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1930-1937: the first β-rays and neutrino theories

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Abstract

The conceptual bases of Fermi's β-ray theory (at its 90th anniversary) are examined, highlighting the innovative drive and inspirational role for the progress that followed just afterwards. Moreover, the three different ideas of the neutrino born from the proposals of Pauli 1930 [1], again Fermi 1933 [2] and Majorana 1937 [3] are discussed, emphasising the interest of the latter for current expectations.

Keywords: Nuclear physics, beta decay, neutrino, Pauli, Fermi, Majorana

1 Introduction

On the 50th anniversary of the discovery of radioactivity induced by neutrons, Edoardo Amaldi wrote a monumental work of review on those very topics [4]. This work contains much valuable and unique material: e.g., there is a famous footnote, in which the origin of the word "neutrino" is recounted. We do not quote its text, relying on the fact that this story is already known, and we deal instead with Amaldi's presentation of an important and closely related aspect, only apparently technical.

In the section entitled "Fermi's paper on beta decay" (page 82) there is a description that leads the modern reader to spontaneous assent, this one:

> his density of interaction Hamiltonian H_{fi} is expressed as the product of 2 four-vectors computed at the same point (contact interaction), one concerning the heavy particles, the other the light particles:
>
> $$H_{fi} = g\left[\left(\bar{\psi}_p \gamma_\mu \psi_n\right)\left(\bar{\psi}_e \gamma^\mu \psi_\nu\right) + \text{h.c.}\right] \tag{1}$$

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However, scrolling the text of the original works, written from 1933 to 1934 [2, 5, 6], it is easy to convince oneself that Fermi uses a Hamiltonian, not a 'Hamiltonian density' as in eq. (1); that his description of nucleons does not rely on the relativistic formalism (and in this way, the symmetry between hadrons and leptons is not emphasised); that neither Dirac γ_μ matrices nor Dirac conjugates are mentioned; that Fermi talks of neutrino emission, not of antineutrinos. Eq. 1 is a modern expression, that in a sense corresponds to those in [2, 5, 6], to which modern theoretical physicists are accustomed, but that does not allow us to understand the difficulties encountered and overcome by Fermi, and that also prevents us from appreciating the value of subsequent theoretical progress.

In view of the fact that Amaldi's review paper has been (and is) influential, and presentations similar or identical to his have since been commonly adopted - see e.g., [7] - we propose to consider a series of questions to prepare ourselves to better appreciate Fermi's work and legacy:

- *What are the objectives and conceptual bases of Fermi's theory? What are its radical innovations?*
- *What was Fermi's theory important for at the time?*
- *In what aspects does Fermi's theory of β decay differ from the modern one?*
- *How do Pauli's, Fermi's and Majorana's ideas on the neutrino compare with each other?*

In the following discussion, we will draw mainly on a recent article [8] prepared on the occasion of the 90th anniversary of Fermi's 'Tentativo', to which we refer the reader interested in detailed information and specific references.

2 Fermi's theory of β rays

2.1 Origin, purpose, basis and innovations

The aim of Fermi's work is to provide an answer to the question *"how is it possible for the nucleus to emit electrons, if there are no electrons in the nucleus?".* The formulation of this question helps us to remind the state of previous knowledge: in the second decade of the 20th century, a somewhat spontaneous opinion gained traction, that the electrons emitted in the β decay must pre-exist in the nucleus. This view is clearly stated in a well-known work of 1920 by Rutherford [9], in which he adheres to a model of a nucleus consisting of protons and electrons. As soon as the neutron is discovered, a new and better model is proposed, where the nucleus contains only protons and neutrons [10, 11, 12] (Iwanenko 1932, Heisenberg 1932 and Majorana 1933); but this urgently raises the question of how to model the emission of β rays.

