Perturbative uncertainties for high-energy tails

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Science & Technology Facilities Council | UK Research and Innovation

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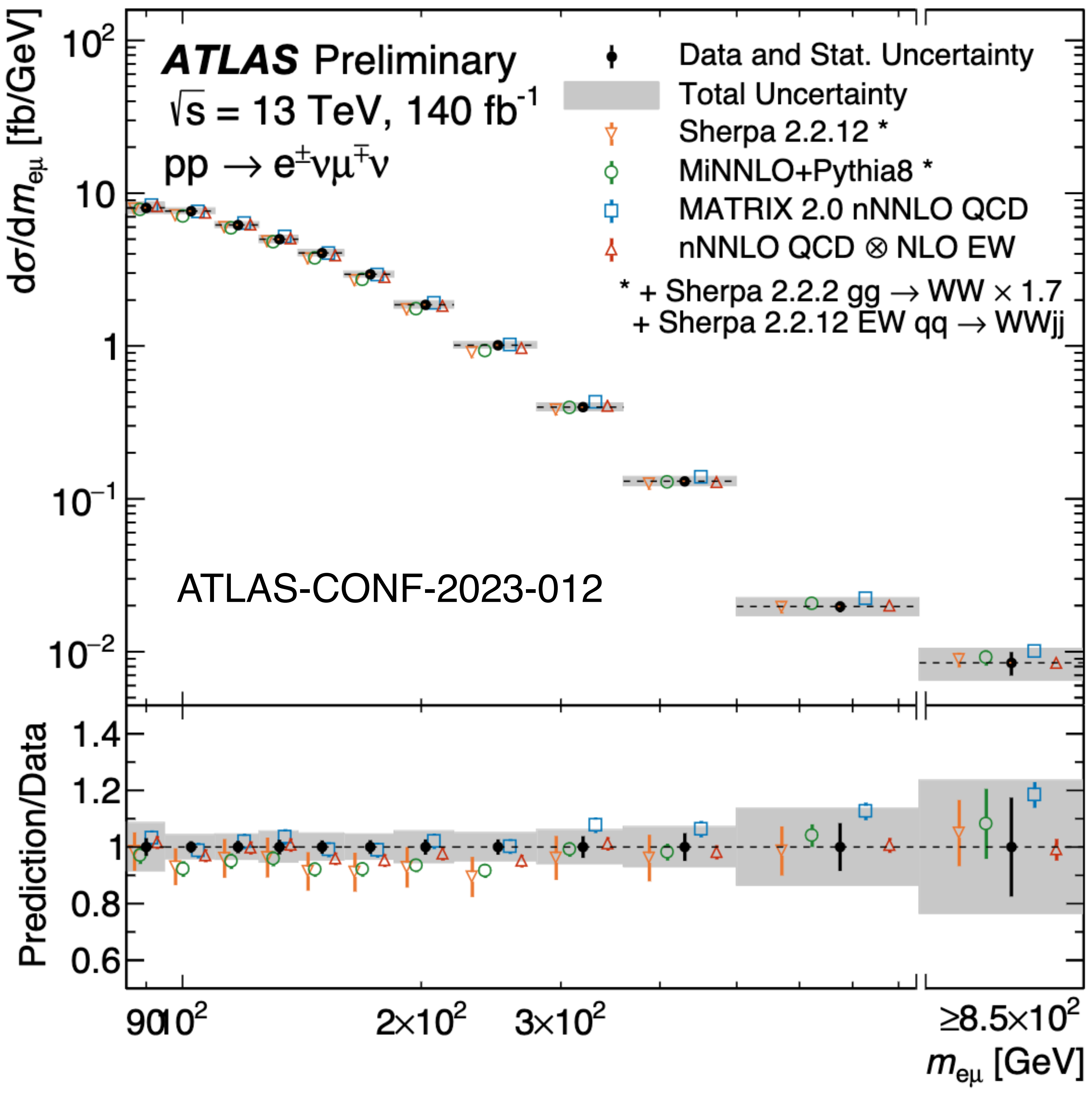
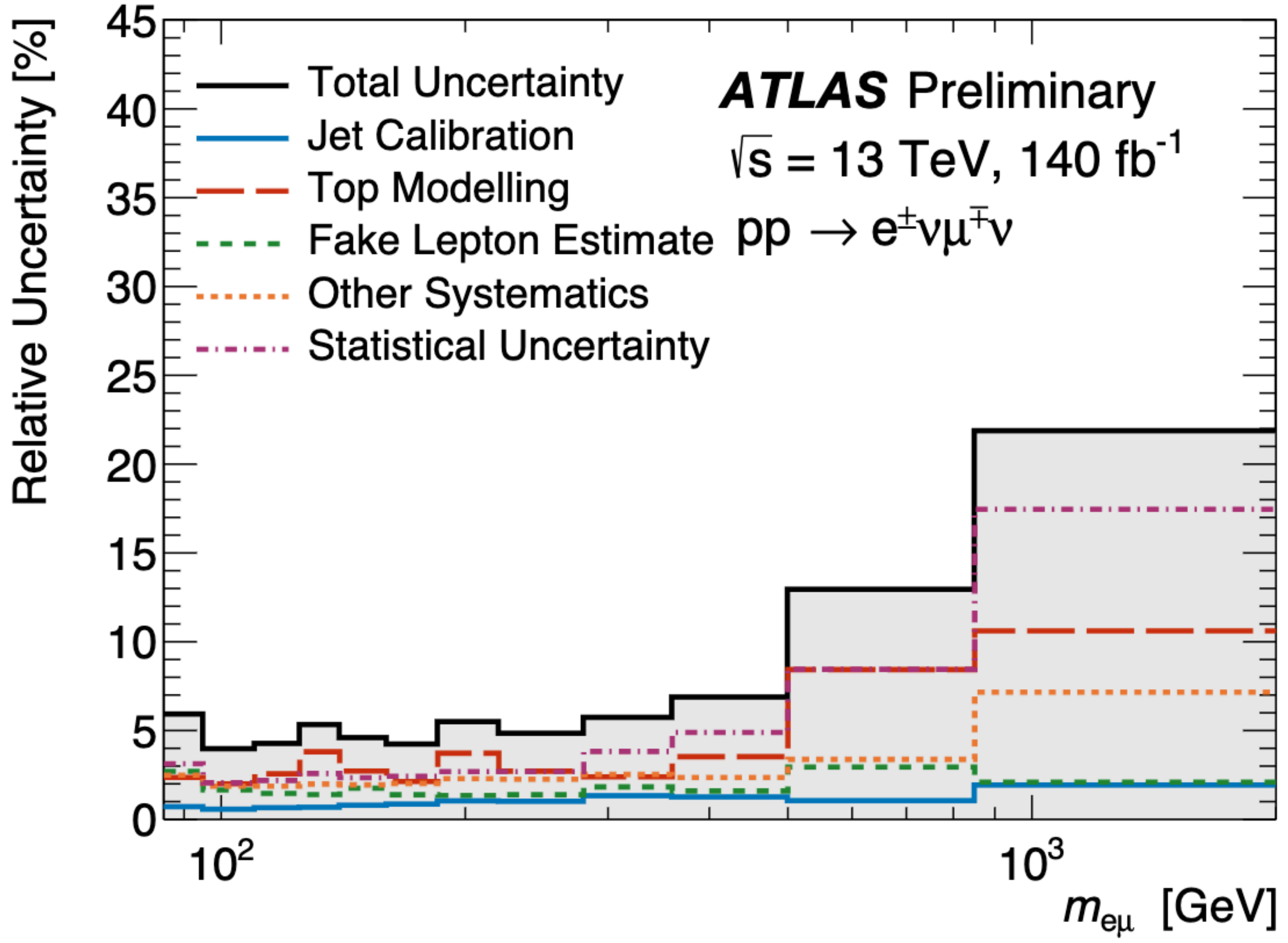
Multiboson tails

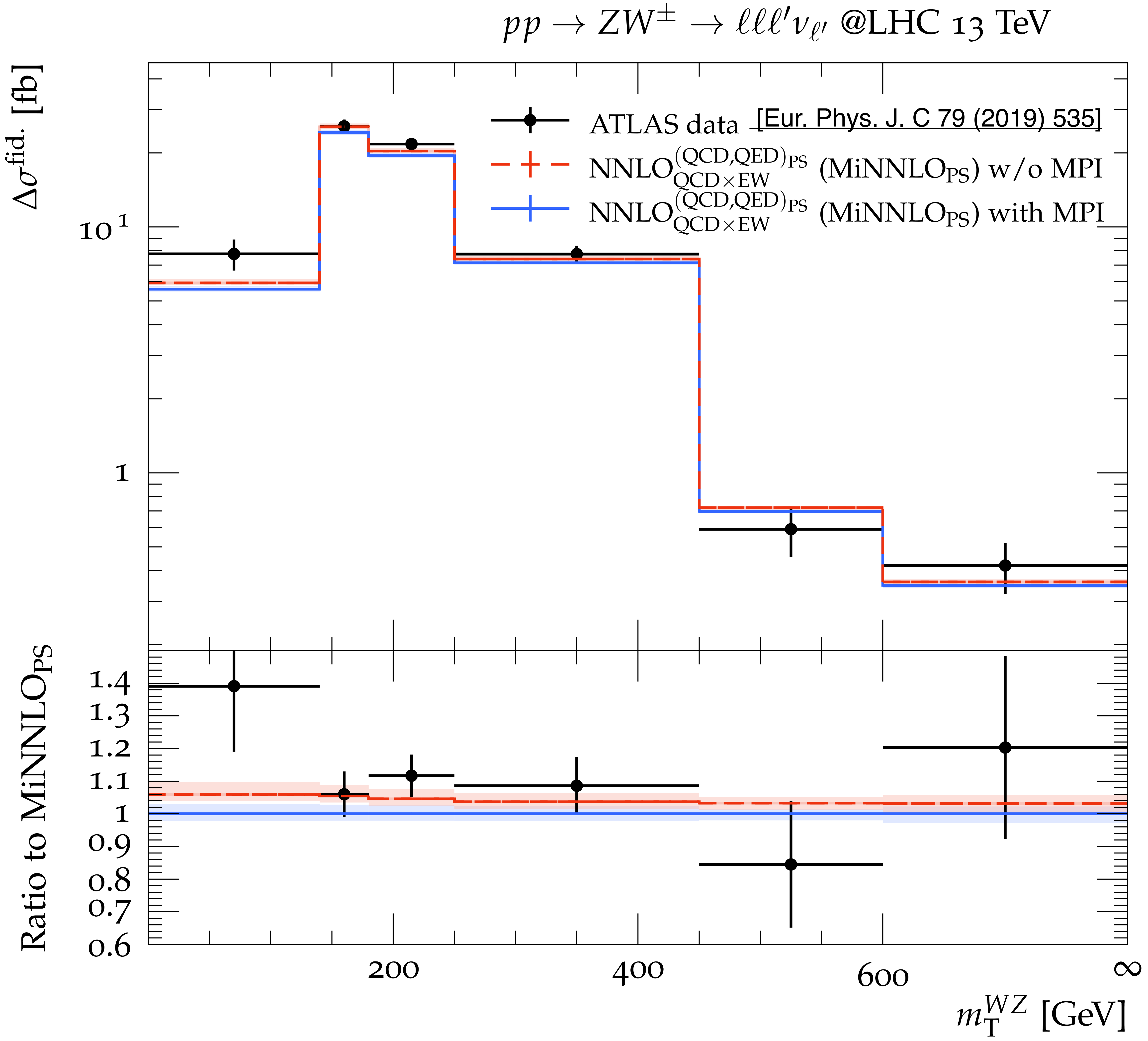
W^+W^-



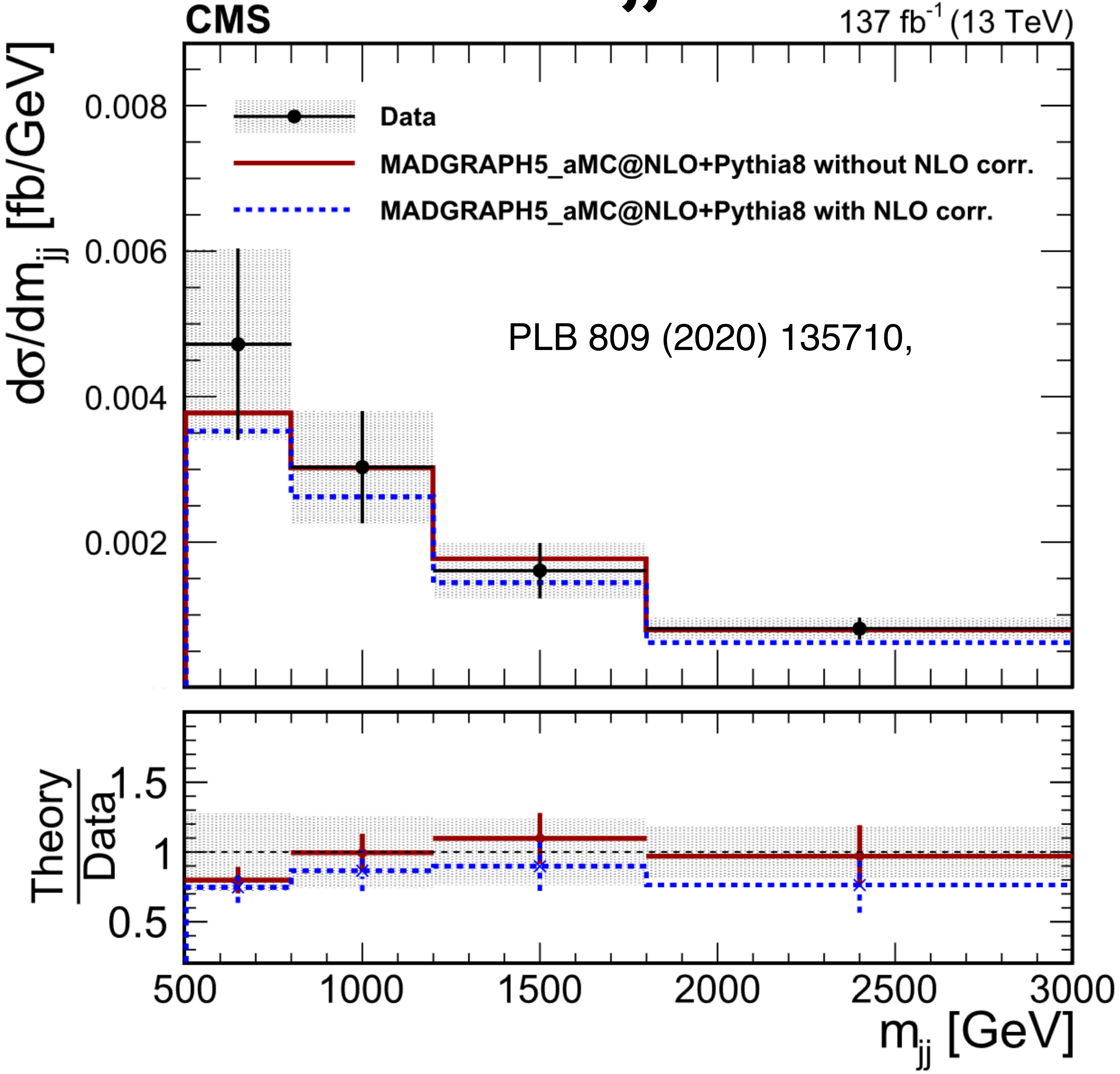


Multiboson tails

[JML, Lombardi, Wiesemann, Zanderighi, Zanoli, '22]



W^+W^+jj



Perturbative expansion

aMC@NLO, Sherpa, Herwig… &
Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, MCFM,
NNLOjet, …

$$d\sigma = d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD NLO EW

$$+\alpha_S^2\, d\sigma_{\mathrm{NNLO}}$$

NNLO QCD

$$+\alpha_S^3\, d\sigma_{\mathrm{N3LO}} + \ldots$$

?

N3LO QCD

scale variation at NNLO

Perturbative expansion

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$$d\sigma = d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD NLO EW

$$+\alpha_S^2\, d\sigma_{\mathrm{NNLO}} + \alpha_{\mathrm{EW}}^2\, d\sigma_{\mathrm{NNLO\,EW}}{}^{?} + \alpha_S \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NNLO\,QCDxEW}}{}^{?}$$

NNLO QCD NNLO EW NNLO QCD-EW

$$+\alpha_S^3\, d\sigma_{\mathrm{N3LO}}{}^{?} + \ldots$$

N3LO QCD

so far only known for DY

scale variation at NNLO

Perturbative expansion

aMC@NLO, Sherpa, Herwig… & Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, MCFM, NNLOjet, …

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_S\, \mathrm{d}\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, \mathrm{d}\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD NLO EW

$$+\alpha_S^2\, \mathrm{d}\sigma_{\mathrm{NNLO}} + \alpha_{\mathrm{EW}}^2\, \mathrm{d}\sigma_{\mathrm{NNLO\,EW}} ? + \alpha_S \alpha_{\mathrm{EW}}\, \mathrm{d}\sigma_{\mathrm{NNLO\,QCDxEW}} ?$$

NNLO QCD NNLO EW NNLO QCD-EW

$$+\alpha_S^3\, \mathrm{d}\sigma_{\mathrm{N3LO}} ? + \ldots$$

N3LO QCD

scale variation at NNLO

scheme variation, e.g. Gmu vs. a(mZ)

+

in case of EW Sudakov dominance: exponentiation

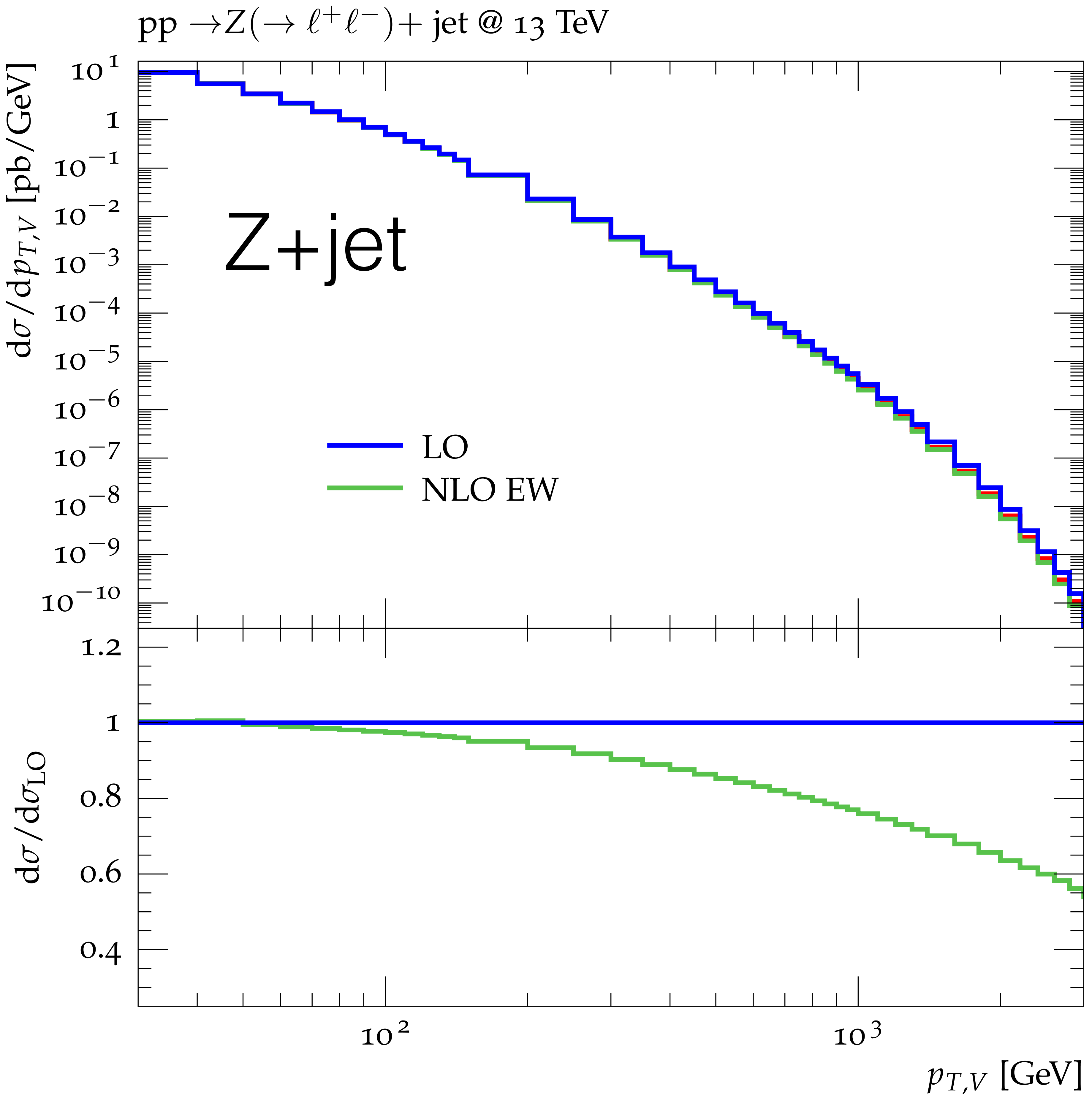
NLO QCD + EW

vs.

