Hierarchical inference of cosmological and population parameters from gravitational wave data with and without electromagnetic counterparts

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Porto - Jun 20, 2024

11th CosWG LISA meeting

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Study funded by the European Union - NextGenerationEU, in the framework of the project "National Centre for HPC, Big Data and Quantum Computing" (HPC CN00000013 – CUP J33C22001170001).

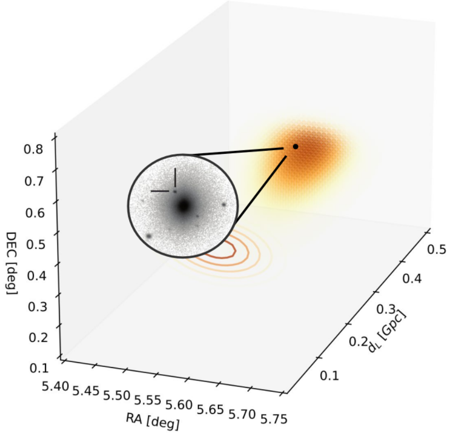
GWs standard sirens

Gravitational Waves (GWs) are **standard sirens**, but we need **external redshift information** to constrain cosmological parameters.

$$h(t) \approx \frac{\mathcal{M}_{\rm det}^{5/3} f_{\rm det}^{2/3}(t)}{d_L} F(\text{angles}) \cos(\Phi(t)), \qquad \mathcal{M}_{\rm det} = (1+z)\frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}, \qquad d_L(z) = (1+z)\int_0^z \frac{dz}{H(z;\lambda_{\rm cosmo})} ..$$

Bright Sirens - with direct EM counterpart (BNS)

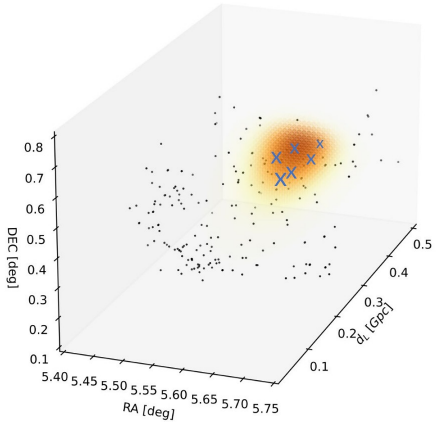
Follow-up observation of the EM counterpart. (Holz et al. 2005; Nissanke et al. 2010).

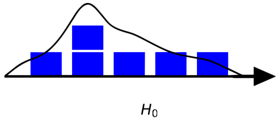


Only **GW170817** (Abbott et al. 2017) out of ~ 180 events so far.

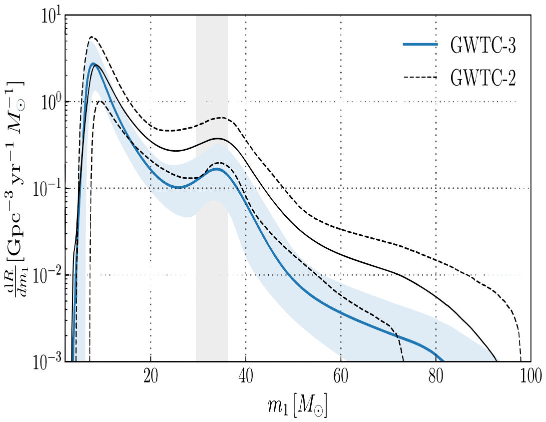
Dark Sirens - without direct EM counterpart (BBH)

Catalog of possible hosts, usually a **galaxy catalog** (Schutz, 1986; Gair et al. 2023).





Features in the source-frame mass distribution (Chernoff et al. 1993; Del Pozzo, 2012; Ezquiaga et al. 2022): **"spectral sirens"**.