Inspired by de Broglie's ideas, Ambarzumian and Iwanenko [13] had suggested already in 1930 that the electron is created in that process, just as happens to a photon spontaneously emitted by an excited atom; the same was further advocated in 1933 by Francis Perrin, for whom the neutrino should also suffer a similar fate [14]. But none of these authors succeeded in producing a quantitative theory, a calculable model.

Fermi, on the other hand, succeeded in this endeavour with the three papers mentioned above, which have the same content. The first of these appeared just 90 years ago, and the other two provide some further details. The model describes the situation in which an atomic nucleus increases its charge by one unit (attributing this to a change of state of a nucleon - from a neutron, to a proton) and at the same time an electron and a neutrino are created. In formulae,

$$(A, Z) \to (A, Z+1) + e + \nu \tag{2}$$

It is well-known that Fermi's description is generally in good agreement with the observational facts and, over time, it has been improved in various aspects, rather than radically modified.

But let's take a closer look at its original structure. The mathematical formalism adopted to deal with relativistic fermions 1) assumes the correctness of the Dirac equation, 2) the interpretation of its spectrum due to Dirac (see below), and 3) exploits the technique of second quantisation developed by Jordan, Klein, Wigner and Fock. This formalism implies using operators

$$\mathbf{\Psi} = \sum_s \psi_s \, \mathbf{a}_s \tag{3}$$

with dimensions square root of a density; the sum is over all possible states s (positive and negative energies); ψ_s are wavefunctions that solve Dirac equation, normalized *à la* Born; $\mathbf{a}_s$ are adimensional annihilation operators that describe the disappearance of a particle in the state s: $\langle 0 \,|\mathbf{a}_s|\, s\rangle = 1$.

How to avoid a disastrous process of creating electrons of negative energy - and in particular, how to prevent atomic electrons from accessing negative energies? The chosen way to go is the one described by Dirac, that we are going to recall. It is assumed that, as a rule, all negative energy fermion states are occupied; this is the hypothesis of the ***Dirac sea***. We reiterate that this formalism is used *only* for electrons and neutrinos. In this way, Fermi

- manages to describe the spin of electrons and neutrinos in the theory;
- does not emphasise the other crucial aspect of Dirac equation - antiparticles;
- relies on the less innovative - but adequate - isospin formalism for nucleons.

Let us emphasise the point we made, as explicitly as possible:

> *Fermi uses quantized fields to deal with electrons and neutrinos, but not the formalism of canonical quantisation*

The original form of Fermi's hamiltonian is the following one,

$$\boldsymbol{H} = g\, Q\, \boldsymbol{\Psi}^t \delta \boldsymbol{\Phi} + \text{h.c.} \tag{4}$$

where
⋆ g denotes Fermi's constant, with units energy per volume;
⋆ Q the dimensionless isospin matrix, which transforms a proton into a neutron;
⋆ $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ the fields of second quantisation of the relativistic particles with spin $1/2$, the electron and the neutrino, which have the same units as the wave functions (root of a density=root of an inverse volume);
⋆ the superscript t denotes the transpose;
⋆ δ an appropriate 4×4 dimensionless matrix which ensures the Lorentz invariance of the expression;
⋆ the Hermitian conjugate term (*h.c.*) in Eq. 4 guarantees the conservation of probability.
The interaction energy $\boldsymbol{H} = \int d^3x\, eV\, \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$ is the model of Fermi's hamiltonian: the electrostatic energy $e\, V(x)$ is replaced by the nuclear energy $g\, \delta^3(x) Q$ and the scalar current $\boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$ by $\boldsymbol{\Psi}^t \delta \boldsymbol{\Phi}$; the expression is given in the limit of nucleons at rest. The *h.c.* term accounts for the decay of the neutron through the irradiation of an electron and a neutrino.

From a conceptual point of view, the main innovation of Fermi's model is that it formally describes the possibility that a particle can be destroyed or created. It is the first time that *particles of matter* are assumed to undergo a similar fate. This constitutes a milestone in modern particle physics, although the formalism adopted (which derives from Jordan, Klein on the one hand and from Dirac's positron theory on the other) does not coincide with the current one. See again [8] for further discussion and just below for subsequent developments.

