PDFs@LHC for EW

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What is the problem?

PDFs are important
They are a large part of the uncertainty budget on extraction of SM parameters from precision data: M_W, $\sin^2\theta_W$, $\alpha_S(M_Z)$

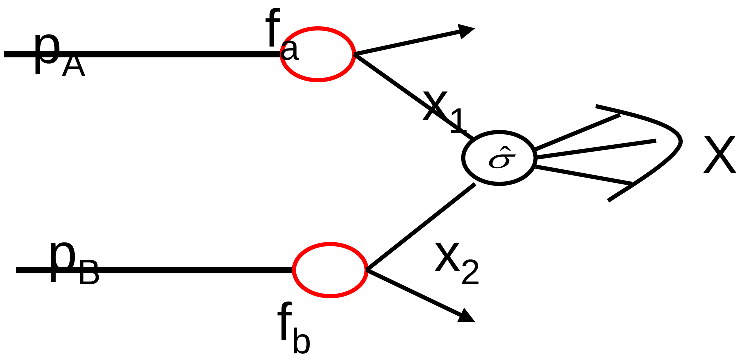
What should we do given that there is a choice of PDFs

Let's just look at some of the similarities and differences and how it impacts our EW determinations
And make a few suggestions

PDFs at the LHC

LHC cross sections are calculated as follows

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$



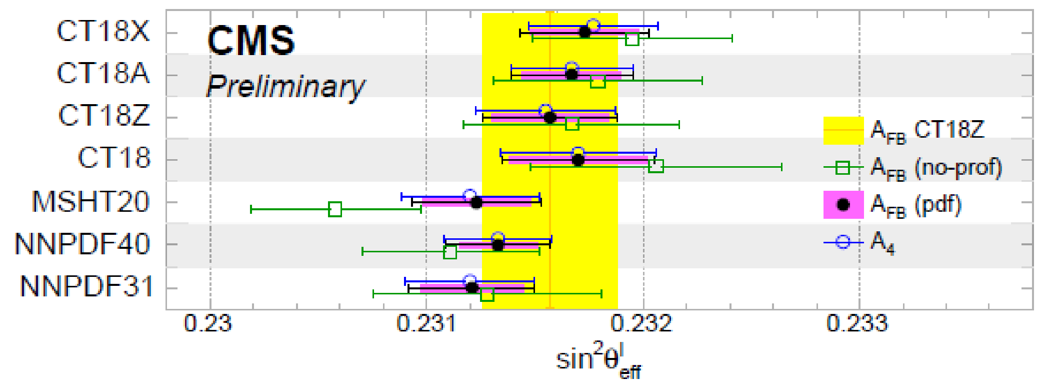
where X could be a Standard Model process like Drell-Yan, W, Z, production etc or could be a new physics process.

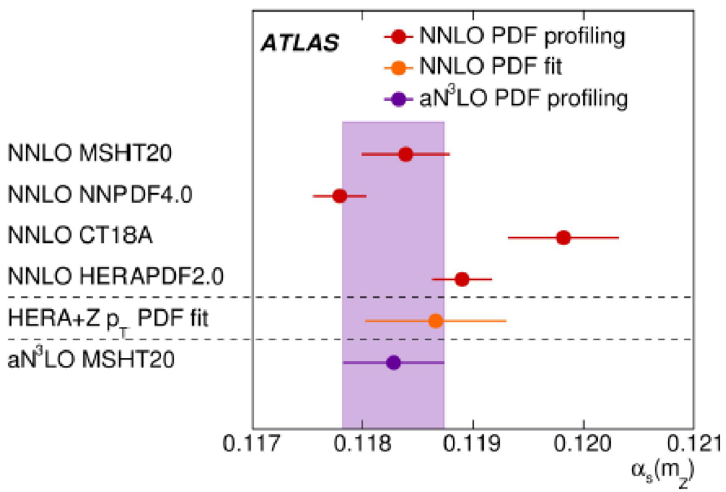
If it is an SM process then the cross sections can be used to improve our knowledge of PDFs. But some SM measurements are also sensitive to SM parameters M_W, $\sin^2\theta_W$ and deviations of these from SM values can point to BSM physics. How to disentangle SM and PDF parameters?

If it is a BSM process then the uncertainties on Parton Distribution Functions (PDFs) will impact how accurately we measure the new cross section

PDF set	p_T^ℓ fit m_W	σ_{tot}	σ_{PDF}	χ^2/n.d.f.	m_T fit m_W	σ_{tot}	σ_{PDF}	χ^2/n.d.f.
CT14	80358.3	$^{+16.1}_{-16.2}$	4.6	543.3/558	80401.3	$^{+24.3}_{-24.5}$	11.6	557.4/558
CT18	80362.0	$^{+16.2}_{-16.2}$	4.9	529.7/558	80394.9	$^{+24.3}_{-24.5}$	11.7	549.2/558
CT18A	80353.2	$^{+15.9}_{-15.8}$	4.8	525.3/558	80384.8	$^{+23.5}_{-23.8}$	10.9	548.4/558
MMHT2014	80361.6	$^{+16.0}_{-16.0}$	4.5	539.8/558	80399.1	$^{+23.2}_{-23.5}$	10.0	561.5/558
MSHT20	80359.0	$^{+13.8}_{-15.4}$	4.3	550.2/558	80391.4	$^{+23.6}_{-24.1}$	10.0	557.3/558
ATLASpdf21	80362.1	$^{+16.9}_{-16.9}$	4.2	526.9/558	80405.5	$^{+28.2}_{-27.7}$	13.2	544.9/558
NNPDF3.1	80347.5	$^{+15.2}_{-15.7}$	4.8	523.1/558	80368.9	$^{+22.7}_{-22.9}$	9.7	556.6/558
NNPDF4.0	80343.7	$^{+15.0}_{-15.0}$	4.2	539.2/558	80363.1	$^{+21.4}_{-22.1}$	7.7	558.8/558

What is our problem?





PDF set	$\alpha_s(m_Z)$	PDF uncertainty
MSHT20 [37]	0.11839	0.00040
NNPDF4.0 [84]	0.11779	0.00024
CT18A [29]	0.11982	0.00050
HERAPDF2.0 [65]	0.11890	0.00027

I won't mention $\alpha_S(M_Z)$ again,
At scale 100 GeV a range 0.116-0.120 is subdominant on the uncertainties of PDFs for DY production

What to do?

Use just the present generation of global fits—well almost

CT18series, MSHT20, NNPDF3.1/4.0 and PDF4LHC21

Let's take a look at what is in them…

But first

Can we afford to ignore PDF fits to restricted data sets?

HERAPDF2.0(Jets) –just HERA DIS and DIS jets (2112.01120)

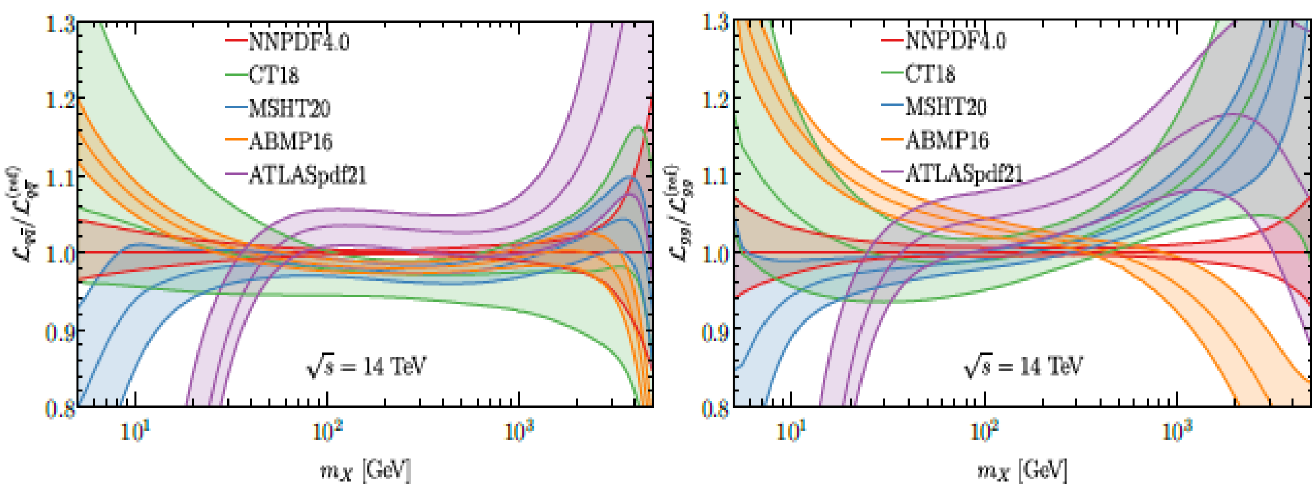
ATLASpdf21—JUST HERA and ATLAS LHC multiple data sets **with interdata set correlations, not completely negligible (2112.11266)**

ABMP16 –JUST DIS, DY and ttbar (updated to ABMPtt with new ttbar 2407.00545)

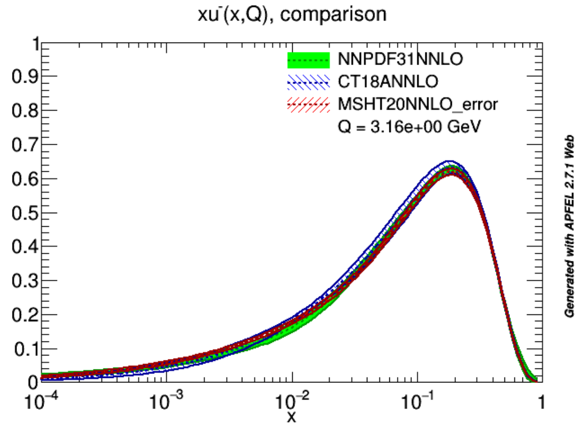
This depends what you want, for example

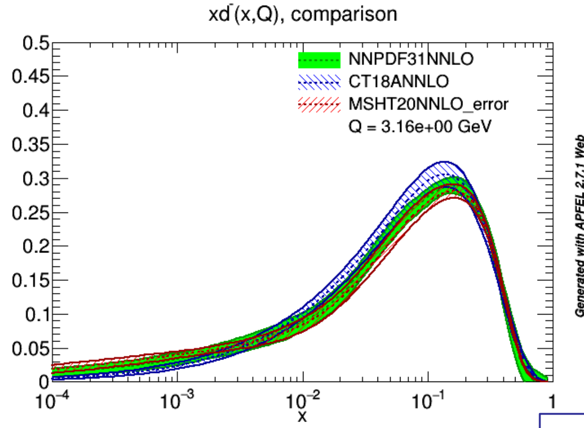
–PDFs with just precision DIS +DY has been suggested, but can one afford to ignore independent information on the gluon?

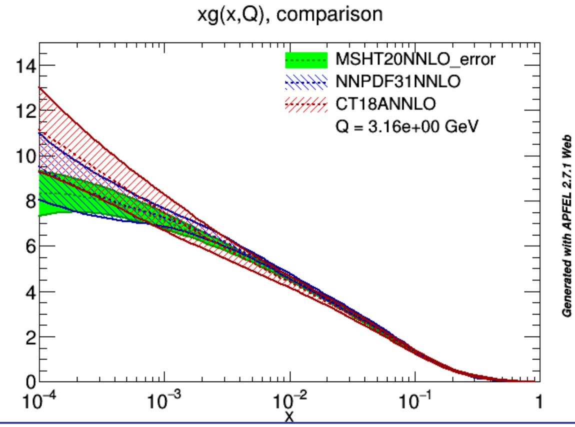
Restricted data set PDFs do present the strongest outliers---both for q-qbar and gluon-gluon induced processes—athough not so much at EW scale



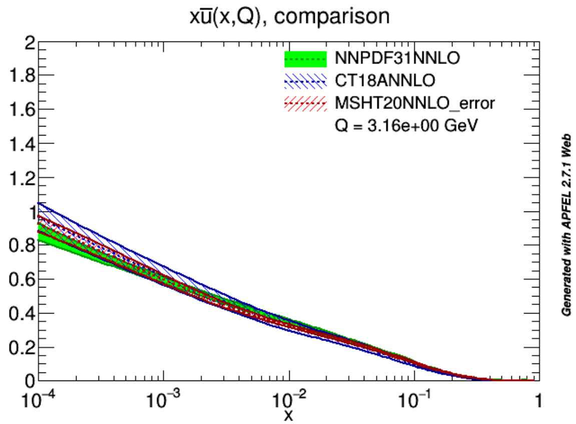
First let’s see how we are doing--- PDF comparisons at NNLO in pQCD