NLO QCD x EW

- sufficient?
- reliable?

EW uncertainties: Sudakov

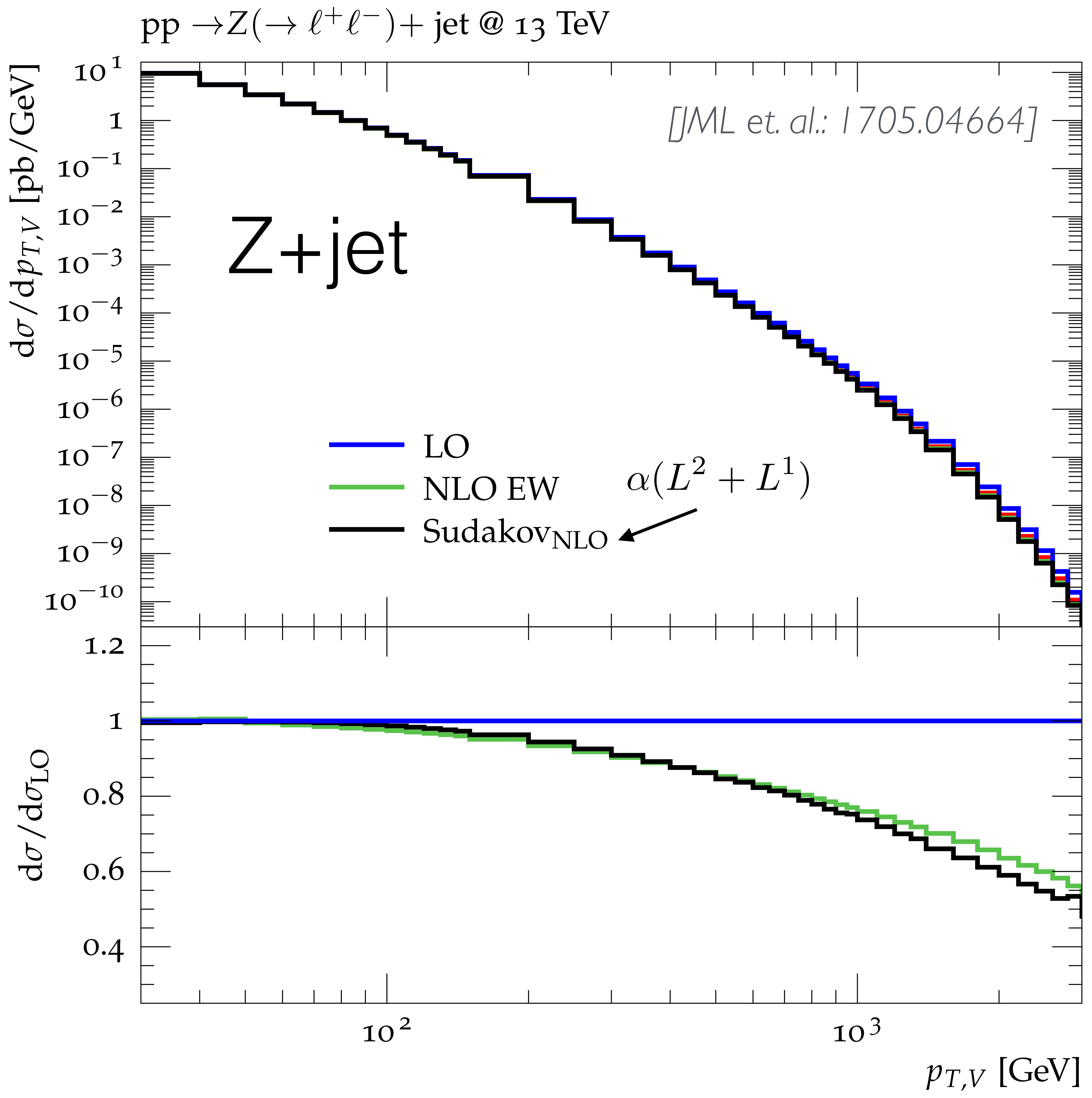


EW corrections become sizeable at large $p_{T,V}$: -30% @ 1 TeV

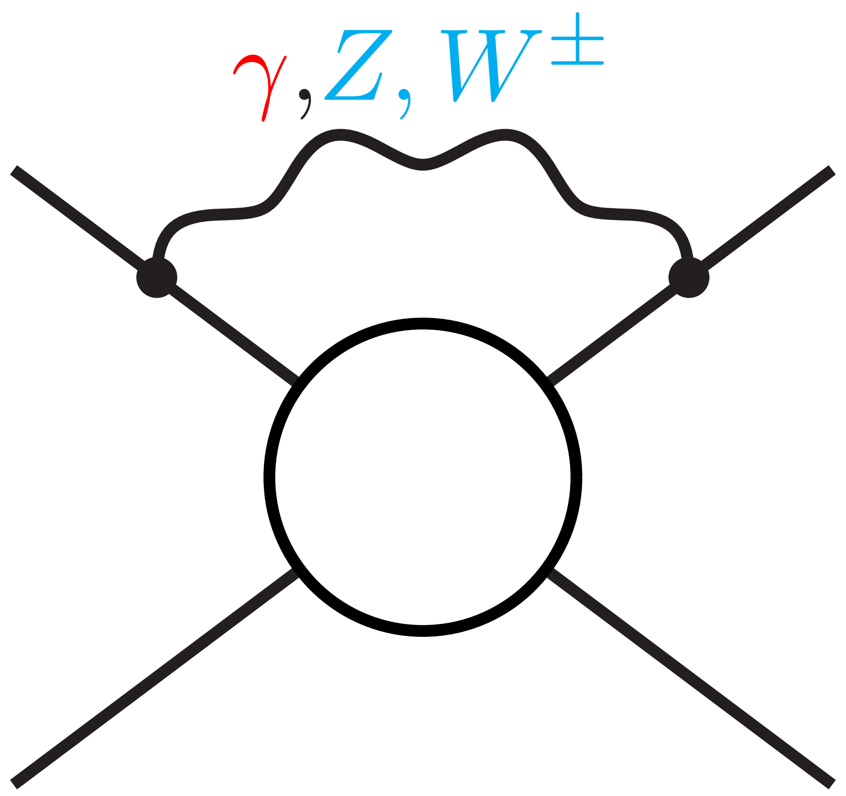
Origin: virtual EW Sudakov logarithms

How to estimate corresponding pure EW uncertainties of relative $\mathcal{O}(\alpha^2)$?

EW uncertainties: Sudakov



Large EW corrections dominated by Sudakov logs

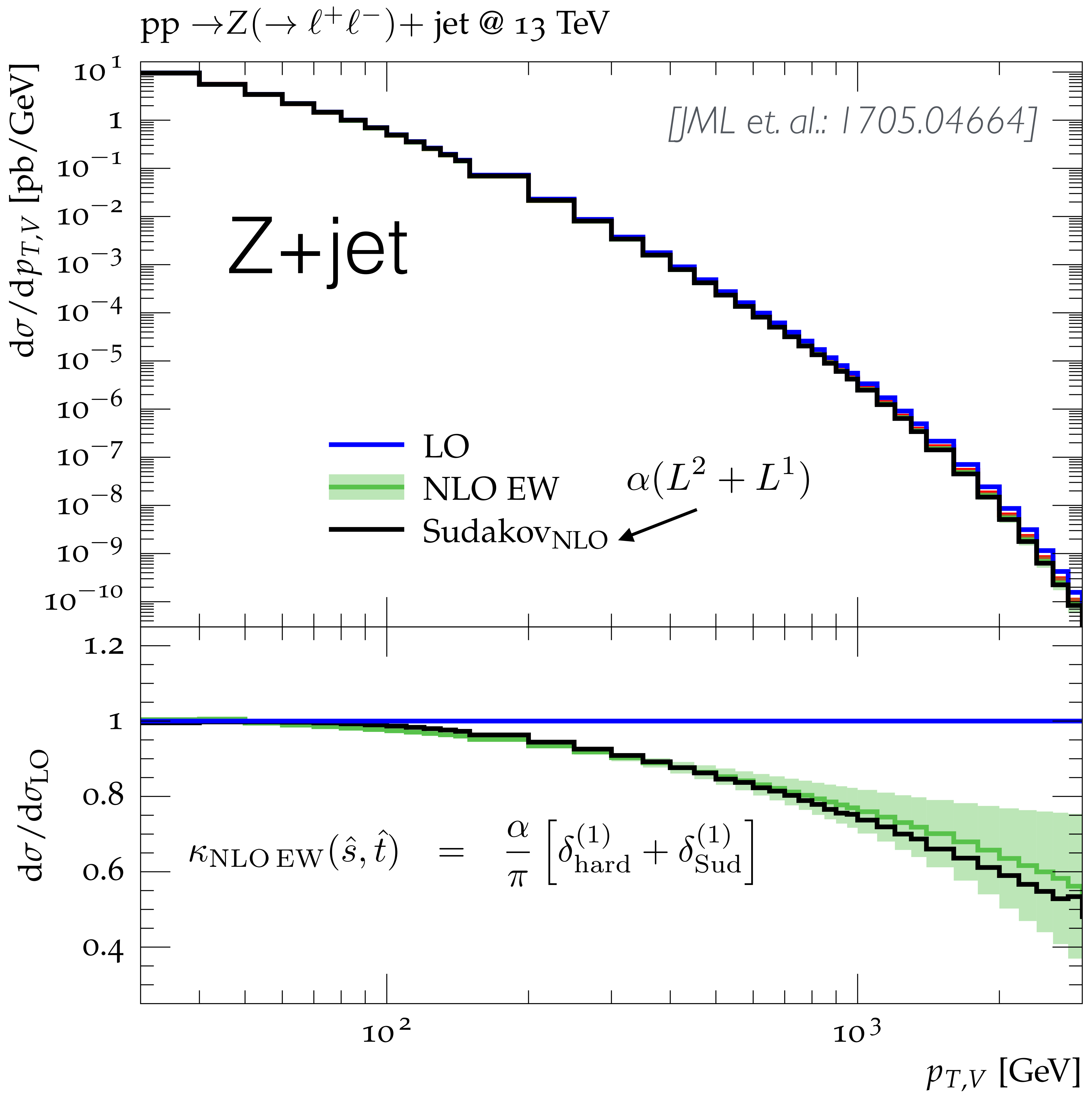


[Ciafaloni, Comelli,'98; Lipatov, Fadin, Martin, Melles, '99; Kuehen, Penin, Smirnov, '99; Denner, Pozzorini, '00]

Universality and factorisation: *[Denner, Pozzorini; '01]*

$$\delta\mathcal{M}^{\text{1-loop}}_{\text{LL+NLL}} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\left\{\frac{1}{2}\sum_{l\neq k}\sum_{a=\gamma,Z,W^\pm} I^a(k)I^{\bar{a}}(l)\ln^2\frac{\hat{s}_{kl}}{M^2} + \gamma^{\text{ew}}(k)\ln\frac{\hat{s}}{M^2}\right\}\mathcal{M}_0$$

EW uncertainties: Sudakov



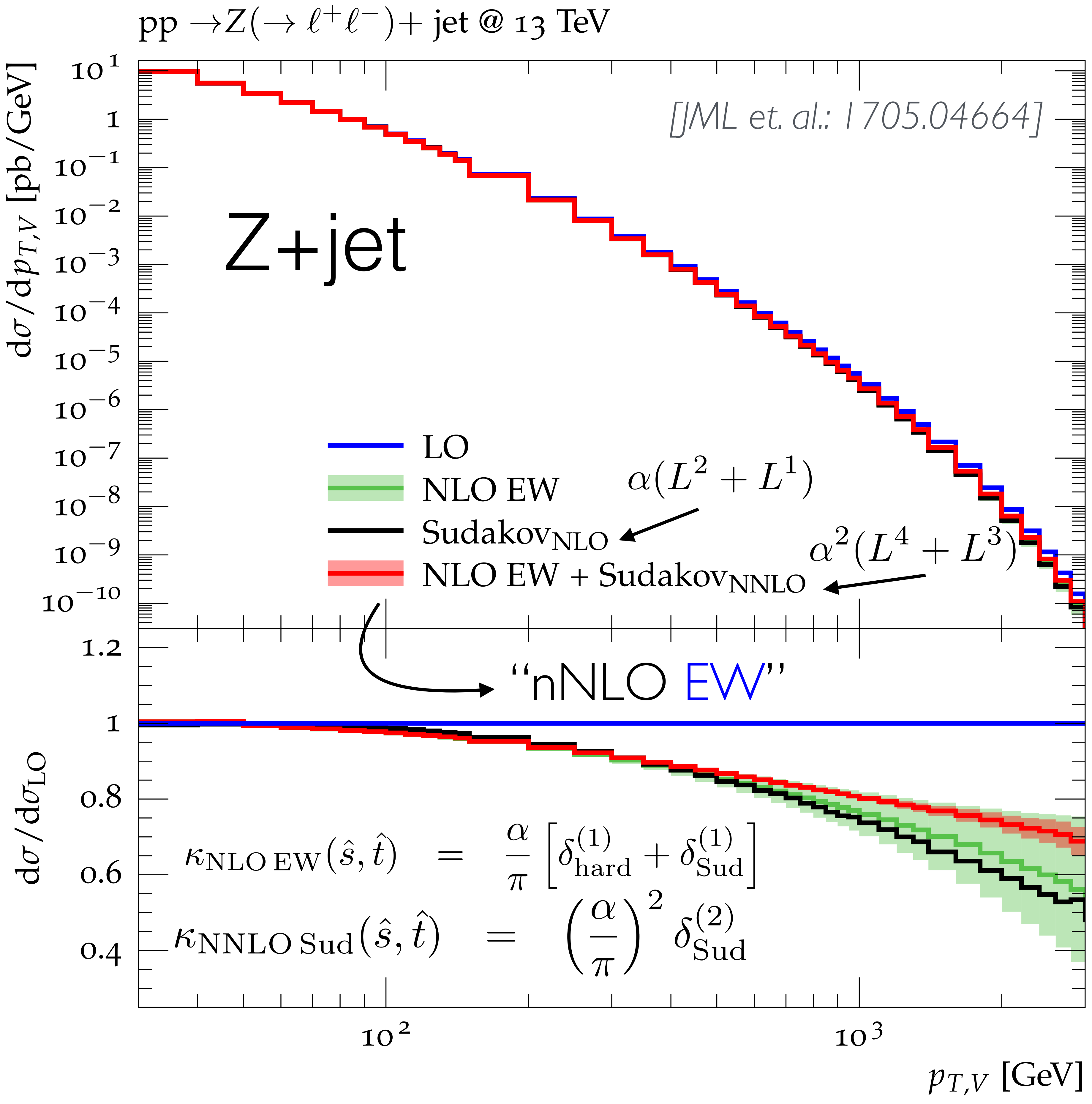
Large EW corrections dominated by Sudakov logs

↓

Uncertainty estimate of (N)NLO EW from naive exponentiation x 2:

$$\Delta^{\rm Sud}_{\rm EW} \approx (k_{\rm NLOEW})^2$$

EW uncertainties: Sudakov



Large EW corrections dominated by Sudakov logs

↓

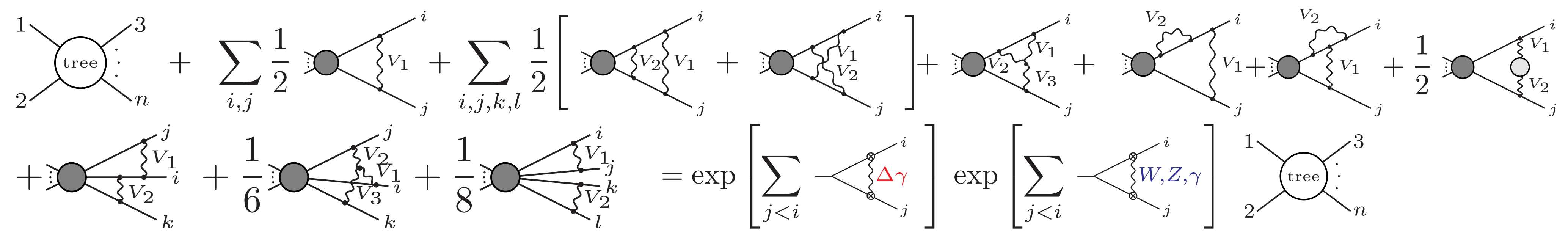
Uncertainty estimate of (N)NLO EW from naive exponentiation x 2:

$$\Delta^{\text{Sud}}_{\text{EW}} \approx (k_{\text{NLOEW}})^2$$

↓

check against two-loop Sudakov logs

[Kühn, Kulesza, Pozzorini, Schulze; 05-07]



