MSHT Updates and Perspective

Lucian Harland-Lang, University College London

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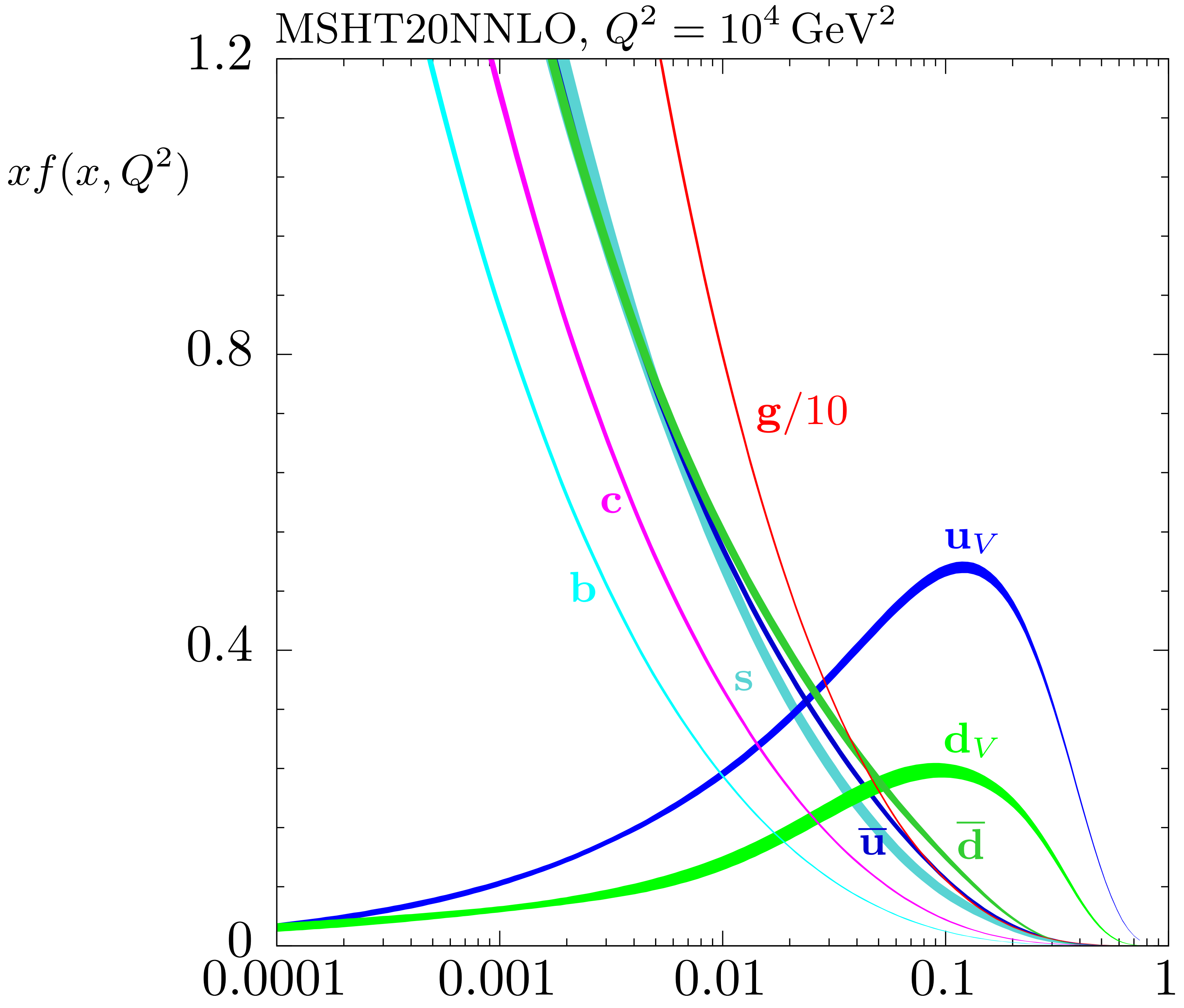
With Robert Thorne and Tom Cridge

MSHT PDFs

- The 'Post-Run I' set from the MSTW, MMHT… group: **MSHT20.**
- Focus on including significant amount of **new data**, higher **precision theory** and on **methodological improvements**.
- Although no official NNLO release since MSHT20, we have been busy! Recent highlights:



★ First global aN3LO PDF analysis. **J. McGowan et al., arXiv: 2207.04739**

★ First global QED and aN3LO PDF analysis. **T. Cridge, LHL, R. Thorne, arXiv: 2312.07665**

★ First global determination of strong coupling at aN3LO. **T. Cridge, LHL, R. Thorne, arXiv: 2404.02964**

★ First closure test of fixed parameterisation & direction comparison to Neural Net approach. **LHL et al., In preparation**

- No time to go through all of these, but will mention some and address some more general questions as well (w.r.t. profiling in particular).

Stress Testing the MSHT Approach

LHL et al., to appear!

Understanding the Fitting Methodology

- Two distinct methodologies on the market to parameterising PDFs: **Neural Nets** (NNPDF) or **Explicit Parameterisation** (CT, MSHT).

✦ **MSHT**: **52** free parameters in terms of Chebyshev polynomials.

$$f_i(x,Q_0):\ A_f x^{a_f}(1-x)^{b_f}\times \longrightarrow \sum_{i=1}^{n}\alpha_{f,i}P_i(y(x))\text{ , CT, MSHT...}$$

$$\searrow\ \mathrm{NN}_i(x)\quad \text{NNPDF}$$

$$u_V(x,Q_0^2)=A_u(1-x)^{\eta_u}x^{\delta_u}\left(1+\sum_{i=1}^{6}a_{u,i}T_i(y(x))\right)$$

$$d_V(x,Q_0^2)=A_d(1-x)^{\eta_d}x^{\delta_d}\left(1+\sum_{i=1}^{6}a_{d,i}T_i(y(x))\right)$$

$$S(x,Q_0^2)=A_S(1-x)^{\eta_S}x^{\delta_S}\left(1+\sum_{i=1}^{6}a_{S,i}T_i(y(x))\right)$$

$$s_+(x,Q_0^2)=A_{s_+}(1-x)^{\eta_{s+}}x^{\delta_S}\left(1+\sum_{i=1}^{6}a_{s_+,i}T_i(y(x))\right)$$

$$g(x,Q_0^2)=A_g(1-x)^{\eta_g}x^{\delta_g}\left(1+\sum_{i=1}^{4}a_{g,i}T_i(y(x))\right)+A_{g-}(1-x)^{\eta_{g-}}x^{\delta_{g-}}$$

$$s_-(x,Q_0^2)=A_{s_-}(1-x)^{\eta_{s-}}(1-x/x_0)x^{\delta_{s-}}$$

$$(\bar d/\bar u)(x,Q_0^2)=A_\rho(1-x)^{\eta_\rho}\left(1+\sum_{i=1}^{6}a_{\rho,i}T_i(y(x))\right)$$

A.D. Martin et al., arXiv: 1211.1215

✦ **NNPDF**: neural net with (in principle) many more free parameters.

- Two important questions to address here:

★ Is such a fixed **parameterisation flexible** enough for LHC precision physics requirements?

★ Are the **PDF uncertainties** appropriate?

PDF comparison

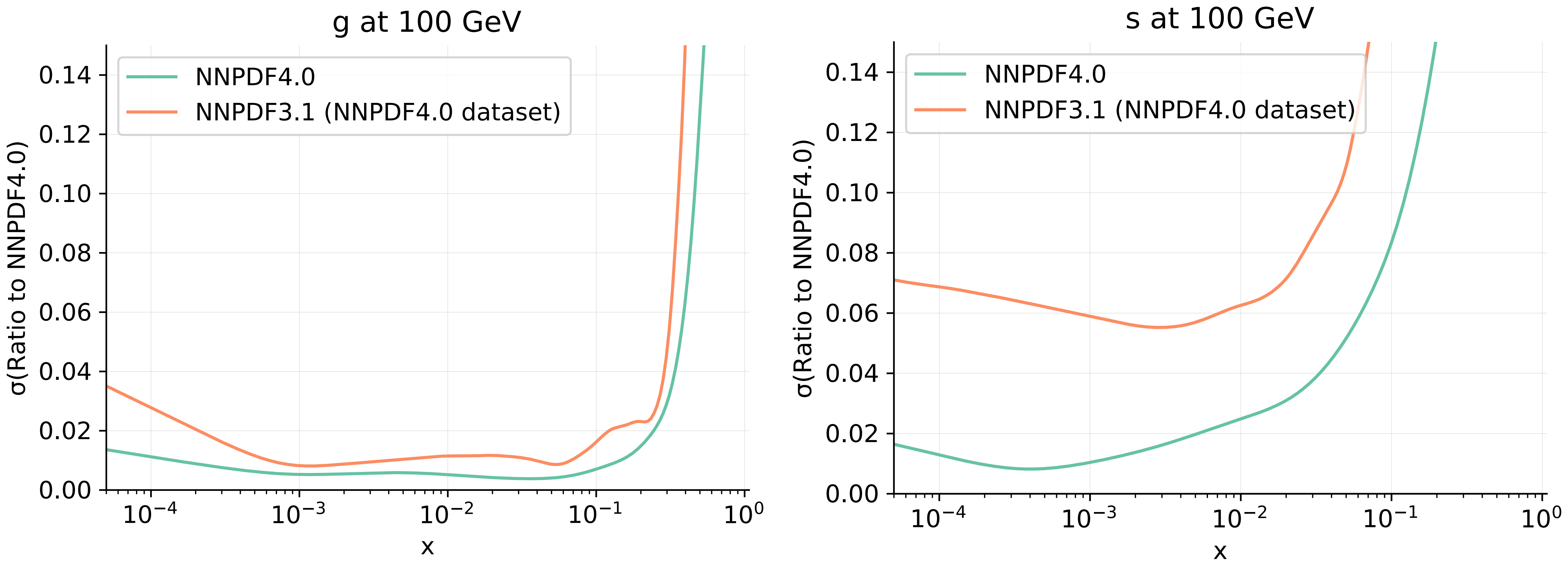
- Comparison/benchmarking of PDFs considered in PDF4LHC21 study. Fit representative subset of global dataset, unified between 3 global fits and with very close theory settings. Find:

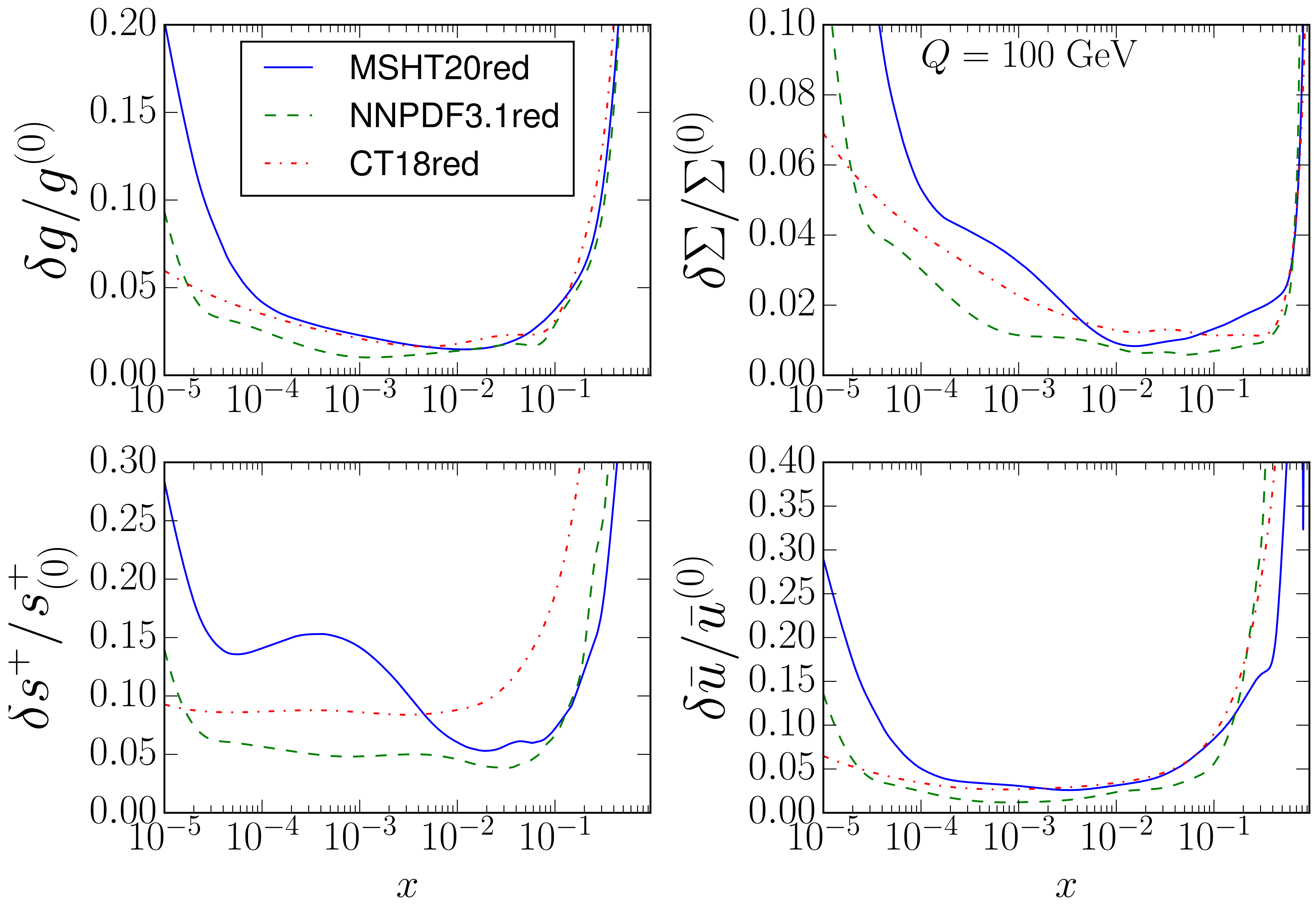
PDF4LHC21, arXiv:2203.05506

  - ✦ Global fits give different errors in PDF4LHC21 benchmarking. NNPDF3.1 in general **smaller errors.**

Benchmark = similar data/settings

- ✦ And **4.0 methodology** gives further errors reduction.





- Different methodologies giving different results. Understanding this clearly important!

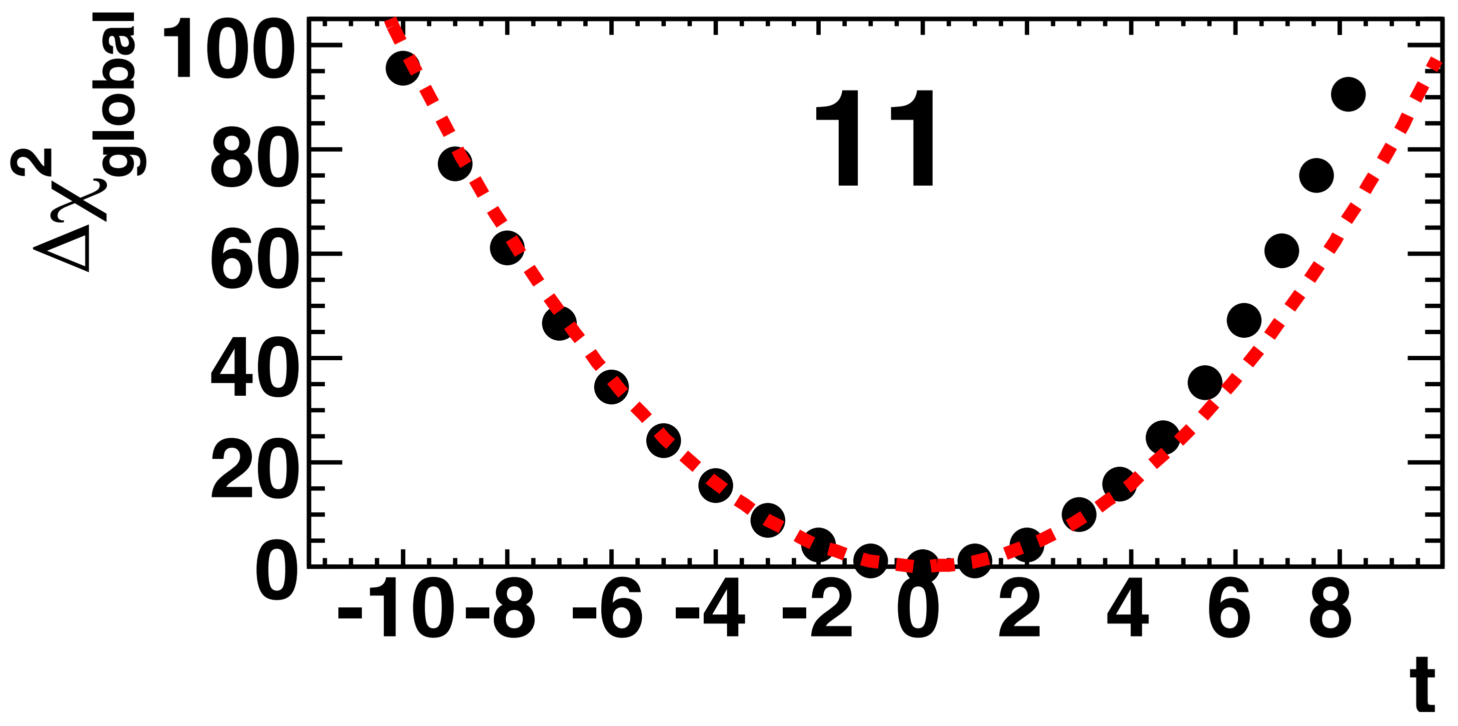
MSHT PDF Uncertainty

$$\chi^2_{\rm global} \sim \frac{(D_{\rm ata} - T_{\rm heory})^2}{\sigma^2}$$

- Find global minimum of χ^2 and evaluate eigenvectors of Hessian matrix at this point.
- Parameter shifts corresponding to given $\Delta\chi^2$ criteria given in terms of these

$$H_{ij} = \frac{1}{2}\frac{\partial^2 \chi^2_{\rm global}}{\partial a_i \partial a_j}\Big|_{\rm min}$$

$$a_i(S_k^\pm) = a_i^0 \pm t\, e_{ik}, \qquad \text{with } t \text{ adjusted to give desired } T = \Delta\chi^2_{\rm global}$$



- $T = 1$: `textbook' criterion for 68% C.L., would apply if:
  - ⋆ Complete statistical compatibility between multiple datasets entering fit.
  - ⋆ Completely faithful evaluation of experimental uncertainties within each dataset.
  - ⋆ Theoretical calculations that match these exactly.

- $T = 1$: `textbook' criterion for 68% C.L., would apply if:

Fixed target, DIS, Tevatron, LHC

  ★ Complete statistical compatibility between multiple datasets entering fit. $N_{\rm dataset} \sim 50 - 60$

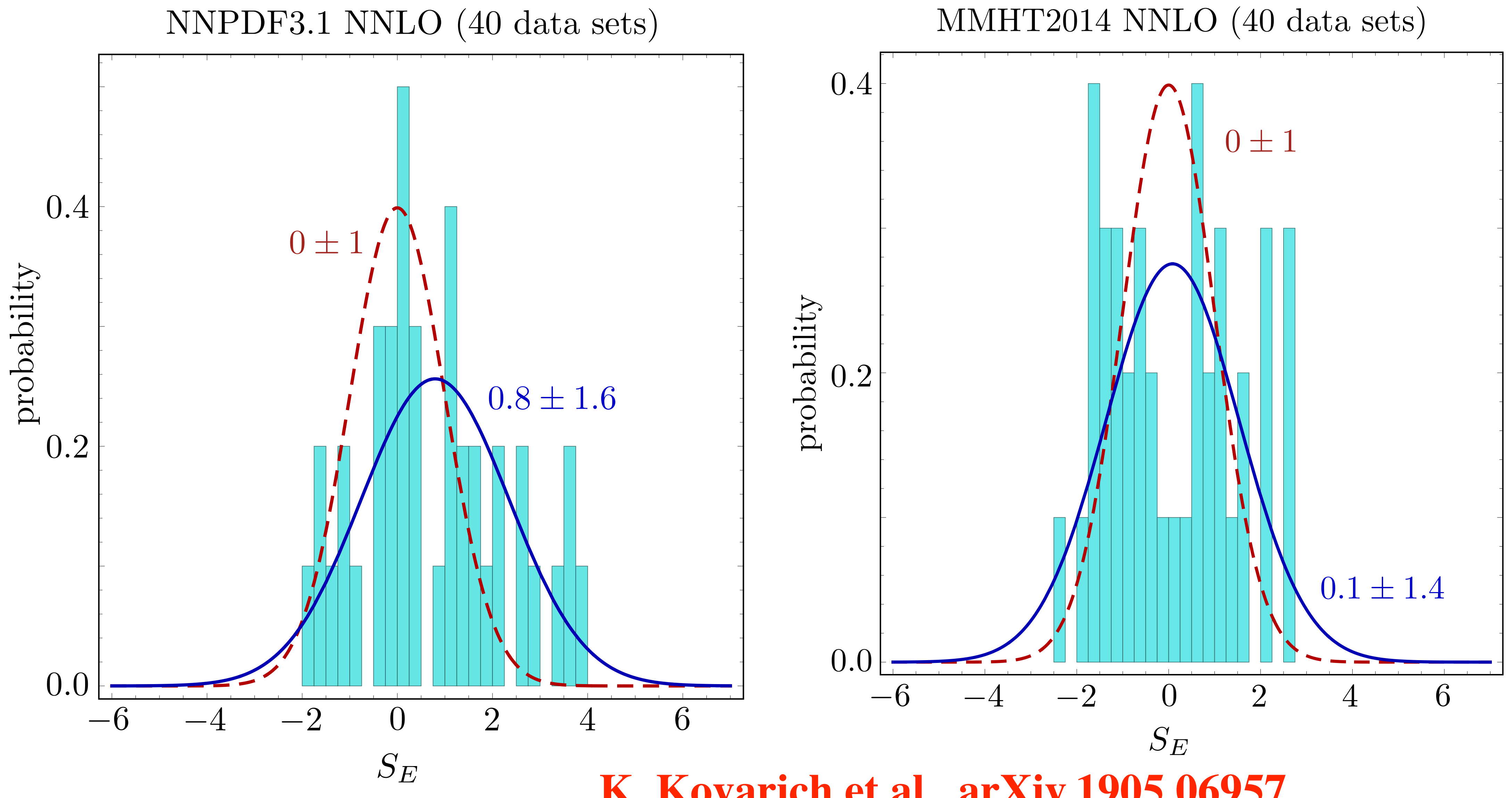
  ★ Completely faithful evaluation of experimental uncertainties within each dataset.

  ★ Theoretical calculations that match these exactly.

- Good evidence that first two points do not always hold, while last point known not be true (though progress towards missing higher order uncertainties made).

G. Watt and R. Thorne, arXiv:1205.4024 M. Yan et al., arXiv.2406.01664
J. Pumplin, arXiv:0909.0268

- Further evidence of this as part of our new study (**Backup**).