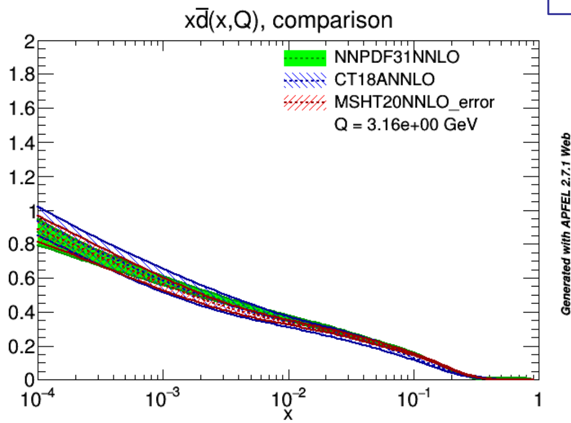


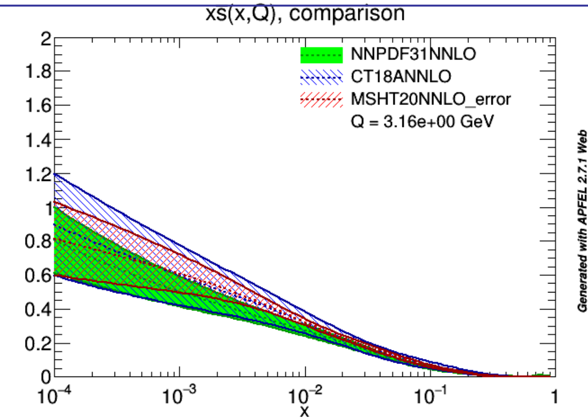




I will come to NNPDF4.0



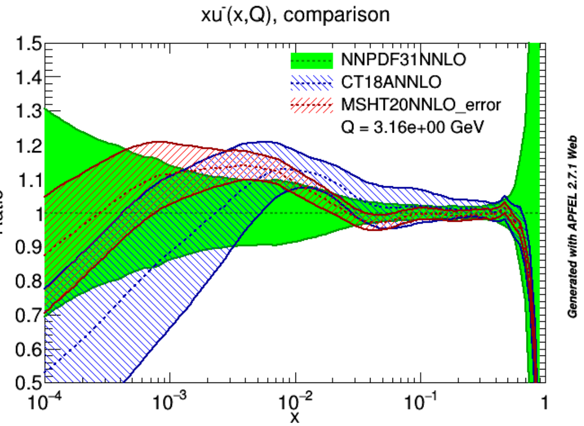


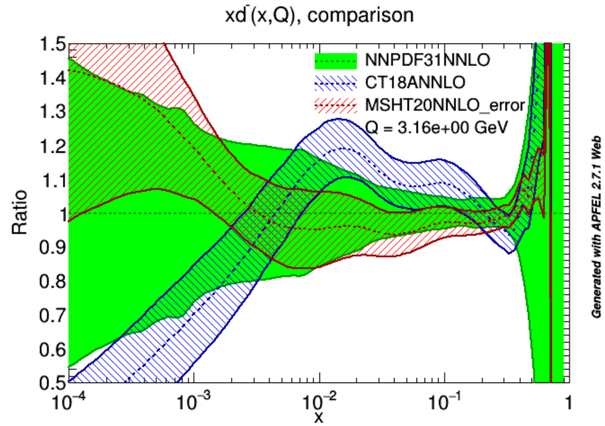


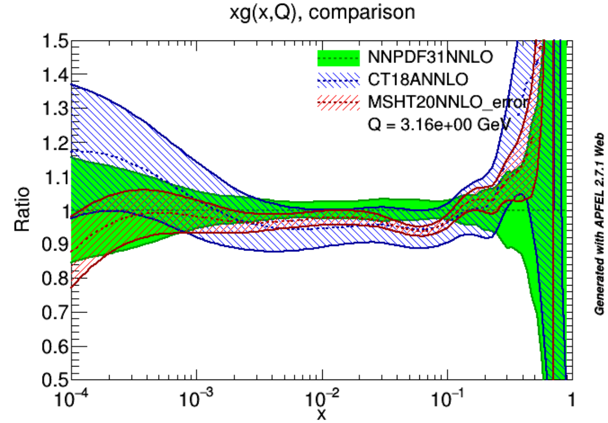
There are differences because of different choices:

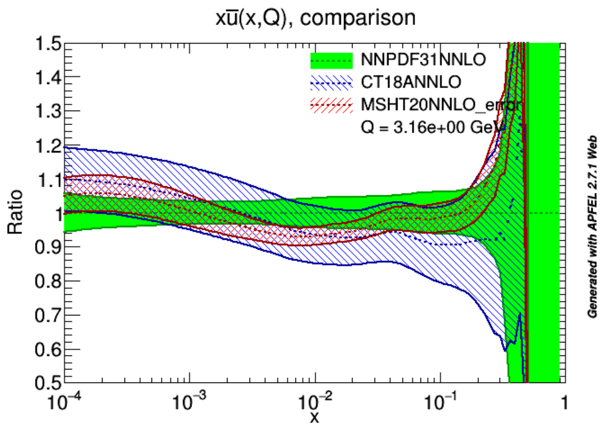
- Exact choice of data sets entering fit and cuts applied to them
- Choice of heavy quark masses, heavy quark schemes
- Choice of starting scale for QCD evolution, choice of parametrisation or neural net
- Perturbative/fitted charm, s-sbar, low-x treatment etce…

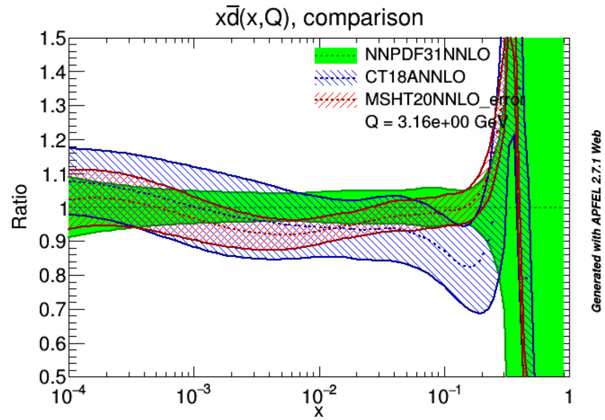
You should not have to worry about this ALL CHOICES are SENSIBLE

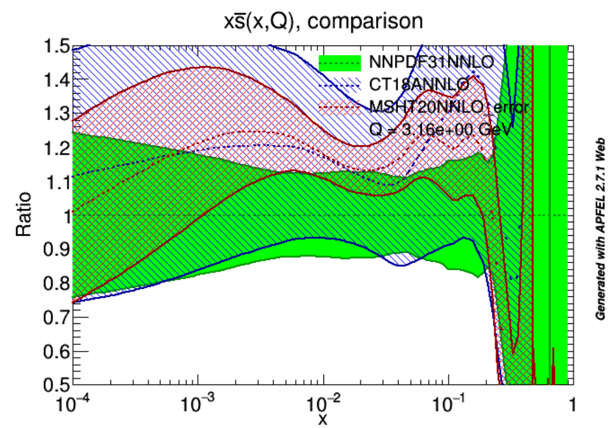










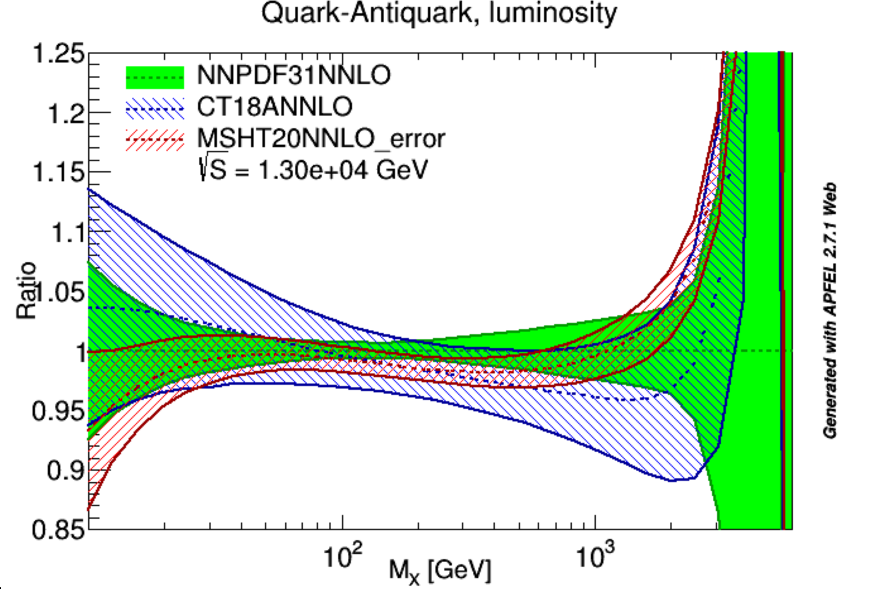


Differences are more obvious in ratio

They are large at small-x and at high-x

BUT they are too big for our liking even where they are best known

One way to see the impact of the uncertainties on the parton distribution functions at the LHC is in terms of parton-parton luminosities, which are the convolution of the purely partonic part of the sub-process cross-section.

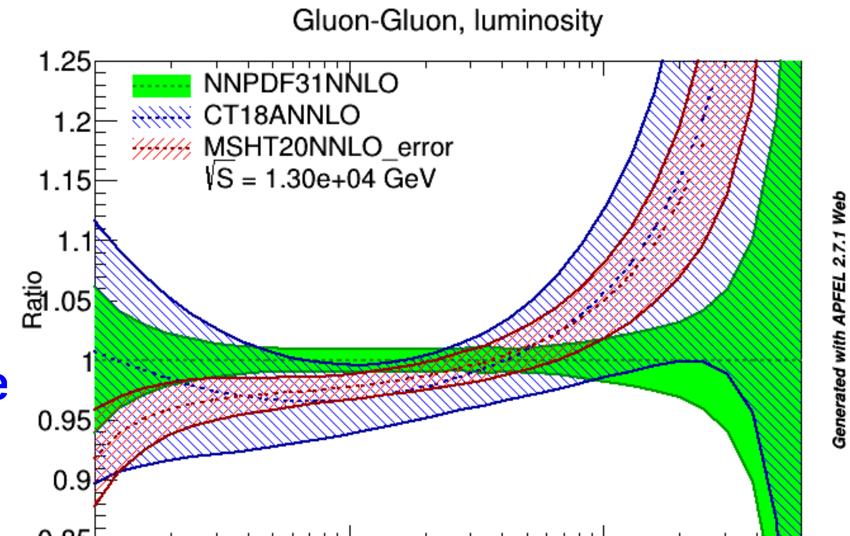


The quark-antiquark and gluon-gluon luminosities for various PDFs are compared here for 13 TeV LHC running in terms of the centre of mass energy of the parton sub- process M_X

Small M_X corresponds to small x and Large M_X to large x

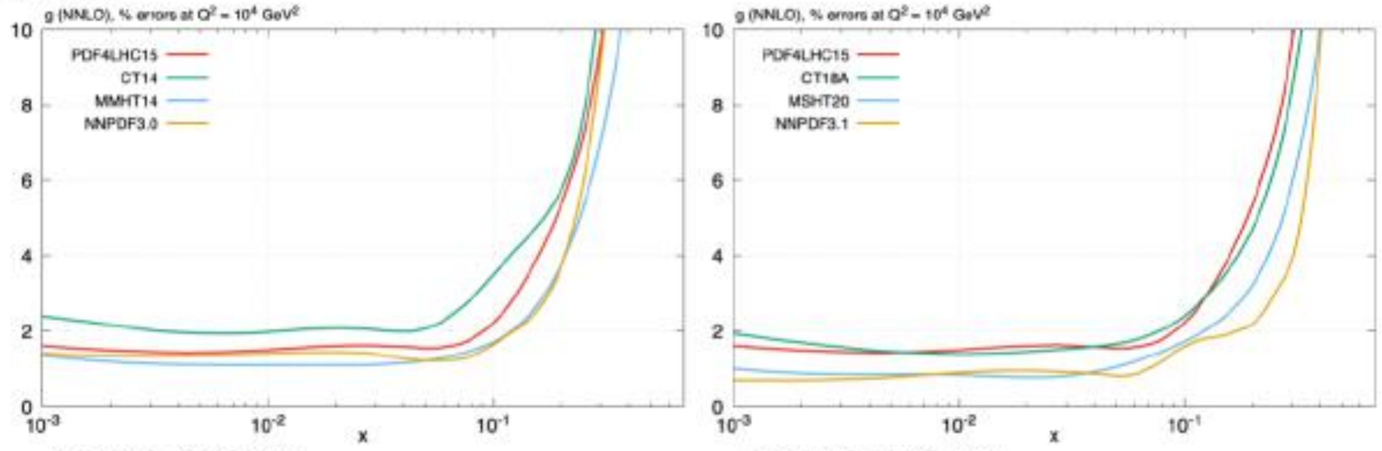
$$\frac{\partial \mathcal{L}_{\Sigma_q(q\bar{q})}}{\partial \hat{s}} = \frac{1}{s}\int_\tau^1 \frac{dx}{x} \sum_{q=d,u,s,c,b} \left[f_q(x,\hat{s}) f_{\bar{q}}(\tau/x,\hat{s}) + f_{\bar{q}}(x,\hat{s}) f_q(\tau/x,\hat{s})\right],$$

So for quark-antiquark production of W or Z bosons ----at Mx ~80,90 GeV
Or for gluon-gluon production of Higgs at ---Mx~125 GeV
the parton-parton luminosities are fairly well known….**but not as well known as we'd like**



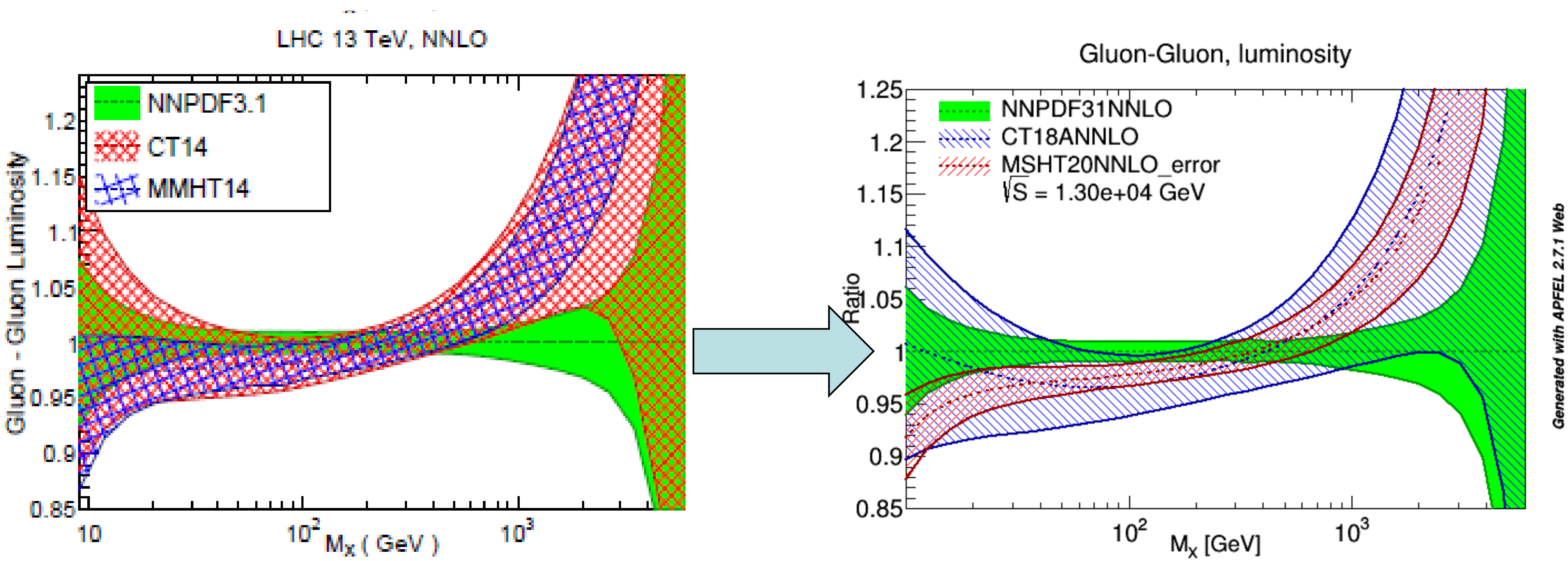
$$\frac{\partial \mathcal{L}_{gg}}{\partial \hat{s}} = \frac{1}{s}\int_\tau^1 \frac{dx}{x} f_g(x,\hat{s}) f_g(\tau/x,\hat{s}),$$

