Gravitational Waves from Nnaturalness

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Hierarchy problem

Higgs mass parameter is sensitive to short distance physics

➡ Naively suggests new physics at the weak scale

- Basic EFT reasoning, "dimensional analysis"
- No other elementary scalars observed in nature
- History: Electron self-energy, Ginzburg-Landau, QCD pions, …

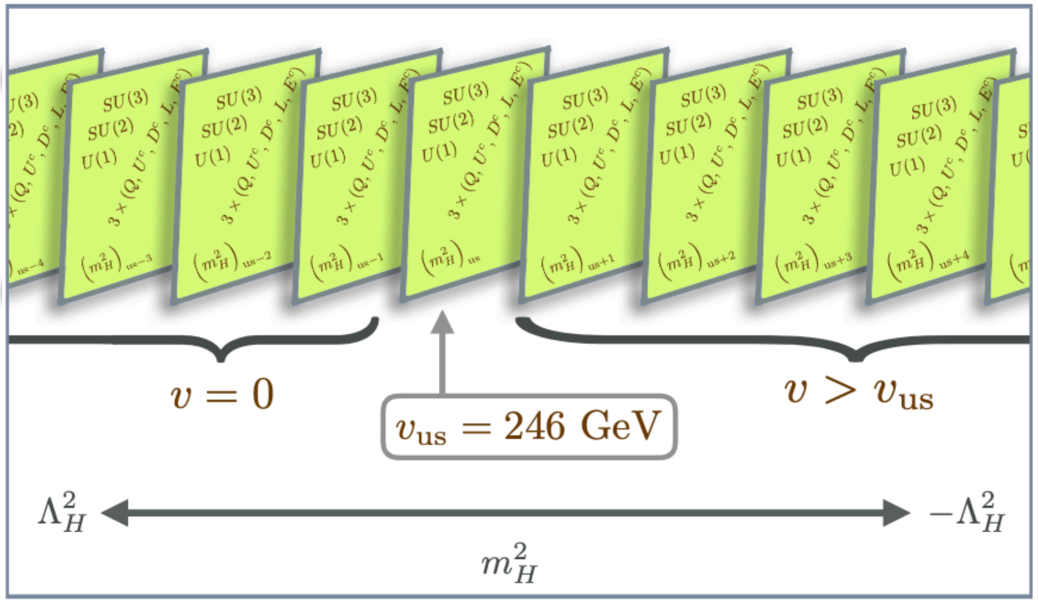
Paradox: No new physics observed at LHC (yet!)

- Maybe there is no problem? Perhaps the question is misguided…
- C.C. a bigger problem - Maybe "Naturalness" is like aether?(!)

Naturalness motivates novel BSM & new signatures
—keep thinking, keep looking!

Nnaturalness

[Arkani-Hamed, Cohen, Tito D'Agnolo, Hook, Kim, Pinner '16]



- Introduce N copies of the Standard Model (SM) with varying Higgs mass parameters over the range of the cutoff of the theory (softly broken sector permutation symmetry)

$$m_{H_i}^2 = -\frac{\Lambda_H^2}{N}\,(2i + r)\,, \qquad -\frac{N}{2} \le i \le \frac{N}{2}.$$

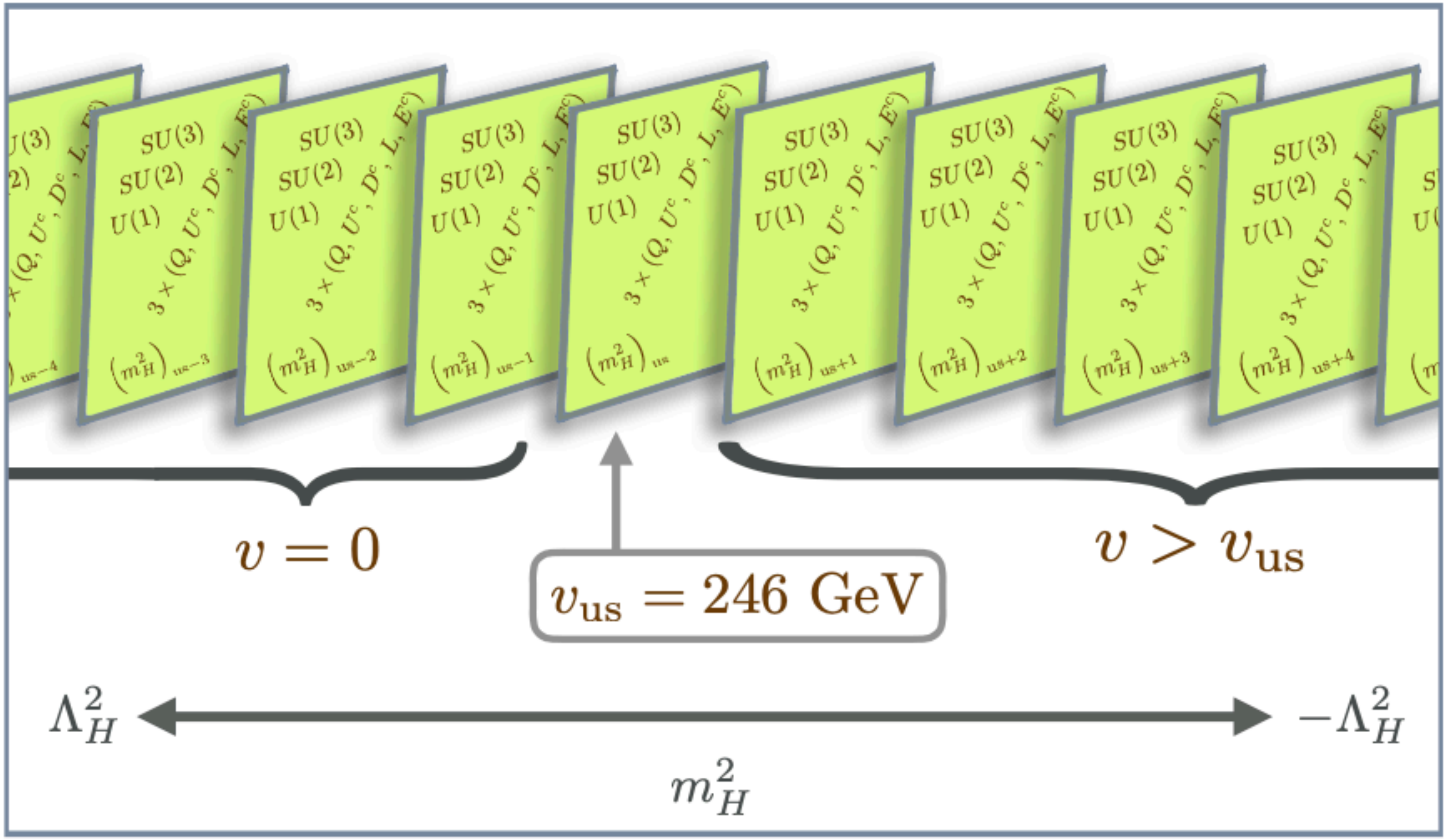
- A "reheaton" ϕ dominates the energy density of the universe following inflation

- If the reheaton is light, it can dominantly decay to the sector with the lightest Higgs mass (see below), which is thus identified with the Standard Model.