From Abbott et al. 2023

Codes for hierarchical inference

The methods can be combined within a **Hierarchical Bayesian Framework** to infer joint constraints on population and cosmology:

$$\mathcal{L}\left(\{\boldsymbol{d}_i\}_{i=1}^{N_{\rm obs}}\middle|\boldsymbol{\lambda}\right) \propto \frac{1}{\xi(\boldsymbol{\lambda})^{N_{\rm obs}}} \times \prod_{i=1}^{N_{\rm obs}} \int {\rm d}\boldsymbol{\theta}\, \frac{p_{\rm GW}(\boldsymbol{\theta}(\boldsymbol{\theta}^{\rm D}, \lambda_{\rm cosmo})|\boldsymbol{d}_i)}{\pi(\boldsymbol{\theta}^{\rm D})\,\det\left|\frac{{\rm d}\boldsymbol{\theta}^{\rm D}}{{\rm d}\boldsymbol{\theta}}\right|(\lambda_{\rm cosmo})}\, p_{\rm pop}(\boldsymbol{\theta}|\boldsymbol{\lambda})$$

Key ingredients:

⇒ $p_{\rm GW}(\boldsymbol{\theta}(\boldsymbol{\theta}^{\rm D}, \lambda_{\rm cosmo})|\boldsymbol{d}_i)$ depends on the GW event and **measurement uncertainties**;

⇒ $\xi(\boldsymbol{\lambda})$ corrects the bias due to **selection effects**;

⇒ $p_{\rm pop}(\boldsymbol{\theta}|\boldsymbol{\lambda})$ is the **population prior**, including mass and rate distributions and galaxy catalogs information.

Mainly three codes:

- ICAROGW 2.0 (Mastrogiovanni et al. 2024),
- GWCOSMO 2.0 (Gray et al. 2023),
- **CHIMERA** (Borghi et al. 2024).

CHIMERA

The Astrophysical Journal, 964:191 (19pp), 2024 April 1 https://doi.org/10.3847/1538-4357/ad20eb

Cosmology and Astrophysics with Standard Sirens and Galaxy Catalogs in View of Future Gravitational Wave Observations

Nicola Borghi[1,2], Michele Mancarella[3,4], Michele Moresco[1,2], Matteo Tagliazucchi[1,2], Francesco Iacovelli[5,6], Andrea Cimatti[1,7], and Michele Maggiore[5,6]

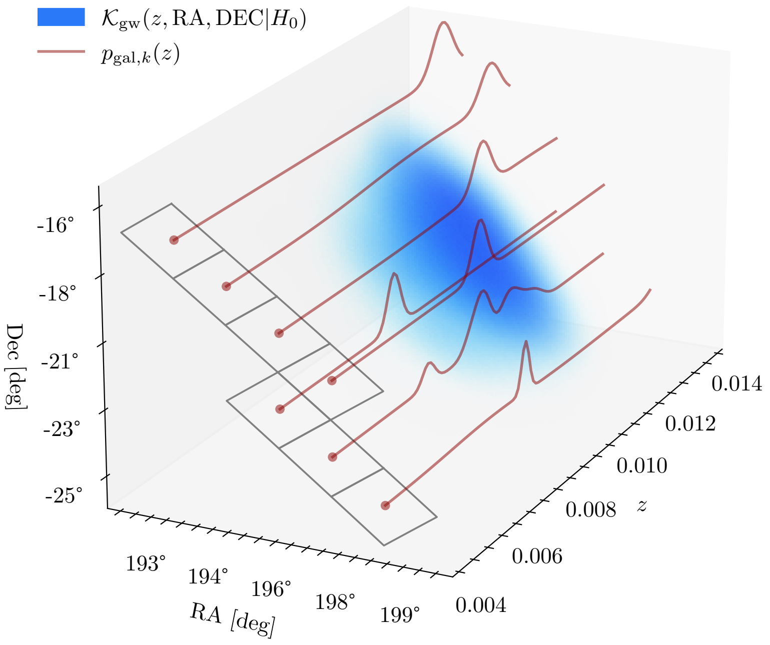
Similar codes MGCosmoPOP (Mancarella et al. 2022), DarkSirensStat (Finke et al. 2021), cosmoLISA (Laghi et al. 2021), . . .

The CHIMERA pipeline

CHIMERA (**C**ombined **H**ierarchical **I**nference **M**odel for **E**lectromagnetic and g**R**avitational-wave **A**nalysis): a novel Python code for the joint inference of cosmology and population properties of GW sources from GW data and galaxy catalogs. Borghi et al. 2024

Workflow:

1. Pre-computation of integration redshift grids and catalog probability given the cosmological priors.
2. For each GW event:
   - **pixelization** of the localisation area;
   - 3D **KDE** estimate of the GW probability, weighted by the mass distribution, within the localization volume;
   - pixel-by-pixel **integration in redshift** of the KDE times the probability of having an host galaxy and sum of all the integrals;
3. Multiplication of the posterior of each event.
4. Monte Carlo approximation of the selection bias.