2.2 Reactions to Fermi's paper and its legacy

Fermi's use of Dirac sea exposed him at the time to the same criticism as Dirac[1]. In addition to these reservations of a general nature, the work will be the subject of a wide-ranging and lively debate. Limiting ourselves for the moment to the main contributions of a critical nature, let us mention for example the specific 1935 proposal by Konopinski and Uhlenbeck [17], which at first seemed superior to Fermi's, but which emerges defeated from the confrontation a few years later. Let us then recall a criticism by Pauli in 1938 [18], centered on the fact that the theory includes the parameter g with canonical dimensions equal to the inverse of a square mass in natural units, a circumstance that entails going

[1] See e.g., [15] and [16]. In his memoirs, Occhialini reiterates that the old guard physicists such as Rutherford and Bohr, but also Chadwick, maintained reservations at least until 1932. From Majorana's correspondence, a feeling of doubt towards Dirac interpretation persisted in 1933. In 1933, Pauli still doubts Dirac's argument for the anti-electron; next year, with Weisskopf he succeeded in quantising a hypothetical spinless particle without resorting to Dirac sea - a procedure Pauli liked to refer to as 'anti-Dirac theory'. See [8] for references.

outside the theory itself with perturbative orders higher than the first; but as is well known today, Fermi's theory is to be thought of as an effective theory and therefore is to be used precisely at the first perturbative order.

In short, Fermi's theory fully hits the mark, in spite of the usage of second quantization based on Dirac sea (or in Fermi's words, the Dirac, Jordan, Klein procedure) and the specific choice of Hamiltonian function, aspects that only apparently are limiting.

To convince oneself of this, one need only recall three important works from various parts of the world [19, 20, 21] inspired by the 'Tentativo' and written soon afterwards, in 1934, by Wick (4 March); Bethe and Peierls (7 April); Yukawa (17 November):

1. *Wick* derives the predictions for β^+ emission and electron capture, using - just like Fermi - the second quantisation formalism. The first process explains observations already obtained by Joliot and Curie, the second (one of the proofs for the existence of the neutrino) will receive experimental confirmation a few years later. See Fig. 1 for a description of the latter process[2].

2. *Bethe and Peierls*, making explicit reference to Fermi 1933, estimate the neutrino-nucleon interaction cross section by means of a brilliant argument. This reaction will be exploited for the first experimental observation of the neutrino. See [22].

3. *Yukawa*, interested in understanding interactions between nucleons, will propose the idea that interactions between nucleons and those between leptons are mediated by a boson with non-zero mass, in order to reproduce Fermi's theory by mimicking even more closely the structure of electromagnetic interactions.

2.3 Subsequent progresses of β decay theory

Nowadays, most particle physicists are aware of certain results of the theory of weak interactions, due to subsequent theoretical developments. For example, it is generally recalled that Gamow and Teller's [23] included the effect of spin in the nucleonic current, which using the current language of γ matrices [24] we attribute to the presence of axial currents; even better known is the much later history of how the $V-A$ structure (chiral interactions) of the charged currents was understood - see e.g., the fine work of review by Weinberg [25]. Among the other recent developments we mention at least the understanding of the conservation of leptonic number in the β interactions, and the thorough examination of the

[2] A description in words is as follows: consider Fermi's reaction $n \to p + e + \nu$ taking place on the Dirac sea of the neutrino, but supposing that there is a hole that we indicate with $|\text{sea} - \nu\rangle$ (to distinguish it from the case when the sea is full). Therefore, the initial state contains a neutron and a anti-neutrino $\bar{\nu}$, and the final one a proton, an electron, a neutrino and a anti-neutrino. However, the newly produced neutrino can fill the hole, and we conclude that the transition $n + \bar{\nu} \to p + e$ is possible. The last step is simply to invert the direction of the arrow getting: $p + e \to n + \bar{\nu}$.

