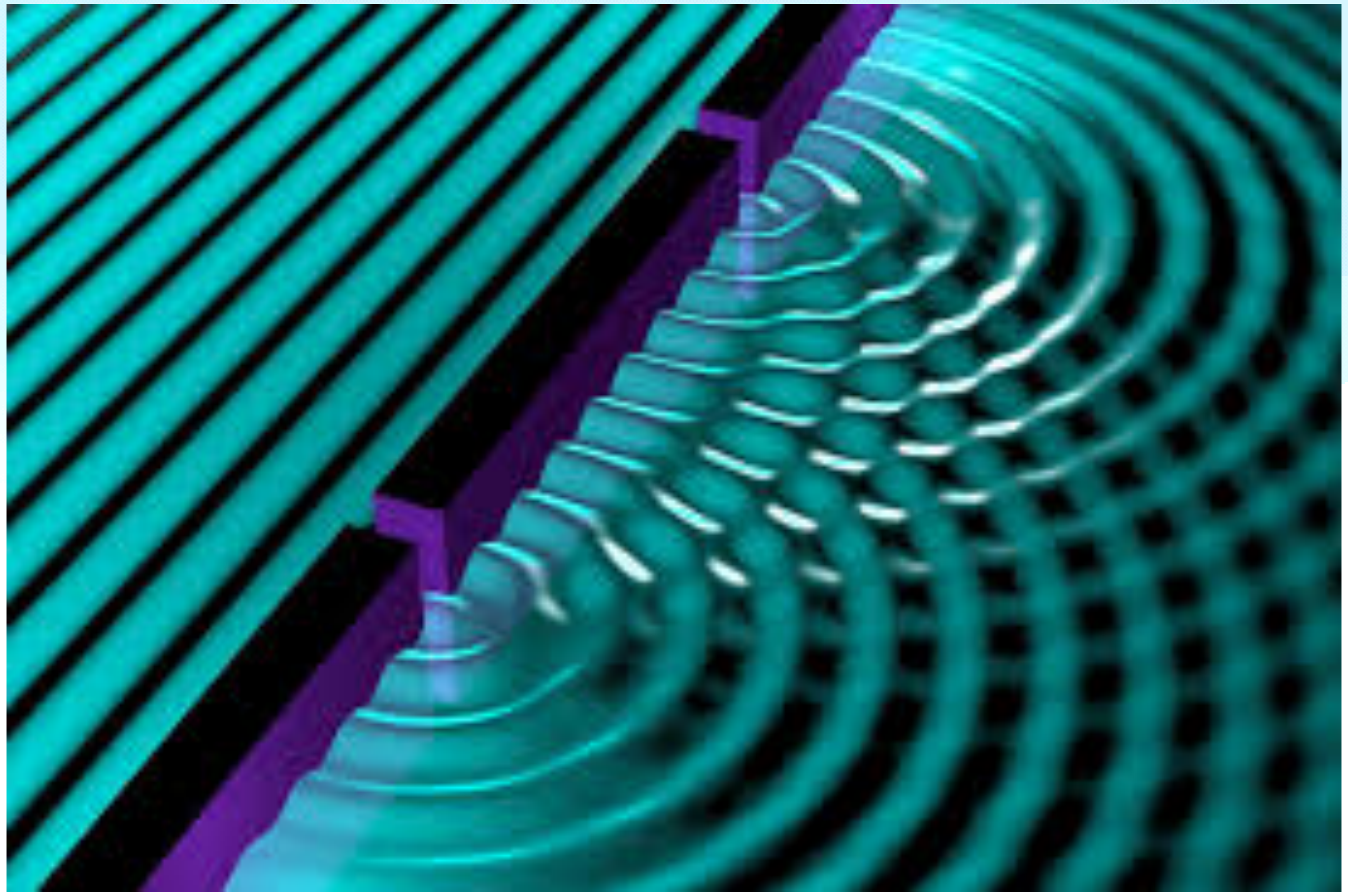
Geometric vs Wave optics

High Frequency: $\omega R_S \gg 1$



Ray description

Low Frequency: $\omega R_S \lesssim 1$



Wave effects

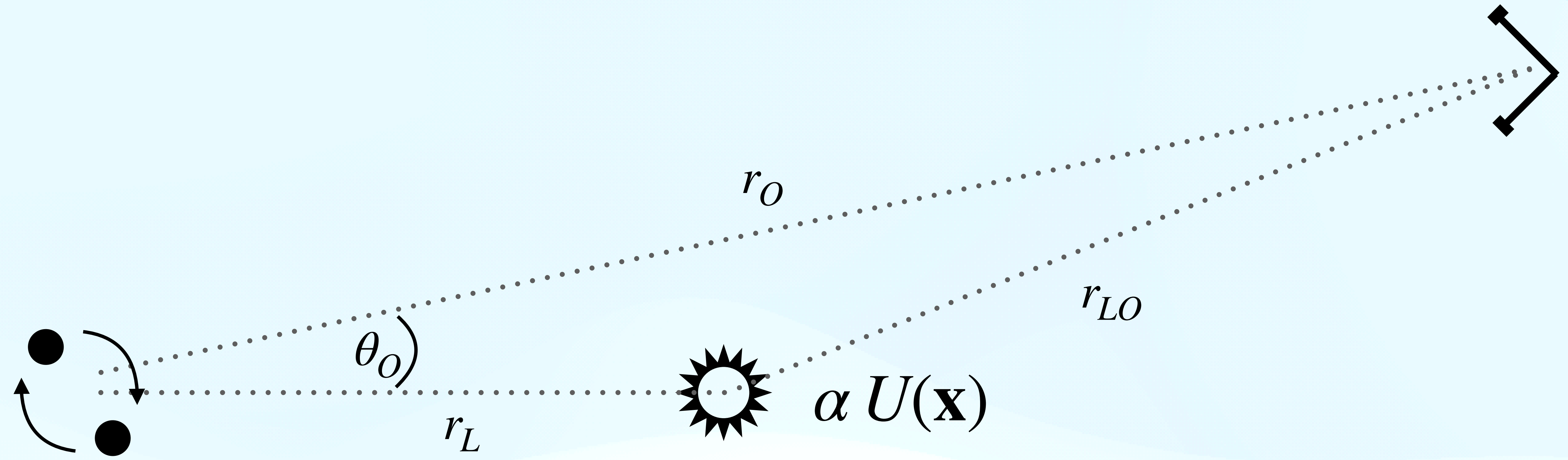
3.4 Wave effects

LISA CosGW: 2204.05434

GWs can be emitted at low frequencies ($w \lesssim 1$), allowing the observation of wave diffractive phenomena. For typical LISA sources, wave optics as in Eq. (15) needs to be considered for lenses with masses $M_L \sim 10^6 - 10^9 M_\odot$, cf. Eq. (13).

Diffraction integral for a scalar wave

Nakamura&Deguchi 1999



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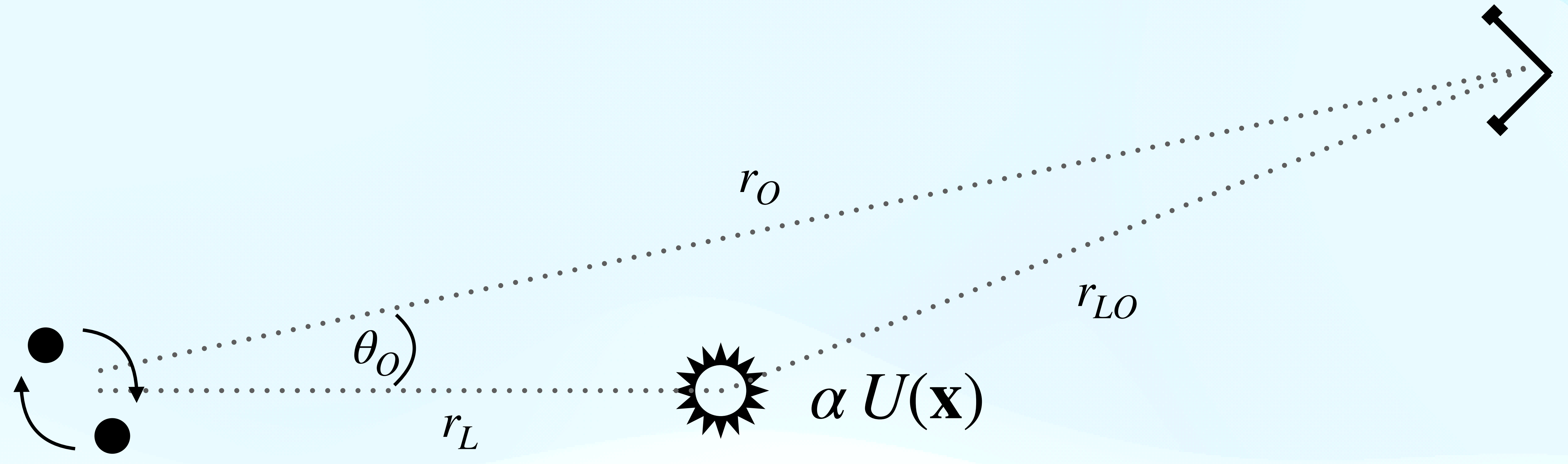

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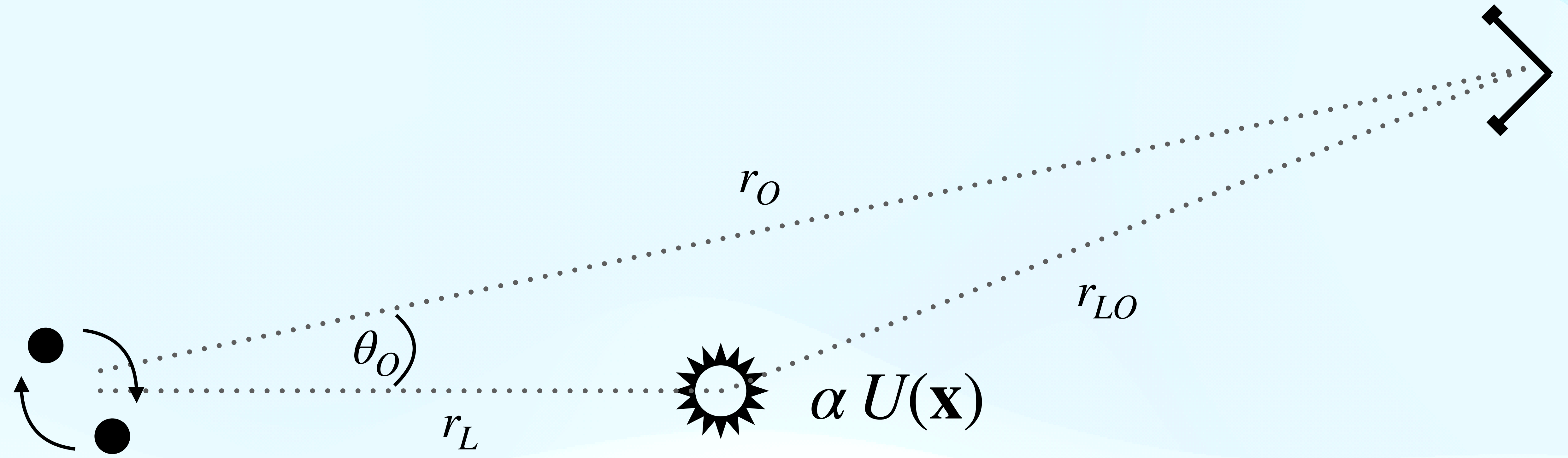

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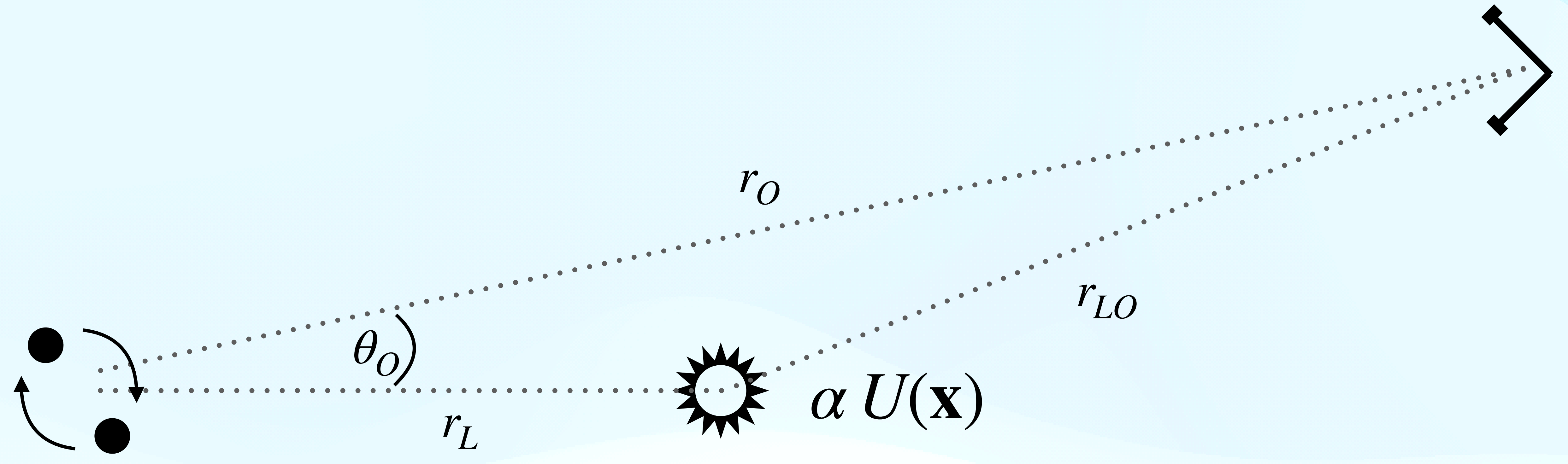

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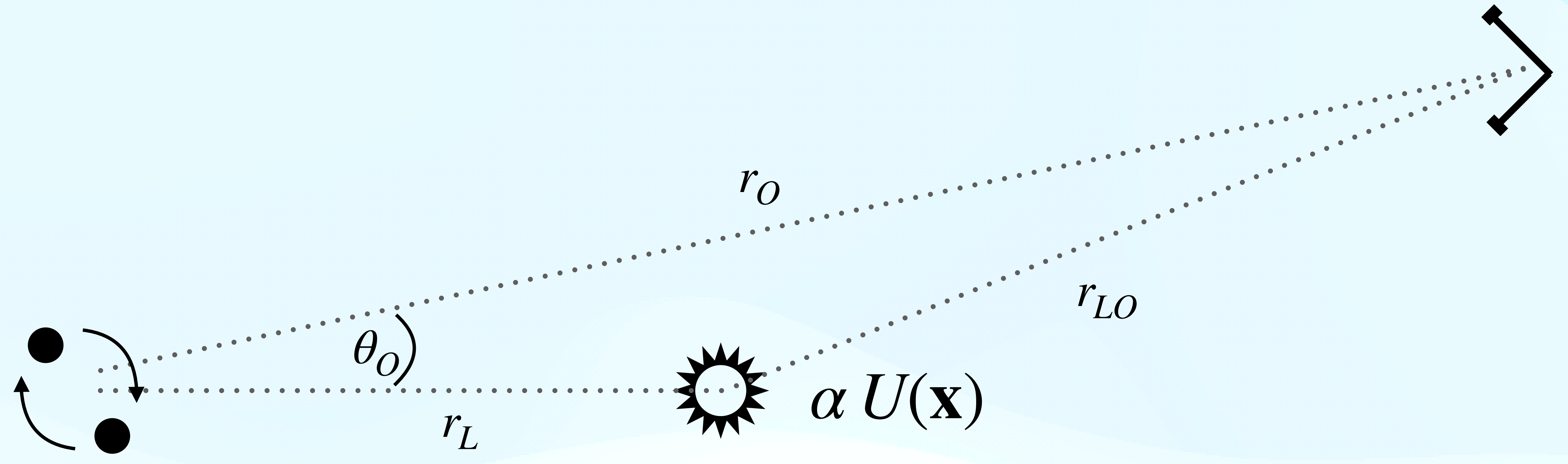

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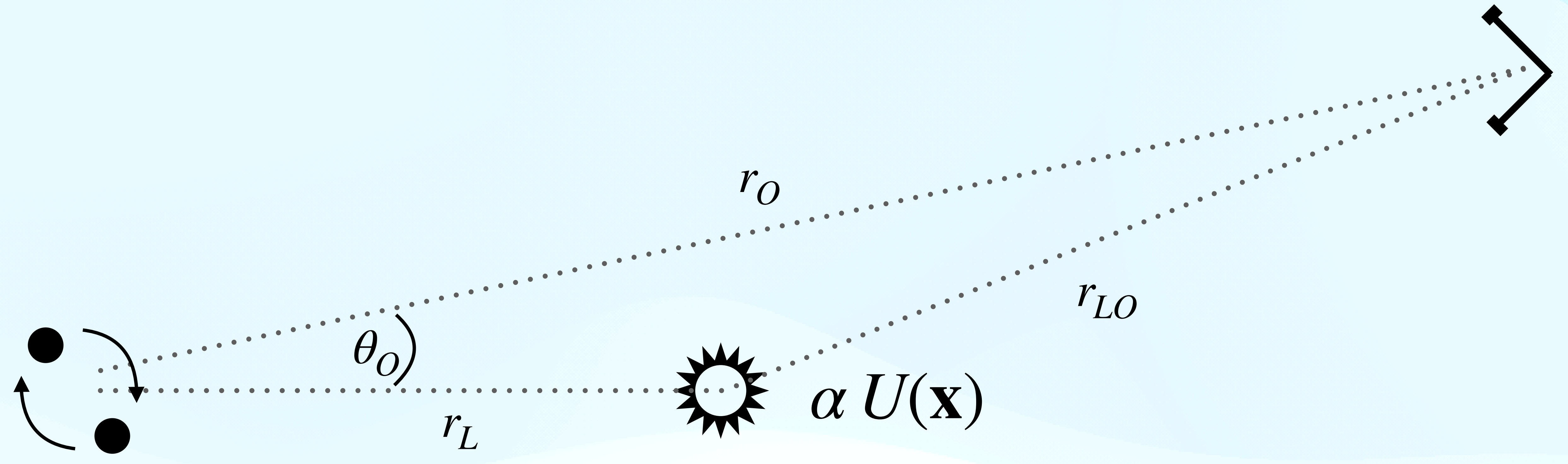

Diffraction integral:

$$F(\vec{r}_O) = \int \mathcal{D}\boldsymbol{\theta}(r)\exp\left\{i\omega\int_0^{r_O} dr\left[\frac{r^2}{2}|\dot{\boldsymbol{\theta}}|^2 - 2\alpha U(r,\boldsymbol{\theta})\right]\right\}$$

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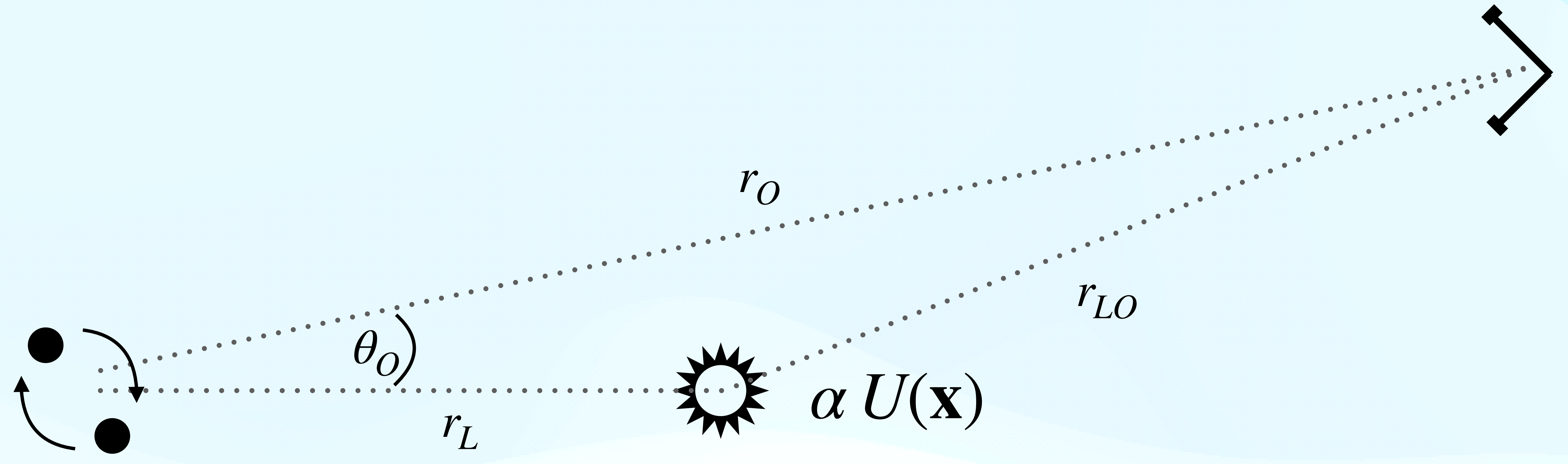

