

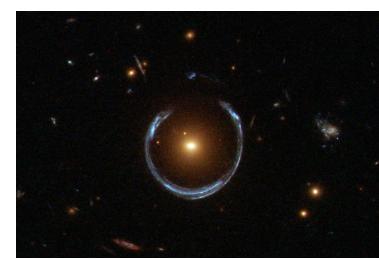
# Gravitational lensing applied to gravitational waves

Martina Toscani,  
PostDoc @ UniMib

Image credit: ESA/Hubble & NASA

June 17th - 21st, 2024

LISA CosWG meeting @ Universidade do Porto

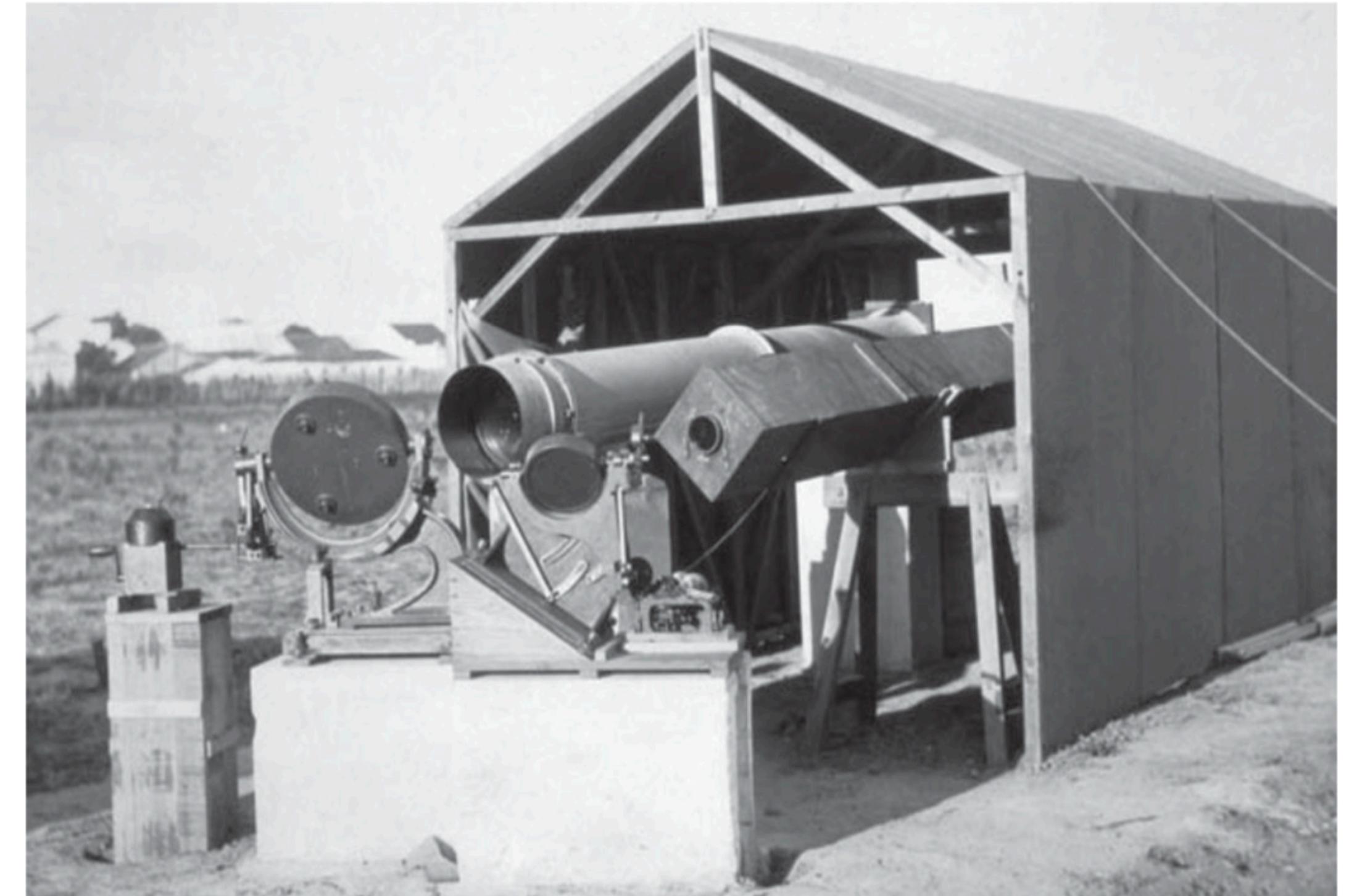


## What is lensing?

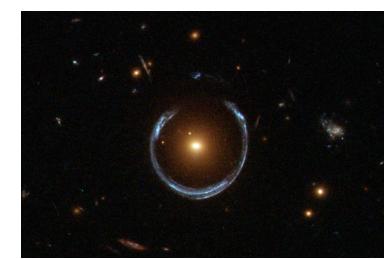
Gravitational field of a massive object (lens) bends the path of the signal from a distant source

GR predicts double displacement wrt Newtonian gravity

Observationally confirmed by Dyson, Eddington and Davidson during the Solar eclipse of 1919



Instruments used for the experiment, picture by Science Museum of London. Longhair 2015.



## What is lensing?

**Shear:** image stretched or compressed without changing the area

**Convergence:** change in the image size

**Magnification factor**  $\mu$

$> 1$ , brighter

$< 1$ , dimmer

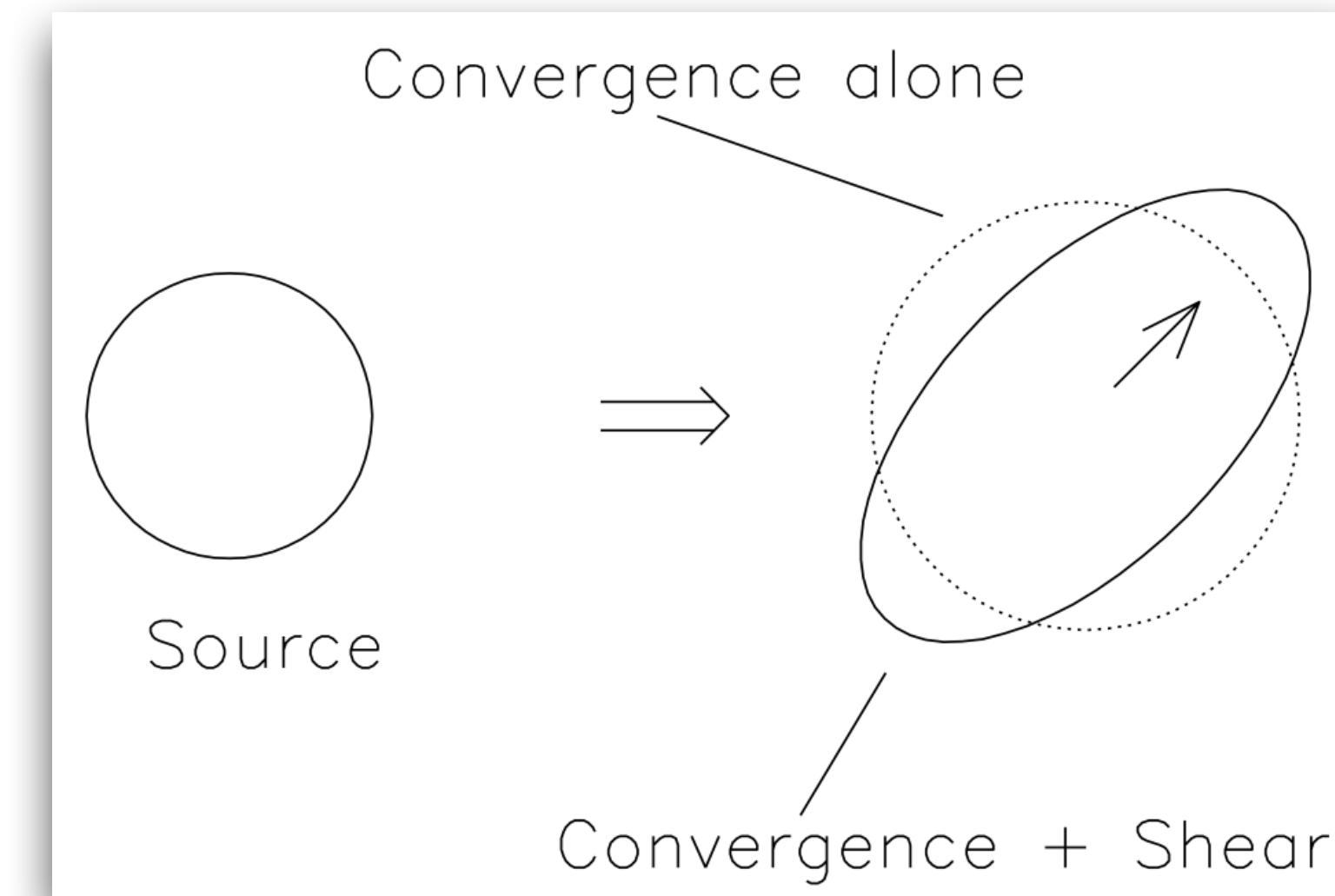
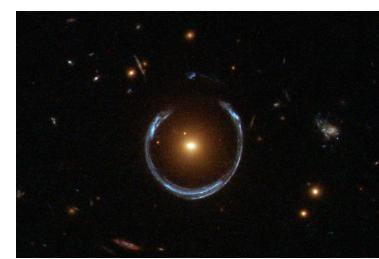