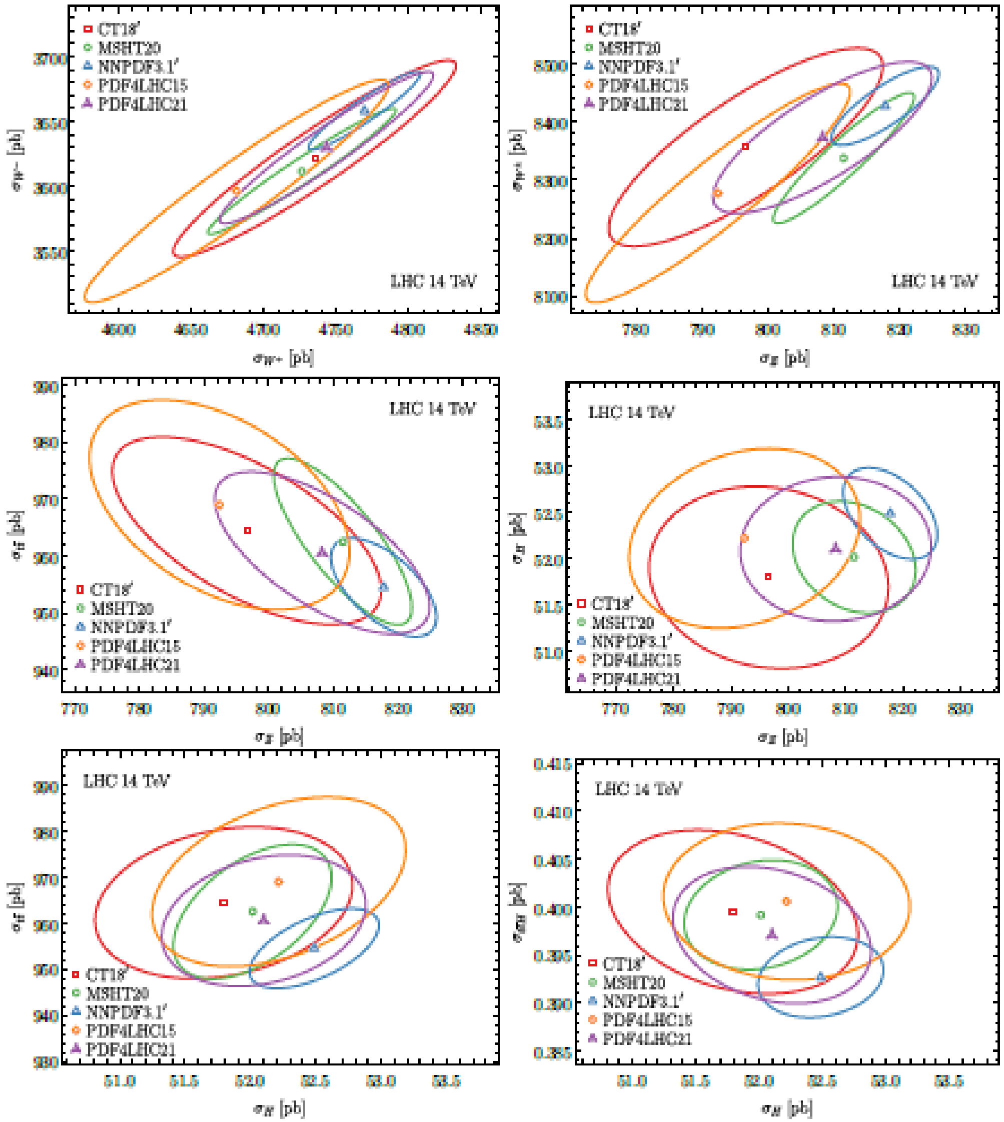
Gluon



IS THERE PROGRESS?

As the uncertainties of each individual PDF decrease with the input of more information, the divergence of the PDFs from each other has increased

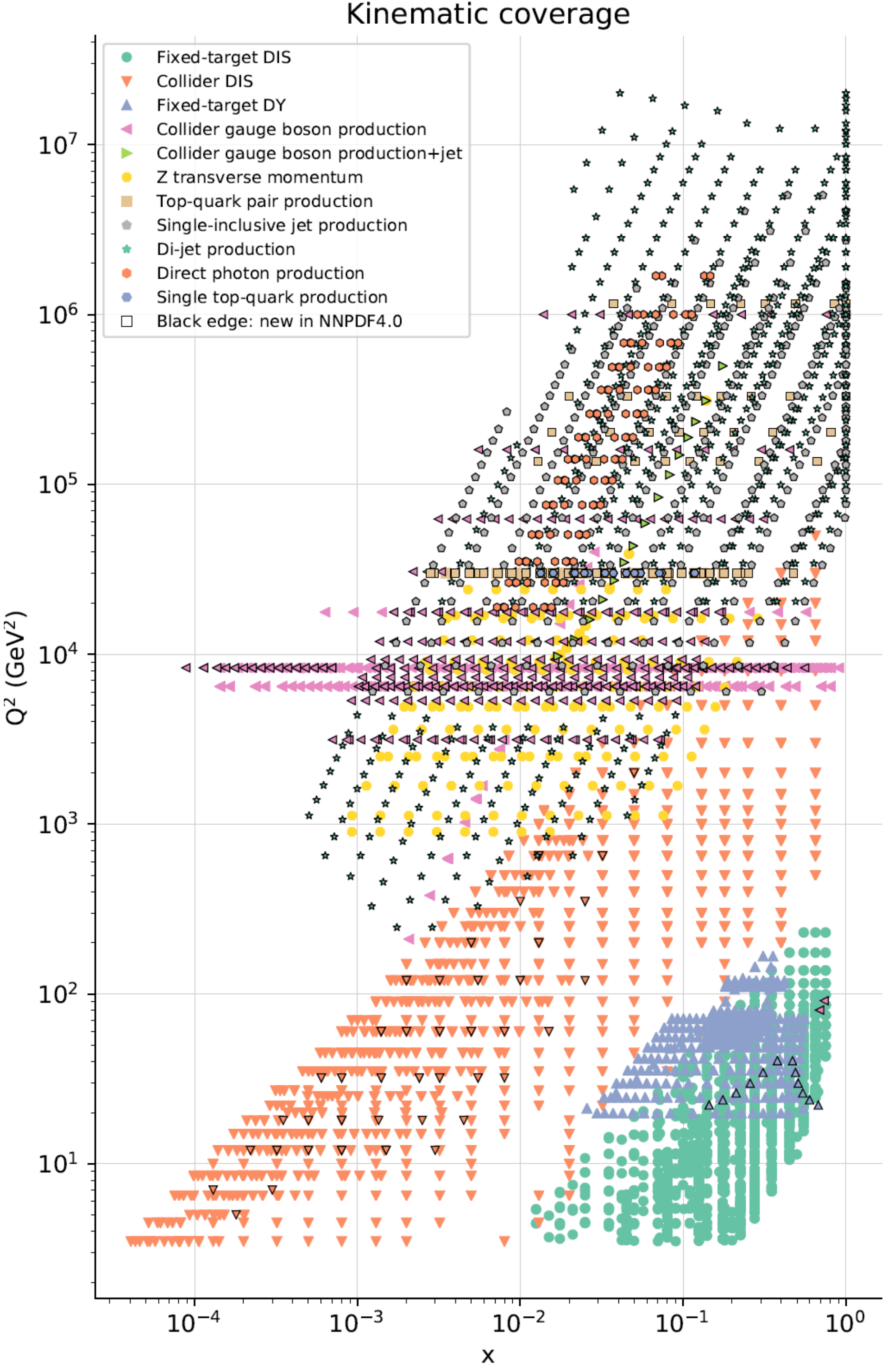




The PDF4LHC group makes combinations of the PDFs from the three main fitting groups NNPDF, CT and MSHT

The PDF4LHC15 combination has now been superseded by the PDF4LHC21 combination (issued in 2022!) arxiv: 2203.05506

There IS an improvement in uncertainty BUT this is not enough to reduce the PDF uncertainty on on LHC measurements of SM parameters such as M_W, sufficiently to compete with the CDF uncertainty- can we do better?



The HERA data are the 'backbone of all PDF fits BUT what could HERA not do?

High-x gluon and sea flavour detail s,c

What other data can we use?

- Drell-Yan data from fixed target DIS and the Tevatron and LHC
- W,Z rapidity spectra from Tevatron and LHC
- Jet pT spectra from Tevatron and LHC
- Top-anti-top differential cross-sections from LHC
- W and Z +jet spectra, or Z pt spectra from LHC
- W and Z +heavy flavours from LHC
- Beware: IS the factorisation theorem proven?-only for DY!
- Beware: there may be new physics at high scale that we 'fit away'
- Further warning, this additional information comes from many different groups– often there is no clarity on the correlations of experimental systematic uncertainties between differing LHC measurements

Now let's consider what goes into the global fits and the PDF4LHC combination in a little more detail

In particular for EW studies, what precision LHC DY data sets are in there

Data set	MSHT20 Dec 2020	CT18 Dec 2019	NNPDF3.1 2017	NNPDF4.0 Sep 2021
ATLAS 7 TeV $W+Z$ 2012	Yes Scale $M_{W,Z}/2$	CT18A,Z Scale $M_{W,Z}$	Yes(Z peak) Scale $M_{W,Z}$	Yes Scale $M_{W,Z}$
ATLAS 8 TeV W^+W^-	Yes	No	No	No
ATLAS 8 TeV Z 3D	2D	No	No	2D
ATLAS 8 TeV high-mass DY	Yes	No	Yes	Yes
ATLAS 8 TeV W^+W^-+ jets	Yes	No	No	Yes
ATLAS 8 TeV Z p_T	Min cuts $M_{ll}.Pt_{ll}$	Max cuts $M_{ll}.Pt_{ll}$	Med cuts $M_{ll}.Pt_{ll}$	Med cuts $M_{ll}.Pt_{ll}$
CMS 8 TeV W [9	Yes	Yes	Yes	Yes
CMS 7 TeV $W+c$	Yes	No	Yes	Yes and 13TeV
LHCb 7+8 TeV $W+Z$ muon	Yes	Yes	Yes	Yes
LHCb 8 TeV $Z \to ee$	Yes	Yes	Yes	Yes and 13 TeV e,μ

Z3D (M_{ll},y_{ll},$\cos\theta_{CS}$) is integrated over CS angle to become 2D to avoid sensitivity to $\sin^2\theta_W$ AND bins for which the cross section is largely or entirely non-zero only at NLO are excluded

- Some data are excluded because EW corrections are too large
- Some data are excluded because of goodness of fit criteria
- Some were simply not in time

A few other points of note

- **NNLO theory is applied mostly via kfactors NNLO/NLO corrections using a fixed PDF** (apart from ttbar data)

K-factors are usually smoothed and an uncorrelated uncertainty is applied for their MC uncertainty ~0.5% for CT ~1.0% for NNPDF. (Such a procedure lowers $\chi 2$ without changing the PDFs)

- K-factors for DY use various programs like FEWZ and DYNNLO, we need to remember that these do not agree with each other perfectly. (Disagreement at~1% level at Z peak)Recent improvements (2405.19714) in each of these came AFTER the PDFs were published.
- **Also the global PDFs do not all use the same scale for DY calculations**

CTEQ say 'The PDF uncertainties related to the choice of QCD scales and the codes for theoretical calculations have not been systematically included in this analysis'

- For NNPDF you can get sets **with Missing Higher Order Uncertainties** evaluated from the usual scale variation by factors of 2, but then fitted, essentially treated as extra systematic uncertainties

- **EW corrections are also applied to all input data sets.**

NOTE that you don't NEED a specific QED PDF unless you need a photon PDF . Photon induced events are often already subtracted by the experimentalists, if not this is corrected.

Data are corrected for EW effects when they are small,
and data are rejected when they are large (eg high mass Drell-Yan or Zpt at high pt)

Data are not always treated 'as published'

In particular with regard to correlated systematic uncertainties

The treatment of correlated systematics is a non trivial issue

- Firstly, is the correlation either 100% or 0% -some decorrelation as a function of kinematics maybe reasonable (e.g for jet data sets and t-tbar data sets. Sometimes with experimenters' input, sometimes more ad hoc)
- Secondly can a two-point systematic really be considered like a Gaussian error?

There are approaches using errors on the errors: Glen Cowan, work by MSHT, see DIS2024 (Reader)

There are approaches by CT authors using a Gaussian Mixture Model 2406.01664

There is the Bayesian Inference with Gaussian processes approach – 2404.07573

Bayesian inference etc 2401.15187

But these are not yet mature/ complete/ sufficient..