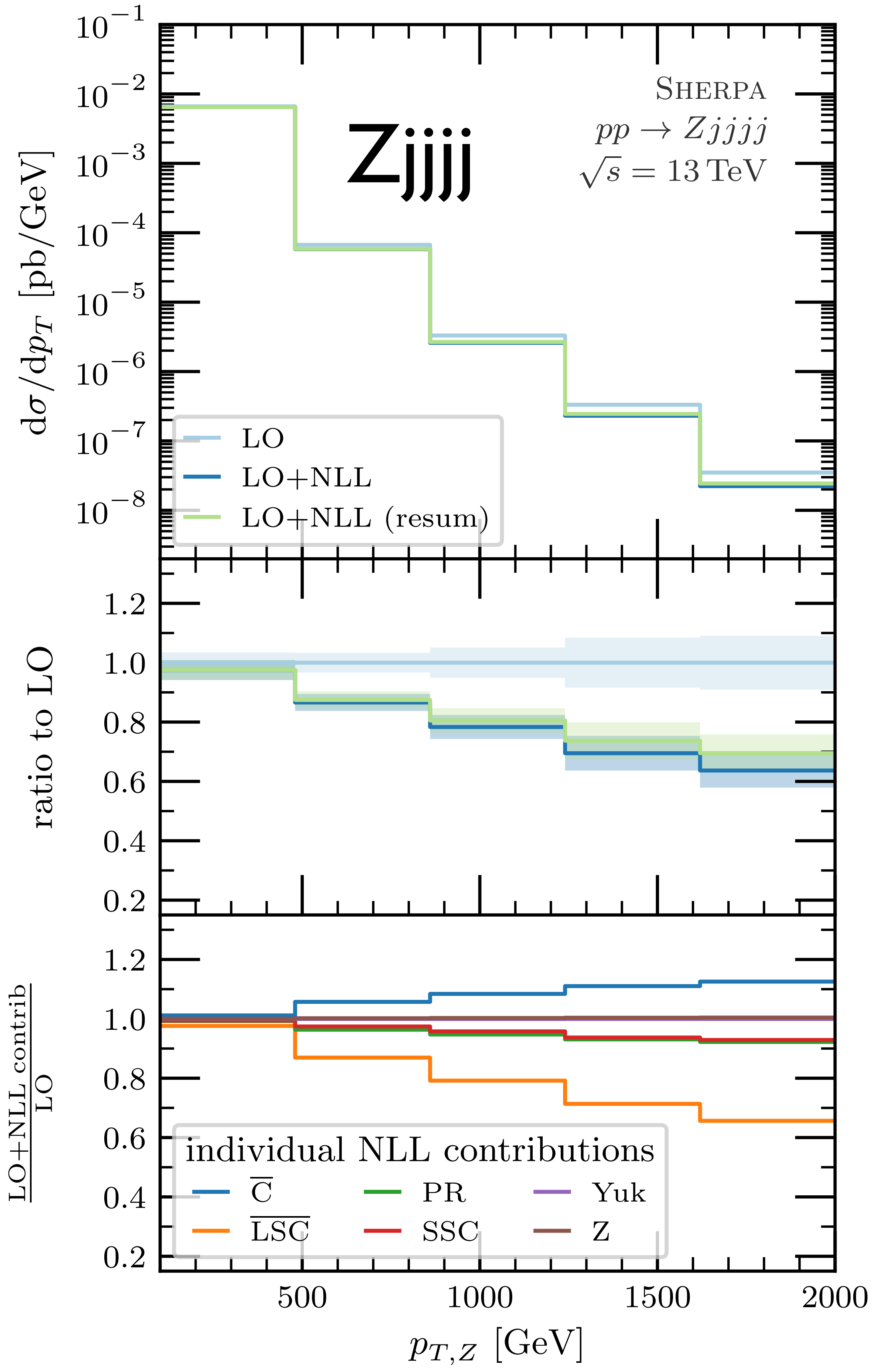
$$\Delta^{\text{hard}}_{\text{EW}} \approx O(1\%)$$

e.g. from scheme variation, e.g. Gmu vs. a(mZ)

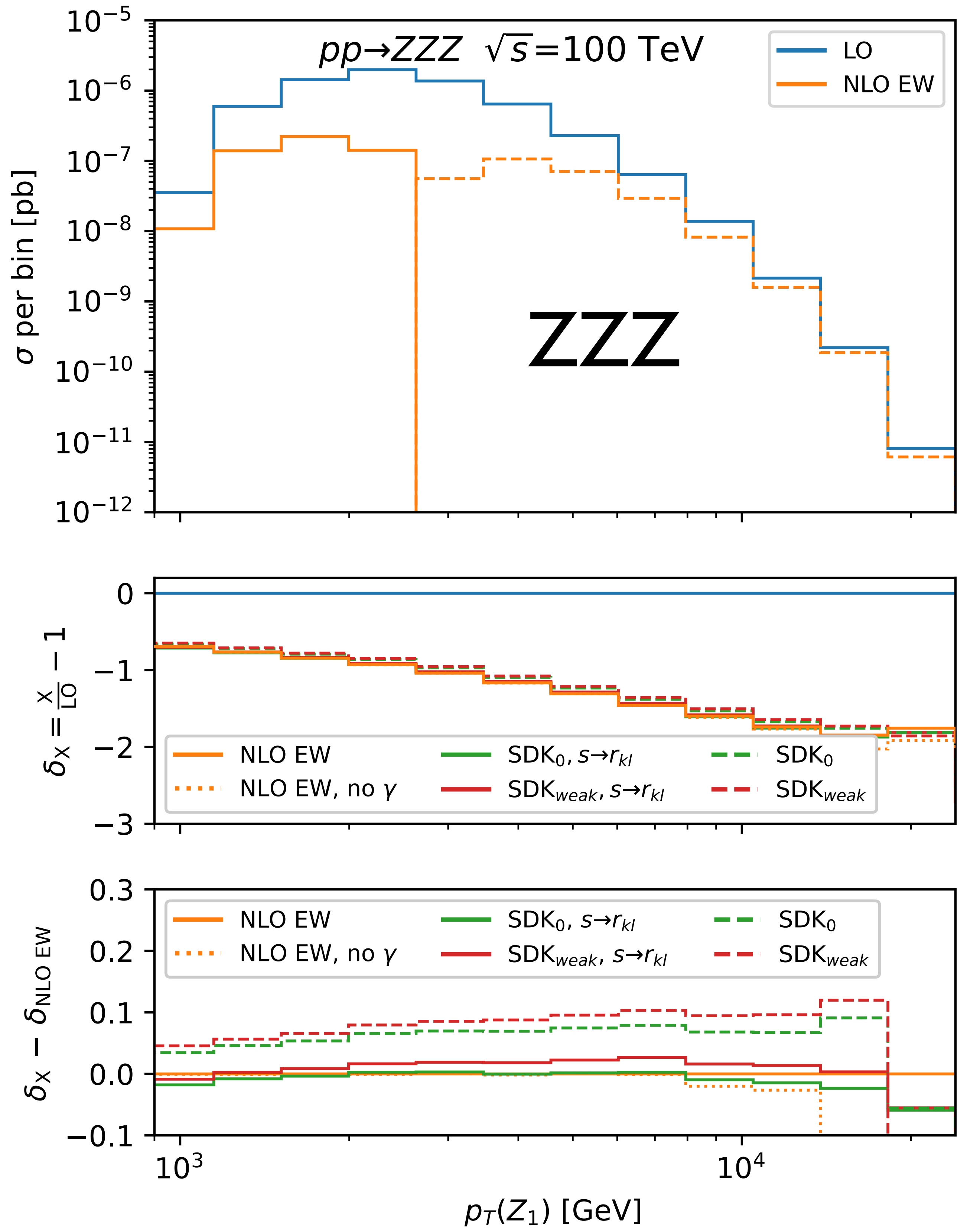
Tools for EW Sudakov corrections

also: alpgen [Chiesa, et. al., '13]

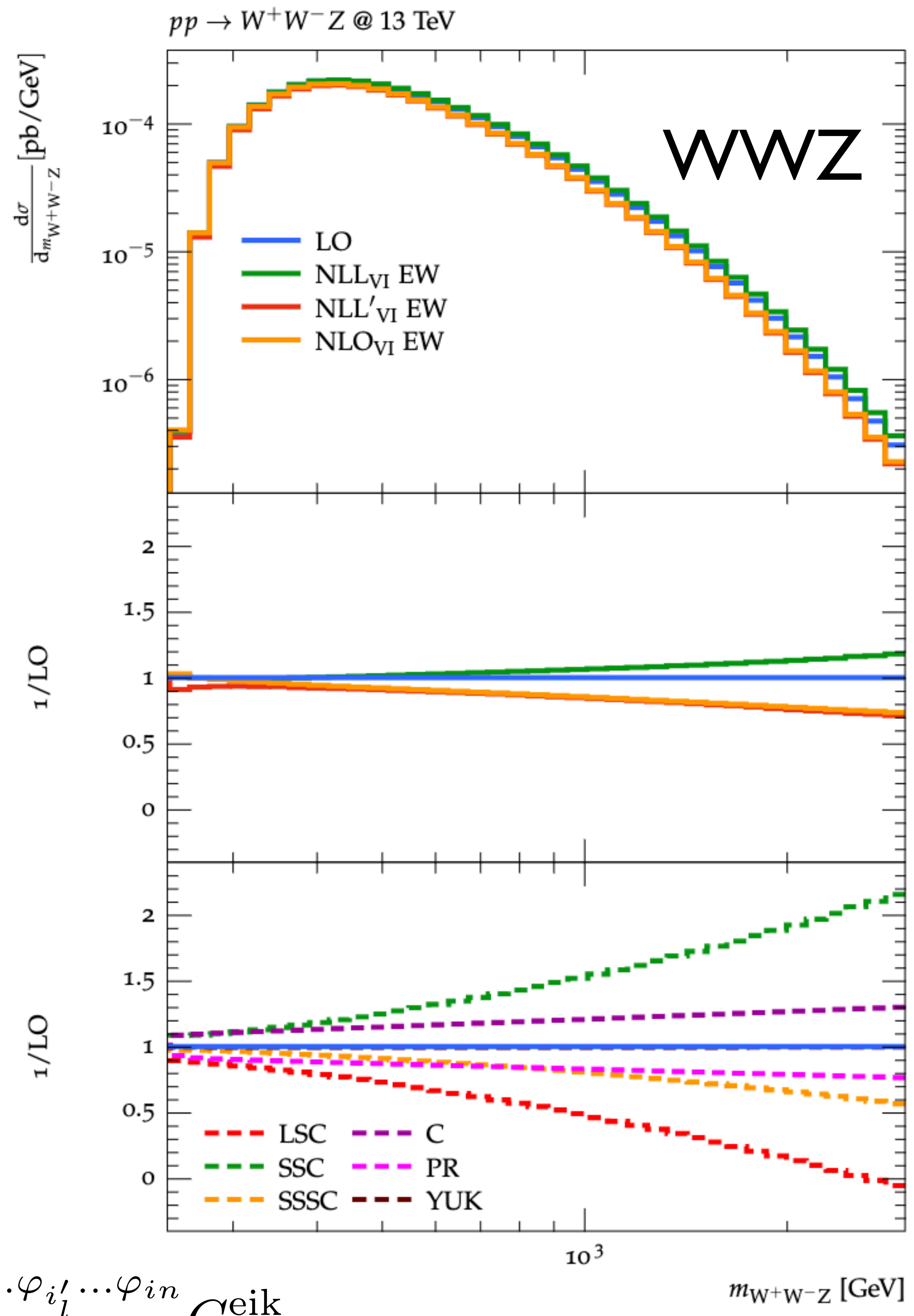
Sherpa
[Bothmann, Napoletano, '20]



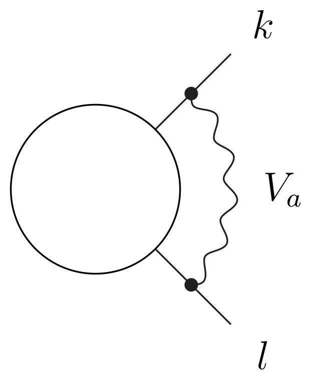
MadGraph5_aMC@NLO
[Pagani, Zaro, '21]



OpenLoops
[JML, Mai, '23]



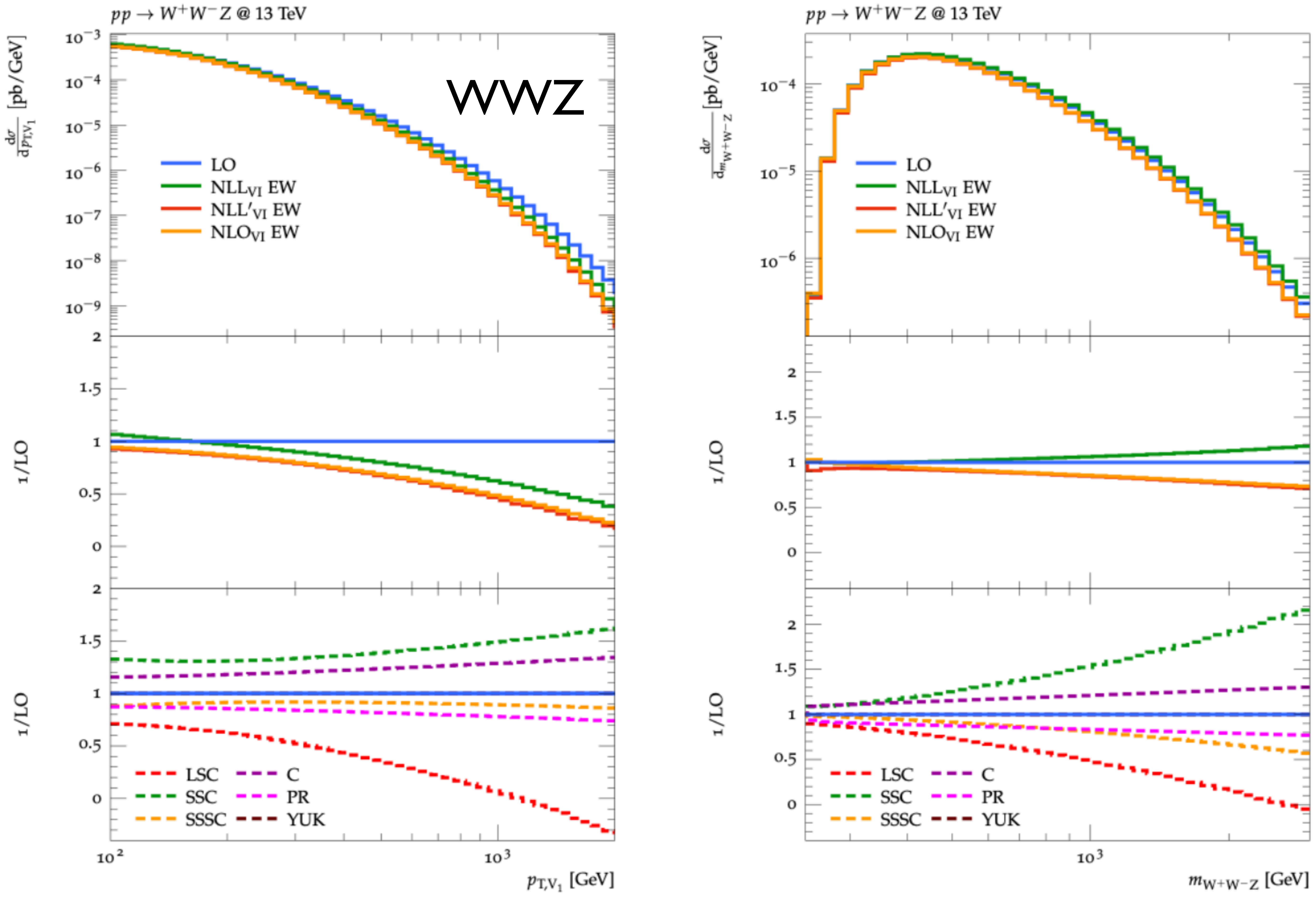
- all based on
  [Denner, Pozzorini, '00, '01]



$$\delta_{kl}^{\mathrm{DL}} \mathcal{M}^{\varphi_{i_1} \cdots \varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \frac{\alpha}{4\pi} \sum_{\varphi_{i'_k}, \varphi_{i'_l}} I^{V}_{\varphi_{i'_k} \varphi_{i_k}} I^{\bar{V}}_{\varphi_{i'_l} \varphi_{i_l}} \mathcal{M}_0^{\varphi_{i_1} \cdots \varphi_{i'_k} \cdots \varphi_{i'_l} \cdots \varphi_{i_n}} C_0^{\mathrm{eik}}$$

$$C_0^{\mathrm{eik}} \equiv \frac{1}{(p_k + p_l)^2} \left[\log^2 \frac{|r_{kl}|}{M_V^2} - 2i\pi \Theta(r_{kl}) \log \frac{|r_{kl}|}{M_V^2} \right]$$

EW Sudakov corrections



1. $$\delta^{\mathrm{DL}}_{kl}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \frac{\alpha}{4\pi}\sum_{\varphi_{i'_k},\varphi_{i'_l}} I^V_{\varphi_{i'_k}\varphi_{i_k}} I^{\bar V}_{\varphi_{i'_l}\varphi_{i_l}} \mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i'_l}\dots\varphi_{i_n}} C_0^{\mathrm{eik}}$$

$$C_0^{\mathrm{eik}} \equiv \frac{1}{(p_k+p_l)^2}\left[\log^2\frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl})\log\frac{|r_{kl}|}{M_V^2}\right]$$

$$\log^2\left(\frac{|r_{jk}|}{m_V^2}\right) = \underbrace{\log^2\left(\frac{s}{m_V^2}\right)}_{\propto\ l^{\mathrm{LSC},V}} + \underbrace{2\log\left(\frac{s}{m_V^2}\right)}_{\propto\ l^{\mathrm{SL},V}}\ \underbrace{\log\left(\frac{|r_{jk}|}{s}\right)}_{\propto\ l^{\mathrm{SSC}}} + \underbrace{\log^2\left(\frac{|r_{jk}|}{s}\right)}_{\propto\ l^{\mathrm{S\text{-}SSC}}}$$