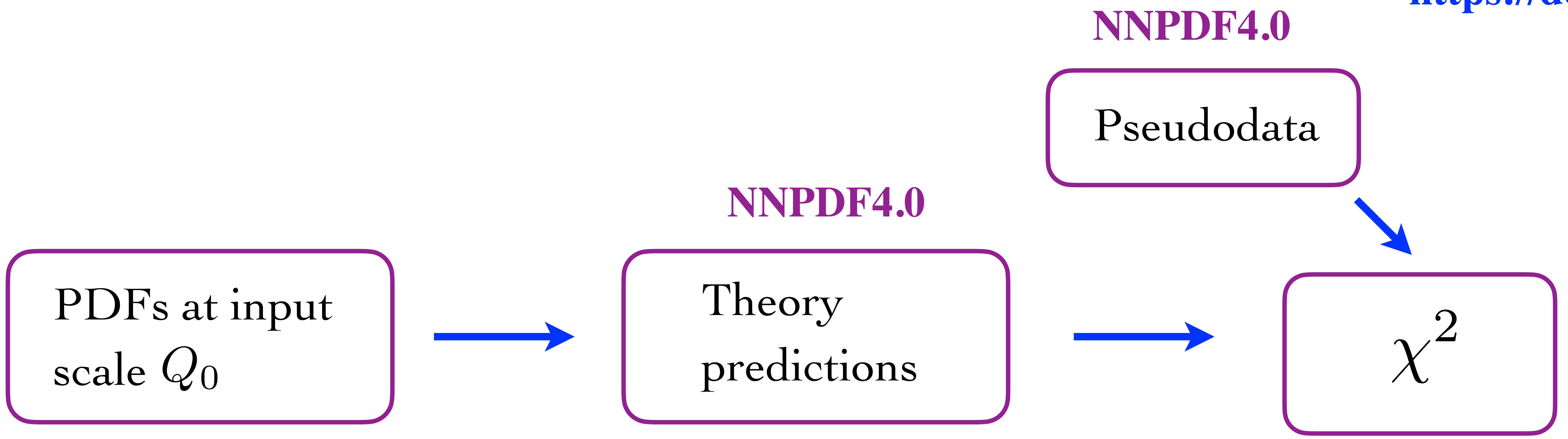
$N_{\rm pts} \sim 4000 - 5000$ **K. Kovarich et al., arXiv.1905.06957**

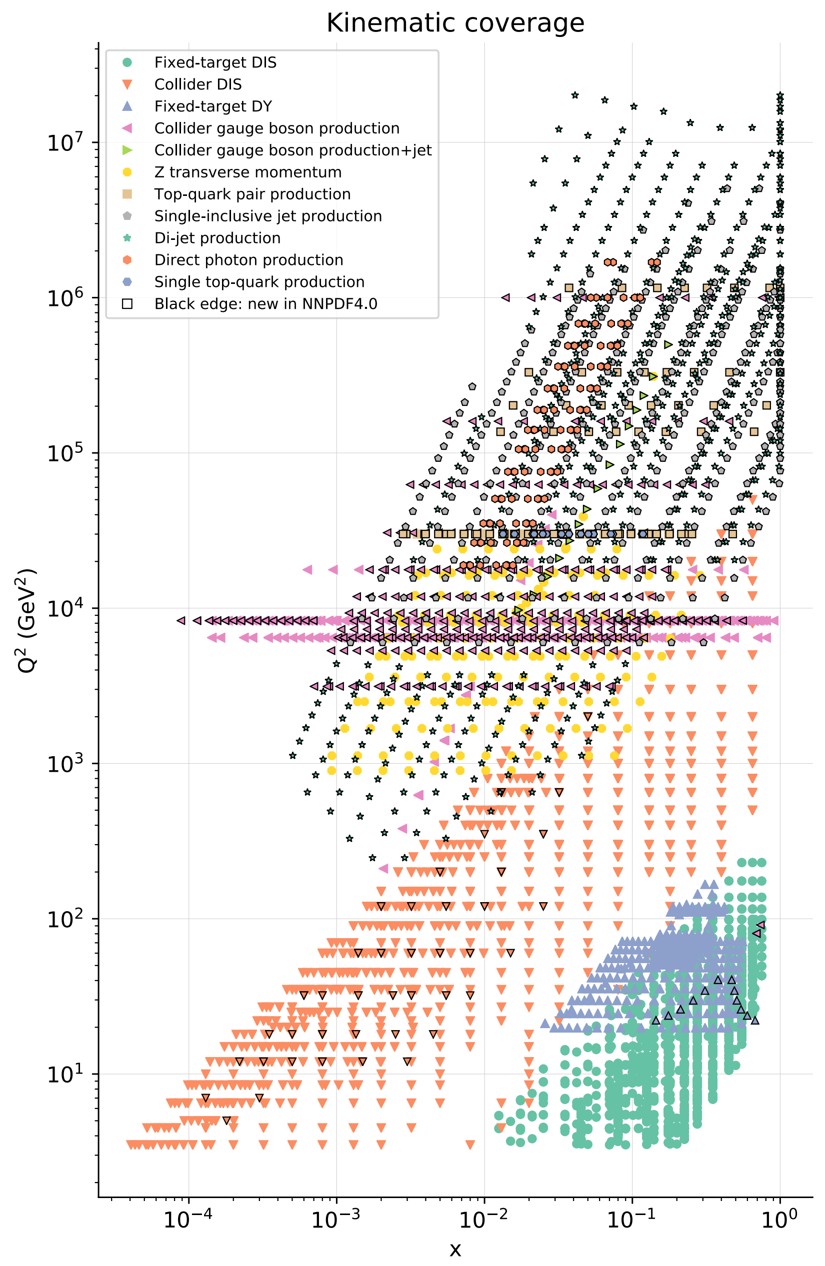
- Given complete statistical compatibility, global PDF fit very constraining. Danger is claimed (high) precision will increasingly not match accuracy with $T = 1$. Motivates enlarged tolerance $T > 1$ (more later).
- Equally possible that parameterisation inflexibility may require this. Does it? To see we will present results of 'closure tests'…

Global Closure Test

- Global Closure Test: generate pseudodata corresponding to global dataset with a particular input PDF set and perform usual MSHT fit to this. Then determine how faithfully underlying input is reproduced.
- To do this we will make use of publicly available NNPDF fitting code.

https://docs.nnpdf.science/



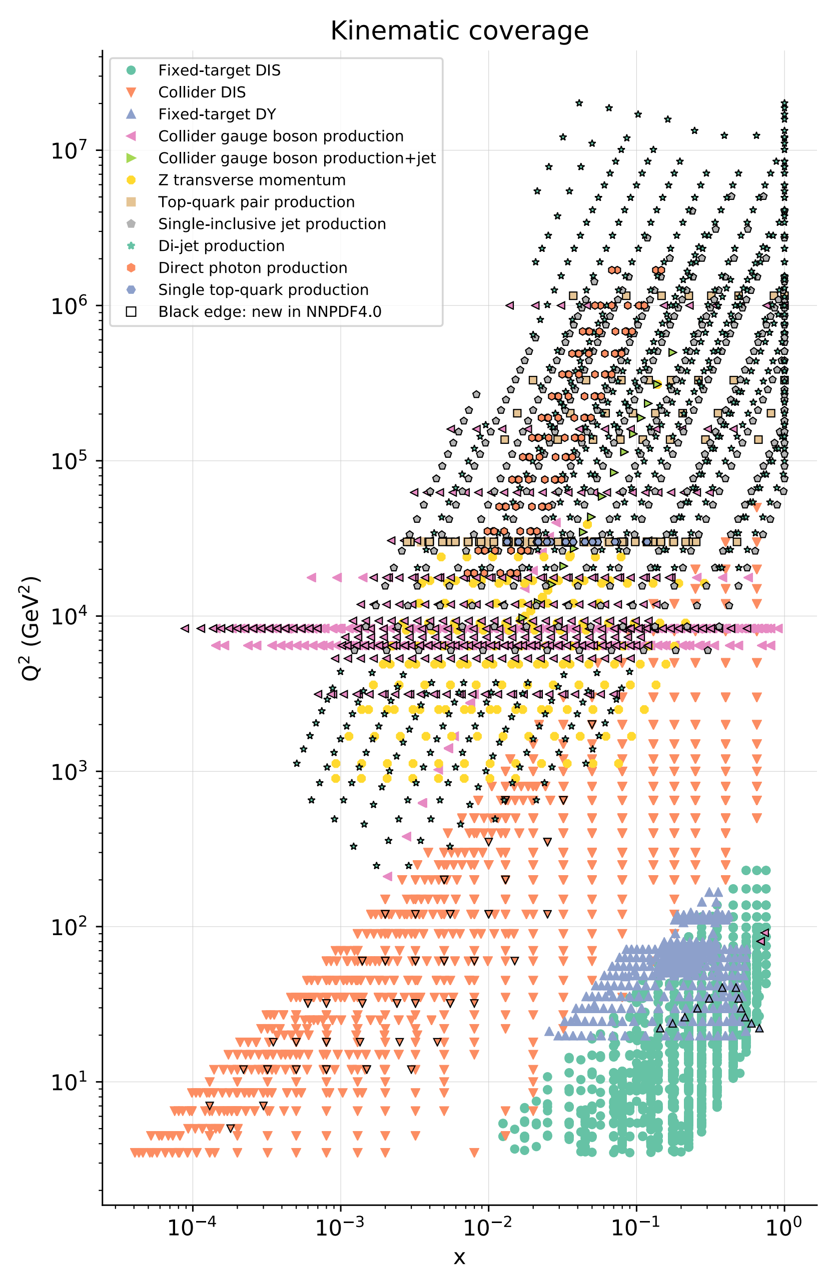


NNPDF, arXiv:2109.02653

- This allows us to evaluate corresponding fit quality with a (MSHT) fixed parameterisation, but to NNPDF data/theory - **only difference** is **input parameterisation.**
- Will use for closure tests (though not essential) - but setting things up in this way will allow direct comparison at level of full fit.

Always NNLO

- For direct comparison will consider perturbative charm - NNPDF4.0pch set as input.
- Then generate unshifted pseudodata for 4.0 global dataset ($N_{\rm pts} = 4627$). In principle exact agreement possible, with $\chi^2 = 0$.
- Then perform fit with default MSHT parameterisation. What do we find?



NNPDF, arXiv:2109.02653

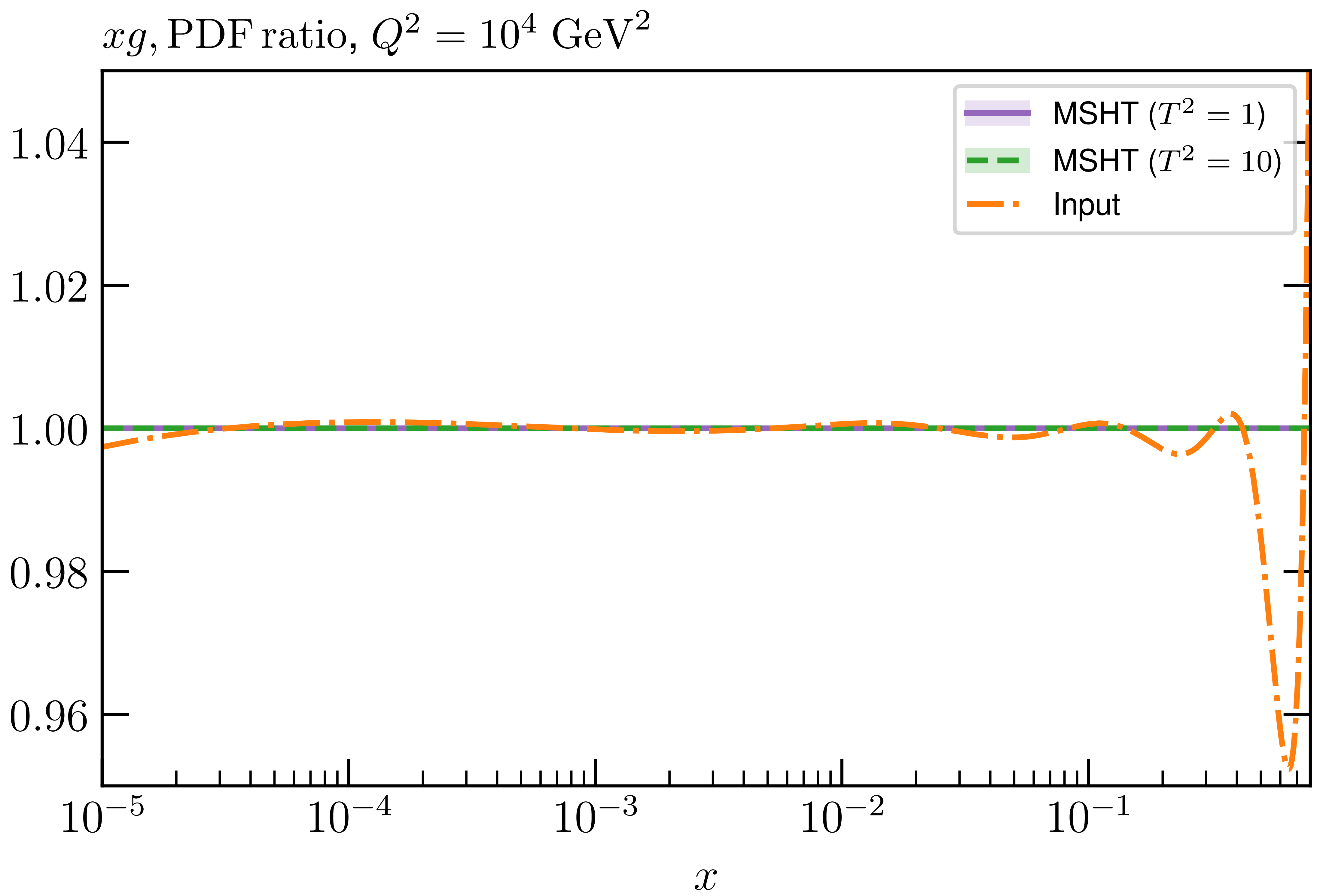
	χ^2	$\chi^2/N_{\rm pts}$
Fit quality:	2.4	0.0005

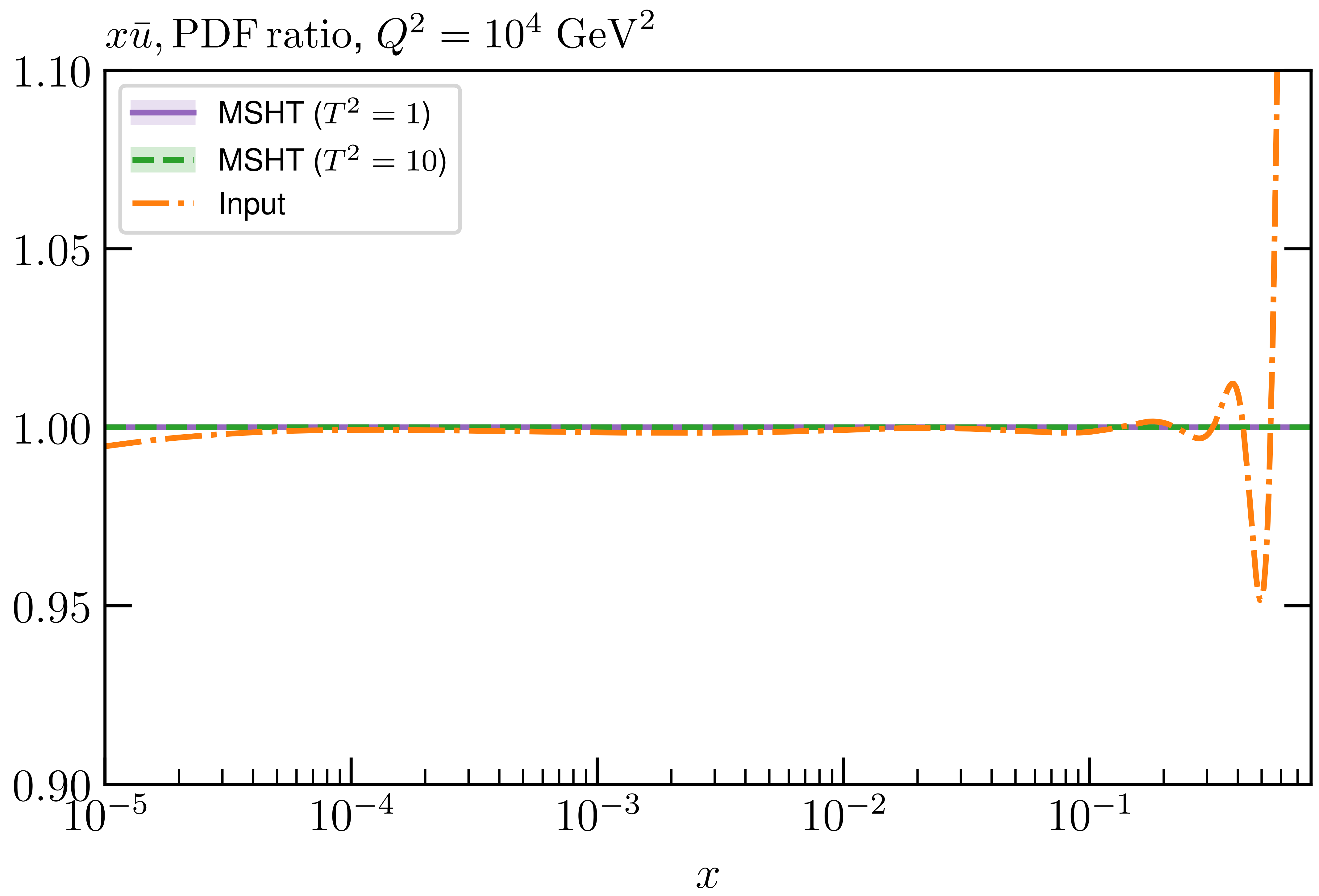
- **Remarkably good!** In fact lower than reported result of NNPDF L0 closure test.

L. Del Debbio, T. Giani and M. Wilson, arXiv:2111.05787

	3.1 meth.	**4.0 meth.**
$\chi^2/N_{\rm pts}$	0.012	0.002

- **Caveat**: only one input set, may well be different (not quite as good) for others. Trend should be similar.
- But apparently no issue with parameterisation inflexibility in this case. But what about PDFs?





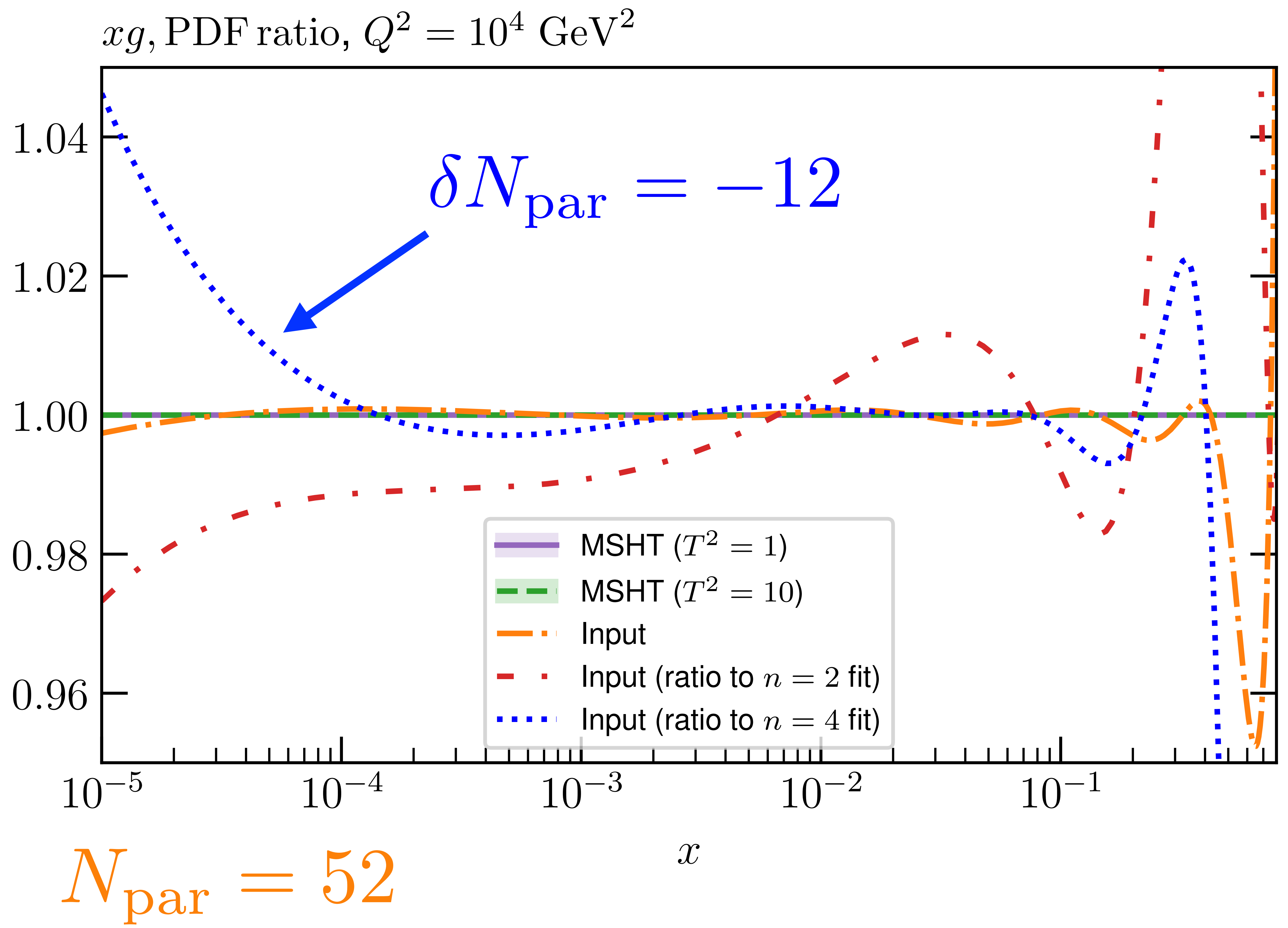
- Ratio of (NNPDF4.0pch) input to fit result, including PDF uncertainties with $T^2 = 1$ and 10 that come from the closure test fit. Latter is ~ result of dynamic tolerance used in MSHT20 (checked here).

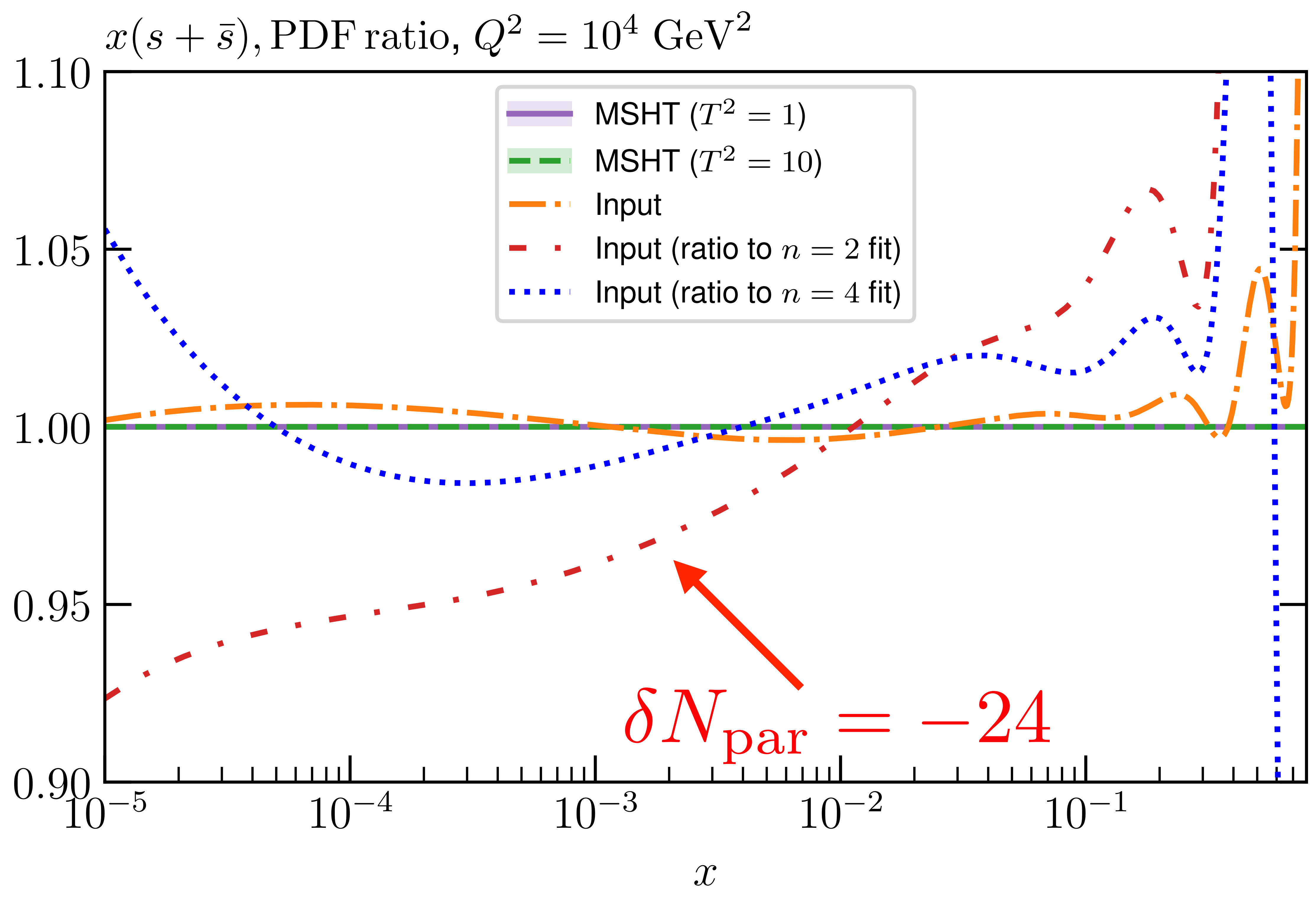
Similar results for other quarks - see backup

★ **Deviation** in general (in data region) **per mille** level and well within the $T^2 = 1$ uncertainties.

⟶ In **data region** input PDF matched very well, and much better than $T^2 = 1$ uncertainties. **No evidence** that the increased tolerance is driven by **parameterisation inflexibility** for MSHT.

✦ Find we pass 'fluctuated' closure test equally well. Agreement less good in very high x extrapolation region, though better with MC replica error propagation (**Backup**).





- Note this level of agreement is not automatic! Need flexible enough parameterisation: **restricting** number of free parameters gives much poorer agreement.

- Is also not coincidental: parameterisation chosen in order to provide 1% precision.

