IL NUOVO CIMENTO **47 C** (2024) 379
DOI 10.1393/ncc/i2024-24379-1

Colloquia: MAYORANA 2023

A discussion of the cross section $\bar{\nu}_e + p \to e^+ + n$

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received 6 August 2024

Summary. — We discuss the interaction cross section $\bar{\nu}_e - p$ due to charged currents, of major importance for neutrino detection. We present the history of its understanding and highlight the aspects necessary for its precise evaluation. We examine the three most recent determinations and, on the basis of the most recent one, tabulate its updated values and assess its uncertainty.

1. – Importance of the IBD cross section

The process $\bar{\nu}_e + p \to e^+ + n$ has been the first by which neutrinos were directly observed [1]. While it can be correctly described as a reaction between electron antineutrinos and protons, it is often called 'inverse beta decay' (IBD) because, in the context of quantised field theory, it shares the same amplitude as the beta decay of the neutron. It is widely used in water- or hydrocarbon-based detectors, which are relatively cheap materials and rich in target protons. Future experiments for detecting antineutrinos from reactor and from gravitationally collapsing supernovae, see *e.g.*, [2-4], will collect very large samples of such events and will require the cross section to be known precisely. This consideration alone has motivated a long-standing interest in its theoretical estimation. In this contribution, we first point out the significant elements for calculating it (sect. **2**), and we do so in an entertaining way, that is, by retracing in broad outline the interesting history of its theoretical understanding. In the second part, after presenting the three most modern and accurate calculations (sect. **3**), we focus on the more recent one. We discuss the most reliable expression of the cross section by estimating what the residual uncertainties are (sect. **3**). The last part (sect. **4**) is devoted to a brief overview of the current status and future prospects.

2. – Brief history of the IBD reaction

In order to highlight what ideas underlie the description of the IBD cross section, and to do so in an agile manner, we take a cursory look at its history. Before it was possible to speak of a cross section, the concept of the neutrino itself had to be developed; then, in about 20 years, scientists moved from the first Hamiltonian theory of beta rays to the modern description of interactions (V-A theory). Since then numerous other advances have occurred, some of which are particularly relevant to the quantitative discussion of the cross section: the understanding of the Cabibbo angle and an adequate description of hadronic interactions.

Evolution of the idea of neutrino. – Let us begin presenting the different ideas of the neutrino, formulated around 1930s:

1) *Pauli 1930* [5] introduces the neutrino as a constituent of the atomic nucleus and assumed that this particle is emitted in β decay([1]).

2) *Fermi 1933-1934* [6] describes neutrinos as relativistic (Dirac) fermions, completely analogous to the electron. Due to the chosen formalism [7] - see also below - antineutrinos and neutrinos are different([2]).

3) *Majorana 1937* neutrino idea [8] consists of the assumption that the neutrino and the antineutrino are the same particle.

4) *Weyl's 1929* relativistic wave equation [9] is simpler than Dirac's one and describes electrons with zero mass. Its relevance to neutrinos will become apparent much later([3]).

As we will see in the next paragraph, the first discussion of the IBD cross section relies on the second type of concept. The importance of the difference between neutrinos and antineutrinos will emerge later, and even later will be understood that it is possible to define, through this difference, a conserved lepton number. What about the 3^{rd} and 4^{th} concepts? Majorana's proposal is explored and temporarily shelved. In the mid-1950s, Weyl's formalism was recognised as valid for the description of neutrinos; its compatibility with Majorana's proposal, accepted today, would only be understood later, slowly and with some hesitation [10].

For completeness, we mention here another important and later evolution in the comprehension of neutrinos, that does not concern us directly: this occurred after it was realised that there are more types of neutrino, a point first discussed as early as 1942 to support Yukawa's theory [11]. In 1962, Sakata and collaborators proposed that the neutrinos that interact with the charged leptons, via weak interactions, are not necessarily mass eigenstates, but possibly superpositions of mass eigenstates. This framework allowed Pontecorvo to reformulate the proposal of 1957 into its modern form, describing what we currently call *neutrino oscillations.*

From the theory of beta rays to the V-A interaction. – The first theory of Fermi of β decay, dating 1933 [6], introduces a constant g which carries dimensions of energy×volume, namely, it is an inverse of a square mass in natural units. This enters the interaction hamiltonian $H = g\,\boldsymbol{\tau}_+\,\boldsymbol{\Psi}^\dagger\delta\boldsymbol{\Phi}^* + \text{h.c.}$ that allows the conversion of a neutron into a proton (the adimensional isospin operator $\boldsymbol{\tau}_+$) and the appearance of a neutral and a charged lepton described by the fields $\boldsymbol{\Phi} = \sum_\sigma \phi_\sigma \boldsymbol{a}_\sigma$ and $\boldsymbol{\Psi} = \sum_s \psi_s \boldsymbol{a}_s$, summed over positive

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([1]) This model has no relativistic characteristics and in particular has no connection with Dirac idea of antimatter.