Image from Narayan & Bartelmann 1995



## What is lensing?

**Shear:** image stretched or compressed without changing the area

**Convergence:** change in the image size

**Multiple images:** production of multiple images of the source

Positive or negative parity

**Magnification factor**  $\mu$

$> 1$ , brighter

$< 1$ , dimmer

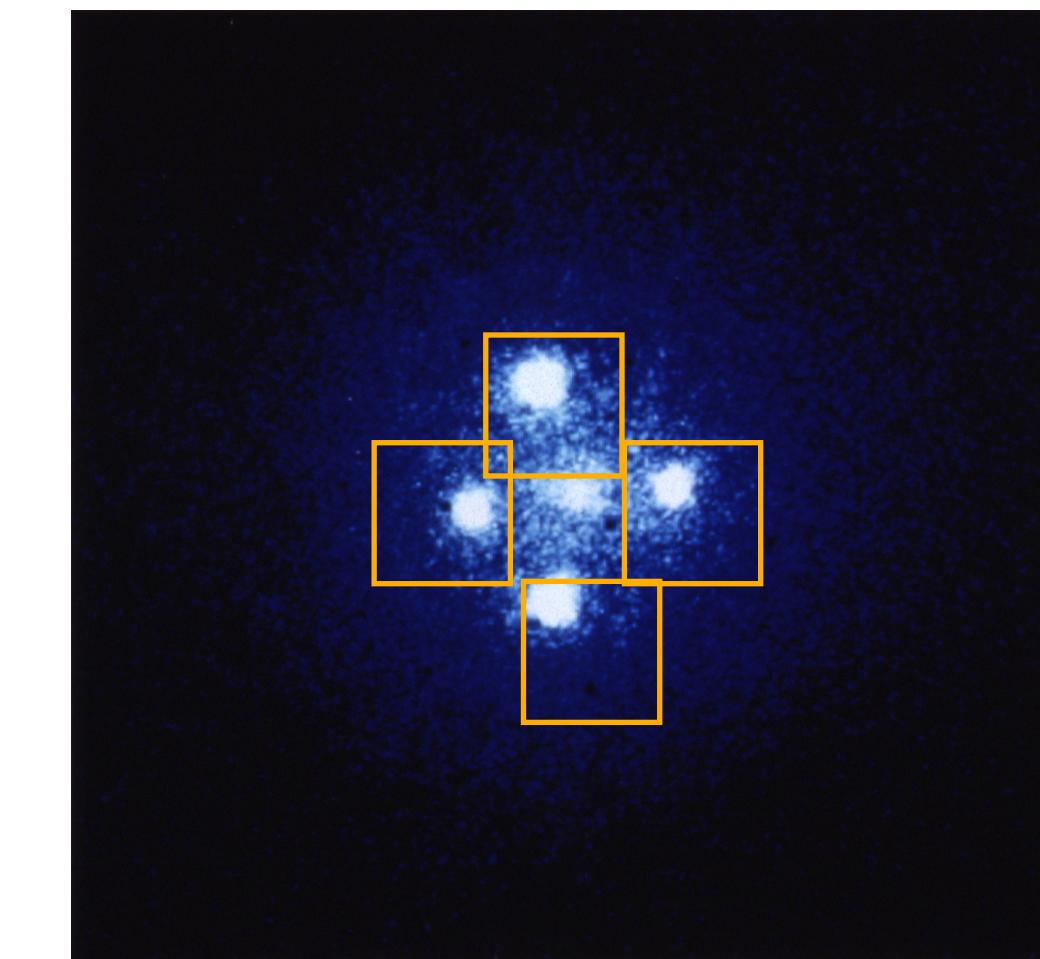
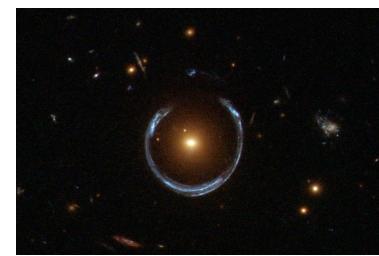