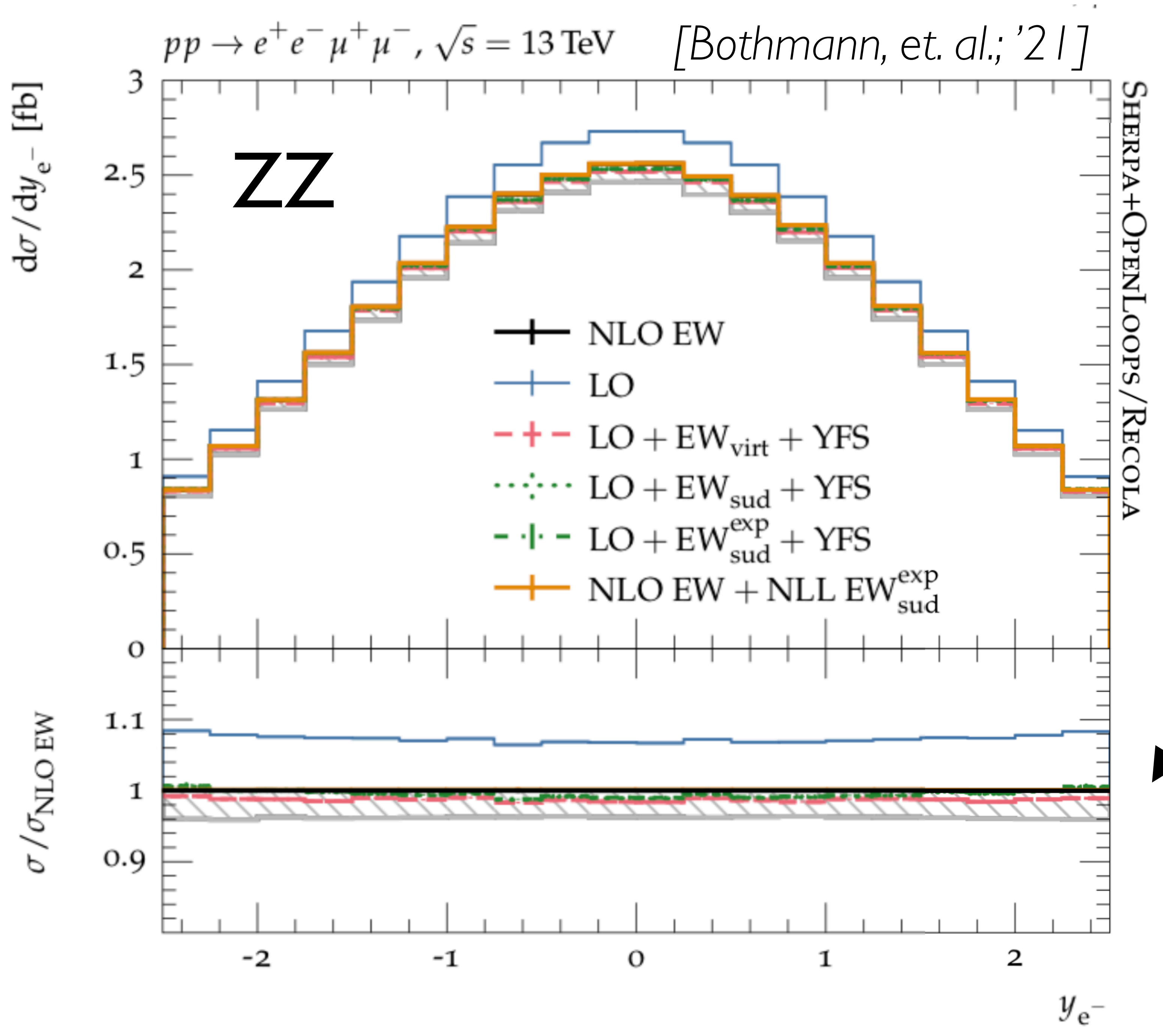
2. $$\hat\delta^{\mathrm{SL}} = \hat\delta^{\mathrm{PR}} + \hat\delta^{\mathrm{COLL}} + \hat\delta^{\mathrm{WFRC}}$$

- Sizeable cancellations between different logarithmic contributions.
- Only partial control of angular-dependent S-SSC contribution in Sudakov approximation

EW uncertainties: hard-coefficient

Scheme variations

e.g. $\{G_\mu, m_W, m_Z\}$ vs. $\{\alpha(m_Z), m_W, m_Z\}$



However: scheme variations mix perturbative and parametric uncertainties!

Estimate hard coefficient

Typical size of hard EW corrections: ~2%

$$\left(\frac{\alpha}{\pi}\right)\delta^{(1)}_{\mathrm{hard}} = 2\% \leftrightarrow \delta^{(1)}_{\mathrm{hard}} = 10$$

Require: $\delta^{(2)}_{\mathrm{hard}} \leq 100\,\delta^{(1)}_{\mathrm{hard}}$

$$\Delta^{\mathrm{hard}}_{\mathrm{EW}} = 1000 \times \left(\frac{\alpha}{\pi}\right)^2 = 0.6\%$$

EW uncertainties: QED radiation

NLOPS EW needs to be **resonance-aware**: [Jezo, Nason, '15]

Conservative estimate of higher-order QED radiation:

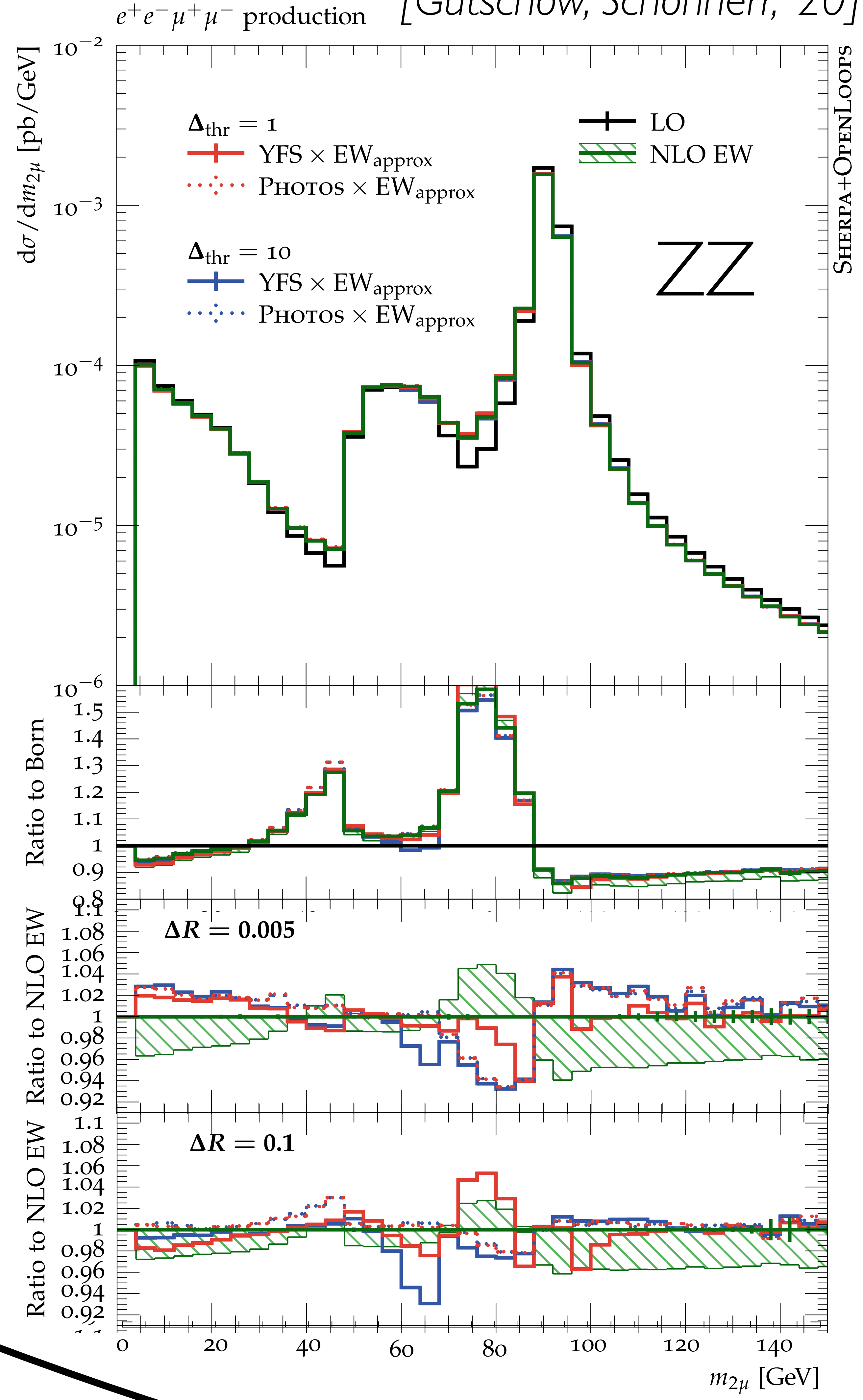
NLO EW

vs.

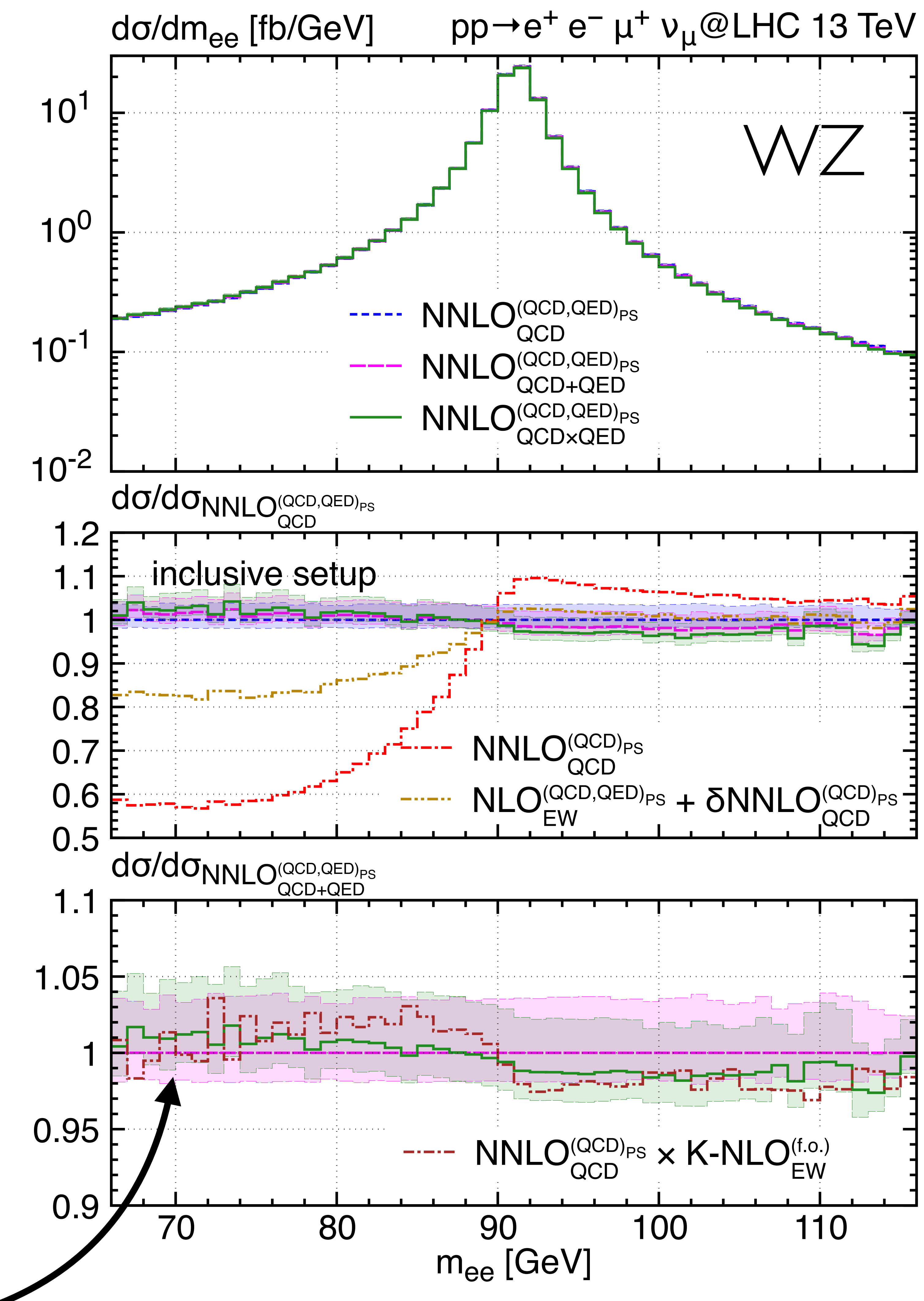
multi-photon radiation (YFS) or QED-PS

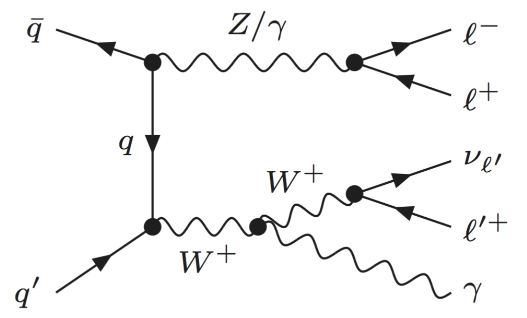
$$\Delta_{\rm EW}^{\rm QED} = |\delta_{\rm EW} - \delta_{\rm EW+PS/YFS}|$$

[Gütschow, Schönherr, '20]



[JML, Lombardi, Wiesemann, Zanderighi, Zanoli, '22]





Mixed QCD-EW uncertainties

Bold estimate:

Consider real $\mathcal{O}(\alpha\alpha_s)$ correction to X production $\simeq$ NLO EW to X+1 jets

and we often observe

$$\left.\frac{d\sigma_{\mathrm{NLO\,EW}}}{d\sigma_{\mathrm{LO}}}\right|_{X+\mathrm{jet}} - \left.\frac{d\sigma_{\mathrm{NLO\,EW}}}{d\sigma_{\mathrm{LO}}}\right|_{X} \lesssim 1\%$$

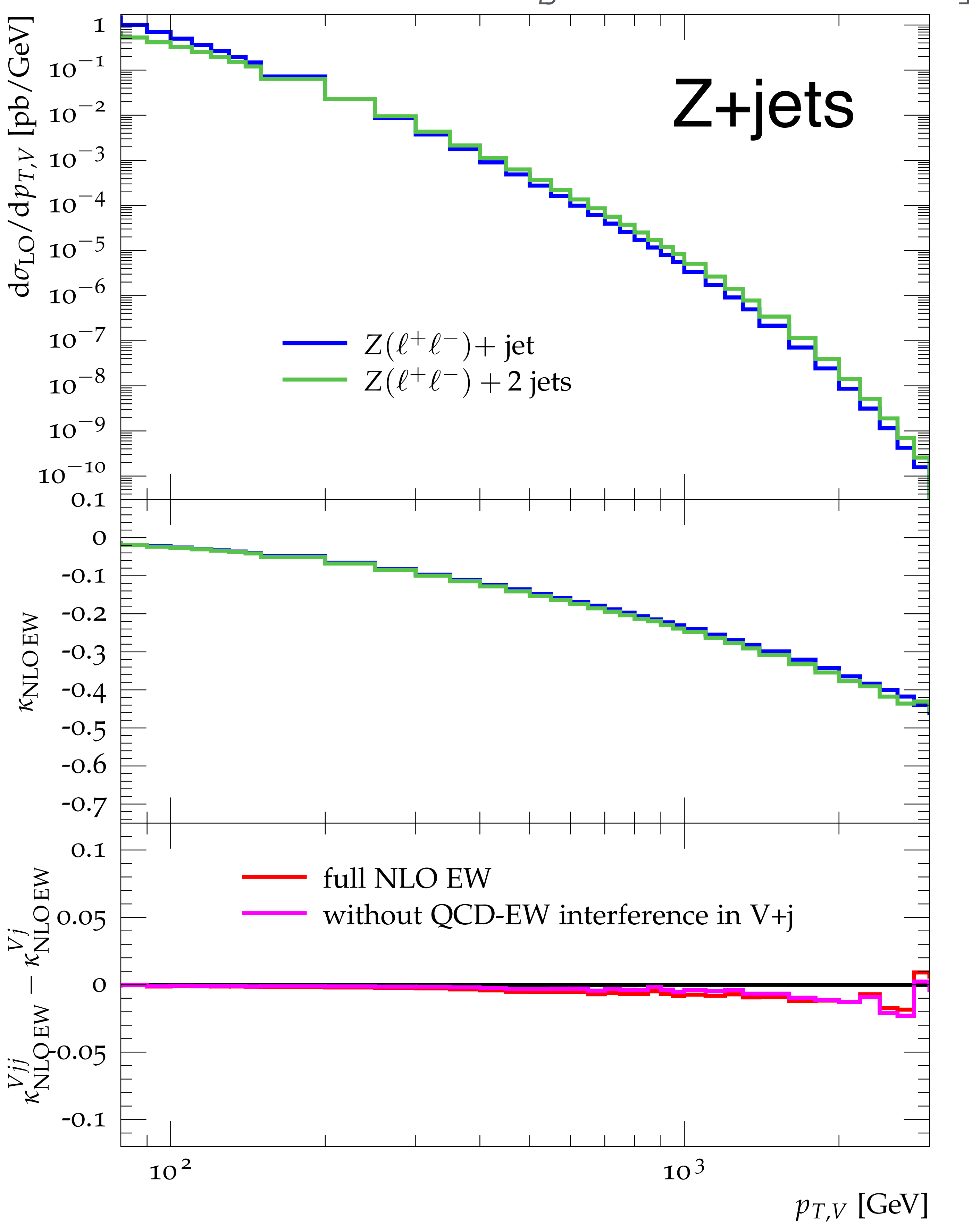
In these cases strong support for
- factorisation
- multiplicative QCD x EW combination
- Consider only such non-factorising effects as uncertainty!?

$$d\sigma_{\mathrm{NNLO\,QCD\times EW}} = d\sigma_{\mathrm{LO}}\left(1+\delta_{\mathrm{QCD}}\right)\left(1+\delta_{\mathrm{EW}}\right)$$

alternative: $\delta_{\mathrm{EW}}^{\mathrm{NLL}} = \delta_{\mathrm{EW}}^{\mathrm{DL}} + \delta_{\mathrm{EW}}^{\mathrm{SL}} + \delta_{\mathrm{EW}}^{\mathrm{non-log}}$