https://github.com/CosmoStatGW/CHIMERA

Mock Catalogs

- **Mock galaxy catalog**: **luminosity-complete** subsample of the **MICE** Grand Challenge light-cone simulation (Fosalba et al. 2015) with a **uniform in comoving volume** density distribution.
  - ⇒ Cut in luminosity corresponds to $\log_{10}(M/M_{\odot}) > 10.5$;
  - ⇒ we associate both spectroscopic ($\sigma_z/(1+z) = 0.001$) and photometric ($\sigma_z/(1+z) = 0.05$) errors on galaxy redshifts;

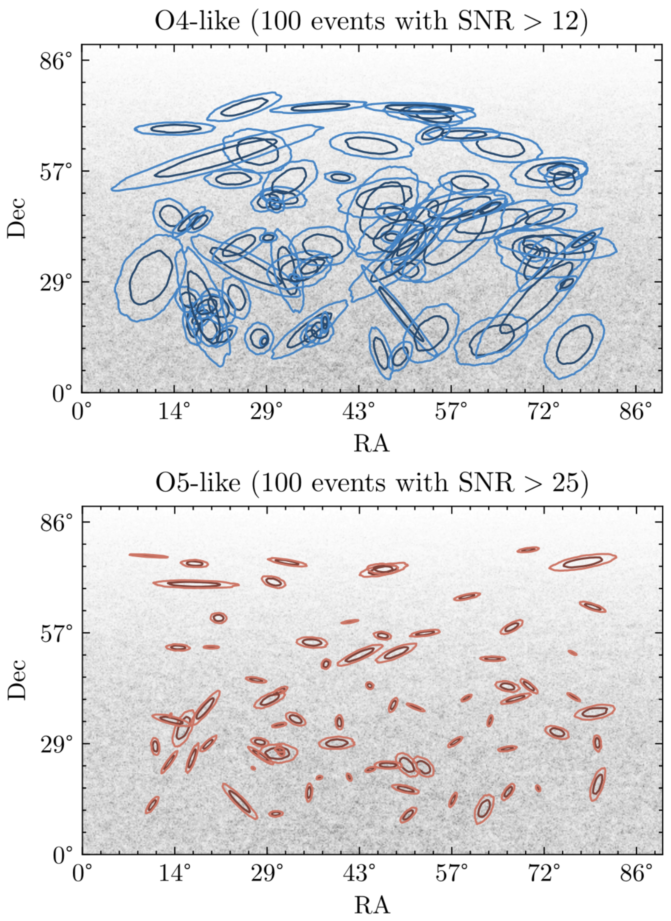
- **Mock GW events** drawn from fiducial population distributions:
  - ⇒ Cosmology: flat ΛCDM with $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{m,0} = 0.25$.
  - ⇒ Mass distribution: Power Law + Gaussian peak
  - ⇒ Rate evolution: Madau-like

Fisher matrix-based computation of SNR and posterior samples using `GWFAST` (Iacovelli et al. 2022). Two configurations considered:

⇒ **O4-like:** L1, H1, Virgo, KAGRA at O4 desing sensitivity.

⇒ **O5-like:** L1, H1, LIGO-India, Virgo, KAGRA at O5 design sensitivity.

Cut in SNR to select the 100 best events over 1 year of observation.