Image from NASA/ESA



# What is lensing?

**Strong lensing:**  
massive lens, (near)  
perfect alignment,  
individual sources

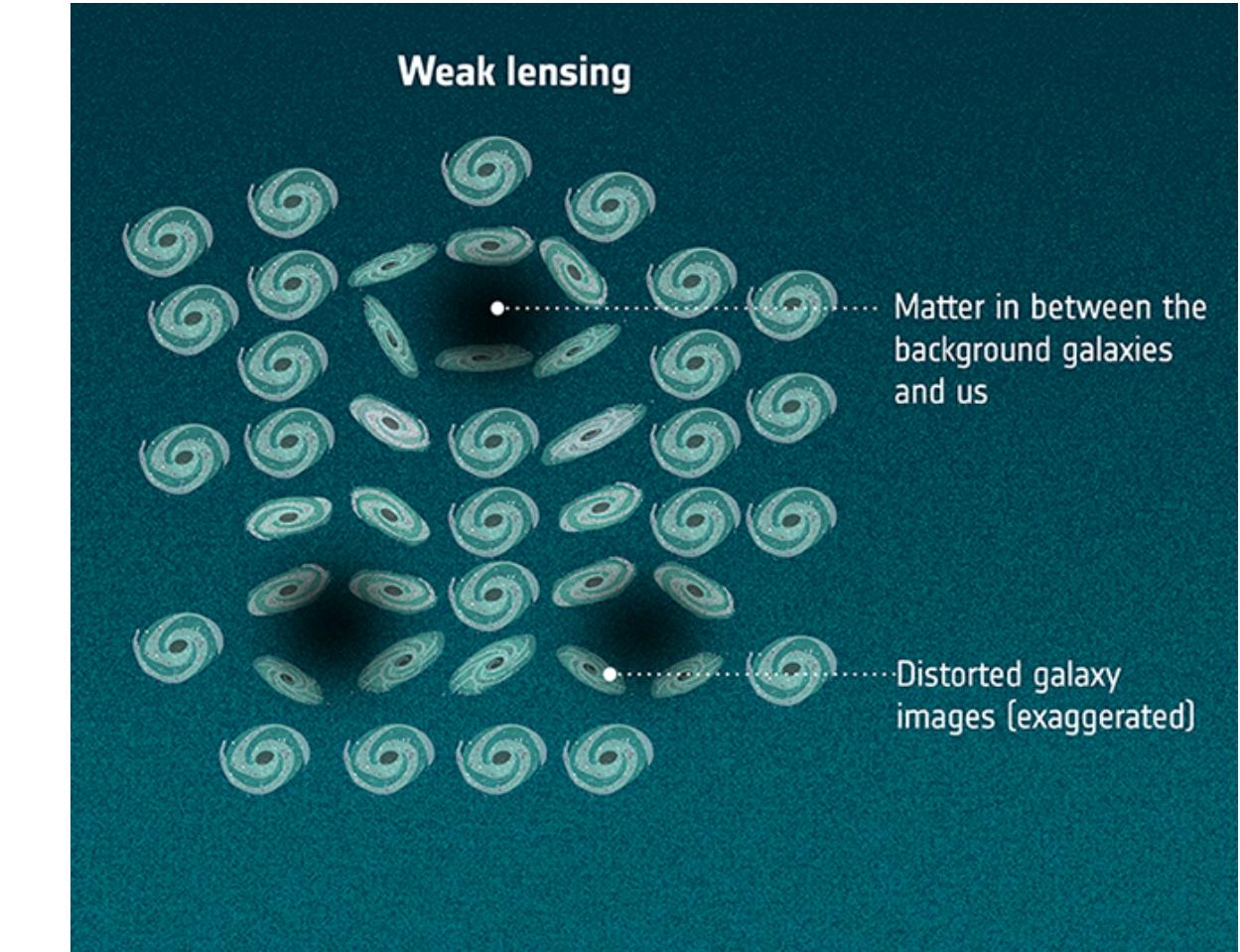
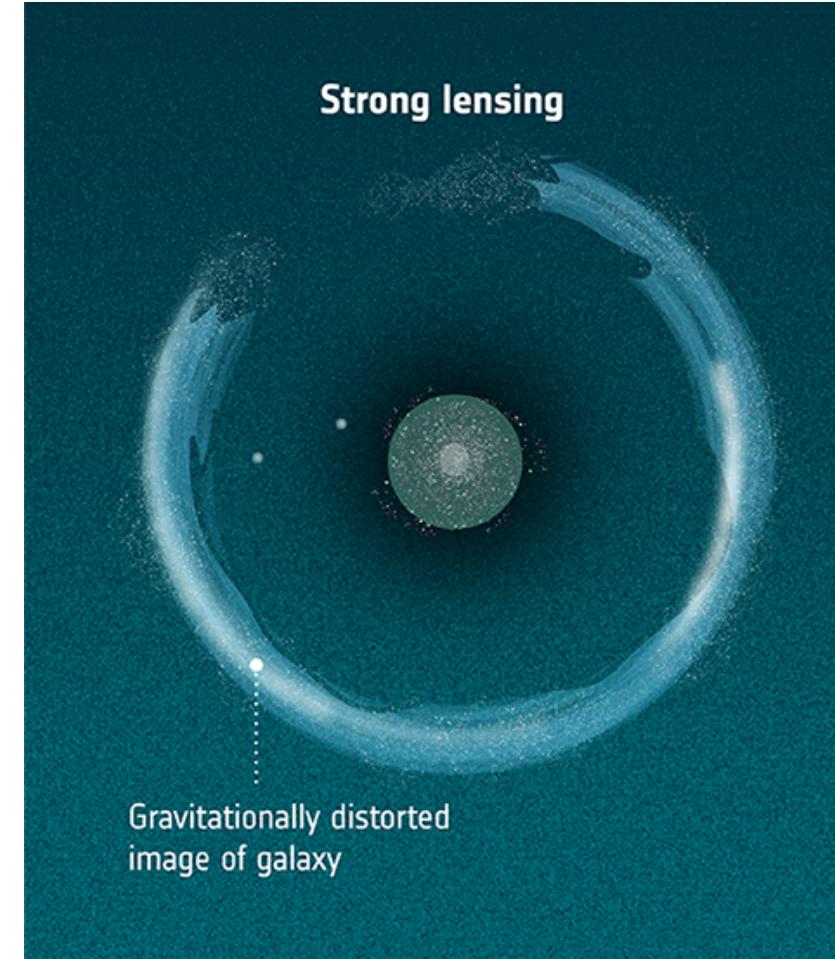
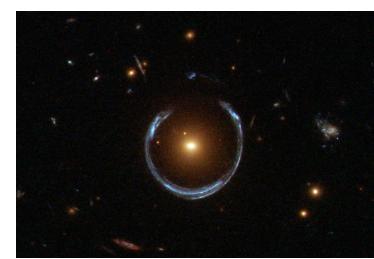
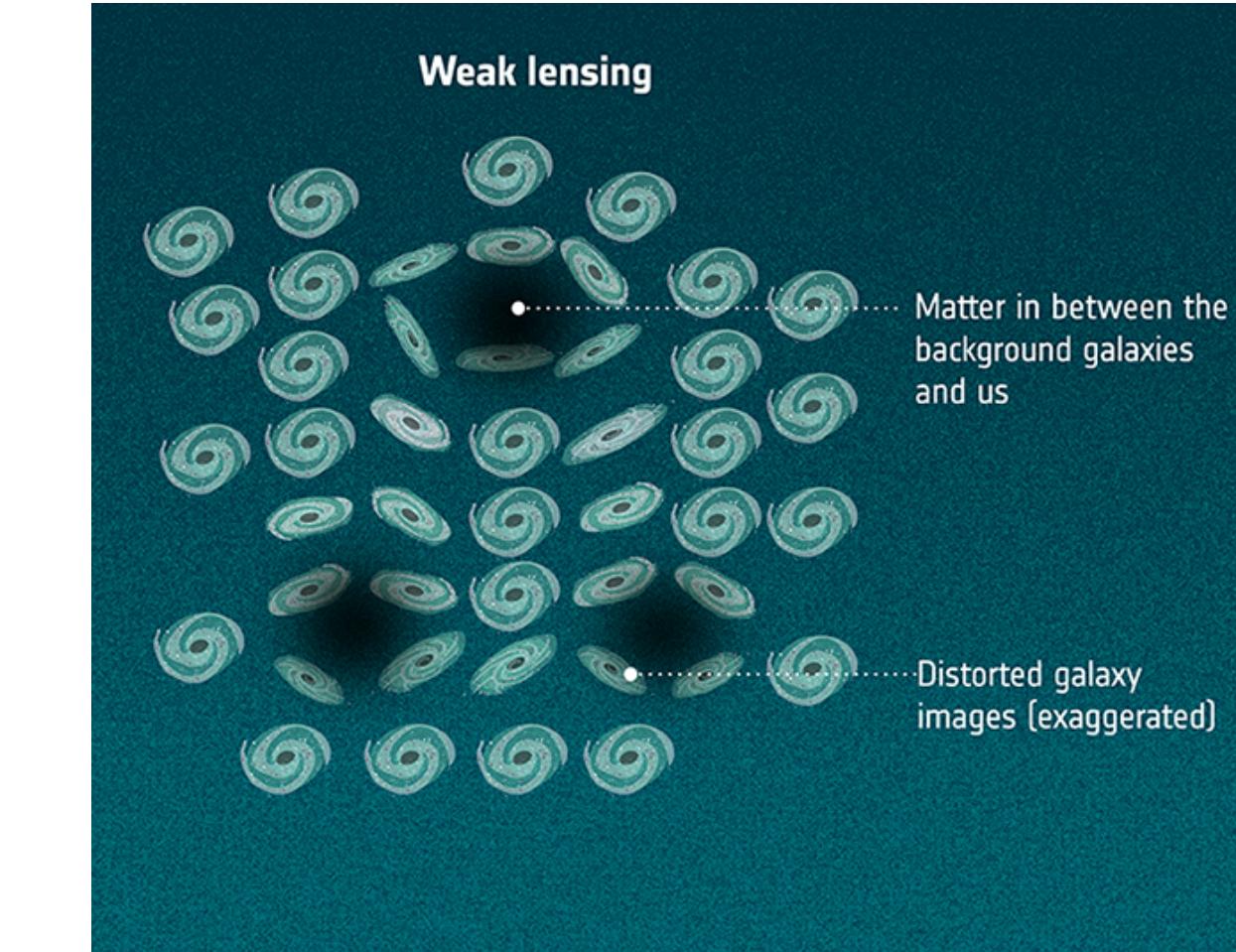
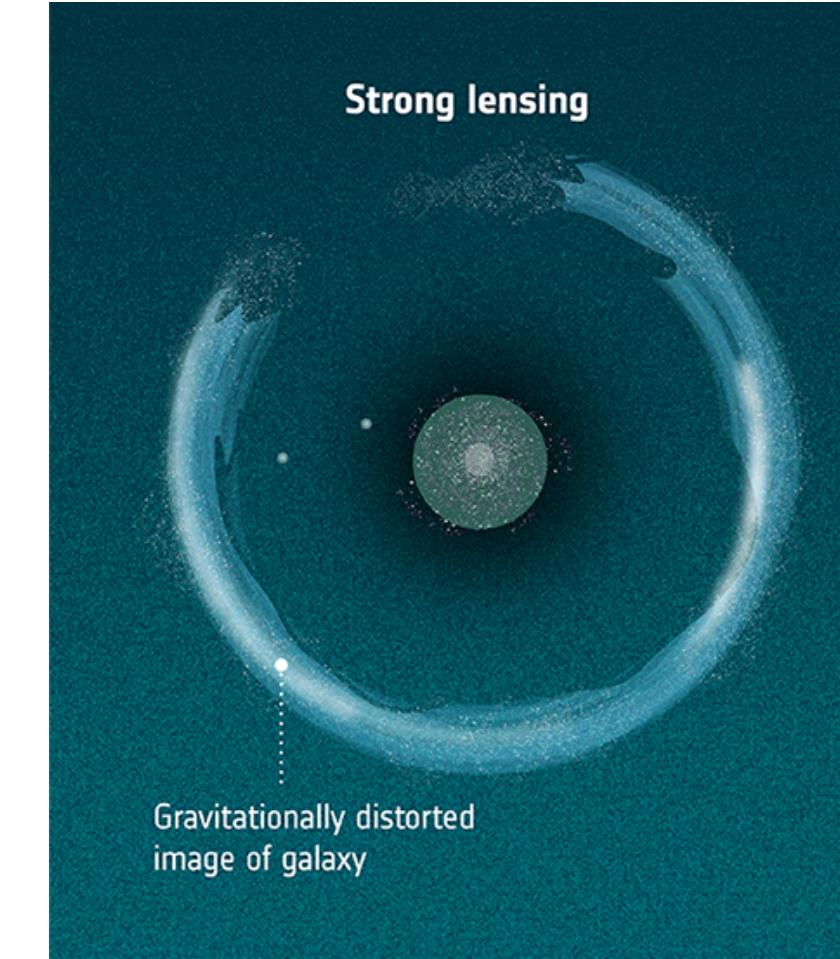