([2]) Fermi's neutrino concept corresponds to what is now called the 'Dirac neutrino'. This term is widespread today, but Fermi does not use it and we do not know any work of Dirac describing such a neutrino concept.

([3]) Weyl's hamiltonian is $H = \pm\vec{\sigma}\,\vec{p}c$; at the time it was set aside because of the peculiar coupling between the momentum $\vec{p}$ and the spin $\vec{S} = \hbar\vec{\sigma}/2$ —polar and axial vectors— which was completely outside the accepted patterns.

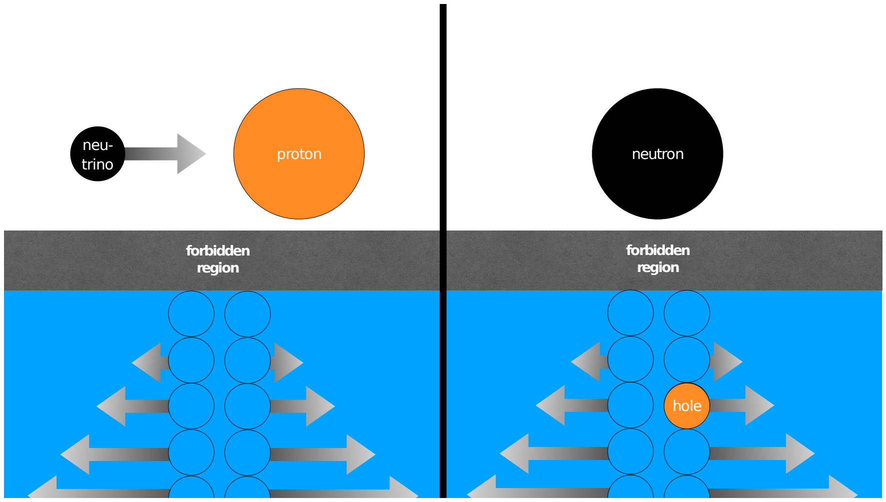

Fig. 1. – Description of IBD cross section in the formalism of second quantization. *Left panel:* Initial state of the process; the neutrino hits the proton, the electron states of Dirac's sea are all occupied. *Right panel:* Final state of the process. The nucleon changed its isospin state and became a neutron; the hole formed in the Dirac sea can be thought of as an anti-electron (=Dirac hole theory).

and negative energies (that form the leptonic current $\boldsymbol{J}_- = \boldsymbol{\Psi}^\dagger \delta \boldsymbol{\Phi}^*$). This theory relies heavily on the old procedure of quantisation (second quantization), based on the existence of Dirac sea of electrons and of neutrinos, but it allows a lot of useful inferences; see *e.g.* [12] and fig. 1 for the description of the IBD cross section in this formalism.

In 1934, Bethe and Peierls [13] observed that the ratio of two quantities $\Gamma_n = \hbar/\tau_n$ and $\sigma_{\bar{\nu}_e p}$ could be estimated roughly by dimensional considerations([4]): thus, from the measurement of τ_n one could get an idea of the size of the IBD cross section. Since $\sigma_{\bar{\nu}_e p}$ turns out to be very small, this argument indicated that it was very difficult to see the antineutrino. This observation inclined Bethe and Peierls toward a pessimistic attitude [13] and they concluded that: "there is no practically possible way of observing the neutrino". But 20 years later, in 1956, Reines and Cowan [1], succeeded in revealing the effects of this reaction. This was achieved by using detectors containing large masses of protons and exposed to copious flux of antineutrinos, emitted by the first nuclear reactors ever built. The existence of antineutrinos had been conclusively demonstrated, as eventually recognised by the 1995 Nobel Prize in Physics awarded to Reines; moreover IBD cross section could be measured.