- Main probes come from cosmology (e.g., $\Delta N_{\rm eff}$); collider probes are remote

The sectors

$$m_{H_i}^2 = -\frac{\Lambda_H^2}{N}(2i + r), \qquad -\frac{N}{2} \leq i \leq \frac{N}{2}.$$



Exotic sectors	Standard Model (SM)	SM-like sectors
$i < 0$	$i = 0$	$i > 0$
$m_{H_i}^2 > 0$	$m_{H_0}^2 = -(88\,\text{GeV})^2$	$m_{H_i}^2 < 0$

Gravitational waves from Nnaturalness

- Conventional wisdom: QCD with $N_f \geq 3$ light quark flavors features a first order chiral symmetry breaking phase transition [Pisarski, Wilczek '84]
  - Argument based on absence of infrared stable fixed points in a renormalization group analysis of the linear sigma model for the quark bilinear order parameter
  - Some lattice studies confirm the claim, while others challenge it, so the issue is not fully settled.
- In the exotic sectors, $i < 0,\ m_{H_i}^2 > 0$, all six quarks are light compared to the QCD confinement scale
- The exotic sectors may thus undergo a first order QCD chiral symmetry breaking phase transitions, which produces an associated stochastic gravitational wave signal [Witten '84] [Hogan '86]
- The key question is whether one can populate the exotic sectors with enough energy density to observe such a signal

Reheaton

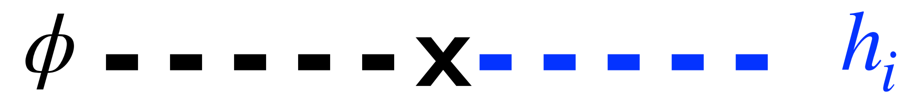
- We consider a real scalar reheaton ϕ for concreteness
- The reheaton couples universally to the Higgs fields of each sector:

$$\mathscr{L}_\phi \supset -a\phi \sum_i |H_i|^2 - \frac{1}{2} m_\phi^2 \phi^2$$

- The cosmology of the model begins following inflation with the reheaton dominating the energy density of the universe
- The reheaton will decay to each sector. The energy density ρ_i stored in each sector i will be proportional to the partial width Γ_i
- The coupling a cancels out in the ϕ branching ratios, but governs the reheat temperature
- We fix $N = 10^4$, which allows for a solution to the little hierarchy problem, $\Lambda_H \sim 10\,\mathrm{TeV}$, and avoids overclosure from stable particles in other sectors

Reheaton decays to SM-like sectors

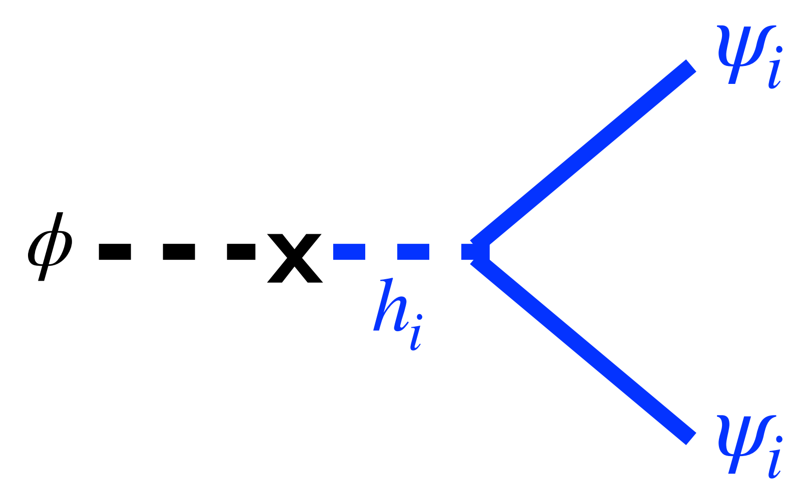
- Following electroweak symmetry breaking, the reheaton mixes with the Higgs boson h_i of each sector



$$\theta_i \simeq \frac{a v_i}{m_{h_i}^2} \approx \frac{a}{m_{h_i}}$$

- The partial decay widths scale as

$$\Gamma_i \propto \frac{1}{m_{h_i}^2}$$

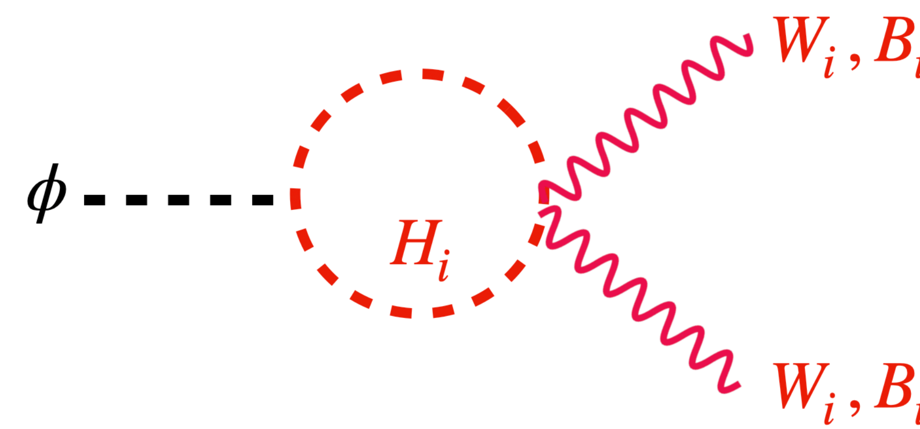


The reheaton width to the SM is typically the largest, implying that the SM can dominate the energy density of the universe following reheating

Reheaton decays to exotic sectors

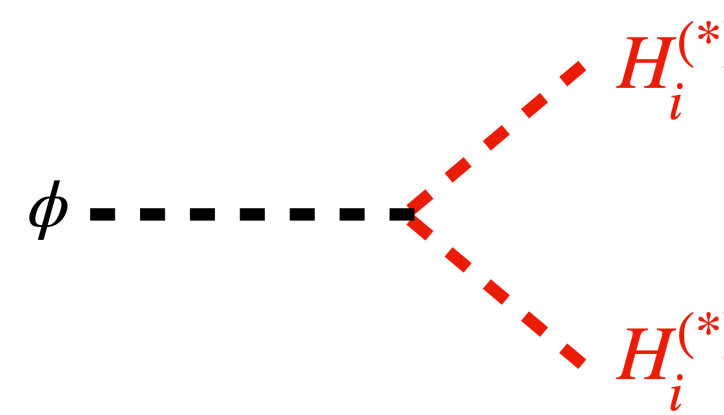
- The small effects of electroweak symmetry breaking in the exotic sector can be neglected as far as reheaton decays are concerned

- For heavier exotic sectors, we expect $m_\phi < m_{H_i}$, so that the reheaton decays via $\phi \to W_i W_i, B_i B_i$, with partial decay width $\Gamma_i \propto 1/m_{H_i}^4$



Energy density stored in the heavier exotic sectors is insignificant

- For the first, lightest exotic sector, as r increased, $m_{H_{-1}} \sim m_\phi/2$, and $\phi \to H_{-1}H_{-1}$ can be sizable



Energy density stored in the first exotic sectors can be substantial

Properties of the first exotic sector

- Higgs squared mass is positive. EW symmetry is unbroken in absence of QCD

- QCD becomes strong at $\Lambda_{-1} \sim \mathcal{O}(100\,\mathrm{MeV})$. All six quarks are very light in comparison to Λ_{-1}. A quark condensate forms, $\langle \bar{q} q \rangle_{-1} \sim 4\pi f_{\pi_{-1}}^3$, spontaneously breaking $SU(6)_L \times SU(6)_R \to SU(6)_V$ chiral symmetry

- This condensate also breaks the weakly gauged $SU(2)_L \times U(1)_Y$ subgroup down to electromagnetism

- The quark condensate triggers a tadpole for the Higgs, inducing a Higgs VEV $\langle H \rangle_{-1} \neq 0$, generating masses for the quarks and leptons

- There are 35 pions. 3 are eaten by the $W^{\pm}, Z$ bosons. The others acquire masses due to explicit chiral symmetry breaking by the quark Yukawa couplings

First exotic sector spectrum

- Higgs doublet, $m_{H_{-1}} \approx \mathcal{O}(100\,\mathrm{GeV})$
- Hadronic resonances $\sim \Lambda_{\mathrm{QCD}_{-1}}$
- Massive gauge bosons:

$$m_{W_{-1}} = (\sqrt{3}/2) g f_{\pi_{-1}}$$
$$m_{Z_{-1}} = (\sqrt{3}/2)\sqrt{g^2 + g'^2} f_{\pi_{-1}}$$