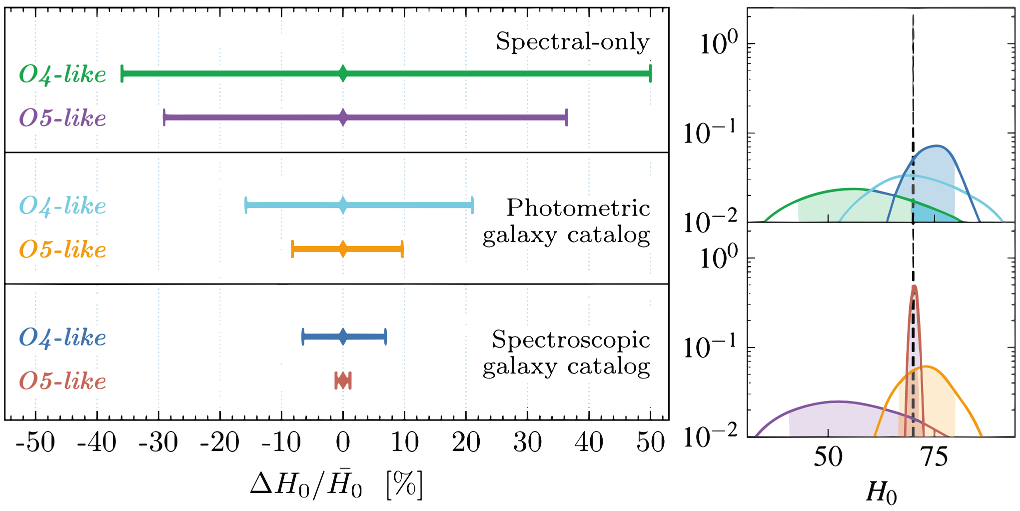


Constraints on H_0

Three MCMC analyses using the EMCEE sampler (Foreman-Mackey et al. 2013) for each GW catalog (6 runs in total):

1. without the galaxy catalog (pure spectral sirens);
2. using the galaxy catalog with photometric errors;
3. using the galaxy catalog with spectroscopic errors.

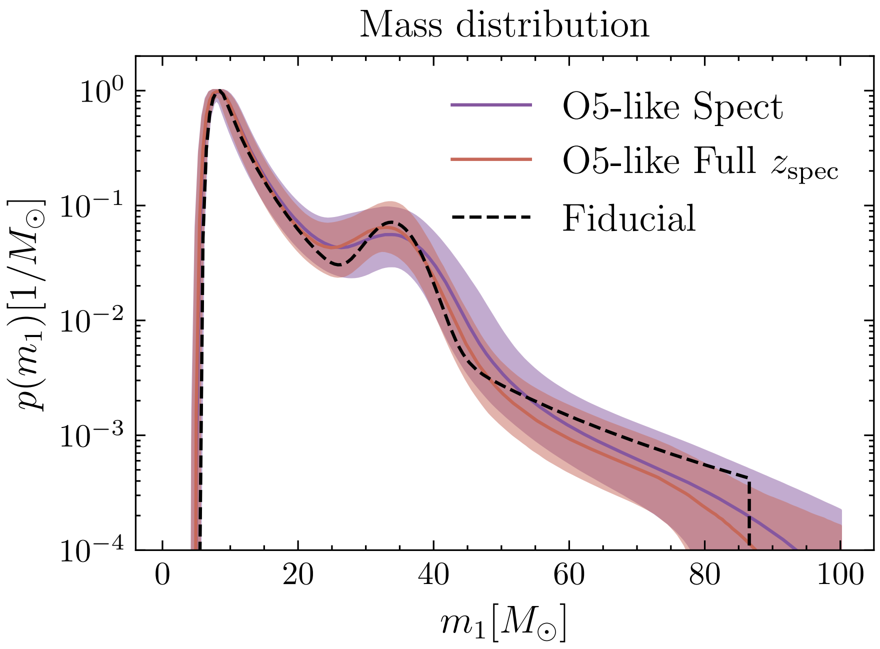
Varying 12 parameters in total (1 for cosmology, 8 for mass distribution, 3 for merger rate)

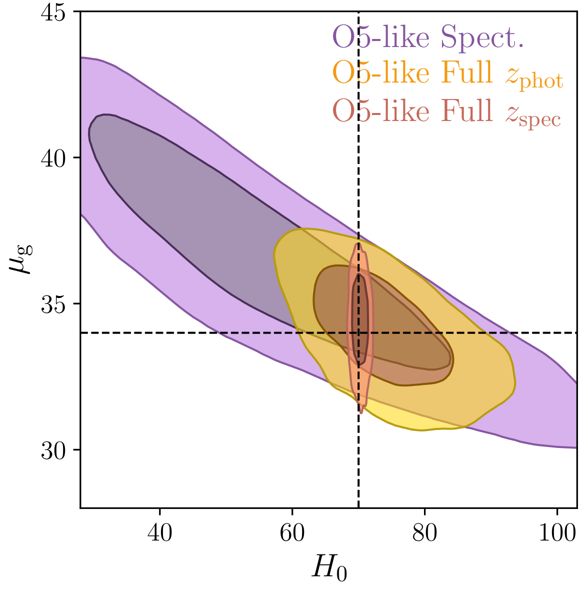


- Possible to achieve **1% accuracy** on H_0 in the O5 configuration with just 100 GW events and a complete spectroscopic galaxy catalog.
- In O5, using a photometric catalog degrades the accuracy to 9%.
- Constraint obtained with O4+spectroscopic catalog (7%) better than those with O5+photometric catalog.
- Spectral sirens cases not competitive with such a number of events (43%, 32%).
- Results robust since obtained while marginalising over population parameters.