Meanwhile, the new fermion quantisation procedure proposed by Majorana [8] (see also [14]) had become popular, making it no longer necessary to assume the existence of the Dirac sea. However, at that moment, Majorana's hypothesis on neutrinos [8] apparently lost its attractiveness, after a null result obtained by Davis([5]) shortly before Reines and Cowan measurement of the IBD cross section [15].

([4]) Their estimate is $\sigma_{\bar{\nu}_e p} \sim \ell^3/(c\tau_n)$ where $\ell = \hbar/(\mu c)$ and μ a mass characteristic of the IBD cross section, as the mass difference between neutrons and protons or the mass of the electron.
([5]) Davis tested that the particle produced in the reactors did not trigger events from $\bar{\nu} + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + \mathrm{e}^-$ [15]. See [10] for a review of a detailed account of the subsequent phase of the discussion.

The last step of this story begins with the realisation that parity is not respected in weak interactions [16, 17], which harmonises well with the idea that the neutrinos are described by Weyl's equation [18-20] and with the ensuing conclusion that weak interactions have a 'vector-axial' (V-A) structure [21, 22] also called chiral. These hypotheses imply that neutrinos are distinguished from antineutrinos by their helicity: Goldhaber's tests [23] confirmed the validity of this conclusion.

At this point we are close to the modern understanding; but the new positions concerning the interactions required to reconsider the conclusions drawn from Davis's experiment [15]. Indeed, in the limit that usually counts in the laboratory, the ultra-relativistic limit, the V-A structure conceals the effects of the neutrino mass, including possible leptonic number violations caused by the Majorana mass. Therefore, Davis' null result supports the view that neutrinos and antineutrinos are different when $E \gg mc^2$, but it is of no use if we want to know what is the value of Majorana's neutrino mass m: very special experimental situations are needed to probe it([6]).

Let us summarise: upgrading Fermi's Hamiltonian as a modern quantum field theory and within V-A structure, neutrinos fit Weyl's ideas and do not contradict in any manner Majorana's hypothesis, which remains attractive for its own reasons, observational and theoretical: see *e.g.* [10, 14, 24].

Further relevant progresses. – There have been many other important advances since that time: they concern 1) the inclusion of radiative corrections, 2) the description of the effects of hadronic interactions, 3) the effects of hadron mixing, and finally 4) the conceptualization of the quark model. We note that some effects of radiative corrections are already considered by Fermi in 1933 [6], and the available calculations [25] are adequate for current needs; furthermore, there is no actual need to rely on quark model concepts to discuss the IBD cross section. Therefore, for the purposes of our discussion, we focus on the second and third aspects.

As far as hadronic interactions are concerned, recall that Yukawa [26] introduces a boson to describe, in a way similar to QED, Fermi's interactions. As it is well known, that boson was subsequently identified with the charged pion. In the 1950s it was realised that there are other possible mediators - generically called resonances([7]) - and some of them are linked to the electromagnetic interactions of the proton and neutron by the hypotheses of 'conserved vector current' (CVC) and 'partially conserved axial current' (PCAC). As far as we are concerned, the outcome is the following [27, 28]: the Lorentz invariant decomposition of the hadronic current includes momentum-dependent form factors for vector and for axial parts $f_i(q^2)$ and $g_i(q^2)$, $i = 1, 2, 3$, whose value in $q^2 = 0$ can be measured and whose evolution to some extent can be constrained.

The last point relevant to the IBD cross section concerns hadronic mixing, signs of which had been seen since the late 1950s. As there were some heated discussions on the occasion of a past Nobel Prize, perhaps it is helpful to recall the papers that are relevant for the point under discussion. The first one [29] postulates that the vector current *matrix elements* includes a mixing between neutrons and Λ-particles to explains certain observations. The second one [30] extends the proposal to the axial matrix elements,

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([6]) See again [10]; but for our purposes, it suffices to note that the search for Majorana's neutrino mass effects involves an ever increasing number of experiments.

([7]) Subsequently these considerations will be connected with the QCD, *i.e.*, with the theory of gluons and quarks, and the resonances will be thought of as bound states of the u-$\bar{d}$ quarks that interact with W^+.

assumed to obey exactly V-A structure. The third paper [31] has a much broader scope [32]: it explores the consequences of CVC and SU(3) flavor symmetry of hadrons for the *operators* (currents) that cause weak interactions. In this way, mathematical predictions for numerous processes are obtained: the matrix elements between hadrons are shown to depend on one angle, the Cabibbo angle, and on another parameter characteristic of axial interactions. This applies, in particular, to the transition between neutrons and protons we are interested in.