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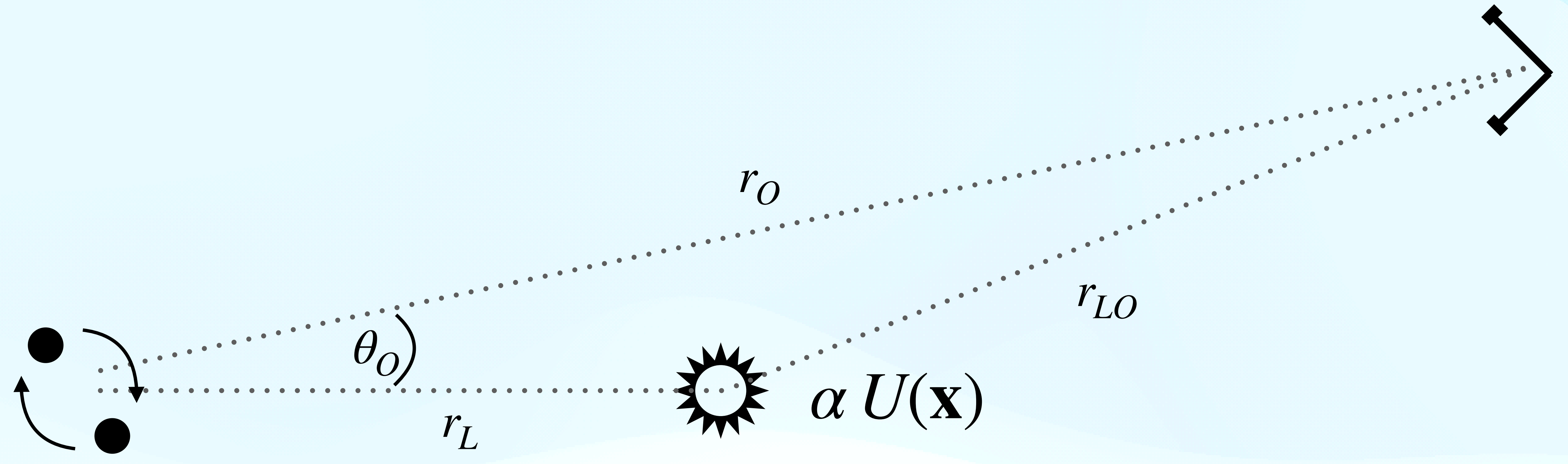

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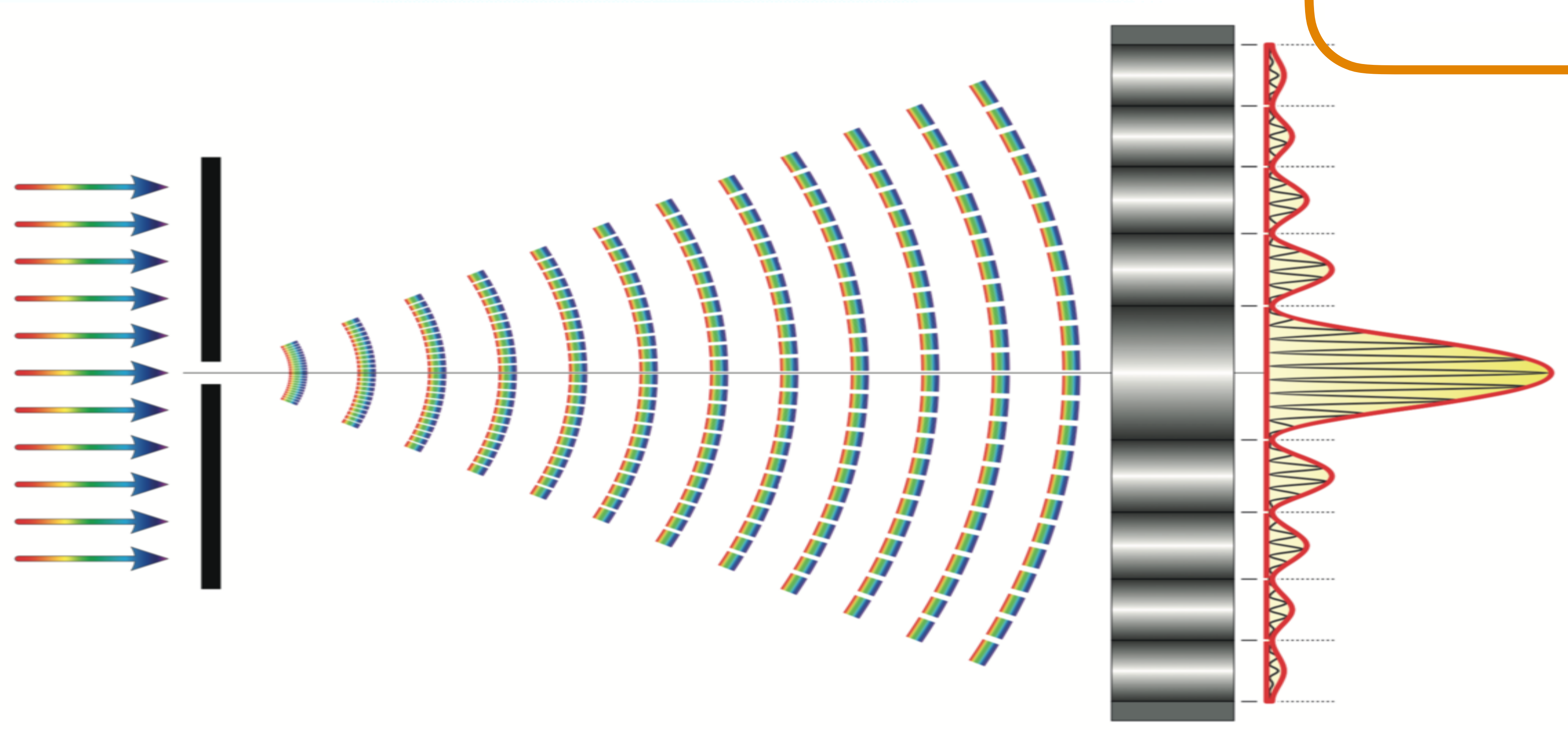

Analogy between wave and quantum effects:
interference between all paths.
Geometric optics = classical limit

Diffraction integral: Pros and Cons

PROs	CONs

Diffraction integral: Pros and Cons

PROs

1. Wave optics effects are frequency dependent

CONs

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2. Easy high frequency limit

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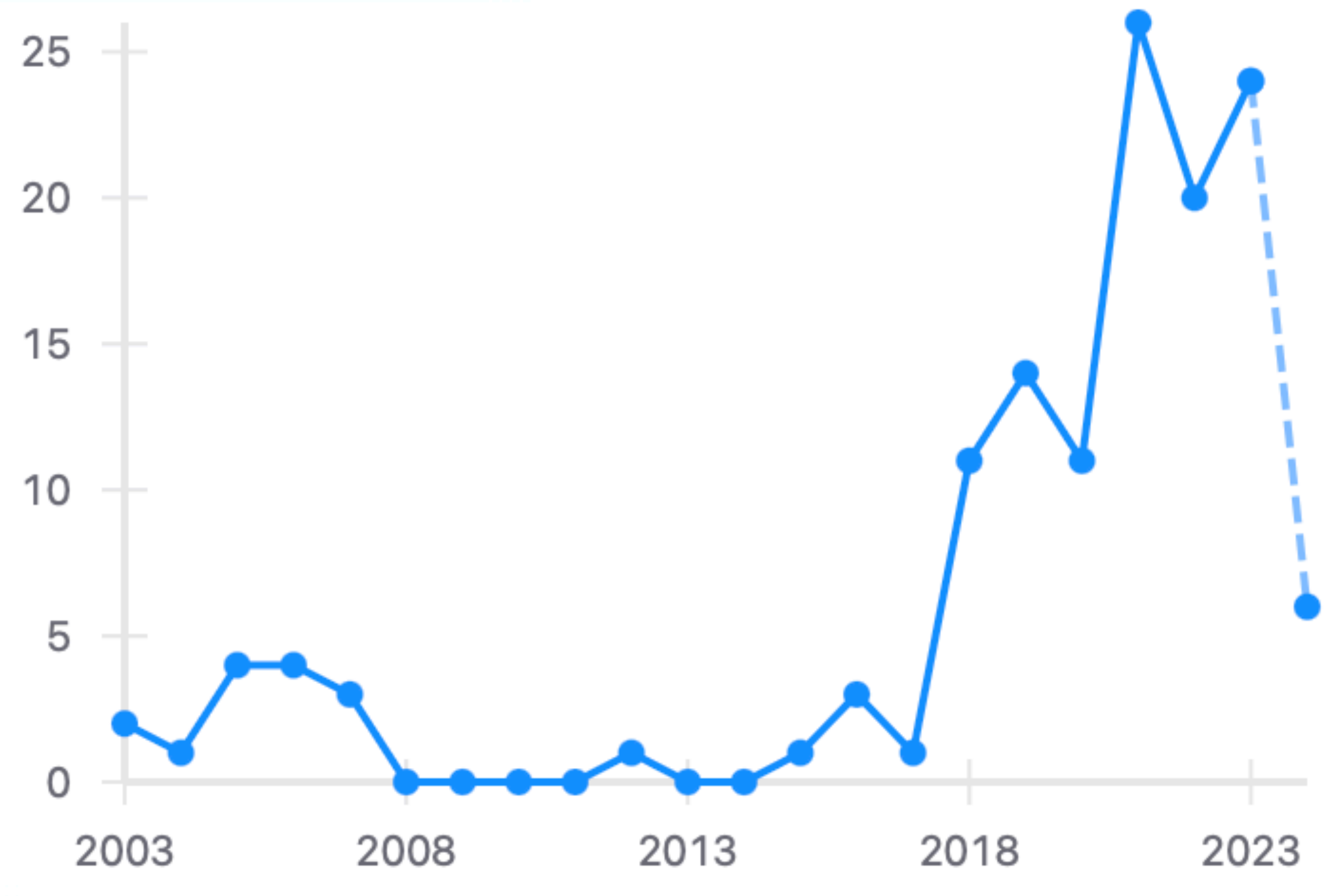
4. Already used for: lens parameter estimation, constraints PBH abundance, matter PS at small scales,...



Citation per year of Nakamura&Deguchi 1999

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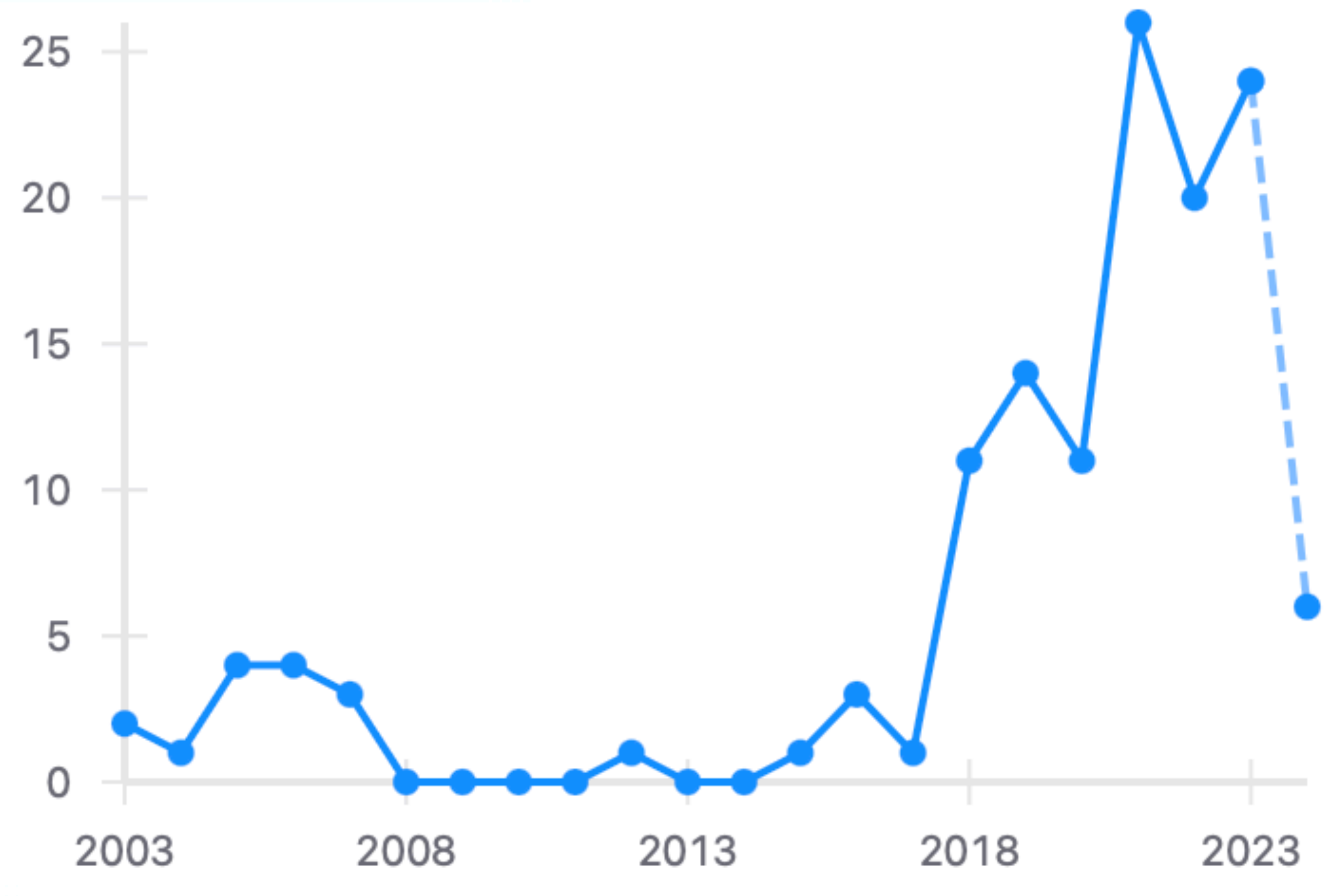

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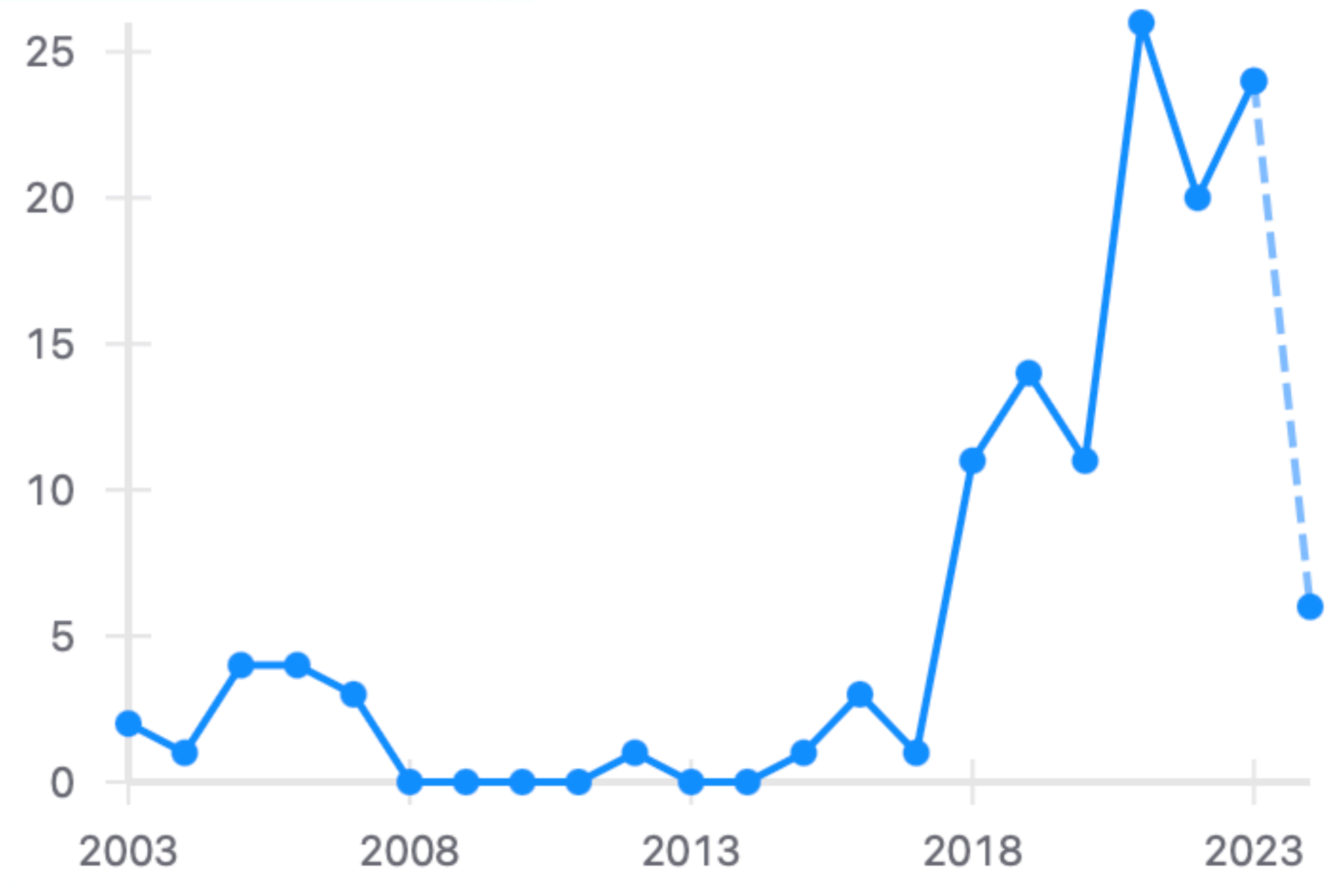