For example the errors on errors approach can take care cases for which $\chi2 / N > (N+ \sqrt{(2N)}) / N$ for particular data sets, and this amounts to an increase of Tolerance $\Delta\chi2=T^2$, of T ~1.2-1.5

which is not enough ---as we shall see

Questions of tolerance

$$\chi^2_{\rm exp}(\boldsymbol{m},\boldsymbol{b}) = \sum_i \frac{\left[m^i - \sum_\alpha \gamma^i_\alpha \mu^i b_\alpha - \mu^i\right]^2}{\left(\delta_{i,\rm stat}\,\mu^i\right)^2 + \left(\delta_{i,\rm uncor}\,\mu^i\right)^2} + \sum_\alpha b^2_\alpha.$$

The usual χ2 fits theoretical parameters ***m*** and experimental nuisance parameters ***b***

Applying a χ2 tolerance of $\Delta\chi 2=T^2 > 1$ is not stupid, nor is it cheating, nor is it simple

MSHT Dynamic tolerance approx. $T^2=10$

Arises from tensions between data sets…

CT, $T^2=100$ for 90%, $T^2=100/1.645$ ~61 for 68% CL

Tolerance also comes from tensions between data sets but also from considering many different parametrisations, hence CT tolerance is larger than MSHT tolerance

It is easy to see how different parametrisations represent different hypotheses.

(We will come to NNPDF)

What is the point?

The point is that $\Delta\chi 2=T^2 = 1$, is the 68%CL parameter setting criteria

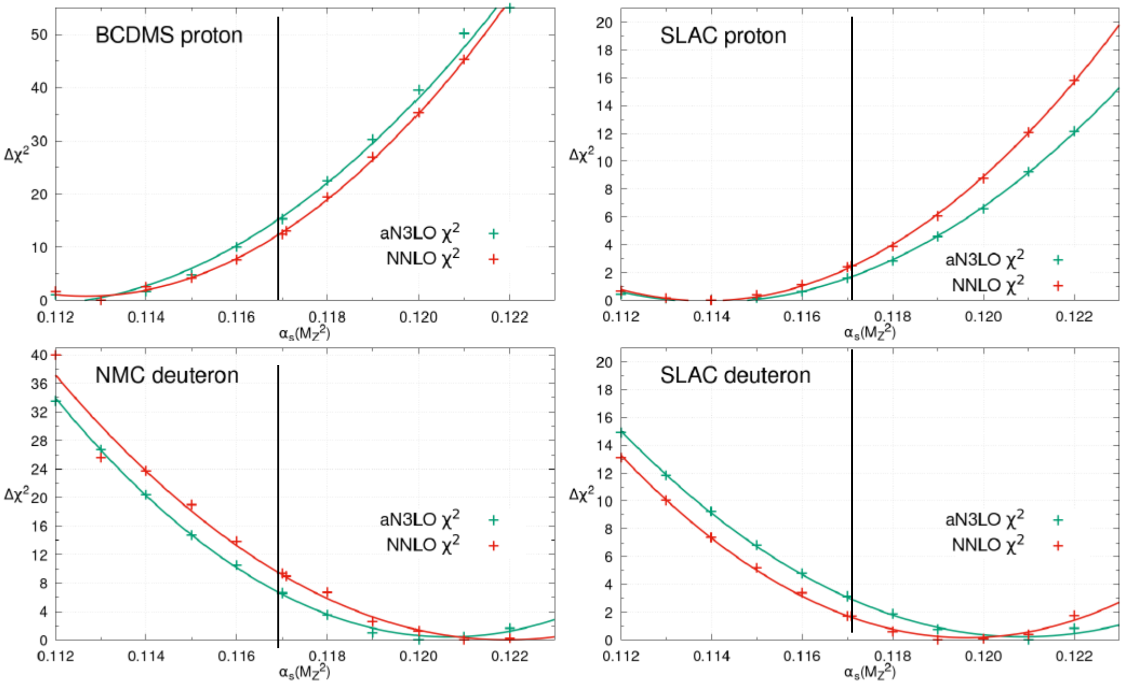
But the criterion for an acceptable hypothesis is $\Delta\chi 2=T^2 = \sqrt{(2N)}$,

where N is no of degrees of freedom. For a whole global fit this can now be ~100.

Nobody is advocating using this blanket figure for the whole data sample---

–but you might want each data set to be fitted within its 68% CL which is roughly a change of $\sqrt{2N}$, from its best fit χ2 value, where N is the number of data points for that data set.

The problem is that the best fit for all the data sets put together is not necessarily the best fit for each data set individually.

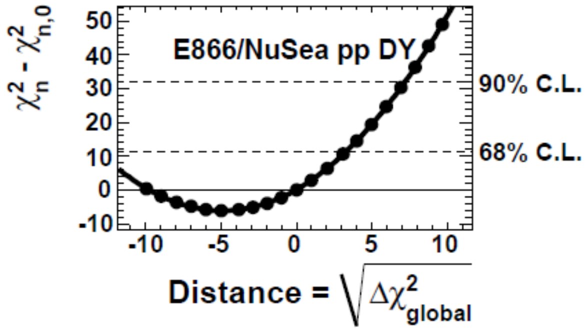


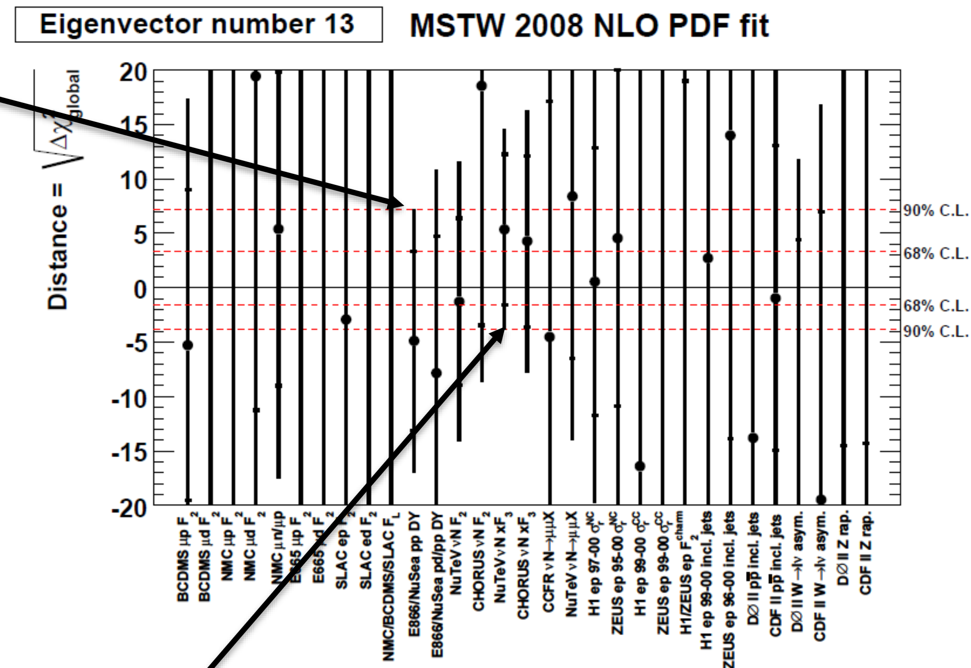
Let me first illustrate the variation of χ2 with a parameter, namely $\alpha_S(M_Z)$, because it is easiest to think in terms parameter, although we must consider instead each orthogonal eigenvector combination of parameters.

Looking at this variation for different data sets we can see that many data sets do not lie within $\Delta\chi2=T^2 = 1$ of the value of all data sets taken together, which is $\alpha_S(M_Z)=0.117$. Some lie above, some below, they ALL lie within $\Delta\chi2=T^2 = 10$.

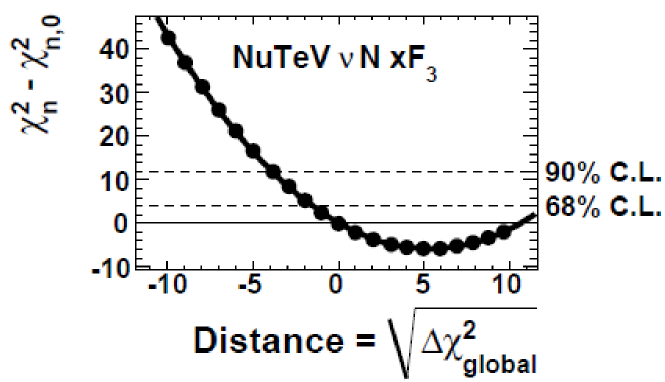
To get the MSHT value of T for this parameter and each data set we compare such curves to the 68 (90)% CL for that data set.
We must do this for each eigenvector and every data set……….

Difference of χ2 for a data set from is value at the global minimum $\chi2_0$ as a function of the change in global χ2





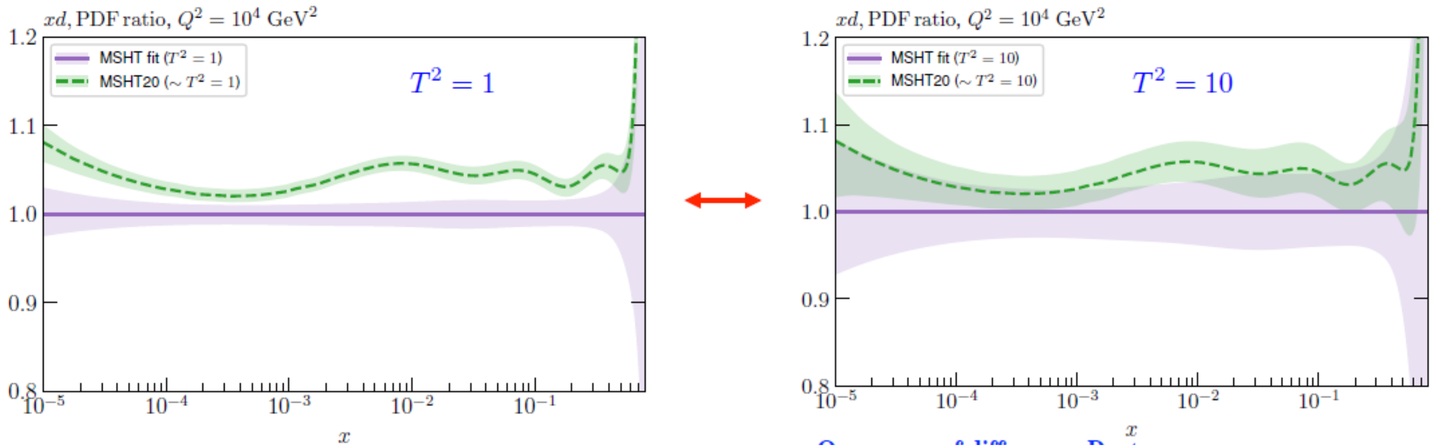
e.g. these two data sets give the upper 68(90)% limit $\Delta\chi2=T^2$ = 3.2 (7)
And lower 68(90)% limit $\Delta\chi2=T^2$ = 1.5(4)
On eigenvector 13 of the MSTW08 fit
T varies with eigenvectors, in general it is larger



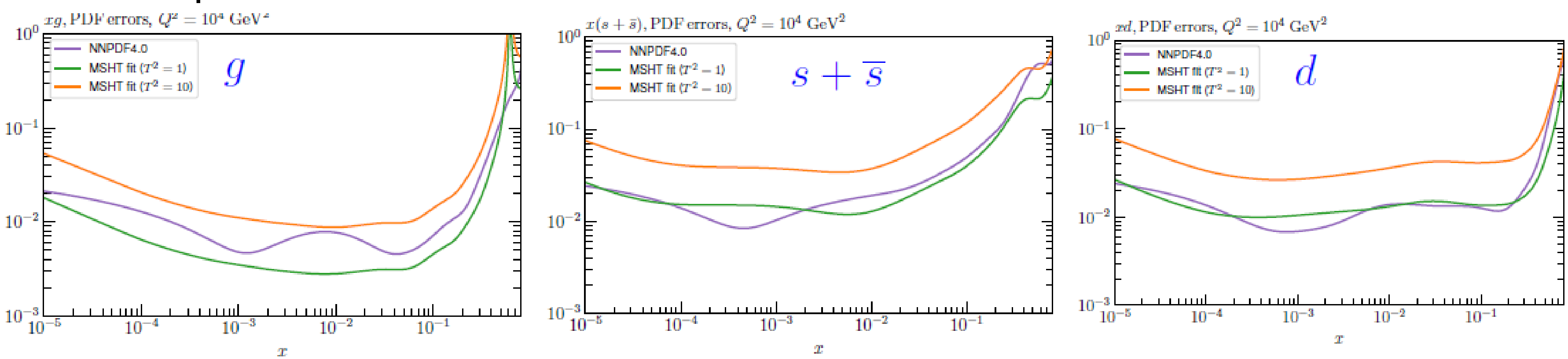
So dynamic tolerance means that each eigenvector has a somewhat different tolerance. For MSHT20 T^2 =10 is good overall approximation

Some amusing tolerance investigations by Lucian Harlan-Lang of MSHT

Use public NNPDF tools to fit NNPDF4.0 data set using NNPDF4.0 theory predictions but with MSHT20 parametrisation, obtaining a very good fit ---even though MSHT parametrisation is not at flexible as the NN (detail: χ2 is actually better than NN)
Compare this MSHT fit to the MSHT20 PDF



Now compare NNPDF4.0 PDF uncertainties to those of MSHT20 for a few PDFs

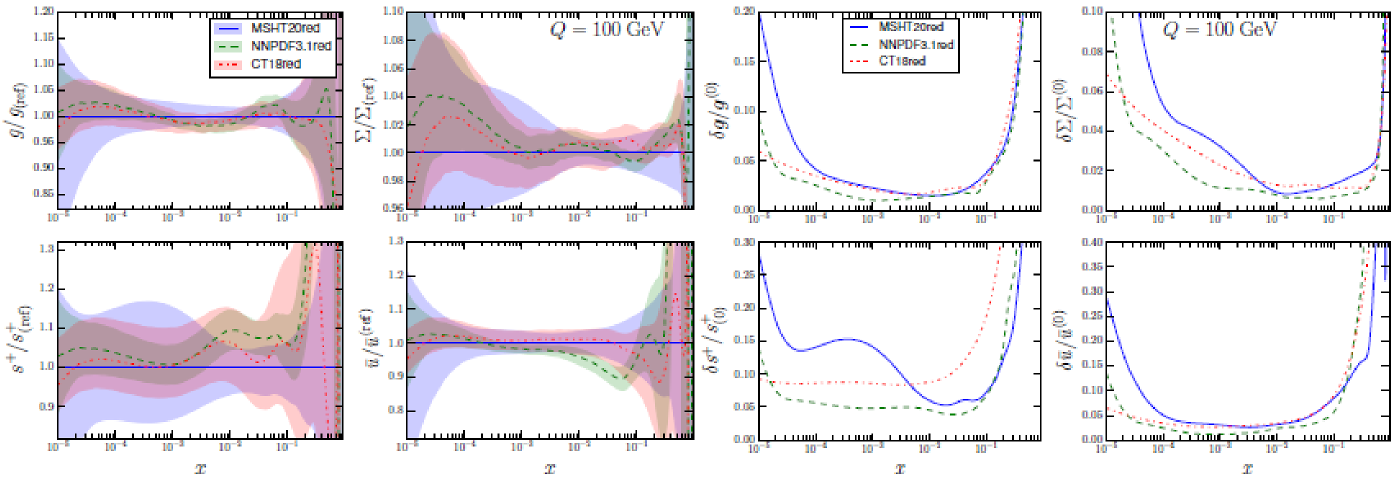


NNPDF4.0 is close to T^2=1, but not always!
And NOTE this means NNPDF3.1 has larger effective T^2. As we shall see

The PDF4LHC group makes combinations of the PDFs from the three main fitting groups NNPDF, CT and MSHT. PDF4LHC21 has NNPDF3.1, CT18 and MSHT20
First try to understand differences by using a common data set and common theory settings for heavy quark masses and alphas

PDF Benchmarking: Reduced Fits

- Use fits to reduced common datasets and common theory settings.