A.D. Martin et al., arXiv: 1211.1215

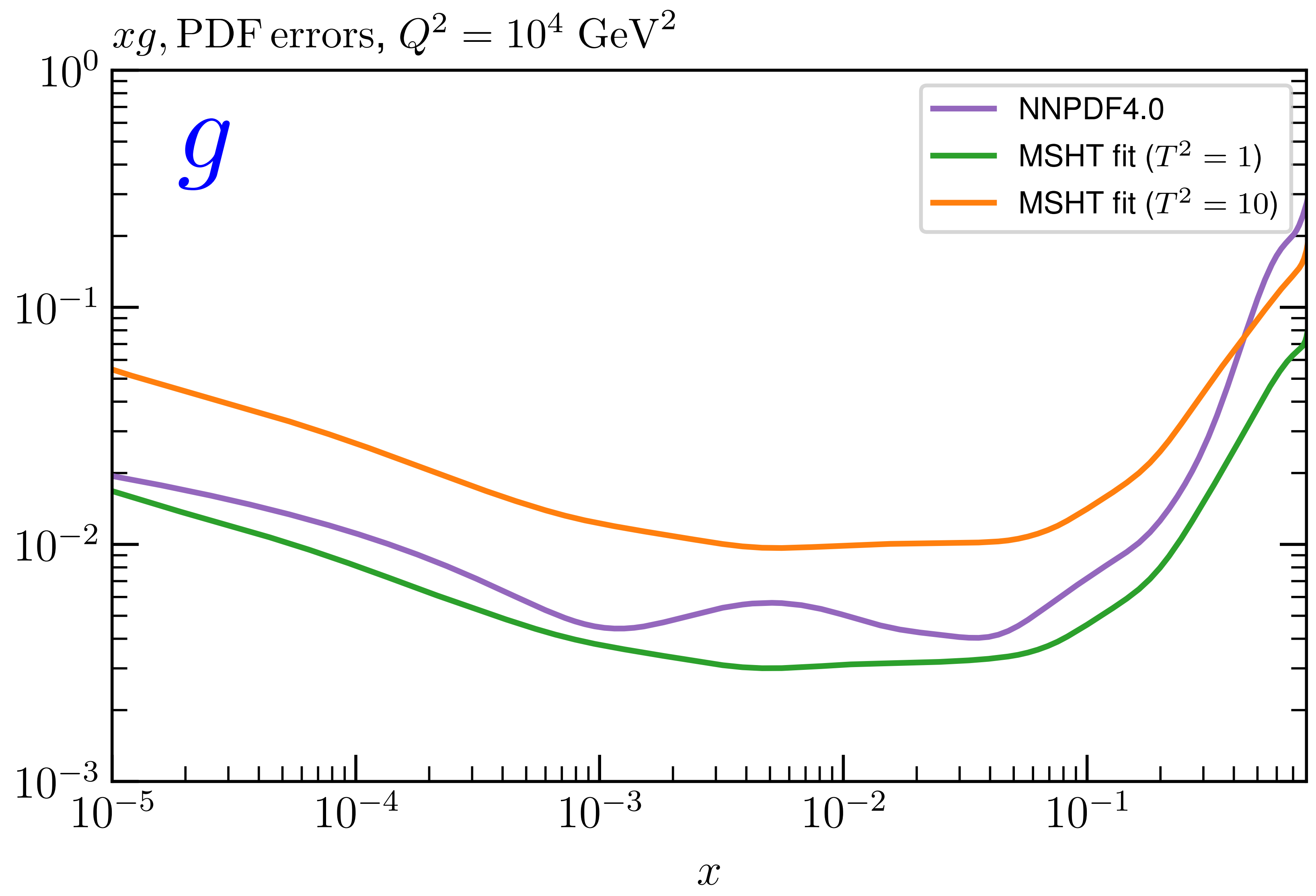
Full Fit: Comparison

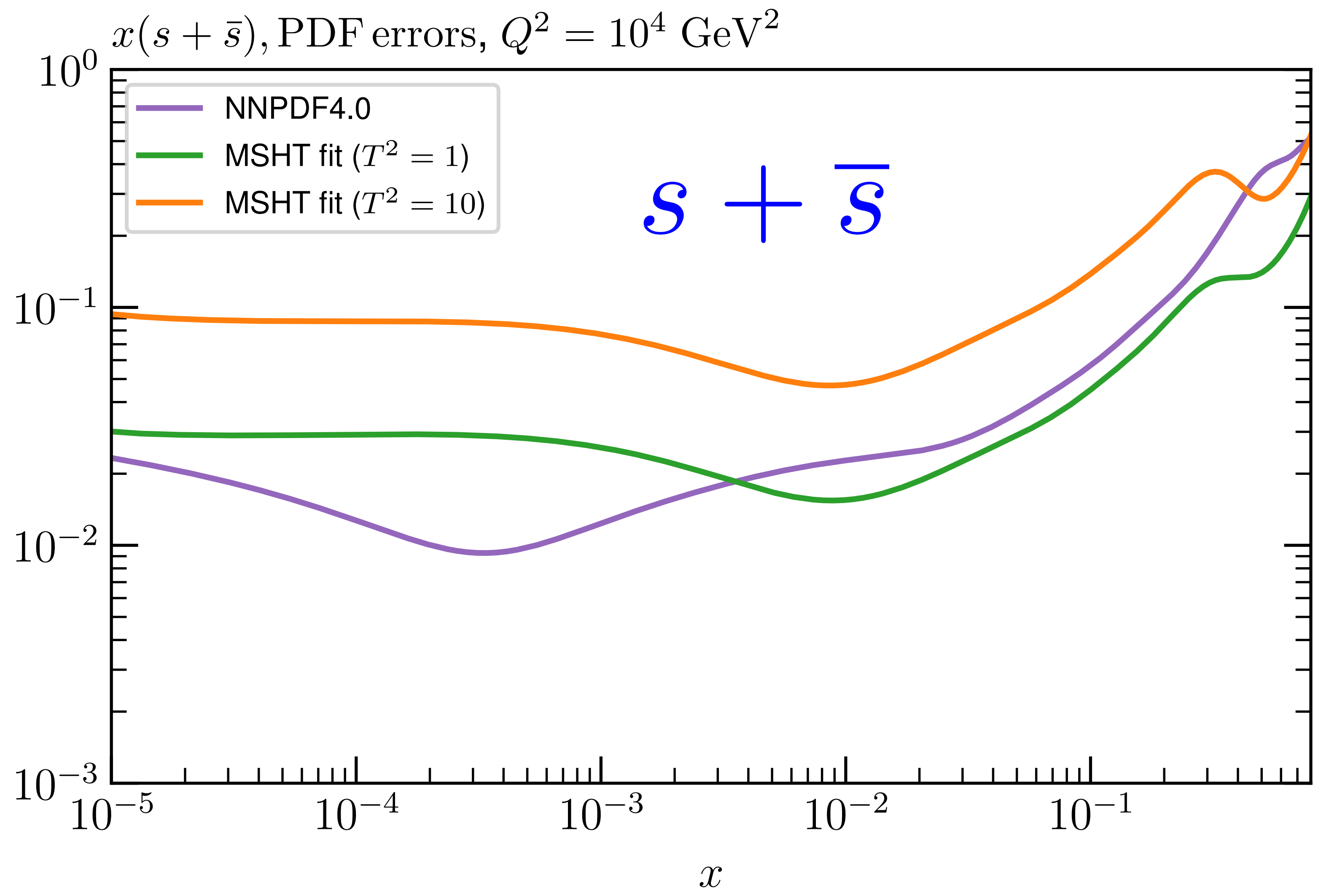
- Can also consider result of fit to real data entering NNPDF4.0 fit with MSHT20 parameterisation. Like-for-like: exactly **same data and theory**, with **only difference** from PDF input **parameterisation**.
- MSHT **fit quality** moderately **improved** w.r.t. NNPDF4.0, i.e. again no evidence for parameterisation inflexibility. At level of PDF uncertainties, NNPDF4.0 rather in line with MSHT but with $T^2 = 1$:

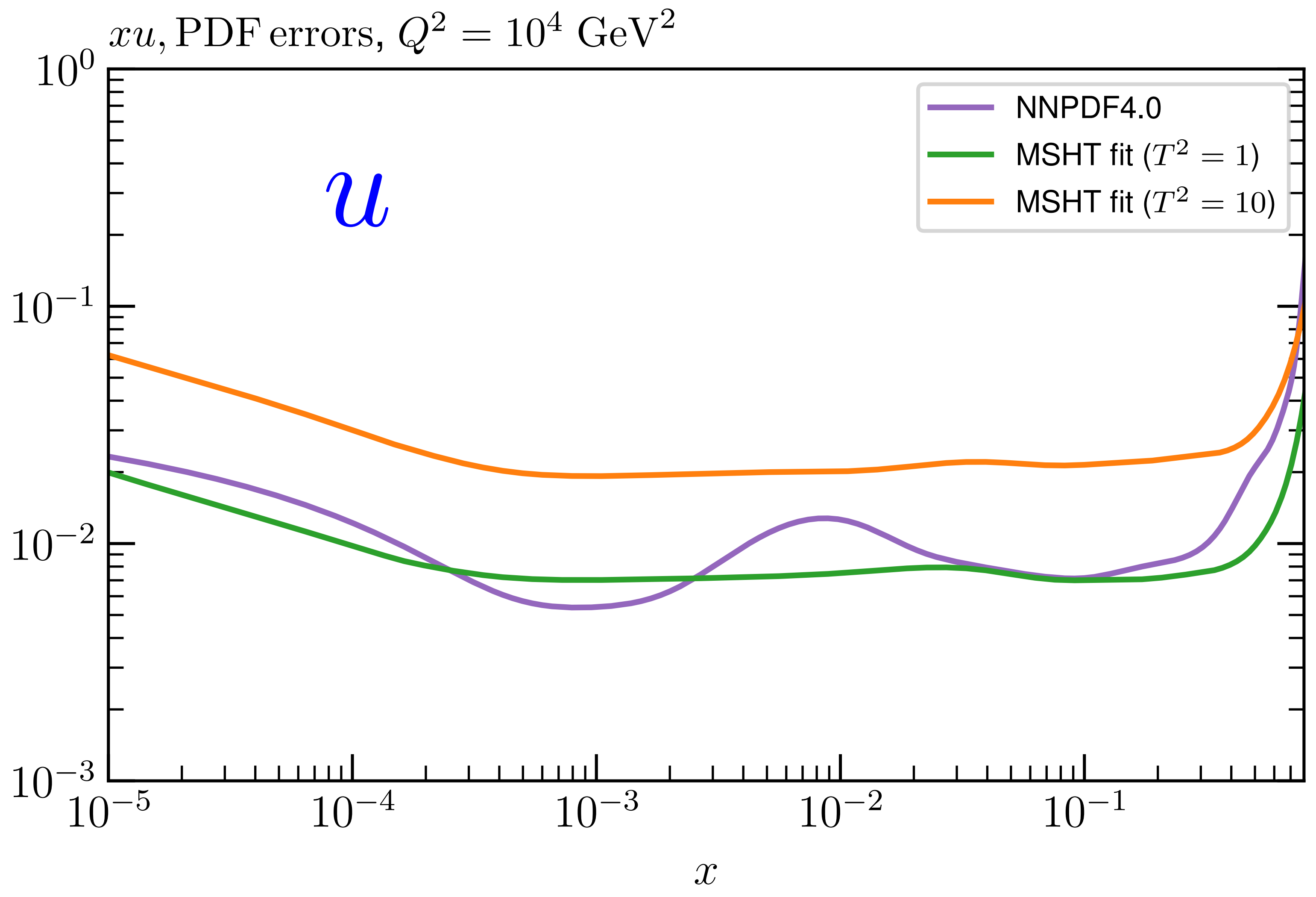
MSHT, $T^2 = 1$
MSHT, $T^2 = 10$
NNPDF4.0

★ Quark flavour decomposition: $\sigma(\text{NNPDF}) \sim \sigma(\text{MSHT}, T^2 = 1)$

★ Gluon: $\sigma(\text{MSHT}, T^2 = 1) \lesssim \sigma(\text{NNPDF}) \lesssim \sigma(\text{MSHT}, T^2 = 10)$







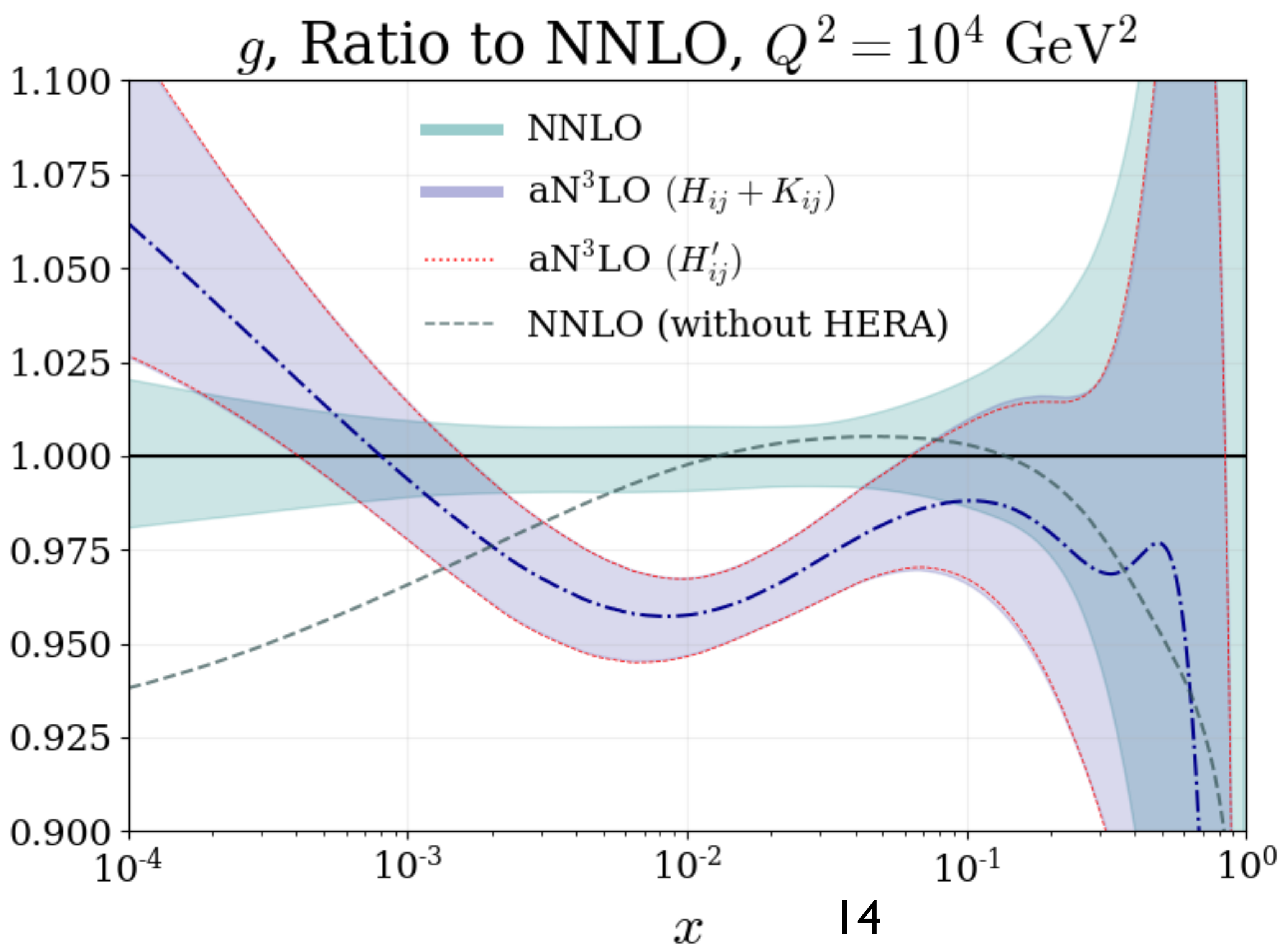
MSHT at Approximate N3LO:
MSHT20aN3LO

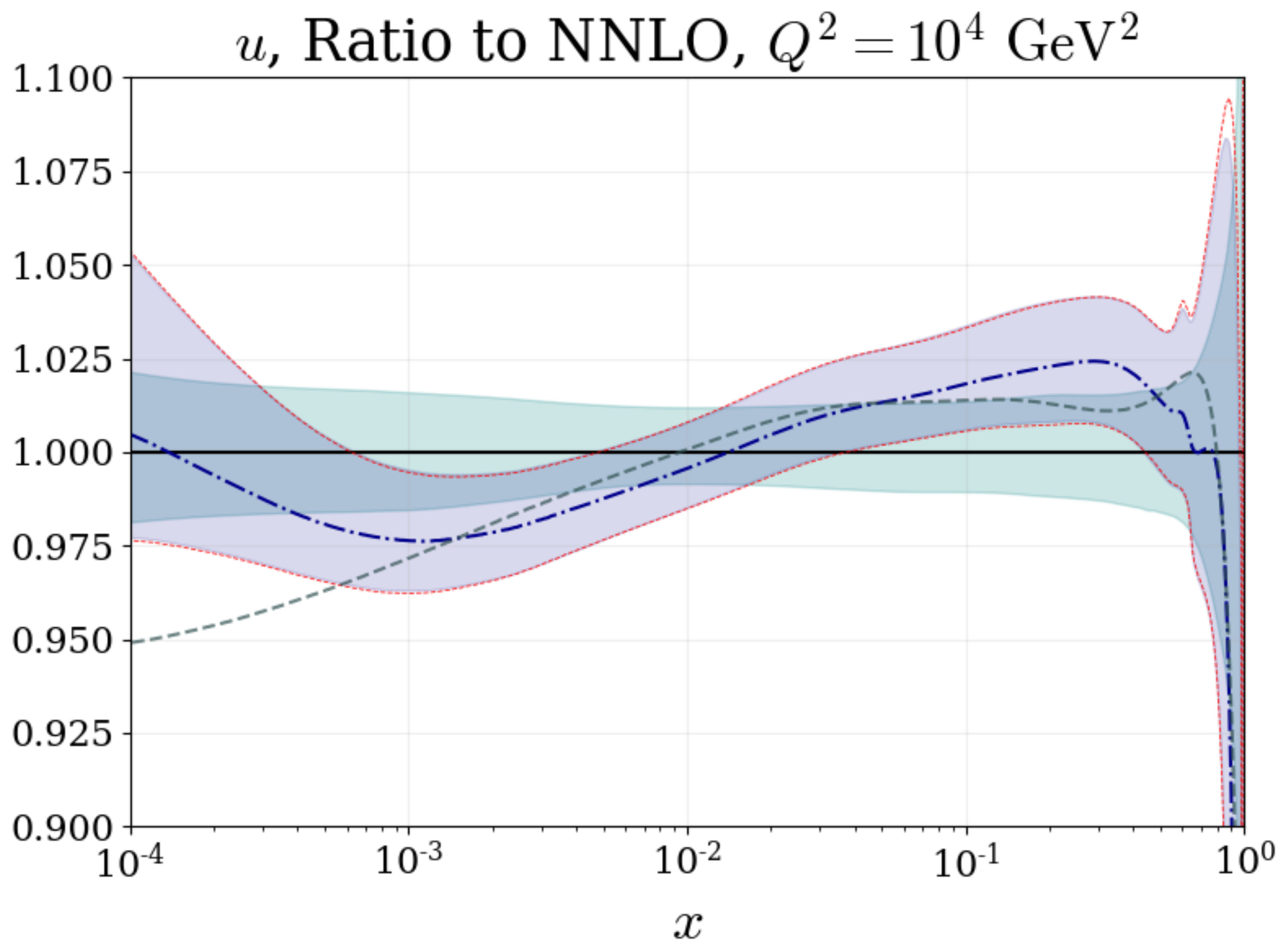
MSHTaN3LO

- First global aN3LO analysis - MSHT20aN3LO. Released ~ 2 years ago.
- Approximate ≠ poorly known! Great deal already known at N3LO about PDF evolution and DIS cross sections. And a lot of new information on splitting functions/heavy flavour transitions since release.
- Main bottleneck to 'real' N3LO is hadronic cross sections. Include via nuisance parameters:
- Clear improvement in fit quality, ~ driven by known N3LO.

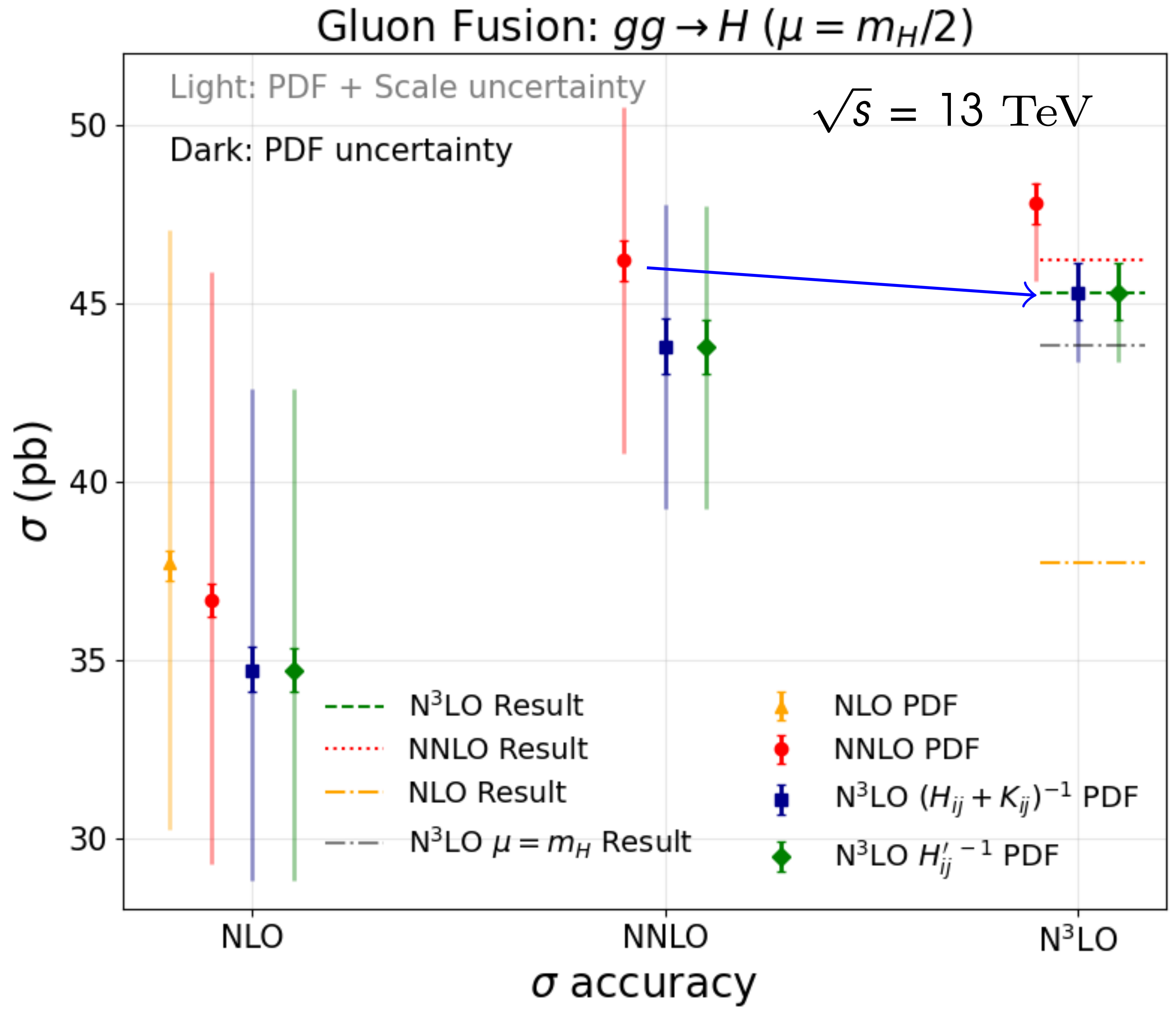
	LO	NLO	NNLO	N^3LO
$\chi^2_{N_{pts}}$	2.57	1.33	1.17	1.14

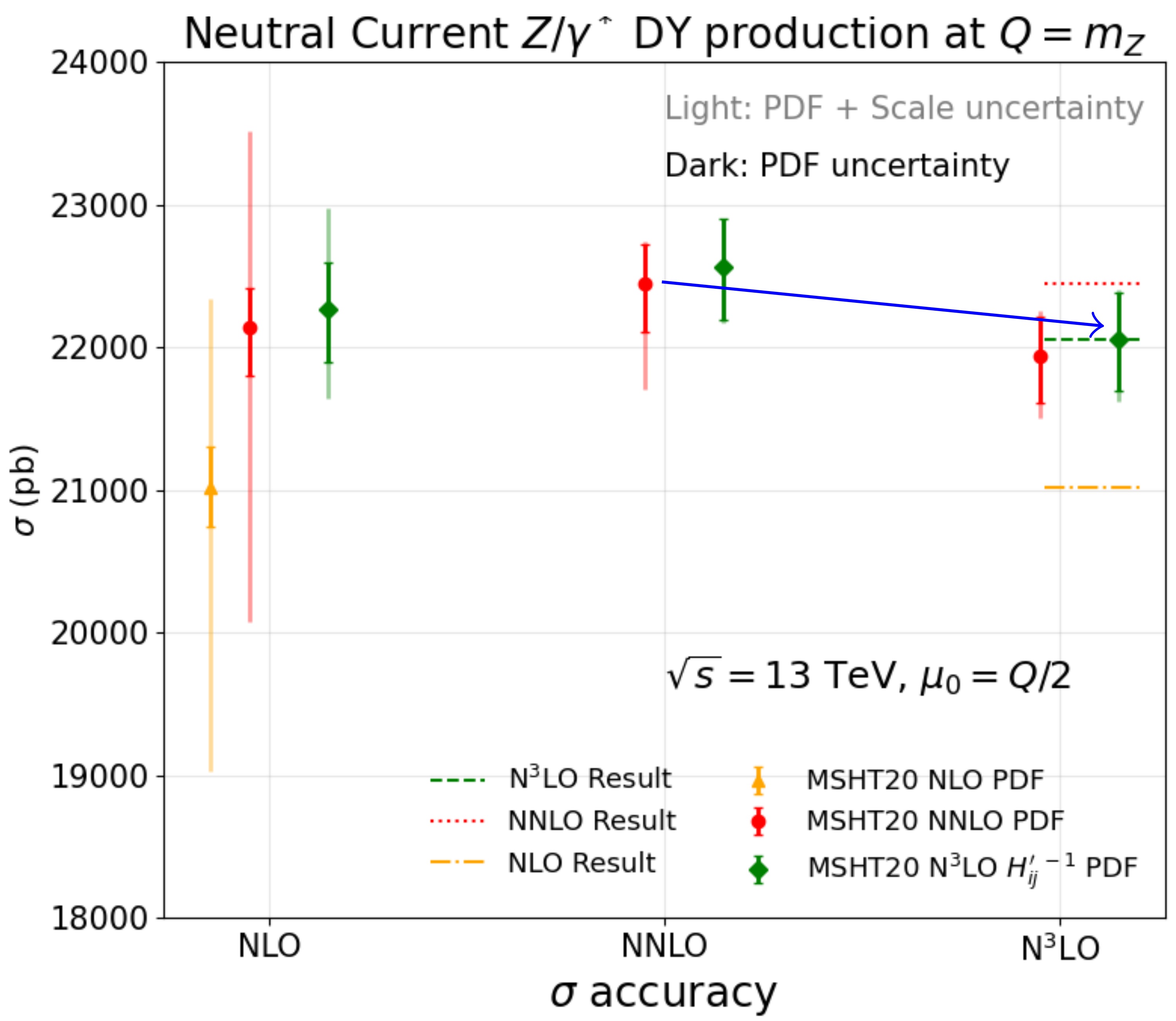
- Evidence that aN3LO reduces tensions between low and high x regions.
- Largest change is in gluon at low and intermediate x. Some change in e.g. quarks at high x.





- Change in gluon corresponds to reduction in e.g. ggH at N3LO - improves stability.