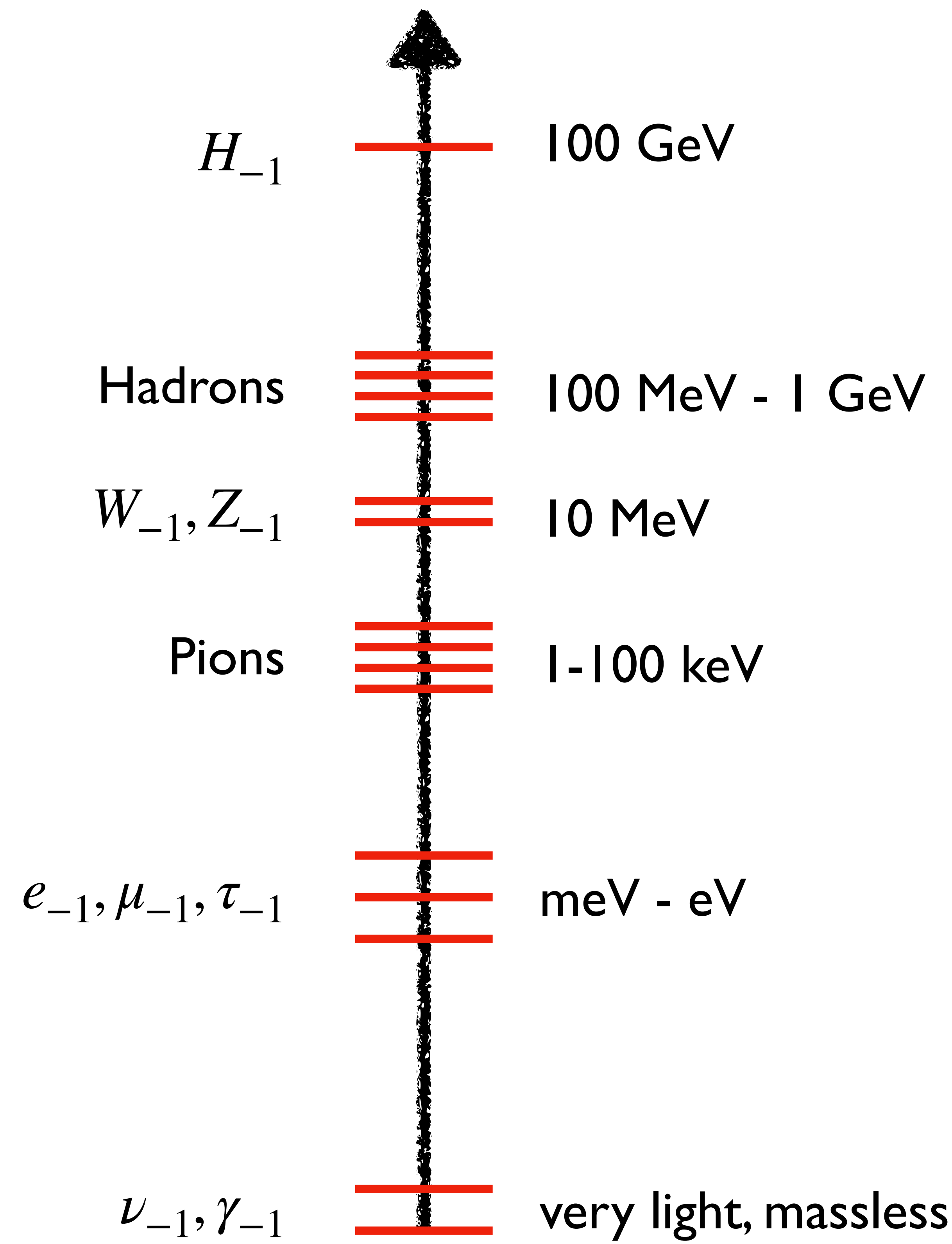
- Pions:

$$m_{\pi_{-1}} = 4\pi\sqrt{y_q y_t}\, f_{\pi_{-1}}^2 / m_{H_{-1}}$$

- Leptons:

$$m_{\ell_{-1}} = 4\pi\sqrt{y_\ell y_t}\, f_{\pi_{-1}}^3 / (2m_{H_{-1}}^2$$

- Neutrinos are extremely light
- Photon is massless



Cosmology overview

- Reheaton decays into all sectors, reheating the universe. Energy density in each sector scales with reheaton partial decay width into that sector, $\rho_i \propto \Gamma_i$.

- Each sector thermalizes within its own sector, with corresponding energy and entropy densities

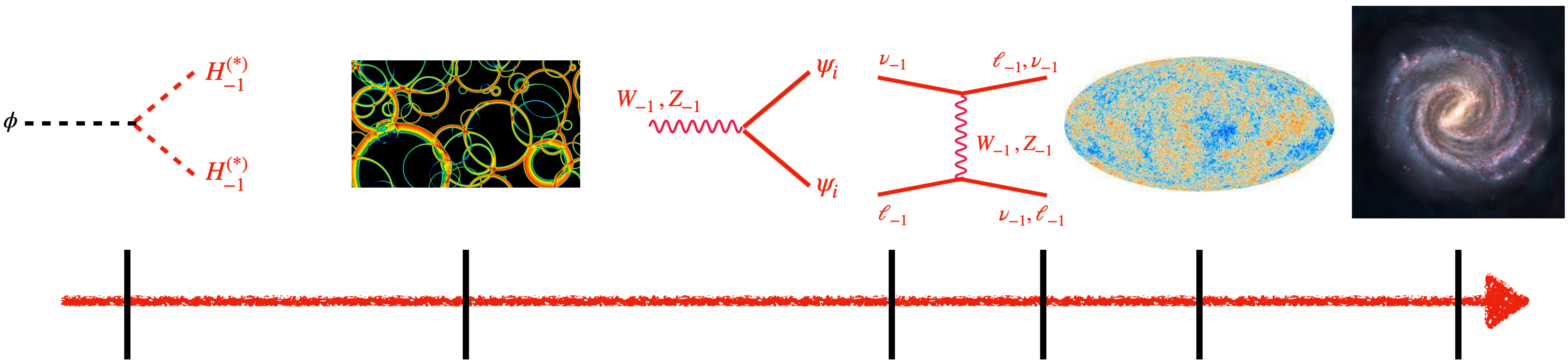
$$\rho_i = \frac{\pi^2}{30} g_{*\rho} \xi_i^4 T^4, \quad s_i = \frac{2\pi^2}{45} g_{*s} \xi_i^3 T^3, \quad \xi_i = T_i/T, \qquad T = \text{SM temperature}$$

- The reheat temperature of SM, $T^{\rm RH}$, is a free parameter, fixed by the coupling a. We must require $\Lambda_{-1} \lesssim T^{\rm RH} \lesssim v$. The temperature ratios are $\xi_i \sim (\Gamma_i/\Gamma_{\rm SM})^{1/4}$

- Cosmology of SM-like sectors is similar to that of the SM. The radiation in these sectors contribute to $\Delta N_{\rm eff}$

- Cosmology in the first exotic sector exhibits some qualitative differences, which we discuss next, though the radiation of the sector also contributes to $\Delta N_{\rm eff}$

Cosmology of the first exotic sector



$T_{\rm RH} \sim 100\,\rm GeV$ | $T_{\rm crit} \sim 100\,\rm MeV$ | $\rm keV$ | $10\,\rm eV$ | $0.1\,\rm eV$ | T_0