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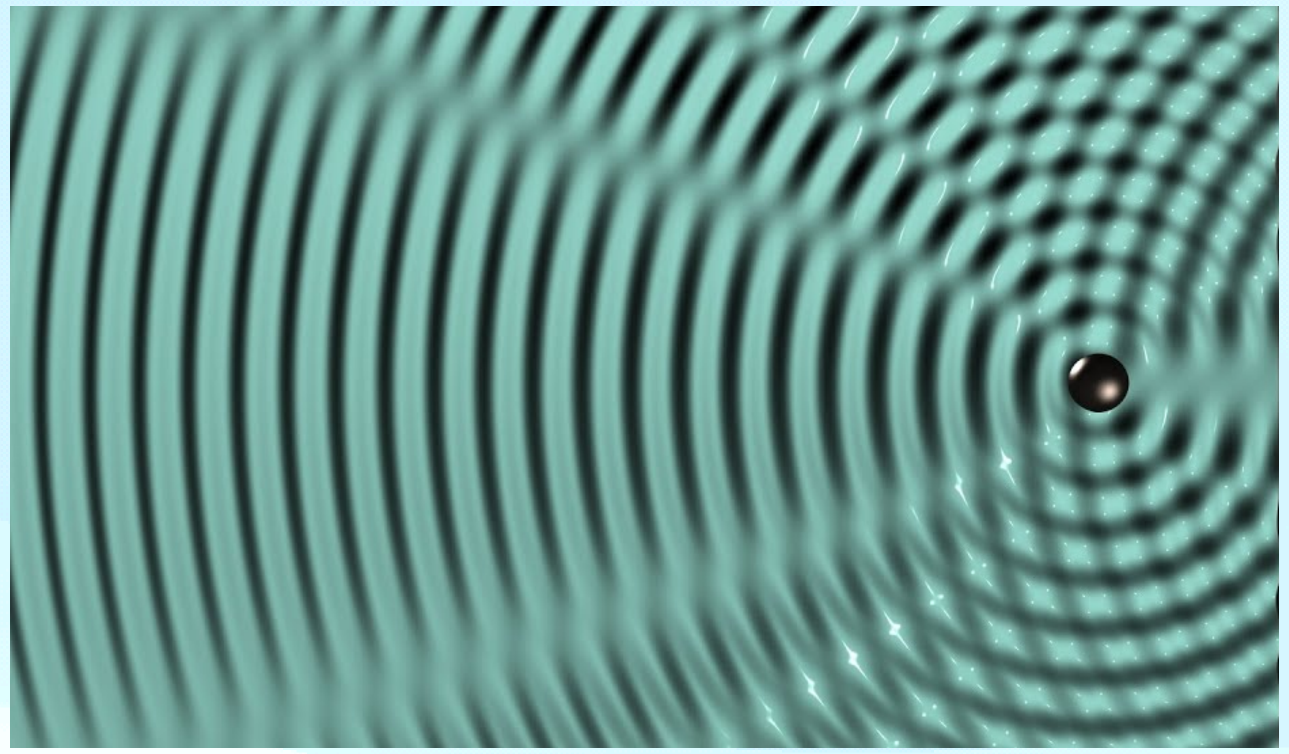
Proper time path integrals

2405.20208

Proper time path integral

Finding a Schrödinger equation without Eikonal approximation



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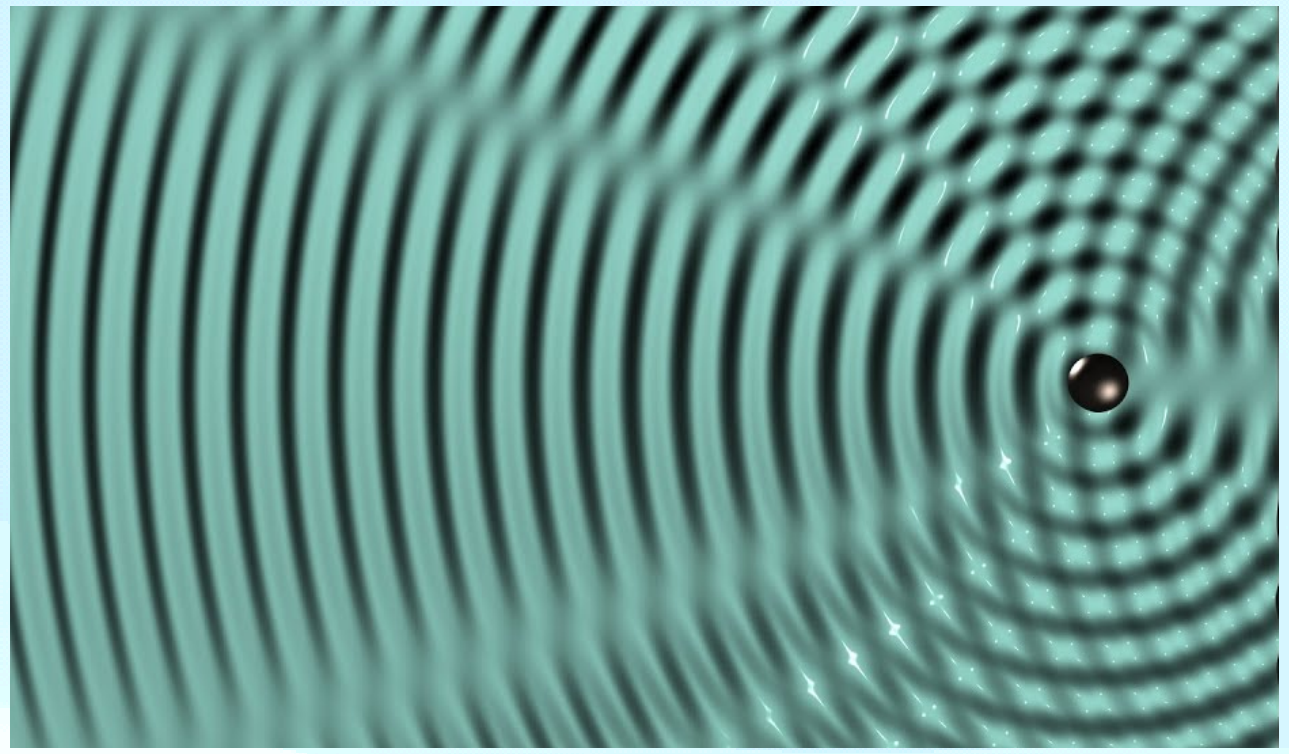

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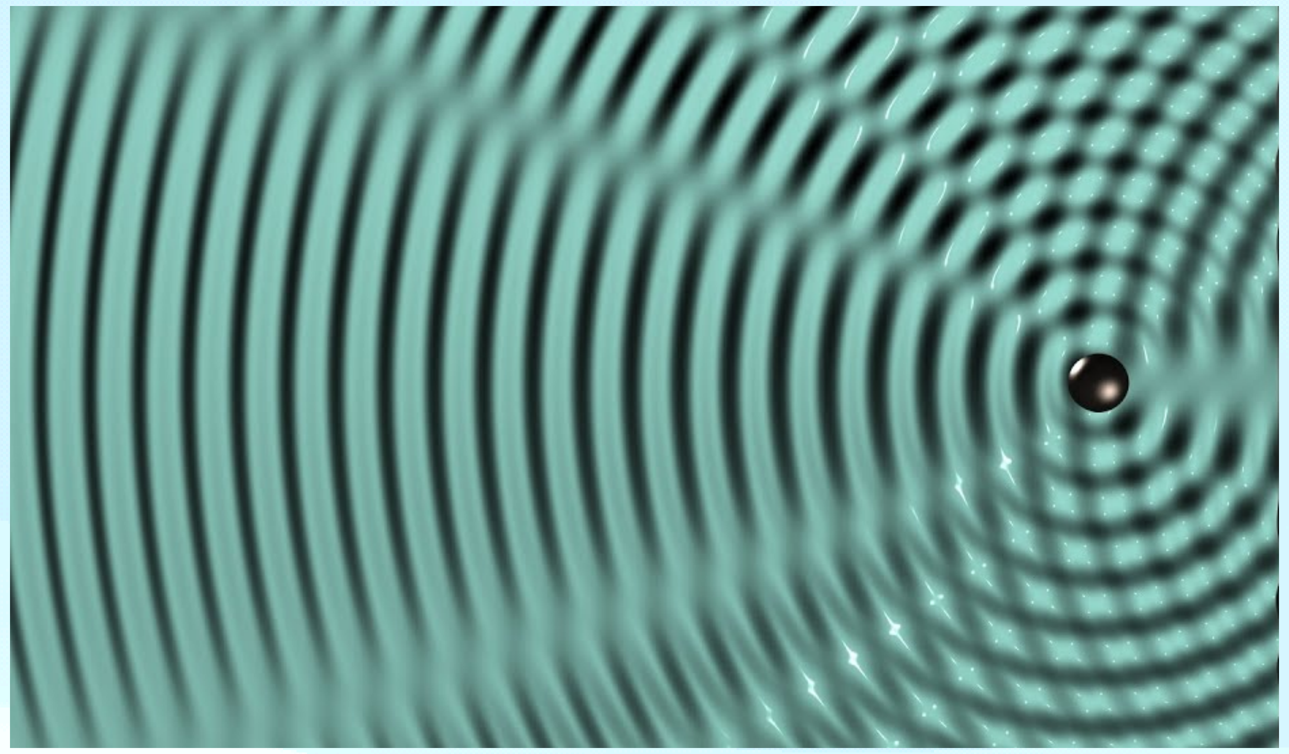

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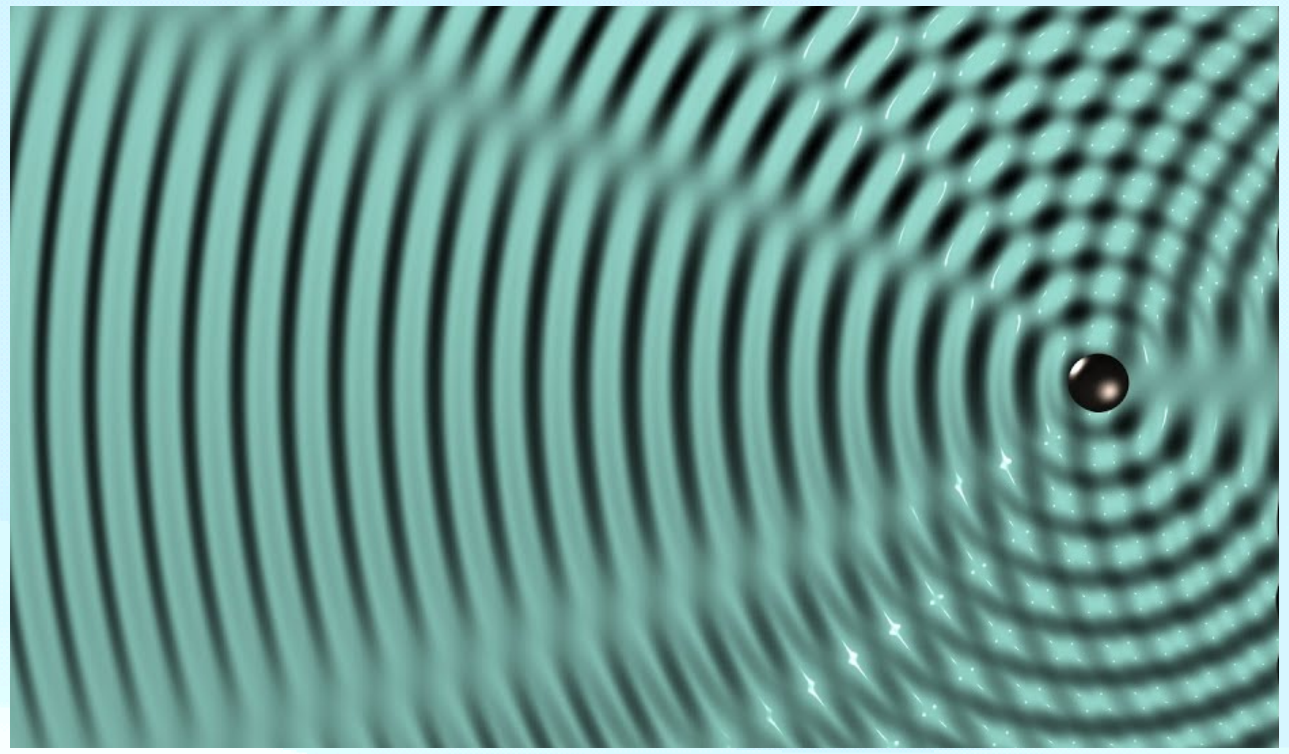

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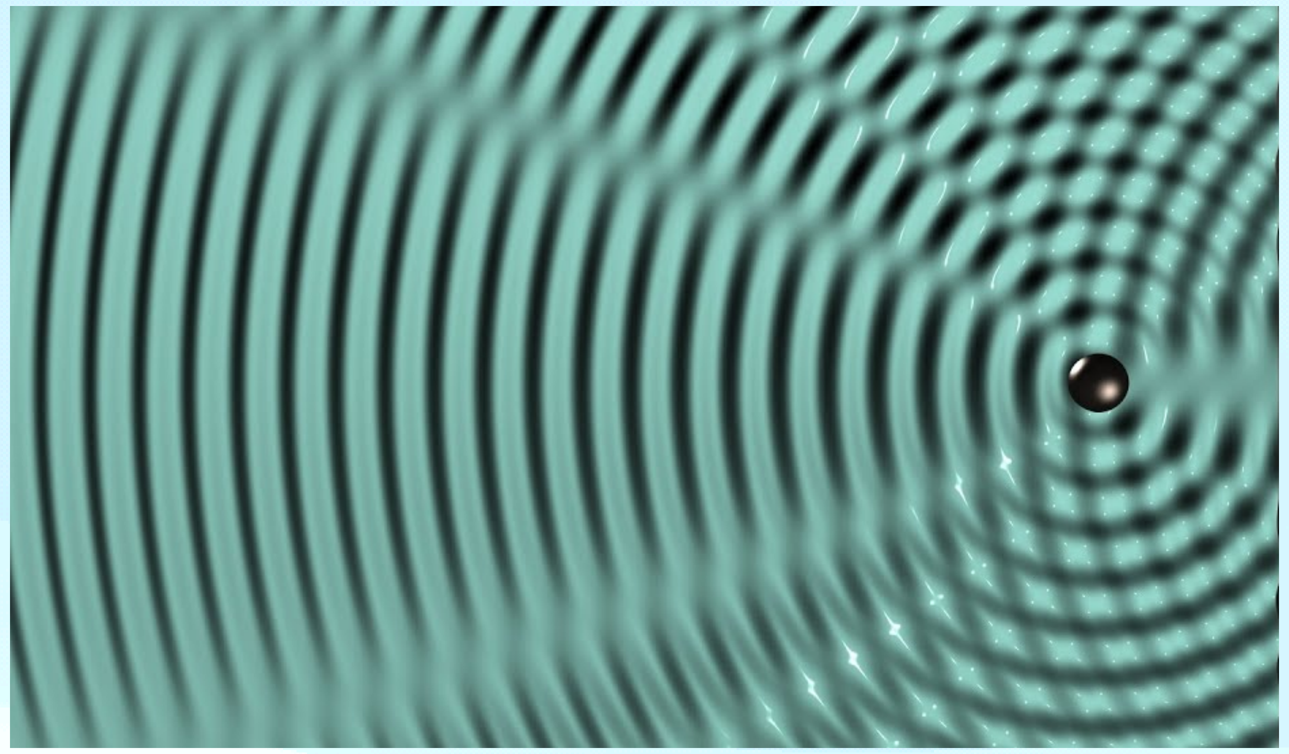

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Sum over paths

Proper time path integral

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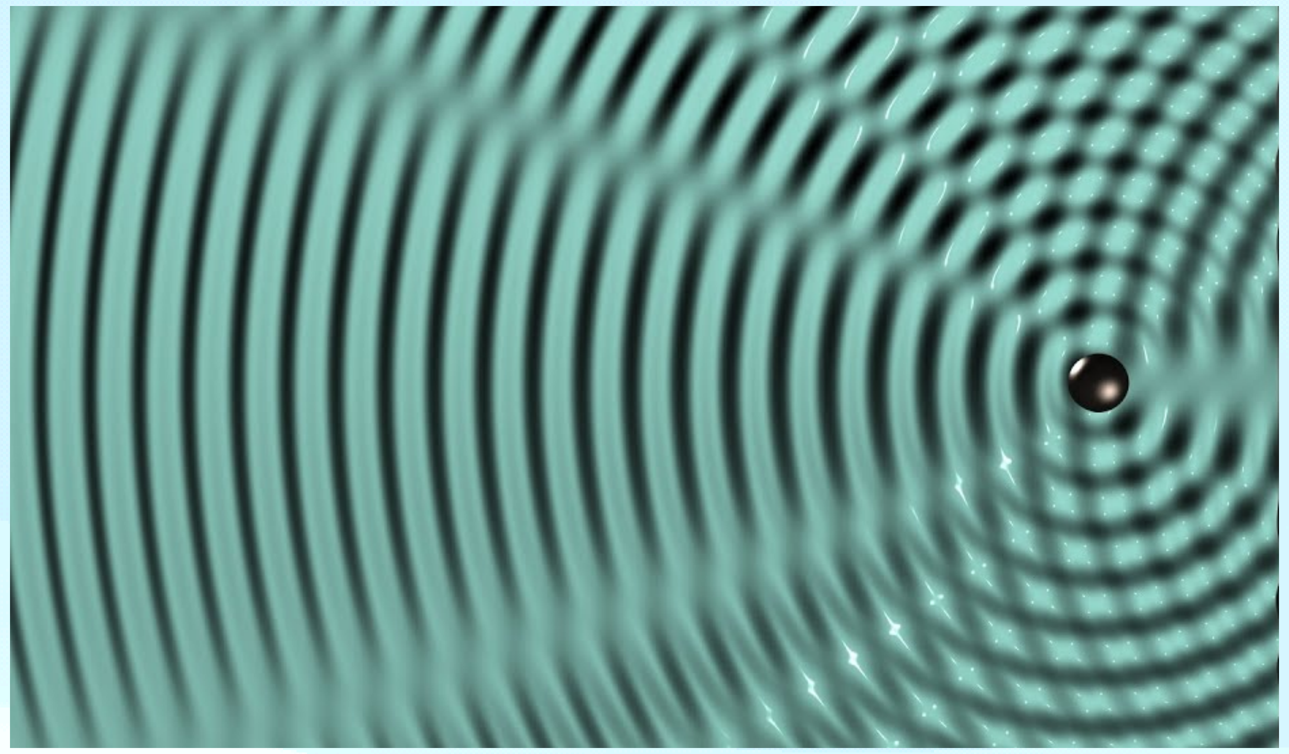

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Particle action

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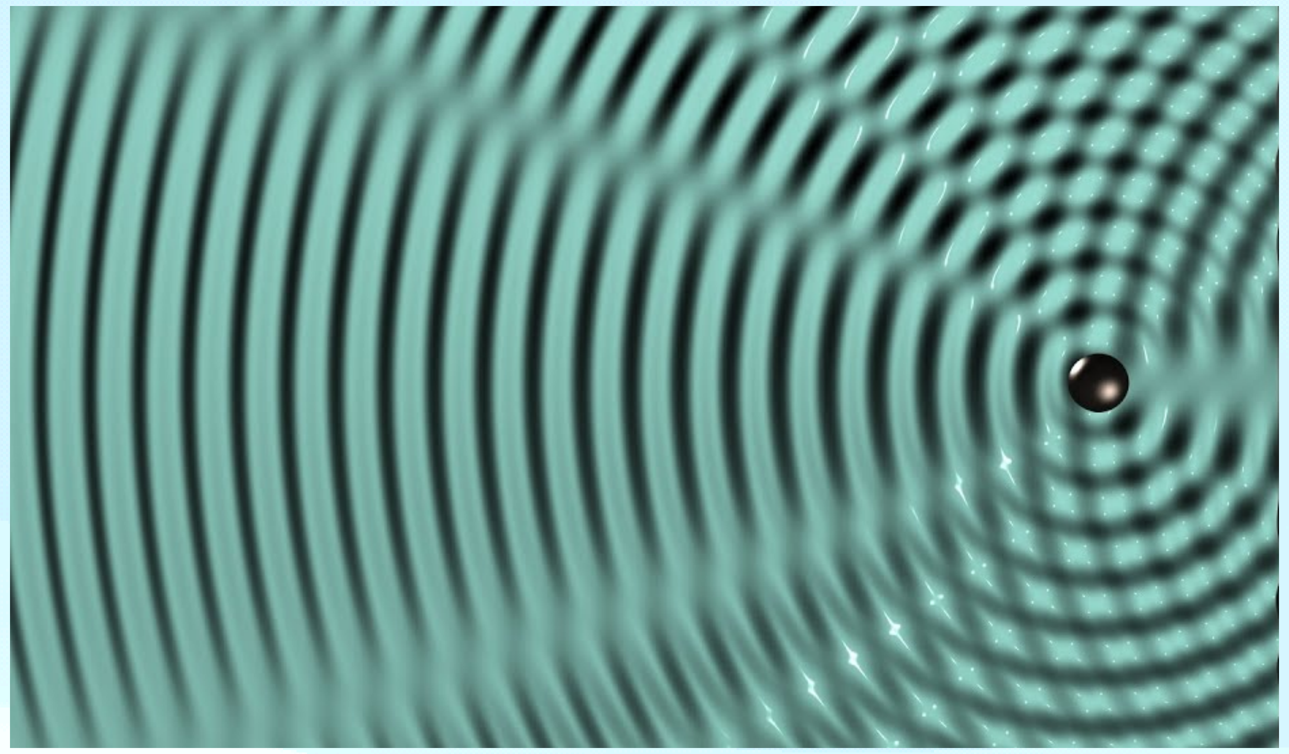