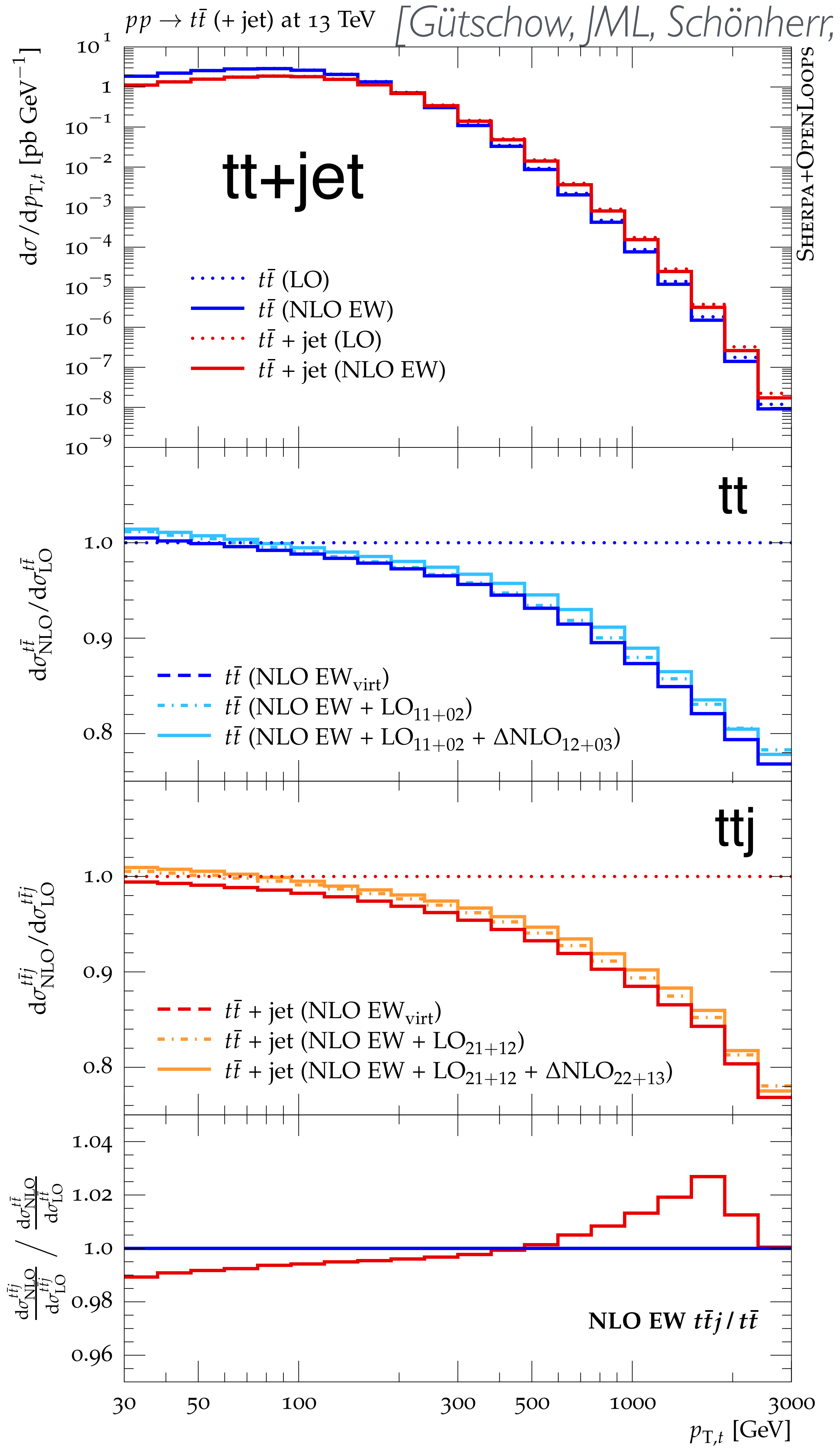
$\rightarrow \mathcal{O}(\alpha_S\alpha) = d\sigma_{\mathrm{LO}}\,\delta_{\mathrm{QCD}}\,(\delta_{\mathrm{EW}}^{\mathrm{SL}} + \delta_{\mathrm{EW}}^{\mathrm{non-log}})$ $\rightarrow \Delta_{\mathrm{QCD-EW}} = \delta_{\mathrm{QCD}}\,(\delta_{\mathrm{EW}}^{\mathrm{SL}} + \delta_{\mathrm{EW}}^{\mathrm{hard}})$

[JML et. al.: 1705.04664]



$p_{T_{j,2}} > 30$ GeV

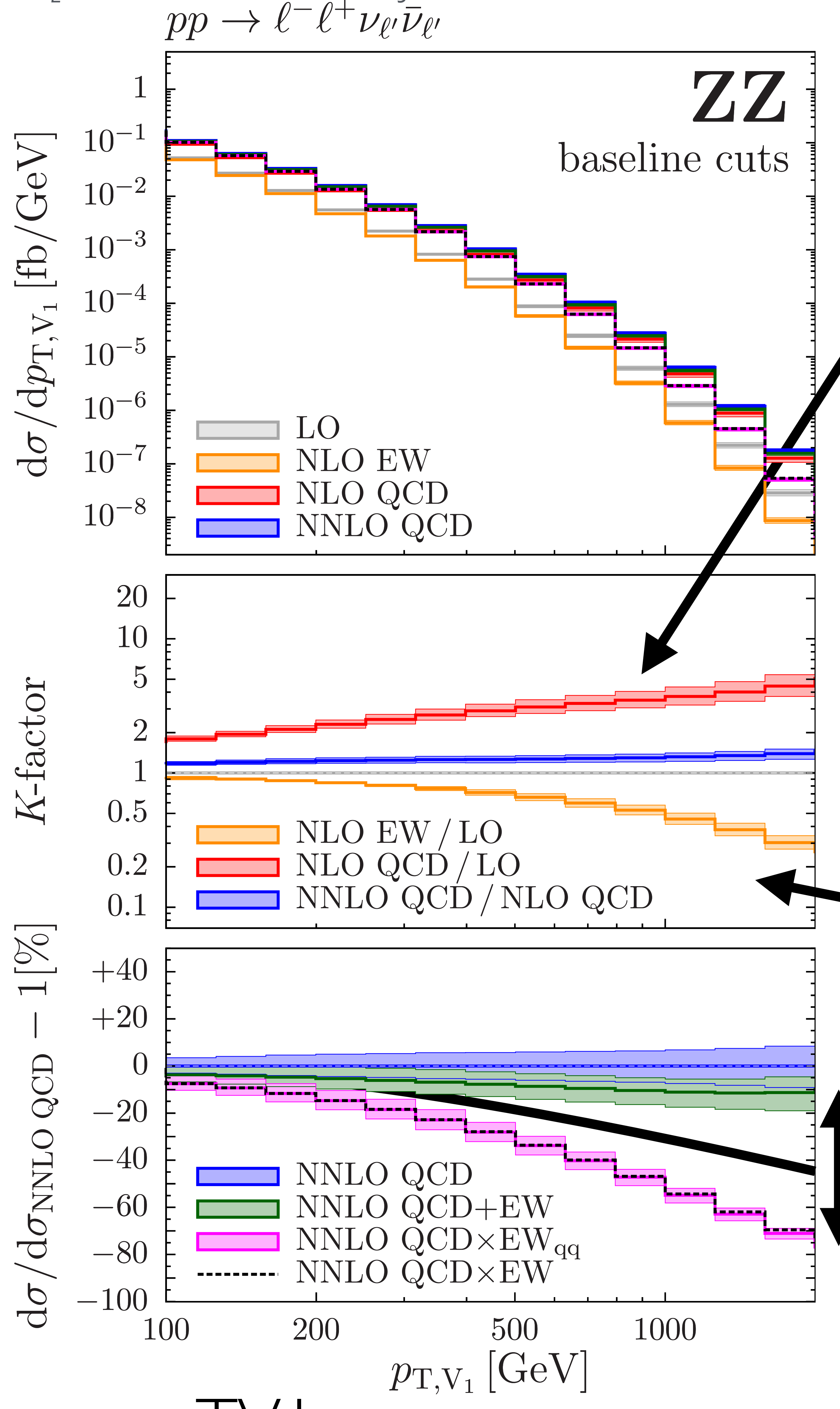
[Gütschow, JML, Schönherr, '18]



$p_{T_j} > 30$ GeV

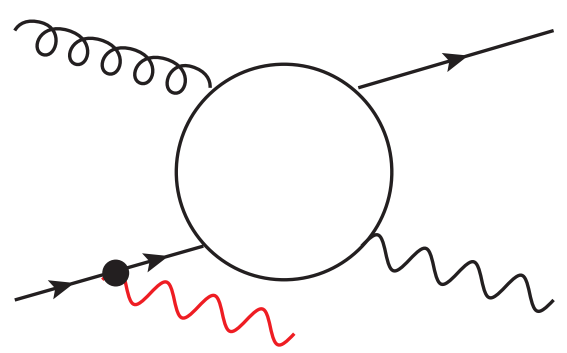
Mixed QCD-EW uncertainties

[M. Grazzini, S. Kallweit, JML, S. Pozzorini, M. Wiesemann; '19]



pTV1

- NLO QCD/LO=2-5! ("giant K-factor")
- at large pTV1: VV phase-space is dominated by V+jet (w/ soft V radiation)



$$\longrightarrow \frac{d\sigma^{V(V)j}}{d\sigma^{LO}_{VV}} \propto \alpha_S \log^2\left(\frac{Q^2}{M_W^2}\right) \simeq 3 \qquad \text{at} \quad Q = 1\,\text{TeV}$$

- NNLO / NLO QCD moderate and NNLO uncert. 5-10%
- NLO EW/LO=-(40-50)%
- Very large difference $d\sigma_{NNLO\,QCD+EW}$ vs. $d\sigma_{NNLO\,QCD\times EW}$
- Problems:
  1. In additive combination dominant Vj topology does not receive any EW corrections
  2. In multiplicative combination EW correction for VV is applied to Vj hard process
- Pragmatic solution I: **take average as nominal and spread as uncertainty**
- Pragmatic solution II: **apply jet veto to constrain Vj toplogoies**

MEPS @ NLO QCD + EW

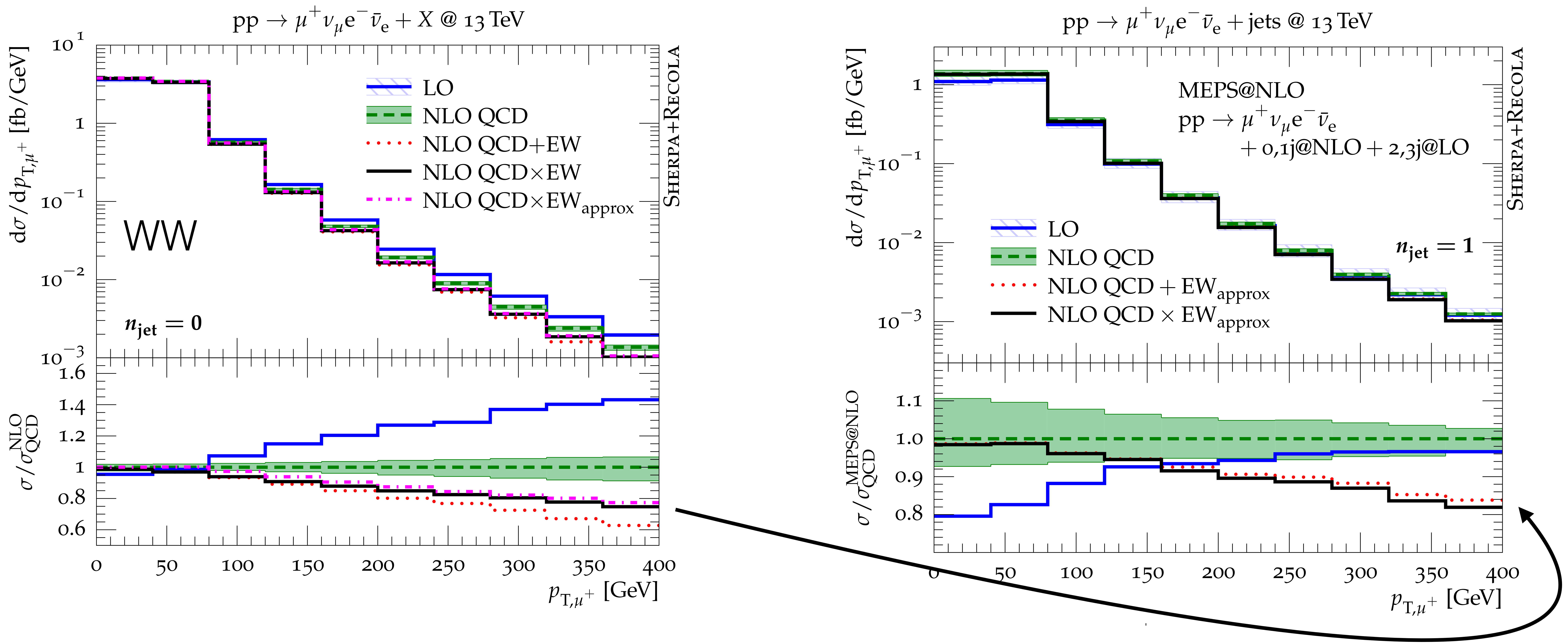
WW(+jet): [Bräuer, Denner, Pellen, Schönherr, Schumann; '20]
ZZ(+jet): [Bothmann, Napoletano, Schönherr, Schumann, Villani; '21]

Used in many ATLAS modern multi-purpose samples: V+jets, VV+jets, tt+jets

[Kallweit, JML, et. al.; '15]

- More rigorous solution: merge VVj incl. approx. EW corrections with VV with Sherpa's MEPS@NLO QCD + EWvirt

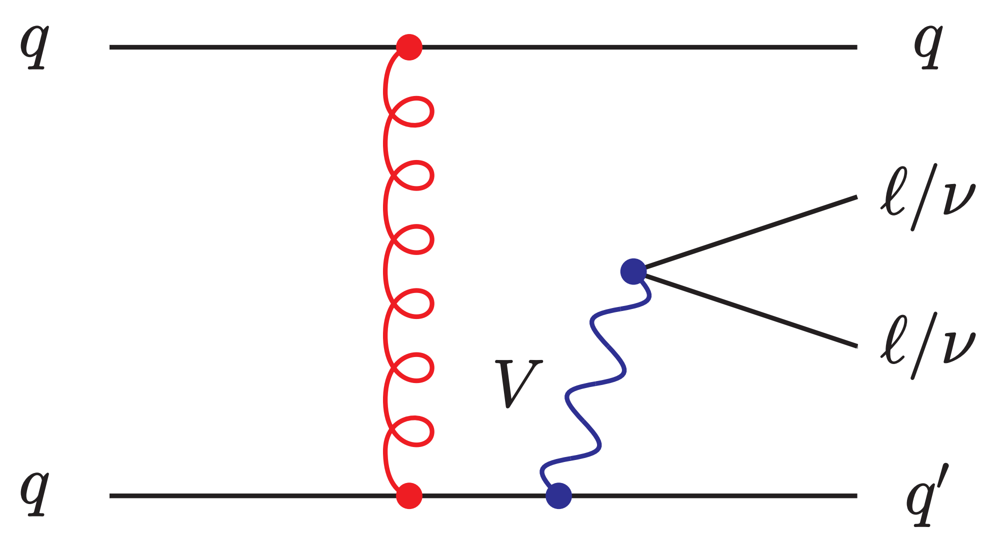
FO | **MEPS@NLO QCD + EWvirt**

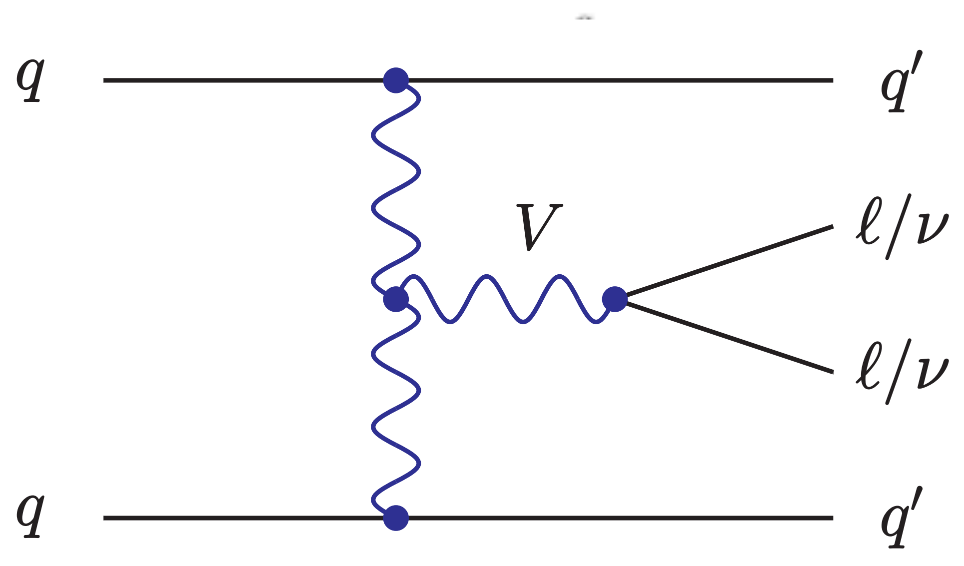


Perturbative expansion: tower of contributions

- For processes with at least 4-quarks there is a tower of LO(NLO) contributions.
- E.g.: multijets, $t\bar{t}+X$, V+jets (VBF-V), VV+jets (VBS-VV),

V+2 jets:





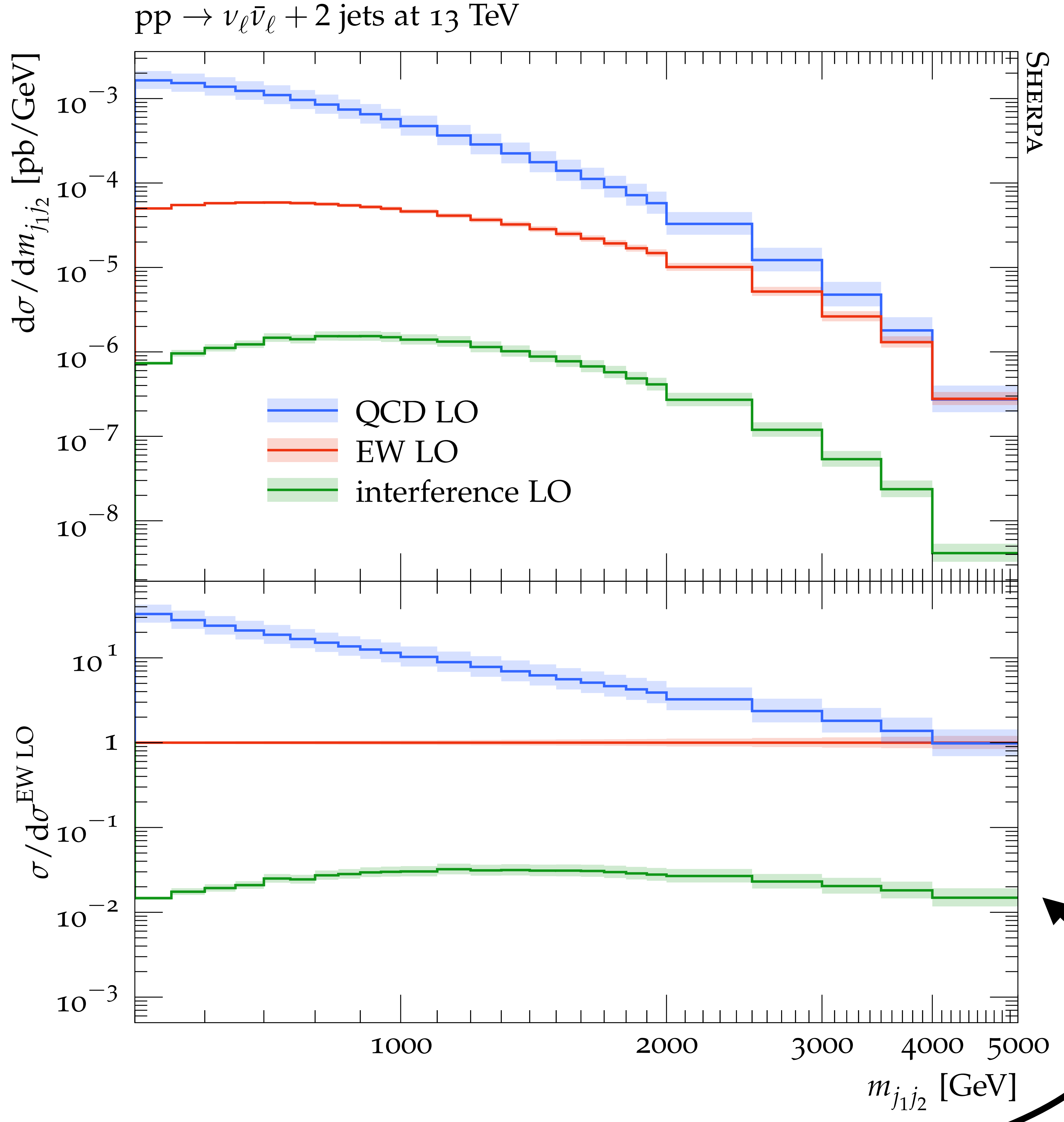
$$\mathrm{d}\sigma = \mathrm{d}\sigma(\alpha_S^2\alpha^2) + \mathrm{d}\sigma(\alpha_S\alpha^3) + \mathrm{d}\sigma(\alpha^4) + \ldots$$ LO