3. – The IBD cross section

The three most recent determinations. – The first modern calculation of the IBD cross section dates back to 1999 and is due to Vogel and Beacom [33]. The authors systematically assessed the effects of nucleon recoil, as well as weak magnetism, whose understanding began in the 1930s and was finalised by Gell-Mann [34]. Vogel and Beacom showed that these effects are rather relevant for the positron angular distribution at the desired accuracy. They adopted an expansion in powers of E_ν/M, where $M = (m_n + m_p)/2$ is the average mass of the nucleon, reliable in the region below $E_\nu < 60$ MeV; they also offered several useful analytical results and discussed the pointing of supernovae through the IBD reaction.

Three years later, Strumia and Vissani [35] produced a fully relativistic calculation based on the 4 known form factors, virtually valid at all energies. The result compares very well with the one of the previous calculation, when all relevant terms are included, and the ease of implementation of the expression is comparable. This paper gives an estimate of the uncertainty: at lower energies it is 0.4%, while at higher energies there is an additional error due to the uncertainties of the form factors whose effect is estimated to be 0.4% $\times(E_\nu/50 \text{ MeV})^2$ for E_ν below about 200 MeV.

Two decades later, Ricciardi, Vignaroli and Vissani [36] improved the assessment of uncertainty in expectations: they verified the insignificance of 'second-class currents', updated the relevant parameter values and performed a number of checks. The first result is obtained by maximising the parameters, while taking phenomenological constraints into account; the others are discussed in the next paragraphs, together with the estimation of the uncertainty in the IBD cross section.

Numerical table of the IBD cross section. – In the last part of this note, entirely based on ref. [36], we overview the current values and uncertainties on the IBD cross section. In this paper the analytical formula of the cross section is given. It is also tabulated using the input values $V_{ud} = 0.97427$, $\lambda = 1.27601$ and $r_A^2 = 0.416$ fm^2. While the first two values are currently the best fit ones, the latter, which corresponds to $M_A = 1060$ MeV, is not - even if it lies within the uncertainty range $r_A^2 = 0.46 \pm 0.16$ fm^2 discussed below. Therefore, the table I presents the IBD cross section, calculated for the set of values $V_{ud} = 0.97427$, $\lambda = 1.27601$ and $r_A^2 = 0.46$ fm^2.

Uncertainties. – The radiative corrections of QED to the cross section are calculated at leading order and included. Next order corrections and other effects such as isospin breaking are estimated to be small. In short, for the accuracy of interest, the leading uncertainties are simply due to three input parameters, and more precisely:

- at lowest energies, the Cabibbo angle and the axial coupling;
- at higher energies, the axial mass M_A, or better (as we will discuss) the axial radius r_A.

Let us discuss the values and uncertainty ranges of the relevant parameters.

Low energy region: First of all, let us summarise the way we treat the two relevant parameters.

The mixing element $V_{ud} = \cos\theta_C$, that multiplies the amplitude of transition, can be probed:

- Directly, from the super-allowed transitions (we use ref. [37]);
- Indirectly, exploiting the unitarity of Cabibbo-Kokayashi-Maskawa (CKM) matrix, using the values of V_{us} and V_{ub} given in [38].

The two results are not in perfect agreement; thus, we include the scale factor $S = \sqrt{\chi^2/(N-1)} = 2.0$ for a conservative estimation of the uncertainty.

The axial coupling $g_1(q^2)$ is usually presented in terms of a ratio with the vector form factor f_1 at zero momentum transfer, namely the axial coupling $\lambda = -g_1(0)/f_1(0)$. There are eight measurements with polarised neutron decay, and the most recent one [39] is very precise. Czarnecki, Marciano and Sirlin [40] suggested to omit pre-2002 values, on behalf of potentially large correction factors not completely under control; we have preferred to include them, but enlarging their error by a factor 2, which implies a larger S. The resulting value $\lambda = 1.2760(5)$ is within 1σ from ref. [39] and agrees with the global average.

Now, once the range of these two parameters is known, we have a *prediction* for the neutron decay lifetime τ_n. On the other hand, this quantity is measured; so in principle this measurement could help us to improve the inferences on the IBD cross section. However, there are two sets of measurements of τ_n that are among them incompatible: the total lifetime measured using trapped ultra-cold neutrons is found to be $\tau_n(\text{tot}) = 878.52 \pm 0.46$ s, but the value deduced using beam neutron and measuring the decay products is about 10 seconds longer: $\tau_n(\text{beam}) = 888.0 \pm 2.0$ s. The data of V_{ud} and λ are perfectly consistent with the former value, and incompatible with the latter. There is no simple theoretical way out; the first suspect becomes an unknown systematic error. Efforts should be made to understand the incompatibility between the two set of τ_n measurements.