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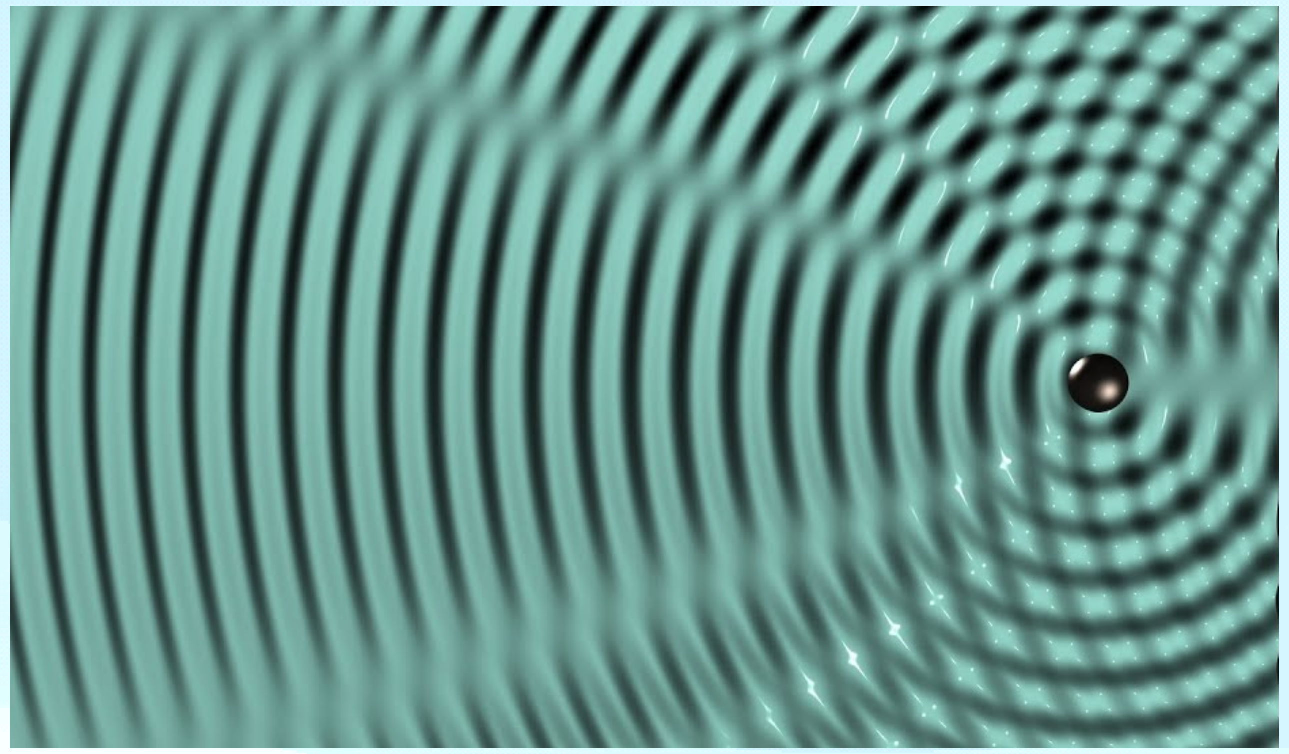

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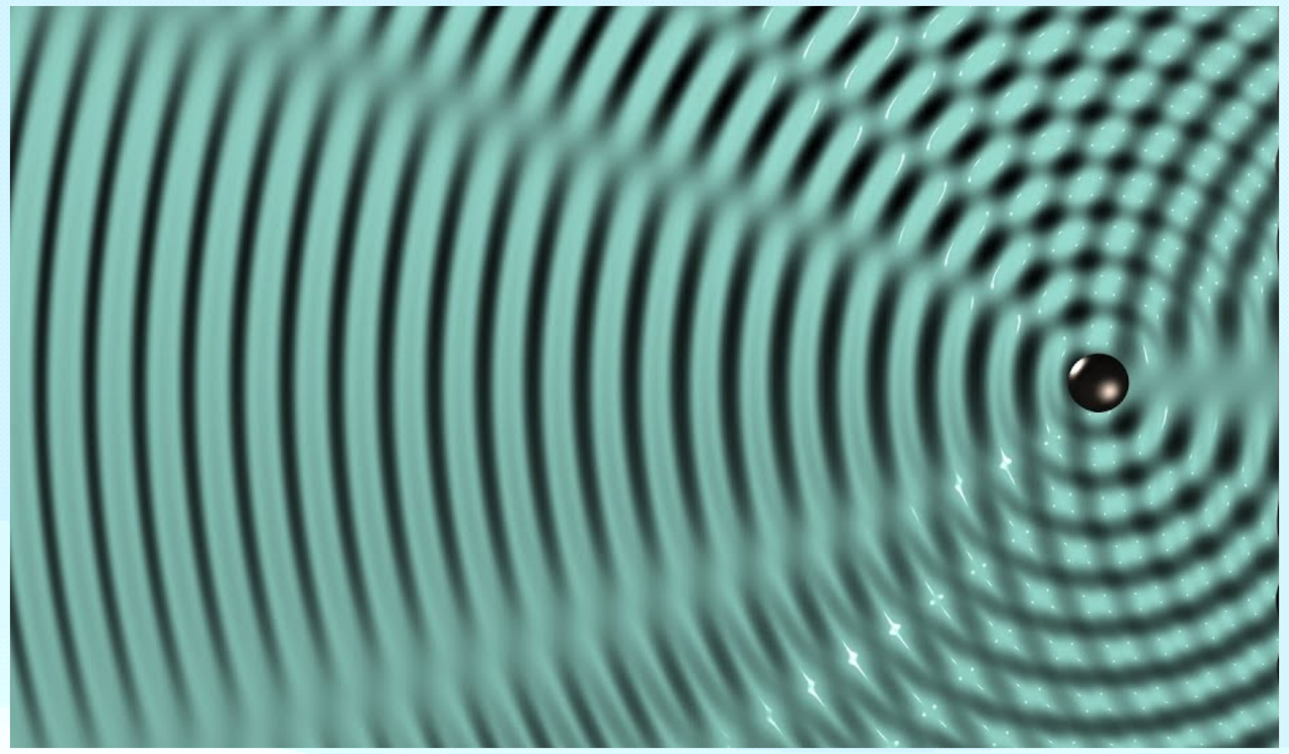
Particle action

Integrate away τ

Sum over paths

Proper time path integral

Finding a Schrödinger equation without Eikonal approximation



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Exact particle-like solution WITHOUT the need of Eikonal approximation

What you can find in 2405.20208:

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   ($\delta\hat{S}/\delta\tau = 0$ and $\delta\hat{S}/\delta\mathbf{x} = 0$)

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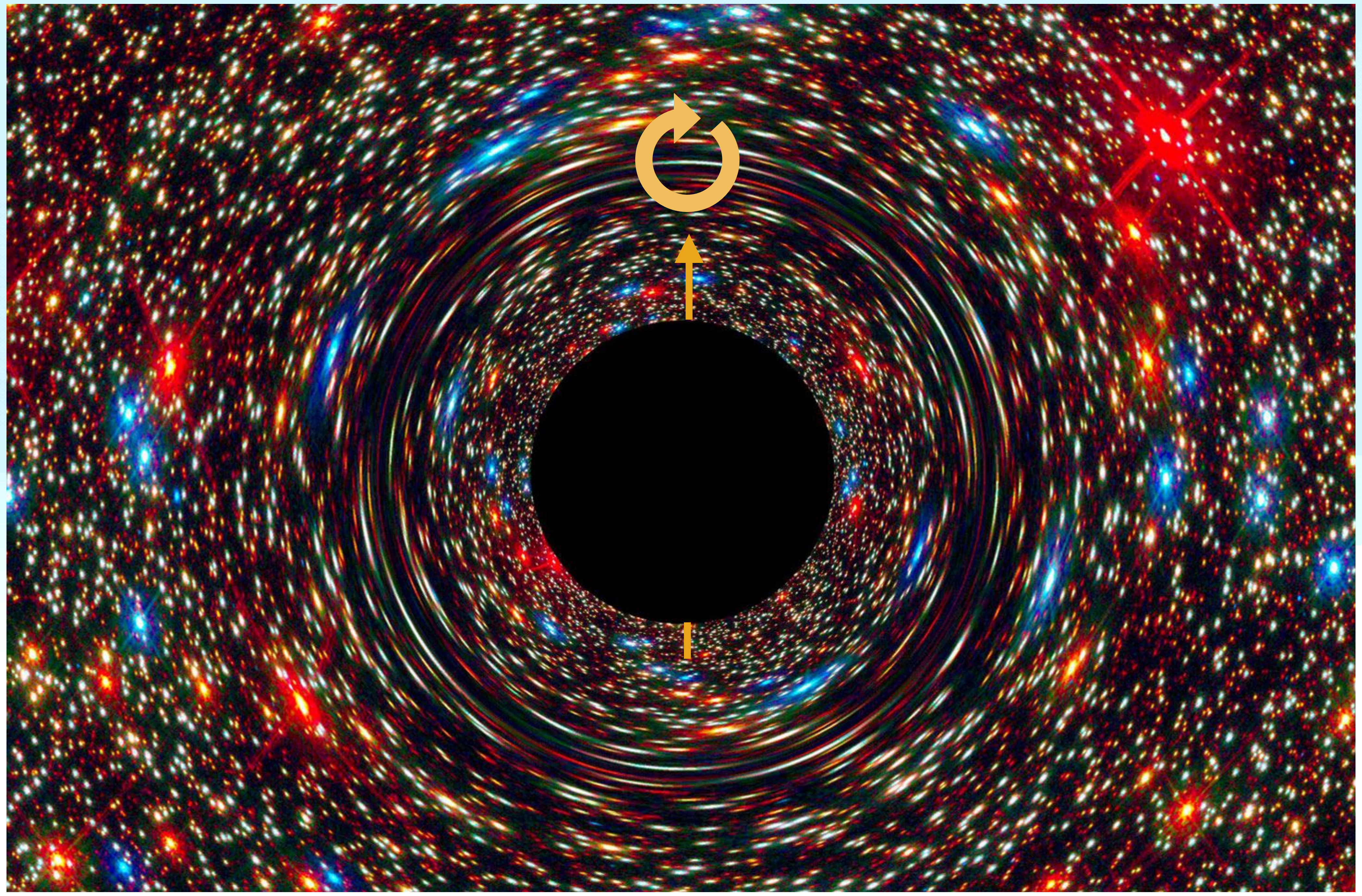
6. Polarization effects

Polarization effects on a Kerr background

S. Teukolsky (1973)



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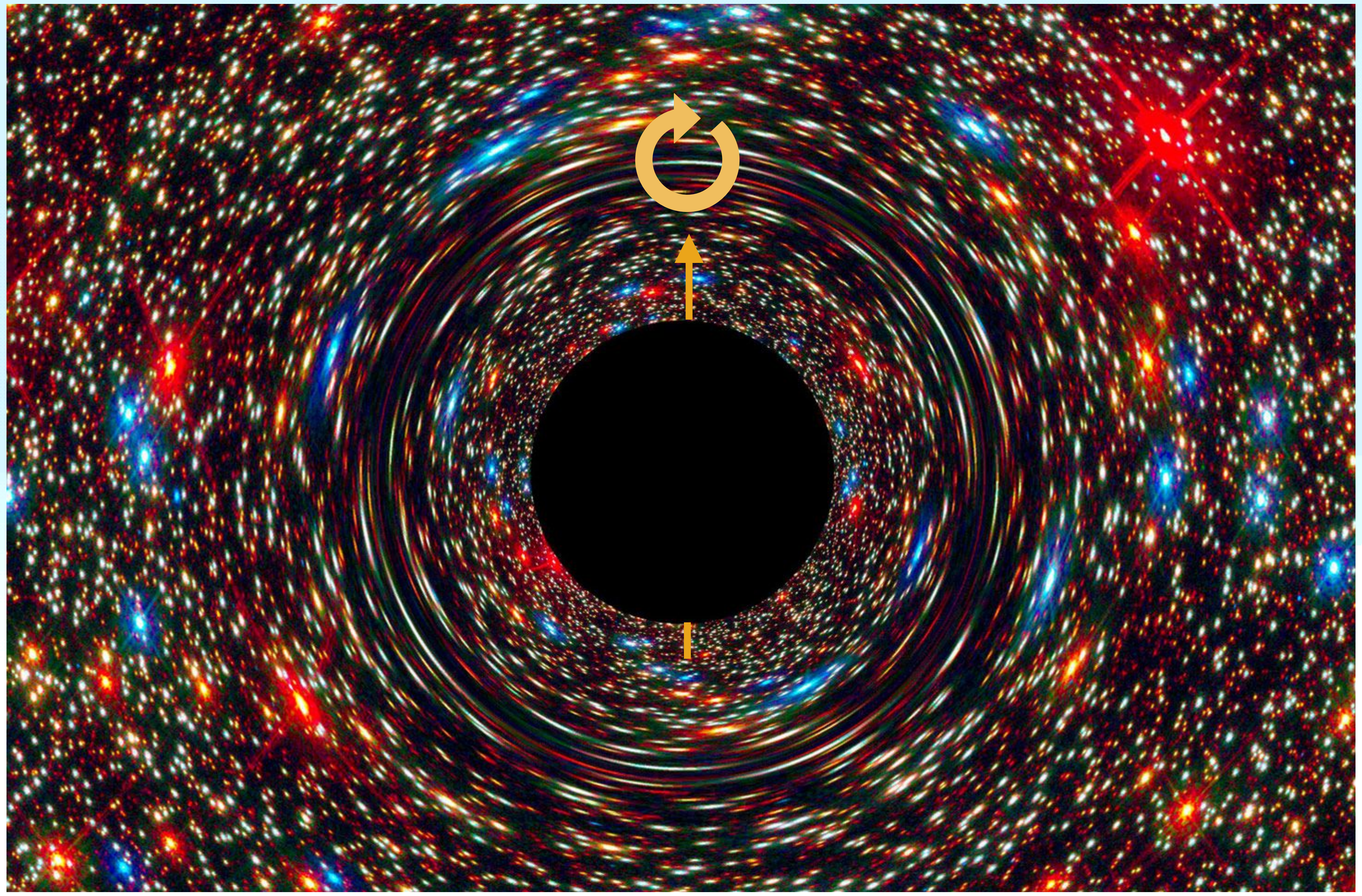

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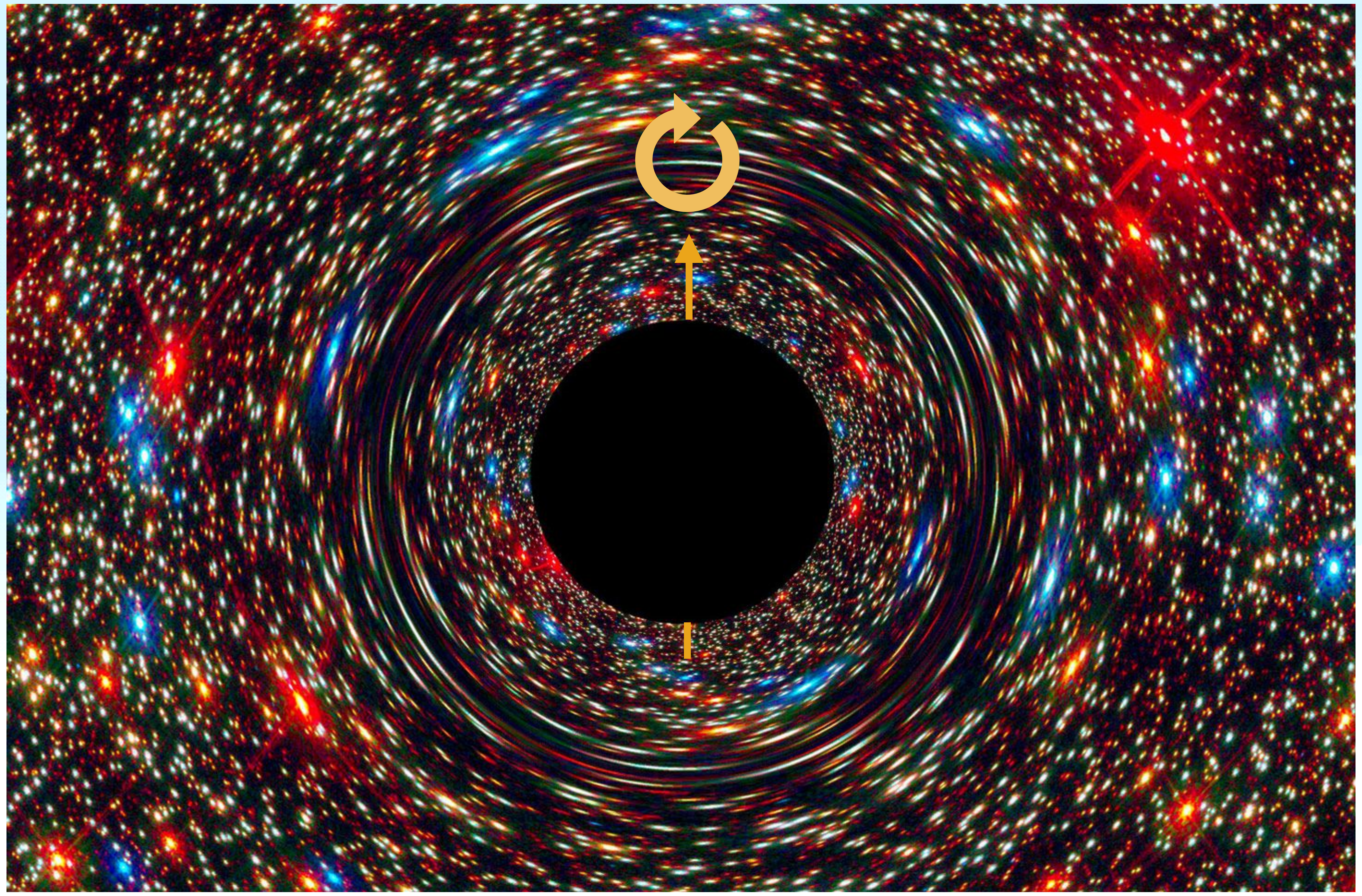