Reheating

QCD phase transition

W, Z, π annihilate, decay

ν decouple

τ, μ annihilate

Constraints from $\Delta N_{\rm eff}$

- The essential prediction of Nnaturalness is the presence of dark radiation from the other sectors, parameterized by $\Delta N_{\rm eff}$:

$$\Delta N_{\rm eff}^{\rm CMB} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\Delta\rho}{\rho_{\gamma,{\rm SM}}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\sum_{i\neq 0}\left[\frac{g_{*\rho,i}^{\rm CMB}}{2}\right](\xi_i^{\rm CMB})^4$$

- CMB provides the strongest bounds on $\Delta N_{\rm eff}$
  - Planck: $\Delta N_{\rm eff} \leq 0.3$
  - Planck + SH0ES $\Delta N_{\rm eff} \leq 0.7$

[Planck, 1502.01589]
[SH0ES, 2012.08534]
[Blinov, Marques-Taveres, 2003.08387]

- In the future, CMB Stage IV will bound $\Delta N_{\rm eff} \leq 0.03$

[CMB-S4, 1610.02743]

$\Delta N_{\rm eff}$ estimate

- Using entropy conservation between $T^{\rm CMB}$ and $T^{\rm RH}$, we can write the contribution to $\Delta N_{\rm eff}^{\rm CMB}$ from the SM-like sectors as

$$\Delta N_{{\rm eff},i>0}^{\rm CMB} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left[\frac{g_{*\rho,{\rm SM}}^{\rm RH}}{2}\right]\left[\frac{g_{*s,{\rm SM}}^{\rm CMB}}{g_{*s,{\rm SM}}^{\rm RH}}\right]^{4/3}\sum_{i>0}\left[\frac{g_{*\rho,i}^{\rm CMB}}{g_{*\rho,i}^{\rm RH}}\right]\left[\frac{g_{*s,i}^{\rm RH}}{g_{*s,i}^{\rm CMB}}\right]^{4/3}\frac{\Gamma_i}{\Gamma_{\rm SM}}$$

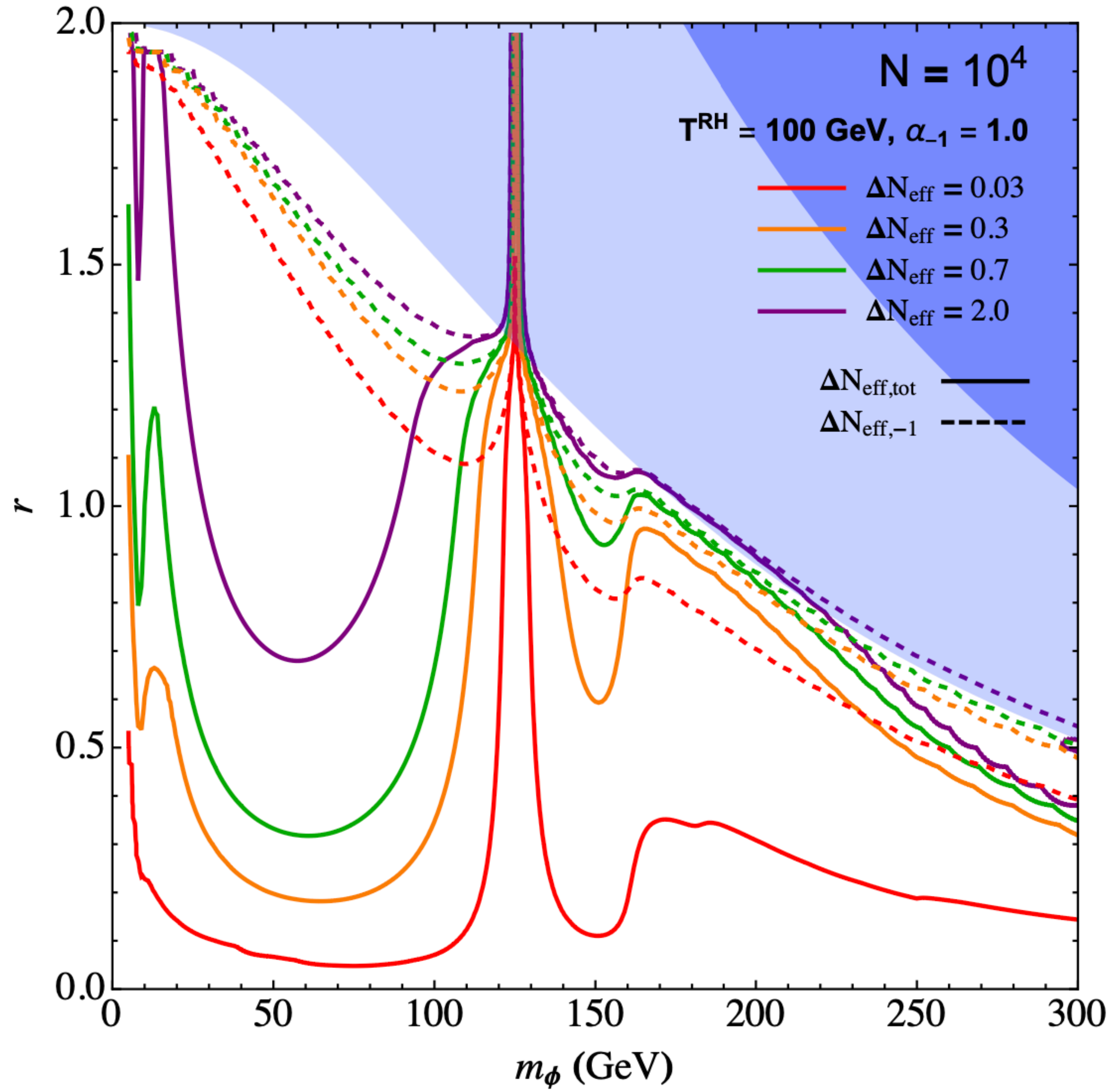
- For the first exotic sector, we must account for the entropy production due to the phase transition, leading to

$$\Delta N_{{\rm eff},-1}^{\rm CMB} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left[\frac{g_{*\rho,{\rm SM}}^{\rm RH}}{2}\right]\left[\frac{g_{*s,{\rm SM}}^{\rm CMB}}{g_{*s,{\rm SM}}^{\rm RH}}\right]^{4/3}\left[\frac{g_{*\rho,-1}^{\rm CMB}}{g_{*\rho,-1}^{\rm RH}}\right]\left[\frac{g_{*s,-1}^{\rm RH}}{g_{*s,-1}^{\rm CMB}}\right]^{4/3} D_{s,-1}^{4/3}\,\frac{\Gamma_{-1}}{\Gamma_{\rm SM}}$$

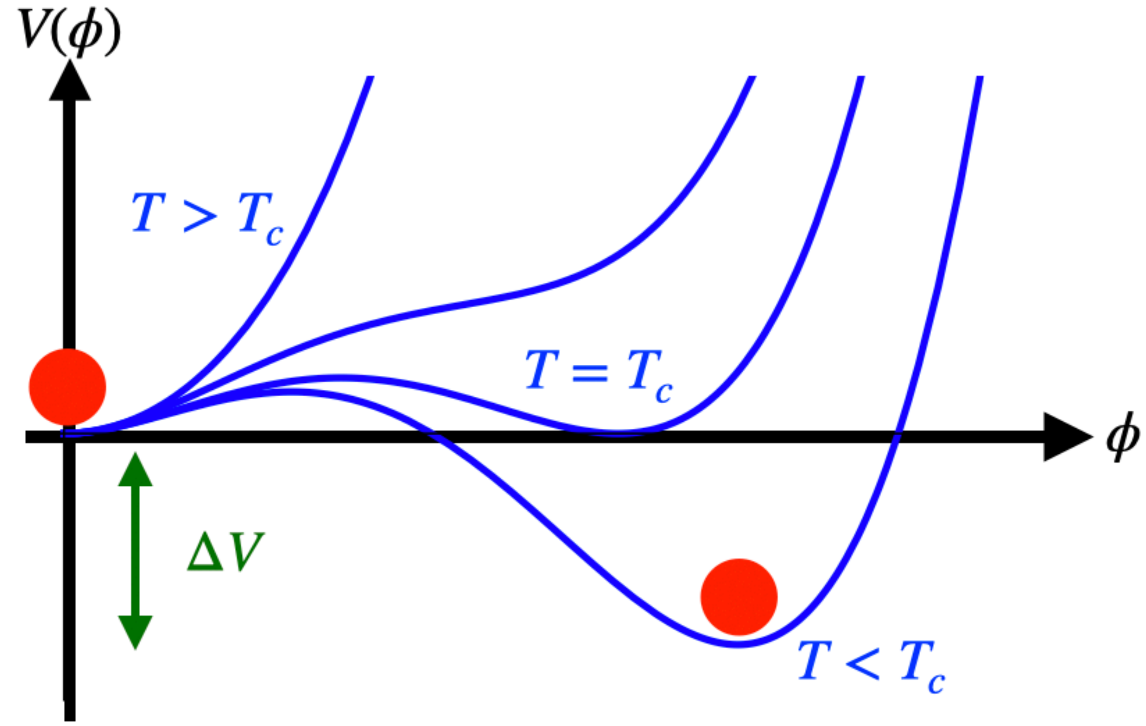
- The key point again is that $\Delta N_{\rm eff}^{\rm CMB}$ is controlled by the reheaton partial widths