Image from ATG under contract with ESA



# What is lensing?

**Strong lensing:**  
massive lens, (near)  
perfect alignment,  
individual sources



**Weak lensing:**  
massive lens,  
moderate/poor  
alignment, statistical  
studies

Image from ATG under contract with ESA

**Microlensing:** compact  
lens, small scale alignment,  
temporary effect

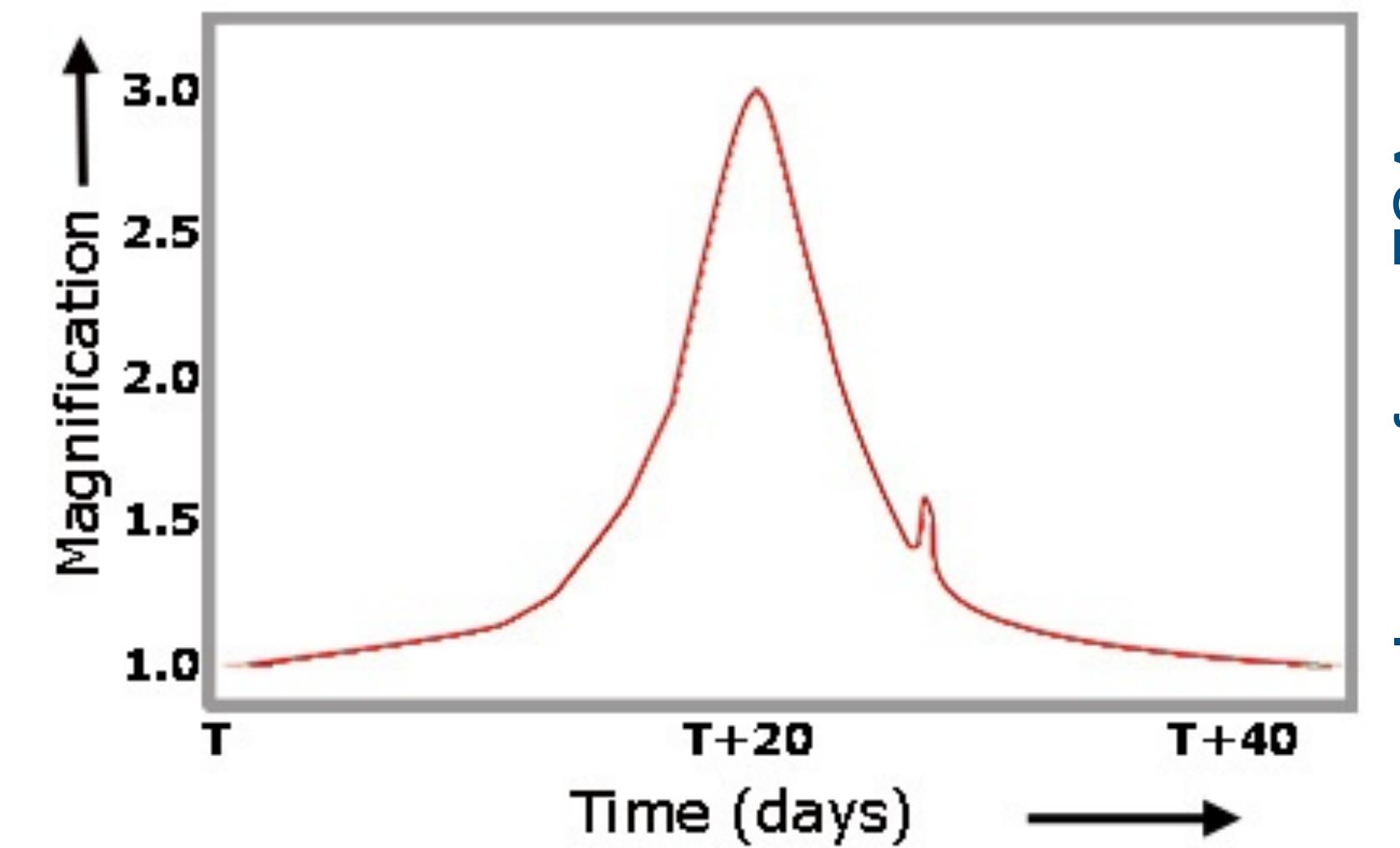
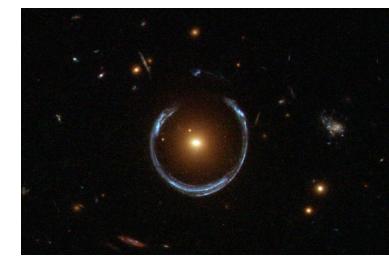
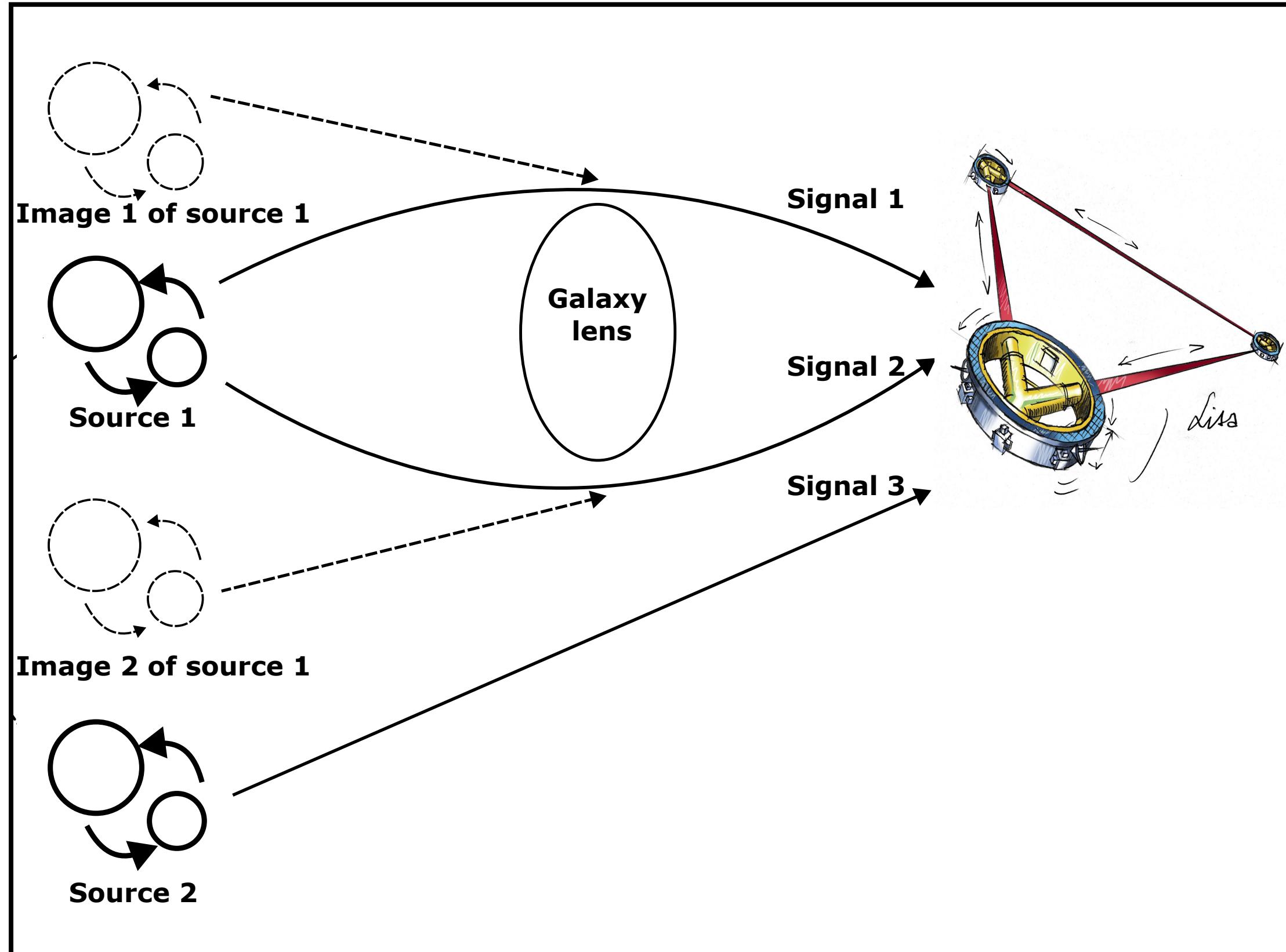


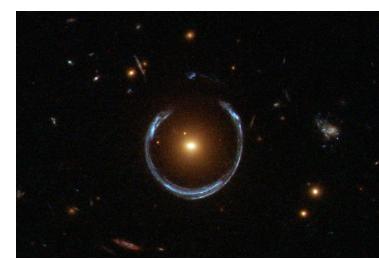
Image from ESA



# Why studying lensing?

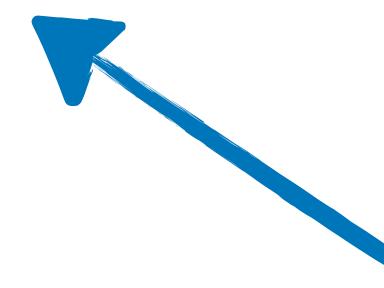
Image from Cheung et al, preprint





## Why studying lensing?

Proof of General Relativity



Understanding the nature of the lens

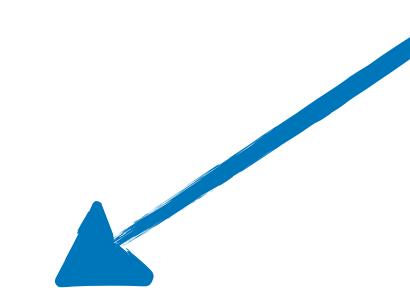
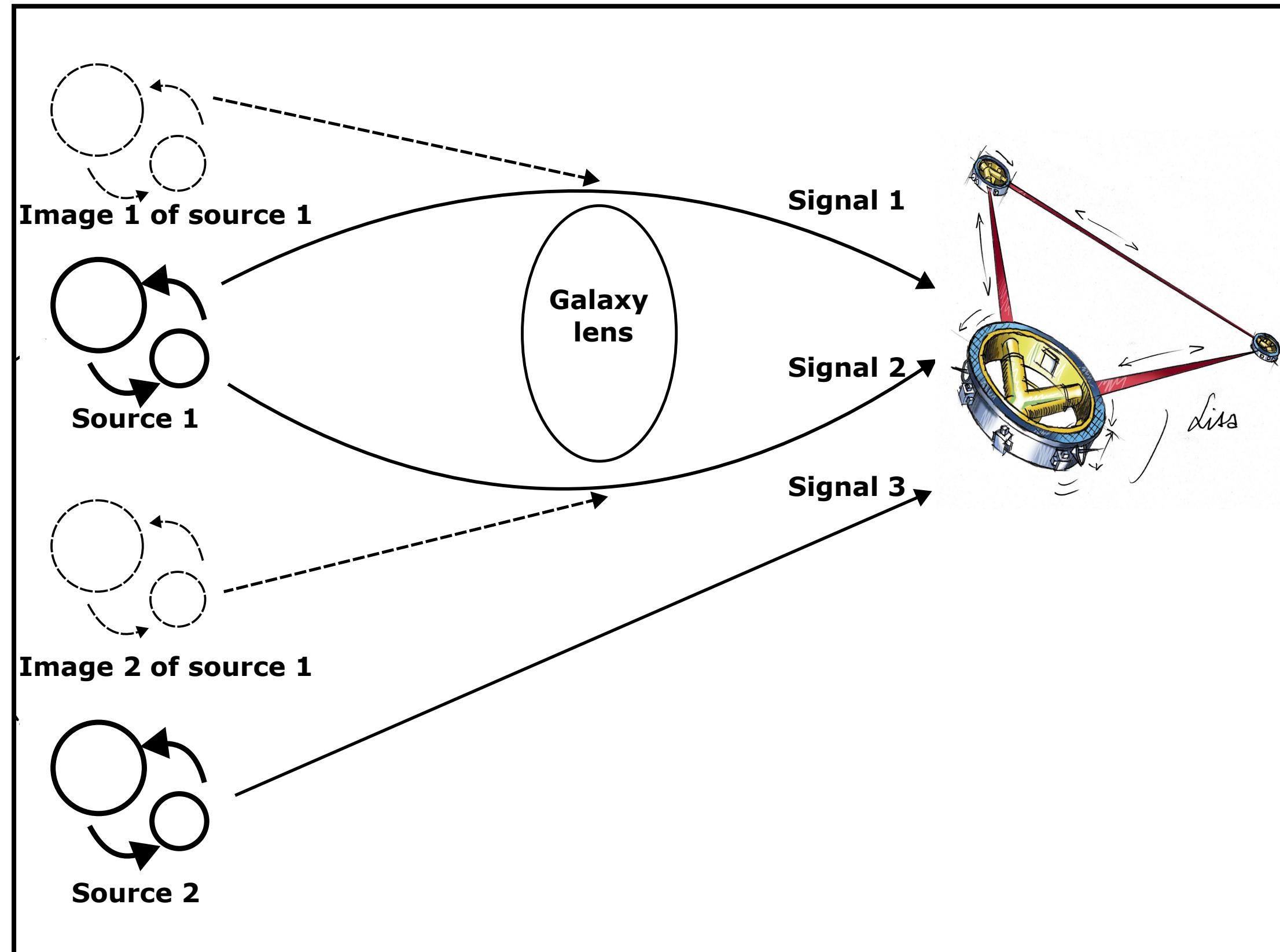
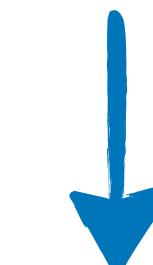