Constraints on population parameters

Constraints on population parameters dominated by the number of events considered, but including galaxies helps in reducing the correlation between H_0 and some population parameters.





Towards 3G detectors - Einstein Telescope

$\sim$ **10^5 CPU hours** for a single MCMC analysis with just 100 GW events.

Computational time **scales linearly** with the number of events.

ET will detect up to $\sim$ **10^5 BBH per year**. Computationally limited.

Main bottleneck: KDE evaluation ($\sim 82\%$ of total time spent)

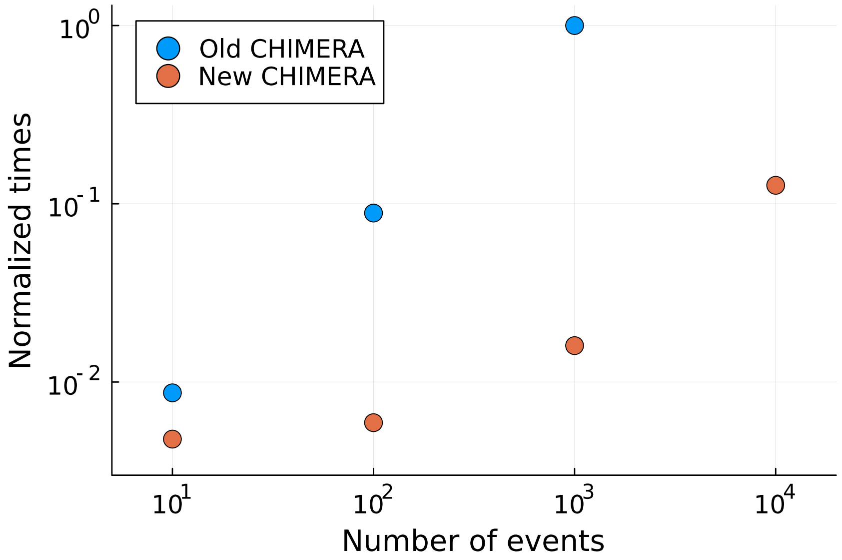
$$t_{\mathrm{KDE}} \sim \mathcal{O}\left(N_{\mathrm{events}} \times N_{\mathrm{pix,event}} \times N_{\mathrm{samples}} \times N_{\mathrm{z-grid}}\right)$$

Improvements:

- 3D KDE factorisation:

$$p_{\mathrm{gw}}(z, \mathrm{RA}, \mathrm{DEC}|\lambda) = p_{\mathrm{gw}}(\mathrm{RA}, \mathrm{DEC}) p_{\mathrm{gw}}(z|\lambda)$$

- KDE with binned data.
- Alternative density estimate algorithms (histogram, ASH, ...).
- Porting on GPU mass, rate and cosmology functions (to be optimised).



Towards 3G detectors - LISA

LISA will (hopefully) detect up to:

- dozens of MBHBs as bright sirens (Colpi et al. 2024);
- thousands of EMRIs as dark sirens (Babak et al. 2017; Colpi et al. 2024).

CHIMERA is ready from the computational point of view.

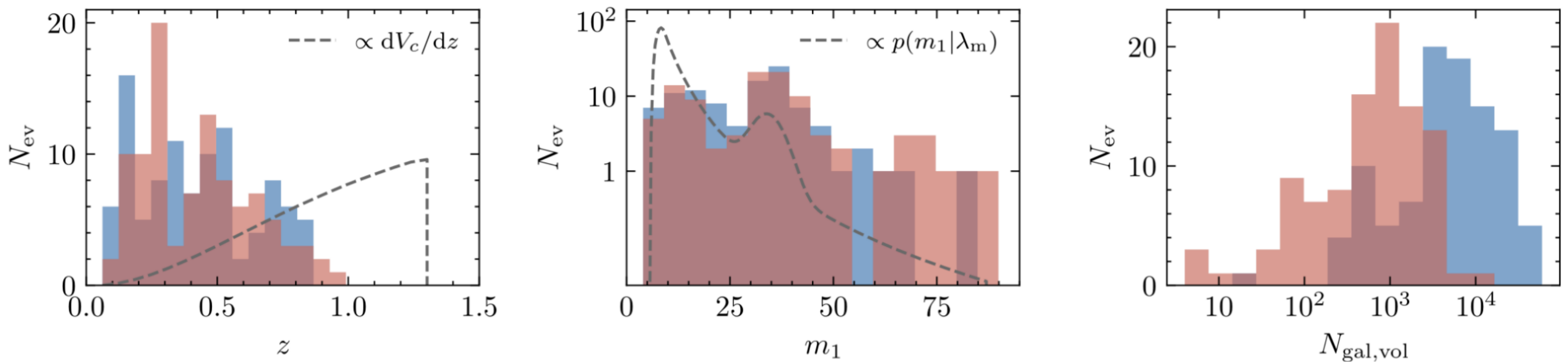
Forecast on constraints on cosmological parameters with EMRIs and a galaxy catalog only (Laghi et al. 2021).