Constraints from $\Delta N_{\rm eff}$

- Parameters below green line are allowed by $\Delta N_{\rm eff}^{\rm CMB}$
- Larger values of r are more constrained (smaller Higgs masses in other sectors)
- Constraints weakened near $m_\phi \sim m_h$ and $m_\phi > 2m_W$
- Light blue region indicates where $\phi \to H_{-1}H_{-1}$ is kinematically allowed
- Note the regions where $\Delta N_{\rm eff}^{\rm CMB}$ is dominated by the first exotic sector (dashed lines)



First order phase transition



- Starting from the unbroken phase, as the Universe cools, the potential develops a new minima away from the origin, separated by a potential barrier

- The phase transition proceeds via bubble nucleation

- Several important milestones:
  - Critical temperature $T^{\rm crit}$: true and false vacua degenerate, separated by barrier
  - Nucleation temperature $T^{\rm nuc} \lesssim T^{\rm crit}$: order one probability for bubble nucleation
  - Percolation temperature $T^{\rm perc} \lesssim T^{\rm nuc}$: substantial fraction in true vacuum

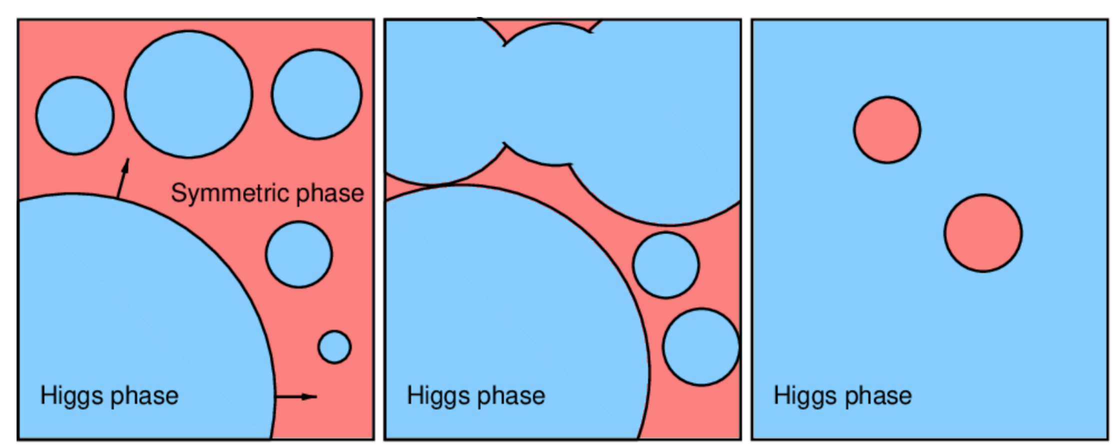


Figure from Hindmarsh et al., 2008.09136

- Phase transition parameters:

$$\alpha_{-1} = \frac{\Delta\theta_{-1}}{\rho_{-1}^{\rm perc}}, \qquad \alpha_{\rm tot} = \frac{\Delta\theta_{-1}}{\rho_{\rm tot}^{\rm perc}} \qquad \frac{\beta}{H} = T_{-1}\frac{d}{dT_{-1}}\frac{S_3}{T_{-1}}\bigg|_{T_{-1}^{\rm nuc}}, \qquad v_{\rm w}$$

strength parameters — duration parameters — wall velocity

Gravitational wave signal estimate

- Gravitational waves are produced through several mechanisms during a first order phase transition, including bubble wall collisions and sound wave in the plasma

[Kosowsky, Turner, Watkins ‘92]
[Hindmarsh, Huber, Rummukainen, Weir ’13]

- The gravitational wave spectrum can be parameterized as follows:

$$\Omega_{\rm GW}^{\rm em}(f_{\rm em}) = \sum_{I={\rm BW,\,SW}} N_I\,\Delta_I(v_{\rm w}) \left(\frac{\kappa_I(\alpha_{-1})\,\alpha_{\rm tot}}{1+\alpha_{\rm tot}}\right)^{p_I} \left(\frac{H}{\beta}\right)^{q_I} s_I(f_{\rm em}/f_{{\rm p},I})$$

$$h^2\,\Omega_{\rm GW}^0(f) = h^2\mathscr{R}\,\Omega_{\rm GW}^{\rm em}\left(\frac{a^0}{a^{\rm perc}}f\right)$$

[see, e.g., Breitbach et al. ’18]

- The spectrum depends crucially on the strength parameters α_{-1}, $\alpha_{\rm tot}$, the duration parameter β/H, the wall velocity $v_{\rm w}$

- In principle these quantities can be computed if one has knowledge of the effective potential. However, in our scenario, this is obscured by the strong QCD dynamics

Scenarios for the phase transition dynamics

- Runaway phase transition:
  - Negative pressure from potential energy difference overcomes friction from plasma. Bubble walls expand at ultra-relativistic speeds
  - Gravitational waves sourced by bubble wall collisions