QCD-mode $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$ interference $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$ VBF-mode

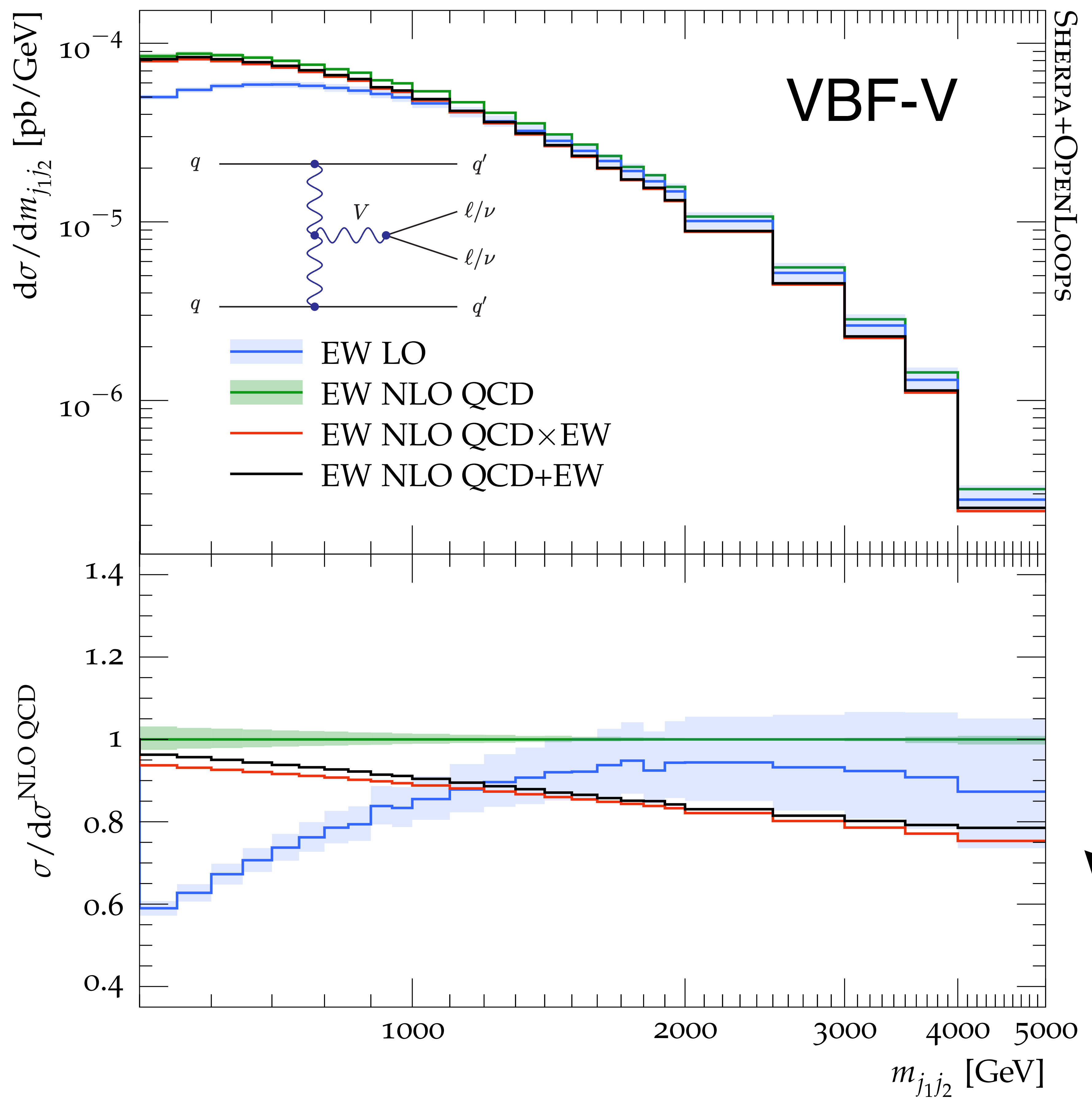
$$\cdots + \mathrm{d}\sigma(\alpha_S^3\alpha^2) + \mathrm{d}\sigma(\alpha_S^2\alpha^3) + \mathrm{d}\sigma(\alpha\alpha^4) + \sigma(\alpha^5)$$ NLO

"NLO QCD" "NLO EW" "NLO QCD" "NLO EW"

VBF-V @ NLO QCD + EW



[JML, Pozzorini, Schönherr, '22]



- If LO interference is small: possible to consider QCD and EW production modes as independent and factorise QCD and EW corrections to the respective processes
- Otherwise, still factorise but consider QCD+EW combination as nominal (and QCD×EW as uncertainty)

Conclusions

- Multiboson tails are becoming precision probes with often <~ 10% uncertainties
- EW uncertainties:
  - Higher-order Sudakov corrections: $\Delta_{\rm EW}^{\rm Sud} = \left(\delta_{\rm Sud}^{(1)}\right)^2$
  - Higher-order hard corrections: $\Delta_{\rm EW}^{\rm hard} \approx 1\%$
  - Higher-order QED radiation: $\Delta_{\rm EW}^{\rm QED} = |\delta_{\rm EW} - \delta_{\rm EW+PS/YFS}|$
- QCD-EW uncertainties:
  - Conservative: difference between add. and multipl. combination: $\Delta_{\rm QCD-EW} = \delta_{\rm QCD}\,\delta_{\rm EW}$
  - More aggressive: $\Delta_{\rm QCD-EW} = \delta_{\rm QCD}\,(\delta_{\rm EW}^{\rm SL} + \delta_{\rm EW}^{\rm hard})$ (applicable when $\delta_{\rm EW} \sim \delta_{\rm EW}^{\rm DL}$)
  - For processes subject to significant QCD radiation: $\Delta_{\rm QCD-EW}^{\rm multi-jet\ merged} = \delta_{\rm QCD}\,\delta_{\rm EW}$
  - X+j @ NLO EW proxy computations might allow for estimate of non-factorising effects
  - Factorisation feasible for processes with small interferences of tower of born orders
- Necessary tools are available:
  - NLO EW in MG5_aMC@NLO / Sherpa / POWHEG
  - NLL EW in Sherpa / MG5_aMC@NLO / OpenLoops
  - NLOPS EW in POWHEG / MEPS NLO EW + YFS in Sherpa

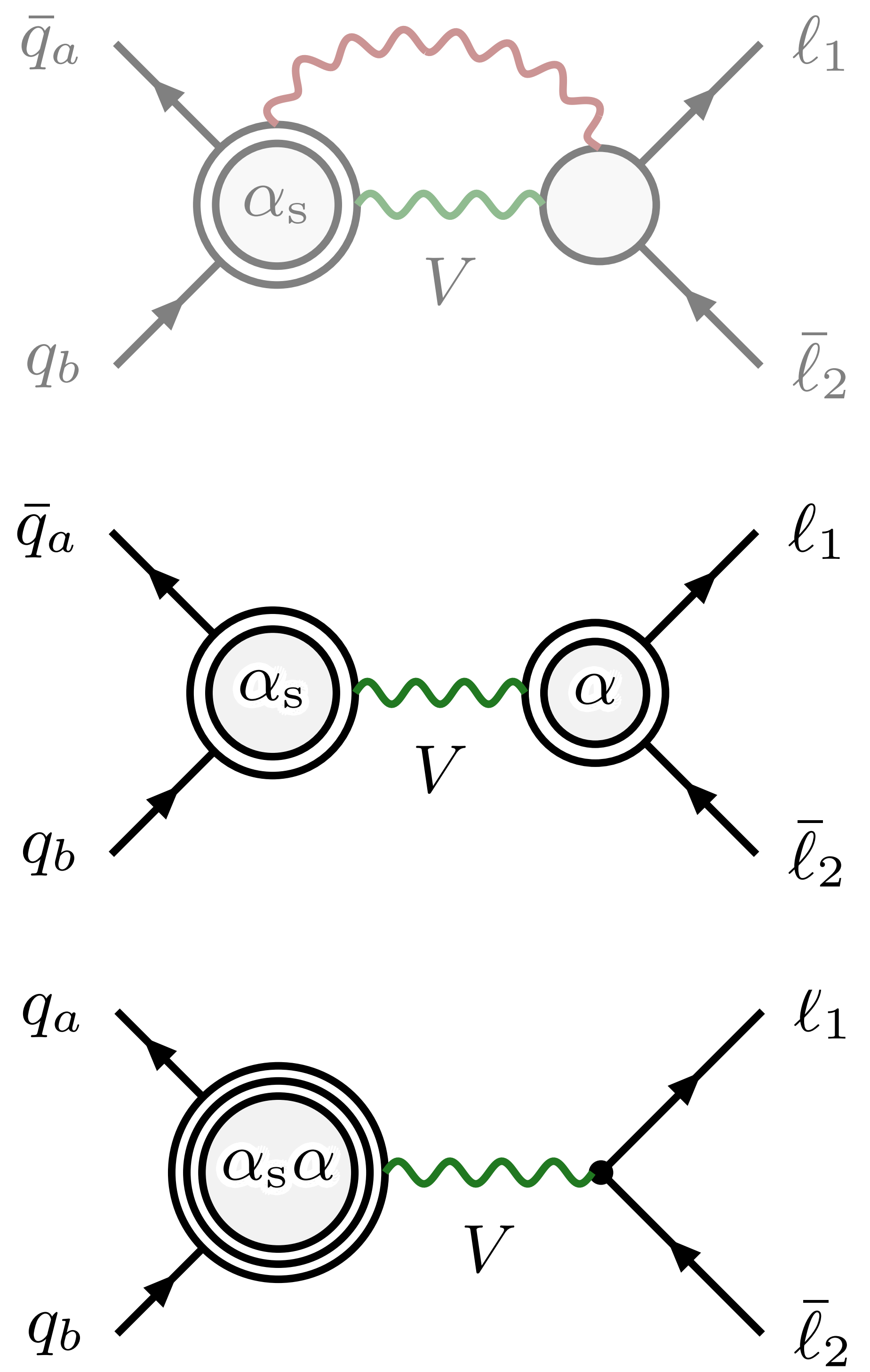
- Plan for recommendation document from WG3 agreed amongst different theorists
- See also:
  - *Electroweak Radiative Corrections for Collider Physics* (Denner & Dittmaier): 1912.06823
  - Les Houches 2023: 2406.00708

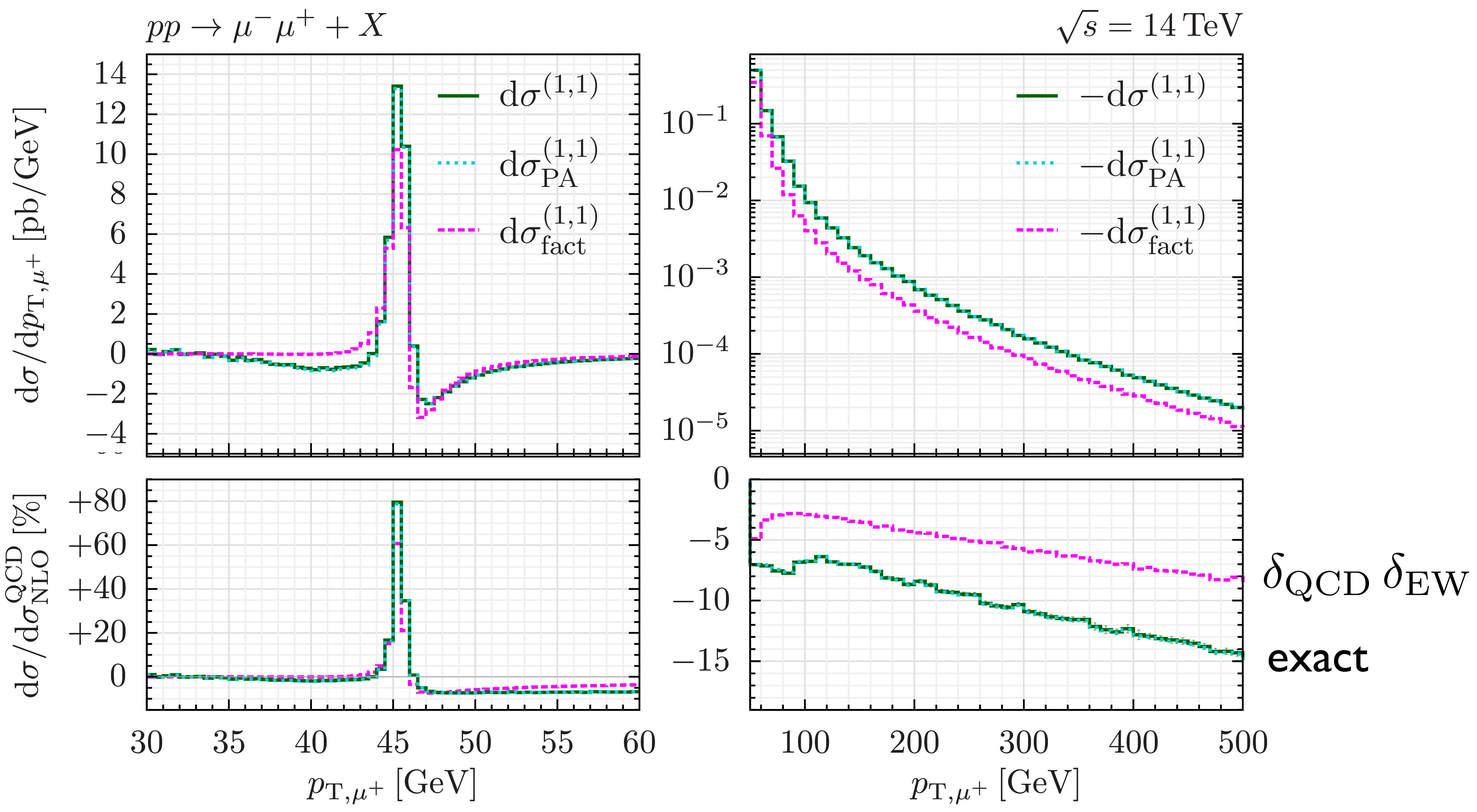
Backup

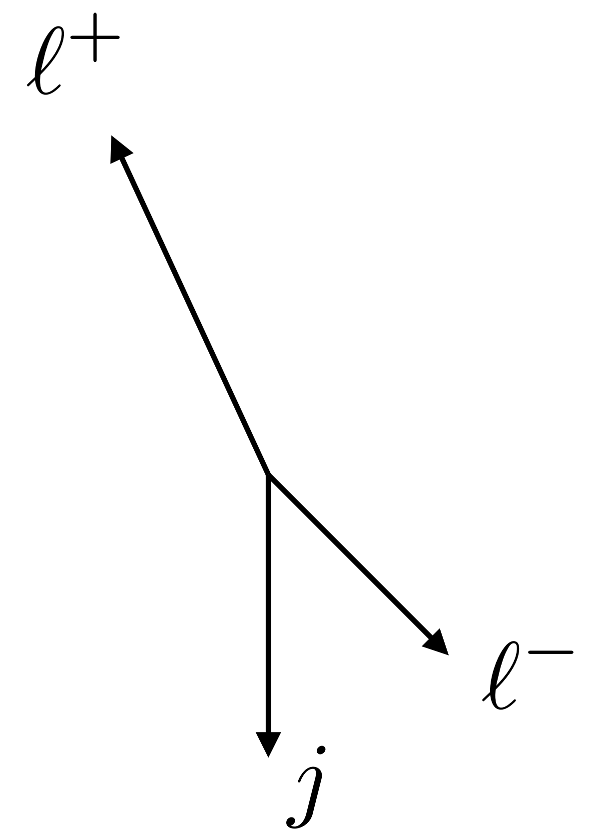
Exact mixed QCD-EW for DY

[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch, '20]
[Behring, Buccioni, Caola, et. al. '20]

[Bonciani, Buonocore, Grazzini, Kallweit et. al. 2 x '21]