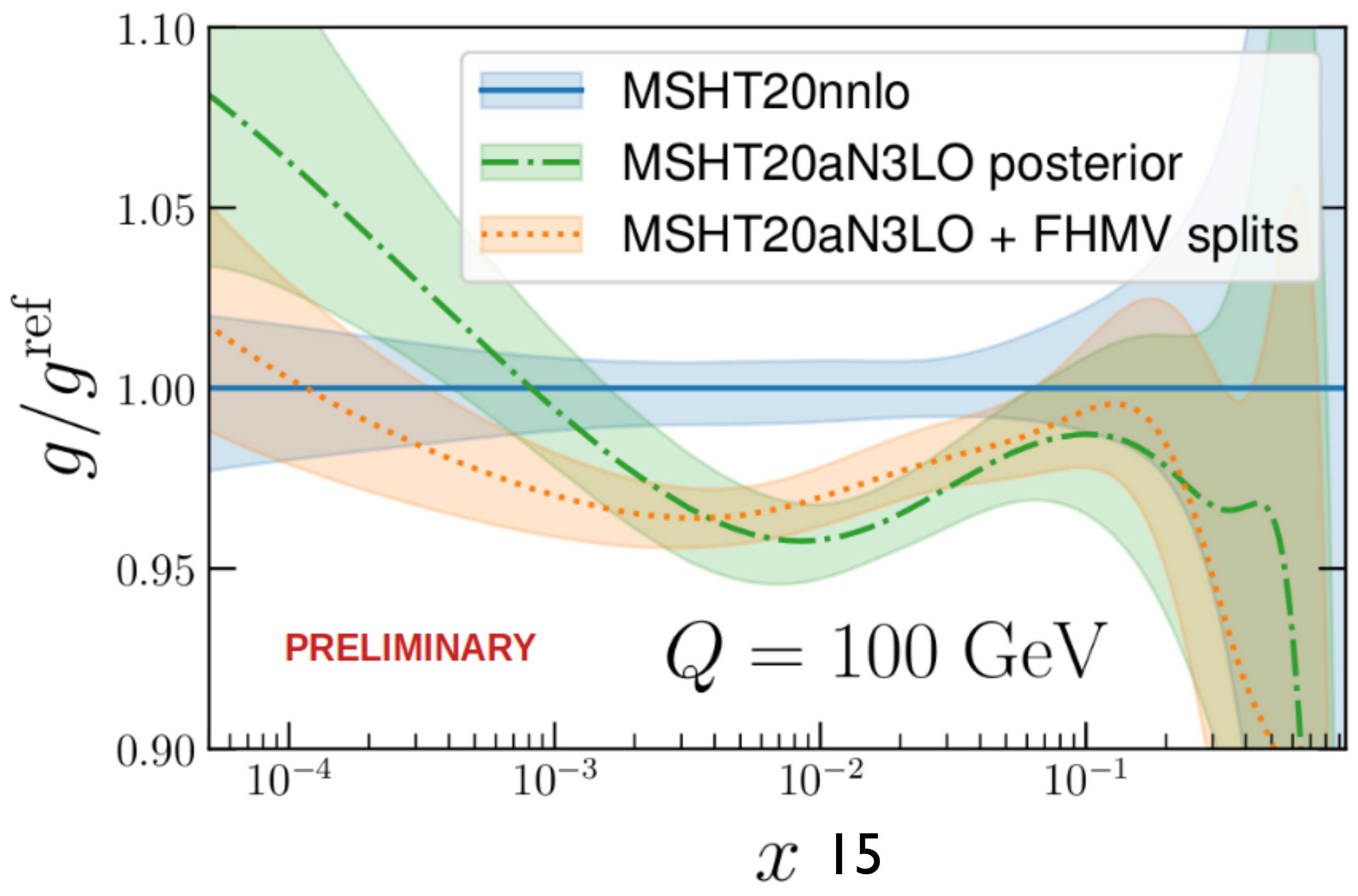


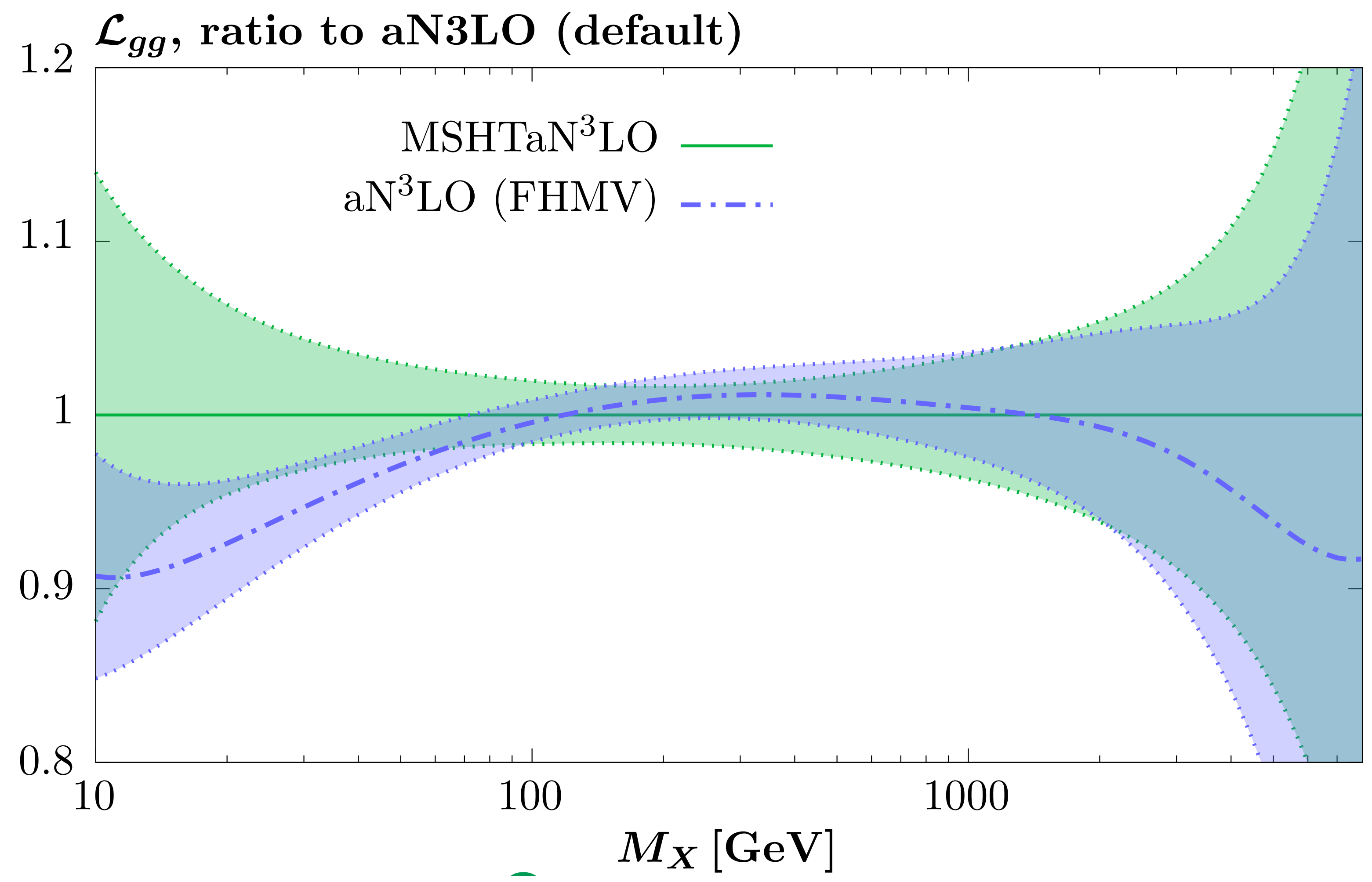


- Since MSHT20aN3LO release, have studied impact of newer splitting function information.

- Some increase in NC DY - again mild improvement in stability.

- Moderate impact, within uncertainties. Find impact on ggH very small!





MSHTaN3LO + QED

- MSHT now also provide **aN3LO + QED** PDFs.

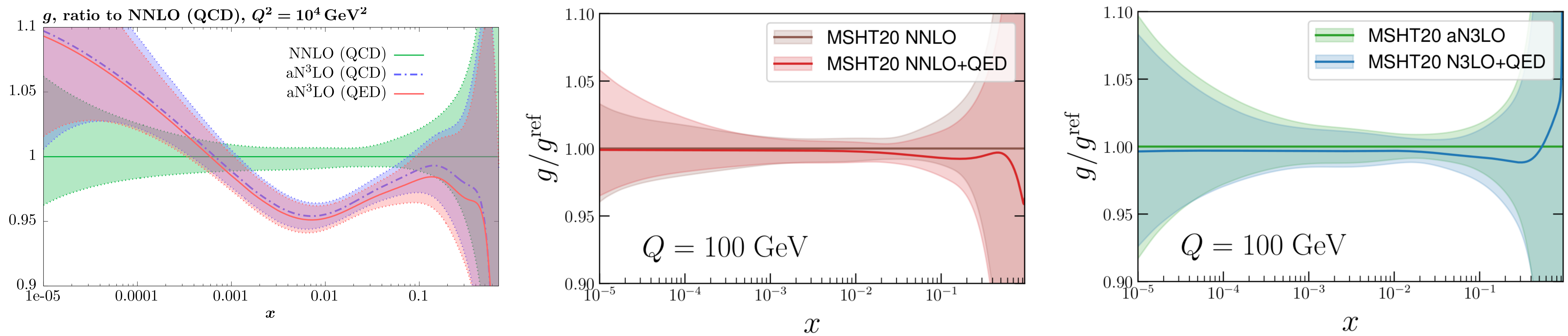
QED $P_{ij} = \frac{\alpha}{2\pi} P_{ij}^{(0,1)} + \frac{\alpha\alpha_S}{(2\pi)^2} P_{ij}^{(1,1)} + \left(\frac{\alpha}{2\pi}\right)^2 P_{ij}^{(0,2)}$

NNLO QCD $+ \frac{\alpha_S}{2\pi} P_{ij}^{(1,0)} + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ij}^{(2,0)} + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ij}^{(3,0)}$

aN3LO QCD $+ \left(\frac{\alpha_S}{2\pi}\right)^4 P_{ij}^{(4,0)}$.

	$\chi^2/N_{\rm pt}$ aN^{3}LO (QED)	$\Delta\chi^2_{\rm aN^3LO}$ QED-QCD	$\Delta\chi^2_{\rm NNLO}$ QED-QCD	$\Delta\chi^2_{\rm QCD,QED}$ aN^{3}LO-NNLO
Total	5323.6/4534	(+3.6)	(+17.3)	(-209.3, -223.1)

- Impact of including QED ~ factorizes from aN3LO. E.g. improvement in fit quality from NNLO to aN3LO remains.

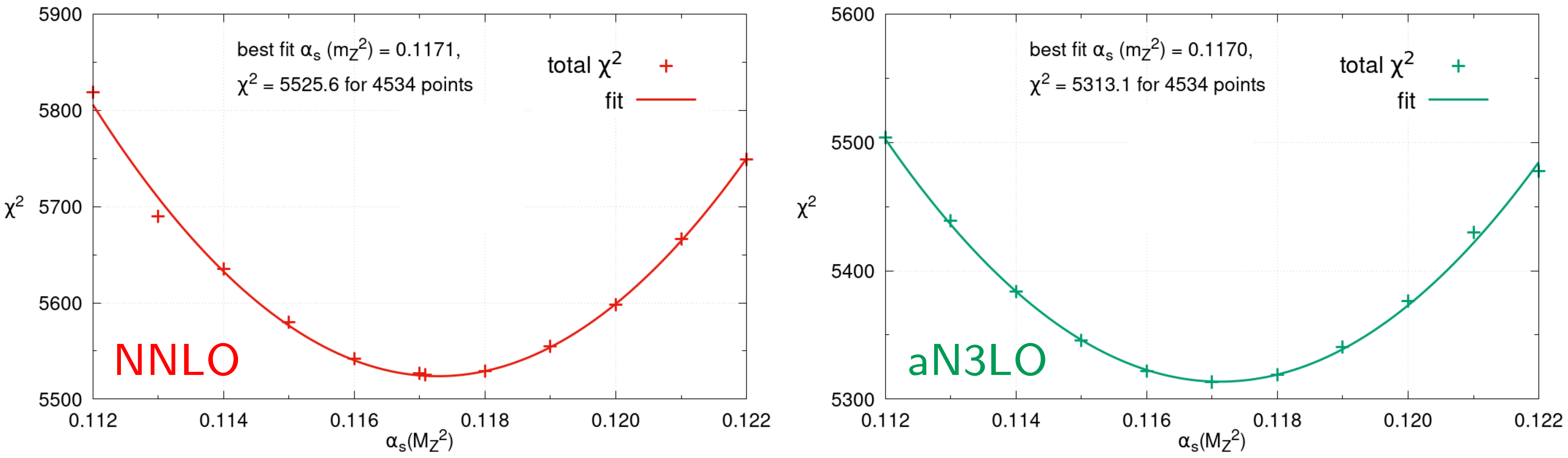


- Impact of QED relatively mild, generally smaller than aN3LO. But not negligible.
- E.g. leads to reduction in gluon (sum rule) and hence ggH at percent level.

The strong coupling at aN3LO

- Determination of α_S and PDFs highly correlated. Only completely consistent way to include impact of a (PDF sensitive) hadronic measurements is via full refit. **S. Forte and Z. Kassabov, arXiv: 2001.04896**
- Recent first extraction of strong coupling in aN3LO global PDF fit.

$$\alpha^{new}_{S,\mathrm{NNLO}}(M_Z^2) = 0.1171 \qquad \alpha^{new}_{S,\mathrm{aN3LO}}(M_Z^2) = 0.1170$$



Nice Quadratic χ^2 profile ✓

0.1180 ± 0.0009

- Very good perturbative convergence to aN3LO, and both consistent with world average.
- Confirmed that more recent aN3LO splitting function information gives v. similar result ($\ll$ uncertainty)
- Looking in more detail…

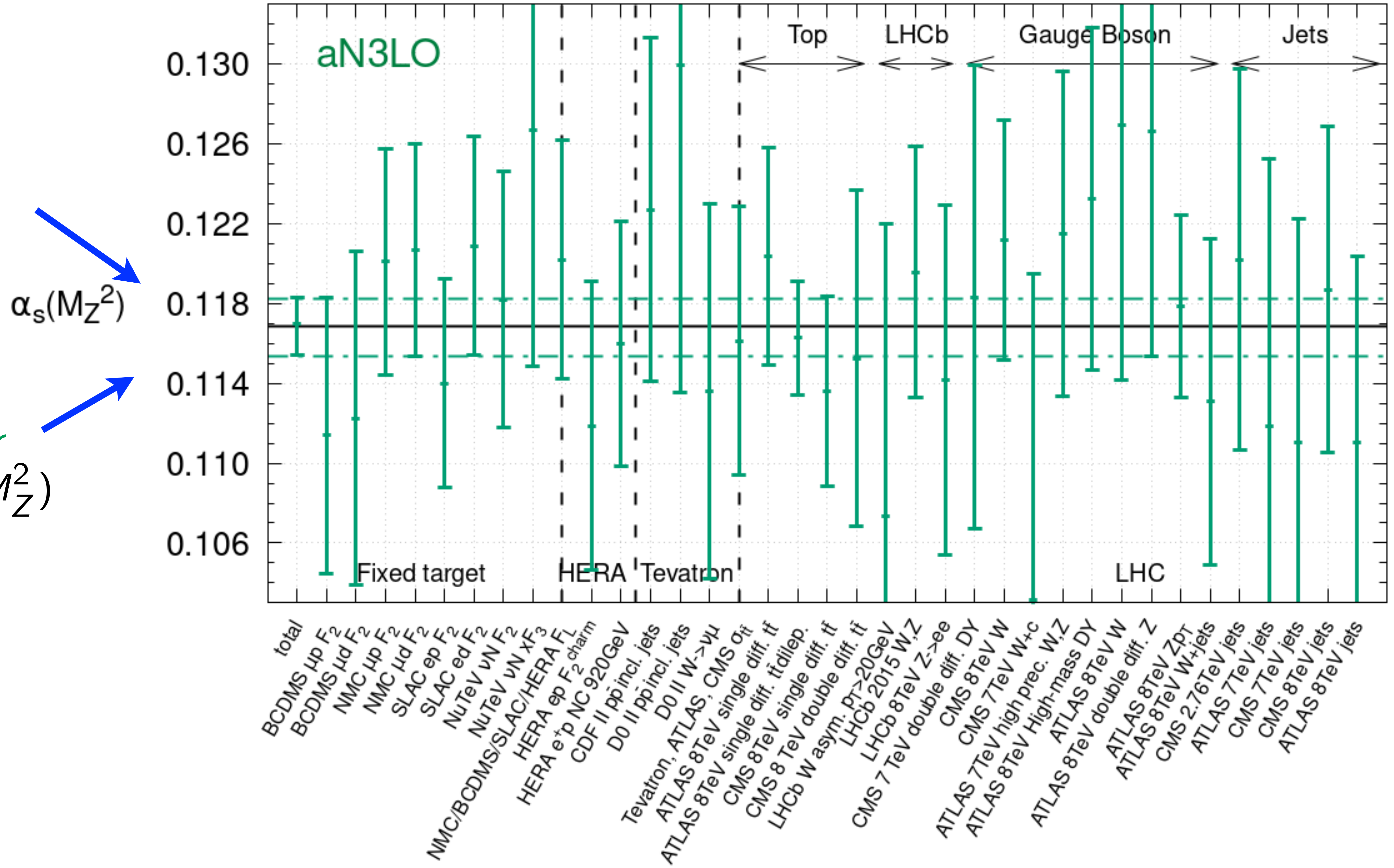
- Find that global χ^2 profile built up of different competing pulls.
- Uncertainty provided using `dynamic tolerance'. Deviation with α_S monitored and limited such that this does not exceed 'hypothesis testing' criterion $\Delta\chi^2 \lesssim \sqrt{2N}$ i.e. remains good according to this measure.

(Consistent with other fixed target DIS (p), and ~ known N3LO)

BCDMSp data strongest constraint upwards: $\Delta\alpha_S(M_Z^2) = +0.0013$.

NMC deuteron, ATLAS 8 TeV Z both give lower bounds of $\Delta\alpha_S(M_Z^2) = -0.0017$.

SLAC deuteron data gives lower bound: $\Delta\alpha_S(M_Z^2) = -0.0016$.



- Putting together and suitably symmetrising, we quote:

$$\alpha_{S,\text{aN3LO}}(M_Z^2) = 0.1170 \pm 0.0016$$

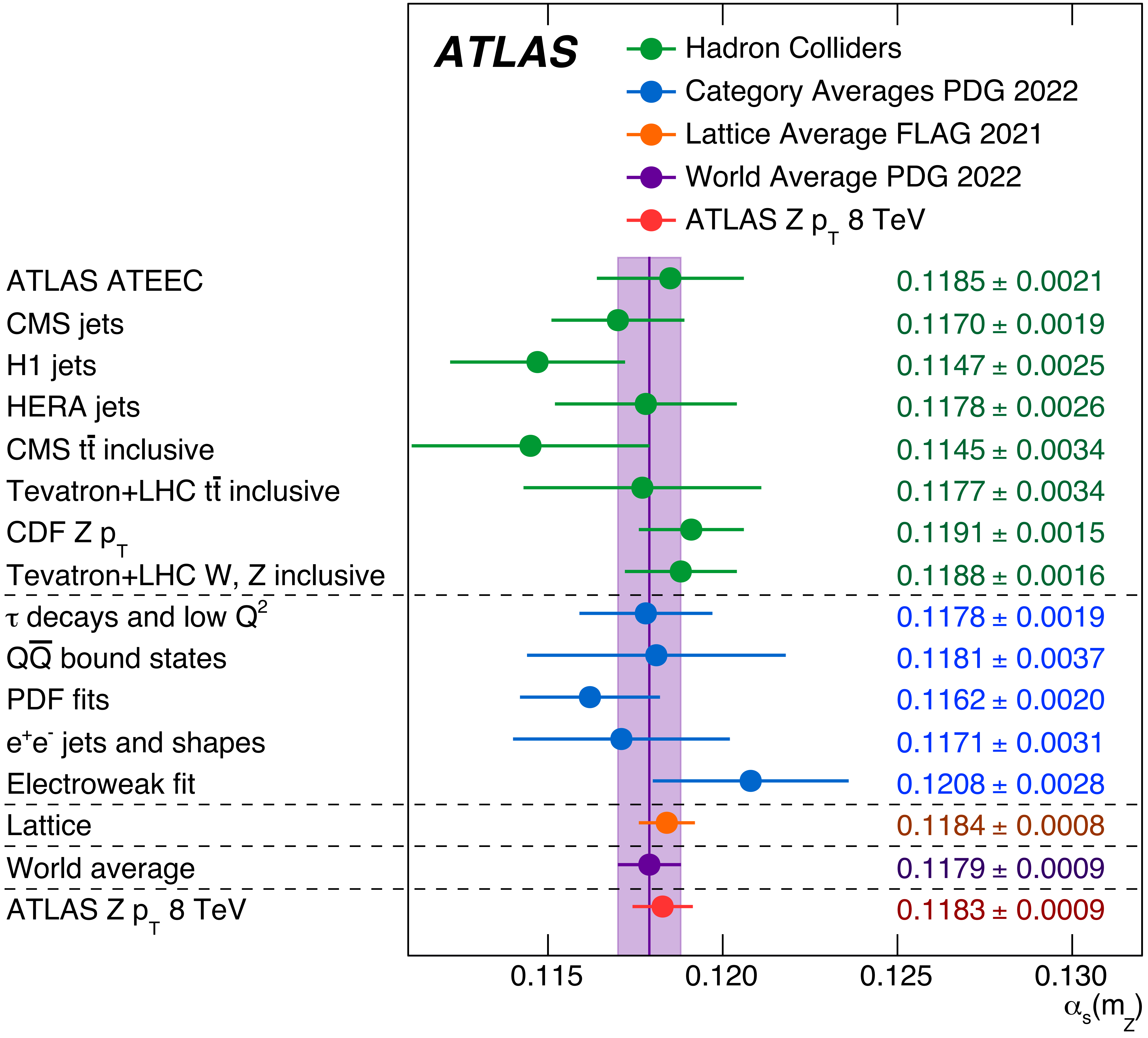
Consistent with (NNLO) World Average of 0.1180 ± 0.0009.

Comparison to other results

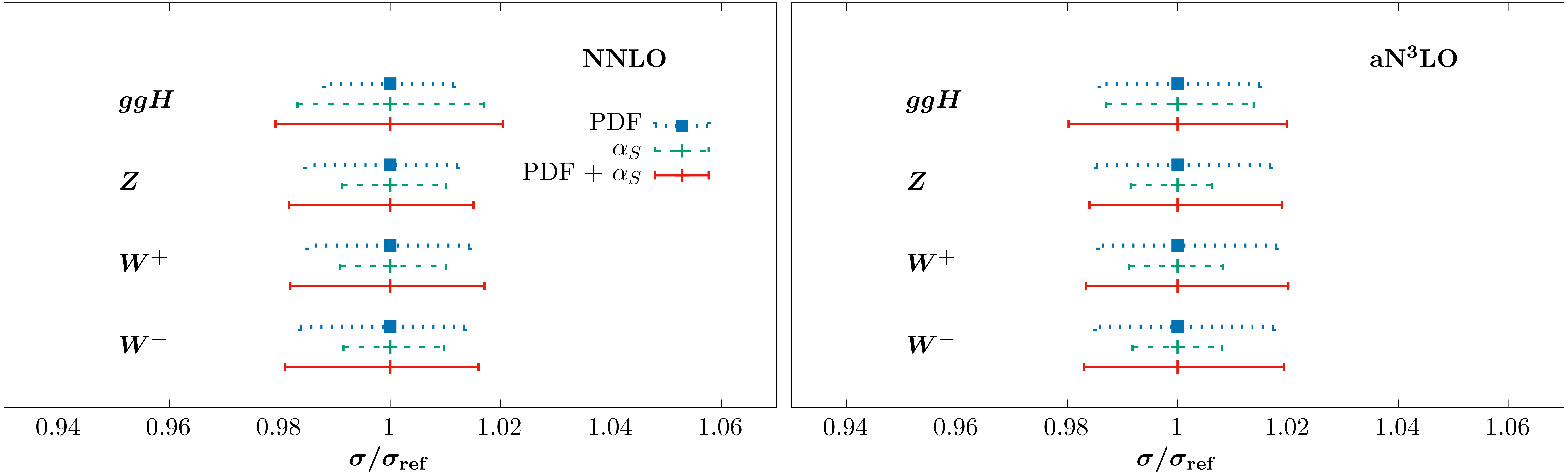
$$\alpha_{S,\mathrm{aN3LO}}(M_Z^2) = 0.1170 \pm 0.0016$$

- Consistent with world average and recent ATLAS measurement.
- Uncertainty larger but similar order.
- If we took $\Delta\chi^2 = 1$ would be factor of ~ 2 smaller, but v. good reasons to believe that is too aggressive.

$\alpha_S = 0.1170 \pm 0.0005$



Cross Sections



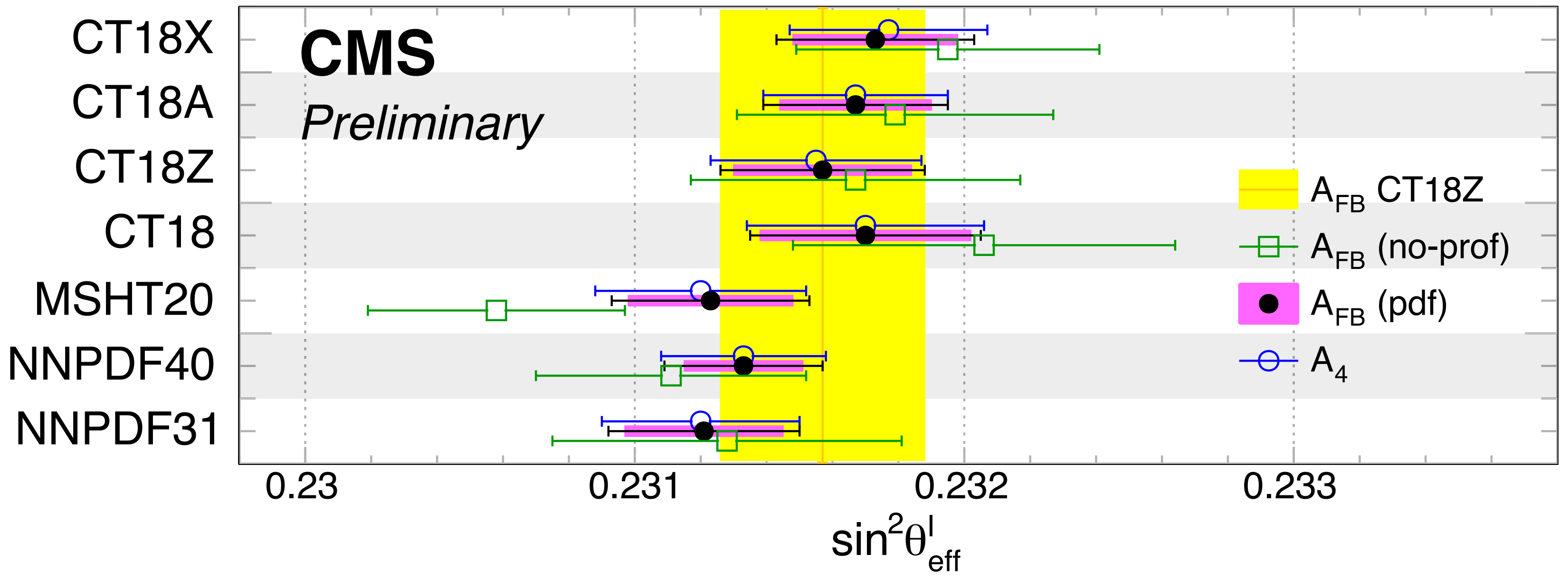
- ★ Impact on cross sections includes α_S variation in matrix elements + PDFs - non-trivial interplay to get final result. Important to treat these together!
- ★ For LHC **Higgs** the anticorrelation between gluon and α_S compensates larger direct uncertainty.
- ★ For **DY** direct α_S uncertainty small, and largest effect from change in PDF.
- ★ Combined PDF + α_S broadly leads to at most moderate increase over PDF uncertainty alone.