Need to build EMRIs population model (parametric or non-parametric) to use them as spectral sirens and in the joint inference formalism implemented in CHIMERA.

$\Longrightarrow$ involved in the **LISA CosWG Collaborative Project on standard sirens**.

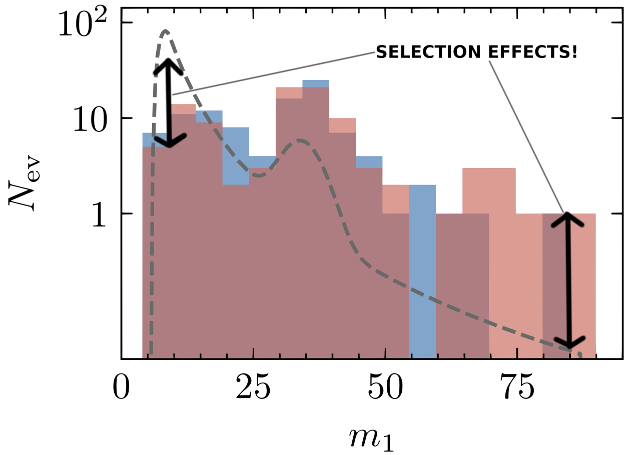
Thank You for Your Attention!

Backup - Mock catalogs properties and selection effects



Selection effects: more massive BBH are more likely to be detected as they produce louder GW signals.

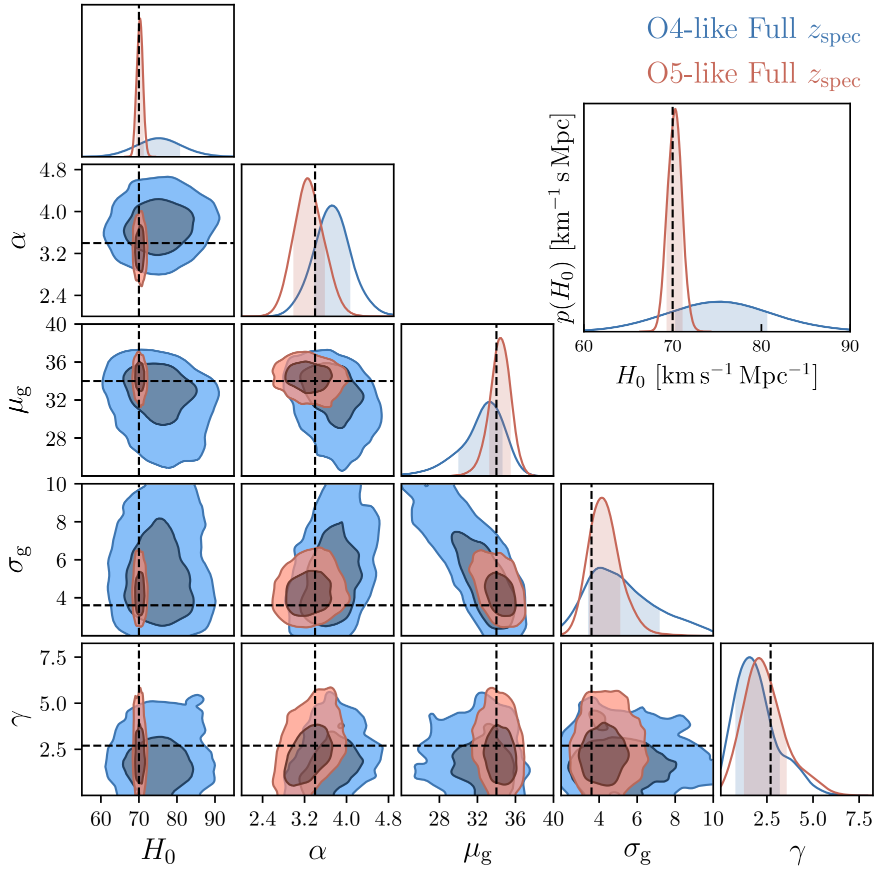
To reconstruct the true underlying population distributions it is necessary to correct this bias.