- Very good agreement within uncertainties, including gluon.
- Similar size uncertainties in data regions, differences outside this, reflecting remaining methodological and other choices.
- Agreement much improved relative to global PDFs.
- Same data and theory settings → consistent PDFs. Smaller remaining differences, e.g. in errors, reflect methodological choices.

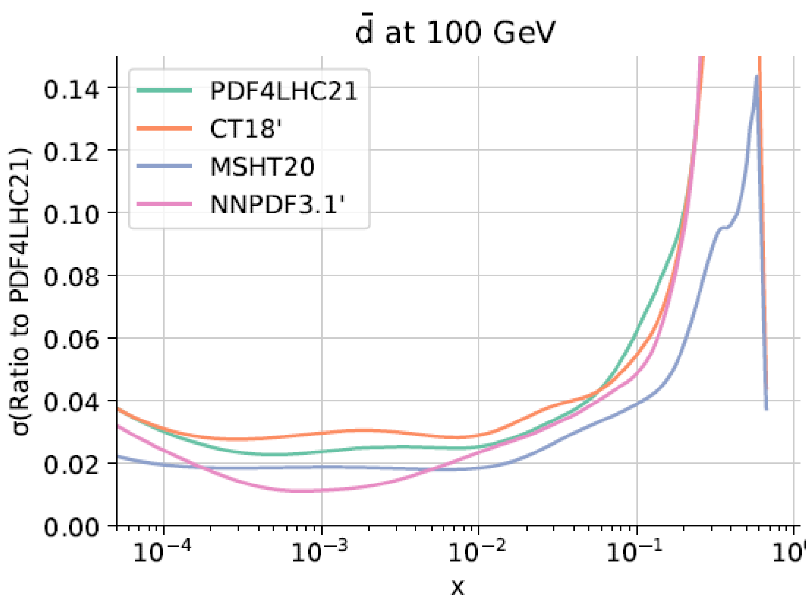
BUT is not recommended to use these reduced fits, greater consistency does not mean greater accuracy—the differences in the main fits are there for a reason!

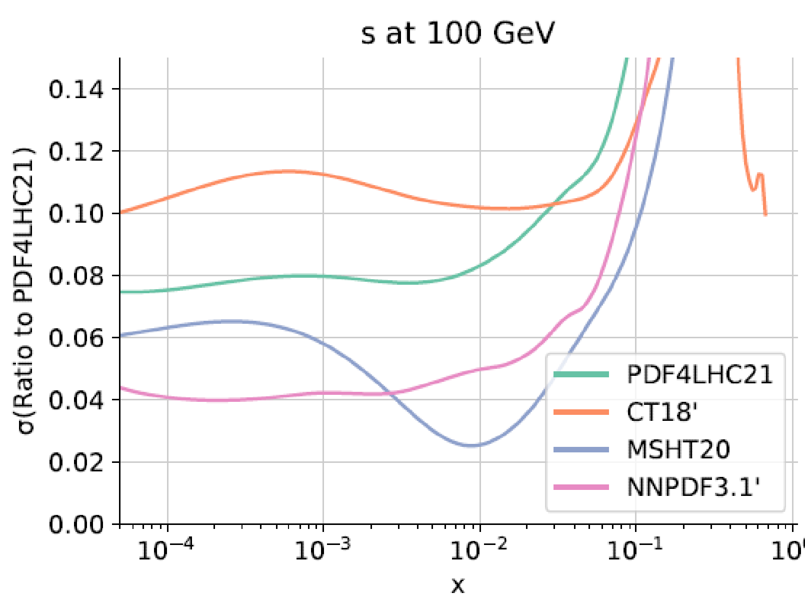
PDF4LHC21 (which was published in 2022)

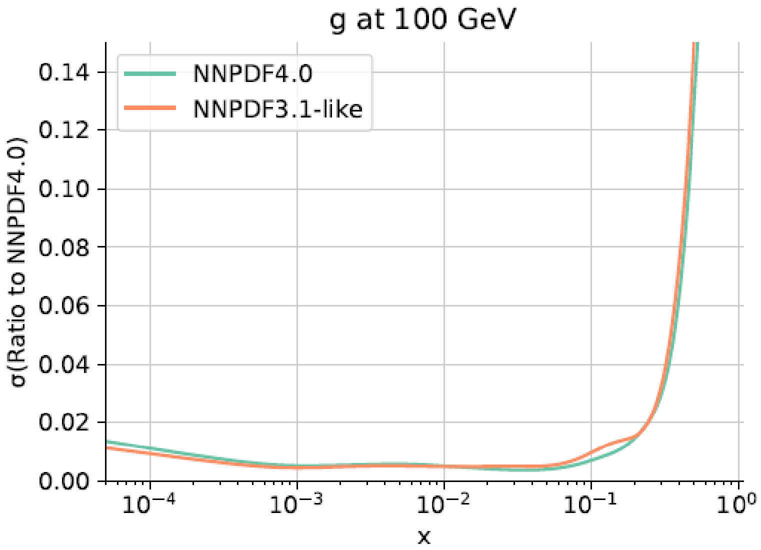
PDF4LHC21 actually combines variants of CT18 and NNPDF3.1 with MSHT20. Variants set heavy quark masses to a common value and have a slight difference in input data sets for NNPDF3.1.

The combination is a statistical combination without correlations between the three input PDFs. Where the three input PDFs are consistent the resulting PDF4LHC uncertainty represents an average of the the PDF set uncertainties—generally closest to, though smaller than, the largest uncertainty of the three, namely CT18. But where there are discrepancies the PDF4LHC uncertainty can be larger than those of any of the individual sets since they include the spread in the central prediction

x



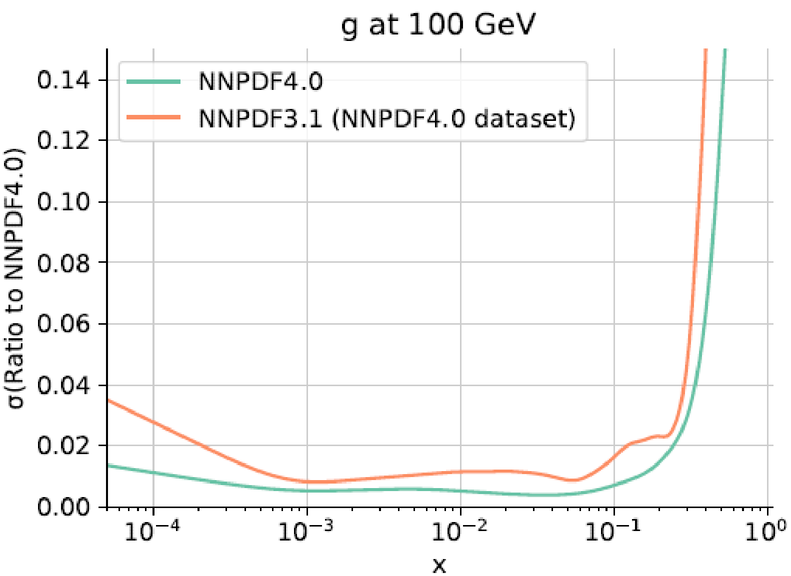




Since the issue of PDF4LHC21 there has been a new PDF set from NNPDF4.0
This has a lot of new data from the LHC

Nevertheless the improvements in uncertainty are not much due to these data, they are more **due to improvements in their procedure**

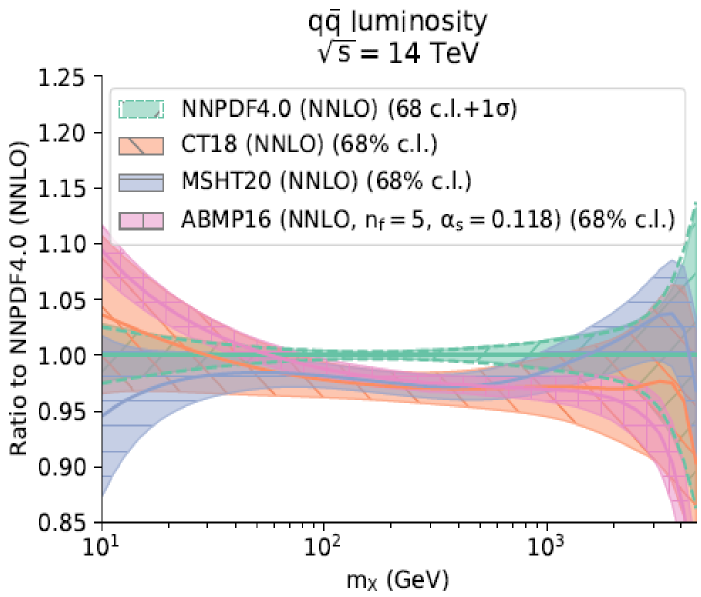
The top plot compares the uncertainties of NNPDF4.0 and 3.1 data sets using the **SAME new methodology**



The bottom plot shows the impact of the methodology on the **SAME new data** set
4.0 shows new methodology and 3.1 here shows old methodology on new data-set

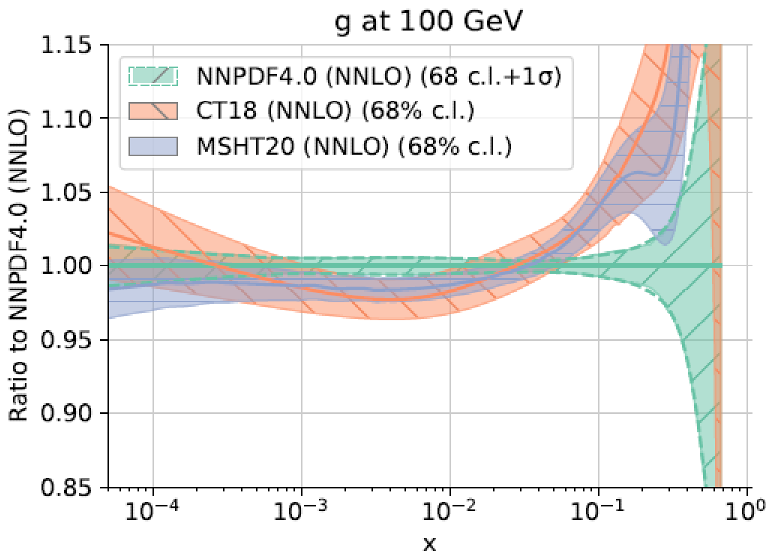
There has been a lot of debate in the PDF community over the new methodology.
But if we just accept it this still does not help much if one is trying to combine with other PDFs MSHT20 and CT18 with different central values

And there are discrepancies, even in the central region
Take a look in ratio to NNPDF4.0



We are not so surprised by differences at high-x, though they can be outside individual PDF set uncertainties

But discrepancies persist at mid –x = EW scale



Discrepancies in low-x gluon brings me to an ‘elephant in the room’
DO we need N3LO?

Do we need N3LO?

- Well ultimately but it is also probably too early for this.
- We only have **approximate N3LO** and we only have it from MSHT and NNPDF so far and they are different
- If they do the same thing they are not different as benchmarking shows (2406.16188)
- But they do NOT do the same thing
  - Need to know:
    - Splitting functions - at 4-loop to evolve PDFs in (x, Q^2):

      $$P(x, \alpha_s) = \alpha_S P^{(0)}(x) + \alpha_S^2 P^{(1)}(x) + \alpha_S^3 P^{(2)}(x) + \alpha_S^4 P^{(3)}(x) + \ldots$$

      Mellin moments, small x, high x limits [11-31].
    - Transition Matrix Elements - at 3-loop to change number of PDF flavours at heavy quark mass (m_h) thresholds.

      $$f_\alpha^{n_f+1}(x, Q^2) = [A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)](x)$$

      Mellin moments, small x, high x limits [32-42].
    - Coefficient Functions for DIS - at 3-loop to determine structure functions.

      $$F_2(x, Q^2) = \sum_{\alpha\in H,q,g;\beta\in q,H} (C_{\beta,\alpha}^{VF,n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2))$$

      Light flavour known, heavy flavour high Q^2 known, approx for low Q^2 [43-45].
    - Hadronic cross-section k-factors - at N3LO.

      $$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots \equiv \sigma_{N3LO} + \ldots$$

      Very little known, PDFs need differential with cuts.
  - Much already known, only a few remaining missing pieces.

MSHT vs NNPDF

Similarities	Differences
Include available N3LO info at time of publication	Own approximations used for each piece
Include theoretical uncertainties for missing pieces	Different methodology for theory uncertainty.