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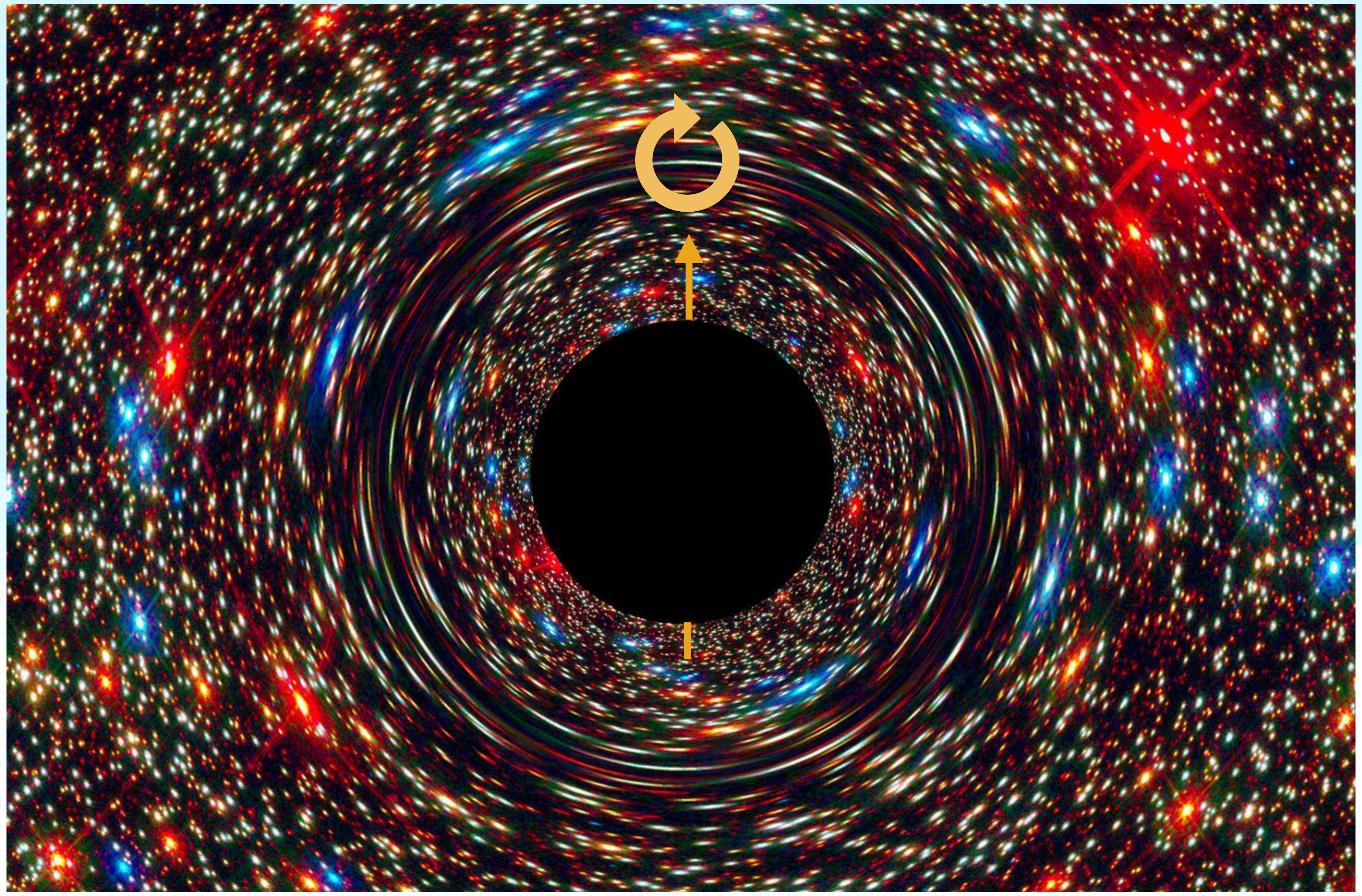

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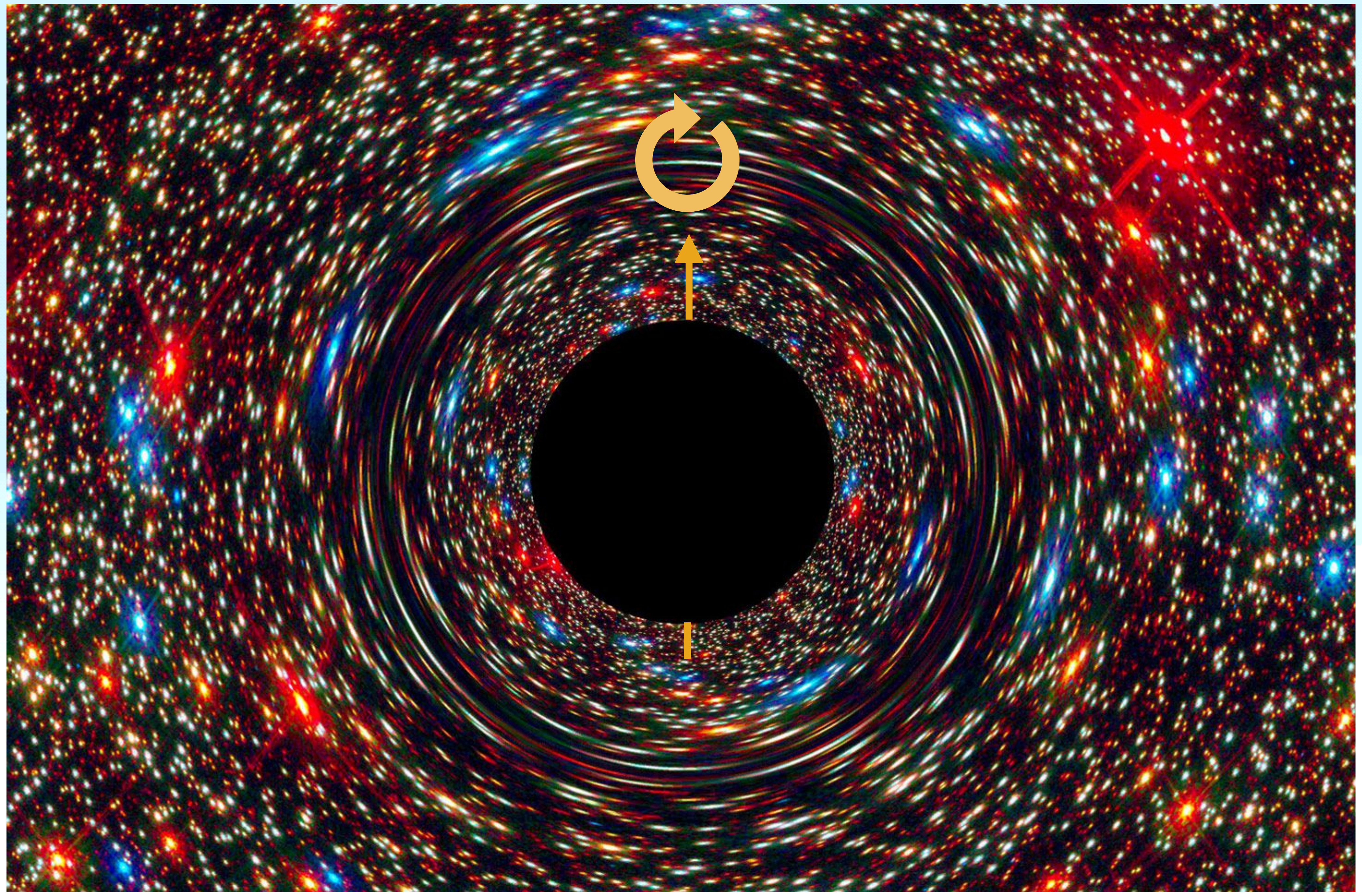
$$\hat{O}[M, a, \omega, s]\, \psi^s_\omega(r, \theta, \varphi) = 0$$

Differential operator



Polarization effects on a Kerr background

S. Teukolsky (1973)

1. Lens = Kerr BH

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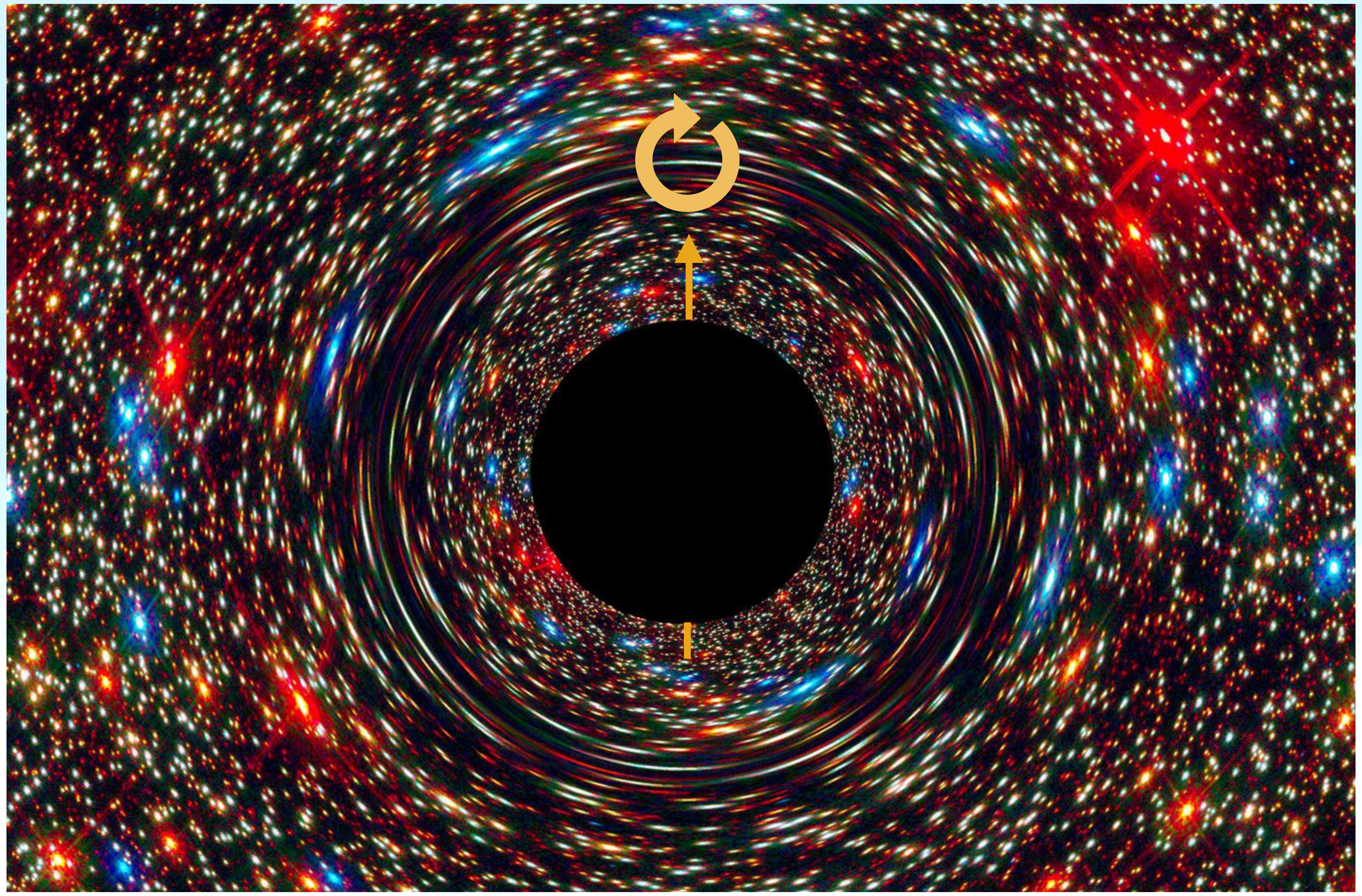
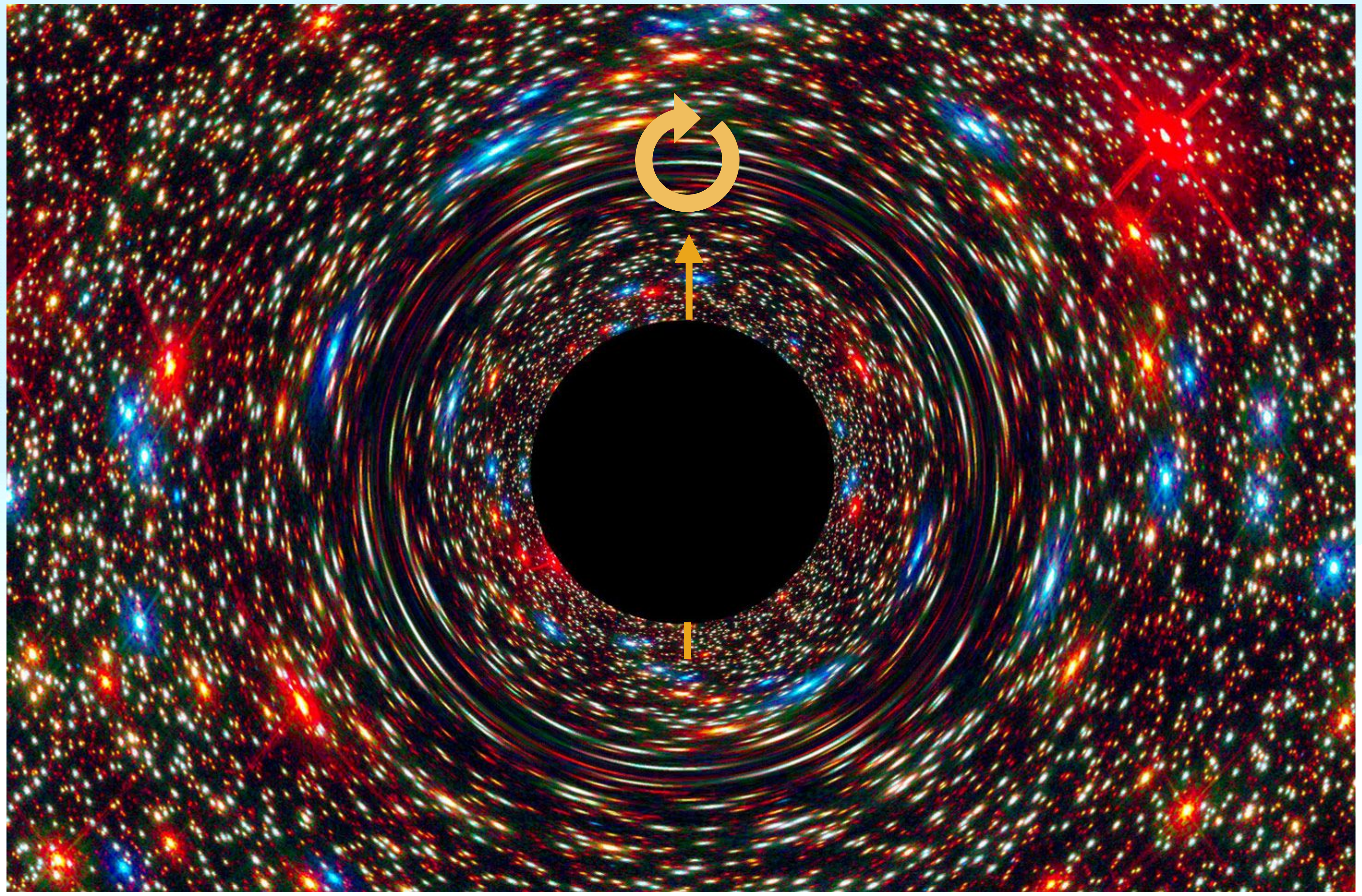
Newman-Penrose scalar, e.g.:
$\psi^{s=2}_\omega \supset \{\ddot{h}_+ \pm i\,\ddot{h}_\times\}$



Polarization effects on a Kerr background

Helmholtz Eq. for radial part



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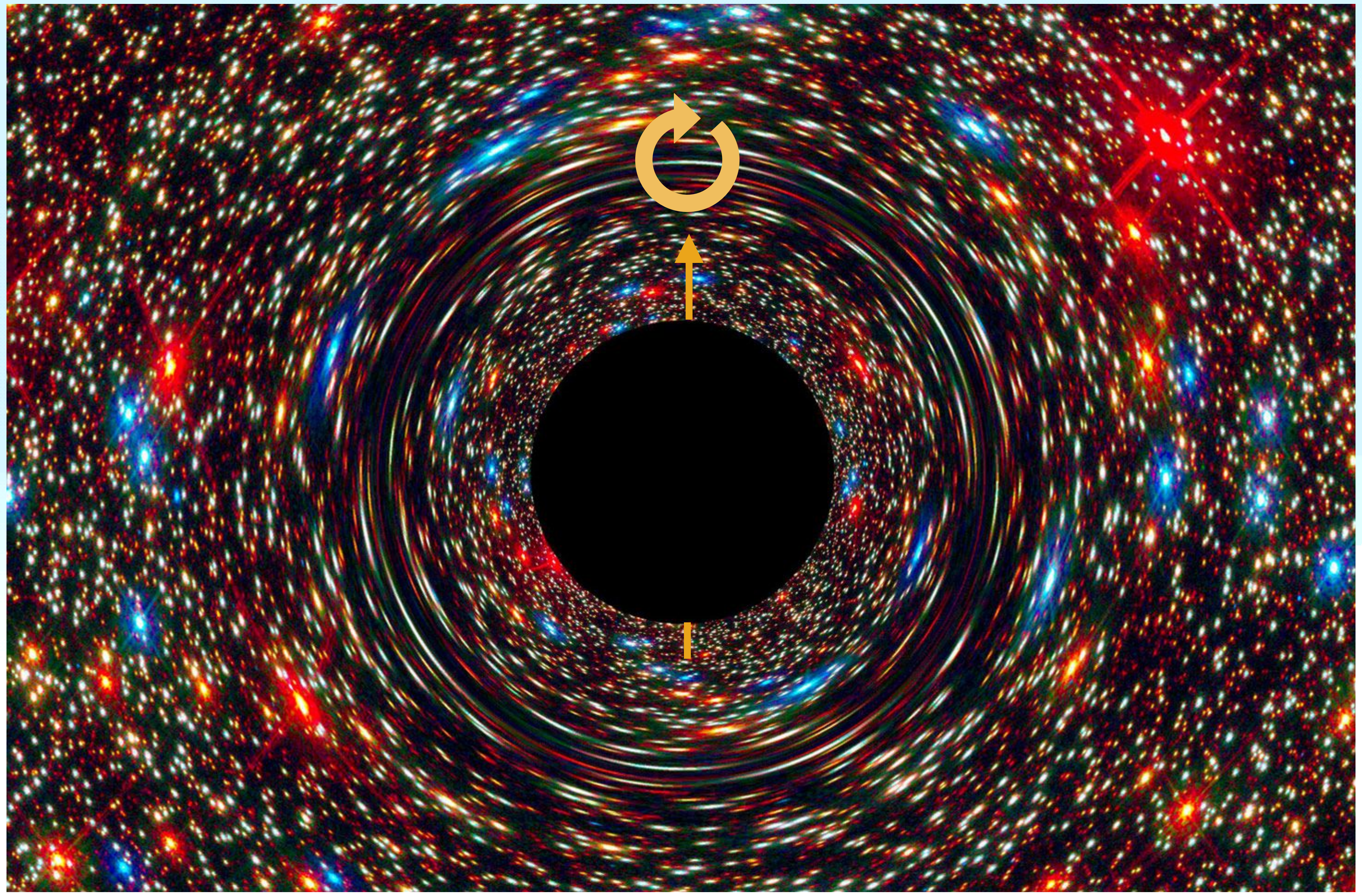