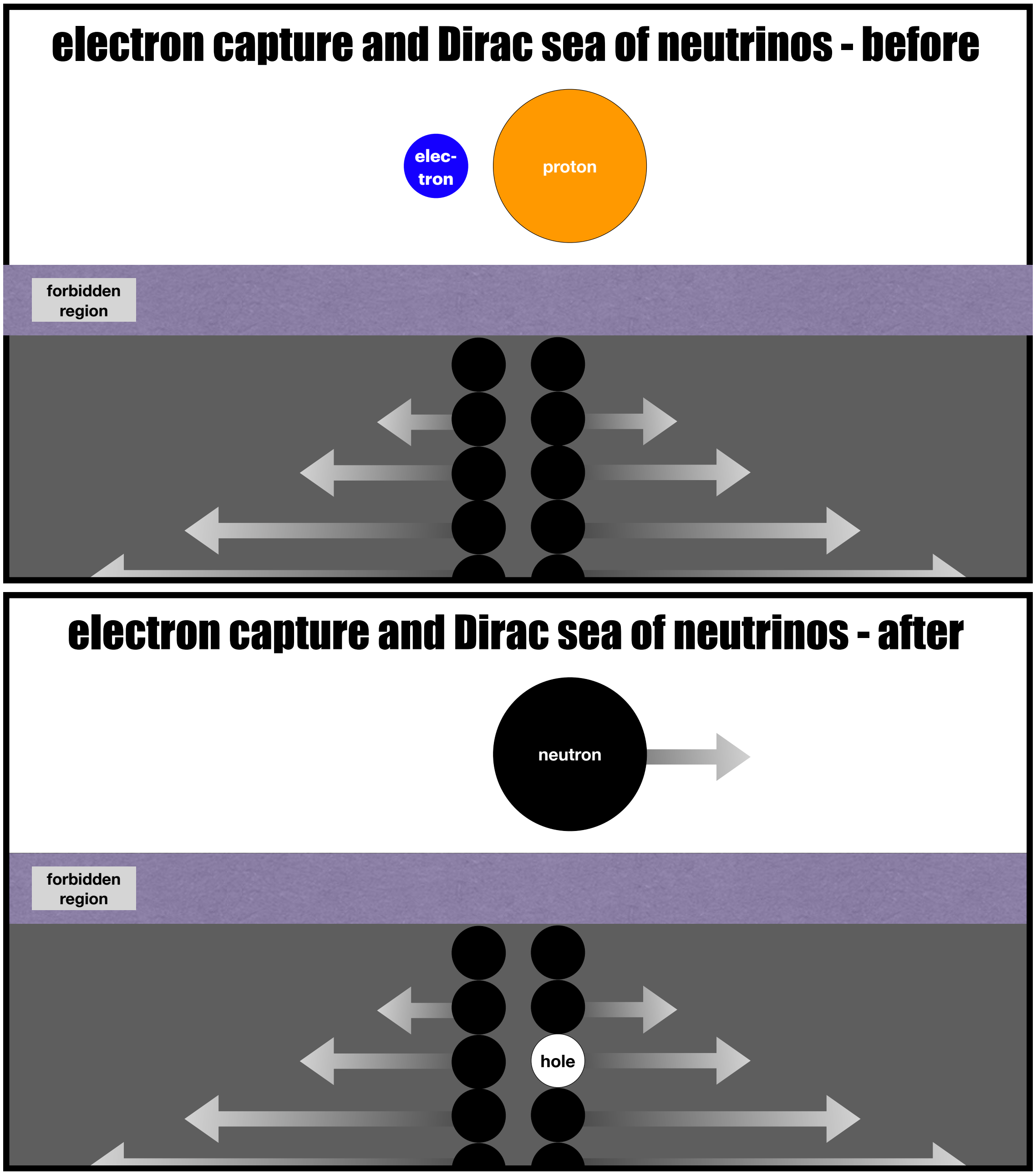

Figure 1: *Description of electron-proton capture in the formalism of second quantization [19], emphasizing the Dirac sea of neutrinos (=region of negative energies).* Panel above: *Initial state of the process; an electron and a proton at rest can be seen; the neutrinos states of Dirac's sea are all occupied.* Panel below: *Final state of the process. The nucleon changed its isospin state and became a neutron; a hole has formed in the Dirac sea, which can be thought of as an antineutrino, moving in the opposite direction of the neutron (=Dirac hole theory).*

structure of currents concluded and completed with the Cabibbo theory. All these advances dovetail with Fermi's theory.

We would just like to point out an important advance that occurred in the late 1930s, which is not sufficiently appreciated today but is the source of many other advances. We refer to a different procedure of quantization of fermionic fields introduced by another of the boys from via Panisperna, Ettore Majorana [3]. Apart from a witty choice of basis for γ matrices used, the new procedure of quantisation of fermions is exactly the one used today, i.e., the 'canonical quantisation'. In the first part of the summary of his work of 1937 (the last one) [3], at page 171, we read

> It is shown how to achieve a full formal symmetrization of the quantum theory of the electron and positron by making use of a new quantisation process. The meaning of the equations of DIRAC equations is quite modified and there is no longer any need to speak of states of negative energy.

To ascertain Fermi's appreciation of this result, let us read his judgement for the chair competition, held in the same year (from [26], preface, page xiii):

> [Majorana] devised a brilliant method for treating the positive and negative electron symmetrically, finally eliminating the need to resort to the extremely artificial and unsatisfactory hypothesis of an infinitely large electric charge spread throughout space, an issue that had been addressed in vain by many other scholars