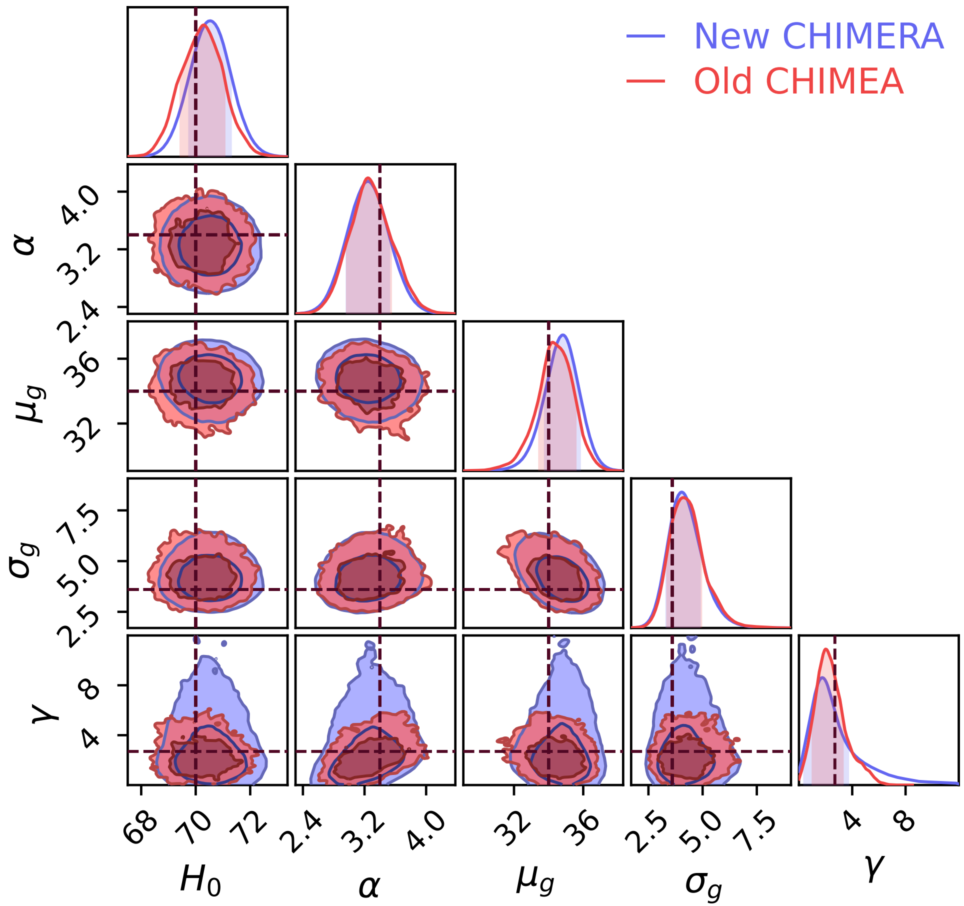
Backup - MCMC parameters and priors

Parameter	Description	Fiducial Value	Prior
	Cosmology (flat ΛCDM)		
H_0	Hubble constant [km/s/Mpc]	70.0	$\mathcal{U}(10.0, 200.0)$
$\Omega_{\mathrm{m},0}$	Matter energy density	0.25	Fixed
	Rate evolution (Madau-like)		
γ	slope at $z < z_p$	2.7	$\mathcal{U}(0.0, 12.0)$
κ	slope at $z > z_p$	3	$\mathcal{U}(0.0, 6.0)$
z_p	peak redshift	2	$\mathcal{U}(0.0, 4.0)$
	Mass distribution (PowerLaw+Peak)		
α	(primary) slope of the power law	3.4	$\mathcal{U}(1.5, 12.0)$
β	(secondary) slope of the power law	1.1	$\mathcal{U}(-4.0, 12.0)$
δ_m	(primary) smoothing parameter [$\mathrm{M}_\odot$]	4.8	$\mathcal{U}(0.01, 10.0)$
m_low	lower value [$\mathrm{M}_\odot$]	5.1	$\mathcal{U}(2.0, 50.0)$
m_high	upper value [$\mathrm{M}_\odot$]	87.0	$\mathcal{U}(50.0, 200.0)$
μ_g	(primary): Gaussian component mean [$\mathrm{M}_\odot$]	34.0	$\mathcal{U}(2.0, 50.0)$
σ_g	(primary): Gaussian component std. dev. [$\mathrm{M}_\odot$]	3.6	$\mathcal{U}(0.4, 10.0)$
λ_g	(primary): fraction of the Gaussian component	0.039	$\mathcal{U}(0.01, 0.99)$