In summary, by propagating the uncertainty factors we find that the cross section is known with $\delta\sigma_{\bar{\nu}_e p}/\sigma_{\bar{\nu}_e p} = 0.1\%$ for low values of electron anti-neutrino energies: this is 4 times better than in [35].

High energy region: Past determinations of the cross section have used the value of the axial mass M_A, which is measured at energies $E_\nu \sim$ GeV or above, assuming that the axial form factor behaves as a double dipole, $g_1(q^2)/g_1(0) = 1/(1 - q^2/M_A^2)^2$. This value is quite precise [41] $M_A = 1014 \pm 14$ MeV and it is supported by electro-production data corresponding to much lower q^2 [42]. On the other hand, this is simply a phenomenological fit; there is in principle no reason why it should work at smaller q^2, and other parametrisations have become recently available. Therefore, for the energy range in which we are interested, we lessen the dependence on the dipole approximation by using simply a linear expansion $g_1/g_1(0) = 1 + q^2 r_A^2/6$. The previous value of the axial mass implies $r_A^2 = 0.455 \pm 0.013$ fm^2, but a determination that does not assume the double dipole has an error larger of about one order of magnitude $r_A^2 = 0.46 \pm 0.12$ fm^2.

Proceeding with this conservative estimation, we find $\delta\sigma_{\bar{\nu}_e p}/\sigma_{\bar{\nu}_e p} = 1.1\%(E_\nu/50\text{ MeV})^2$ in the region above $\sim$ 10 MeV; which is actually 3 times larger than in [35].

TABLE I. – *Numerical values of the IBD cross section $\bar{\nu}_e p \to e^+ n$ as a function of neutrino energy by fixing the input parameters at $V_{ud} = 0.97427$, $\lambda = 1.27601$ and $r_A^2 = 0.46$ fm^2. Left part, low energy region. Right part, high energy region.*