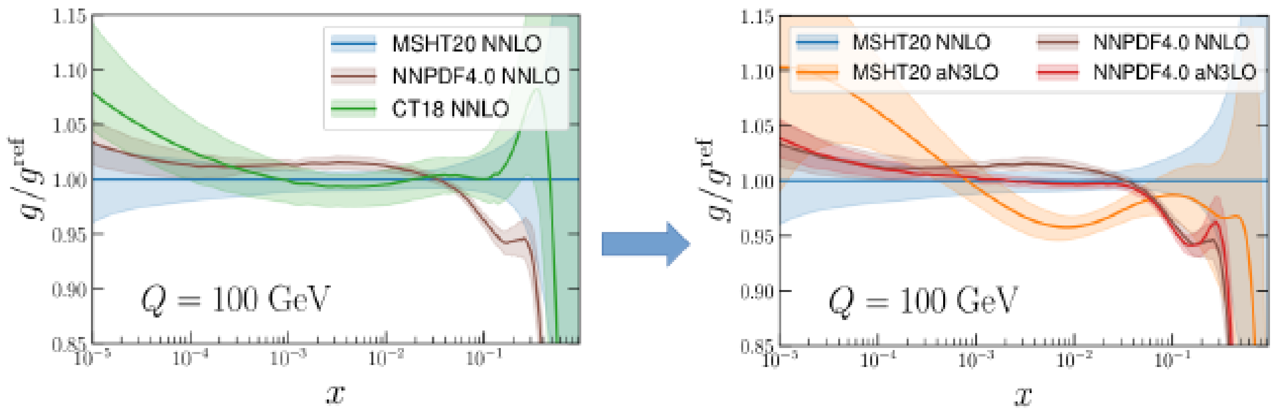
Both go beyond MHOU's to Incomplete Higher Order Uncertainties IHOU's but they estimate them differently

Note uncertainties can be increasing not decreasing here

Both groups represent the unknown pieces in term of sets of sensibly chosen basis functions. NNPDF vary the set of basis functions chosen in order to estimate the uncertainties.

The approach of MSHT is different in that the data has some say in determining the uncertainties. For example, at small-x, a parameter is chosen to be the coefficient of the most divergent of the set of basis functions. There is a prior based on all available knowledge and this is modified in a fit to data to produce a posterior value of the parameter for each splitting function. Hence the posterior can absorb not only N3LO corrections but other missing contributions of experimental or theoretical origin. Perhaps most obviously low-x, $\ln(1/x)^n$ terms

- Largest effect on the gluon PDF.

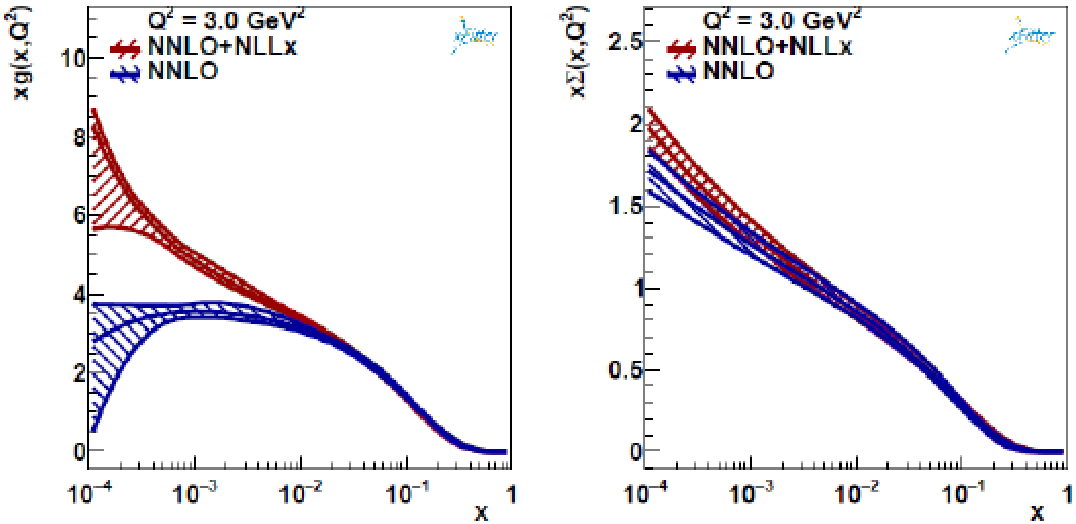


- NNLO - gluon PDFs differ by up to 2-3 % in Higgs region.
- aN3LO - gluon PDFs differ by up to 4-5 % in Higgs region.
- NNLO vs aN3LO - MSHT and NNPDF both see dip (2 and 5% respectively) in gluon at $x \sim 10^{-2}$ from aN3LO effects.

- Both groups see improved χ2
- Both see perturbative convergence
- **BUT why might this strong effect on the MSHT gluon actually be right?**

LOW-x PHYSICS

There has long been an issue that at low-x one should probably be **resuming ln(1/x) terms as well as ln(Q^2) terms –this is BFKL resummation** and is beyond DGLAP
This has been done by NNPDF- **NNPDF3.1sx** 1710.05935
And on the HERAPDF using xFitter 1802.00064 (using HELL, Bonvini 1805.08785)



What does it do?
It turns blue into red– dramatic change on the low-x gluon

Reminds me of the aN3LO effect

But there is another thing one needs to consider at low-x– **high density effects** when the gluon gets large such that gluons may recombine, as well as split, and this may lead to gluon saturation. CT have modelled this with an x dependent scale for DIS in CT18X and CT18Z
Scale is not Q^2 BUT

$$\mu^2_{DIS,X} = 0.8^2 \left(Q^2 + \frac{0.3\ GeV^2}{x^{0.3}} \right)$$

This also enhances the low-x gluon—-though not as extremely
The problem is that a change in the low-x gluon has knock on effects at higher x which can feed into the DY processes

So back to our problem of what PDF to use with our precision EW parameter determinations.

You might want to improve the PDF with your new EW data, assuming it is not in the fit– (if it is one must consider correlations)

Profiling varies nuisance parameters for the new data and for the eigenvectors of the PDF fit prediction
This assigns a different weight to each eigenvector variation such that a new PDF set can be constructed

$$\chi^2(\boldsymbol{b}_{\text{exp}}, \boldsymbol{b}_{\text{th}}) = \sum_{i=1}^{N_{\text{data}}} \frac{\left(\sigma_i^{\text{exp}} + \sum_\alpha \Gamma_{i\alpha}^{\text{exp}} b_{\alpha,\text{exp}} - \sigma_i^{\text{th}} - \sum_\beta \Gamma_{i\beta}^{\text{th}} b_{\beta,\text{th}}\right)^2}{\Delta_i^2} + \sum_\alpha b_{\alpha,\text{exp}}^2 + \sum_\beta b_{\beta,\text{th}}^2 .$$

$$f_0' = f_0 + \sum_\beta b_{\beta,\text{th}}^{\text{min}} \left(\frac{f_\beta^+ - f_\beta^-}{2}\right]$$

where f_0 is the orginal central set and f_β^+ and f_β^- are the up and down eigenvectors

Profiling like this assumes T=1 for the new data set and thus assigns it a very large weight with respect to data in the fit, since the CT tolerance is usually $T^{2\sim}61$ and MSHT is T^2~10. Also T should really be a function of β for dynamic tolerance.
Can use ePump for profiling with Tolerance.

For NNPDF replicas **reweighting** is more appropriate than profiling (although you can use the equivalent Hessian set)

$$\langle \mathcal{O}(\text{PDF})\rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{O}(\text{PDF}_k) \longrightarrow \langle \mathcal{O}^{\text{new}}(\text{PDF})\rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} w_k \mathcal{O}(\text{PDF}_k)$$

Where the weights are given by

$$w_k = \frac{(\chi_k^2)^{\frac{1}{2}(N_{\text{data}}-1)} \exp^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} (\chi_k^2)^{\frac{1}{2}(N_{\text{data}}-1)} \exp^{-\frac{1}{2}\chi_k^2}},$$

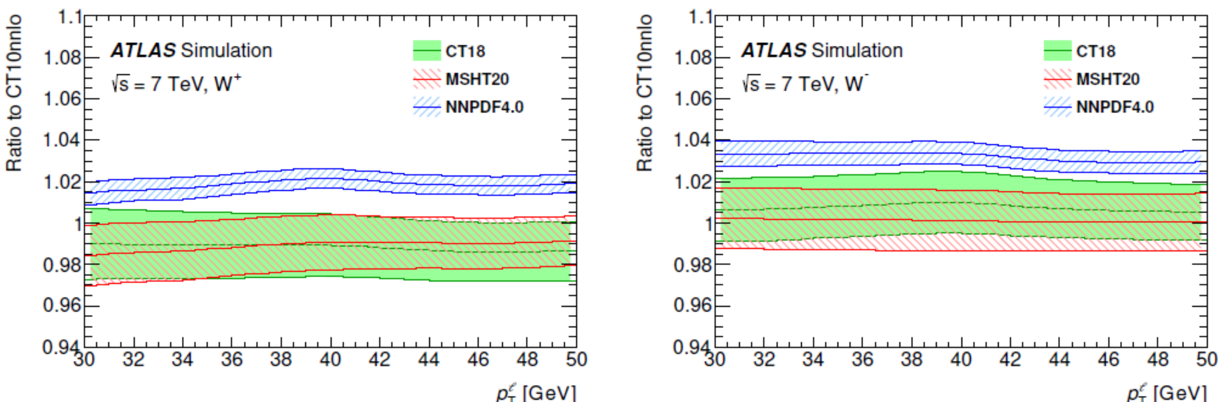
First the ATLAS W –mass determination

PDF set	p_T^ℓ fit m_W	σ_{tot}	σ_{PDF}	χ^2/n.d.f.	m_T fit m_W	σ_{tot}	σ_{PDF}	χ^2/n.d.f.
CT14	80358.3	$^{+16.1}_{-16.2}$	4.6	543.3/558	80401.3	$^{+24.3}_{-24.5}$	11.6	557.4/558
CT18	80362.0	$^{+16.2}_{-16.2}$	4.9	529.7/558	80394.9	$^{+24.3}_{-24.5}$	11.7	549.2/558
CT18A	80353.2	$^{+15.9}_{-15.8}$	4.8	525.3/558	80384.8	$^{+23.5}_{-23.8}$	10.9	548.4/558
MMHT2014	80361.6	$^{+16.0}_{-16.0}$	4.5	539.8/558	80399.1	$^{+23.2}_{-23.5}$	10.0	561.5/558
MSHT20	80359.0	$^{+13.8}_{-15.4}$	4.3	550.2/558	80391.4	$^{+23.6}_{-24.1}$	10.0	557.3/558
ATLASpdf21	80362.1	$^{+16.9}_{-16.9}$	4.2	526.9/558	80405.5	$^{+28.2}_{-27.7}$	13.2	544.9/558
NNPDF3.1	80347.5	$^{+15.2}_{-15.7}$	4.8	523.1/558	80368.9	$^{+22.7}_{-22.9}$	9.7	556.6/558
NNPDF4.0	80343.7	$^{+15.0}_{-15.0}$	4.2	539.2/558	80363.1	$^{+21.4}_{-22.1}$	7.7	558.8/558

NNPDF are low

A profiled likelihood analysis is used determining M_W together with nuisance parameters for experimental or modelling uncertainties on the data for m_T or p_T^l. PDF uncertainties are part of the modelling uncertainty.

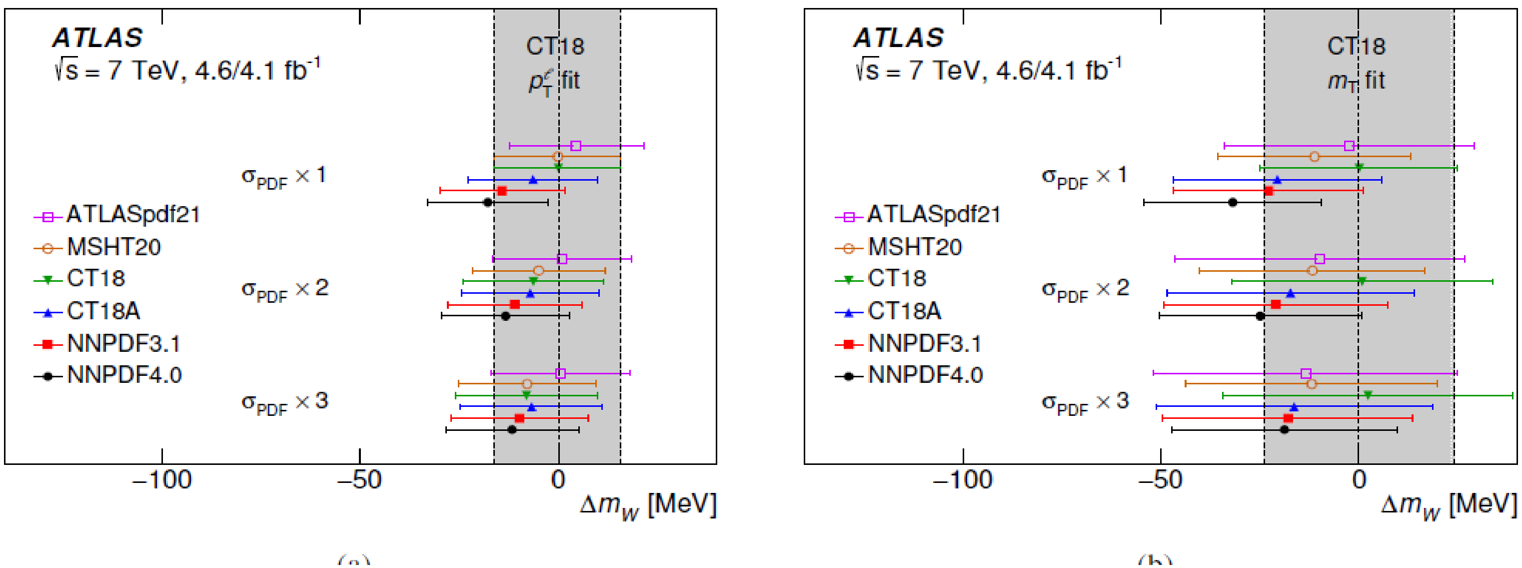
The previous M_W determination was done using CT10, and compared to MMHT14 and CT14. The present one uses many more PDFs



Here's a comparison of $W^\pm$ p_T^l spectra for new and old PDFs

A study is made of the influence of PDF uncertainties by increasing them by factors 1-3 resulting in reduced model dependence