Profiling: Some Comments

PDF Profiling

- PDF profiling common technique in experimental analyses.
- Allows impact of data on (Hessian) PDFs to be accounted for simultaneously with (e.g.) EW precision observable.

CMS PAS SMP-22-010



- Aim of this is (or should be) to approximate what would happen if data were included in global fit:

$$\chi^2_{\text{global, new}} = \chi^2_{\text{new}} + \chi^2_{\text{global, old}}$$

To be precise: T depends on the eigenvector for MSHT (omit for simplicity).

$$\sum_{i,j=1}^{N_{\text{dat}}} \left(D_i - t_i - \sum_k \Gamma^{\text{PDF}}_{i,k} \beta_{k,\text{PDF}} \right) C^{-1}_{ij} \left(D_j - t_j - \sum_m \Gamma^{\text{PDF}}_{i,m} \beta_{m,\text{PDF}} \right) + T^2 \sum_k \beta^2_{k,\text{PDF}}$$

New data — **Data already in fit**

- $\beta_{k,\text{PDF}}$: nuisance parameter. $\beta_{k,\text{PDF}} = 0$ is previous best fit and refitting will give $\beta_{k,\text{PDF}} \neq 0$.
- Refitting is a balance between new data and existing data in fit. **Key point:** PDF eigenvectors are **defined** such that taking a given $\beta_{k,\text{PDF}} \to 1$ gives $\Delta\chi^2_{\text{global,old}} = T^2$.
- This is **not optional**: it is what we would e.g. get by passing a given eigenvector through our fitting code.

$$\chi^2_{\text{global, new}} = \chi^2_{\text{new}} + \chi^2_{\text{global, old}}$$

$$\sum_{i,j=1}^{N_{\text{dat}}} \left(D_i - t_i - \sum_k \Gamma^{\text{PDF}}_{i,k} \beta_{k,\text{PDF}} \right) C^{-1}_{ij} \left(D_j - t_j - \sum_m \Gamma^{\text{PDF}}_{i,m} \beta_{m,\text{PDF}} \right) + T^2 \sum_k \beta^2_{k,\text{PDF}}$$

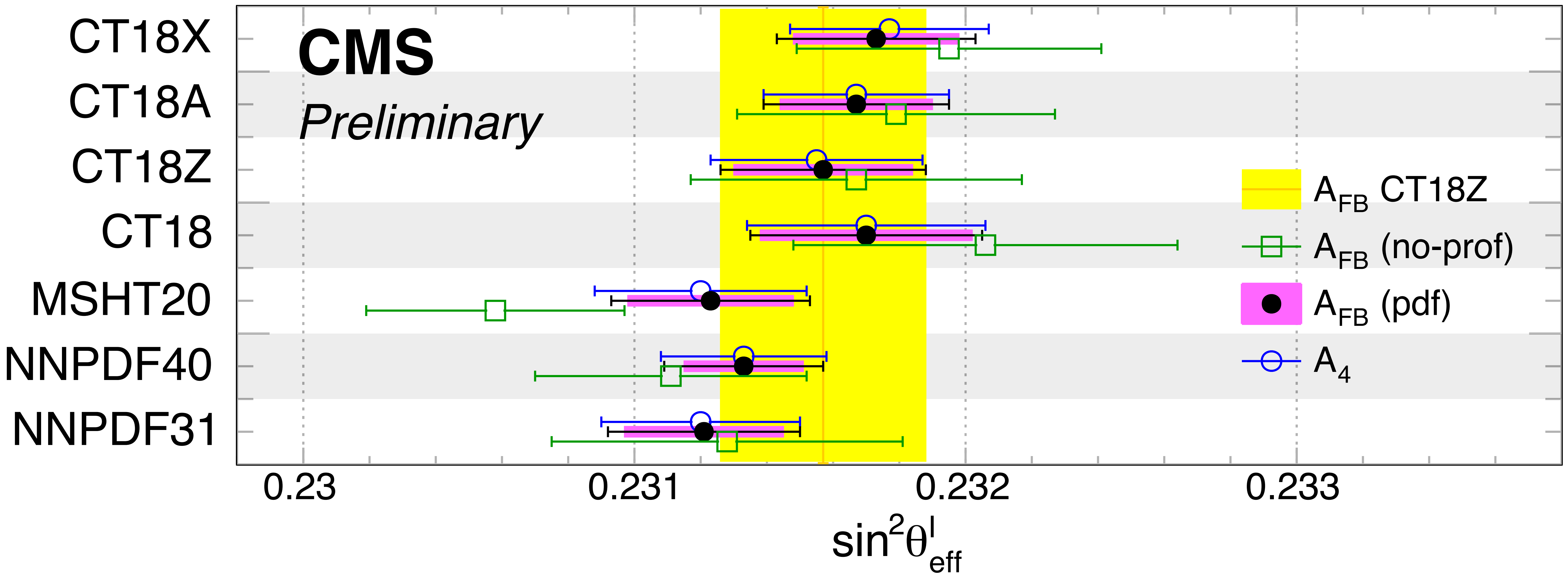
New data **Data already in fit**

- **Key point:** PDF eigenvectors are **defined** such that taking $\beta_{k,\text{PDF}} \to 1$ gives $\Delta\chi^2_{\text{global,old}} = T^2$.
- So if new data prefers e.g. $\beta_{k,\text{PDF}} \to 1$ this to be balanced against this deterioration in fit to remaining data.
- To best of our knowledge, all LHC analyses where profiling is performed instead take:

$$\sum_{i,j=1}^{N_{\text{dat}}} \left(D_i - t_i - \sum_k \Gamma^{\text{PDF}}_{i,k} \beta_{k,\text{PDF}} \right) C^{-1}_{ij} \left(D_j - t_j - \sum_m \Gamma^{\text{PDF}}_{i,m} \beta_{m,\text{PDF}} \right) + \cancel{T^2} \sum_k \beta^2_{k,\text{PDF}}$$

- I.e. as if taking $\beta_{k,\text{PDF}} \to 1$ only leads to $\Delta\chi^2_{\text{global,old}} = 1$, which is incorrect. Corresponds to **down-weighting** the impact of other data in the fit (much of it from the LHC).
- Note this issue already enters at the level of pull of new data on central value of PDFs, i.e. it is **independent** of question of how final PDF uncertainties should be defined in analysis - although also relevant there!

Uncertainty Correlations (Brief Comments)



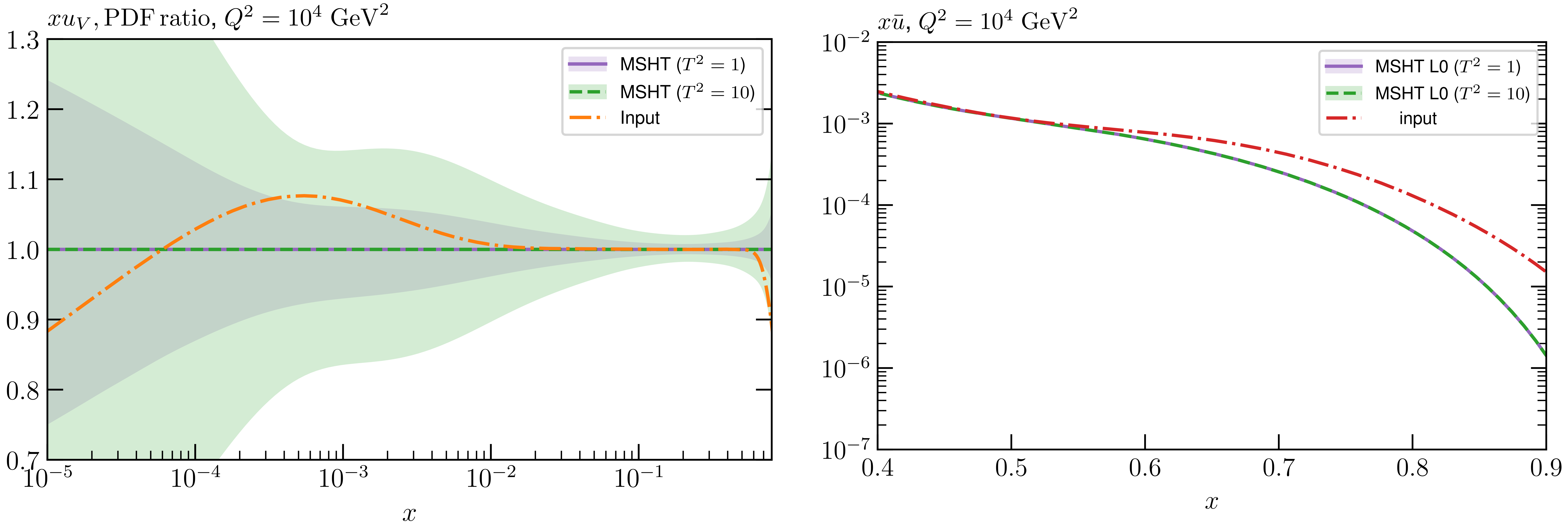
- Ongoing proposal to evaluate the correlations between the PDF uncertainties of different groups - aim to arrive at better estimate of eventual uncertainty. Two brief remarks/questions:
  - ★ If PDF profiling/reweighting is becoming the default, how would such a study sit within this?
  - ★ Deeper question of tolerance, comparisons between methodologies may be key.

Summary/Outlook

- ⋆ Have presented here the first global closure test of the MSHT20 fitting approach: parameterisation inflexibility not observed to be major contribution to error budget.
- ⋆ But I have tried to motivate why an enlarged error definition is nonetheless needed in the complex environment of a global PDF fit.
- ⋆ At level of errors $\sigma(\text{NNPDF}) \sim \sigma(\text{MSHT}, T^2 = 1)$ in general with some exceptions - gluon larger though less than $T^2 = 10$ (~MSHT20 default).
- ⋆ Have had to be very brief here - full paper out very soon with all the details. Stay tuned!
- ⋆ Approximate N3LO PDFs very well advanced - a lot is known and these improve accuracy of results along with missing higher order uncertainties.
- ⋆ First strong coupling determination at aN3LO - perturbative convergence has been reached.
- ⋆ Finally: care needed to correctly profile PDFs with tolerance in error definition!

Thank you for listening!

Backup

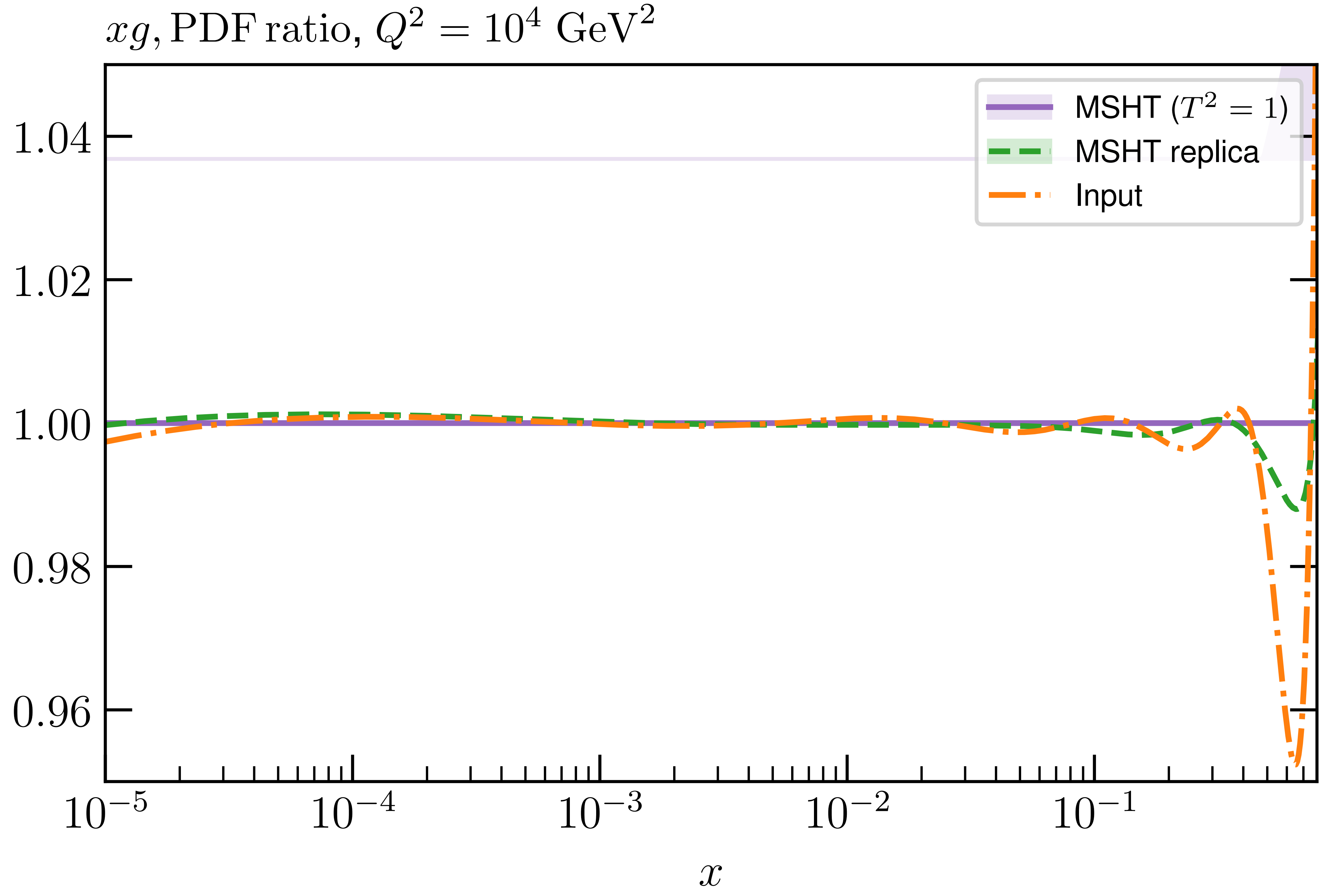


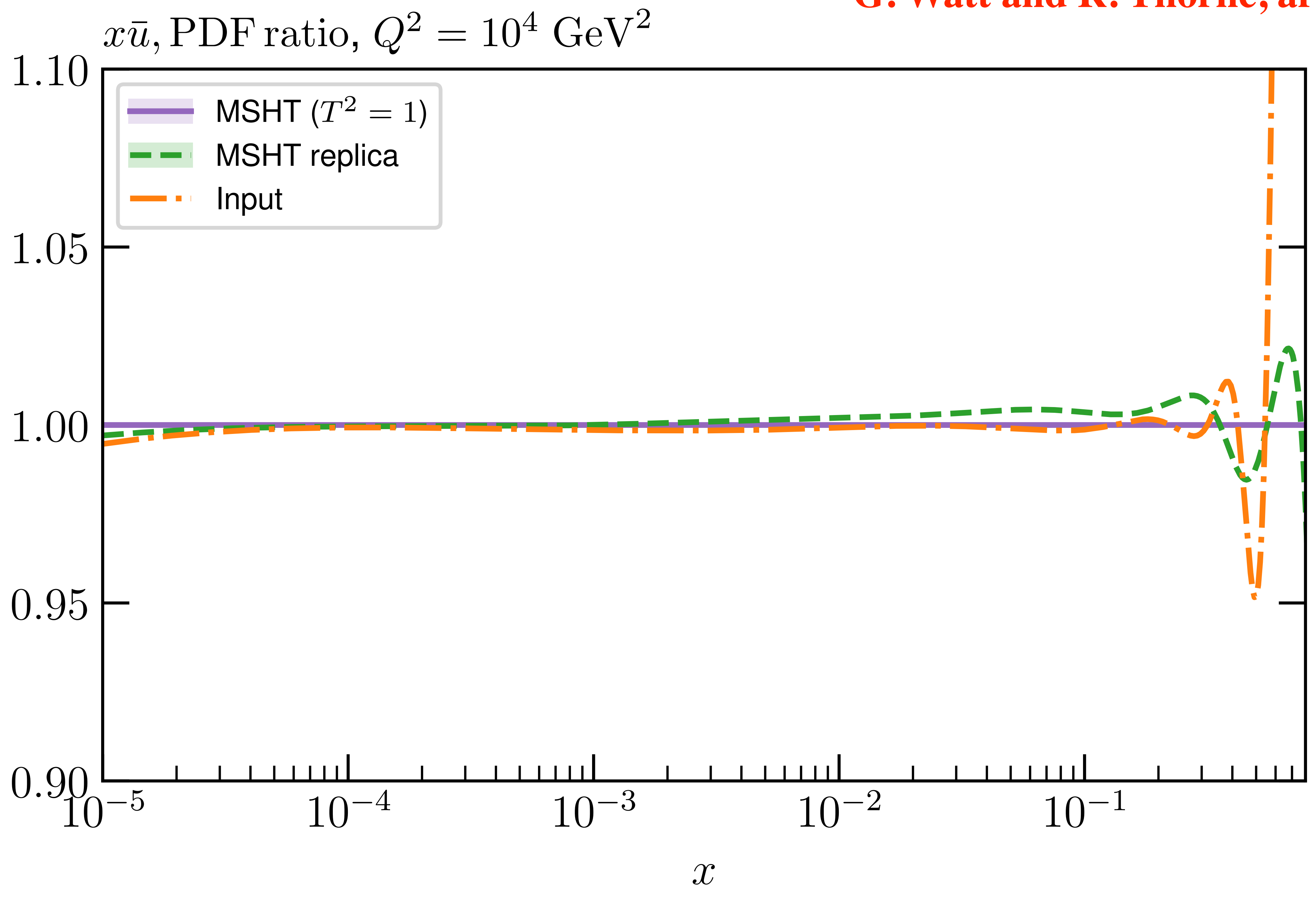
- In less well constrained regions deviation larger, e.g for u_V, d_V at low and high x and the $\bar{u}, \bar{d}$ at high x .
- Hence in extrapolation region input not always consistent within uncertainties
- As ~ outside data region not inconsistent (errors driven by data), but indicates more conservative error definition in these regions may be desirable (as tends to happen in NN approach).
- Though arguably no 'right' answer in true extrapolation region (too conservative vs. over-conservative).

Global Fluctuated Closure Test

- Exactly the same closure test settings, but fluctuate pseudodata according to experimental uncertainties. Fit quality $\chi^2 \sim N_{\text{dat}} \pm \sqrt{2N_{\text{dat}}}$ expected (and found).
- Test faithfulness of MSHT parameterisation by producing MC replica set - perform 100 replica fits.
- Error propagation used by NNPDF. Shown to be equivalent to Hessian $T^2 = 1$ w. fixed parameterisation.