Backup - MCMC corner plot



Backup - Validation of the improved CHIMERA



Backup - Population priors I

$p_{\text{pop}}(\boldsymbol{\theta}|\boldsymbol{\lambda})$ is the **population prior** and can be written as:

$$p_{\text{pop}}(\boldsymbol{\theta}|\boldsymbol{\lambda}) = p(m_1, m_2|\lambda_{\text{m}}) \frac{p_{\text{gal}}(z, \hat{\Omega}|\lambda_{\text{c}})\, p_{\text{rate}}(z|\lambda_{\text{r}})}{\int \mathrm{d}z\, \mathrm{d}\hat{\Omega}\, p_{\text{gal}}(z, \hat{\Omega}|\lambda_{\text{c}})\, p_{\text{rate}}(z|\lambda_{\text{r}})}$$

⇒ $p(m_1, m_2|\lambda_{\text{m}})$ is the probability of having m_1, m_2 given a mass distributions;

⇒ $p_{\text{rate}}(z|\lambda_{\text{r}}) \propto \psi(z; \lambda_{\text{rate}})/(1+z)$ is the probability of having a merger at redshift z;

⇒ $p_{\text{gal}}(z, \hat{\Omega}|\lambda_{\text{c}})$ is the probability that there is a galaxy (*host*) at $(z, \hat{\Omega})$ and is constructed from a galaxy catalog and takes into account the **completeness**

Rate evolution: Madau

$$\psi(z; \lambda_{\text{rate}}) = \frac{(1+z)^{\gamma}}{1 + \left(\frac{1+z}{1+z_p}\right)^{\gamma+\kappa}},$$

Backup - Population priors II

Mass distribution: power law + Gaussian peak

$$p(m_1, m_2|\lambda_{\rm mass}) = p(m_1|\lambda_{\rm mass})\, p(m_2|m_1, \lambda_{\rm mass})\,,$$

where probability of the primary BH mass is given by

$$p(m_1|\lambda_{\rm mass}) \propto \left[(1-\lambda_{\rm p})\,\mathcal{P}(m_1) + \lambda_{\rm p}\,\mathcal{G}(m_1)\right]\,\mathcal{S}(m_1)\,.$$

Here, $\mathcal{P}(m_1) \propto m_1^{-\alpha}$ is a power-law truncated in the domain $m_1 \in [m_{\rm low}, m_{\rm high}]$, $\mathcal{G}(m) \propto \mathcal{N}(\mu_{\rm g}; \sigma_{\rm g}^2)$ is the Gaussian component, and $\mathcal{S}(m_1) \in [0, 1]$ is a smoothing piece-wise function defined as Abbott et al. 2021:

$$S\left(m_1 \mid m_{\rm low}, \delta_m\right) = \begin{cases} 0 & (m_1 < m_{\rm low}) \\ \left[f\left(m_1 - m_{\rm low}, \delta_m\right) + 1\right]^{-1} & (m_{\rm low} \leqslant m_1 < m_{\rm low} + \delta_m)\,, \\ 1 & (m_1 \geqslant m_{\rm low} + \delta_m) \end{cases} \quad \text{with} \quad f\left(m', \delta_m\right) = \exp\left(\frac{\delta_m}{m'} + \frac{\delta_m}{m' - \delta_m}\right)$$

The secondary BH mass is modeled by a power-law with an index β in the domain $m \in [m_{\rm low}, m_1]$.

$$p(m_2|m_1, \lambda_{\rm mass}) \propto \begin{cases} m_2^{-\beta} & (m_{\rm low} \leqslant m_2 \leqslant m_1) \\ 0 & \text{otherwise} \end{cases}$$

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