Image from Cheung et al, preprint



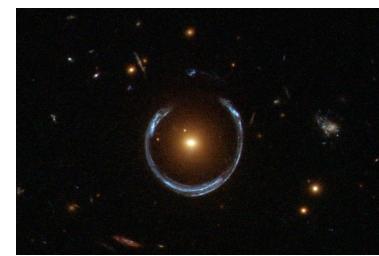
Constrains cosmological parameters



Understanding GWs propagation in strong gravity



Observing faint sources



# Lensing in the context of LISA

$$10^{-4}\text{Hz} \lesssim f \lesssim 10^{-1}\text{Hz}$$

Diffraction becomes relevant for

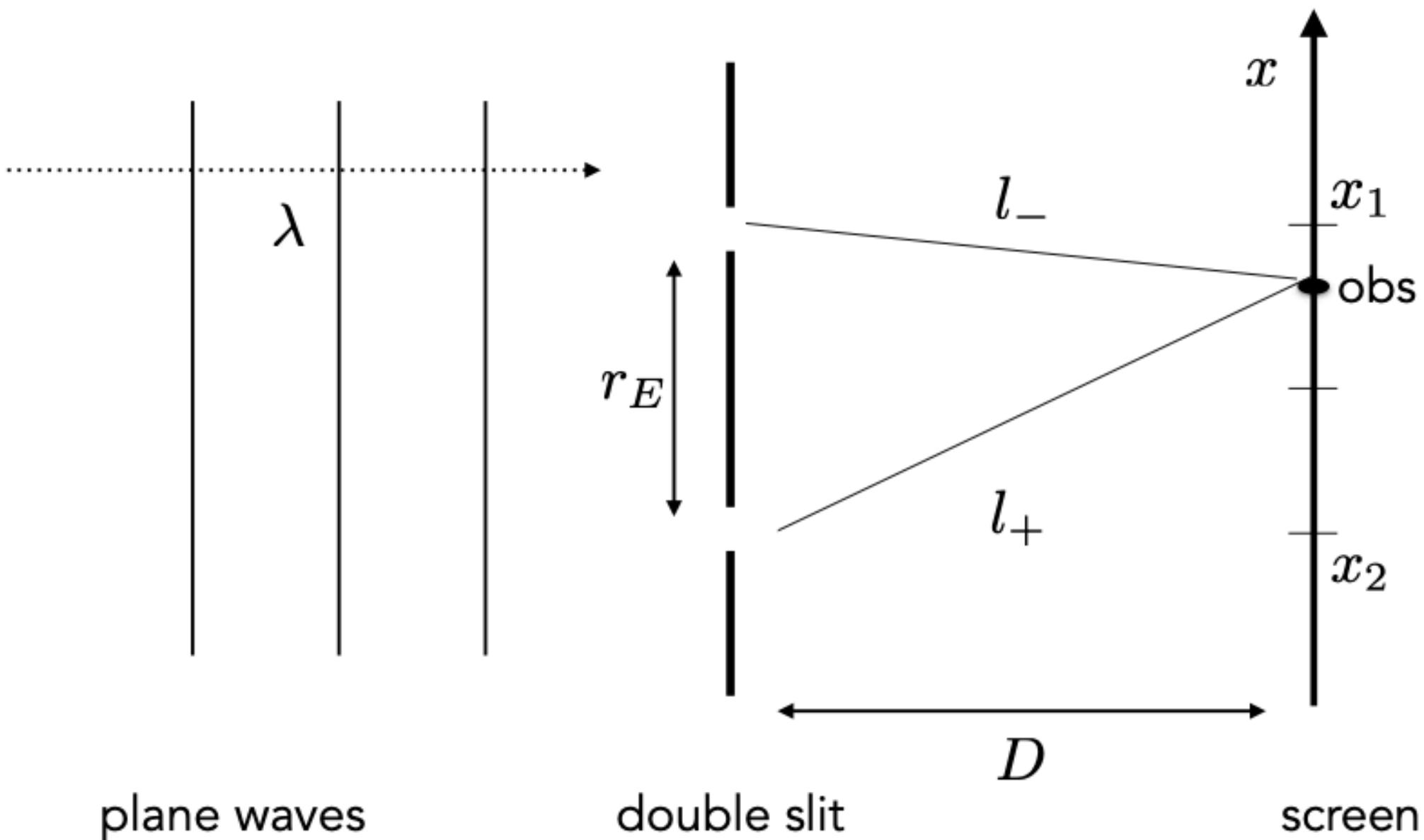


Image from Cusin & Lagos 2020

$$\lambda \gtrsim R_{s,c} \rightarrow M_L \lesssim 10^8 M_\odot \left( \frac{f}{\text{mHz}} \right)$$

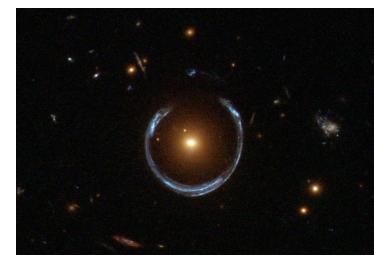


## Geometrical optics

Lensing by galaxy  
and galaxy clusters

## Wave optics

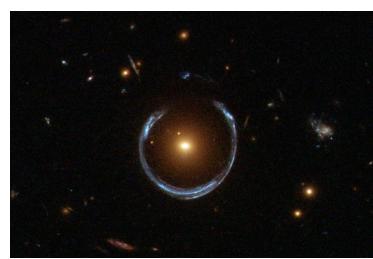
Lensing by black  
holes



## Strongly lensed MBHBs

Dai 2007, Sereno et al. 2010, Sereno et al. 2011, Ezquiaga et al. 2020, Goyal et al. 2020, Hannuksela et al. 2020, Cusin and Tamanini 2021, Wang 2021, Vijaykumar 2022, Wempe et al. 2022, Toscani et al. 2023, Toscani et al. 2024...

- Loudest sources
- Cosmological distances



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- Loudest sources

Up to a few in LISA (Sereno et al 2010)

Multiple near-identical images

Difference only in

- amplitude
- overall phase
- arrival time

Same sky location

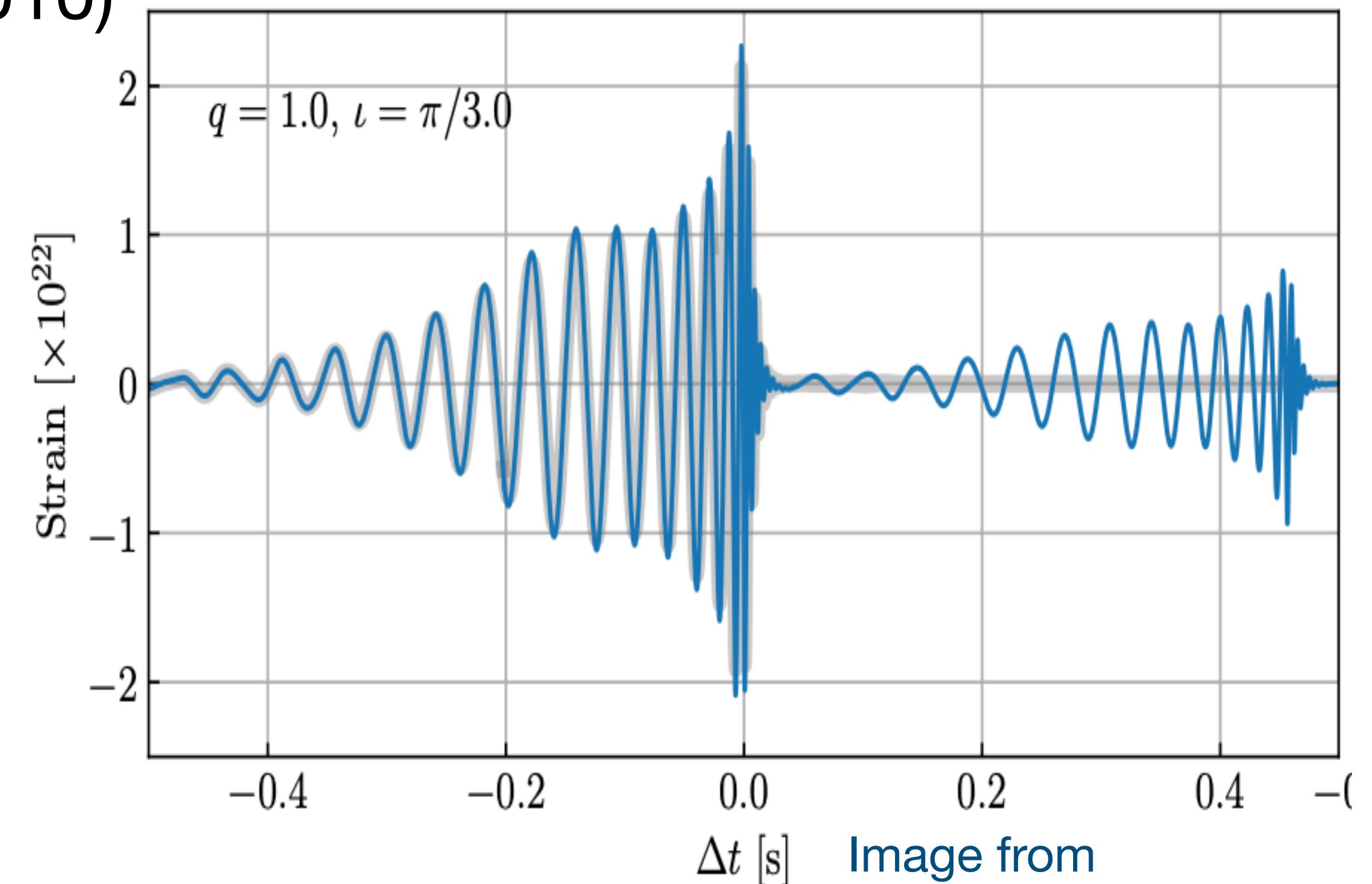
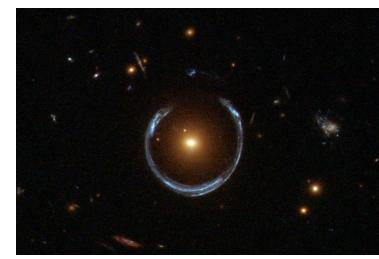