$$v_{\rm w} = 1, \quad \kappa_{\rm BW} = 1, \quad \kappa_{\rm SW} = 0,$$

$$(\alpha_{-1}, \beta/H) = (10, 3),\ (5, 10),\ (1, 10^3).$$

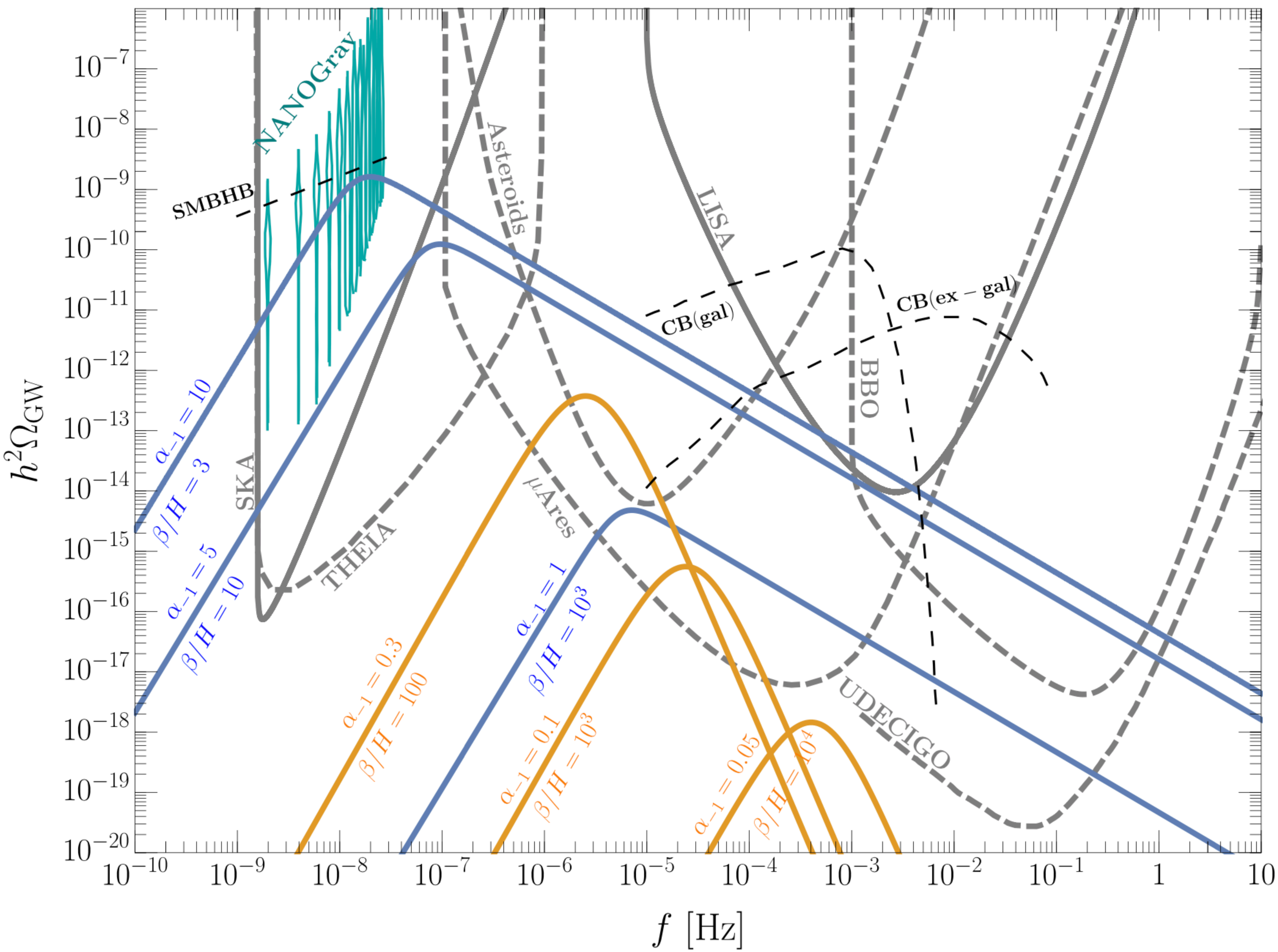
- Non-runaway phase transition:
  - Frictional pressure from plasma causes the bubble wall to reach a terminal velocity near the plasma sound speed.
  - Latent heat is transferred into coherent plasma motion
  - Gravitational waves sourced by sound waves

$$v_{\rm w} = \frac{1}{\sqrt{3}}, \quad \kappa_{\rm BW} = 0, \quad \kappa_{\rm SW} = \frac{\alpha_{-1}^{2/5}}{0.017 + (0.997 + \alpha_{-1})^{2/5}},$$

[Espinosa, Konstandin, No, Servant, '10]

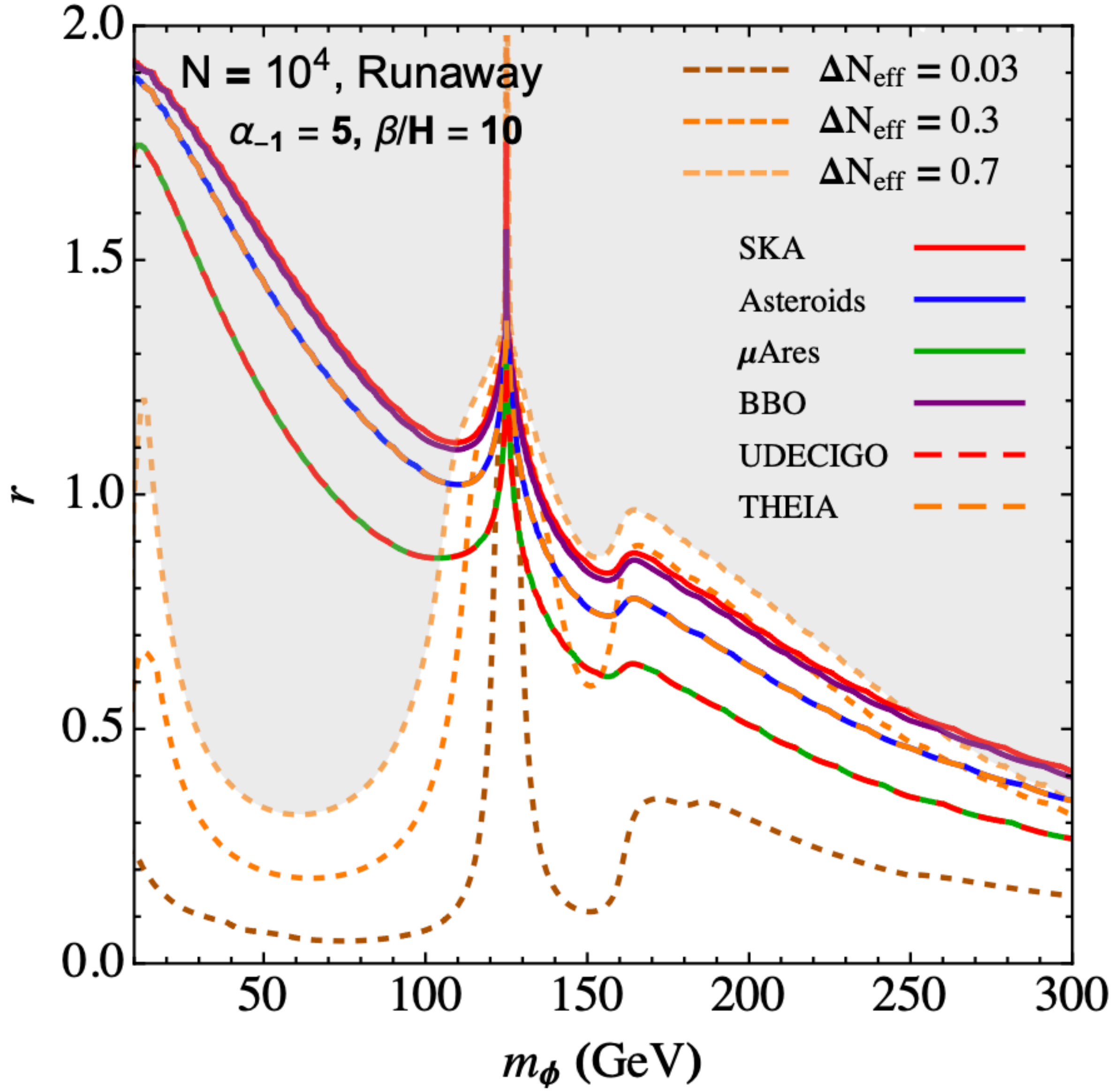
$$(\alpha_{-1}, \beta/H) = (0.3, 10^2),\ (0.1, 10^3),\ (0.05, 10^4).$$