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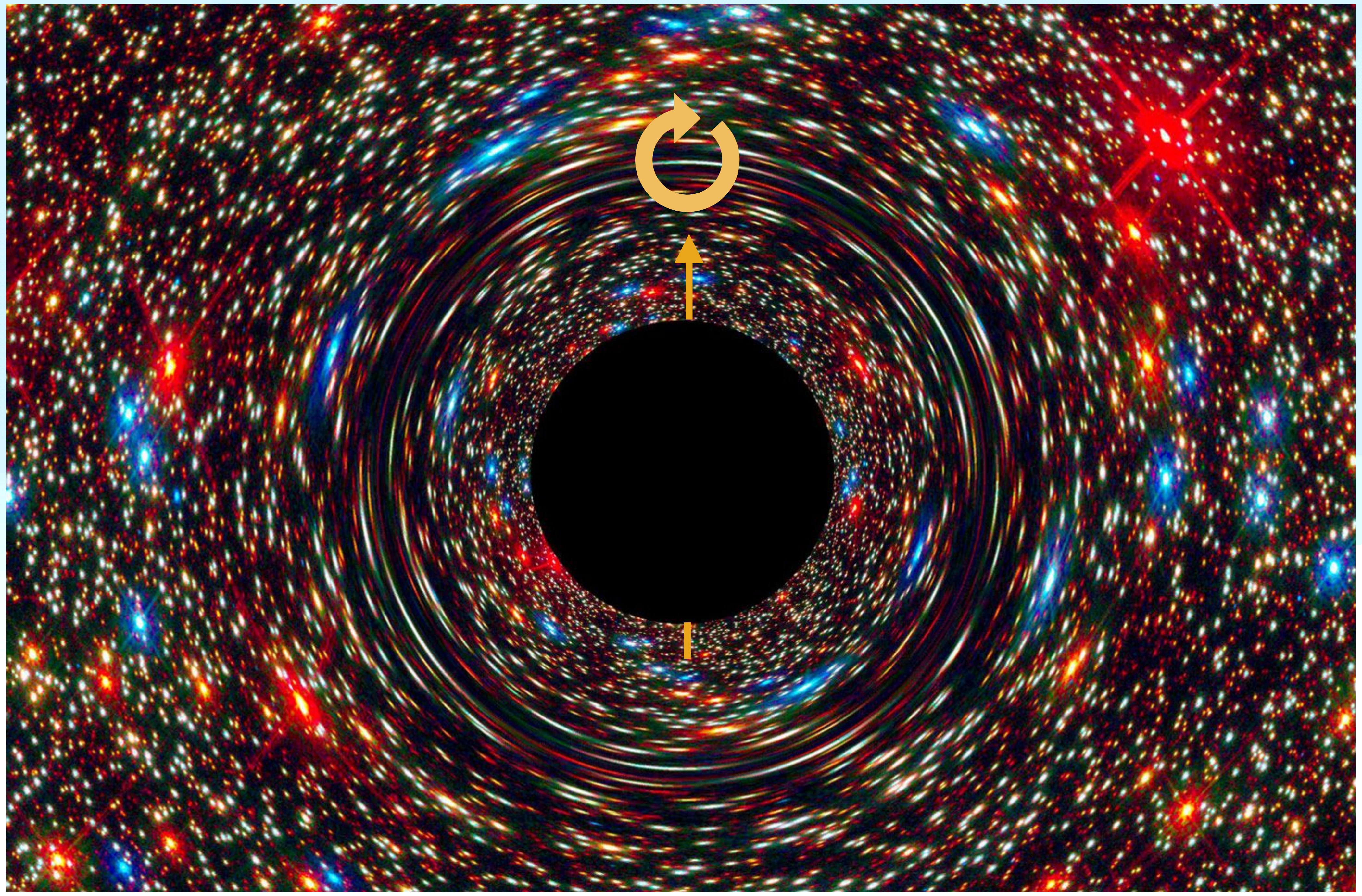

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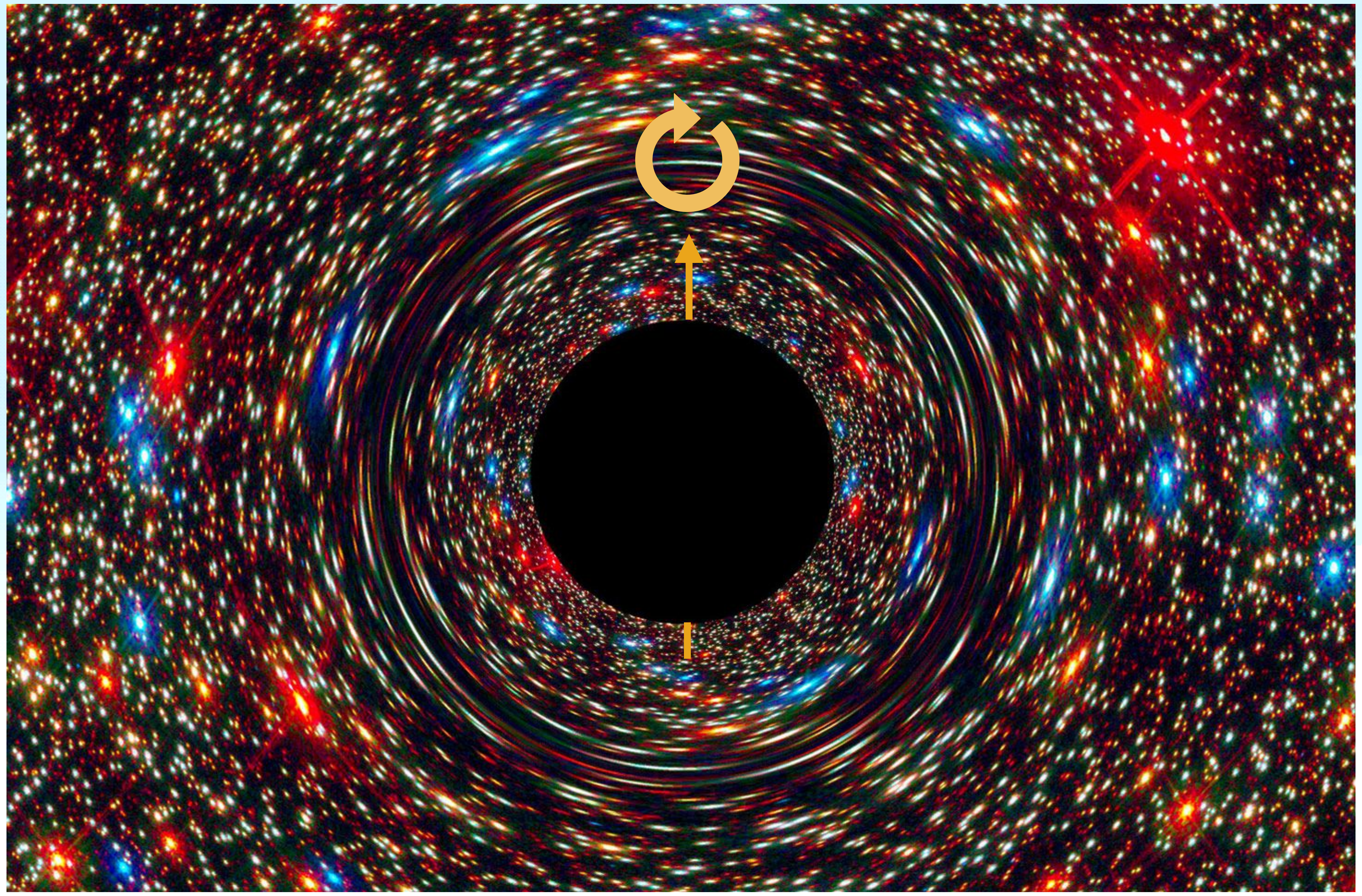

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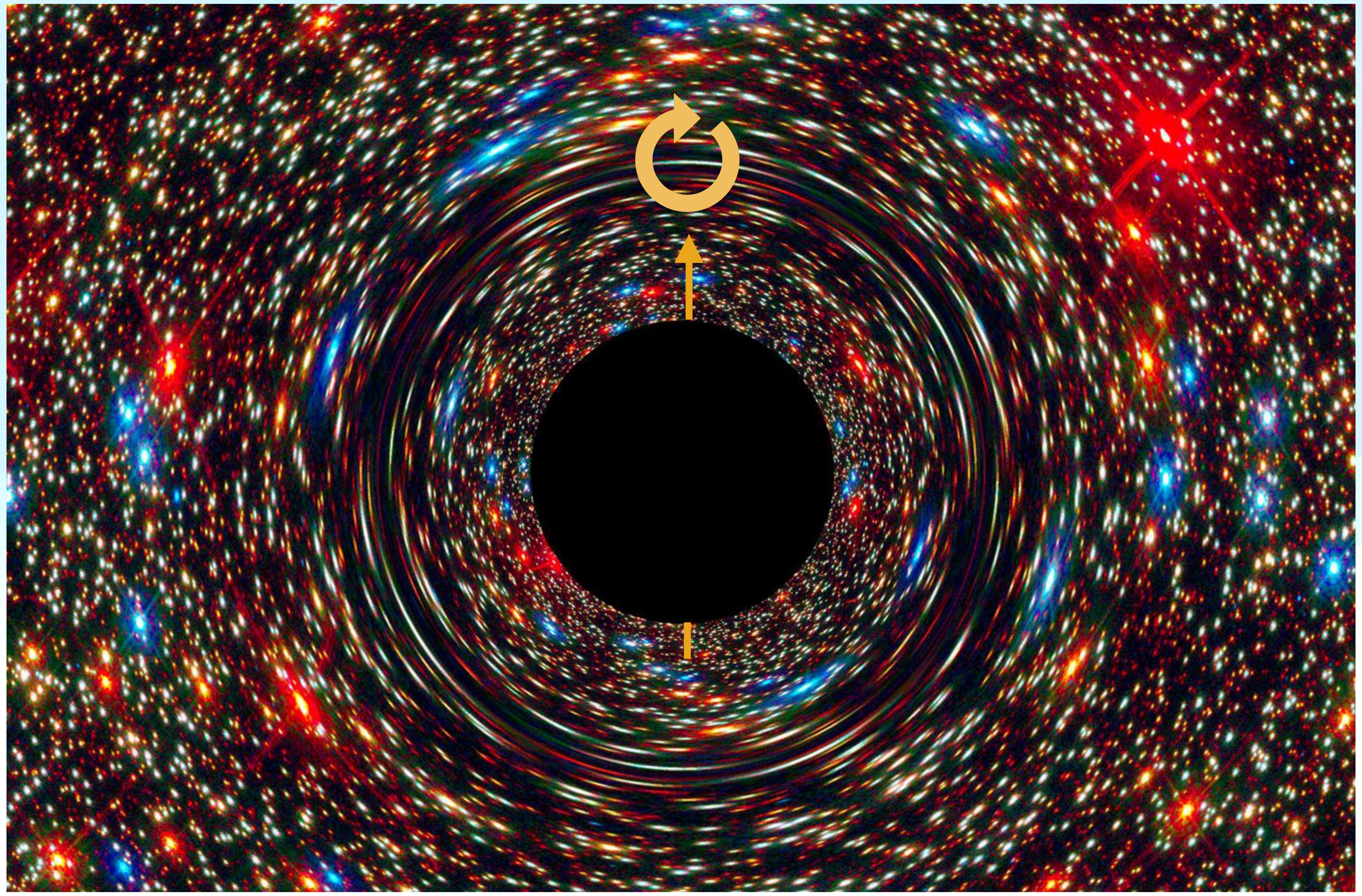

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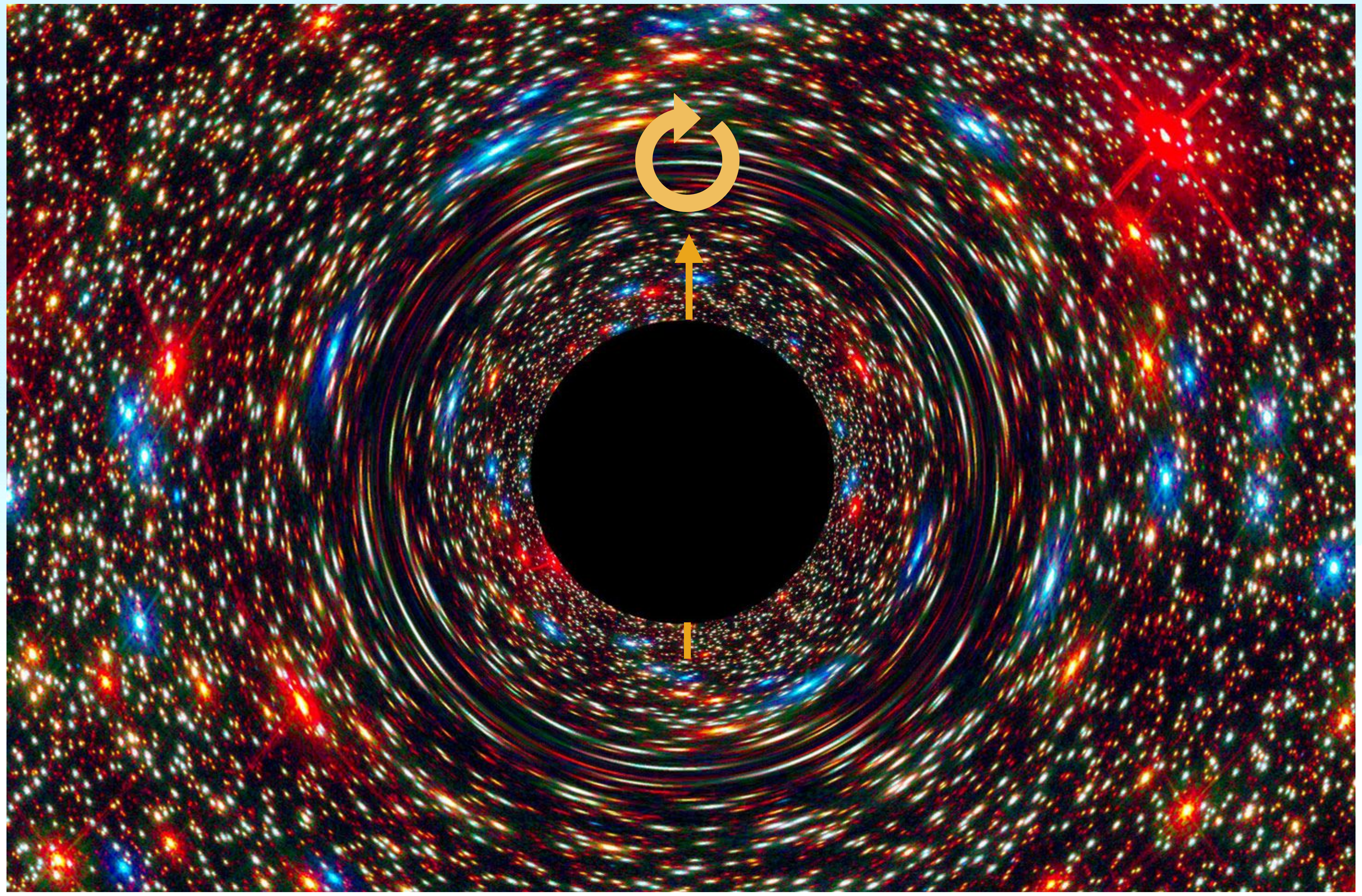

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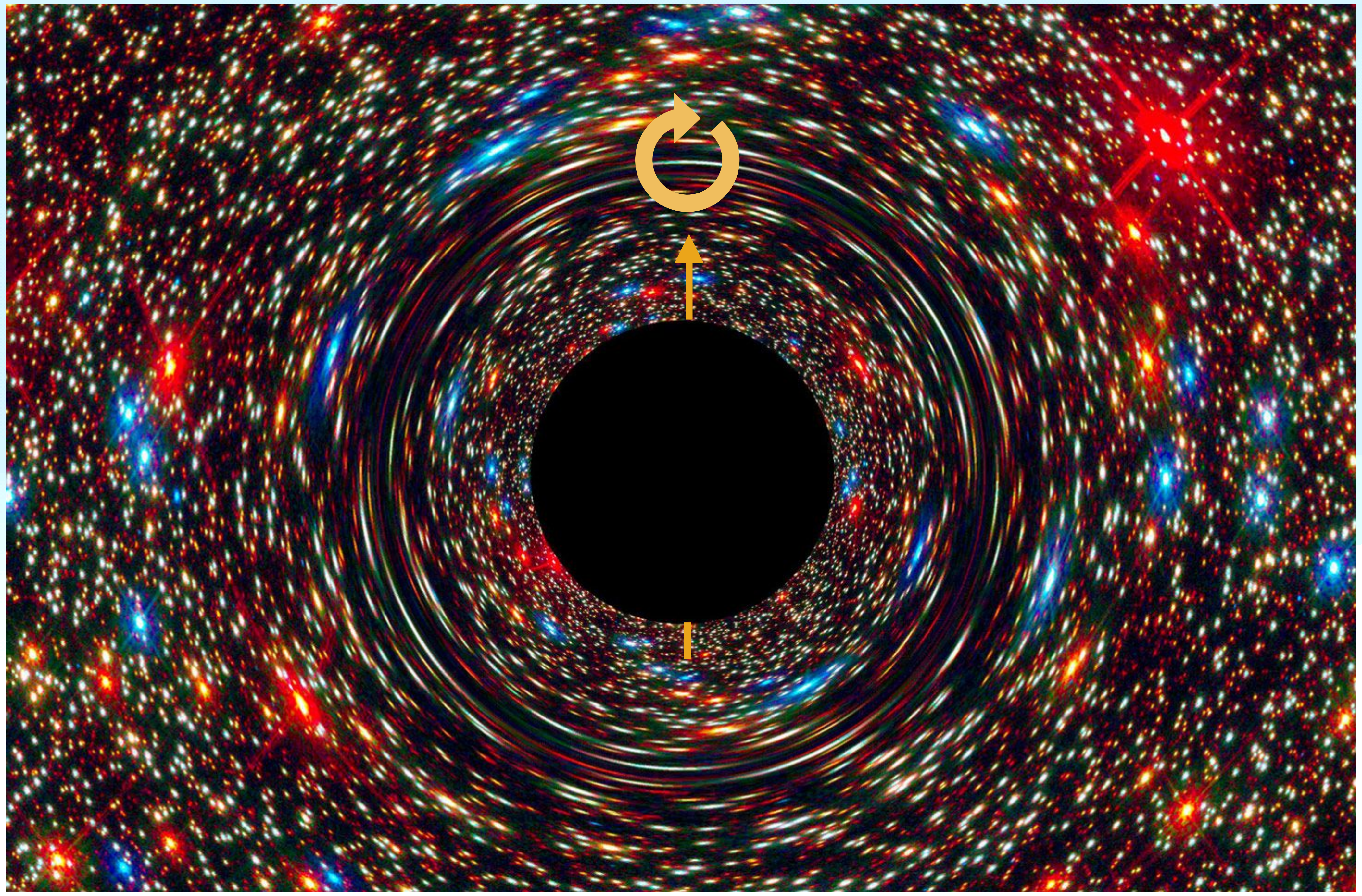

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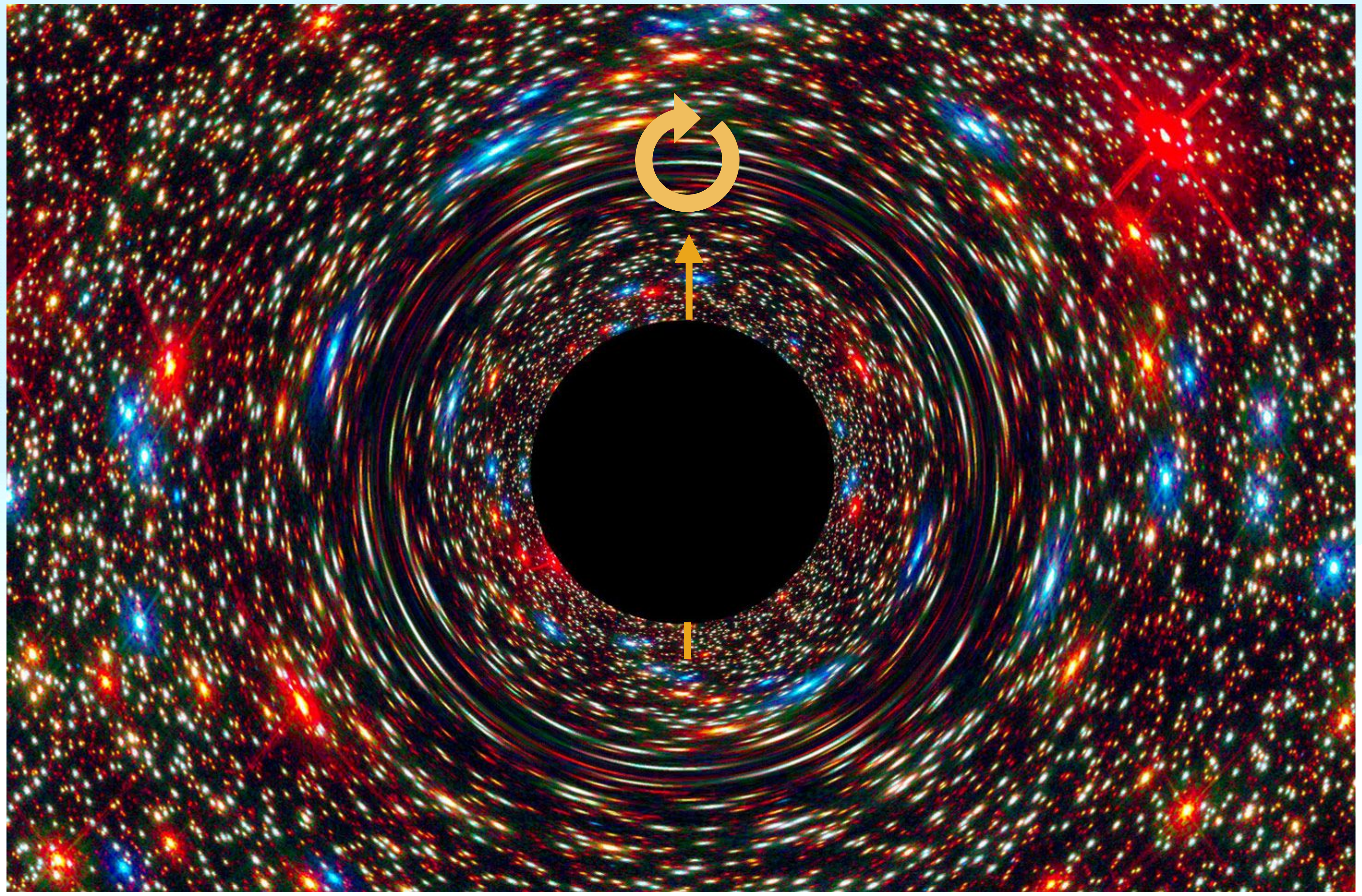