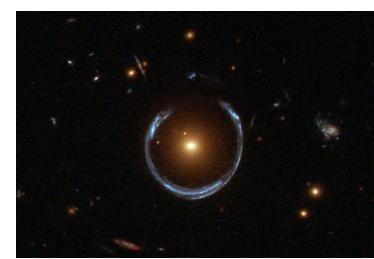
Image from  
Ezquiaga et al. 2020



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$$h \propto \frac{\sqrt{\mu}}{d_L}$$



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$$h \propto \frac{\sqrt{\mu}}{d_L}$$

Selection effects shift mean magnification and introduce distance bias

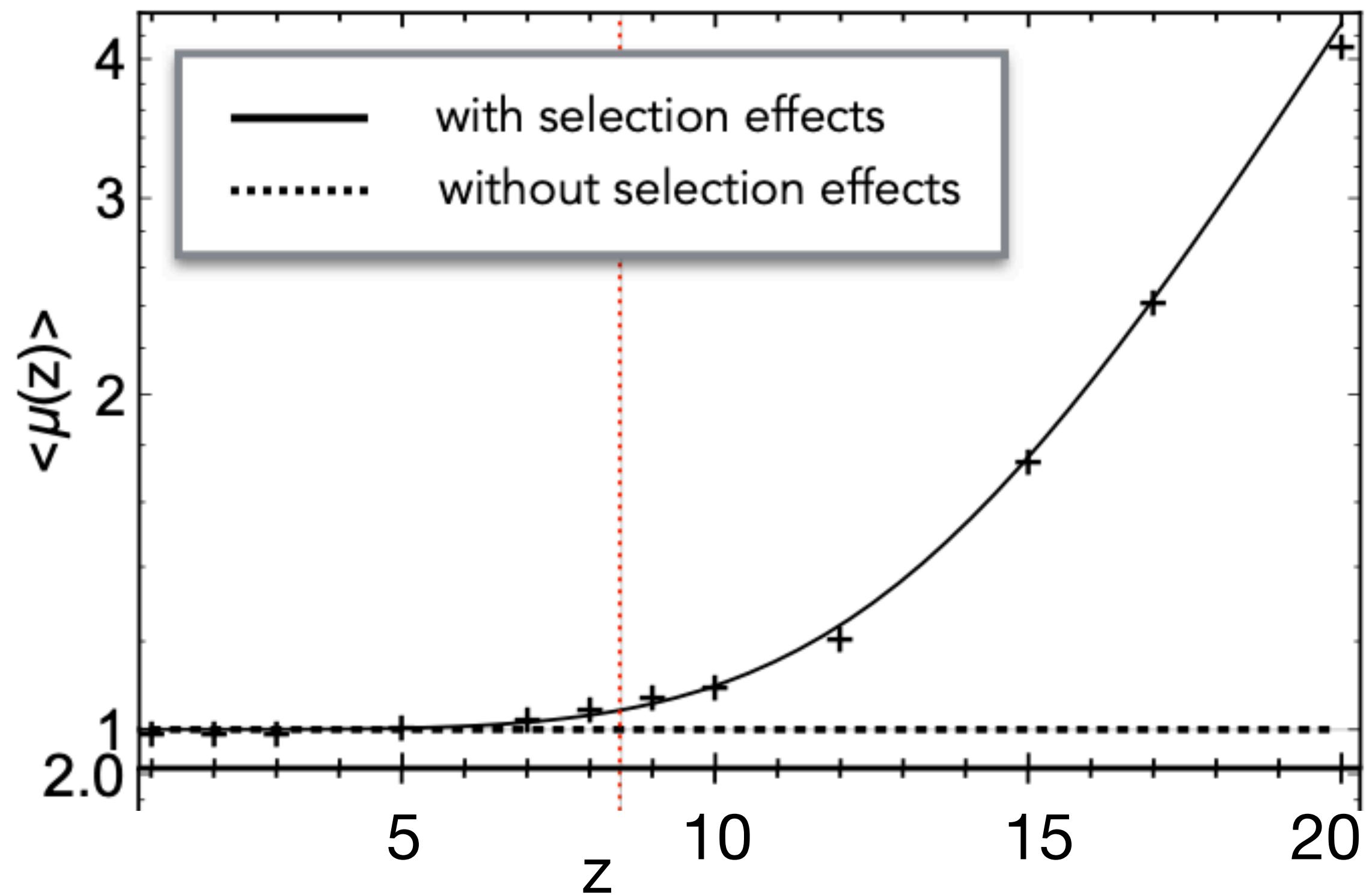
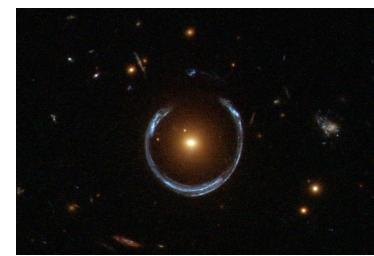


Image from Cusin & Tamanini 2021



## Strongly lensed MBHBs

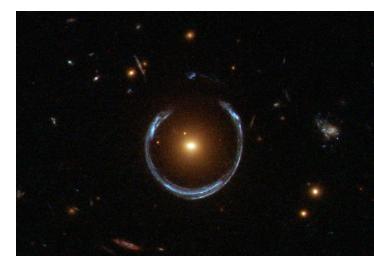
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Cosmological probes: time delay analysis, strong lensing statistics

- time delay analysis → Hubble constant

- strong lensing statistics → Dark matter distribution

No need for EM counterparts



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Host galaxy will be lensed



Follow up EM observations

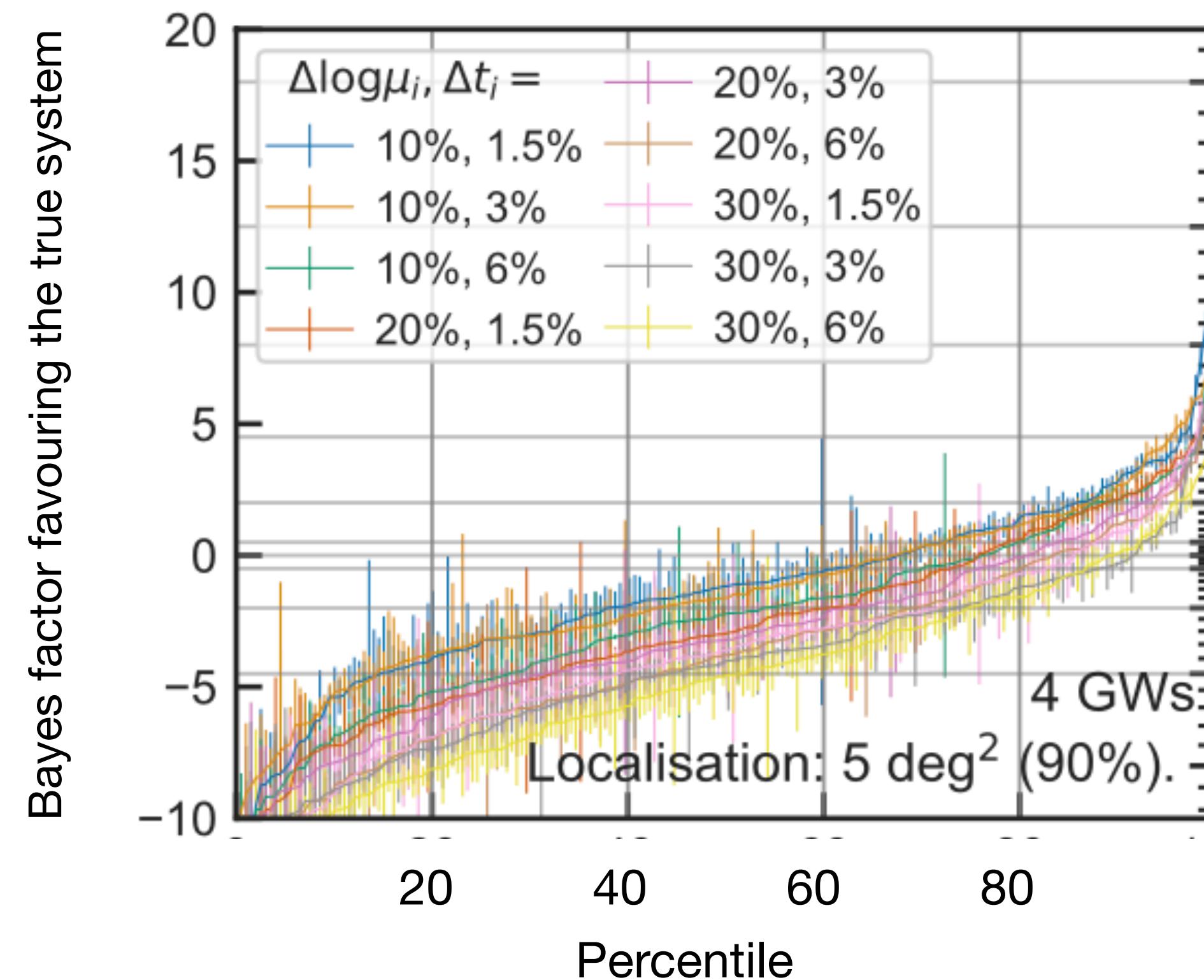
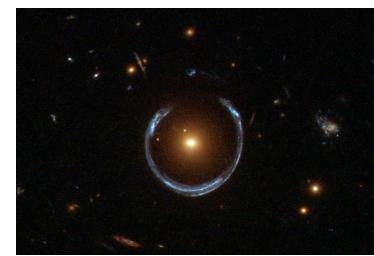


Image from Wempe et al. 2022

2 detected GWs: correct identification for 10ish% events

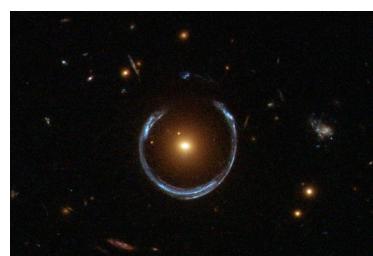
4 detected GWs: correct identification for 30ish% events



## Strongly lensed EMRIs

Dai 2007, Sereno et al. 2010, Sereno et al. 2011, Ezquiaga et al. 2020, Goyal et al. 2020, Hannuksela et al. 2020, Cusin and Tamanini 2021, Wang 2021, Vijaykumar 2022, Wempe et al. 2022, Toscani et al. 2023, Toscani et al. 2024...