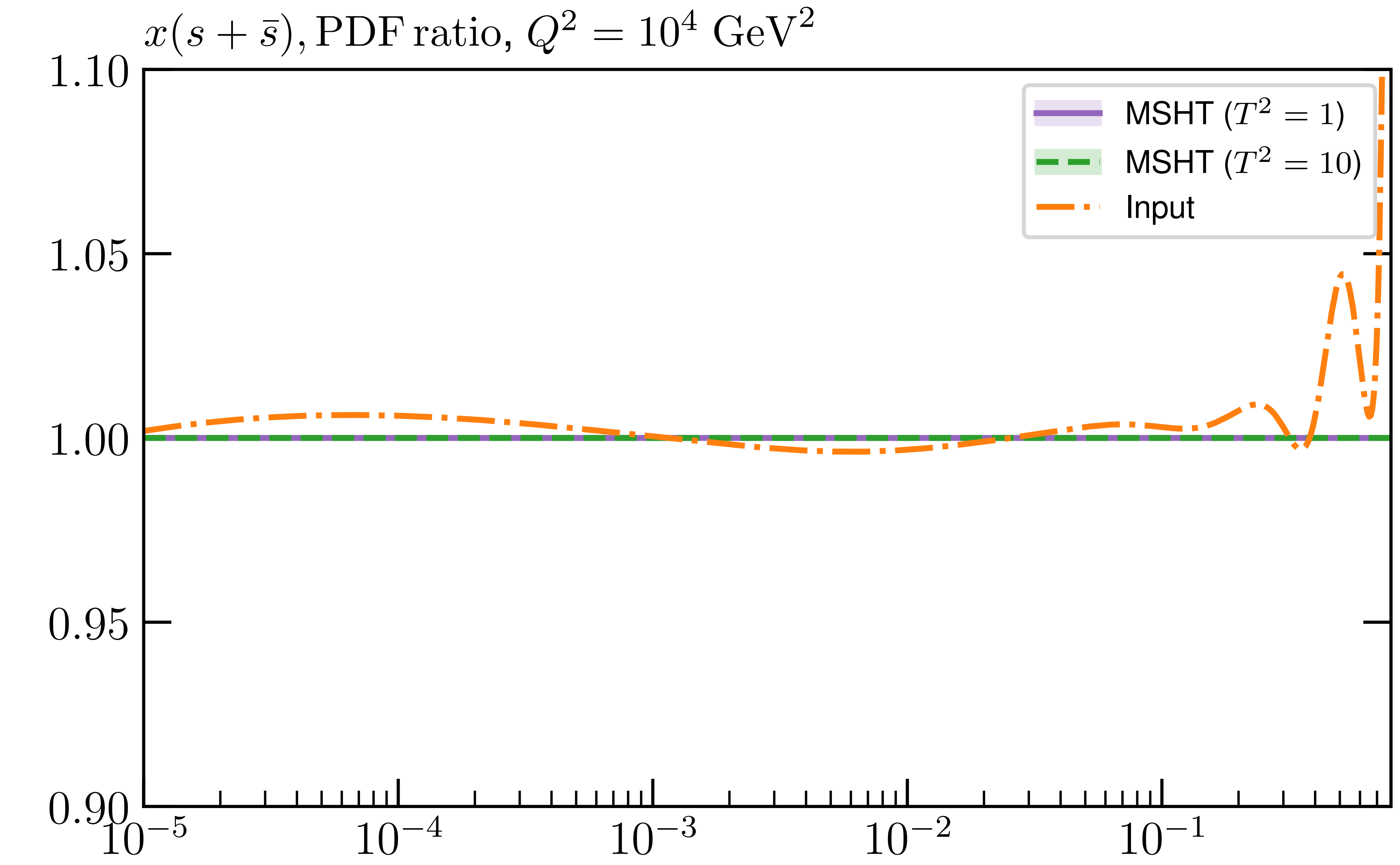
G. Watt and R. Thorne, arXiv:1205.4024

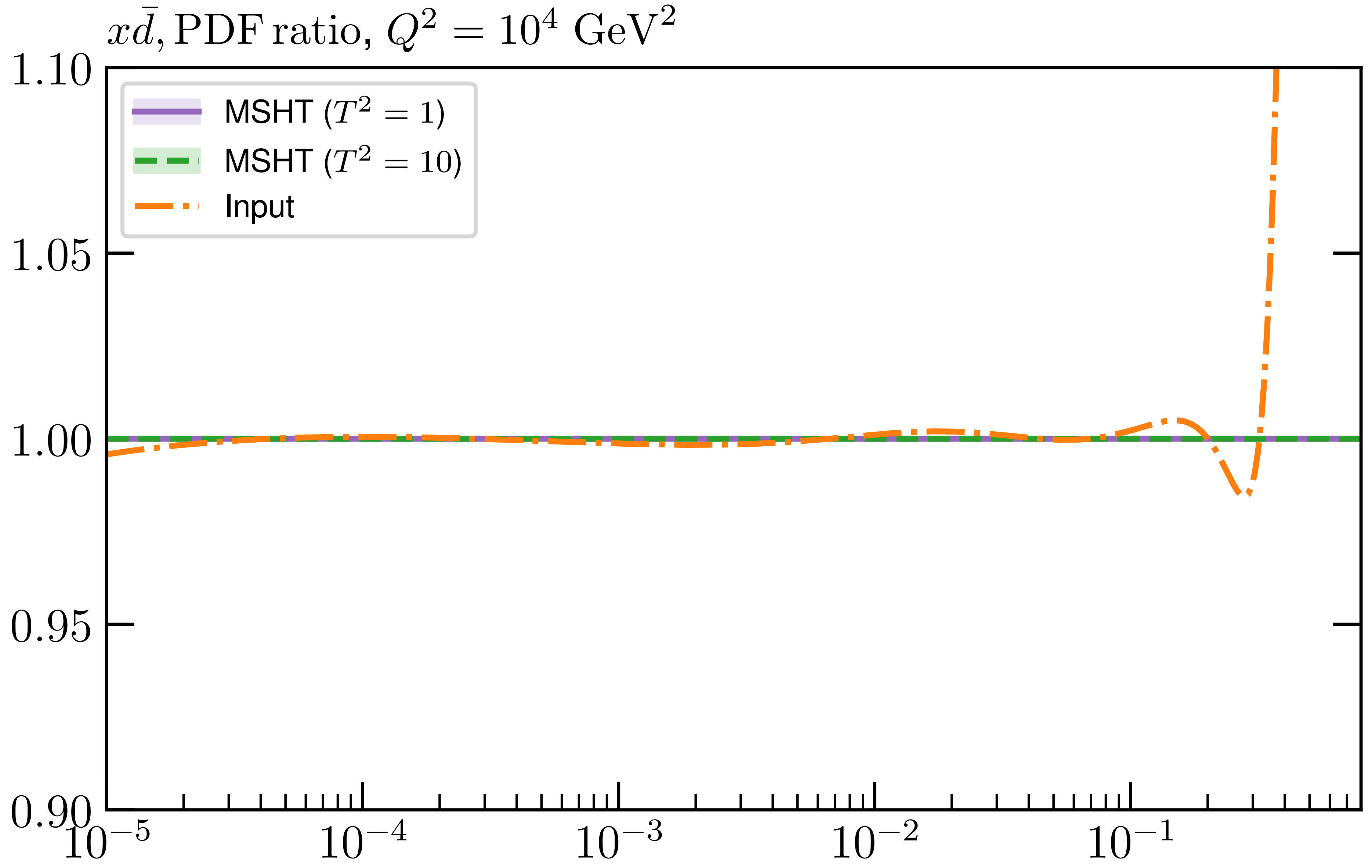


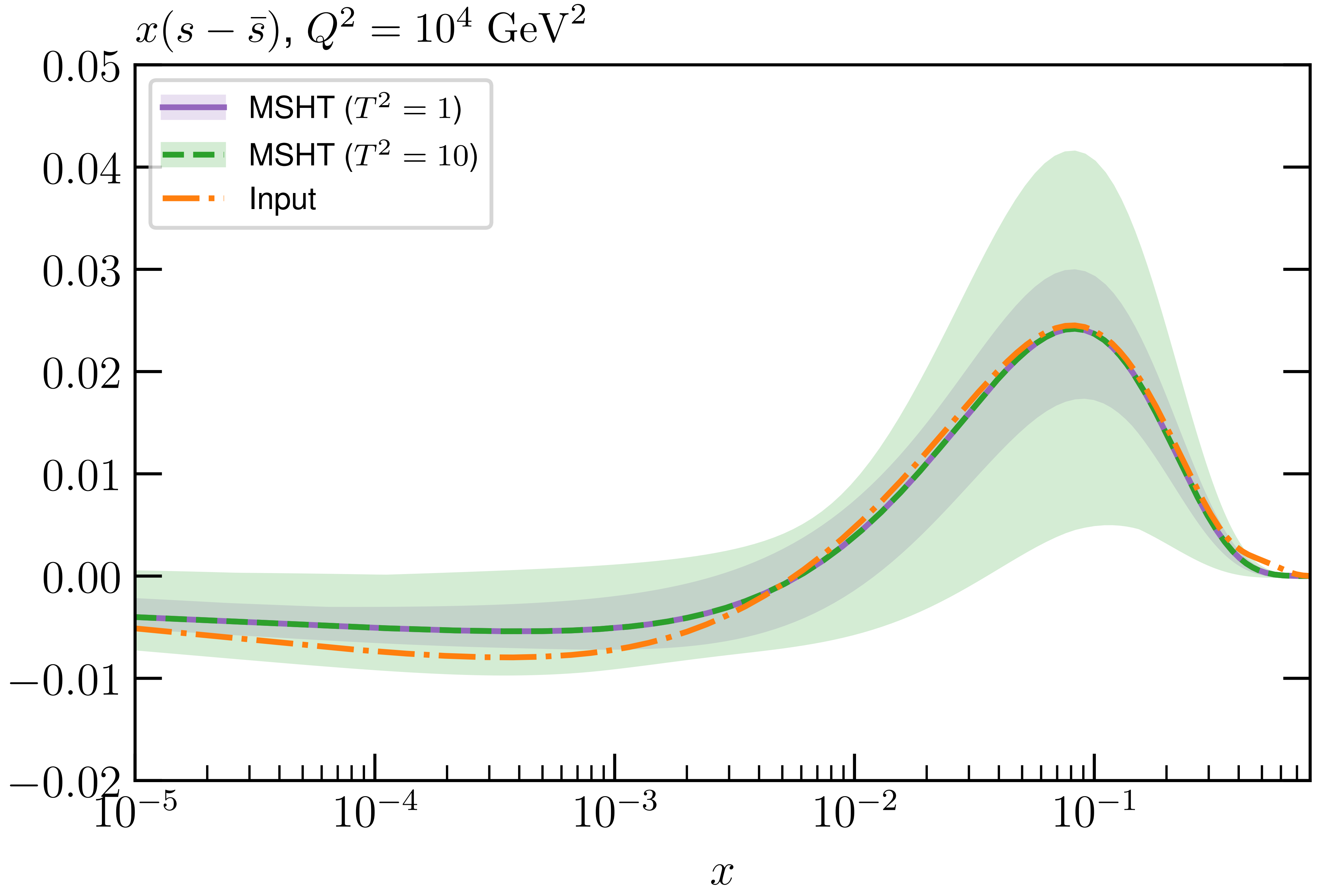


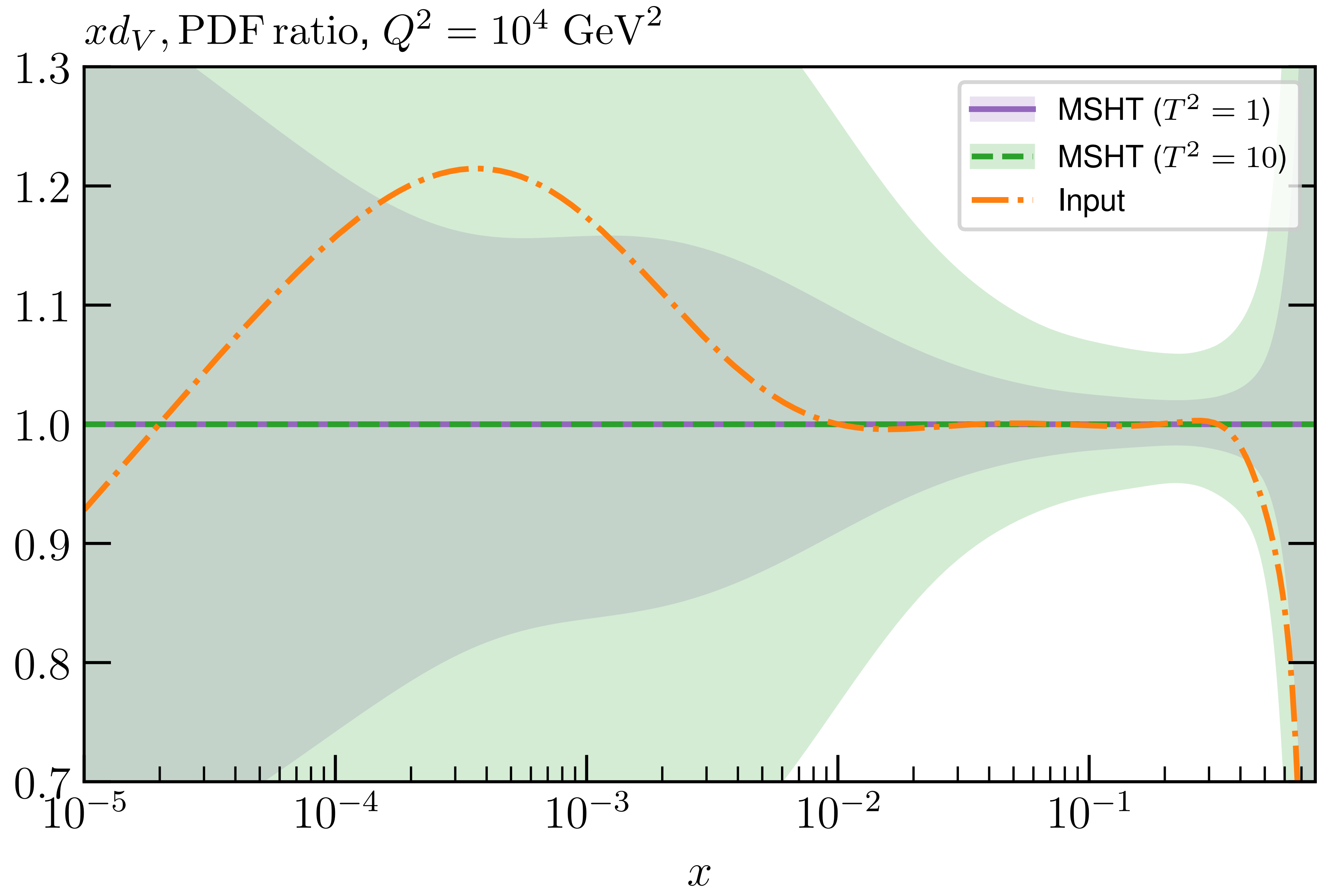
- Encouraging agreement between MC replica and Hessian uncertainties. Would not expect if issues with parameterisation inflexibility. PDF uncertainties more representative at high x .

Global Closure PDFs

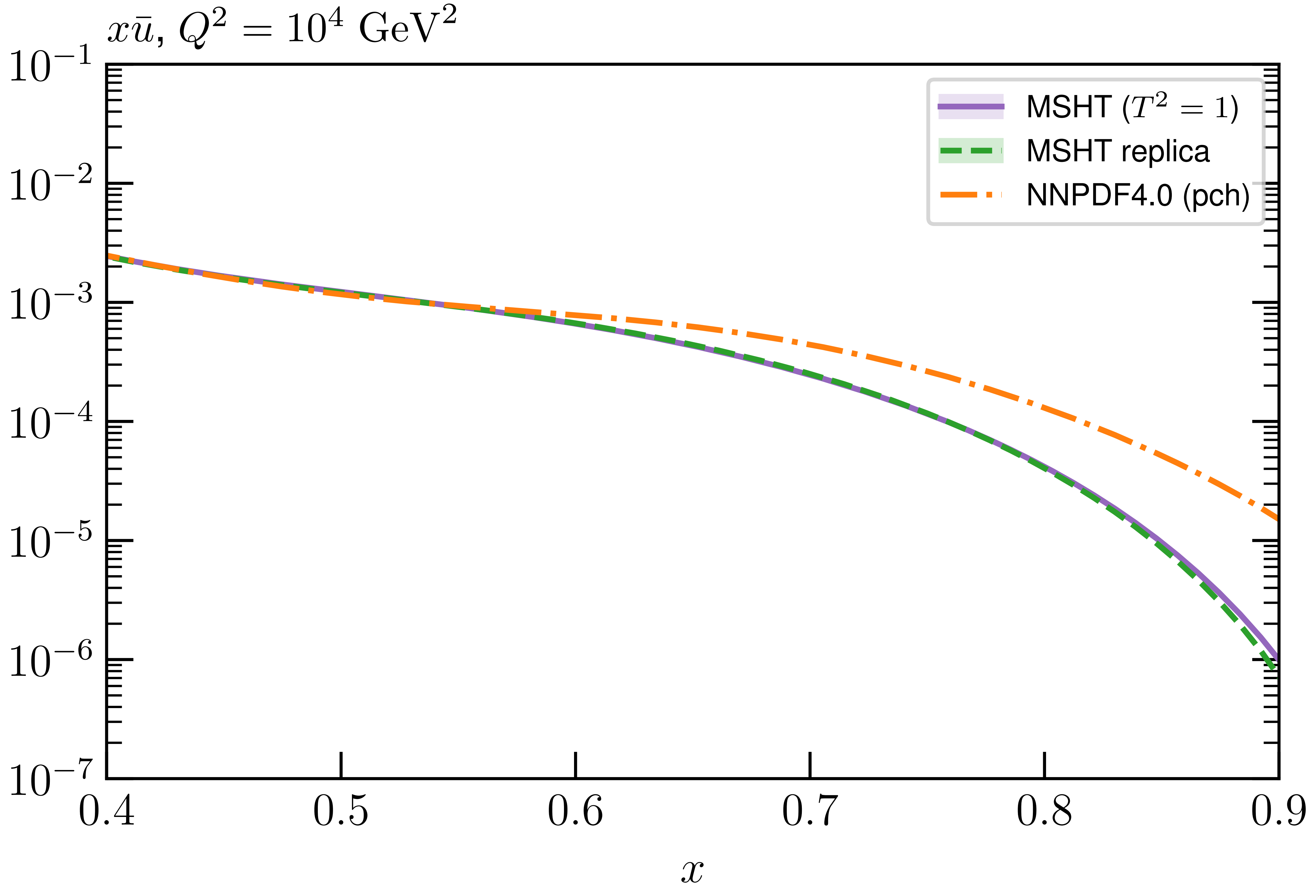


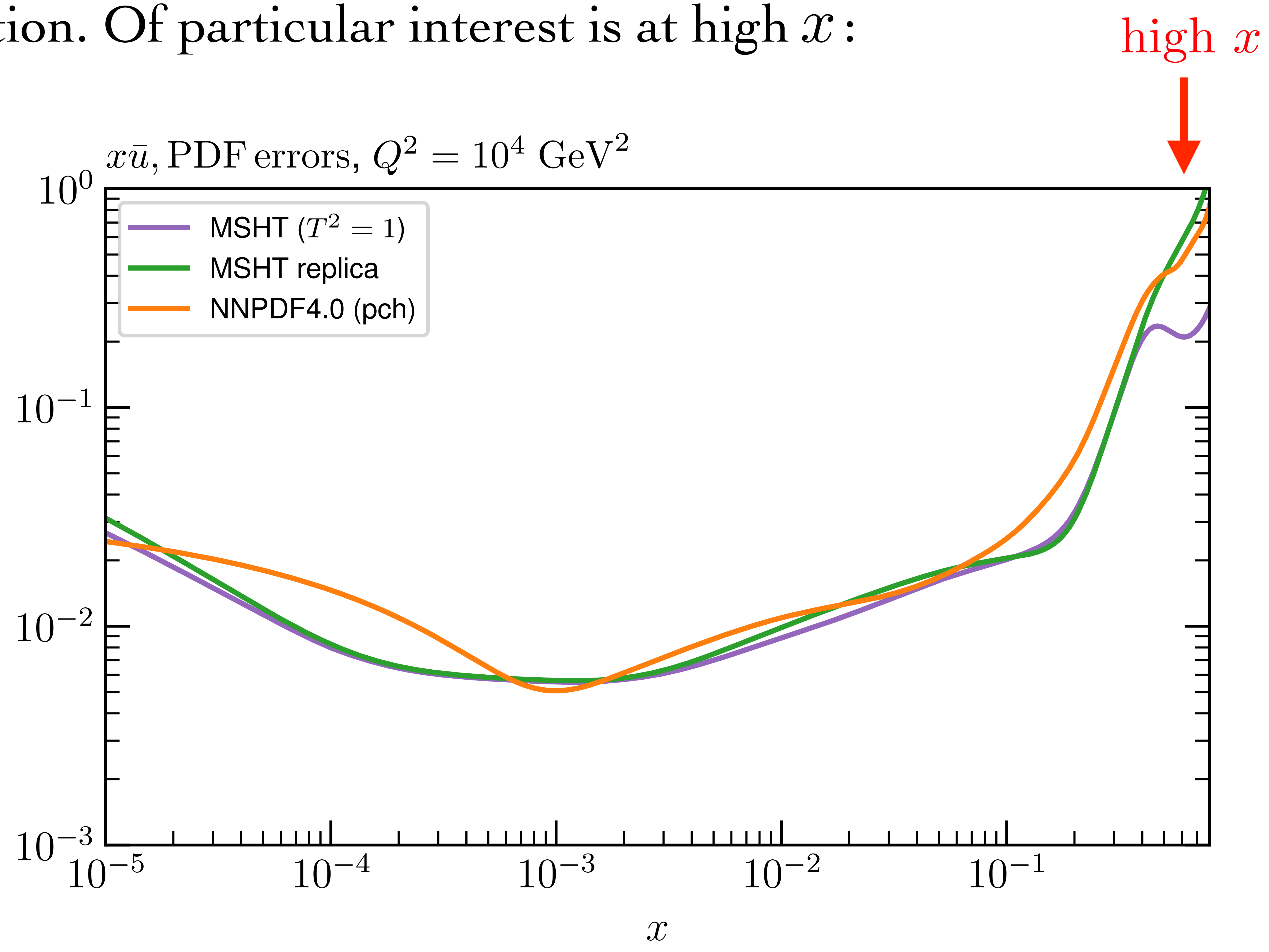






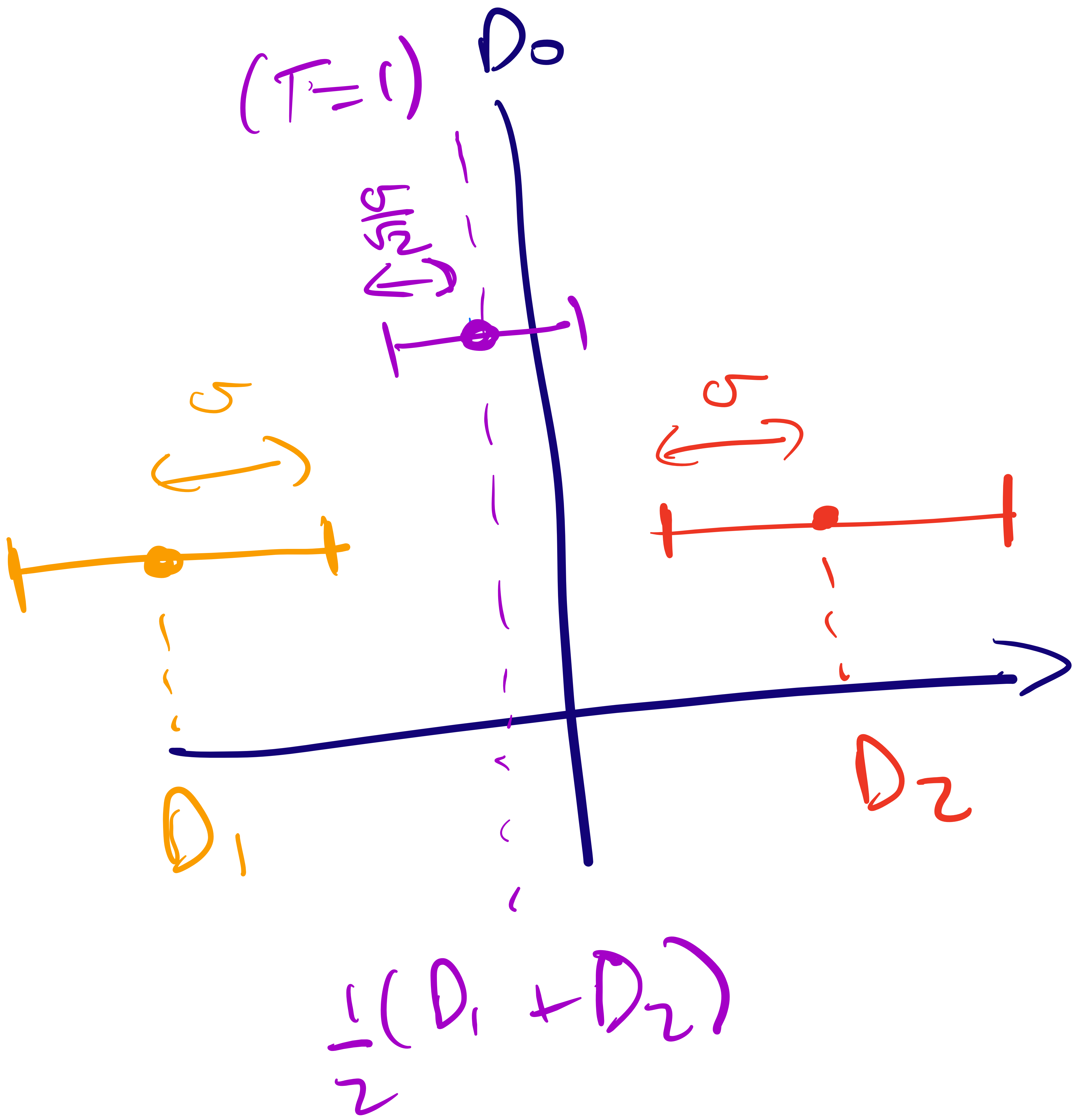
- However exact agreement between Hessian and MC replica approach only expected in exact Gaussian approximation. Away from this can see some deviation. Of particular interest is at high x:





- MC replica uncertainty much larger here - helps improve matching with input set.
- Much more in line with NNPDF uncertainty. Perhaps MC replica propagation (rather than NN) playing (most?) significant role here?

Tolerance: Toy Model



$$t_0 = \frac{1}{2}(D_1 + D_2)$$

$$t = t_0 + \Delta t$$

$$\Delta t = \pm \frac{\sigma}{\sqrt{2}}$$

Independent of particular values of $D_{1,2}$

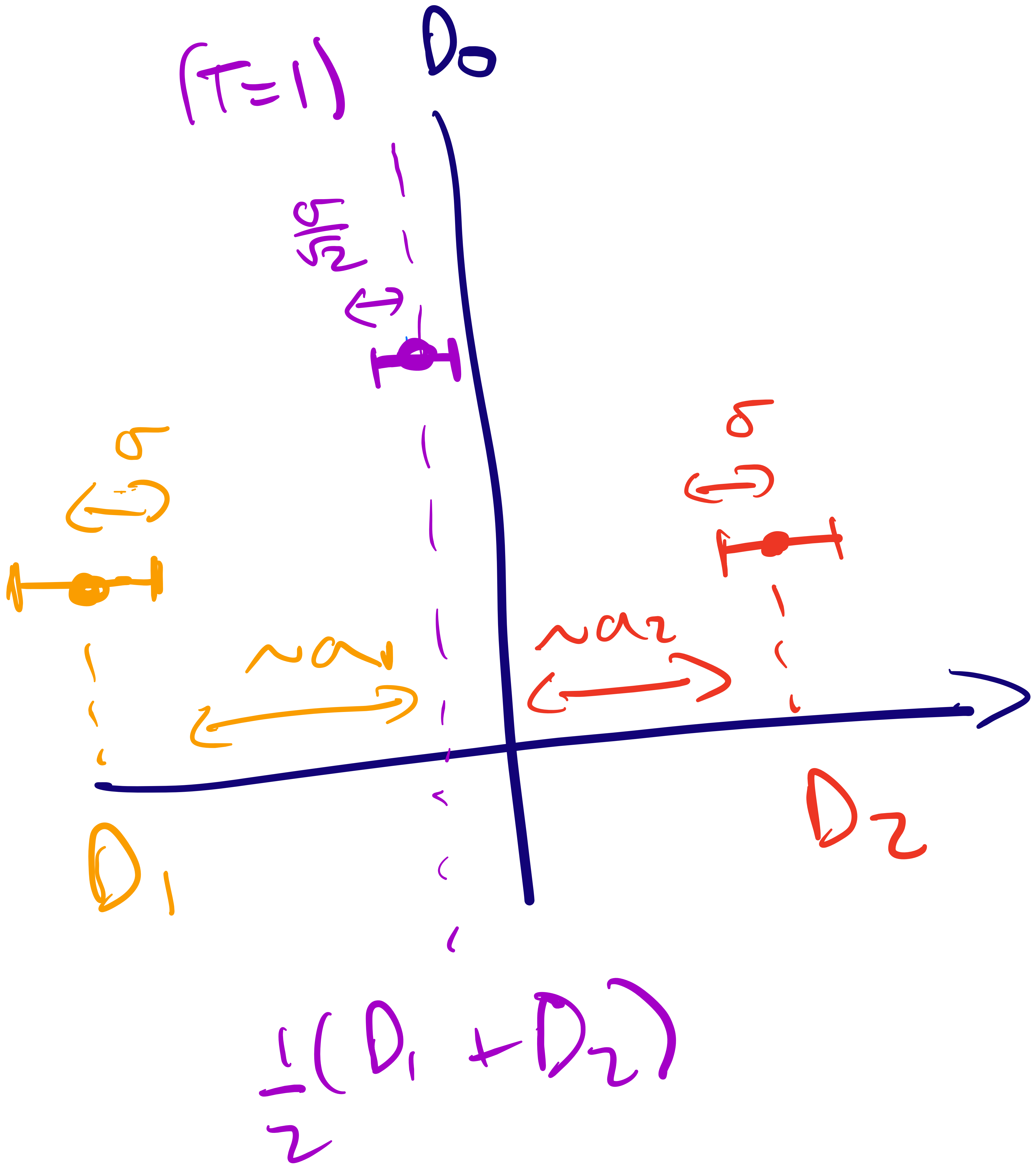
For **consistent** case

$$D_{1,2} = D_0 + \sigma\delta_{1,2}\ ,$$

this is **correct**.