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Angular momentum (decomposition)

Spin dependent terms: Negligible in $\omega \gg 1$ limit

Summary

What's next

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1. PPTI: wave-optics description without eikonal approximation (no frequency lower bound)

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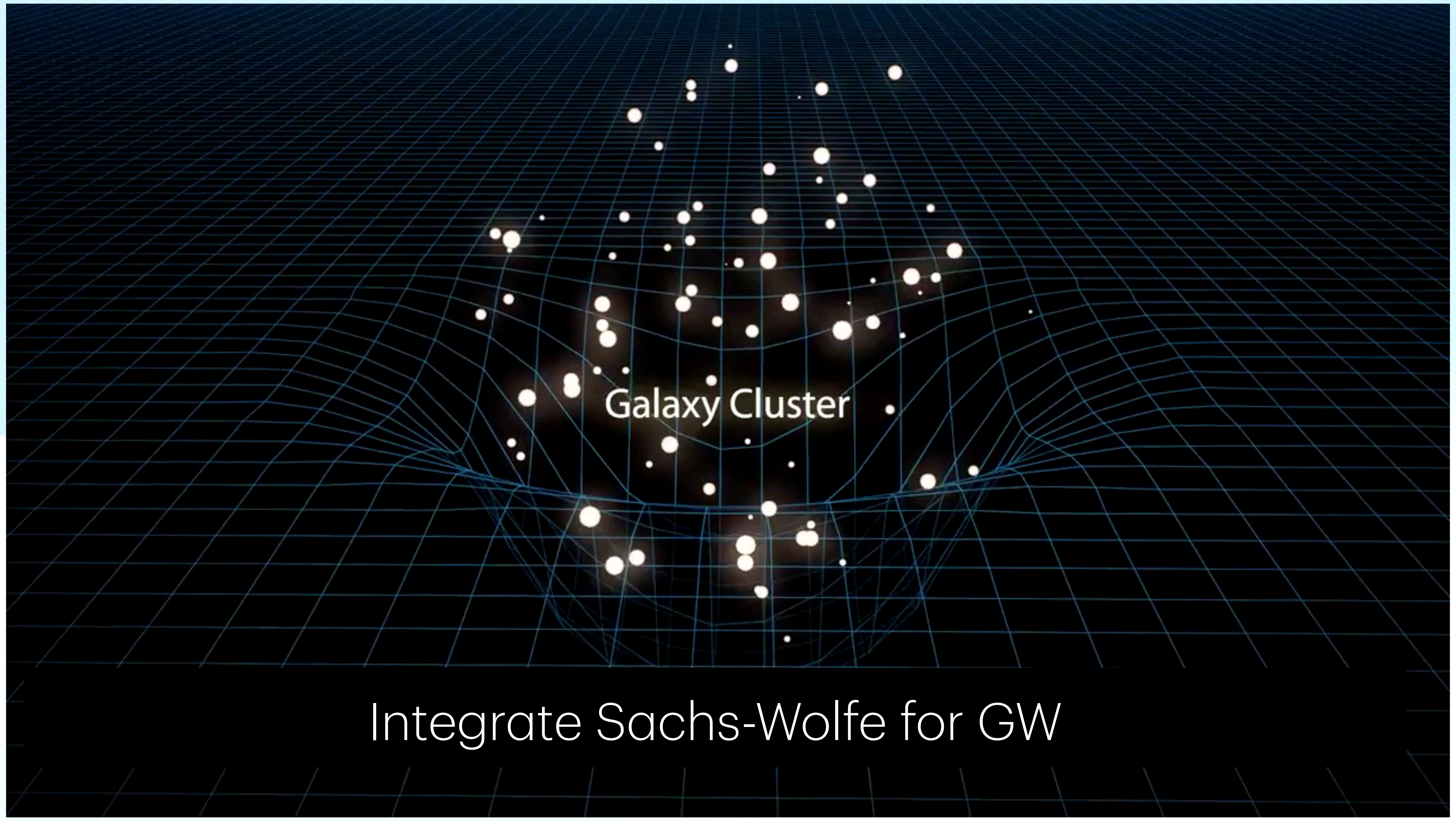
Thank you!

Optical regimes

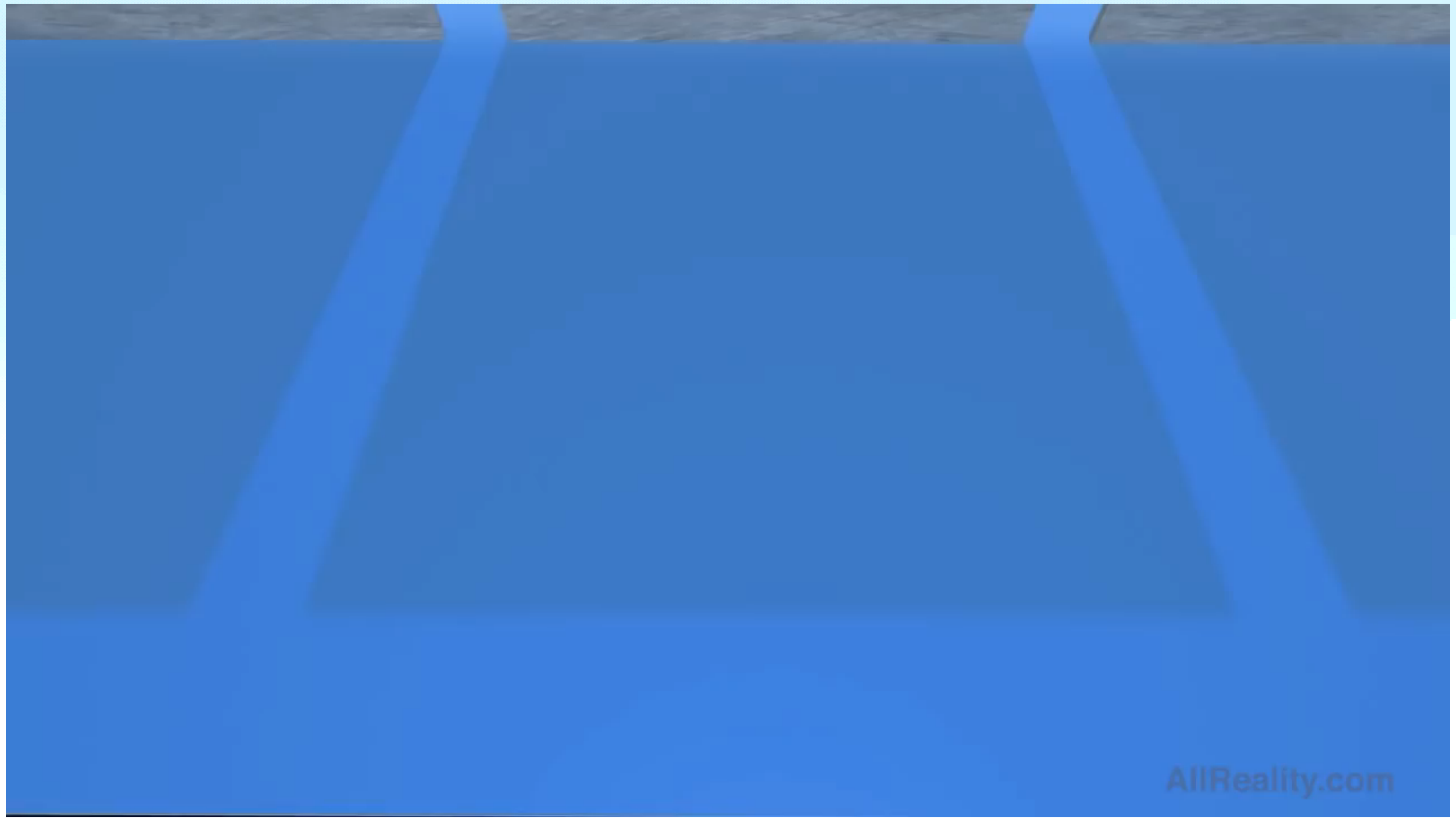
Geometric vs Wave optics

High Frequency: $\omega R_S \gg 1$



Ray description

Low Frequency: $\omega R_S \lesssim 1$



Wave effects

Optical regimes

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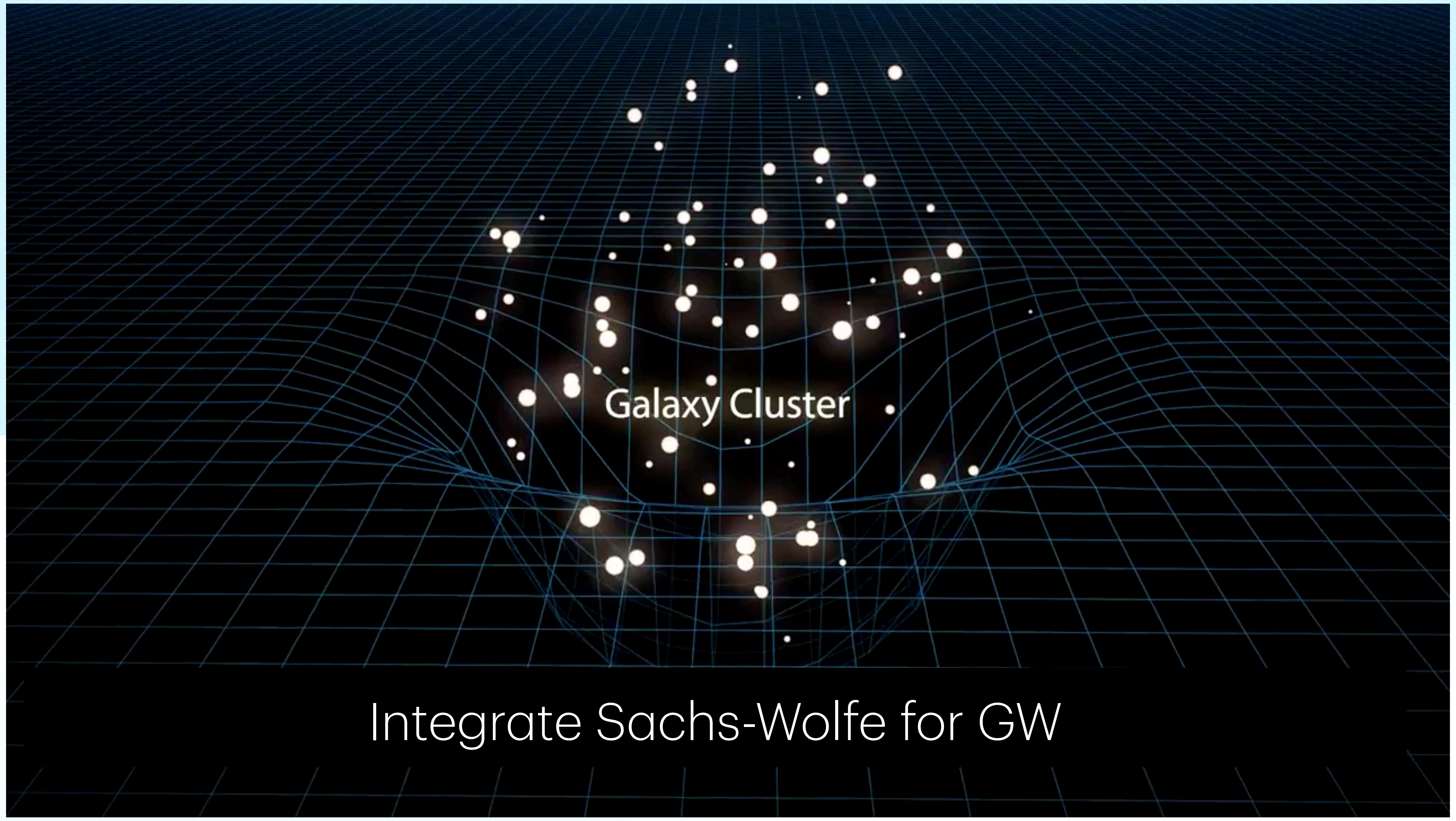

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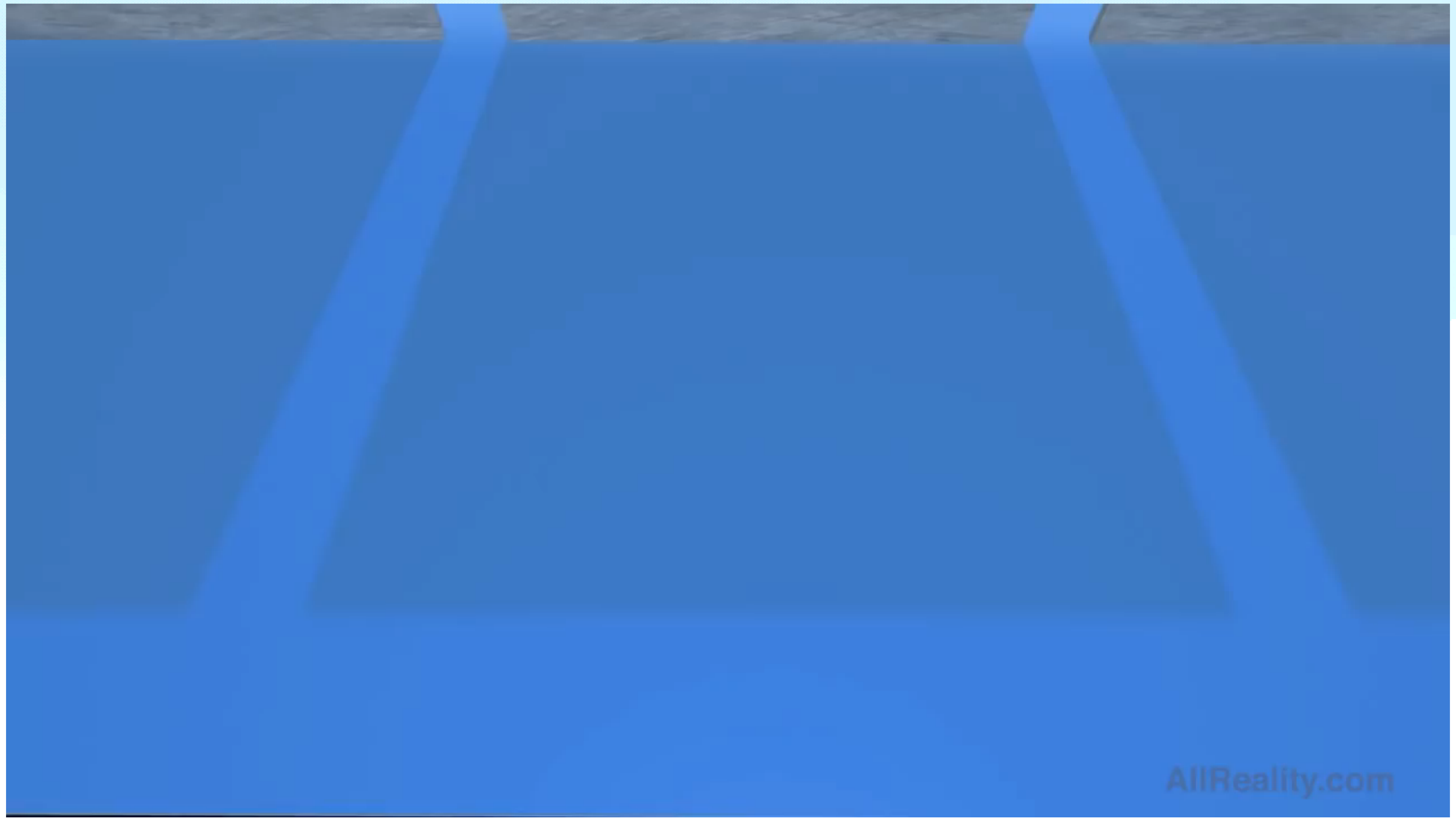

Wave effects

3.4 Wave effects

LISA CosGW: 2204.05434

GWs can be emitted at low frequencies ($w \lesssim 1$), allowing the observation of wave diffractive phenomena. For typical LISA sources, wave optics as in Eq. (15) needs to be considered for lenses with masses $M_L \sim 10^6 - 10^9 M_\odot$, cf. Eq. (13).