E_ν MeV	$\sigma_{\bar{\nu}_e p}$ 10^{-41}cm^2	E_ν MeV	$\sigma_{\bar{\nu}_e p}$ 10^{-41}cm^2	E_ν MeV	$\sigma_{\bar{\nu}_e p}$ 10^{-41}cm^2	E_ν MeV	$\sigma_{\bar{\nu}_e p}$ 10^{-41}cm^2	E_ν MeV	$\sigma_{\bar{\nu}_e p}$ 10^{-41}cm^2	E_ν MeV	$\sigma_{\bar{\nu}_e p}$ 10^{-41}cm^2
1.9	0.00190183	5.3	0.148898	8.7	0.497688	2.	0.00331709	35.	8.42244	68.	25.8417
2.0	0.00331709	5.4	0.156354	8.8	0.510838	3.	0.026518	36.	8.86217	69.	26.4317
2.1	0.00484224	5.5	0.163984	8.9	0.524148	4.	0.0680329	37.	9.30948	70.	27.0238
2.2	0.00652674	5.6	0.171788	9.0	0.537618	5.	0.127581	38.	9.76414	71.	27.6179
2.3	0.00838532	5.7	0.179765	9.1	0.551247	6.	0.204734	39.	10.2259	72.	28.2138
2.4	0.0104239	5.8	0.187916	9.2	0.565036	7.	0.299068	40.	10.6946	73.	28.8114
2.5	0.0126452	5.9	0.196239	9.3	0.578983	8.	0.410165	41.	11.1699	74.	29.4107
2.6	0.0150505	6.0	0.204734	9.4	0.593089	9.	0.537618	42.	11.6517	75.	30.0115
2.7	0.0176403	6.1	0.213402	9.5	0.607353	10.	0.681027	43.	12.1398	76.	30.6138
2.8	0.0204149	6.2	0.22224	9.6	0.621774	11.	0.840001	44.	12.6338	77.	31.2174
2.9	0.0233742	6.3	0.23125	9.7	0.636352	12.	1.01415	45.	13.1338	78.	31.8223
3.0	0.026518	6.4	0.24043	9.8	0.651088	13.	1.20311	46.	13.6393	79.	32.4284
3.1	0.0298461	6.5	0.24978	9.9	0.665979	14.	1.4065	47.	14.1504	80.	33.0356
3.2	0.0333582	6.6	0.2593	10.0	0.681027	15.	1.62395	48.	14.6666	81.	33.6438
3.3	0.037054	6.7	0.268989	10.1	0.696231	16.	1.85512	49.	15.188	82.	34.2529
3.4	0.040933	6.8	0.278847	10.2	0.711589	17.	2.09964	50.	15.7143	83.	34.863
3.5	0.0449949	6.9	0.288873	10.3	0.727103	18.	2.35718	51.	16.2452	84.	35.4737
3.6	0.0492392	7.0	0.299068	10.4	0.742771	19.	2.6274	52.	16.7808	85.	36.0853
3.7	0.0536656	7.1	0.30943	10.5	0.758593	20.	2.90997	53.	17.3207	86.	36.6974
3.8	0.0582736	7.2	0.319959	10.6	0.774569	21.	3.20455	54.	17.8648	87.	37.3101
3.9	0.0630629	7.3	0.330655	10.7	0.790698	22.	3.51084	55.	18.413	88.	37.9234
4.0	0.0680329	7.4	0.341518	10.8	0.80698	23.	3.82851	56.	18.9651	89.	38.5371
4.1	0.0731833	7.5	0.352546	10.9	0.823414	24.	4.15727	57.	19.5209	90.	39.1511
4.2	0.0785136	7.6	0.36374	11.0	0.840001	25.	4.4968	58.	20.0803	91.	39.7655
4.3	0.0840233	7.7	0.3751	11.1	0.85674	26.	4.84681	59.	20.6432	92.	40.3802
4.4	0.0897122	7.8	0.386624	11.2	0.873629	27.	5.20701	60.	21.2093	93.	40.995
4.5	0.0955797	7.9	0.398312	11.3	0.89067	28.	5.57712	61.	21.7787	94.	41.6101
4.6	0.101625	8.0	0.410165	11.4	0.907862	29.	5.95686	62.	22.351	95.	42.2252
4.7	0.107849	8.1	0.422181	11.5	0.925204	30.	6.34595	63.	22.9263	96.	42.8404
4.8	0.114249	8.2	0.43436	11.6	0.942696	31.	6.74412	64.	23.5043	97.	43.4555
4.9	0.120827	8.3	0.446702	11.7	0.960337	32.	7.15111	65.	24.085	98.	44.0707
5.0	0.127581	8.4	0.459206	11.8	0.978128	33.	7.56666	66.	24.6682	99.	44.6857
5.1	0.134511	8.5	0.471872	11.9	0.996067	34.	7.99052	67.	25.2538	100.	45.3006
5.2	0.141617	8.6	0.4847	12.0	1.01415						

4. – Overview

The cross section of the IBD retain its importance for present and future observations. Generally speaking, it seems to be quite well understood. Second class currents are not expected to give a significant contribution. To perform its maintenance for the present needs, all we need is a set of consolidated theoretical concepts (that we have thoroughly overviewed) and, most crucially, we need reliable measurements of the key parameters.

In the range of energies relevant for the detection of reactor and supernova electron antineutrinos, the cross section depends critically upon V_{ud}, λ and r_A. We have estimated the current uncertainties with a conservative procedure. The uncertainty related to the first two parameters are small and plays a role at low energies; it should be added to the one related to the third parameter, which becomes important at higher energies instead. When $E_\nu = 15$ MeV, the two factors affect the knowledge of the cross section to the same extent.

Note that neutrinos with different energies are detected in different experiments. There is a low energy region that includes geoneutrinos (which extend up to about 2.5 MeV) and reactor neutrinos (which end at $\sim$10 MeV); there is a region of higher energies that includes neutrino fluxes from supernovae (up to 50 MeV) - their energy in the interior of the star, during gravitational collapse, is even higher.

How to clarify/improve/progress? We need to address the reason of discrepancy in τ_n measurements. Even more significant, for the quantitive impact, it is to decrease the uncertainty due to r_A–we need to refine the description of the axial form factor in the 100 MeV range.

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We are grateful to Shota Izumiyama for pointing out our inaccuracy in the choice of r_A^2. With partial support of the INFN research initiative ENP and of the grant number 2022E2J4RK, PANTHEON: Perspectives in Astroparticle and Neutrino THEory with Old and New Messengers, part of PRIN 2022 of the Italian Ministero dell'Università e della Ricerca.

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