This is shown below for both p_T^l and M_W fits compared to **the baseline CT18 which is chosen as a) conservative b) not including the 7 TeV W data used in the fit**



Finally the results for the two spectra, using CT18, are combined accounting for correlations

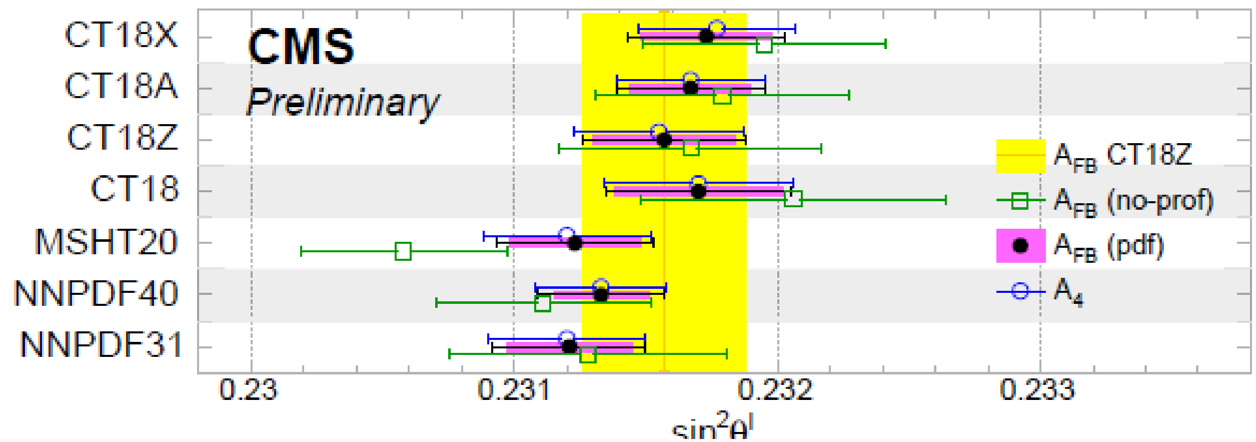
$$m_W = 80366.5 \pm 9.8\ (\text{stat.}) \pm 12.5\ (\text{syst.})\ \text{MeV} = 80366.5 \pm 15.9\ \text{MeV},$$

And the PDF uncertainty contributes ~6 MeV to the systematic uncertainty of the combination--- is this conservative enough?

The CMS $\sin^2\theta_W$ determination

What is measured is A_{FB} and A_4 fir the Z at 13TeV, which is thus not in any PDF fit so far (NOTE 8 TeV could also be used because the relevant FB information is also not in the global fits so far)

Various PDFs are used AND they are 'improved' by profiling



$$\sin^2\theta^{\ell}_{\text{eff}} = 0.23157 \pm 0.00010(\text{stat}) \pm 0.00015(\text{syst}) \pm 0.00009(\text{theo}) \pm 0.00027(\text{pdf})$$

CT18Z profiled is used for the quoted result, because it is in the middle of the others and has a spread which covers all central values (after profiling)

In my opinion you may as well use PDF4LHC21 as just pick CT18Z or CT18.
'BUT the PDF4LHC is not supposed to be used for precision measurements' ?
This meant that you should not rely on PDF4LHC alone.
But if you have looked at every reasonable PDF and you need an average with a reasonable uncertainty, then I think it's more objective
Also I would profile it with a T^2=10 tolerance…. **BUT CAN IT BE IMPROVED**

For several years now there has been a request to make a PDF4LHC combination which accounts for correlations….

Requests & proposals – Correlations between PDF sets

Proposal to evaluate correlations between PDF sets, originating from common experimental inputs, using coherently-generated pseudo-experiments

Use the xFitter framework to generate pseudo-experiments fluctuating the statistical and systematic experimental uncertainties, taking into account correlations, for an inclusive sample of data (covering all the data used for the various PDF fits)

For each generated pseudo-experiment, select the data points used by each PDF fitting group and re-do the corresponding fit
(Only the nominal fit has to be determined at this stage, not the eigenvectors)

(After validation and cross-checks – see backup:)
Use the ensemble of fitted pseudo-experiments to determine correlations between the uncertainties of various PDF sets

"PDF benchmarking proposal for precision Drell-Yan" (PDF4LHC meeting, CERN, 2018)
"PDF benchmarking discussion" (LHC EW Precision sub-group workshop, IPPP Durham UK, 2019)
"PDF benchmarking report" (LHC Electroweak WG meeting, CERN, 2019)

It is not a perfect world, currently I am unsure this will ever happen!

Summary/ Things to think about.

Nothing that anybody is doing is daft. It is not a perfect world

- PDF improvement is not just a matter of more data
- Consistency of data matters
- Knowledge of common systematic uncertainties matters
- Real data are always more problematic than pseudo-data projections

- Differences in the PDFs are not just about choice of data set—PDF4LHC reduced data sets still give some differences in PDFs--
- There are irreducible methodological differences between the PDFs
- Sometime this is just a matter of model choices that can be made consistent------- heavy quark masses, $\alpha_s(M_Z)$.
- But sometimes the choices are made for ‘ideological reasons’—parameterisations, NNs, heavy quark treatment/intrinsic charm, strange≠antistrange
- Greatest differences in definitions of how to set uncertainties – choice of $\chi2$ tolerance /NN method.
- **PDF4LHC combines MSHT,CT, NNPDF at NNLO, it is the best we can do for now**
- **BUT it could be improved (?)**

- MHOU, N3LO, IHOU, Ln(1/x) resummation, recombination/saturation

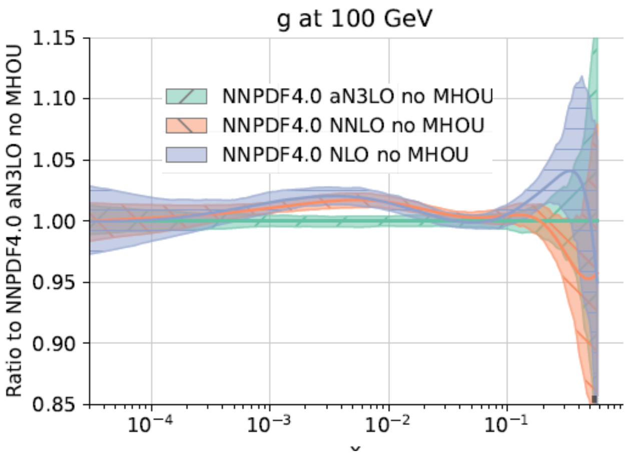
One day we are going to have to consider all these,
– there are consequences at the EW scale

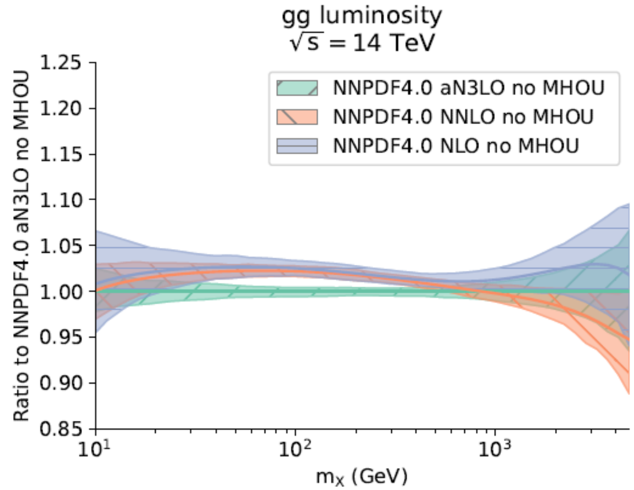
- Improved methods of PDF determination and uncertainty determination are coming along

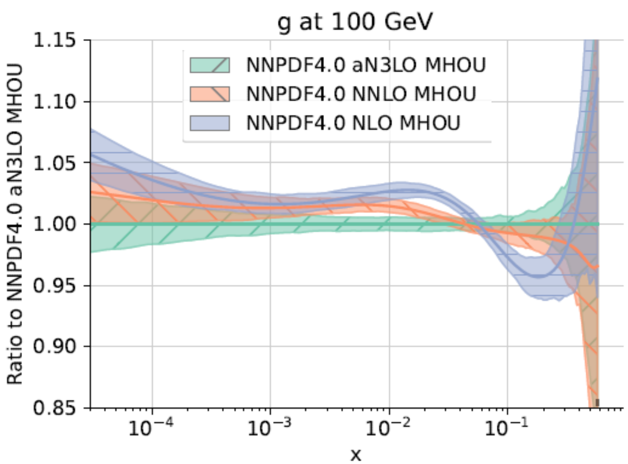
Back-up

EW and PI corrections

Data	Observables	Size of EW (and PI) corrections	Ref.	Data included in the CT18(Z)?	EW corrections included in the fits
Inclusive jet	$p_T \sim 1.4$ TeV, central	8%	[131]	Yes	Yes
$t\bar{t}$	$p_T^t \sim 500$ GeV	-5%	[135]	Yes	No
$W^+(W^-)$		-0.4(0.3)%	[38]	CT18Z	Yes
DY low-mass	$46 < M_{\ell\bar{\ell}} < 66$ GeV central	+1.5%(PI) +6%(EW)			
DY Z-peak	$66 < M_{\ell\bar{\ell}} < 116$ GeV central (forward)	<0.1%(PI)-0.3(-0.4)%(EW)			
DY high-mass	$116 < M_{\ell\bar{\ell}} < 150$ GeV central (forward)	+1.5%(PI)-0.5(-1.2)% (EW)			
high-mass Drell-Yan	$M_{\ell\bar{\ell}} \sim 1$ TeV	+5%(PI)-3%(EW)	FEWZ	No	–
Z p_T	$p_T \sim m_Z$	about -5%	[134]	Yes	No
	$p_T \sim 1$ TeV	about -30%	[134]	No	–


g at 100 GeV
NNPDF4.0 aN3LO no MHOU
NNPDF4.0 NNLO no MHOU
NNPDF4.0 NLO no MHOU
Ratio to NNPDF4.0 aN3LO no MHOU


gg luminosity
$\sqrt{s} = 14$ TeV
NNPDF4.0 aN3LO no MHOU
NNPDF4.0 NNLO no MHOU
NNPDF4.0 NLO no MHOU
Ratio to NNPDF4.0 aN3LO no MHOU
m_X (GeV)

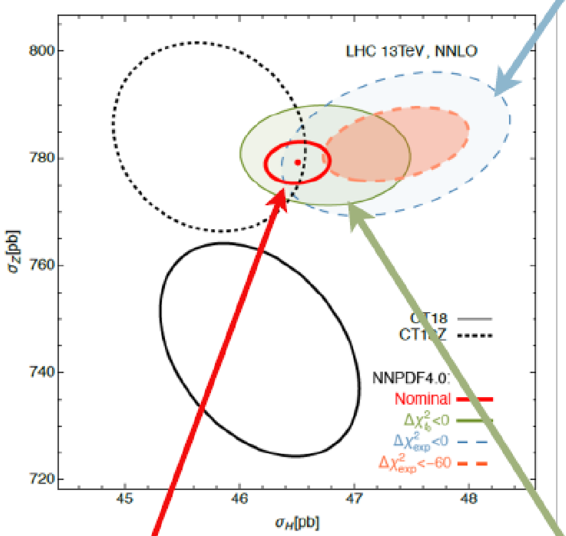

g at 100 GeV
NNPDF4.0 aN3LO MHOU
NNPDF4.0 NNLO MHOU
NNPDF4.0 NLO MHOU
Ratio to NNPDF4.0 aN3LO MHOU
x

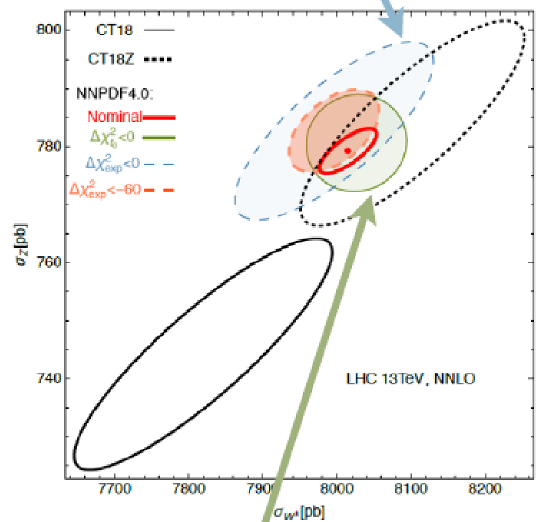
Monte-Carlo sampling sensitivity for PDFs

Debate on NNPDF4.0 uncertainties

Regions containing (very) good solutions according to the experimental form of χ^2

(is used in χ^2 summary tables of the NN4.0 article, was a default in the NN4.0 public code)





CT criticism
ArxiV:2205.10444

Nominal NN4.0 Hessian or MC 68%cl

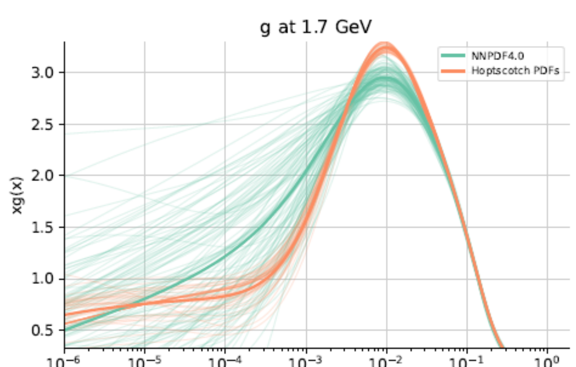
Region containing good solutions according to the most recent t_0 form of χ^2
(used to train NN4.0 replicas)

NNPDF prior disfavors some of the most displaced hopscotch replicas.
[M. Ubiali, HP2 2022 workshop, Durham, 2022/09]

The hopscotch replicas pass CT18 criteria for good individual solutions.