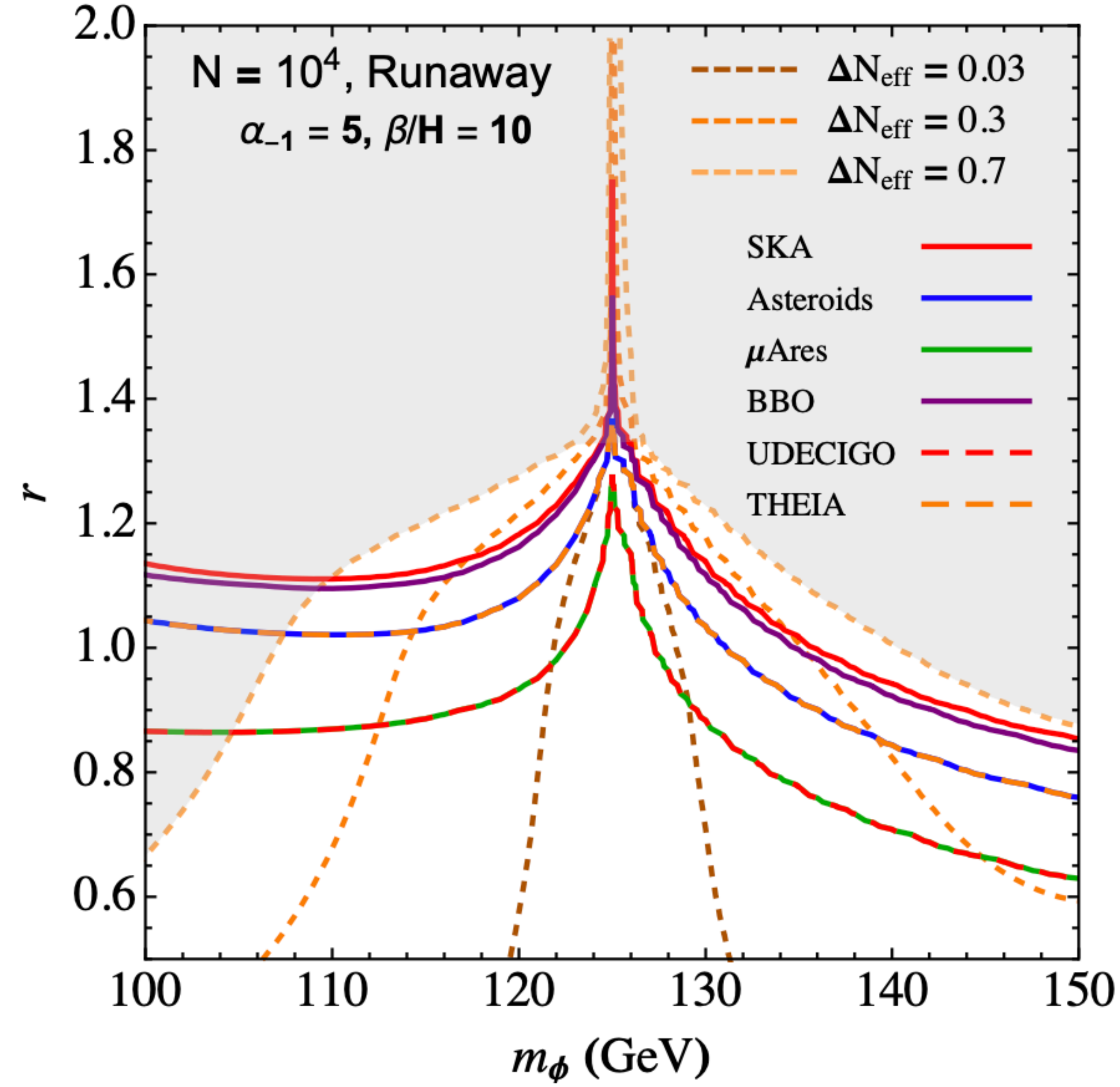
Gravitational wave spectra



Gravitational waves from Nnaturalness

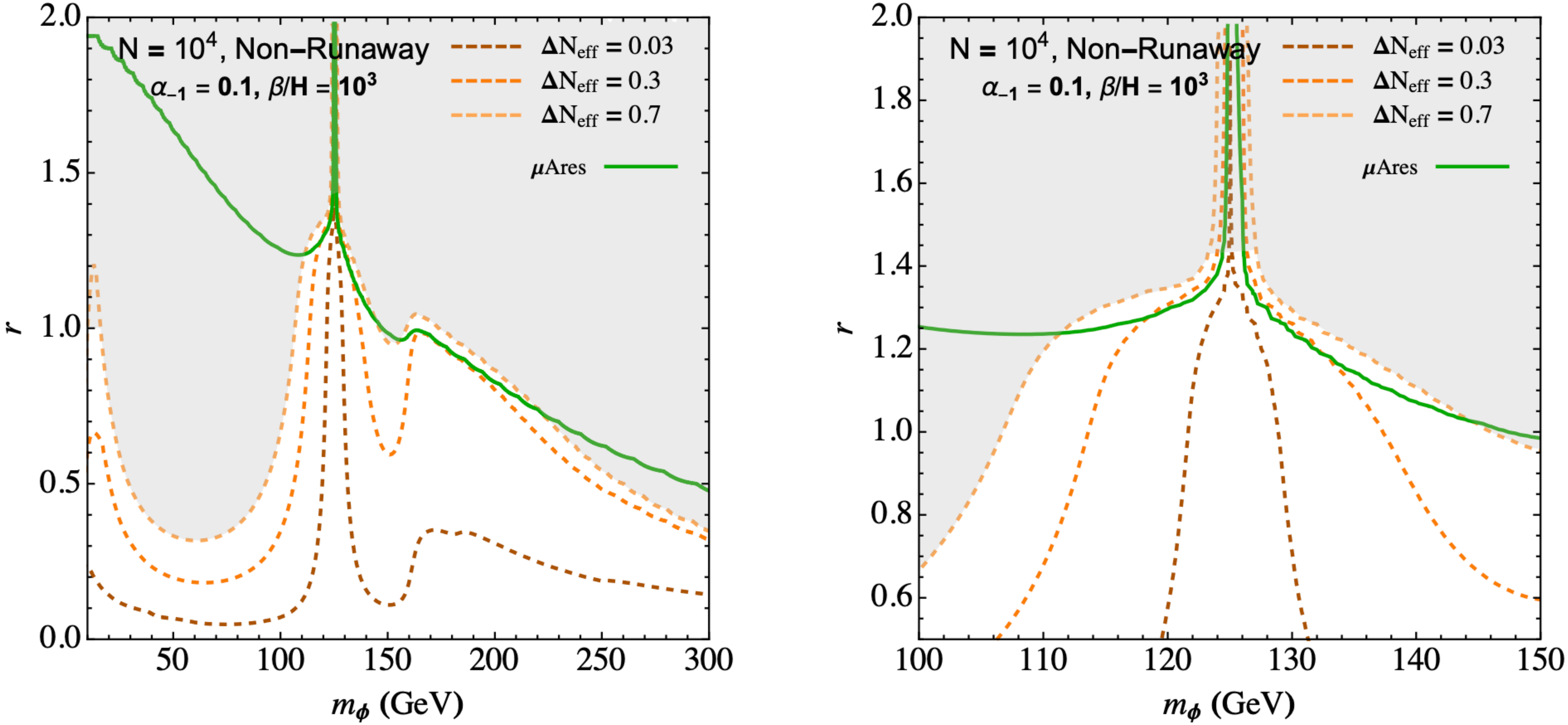
Runaway scenario, $\alpha_{-1} = 5, \quad \beta/H = 10$