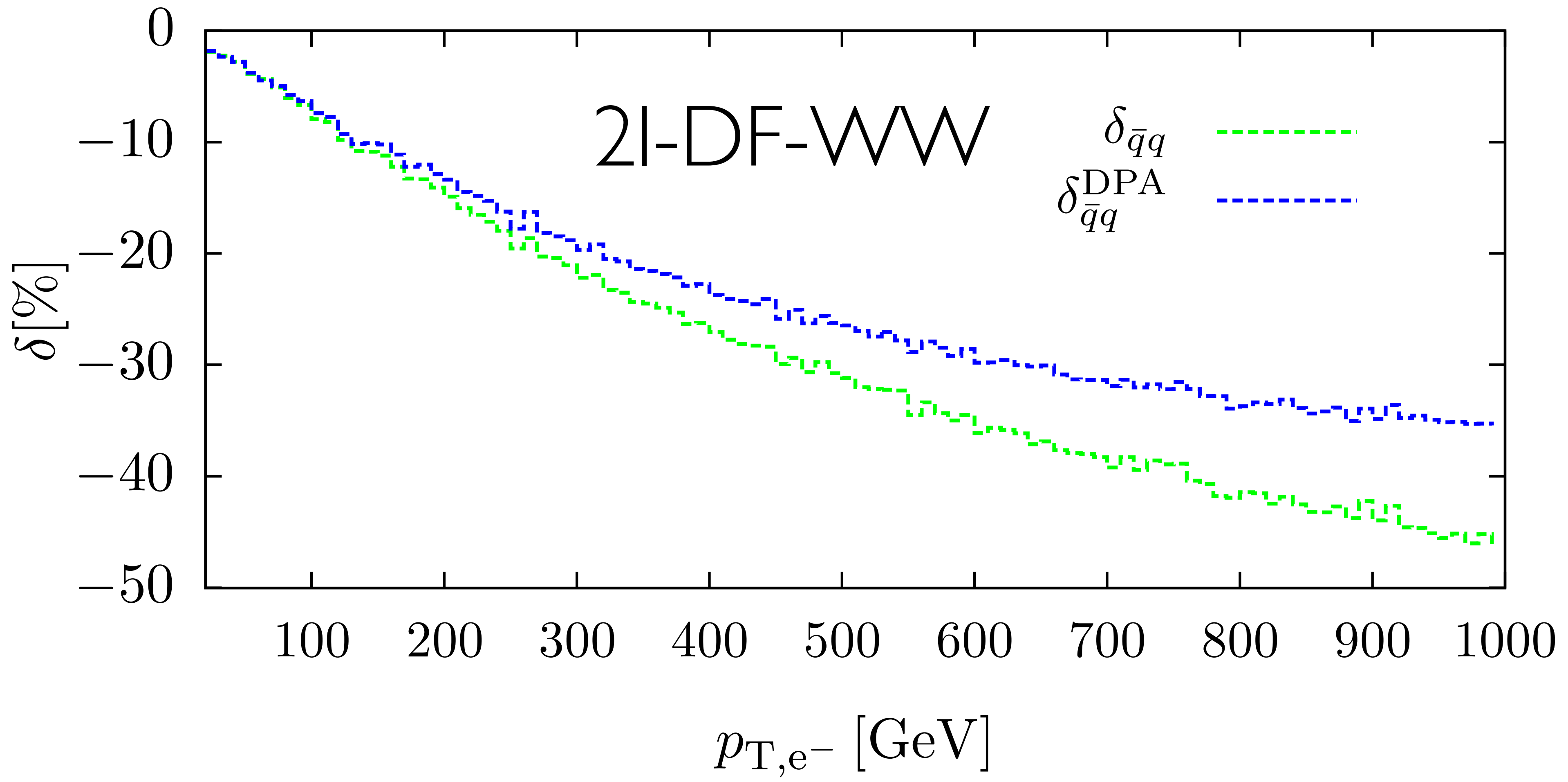


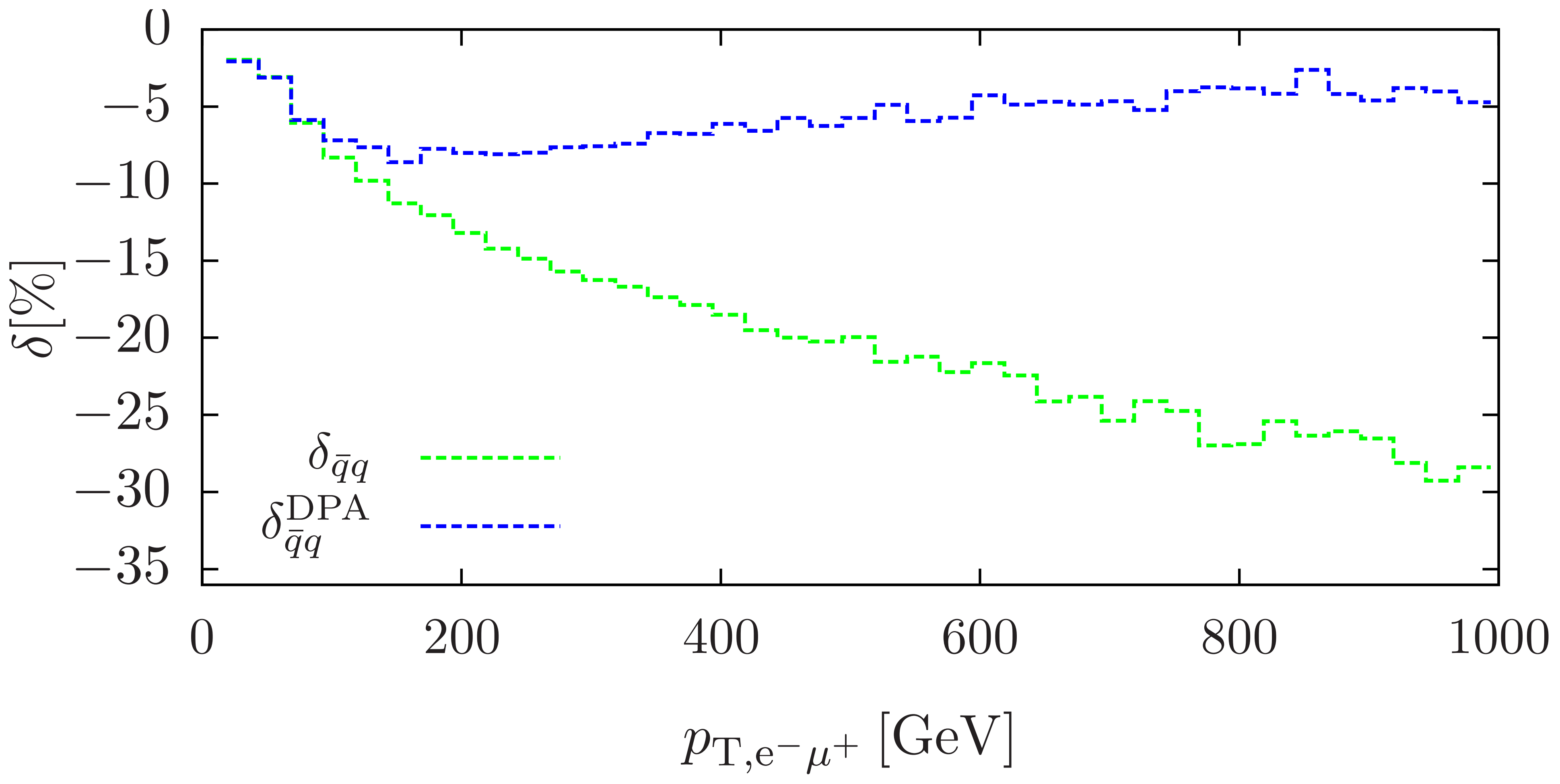


- pole approximation vs. full computation: agree below the percent level
- Comparison against naive factorised NLO QCD x NLO EW ansatz: fail at the 5-10% level
- At large p_{T,μ^+} in DY: sizeable contributions from $pp \to Vj$ which receives larger EW corrections

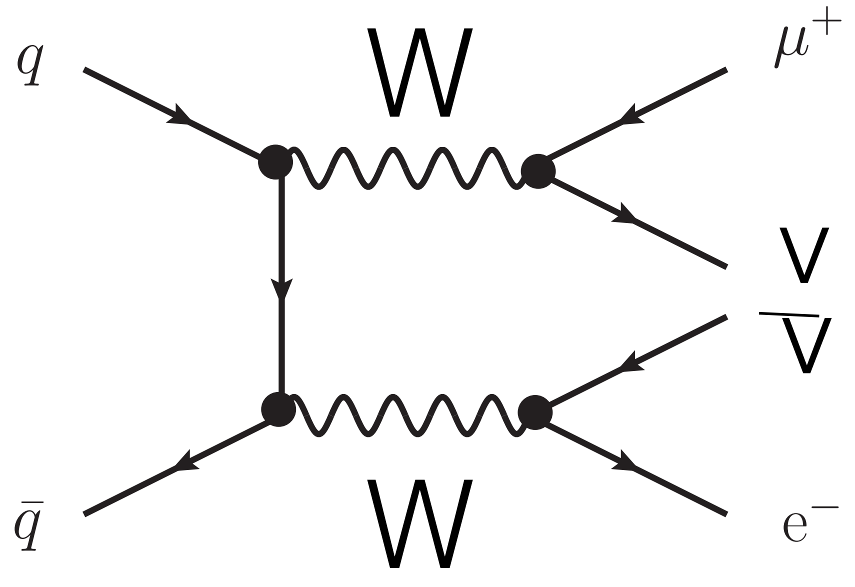
The need for off-shell computations: VV

[Biedermann, M. Billoni, A. Denner, S. Dittmaier, L. Hofer, B. Jäger, L. Salfelder ;'16]

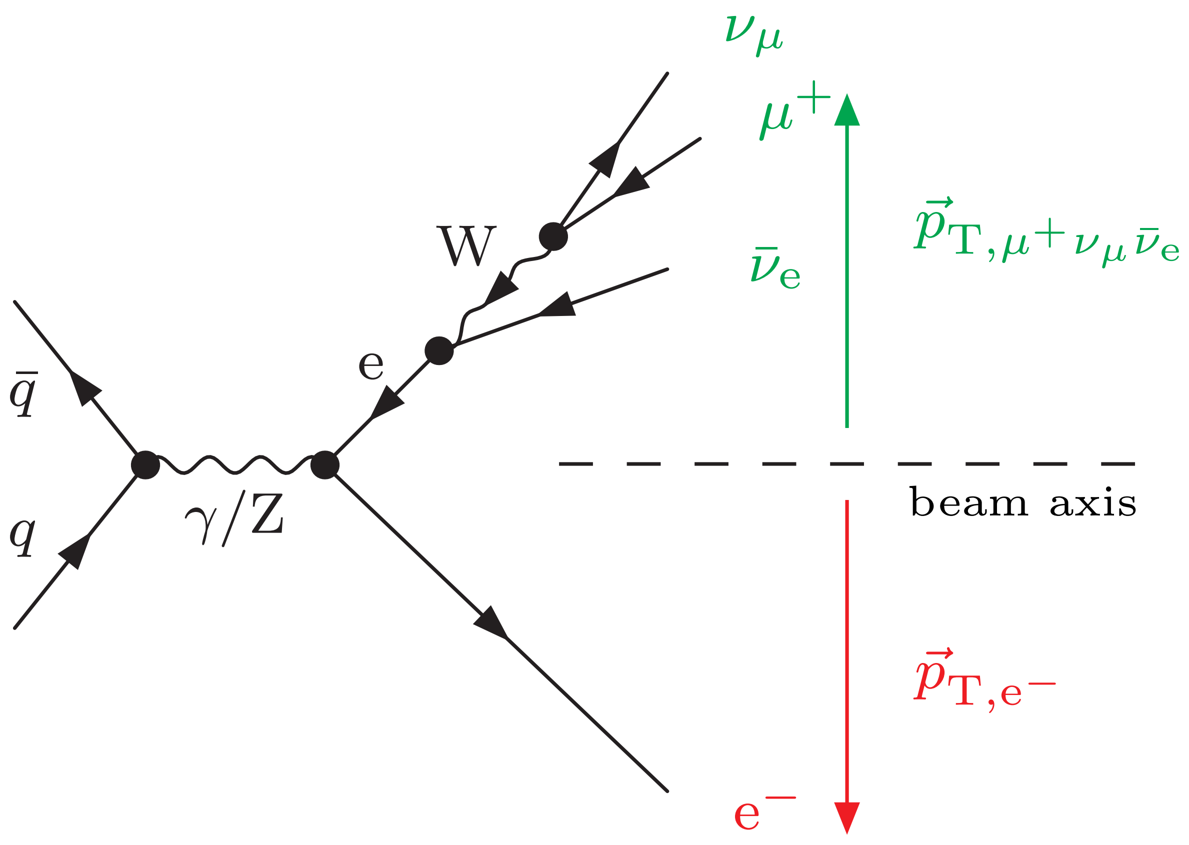




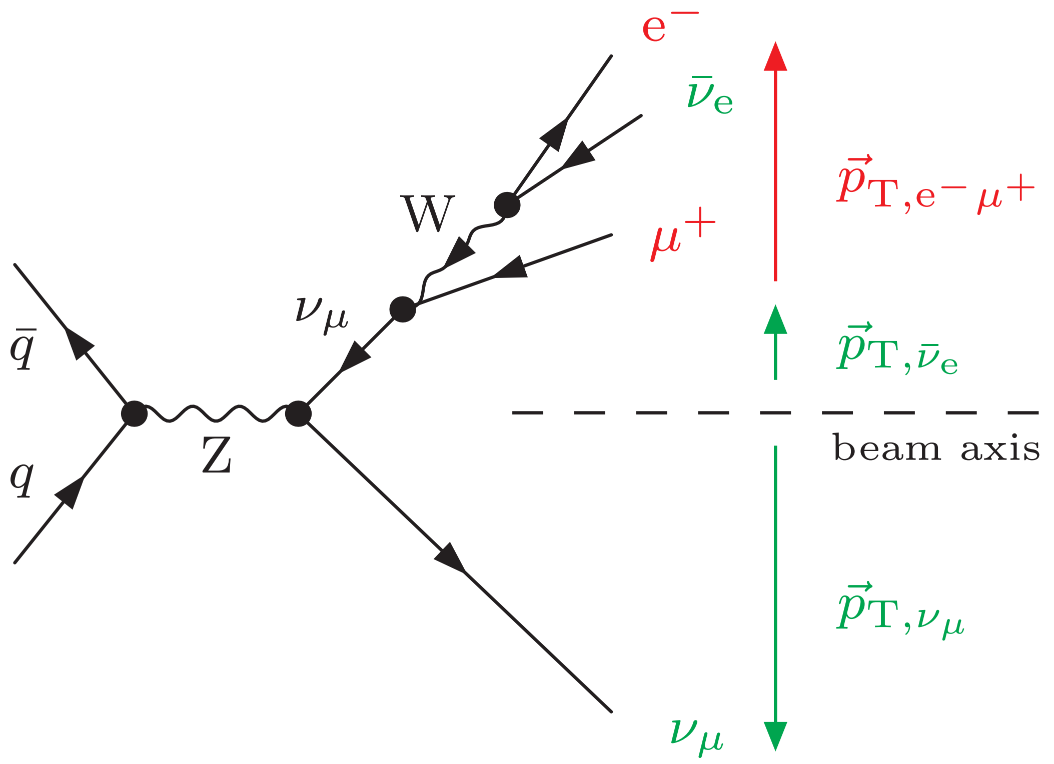
$$pp \to V(\to \ell\bar{\ell})V'(\to \ell'\bar{\ell}')$$



VS.



$$pp \to \ell\bar{\ell}\ell'\bar{\ell}'$$



➡ sizeable differences in fully off-shell vs. double-pole approximation in tails

Perturbative expansion

aMC@NLO, Sherpa, Herwig… &
Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, MCFM, NNLOjet, …

$$d\sigma = d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD — NLO EW

$$+\alpha_S^2\, d\sigma_{\mathrm{NNLO}} + \alpha_{\mathrm{EW}}^2\, d\sigma_{\mathrm{NNLO\,EW}}{}^{?} + \alpha_S \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NNLO\,QCDxEW}}{}^{?}$$

NNLO QCD — NNLO EW — NNLO QCD-EW

$$+\alpha_S^3\, d\sigma_{\mathrm{N3LO}}{}^{?} + \ldots$$

N3LO QCD

scale variation at NNLO

scheme variation, e.g. Gmu vs. a(mZ)

+

in case of EW Sudakov dominance: exponentiation

NLO QCD + EW
vs.
NLO QCD x EW

Combination of QCD and EW corrections

- full calculations of $\mathcal{O}(\alpha\alpha_s)$ out of reach
- Approximate combination: MEPS@NLO including (approximate) EW corrections
- key: QCD radiation receives EW corrections!
- strategy: modify MC@NLO B-function to include NLO EW virtual corrections and integrated approx. real corrections = VI

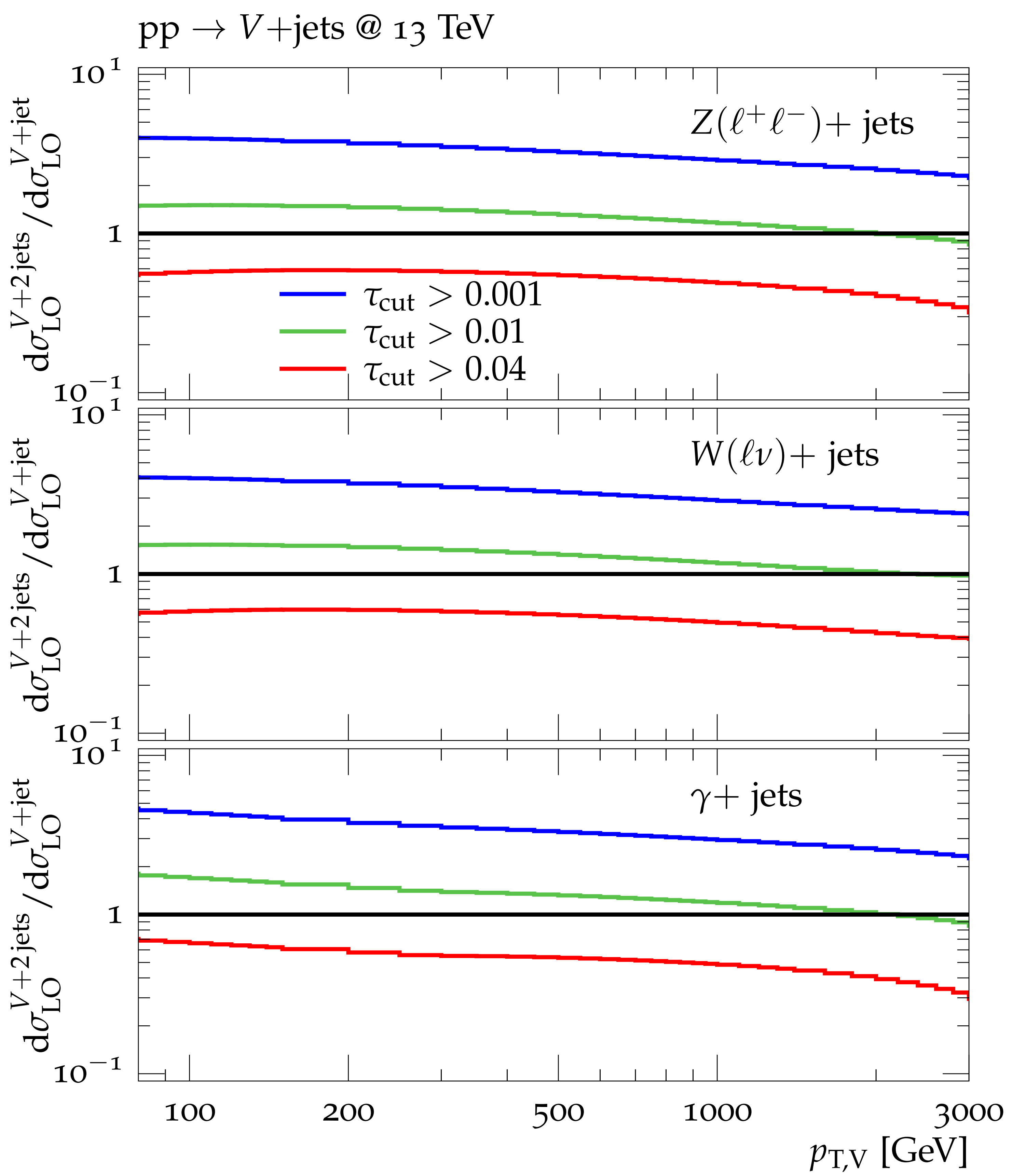
$$\overline{\mathrm{B}}_{n,\mathrm{QCD+EW_{virt}}}(\Phi_n) = \overline{\mathrm{B}}_{n,\mathrm{QCD}}(\Phi_n) + \mathrm{V}_{n,\mathrm{EW}}(\Phi_n) + \mathrm{I}_{n,\mathrm{EW}}(\Phi_n)$$