$\delta_{1,2}$: univariate Gaussian

Tolerance: Toy Model



$$t_0 = \frac{1}{2}(D_1 + D_2)$$

$$t = t_0 + \Delta t$$

$$\Delta t = \pm \frac{\sigma}{\sqrt{2}}$$

Independent of particular values of $D_{1,2}$

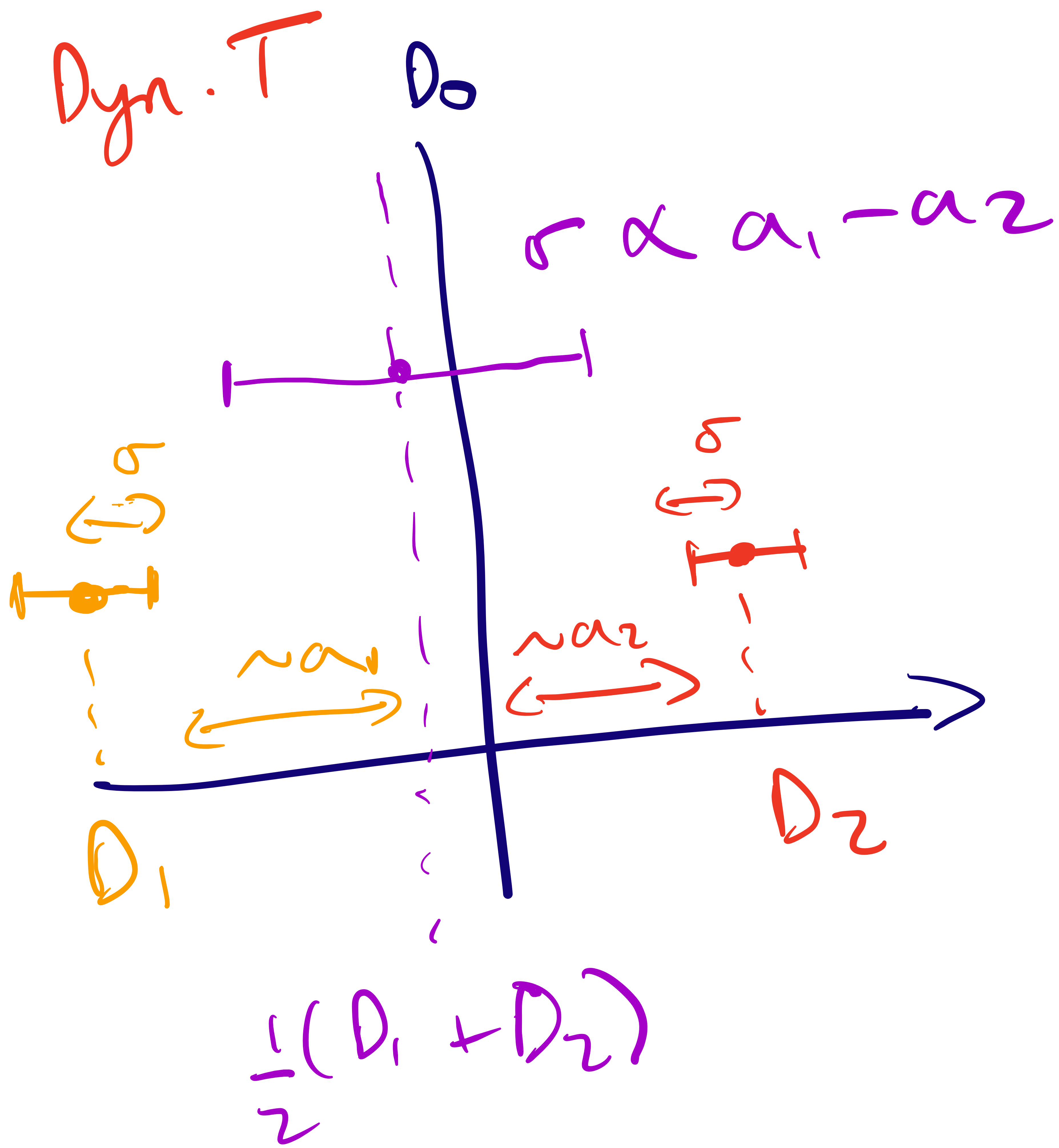
For **inconsistent** case

$$D_{1,2} = a_{1,2} + (D_0 + \sigma\delta_{1,2}) \ ,$$

this is **incorrect**.

$\delta_{1,2}$: univariate Gaussian

Tolerance: Toy Model



$$t_0 = \frac{1}{2}(D_1 + D_2)$$

$$t = t_0 + \Delta t$$

Applying **dynamic tolerance instead** find

$$\Delta t \propto a_1 - a_2$$

i.e. larger spread to account for tension.

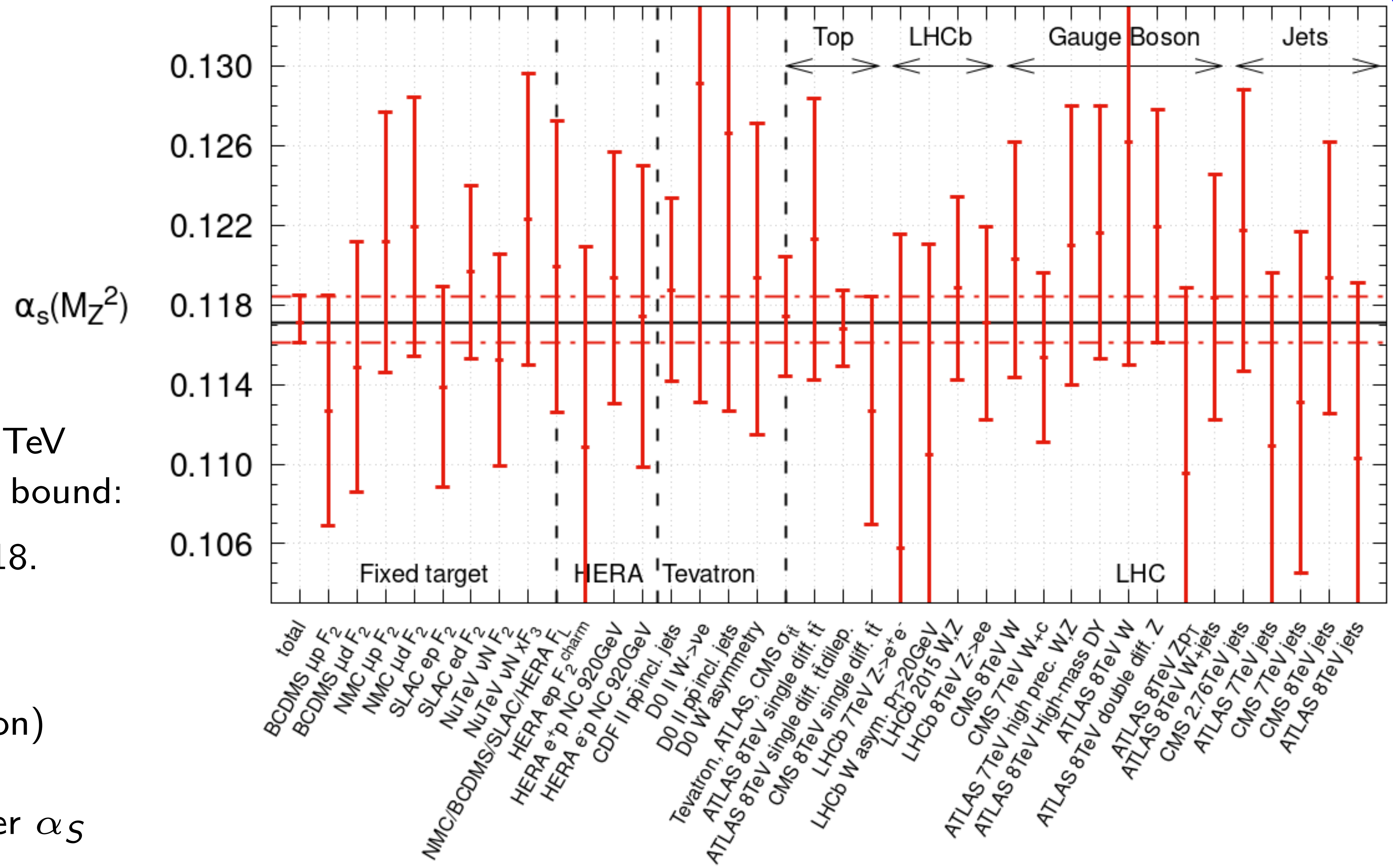
MSHT20 α_S bounds - NNLO

Consistent with α_S bounds seen in previous studies, and between orders (NNLO and aN3LO).

BCDMSp data strongest constraint upwards: $\Delta\alpha_S(M_Z^2) = +0.0014$.

SLACp and ATLAS 8TeV Zp_T both give upper bound: $\Delta\alpha_S(M_Z^2) = +0.0018$.

CMS/ATLAS (dilepton) $t\bar{t}$ single diff. would give lower/same upper α_S bound, but not used.



ATLAS 8 TeV Z data gives lower bound: $\Delta\alpha_S(M_Z^2) = -0.0010$.

NMC deuteron, ATLAS 8 TeV High Mass DY give lower bounds of $\Delta\alpha_S(M_Z^2)$ -0.0017, -0.0018.

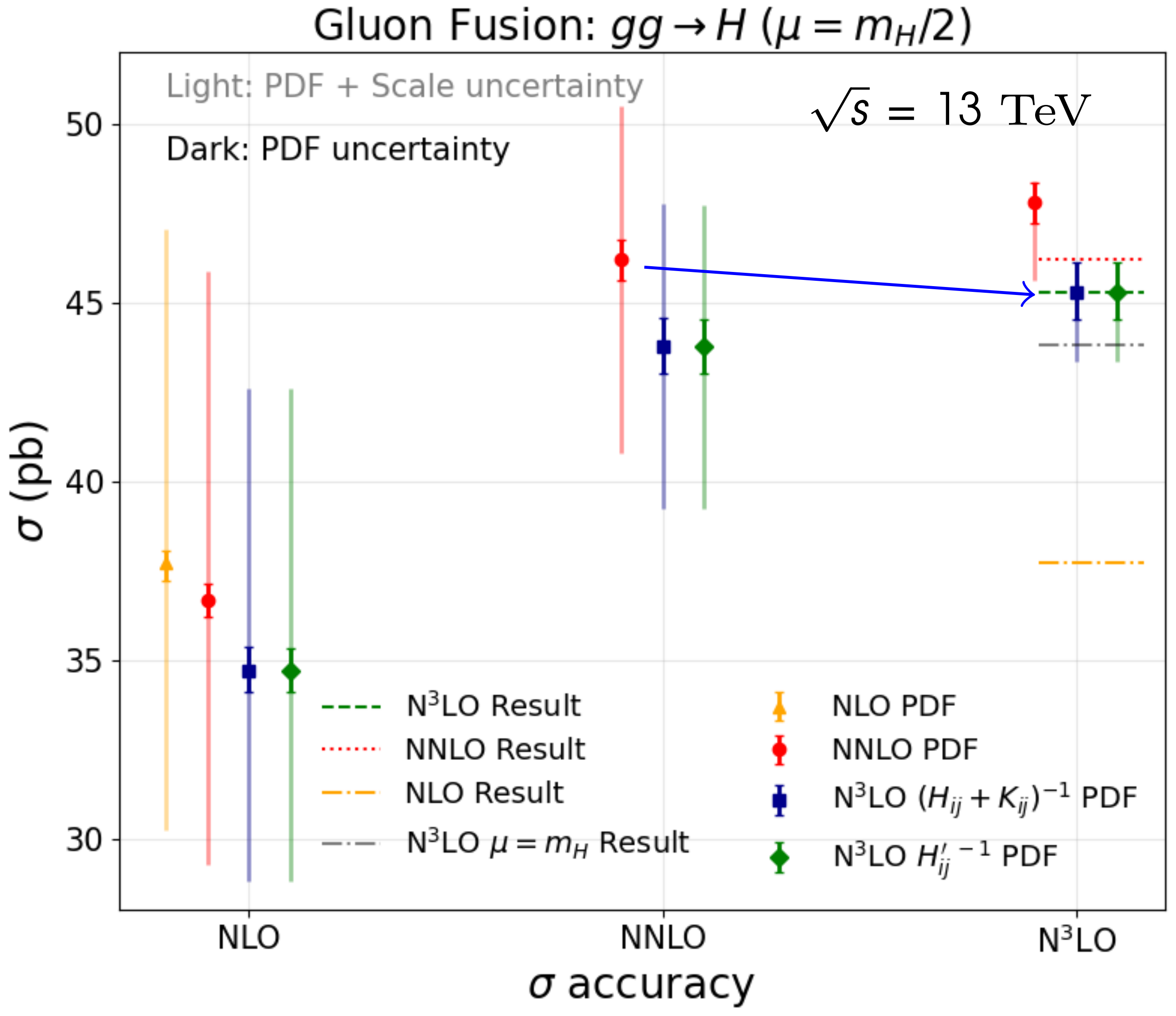
- Therefore upper/lower bounds are +0.0014/-0.0010 at NNLO.

$$\alpha_{S,\mathrm{NNLO}}(M_Z^2) = 0.1171 \pm 0.0014$$

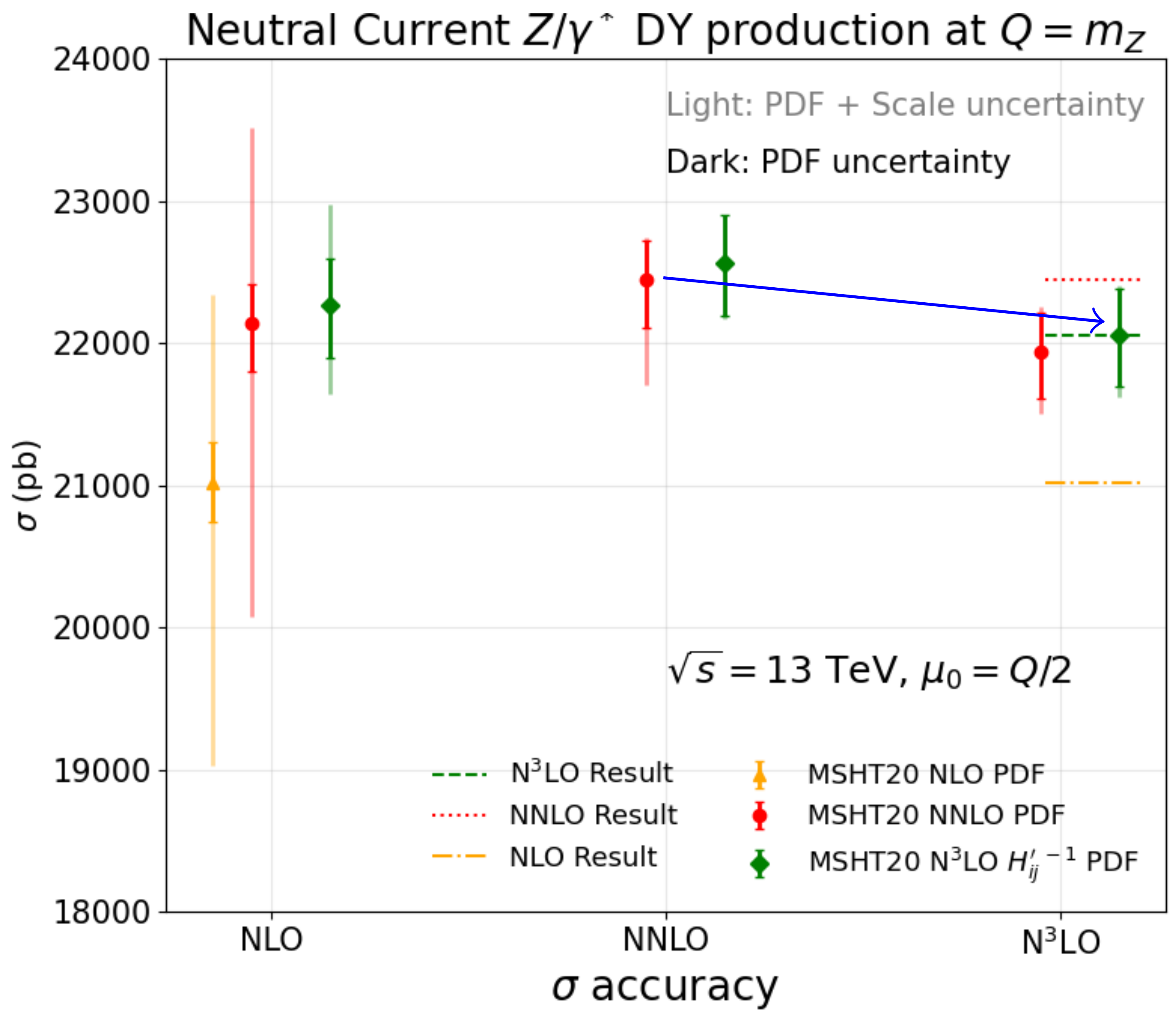
Consistent with World Average of 0.1180 ± 0.0009.

With thanks to T. Cridge

- Change in gluon corresponds to reduction in e.g. ggH at N3LO - improves stability.

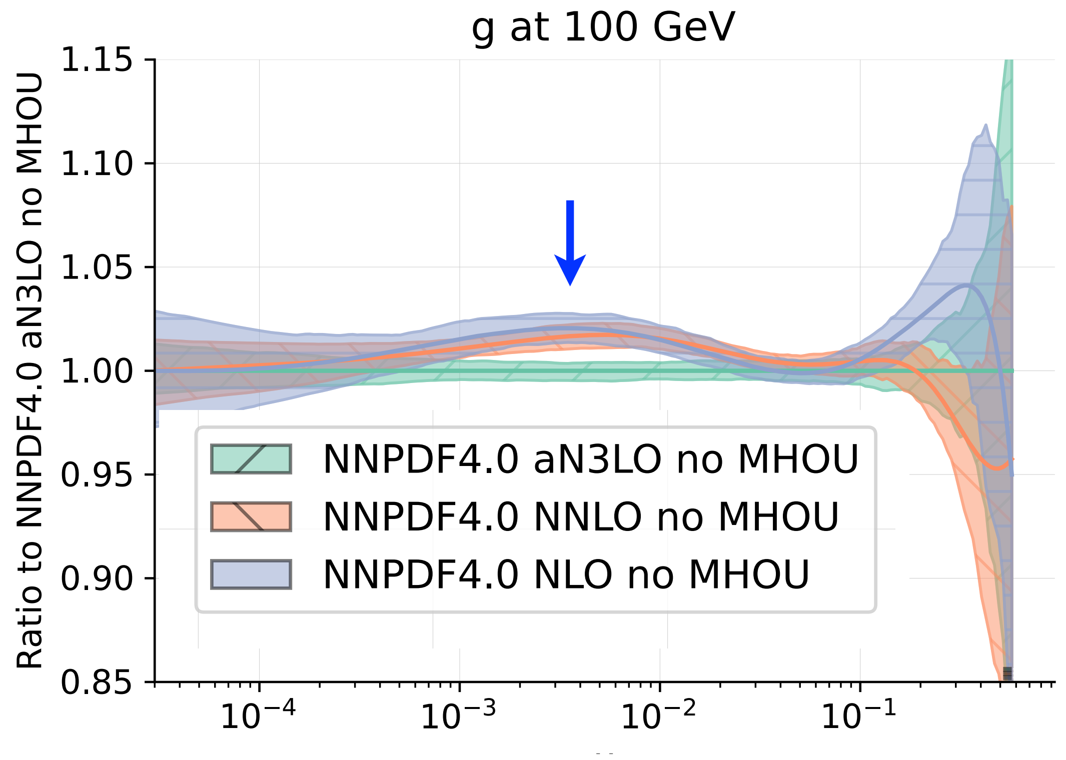


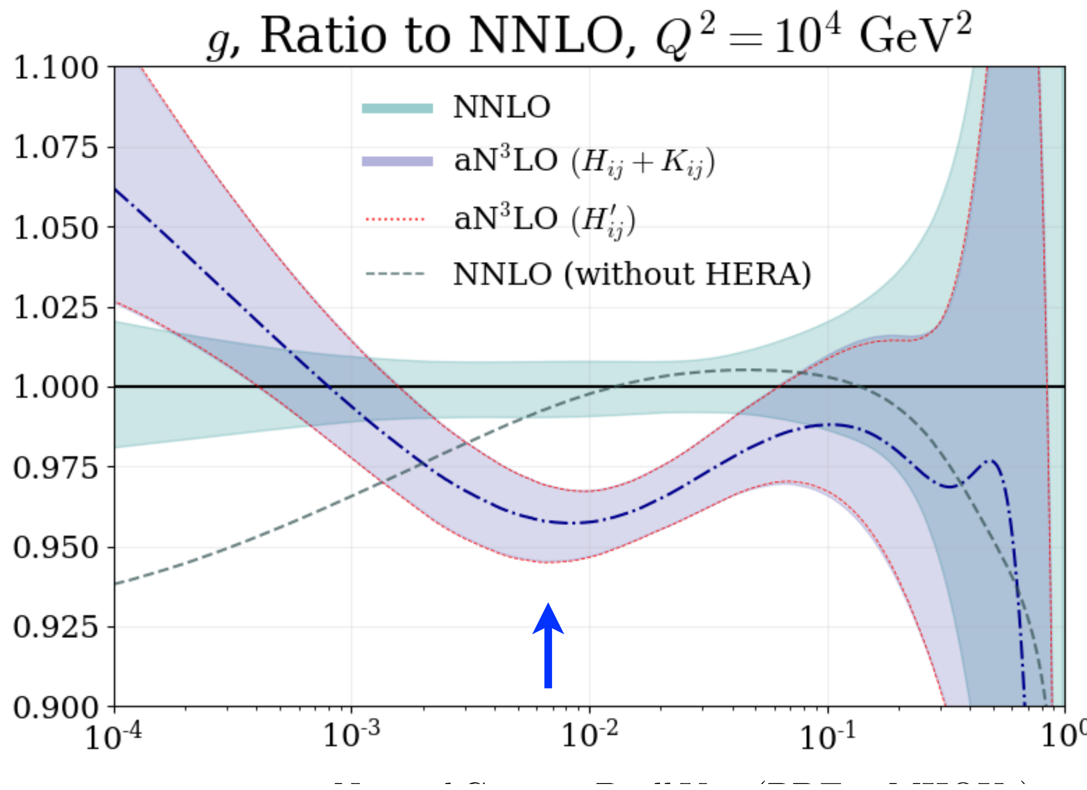
- Some increase in NC DY - again mild improvement in stability.



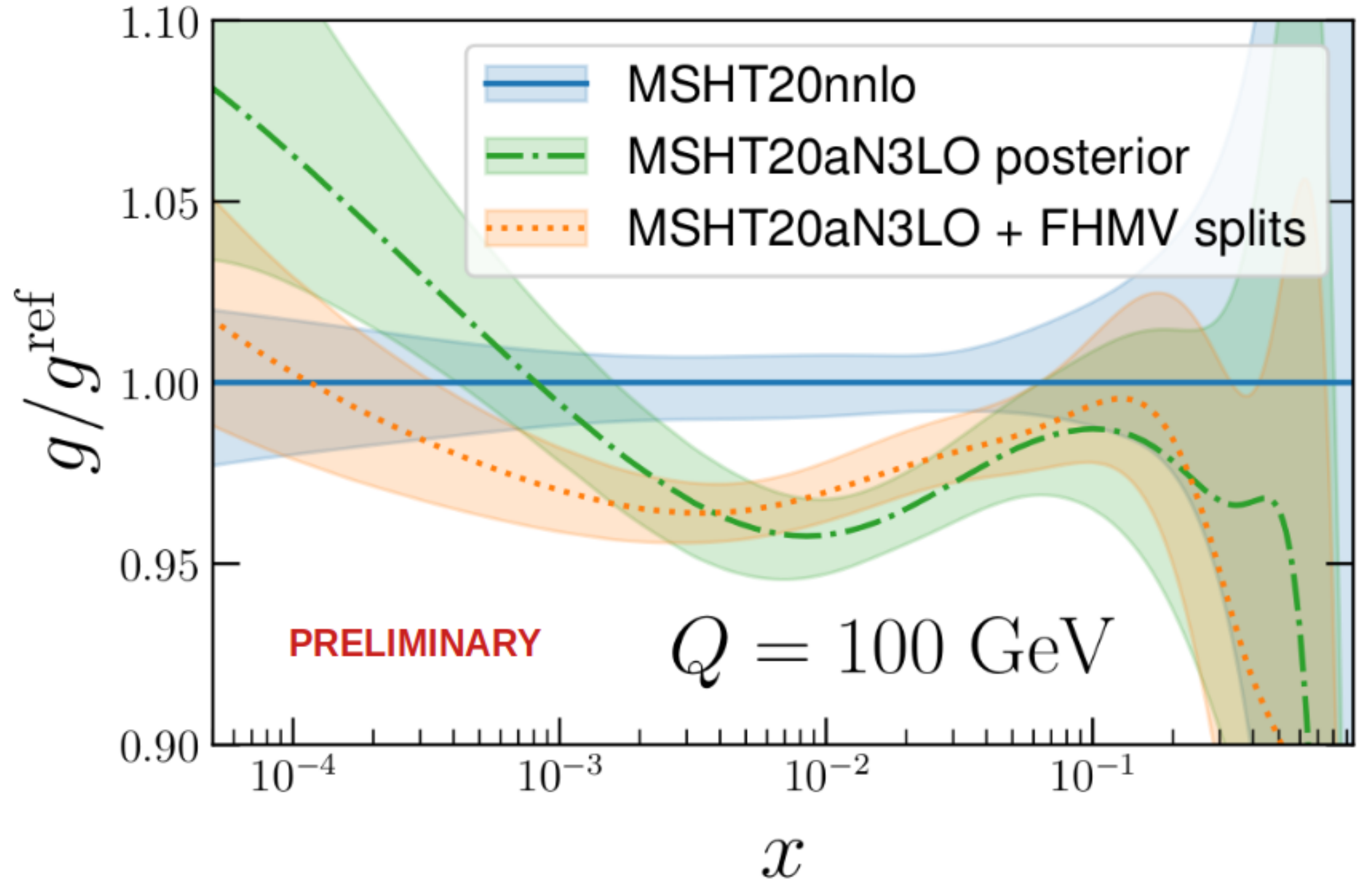
Stay tuned!

- NNPDF have also produced aN3LO fit. Gluon qualitatively similar, but change smaller:





- Has lead to detailed benchmarking of evolution. MSHT: updating to latest result has mild impact.