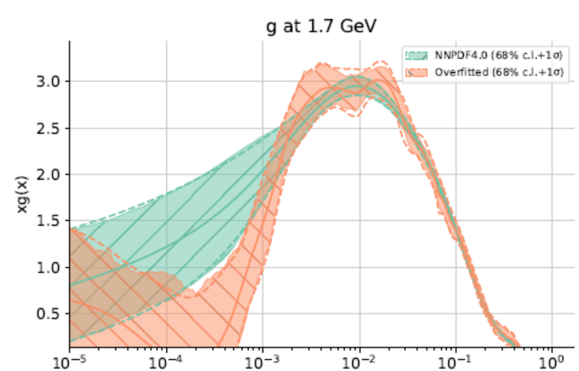
Observation 1

A comparison between NNPDF replicas and HS PDFs reveals a peculiar kink in HS PDFs



Observation 2

An overfitted gluon can be obtained with $\chi^{2(0,c)}$ smaller than NNPDF4.0 by $0.08 \times N_{\rm dat}$

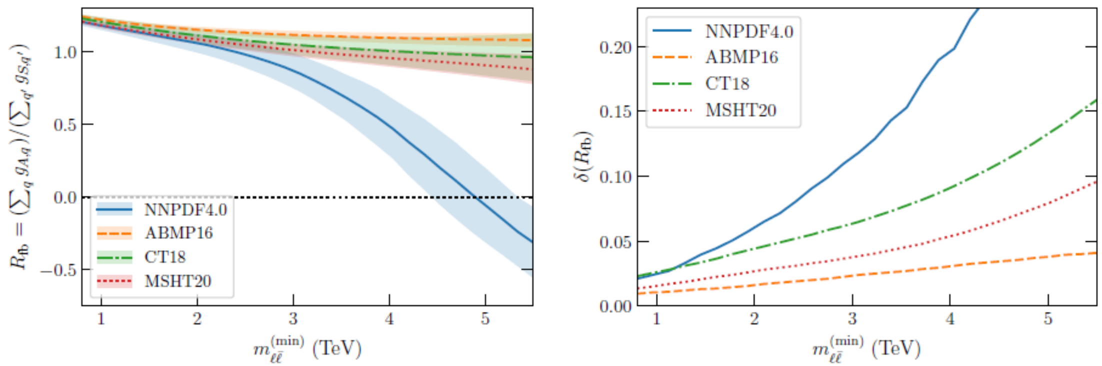


NNPDF reply at PDF4LHC2022… but it goes on

Comparisons at very high-x / High scale
AFB is very different for NNPDF4.0
NNPDF4.0 uncertainties remain large/largest beyond the current data region– but not large enough to cover this

Positive or negative asymmetry?

$$A_{\mathrm{fb}}(\cos\theta^*) \stackrel{\mathrm{LO}}{=} \frac{\cos\theta^*}{1+\cos^2(\theta^*)}\frac{g_A}{g_{S,q'}} \qquad g_{A,S} \propto \int \frac{dm_{\ell\bar{\ell}}}{m_{\ell\bar{\ell}}}\frac{dx_1}{x_1}\mathcal{L}_{\mathcal{A},\mathcal{S}}(m_{\ell\bar{\ell}}, x_1)$$

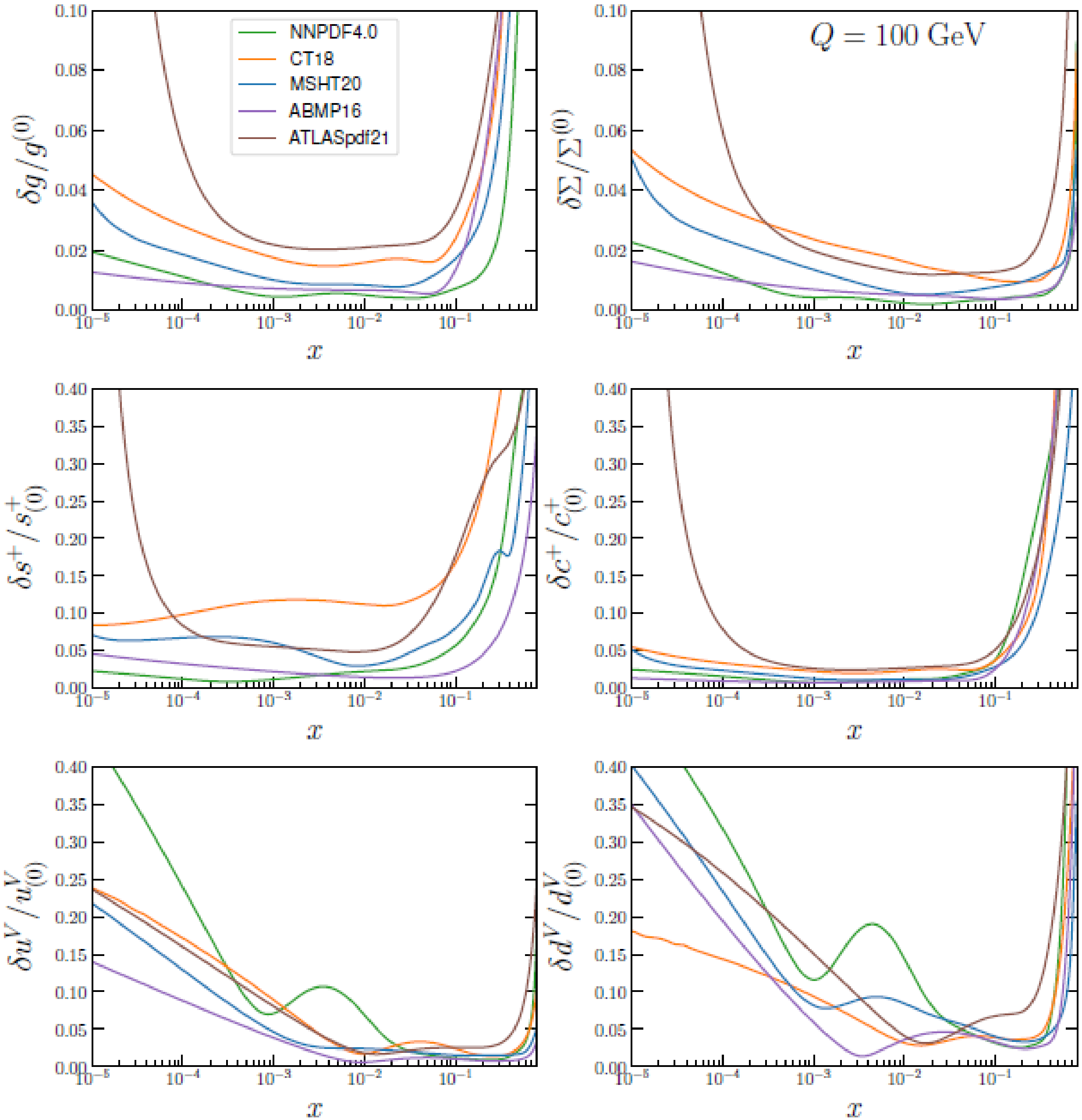


As the scale increases, larger x is probed

ABMP16, CT18 and MSHT20:
approximately scale independent

NNPDF: the coupling depends on the scale, larger uncertainty

Expectation: for NNPDF4.0, A_{fb} vanishes at large $m_{\ell\bar{\ell}}$



But first look at uncertainties

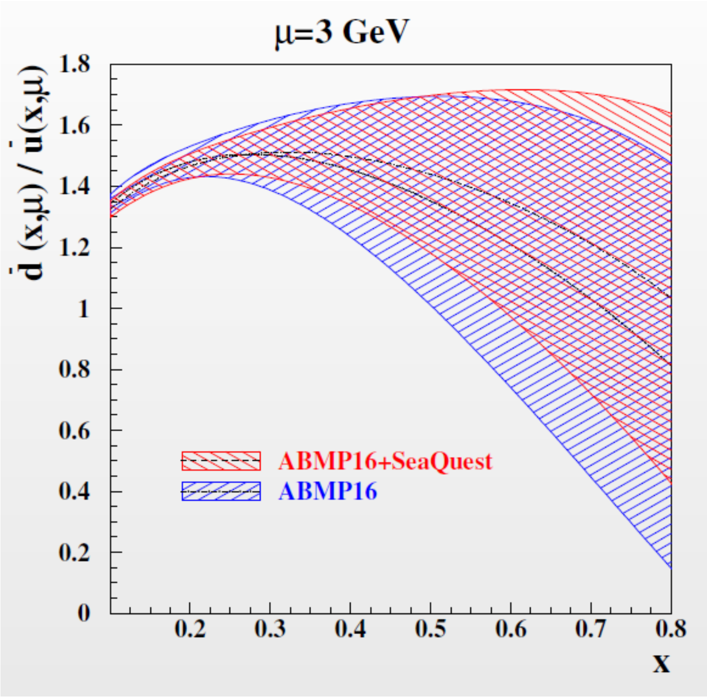
NOTE ABMP16 is relatively small in regions where similar amounts of data are used, because Δχ2=1 is used rather than a higher tolerance

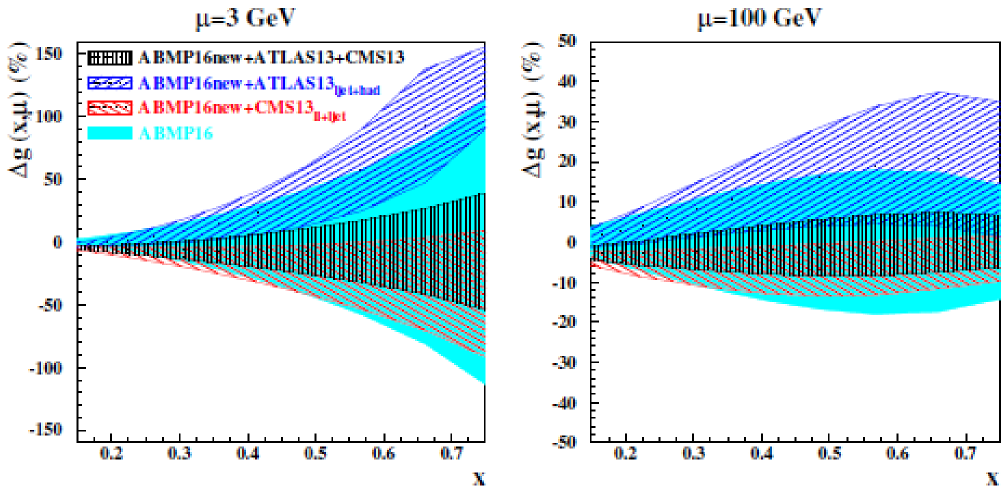
ATLASpdf21 is larger at low and small x because less data are used

CT18 is often the larger of CT, MSHT because of a larger tolerance than MSHT

NNPDF4.0 has generally very small uncertainties in the data region--- new procedure, positivity, integrability etc..

ABMP uses DIS,Dy and ttbar data, not jets
Garzelli shows newer DY data from Seaquest is compatible with ABMP16
ABMtt shown at ggf meeting improves ABMP16 gluon uncertainties





Determinations of $\alpha_s(m_Z)$ at hadron colliders are usually affected by significant correlations between $\alpha_s(m_Z)$ and the PDFs, especially the gluon PDF [65]. The dependence of the PDFs on the value of $\alpha_s(m_Z)$ is accounted for by using corresponding α_s-series of PDF sets, which are provided for seven fixed values of $\alpha_s(m_Z)$ in the range $0.114 < \alpha_s(m_Z) < 0.120$. At each value of $\alpha_s(m_Z)$, the PDF uncertainties are Hessian profiled and the χ^2 function is minimised by solving a system of linear equations, according to Eq. (1) [66], whereas the different values of χ^2 as a function of $\alpha_s(m_Z)$ are minimised through a polynomial interpolation to determine $\alpha_s(m_Z)$.

The PDF set used in the predictions is the approximate N^{3}LO MSHT20 PDF set [59], which is the only PDF set currently available at this order. The PDFs are interpolated with LHAPDF [60] at the factorisation scale μ_f, and evolved backwards using the N^{3}LO solution of the evolution equation. The number of active flavours is set to five in all the coefficients entering the calculation, and in the evolution of the PDFs. The charm- and bottom-quark PDFs are asymptotically switched off in the backward evolution when approaching their corresponding thresholds.

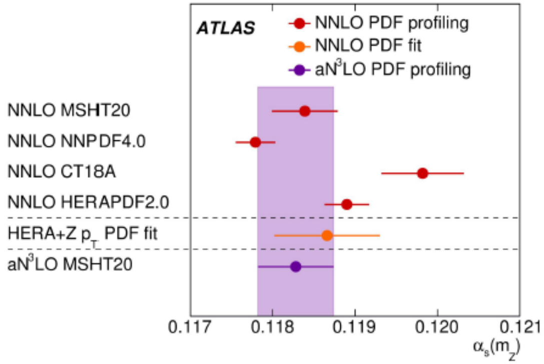
The determination of $\alpha_s(m_Z)$ is repeated at a lower order, N^{3}LL+N^{3}LO, with the MSHT20, CT18A, NNPDF4.0 and HERAPDF2.0 NNLO PDF sets. The spread of the fitted values of $\alpha_s(m_Z)$ is ± 0.00102, driven by the difference between CT18A and NNPDF4.0. While these PDF sets are not appropriate for the present measurement given their lower theoretical accuracy, this study provides a conservative estimate of the residual PDF model dependence of the result, demonstrating that the achievable accuracy is excellent compared to that of other methods of extracting $\alpha_s(m_Z)$. At this order, in addition to the Hessian profiling

Examples – α_S from Z boson $d\sigma/dp_T$ peak (ATLAS)

Different NNLO PDF sets have a spread of ±0.00102, driven by the NNPDF4.0-CT18A difference (with CT14 the spread would be a factor of 2 smaller). CT18 is not compatible with the rest of PDFs within PDF uncertainties

Adding HERA data to the fit (counted twice), the spread is reduced to ±0.00016, around a central value of 0.11804

Would be interesting/possible to compare the nominal CT18 with a CT18 fit to only HERA data?



PDF set	$\alpha_s(m_Z)$	PDF uncertainty
MSHT20 [37]	0.11839	0.00040
NNPDF4.0 [84]	0.11779	0.00024
CT18A [29]	0.11982	0.00050
HERAPDF2.0 [65]	0.11890	0.00027