- Excellent probes for strong field gravity
- Uncertain detection rates



# Strongly lensed EMRIs

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- Excellent probes for strong field gravity
- Uncertain detection rates

Up to 40 in LISA (Toscani et al. 2024)

Many observational cycles

Signal lasts longer than typical lensing time delay

Multiple images superimpose

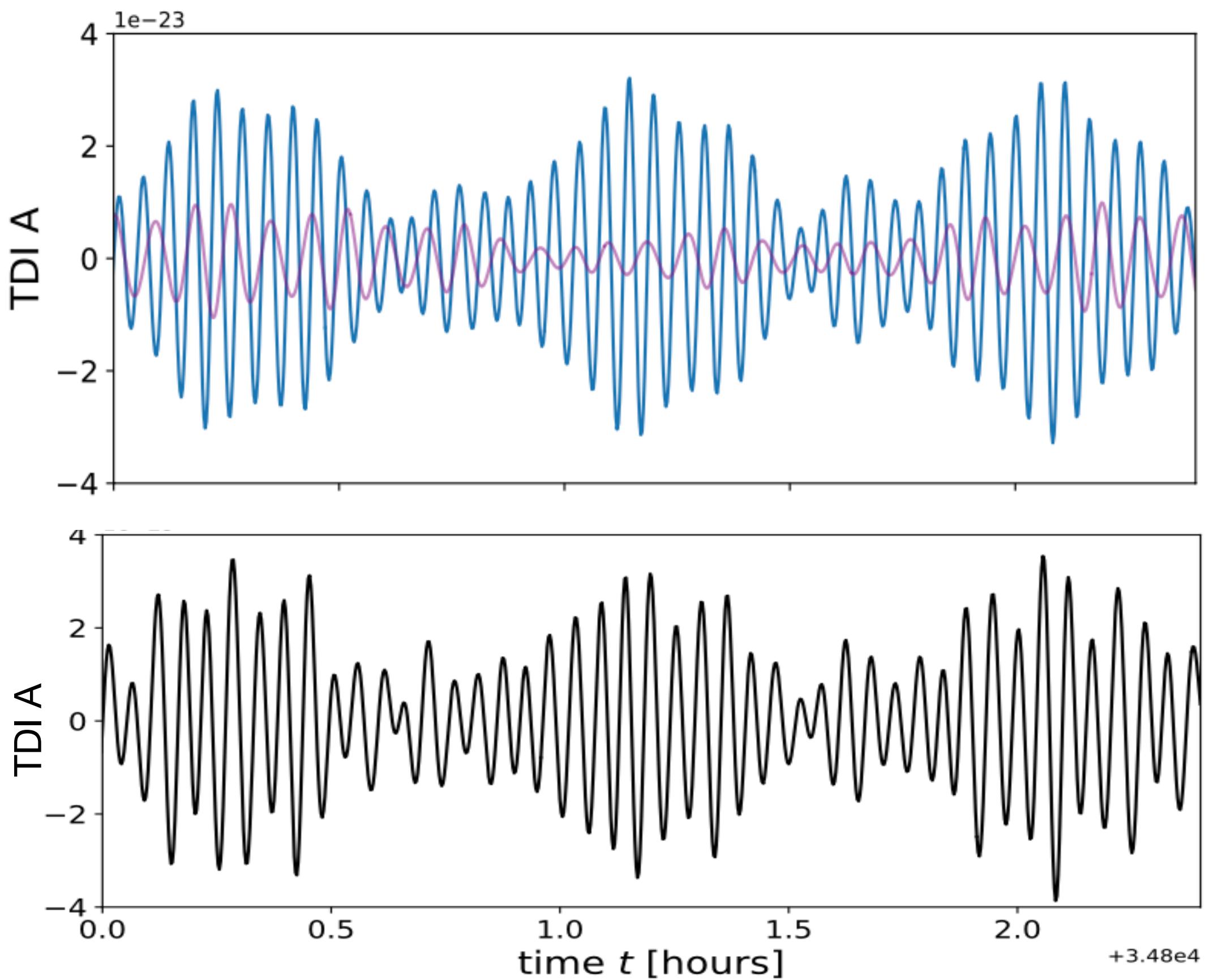
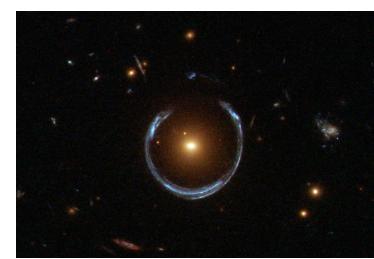


Image from Toscani et al. 2024



# Strongly lensed EMRIs

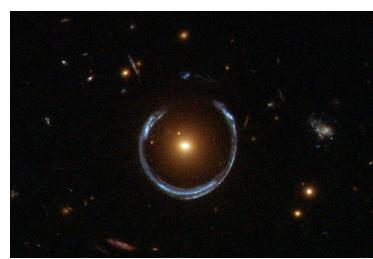
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Inject an unlensed EMRI waveform in the LISA data stream

Time shift operation

Maximise noise-weighted inner product

Inject a lensed EMRI waveform



# Strongly lensed EMRIs

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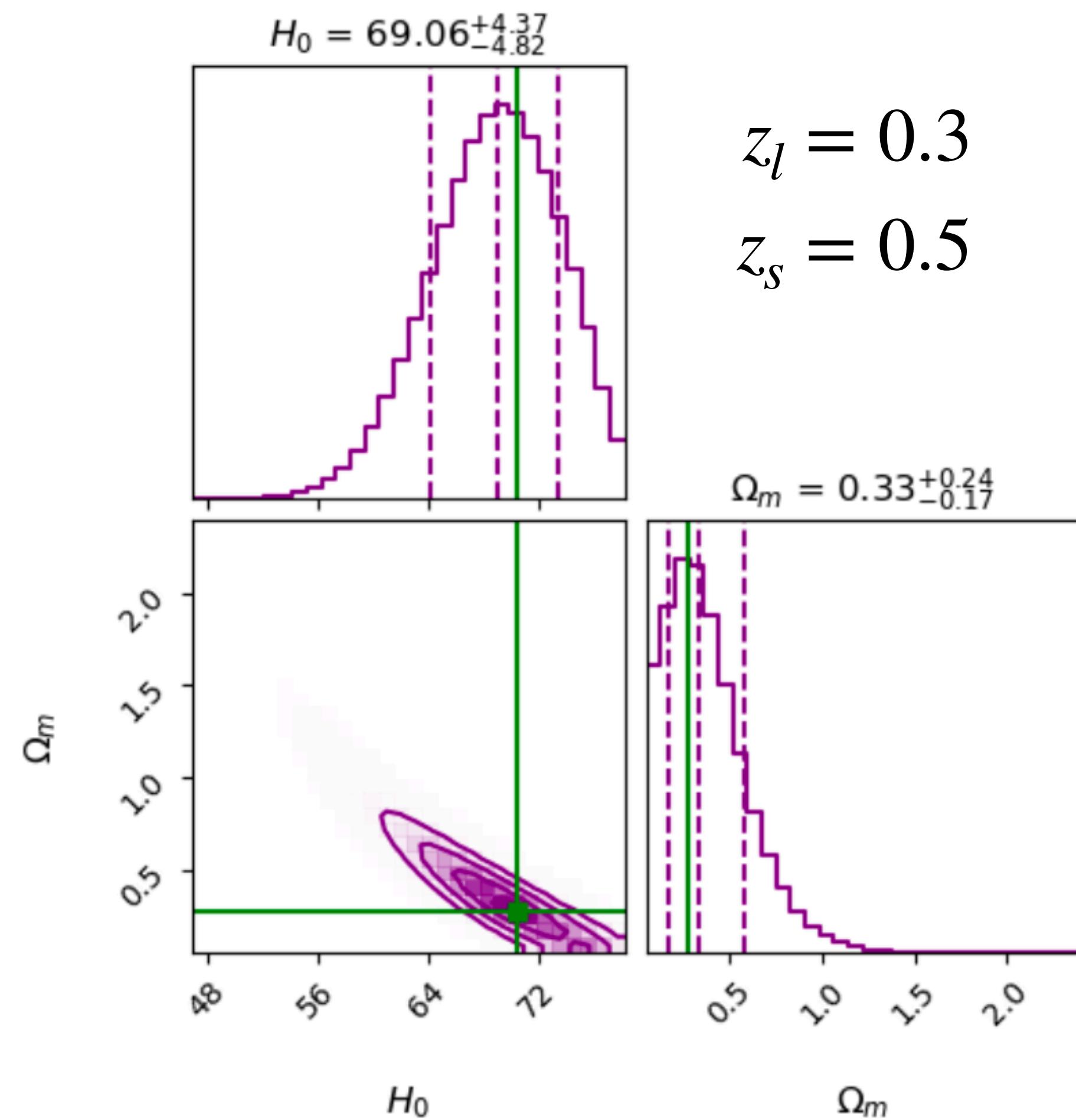
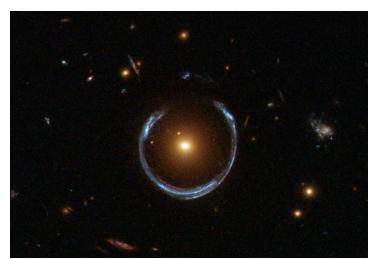