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Geometric vs Wave optics

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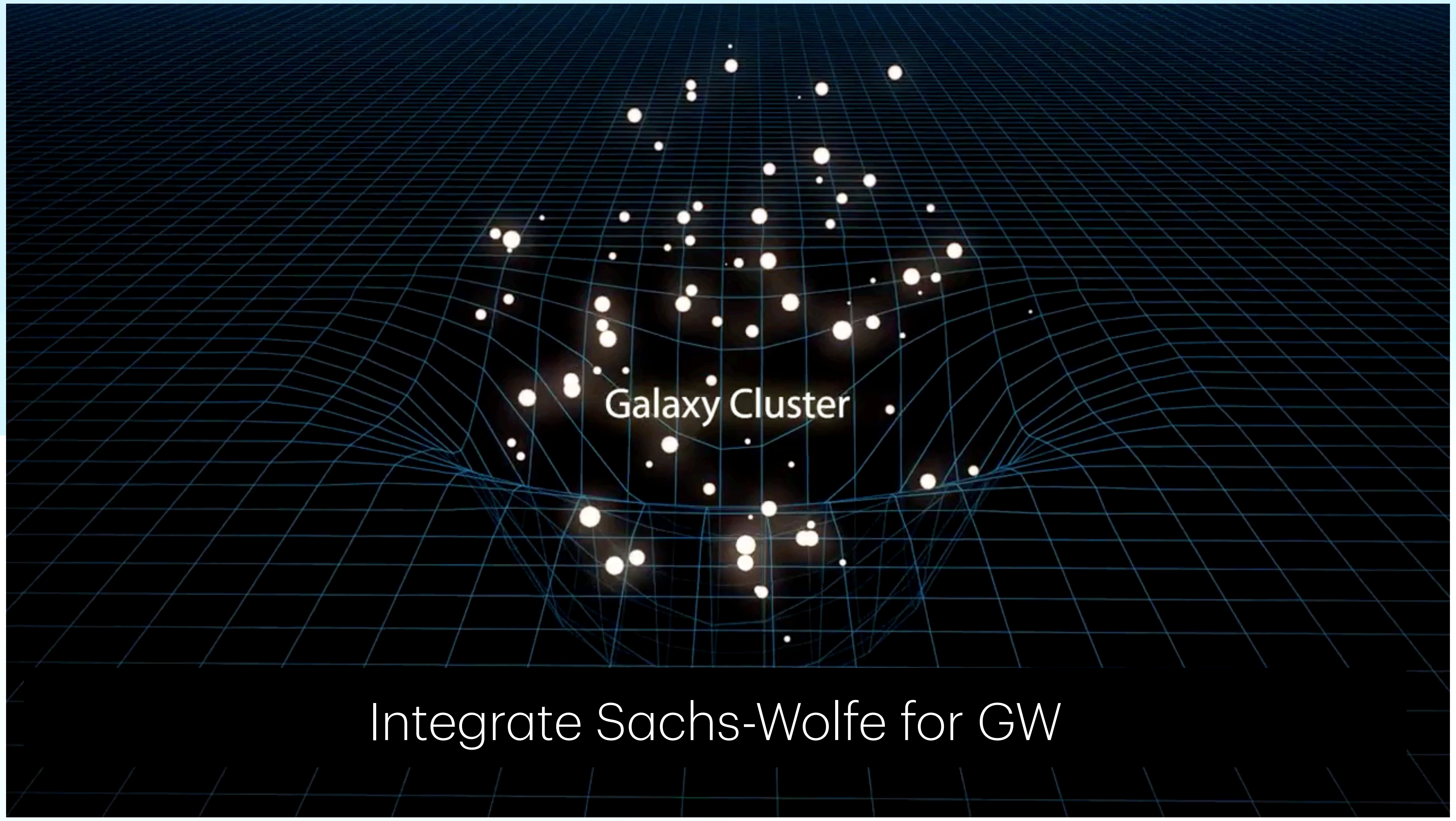

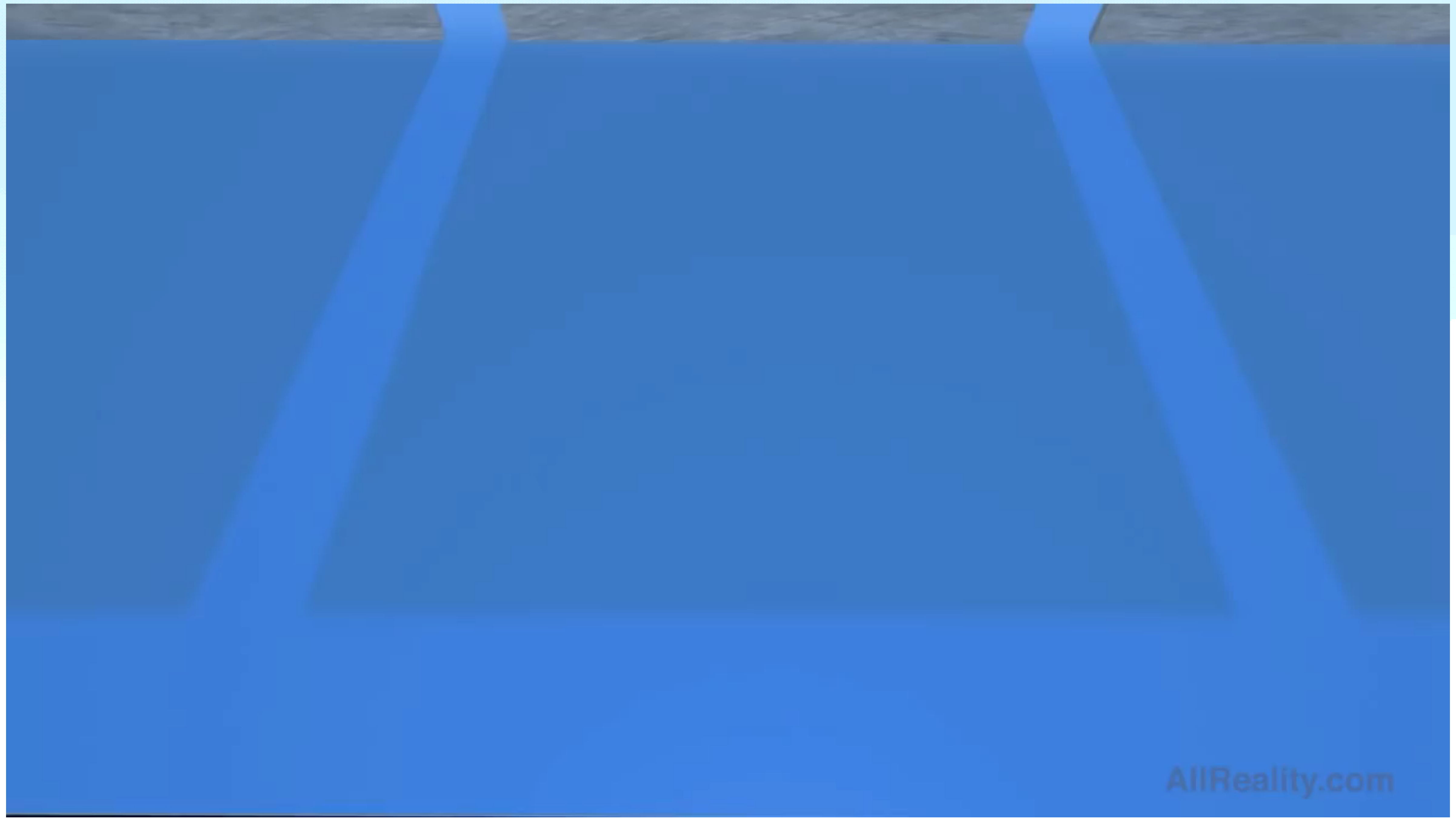

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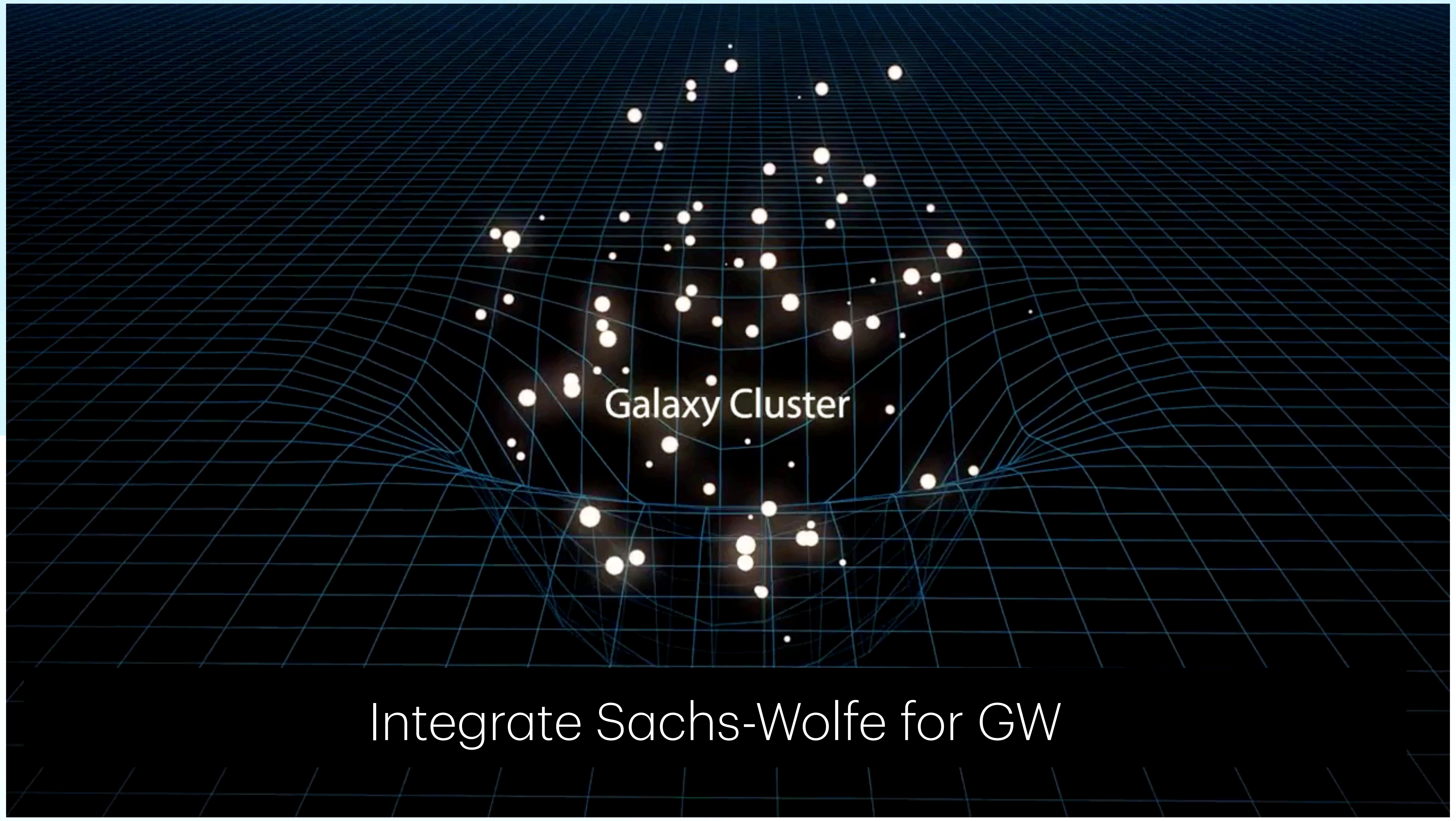

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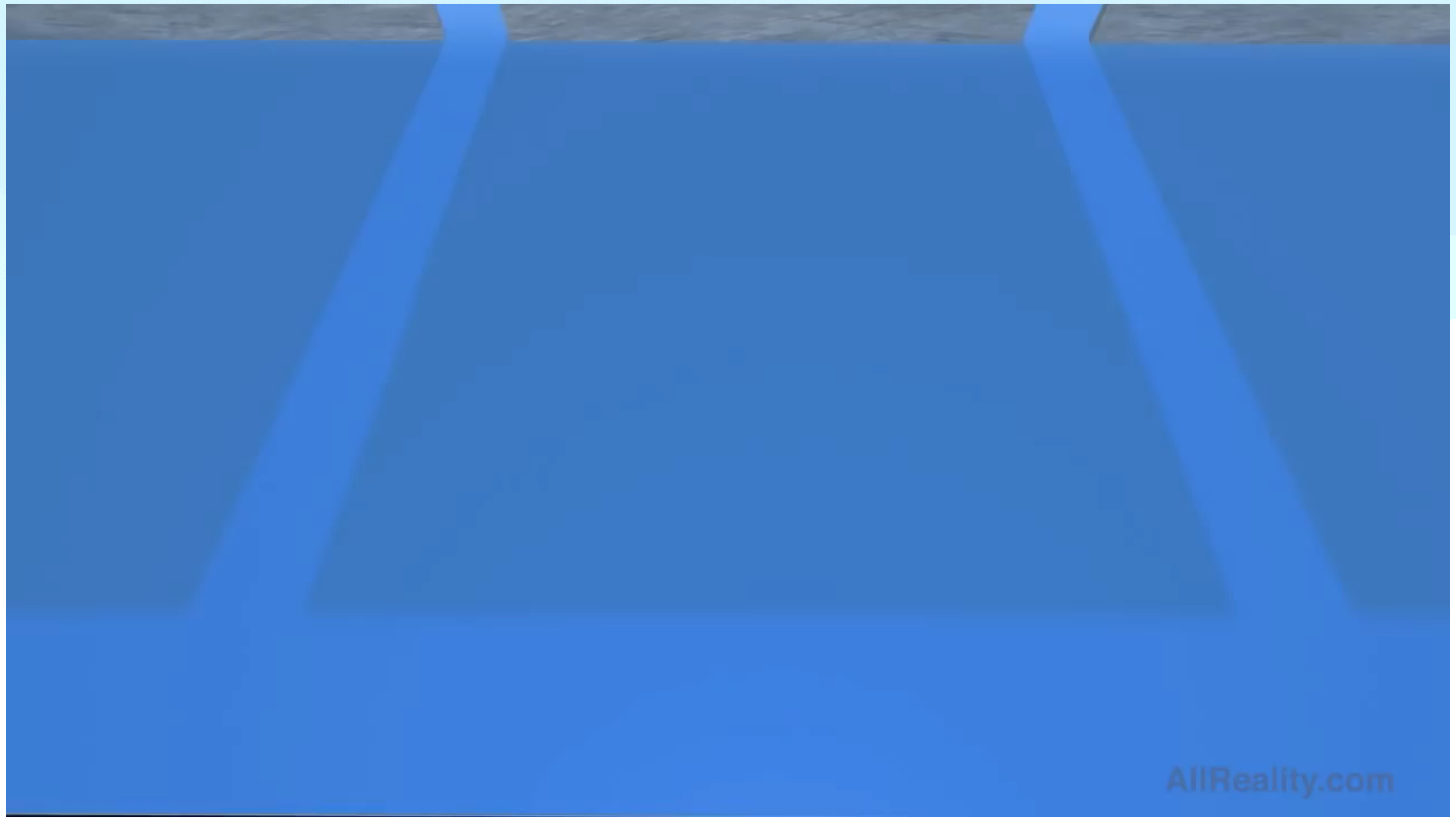

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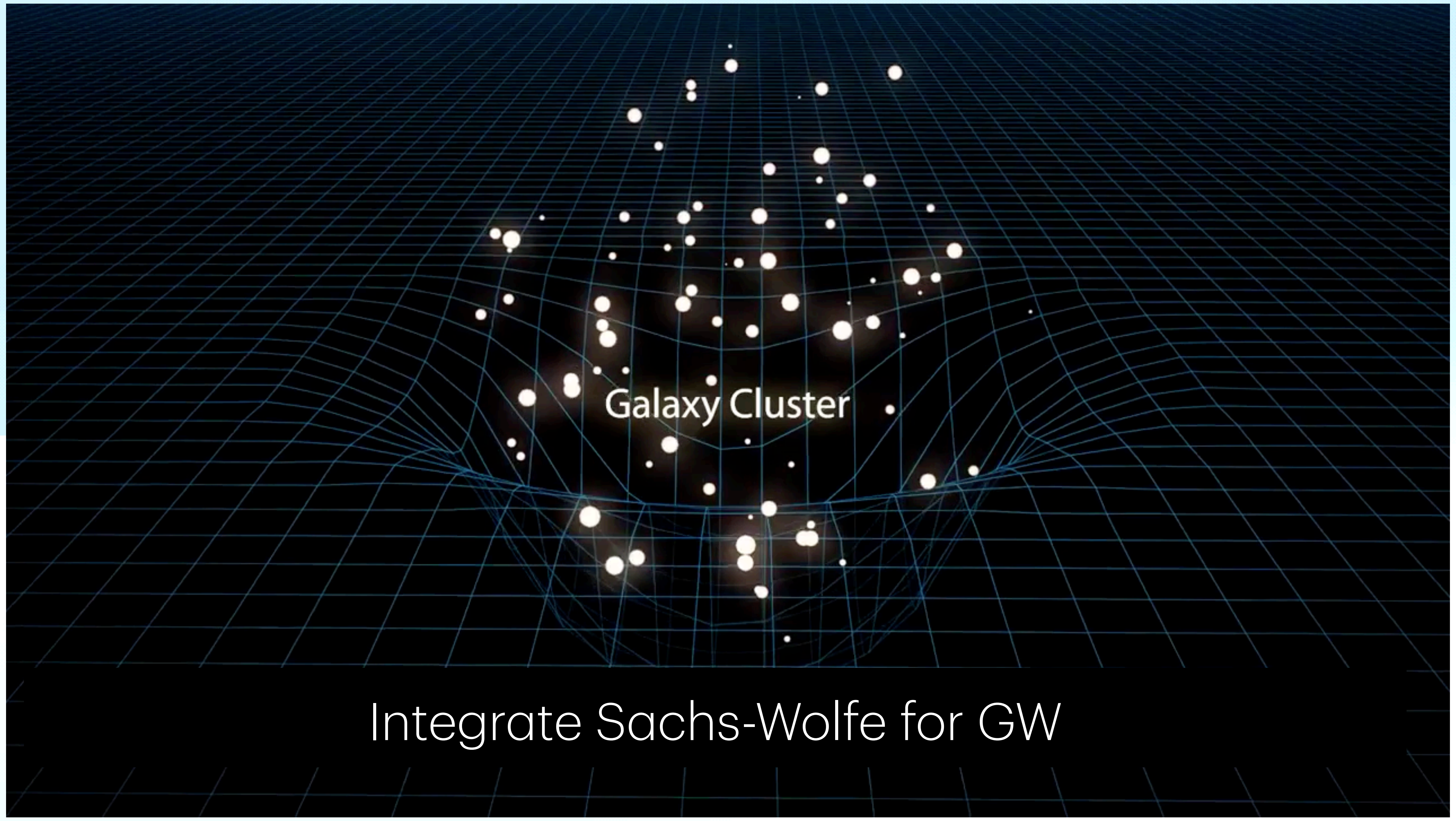

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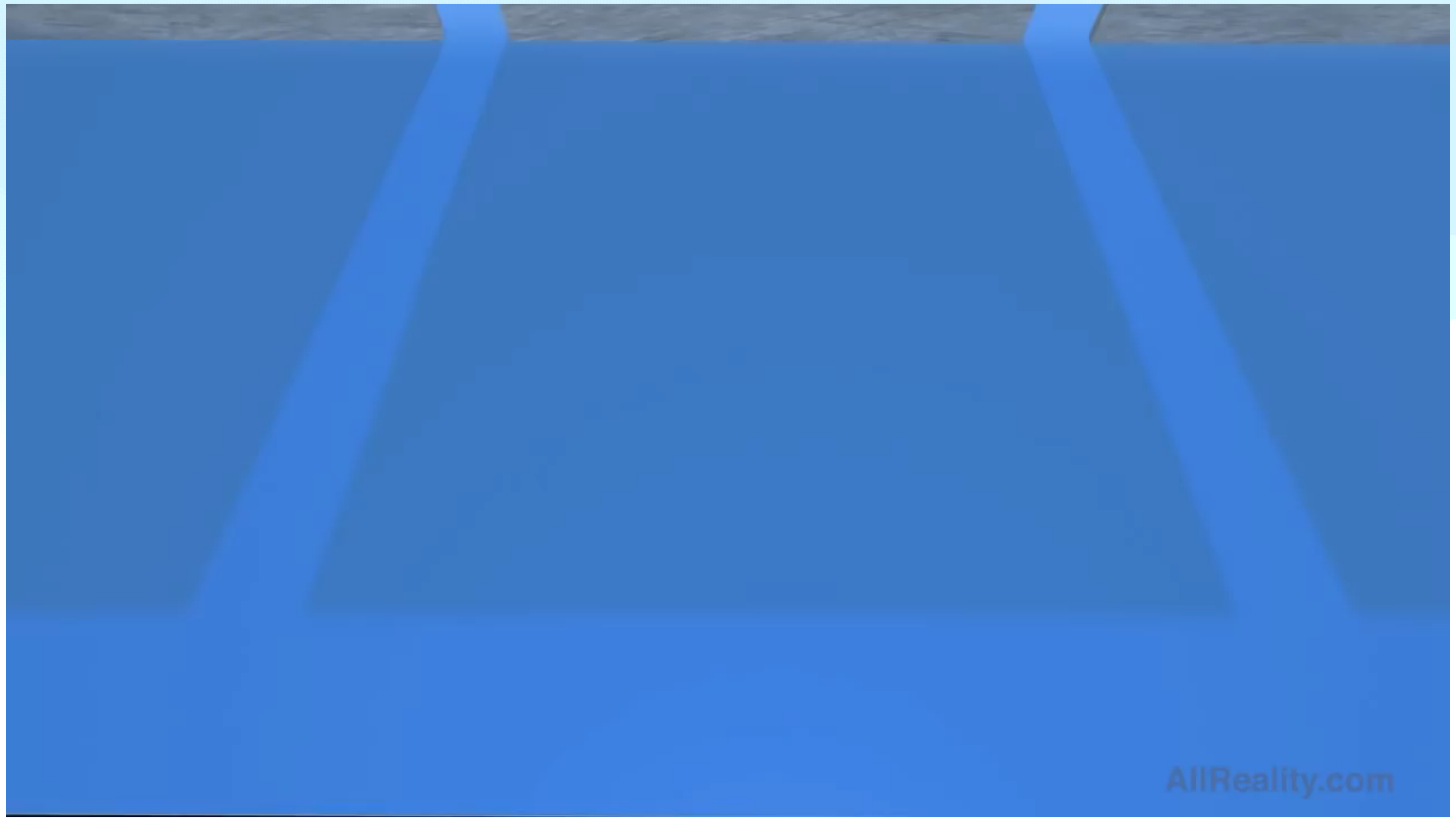

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