Gravitational waves from Nnaturalness

Non-runaway scenario, $\alpha_{-1} = 0.1, \quad \beta/H = 1000$



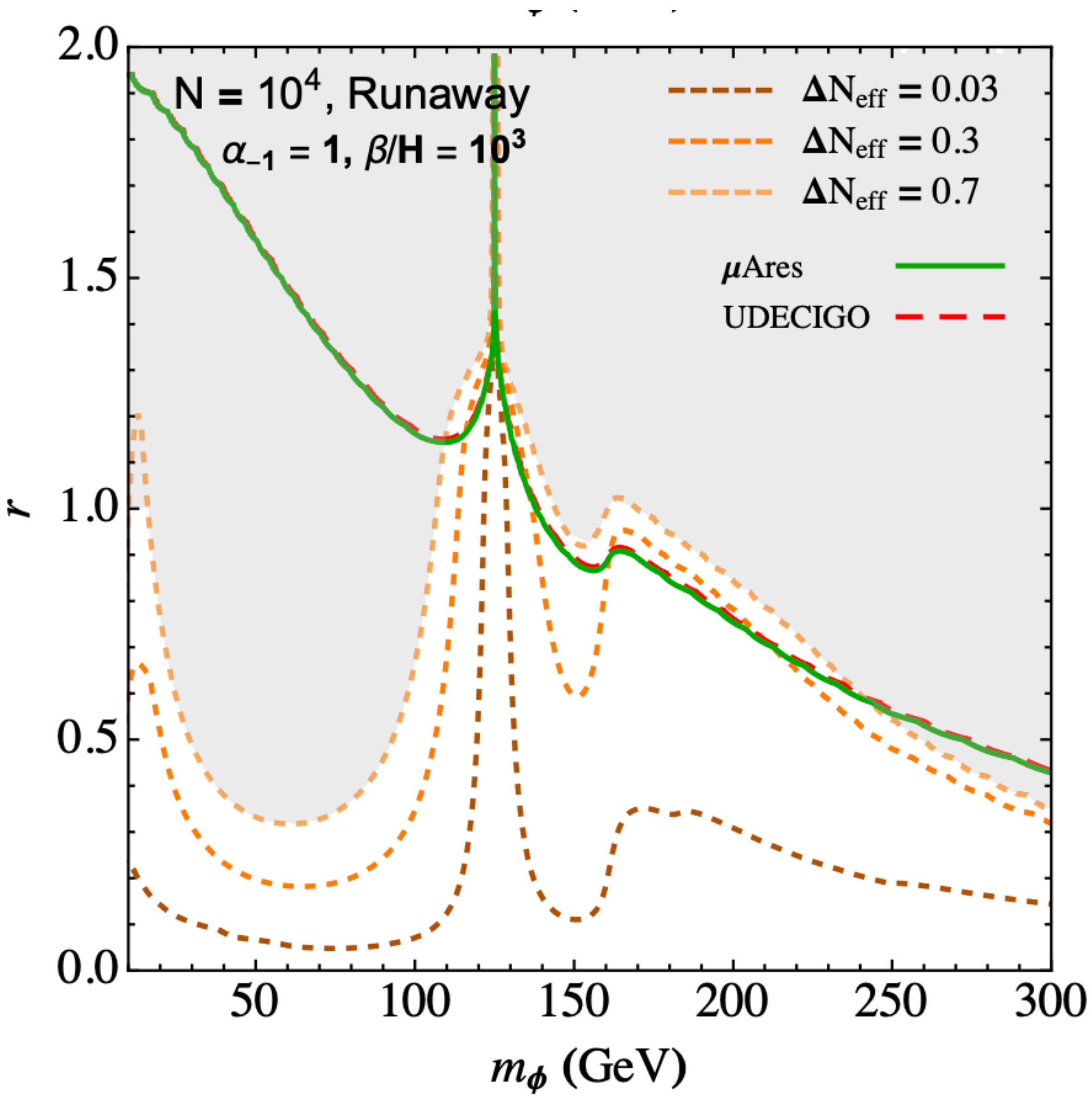
Outlook

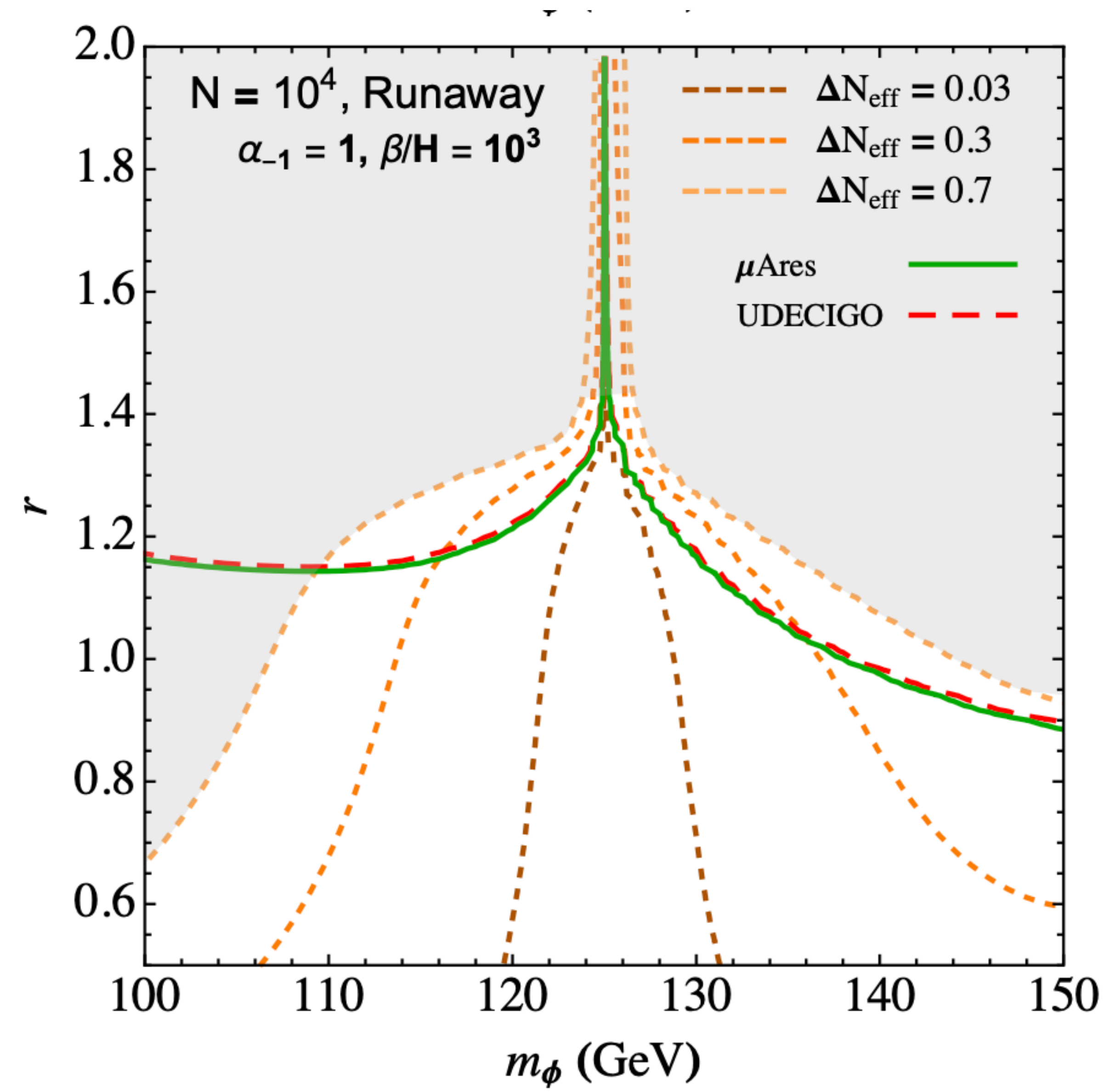
- Nnaturalness provides a novel approach to the electroweak hierarchy problem
- The main probes of the scenario are from cosmology, particularly $\Delta N_{\rm eff}$
- In certain regions of parameter space the first exotic sector may receive a sizable fractional energy density
- Depending on the strongly-coupled QCD phase transition dynamics of this sector, the associated stochastic gravitational wave signal may be detectable by several future experiments
- Open questions & avenues for further study:
  - Further scrutiny of the exotic sector QCD phase transition (lattice, models)
  - Fermionic reheaton models
  - Precision analysis of cosmological perturbations and impact on CMB
  - Novel remnants of phase transition (quark nuggets, primordial black holes)

Backup

Gravitational waves from Nnaturalness

Runaway scenario, $\alpha_{-1} = 1, \quad \beta/H = 1000$





Gravitational waves from Nnaturalness

Non-runaway scenario, $\alpha_{-1} = 0.3, \quad \beta/H = 100$