Image from Toscani et al. 2024



## Weak lensing by LSS

Cutler 2009, Shang 2010, Mpetha 2022,  
Balaudo et al. 2022

Large scale structures will induce small perturbations in the path of the propagating wave

Weak lensing will introduce some scatter in the measure of the luminosity distance

Combining bright and dark sirens with weak lensing gives better constraints

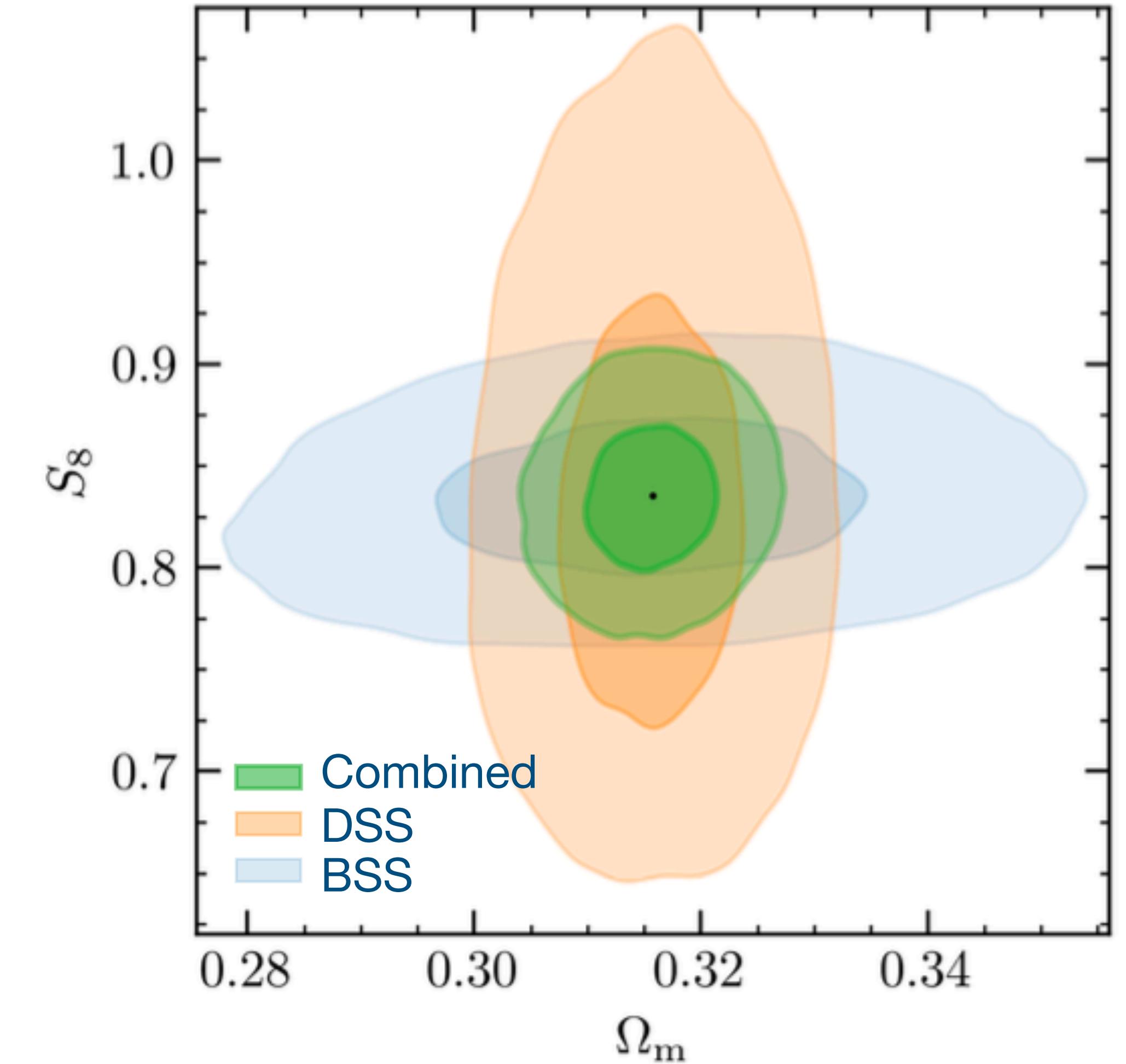
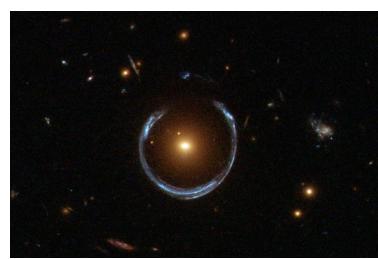


Image from Mpetha et al. 2022



## Wave effects

$$M_L \lesssim 10^8 M_\odot \left( \frac{f}{\text{mHz}} \right)$$

Lenses like BHs, stars..

Oscillatory behaviour due to interference of multiple images

Takahashi & Nakamura 2003, Gao et al.  
2022, Caliskan et al. 2022, Garoffolo  
2023

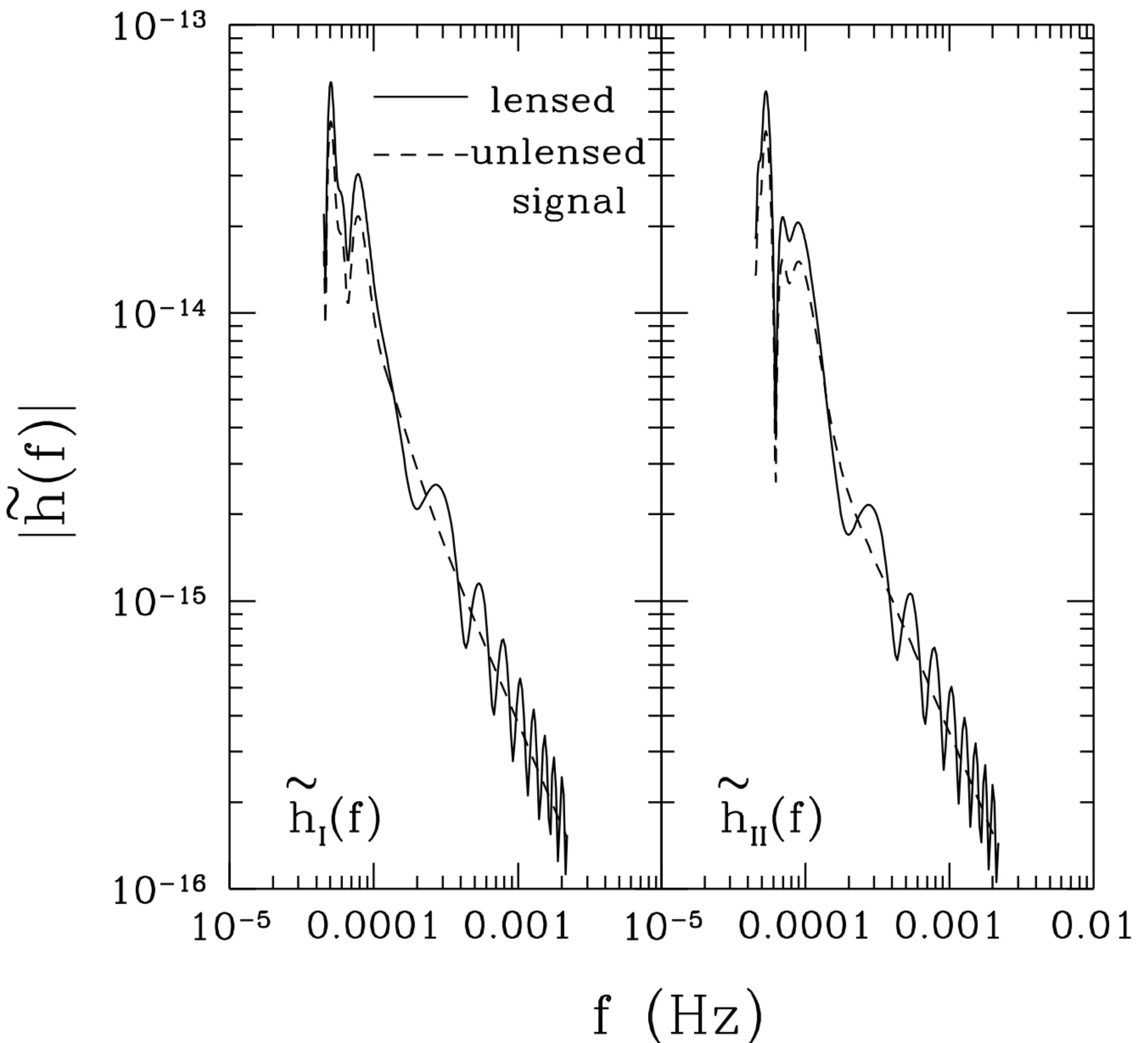
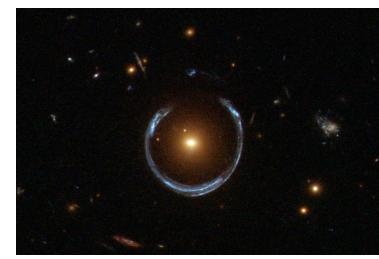


Image from Takahashi & Nakamura 2003



## *Conclusions*

In the upcoming years we will see lensed GWs :-)!!

Important to recognise lensing effect to properly reconstruct astrophysical properties of the source population

Useful to constrain cosmological parameters

*The End*

Any questions ... ?