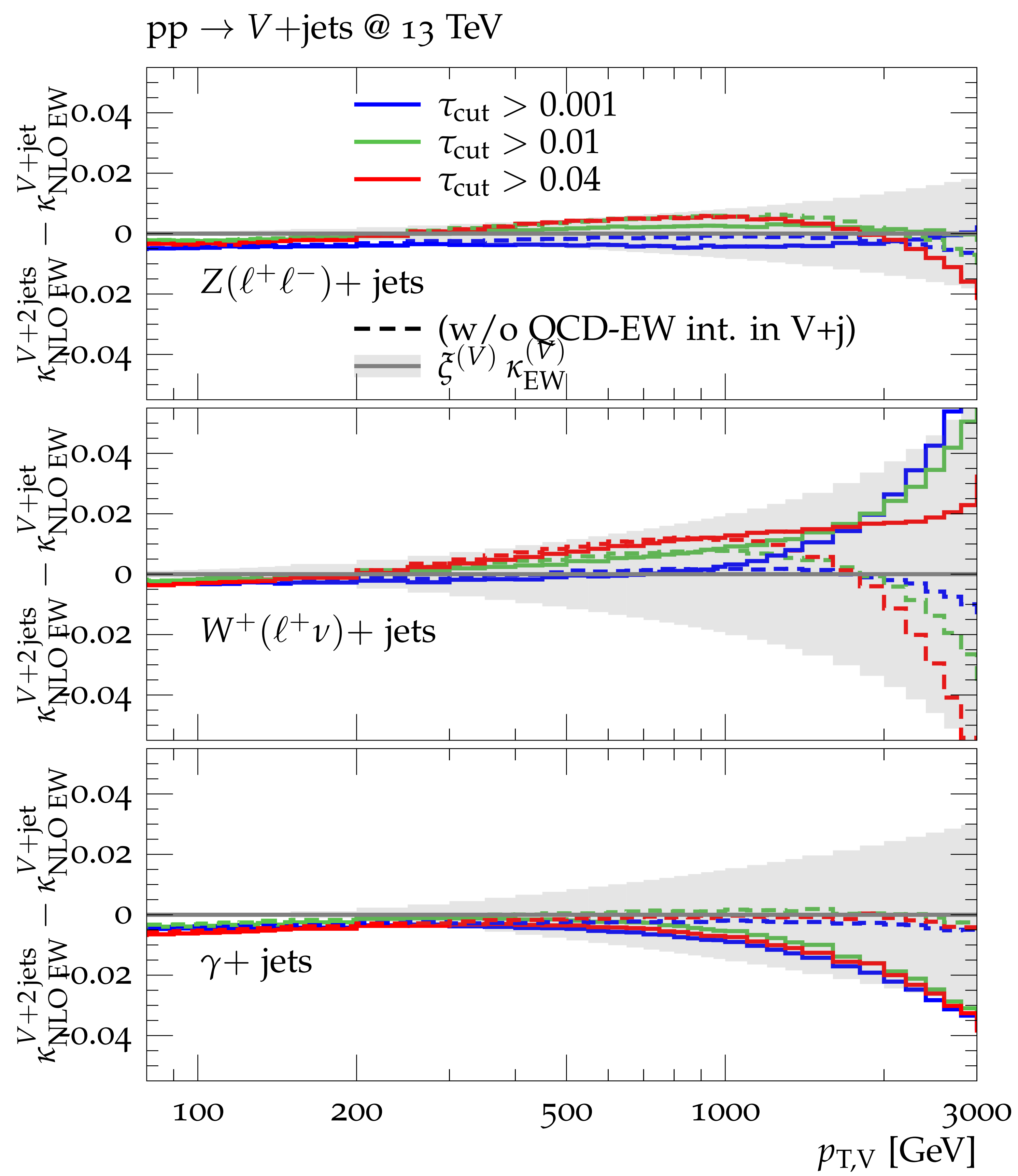
exact virtual contribution (→ $\mathrm{V}_{n,\mathrm{EW}}$)

approximate integrated real contribution (→ $\mathrm{I}_{n,\mathrm{EW}}$)

Mixed QCD-EW uncertainties

Estimate of non-factorising contributions





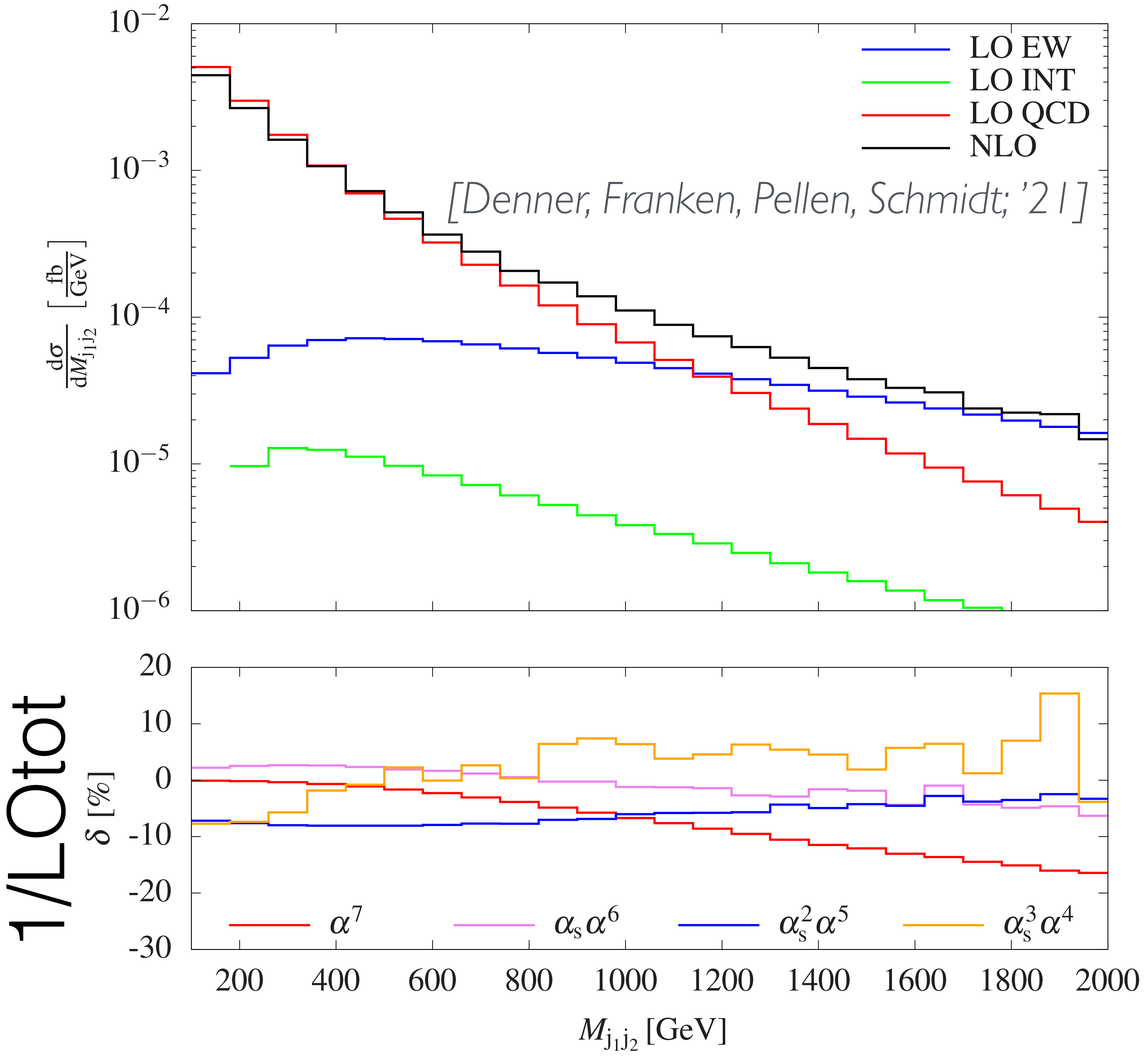
N-jettiness cut ensures approx. constant ratio V+2jets/V+jet

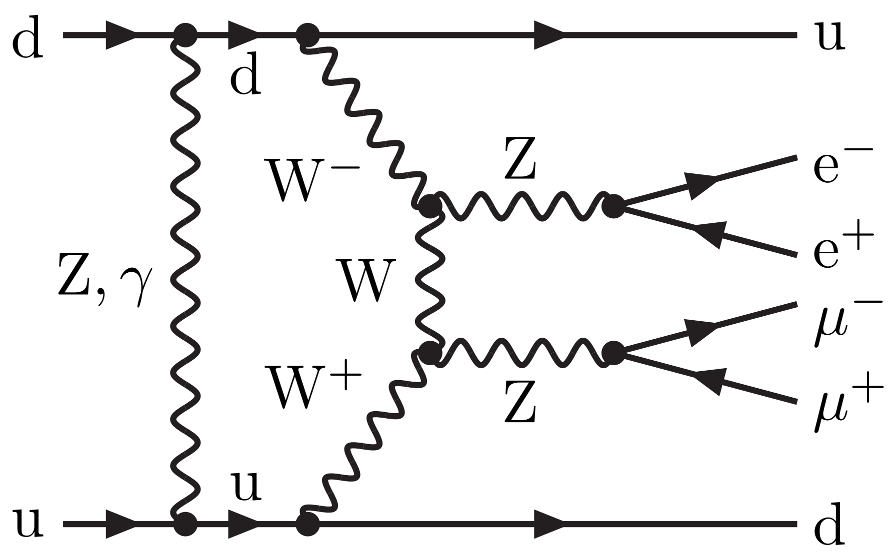
$$\tau_1 = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i \sqrt{\hat{s}}} \right\}$$

VBS @ NLO QCD + EW

- QCD and EW **ss-WWjj** at NLO QCD+EW: *[Biedermann, Denner, Pellen '16+'17]*
- EW **WZjj** at NLO QCD+EW: *[Denner, Dittmaier, Maierhöfer, Pellen, Schwan, '19]*
- QCD and EW **ZZjj** at NLO QCD+EW: *[Denner, Franken, Pellen, Schmidt, '20+'21]*
- EW **WWjj** at NLO QCD+EW: *[Denner, Franken, Schmidt, Schwan, '22]*

EW ZZ+2jets @ NLO QCD + EW





- 2 → 6 particles at NLO EW !

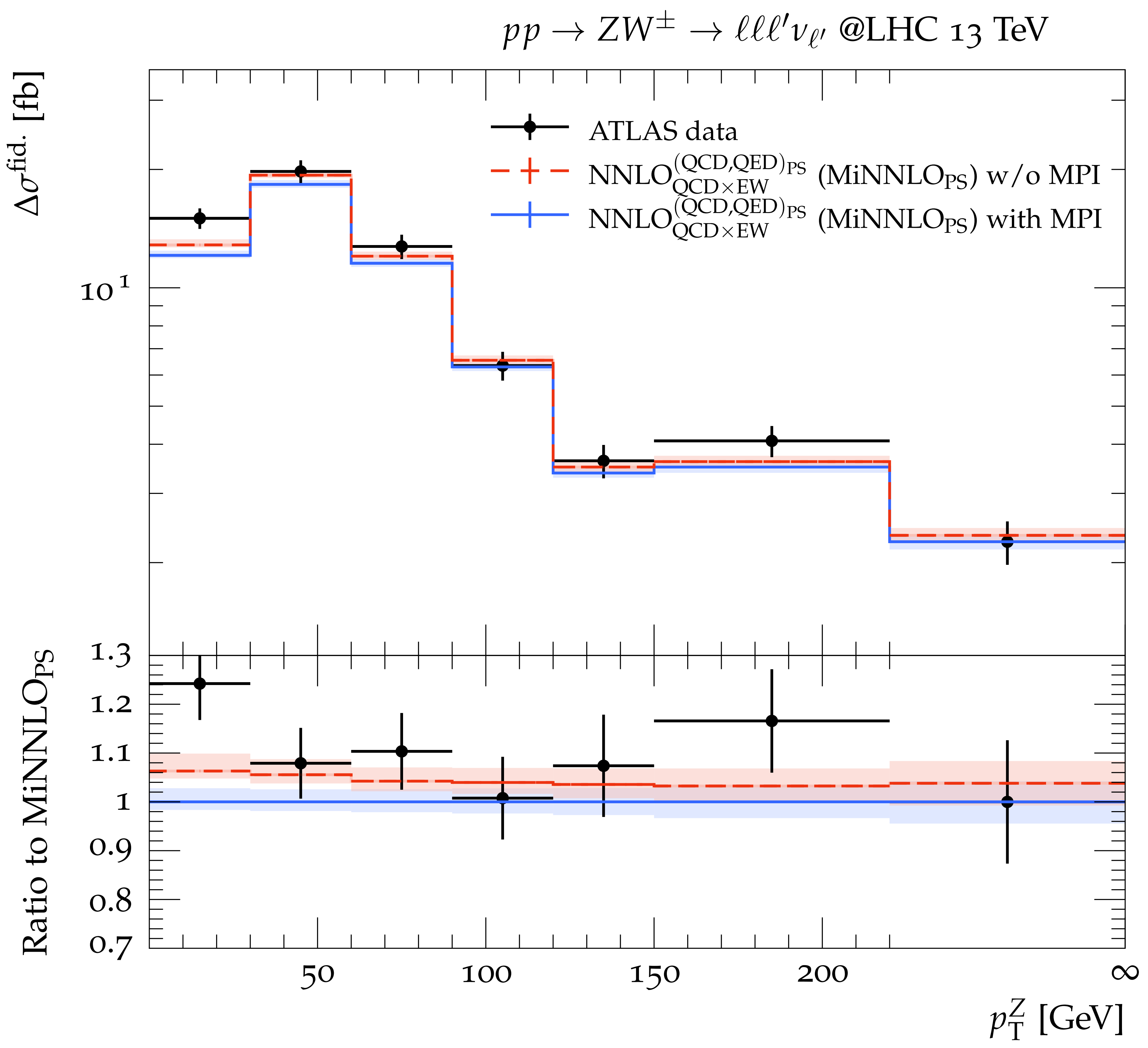
Order	$\mathcal{O}(\alpha^6)+\mathcal{O}(\alpha^7)$	$\mathcal{O}(\alpha^6)+\mathcal{O}(\alpha_s\alpha^6)$	$\mathcal{O}(\alpha^6)+\mathcal{O}(\alpha^7)+\mathcal{O}(\alpha_s\alpha^6)$
$M_{j_1j_2} > 500$ GeV			
σ_{NLO}[fb]	0.06069(4)	0.07375(25)	0.06077(25)
δ[%]	−17.6	0.1	−17.5

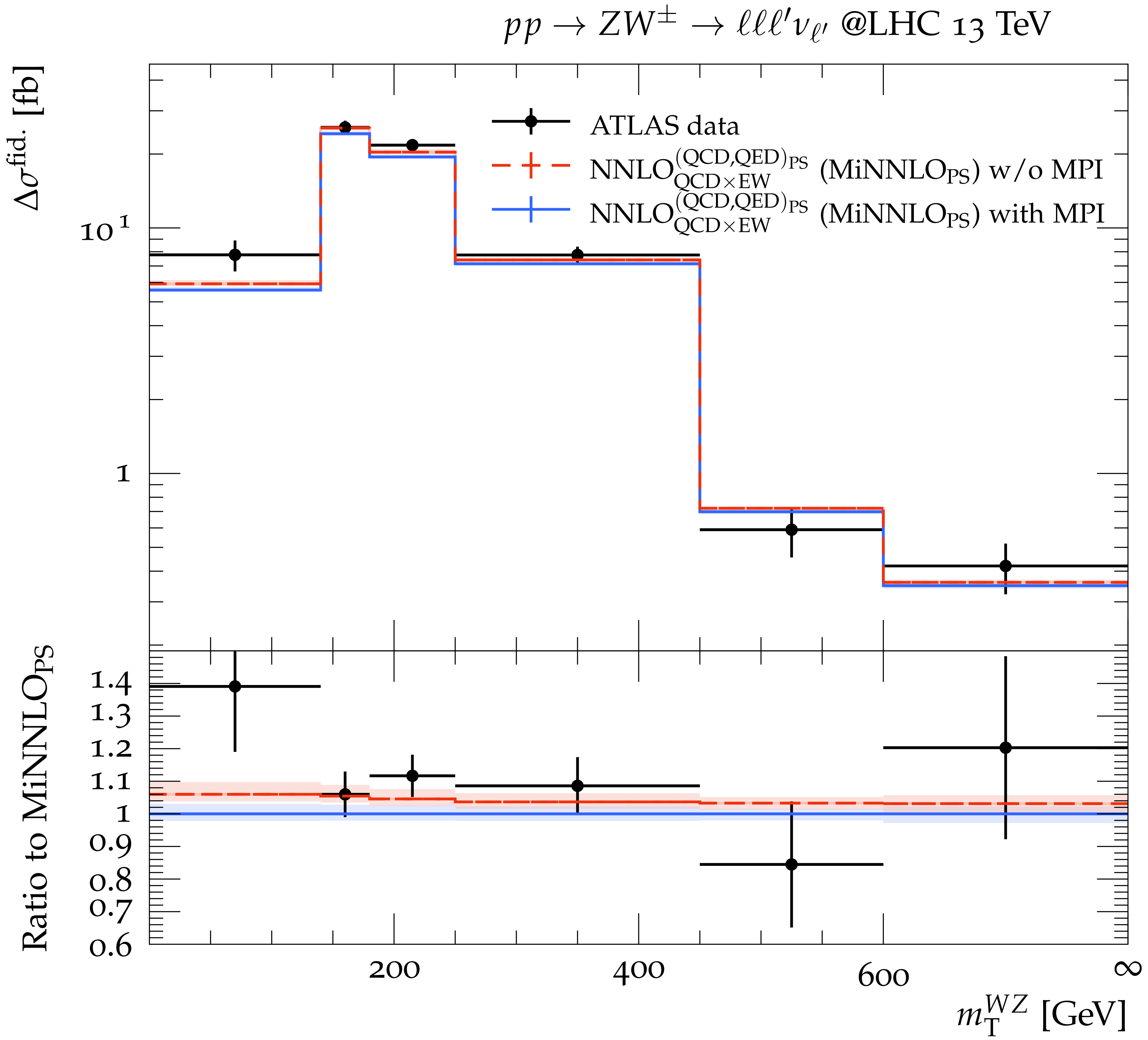
- In the VBS phase-space EW mode receives:
  - very small QCD corrections (percent level)
  - O(20%) EW corrections
- Always measure also combined QCD-mode + EW-mode fiducial xsections!

MiNNLOPS QCD + NLOPS EW

[JML, Lombardi, Wiesemann, Zanderighi, Zanoli, '22]

for NLOPS QCD + EW also *[Chiesa, Re, Oleari '20]*





[JML, Lombardi, Wiesemann, Zanderighi, Zanoli, '22]

- Percent level precision in MiNNLOPS QCD + NLOPS EW predictions