If the terms were used literally, only from this moment on would it be legitimate to speak of a "vacuum state" rather than a "ground state". In more, evocative terms we can say that it was Majorana who showed the world *how to empty the Dirac sea*[3]. But an unsuspecting reader, who believed Amaldi's 1984 notations to be the original ones, would not even notice this breakthrough; losing sight of the context, he would no longer be able to truly understand Fermi's work, let alone Majorana's.

3 Pauli, Fermi and Majorana: three ideas on the neutrino compared

In this last section, we address one last conceptual point, and discuss the three different ideas of the neutrino that were formulated in the 1930s:

1. *Pauli 1930* [1] introduced the neutrino as a constituent of the atomic nucleus in 1930 and assumed that this particle is emitted in β decay. This model has no relativistic characteristics and in particular has no connection with Dirac idea of antimatter.

[3] It should be stressed that the Dirac sea hypothesis, unattractive from a physical point of view and now abandoned, is accompanied by relatively simple and almost spontaneous expressions for the second quantization fields, Eq. 3. For this reason it maintains a certain interest in learning paths: it allows us to appreciate how we arrived at modern quantized field theory.

2. *Fermi 1933-1934* [2, 5, 6], on the other hand, describes neutrinos that are relativistic fermions, completely analogous to the electron. Given the formalism adopted - which requires a Dirac sea of neutrinos with negative energy - antineutrinos exist and are quite distinct from neutrinos: see again Fig. 1. (In other words, such a neutrino concept corresponds closely to what is now called the 'Dirac neutrino'. Although this term is widespread today, Fermi does not use it and there is no work by Dirac describing such a neutrino concept.)

3. Finally, *Majorana 1937* neutrino idea [3] is still different, and consists of the assumption that the neutrino and the antineutrino are the same particle. A similar identification applies for example to the photon, which however, unlike the neutrino, is not a particle of matter.