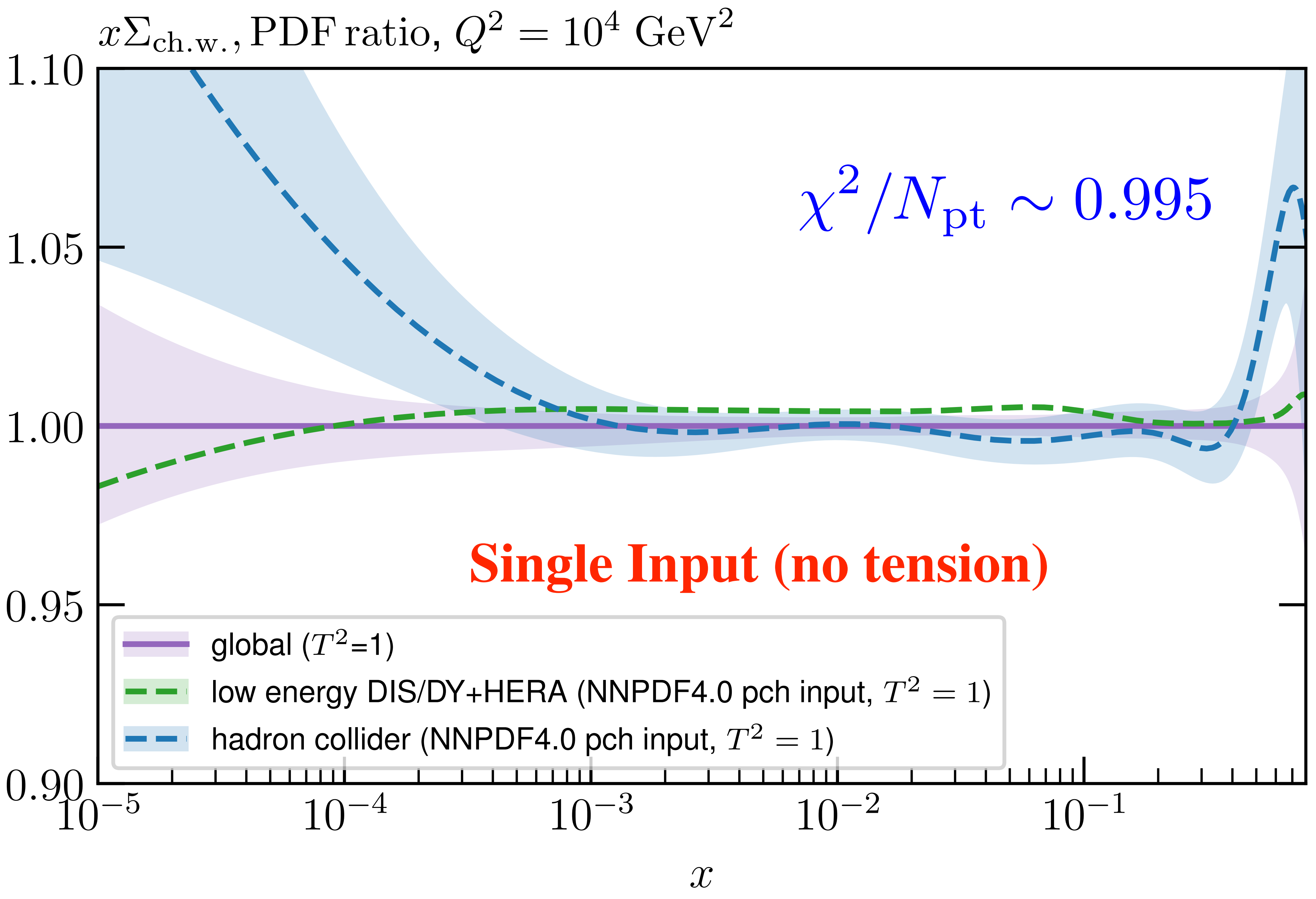


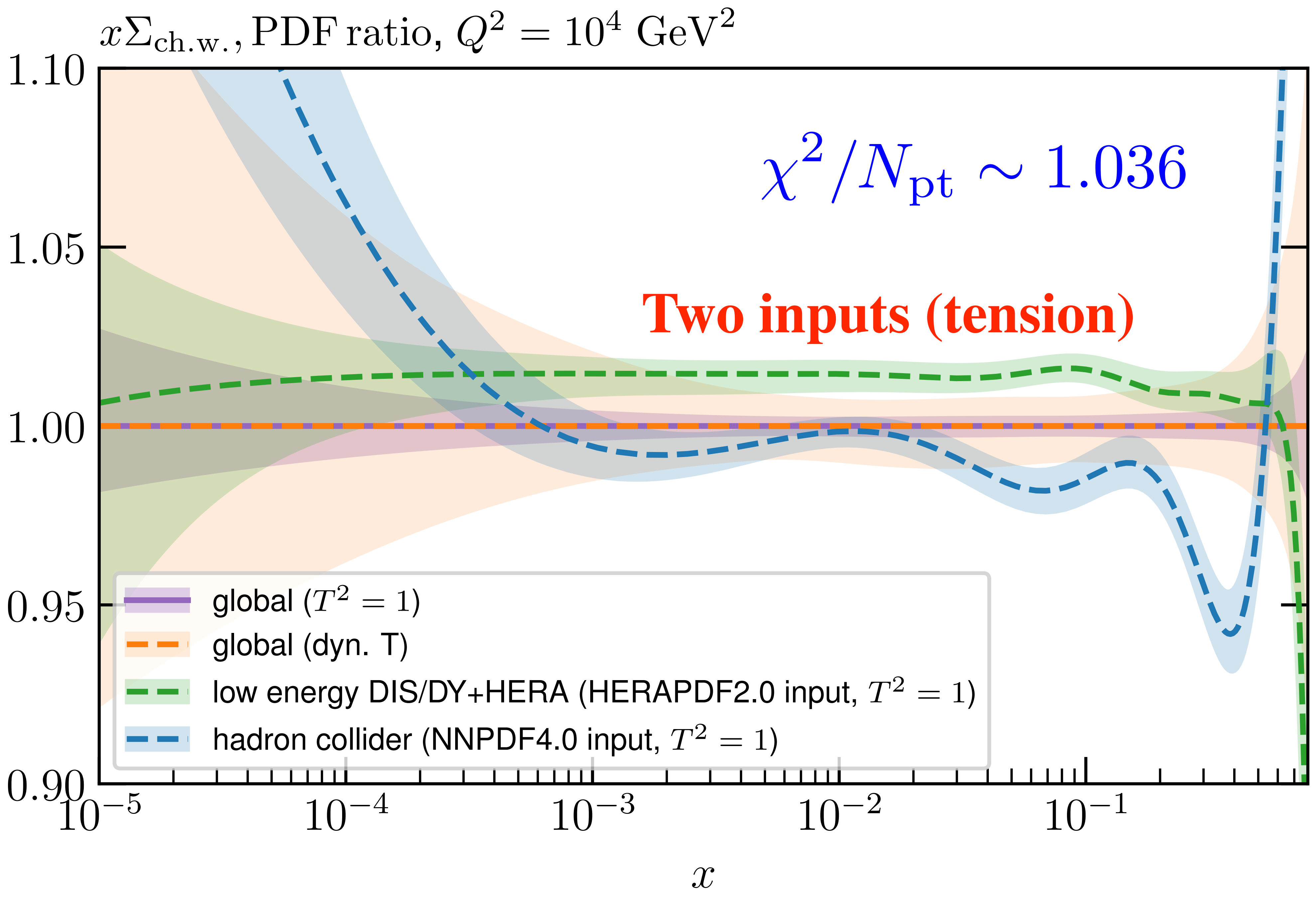
Tolerance (Again)

See also G. Watt and R. Thorne, arXiv:1205.4024

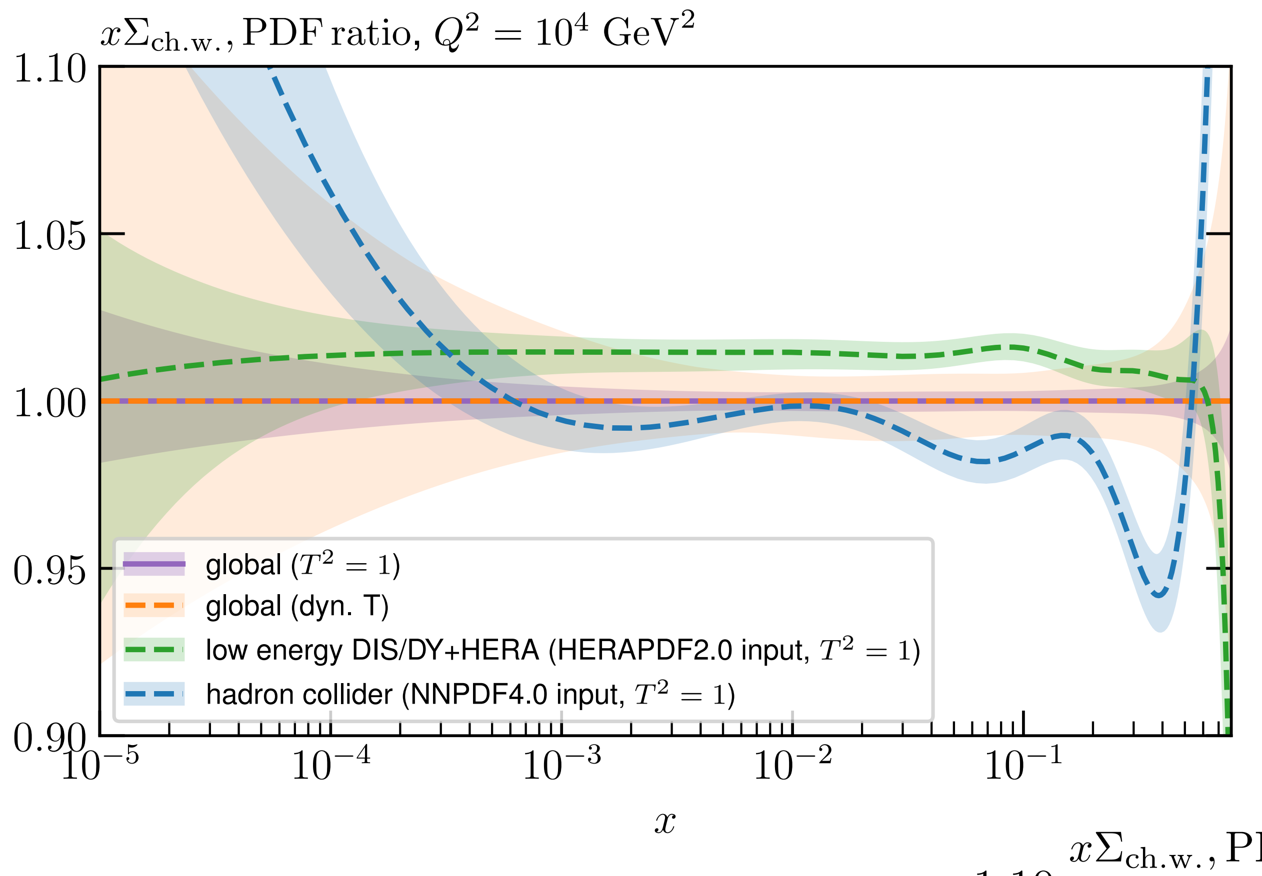
- Can also use closure test to motivate need for tolerance. Generate:
  - ★ **Fixed-Target DY + DIS** data with **HERAPDF2.0** input.
  - ★ **Hadron Collider** data with **NNPDF4.0** (pch) input.

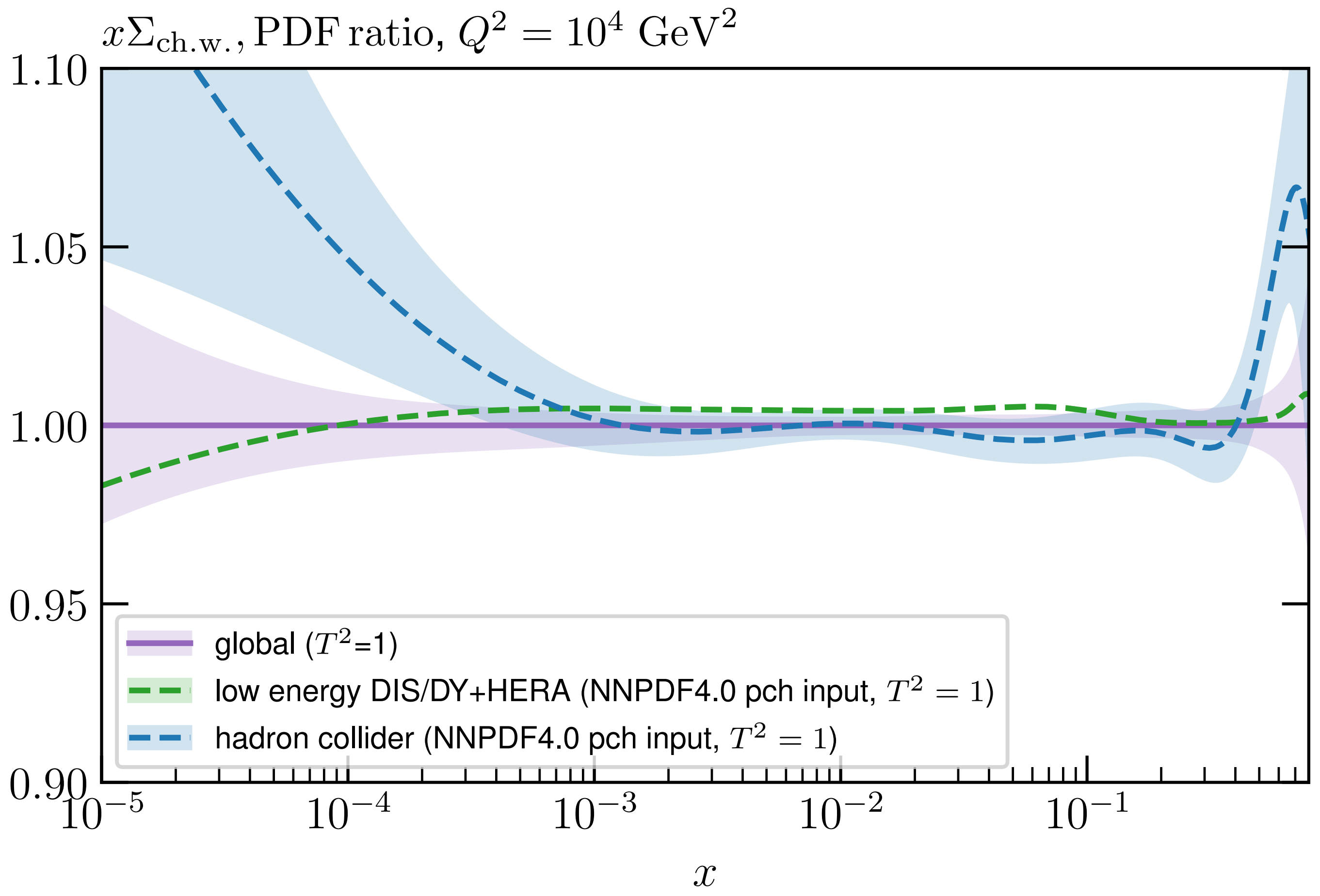
- Inputs are indeed in tension for various PDFs - simply model of incompatibility in fit. What do we find?

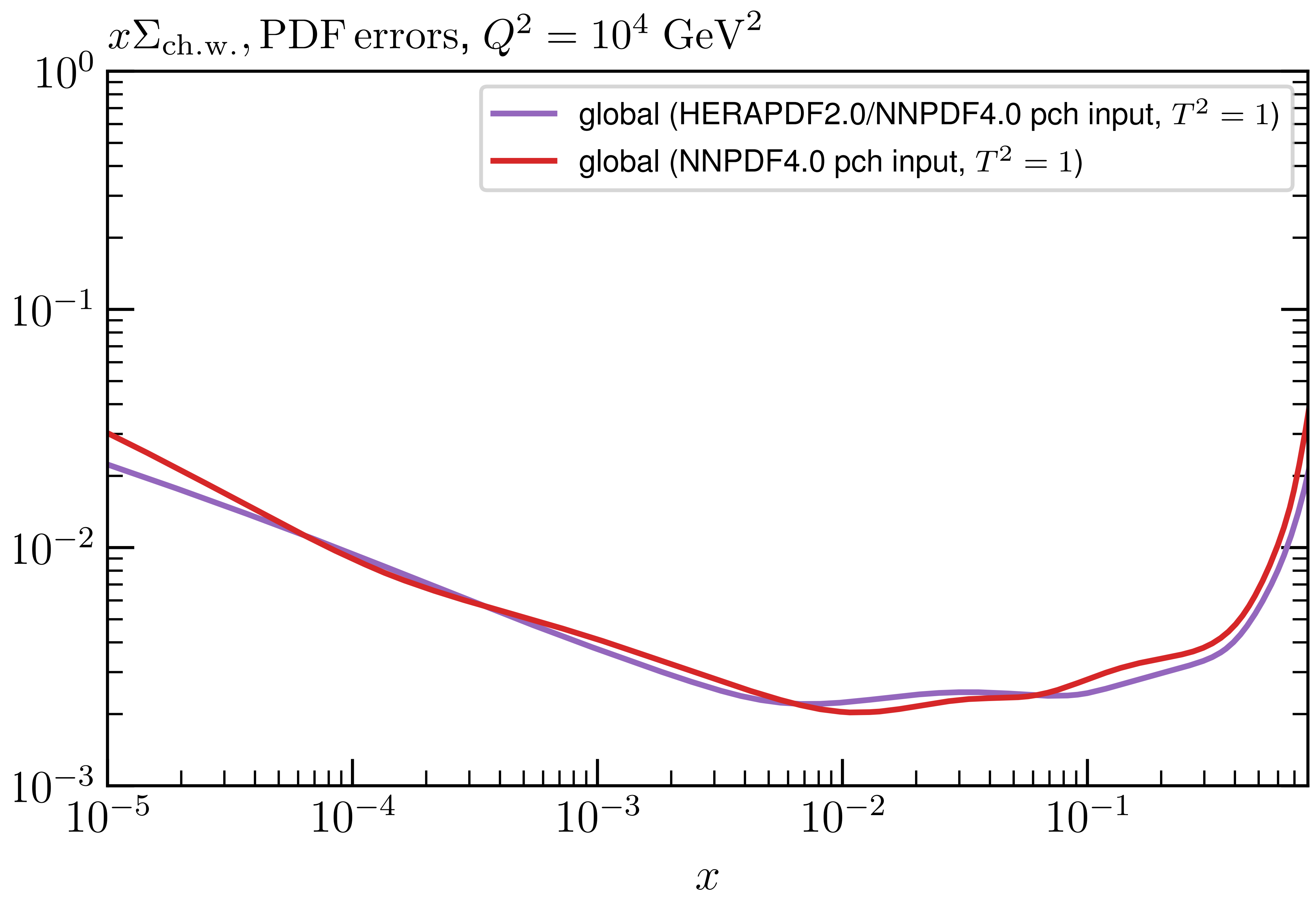




- Fit including tension lies ~ in the middle where tension appears. The $T = 1$ error clearly too small - enlarged MSHT tolerance does rather better. Crucially the $T = 1$ error v. similar between left + right…







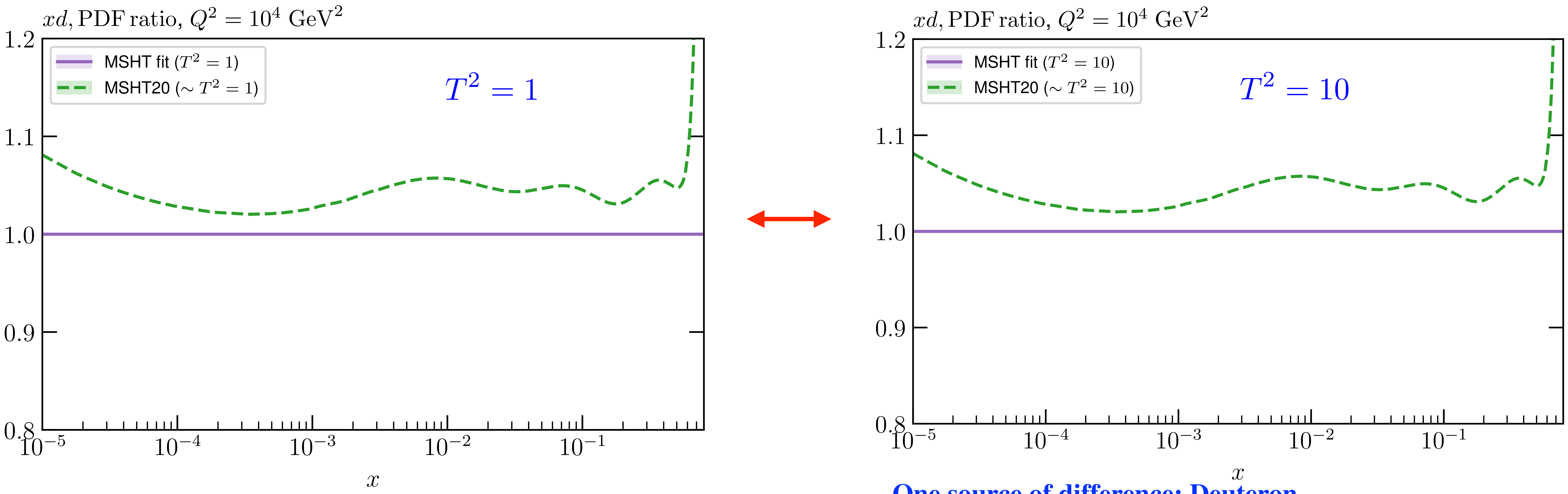
Backup

- This effect is completely expected. Can show in simple toy model: PDF uncertainties driven by the quoted experimental (theoretical) uncertainties whether underlying fit is self—consistent or not.
- Naive application of $T = 1$ criterion in such a scenario will lead to overly **aggressive errors**.

Tolerance (and Again)

Stay tuned for more!

- Final indication here. Perform fit to real NNPDF4.0 data + theory but with MSHT20 parameterisation.
- Compare to public MSHT20 fit: only difference due to differing data + theory.

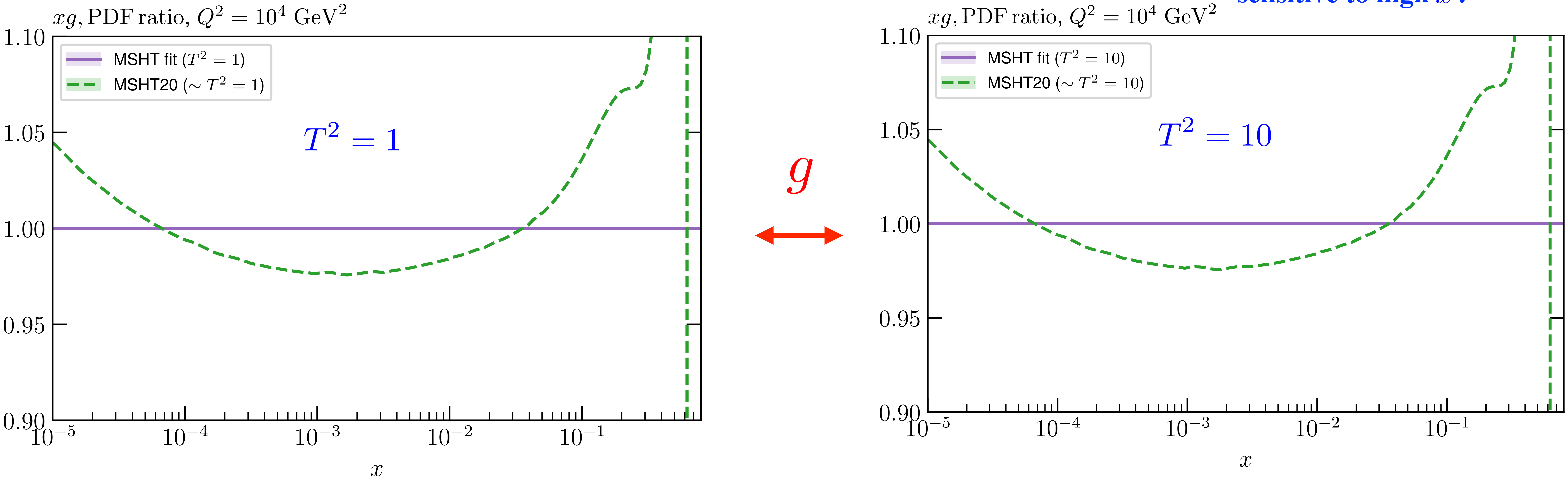


One source of difference: Deuteron Corrections

Tolerance (and Again)

- Final indication here. Perform fit to real NNPDF4.0 data + theory but with MSHT20 parameterisation.
- Compare to public MSHT20 fit: only difference due to differing data + theory.

One source of difference: LHC data sensitive to high x .



- Results arguably speak for themselves!
- Aside: MSHT parameterisation also performs very well vs. NN one. NNPDF uncertainties broadly $\sim T = 1$.

Stay tuned for more!

See LHL, DIS2024

N3LO - What do we know?

- **Approximate** $\neq$ **poorly known**!

$$P(x, \alpha_s) = \alpha_S P^{(0)}(x) + \alpha_S^2 P^{(1)}(x) + \alpha_S^3 P^{(2)}(x) + \alpha_S^4 P^{(3)}(x) + \dots$$

★ **Splitting functions:** a wealth of information. Moments & various limits, with much recent further progress.

G. Falcioni et al., arXiv:2307.04158, arXiv:2302.07593

Emanuele Nocera, Forward Physics and QCD at the LHC and EIC, Bad Honnef 23

Splitting Functions

Singlet (P_{qq}, P_{gg}, P_{gq}, P_{qg})
- large-n_f limit [NPB 915 (2017) 335; arXiv:2308.07958]
- small-x limit [JHEP 06 (2018) 145]
- large-x limit [NPB 832 (2010) 152; JHEP 04 (2020) 018; JHEP 09 (2022) 155]
- 5 (10) lowest Mellin moments [PLB 825 (2022) 136853; ibid. 842 (2023) 137944; ibid. 846 (2023) 1

Non-singlet ($P_{NS,v}$, $P_{NS,+}$, $P_{NS,-}$)
- large-n_f limit [NPB 915 (2017) 335; arXiv:2308.07958]
- small-x limit [JHEP 08 (2022) 135]
- large-x limit [JHEP 10 (2017) 041]
- 8 lowest Mellin moments [JHEP 06 (2018) 073]

$$F_2(x, Q^2) = \sum_{\alpha \in H,q,g;\beta \in q,H} (C^{VF,n_f+1}_{\beta,\alpha} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2))$$

★ **DIS:** massless coefficient functions known (+ massive high Q^2). Massive low Q^2 approx. known.

$$f_\alpha^{n_f+1}(x, Q^2) = [A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)](x)$$

★ **Heavy Flavour:** again wealth of information. Moments & various limits, with much recent progress.

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \dots \equiv \sigma_{N3LO} + \dots$$

★ **Hadronic Cross Sections:** while much progress made, thus far not useable in PDF fits.

- First three ingredients now largely known with sufficient precision to give close to a N3LO fit. Final ingredient clearly the bottleneck for that - approximation + uncertainty required.