Here is how Majorana concludes the summary of his work (again in [3], p.171)

> there is no longer reason [...] to assume for any other type of particles, particularly neutral ones, the existence of «antiparticles» corresponding to «vacua» of negative energy.

where we note the statement on neutral particles which makes implicit reference to neutrinos. (The reference is made explicit in the text.)

We conclude by remarking that the structure of the "standard model" of the electroweak interactions - and in particular, the chiral nature of the charged-currents weak interactions and the way in which neutrinos are included - suggests that Majorana's hypothesis is realised in nature, albeit in a quite specific way: the neutrino and the antineutrino, which we know to be different from each other when they move in ultra-relativistic motion, manifest themselves as *the same particle in the system at rest.* Fig. 2 better illustrates the physical content of this statement. This hypothesis on the nature of the neutrino is the subject of lively experimental investigations in laboratories all over the world[4].

For a more detailed discussion and further references, we refer the reader to [8]; for the modern developments of Fermi's theory, see [30].

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[4]For two recent reviews on this subject, see [27] and [28]. For an interesting discussion of the influence of Hermann Weyl's ideas in neutrino physics, see [29].

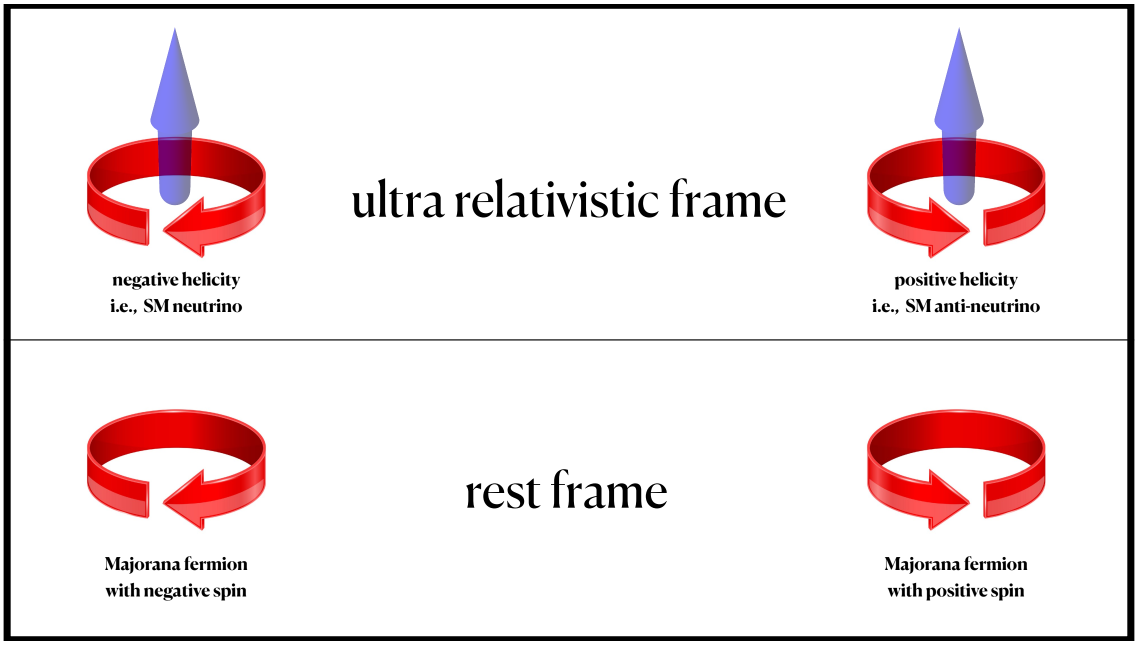

Figure 2: *Illustration of the concept of (neutrino with) Majorana mass in the context of the electroweak/*V-A *theory/standard model. The projection of spin onto the momentum of the particle - helicity - makes it possible to univocally tell neutrinos from antineutrinos in the ultra-relativistic limit. But in the rest system - which for massive fermions exists - the two states are identical, up to the orientation of the spin.*

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