Nuclear Equation of State: from Laboratory to Neutron Stars

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Pre-workshop school

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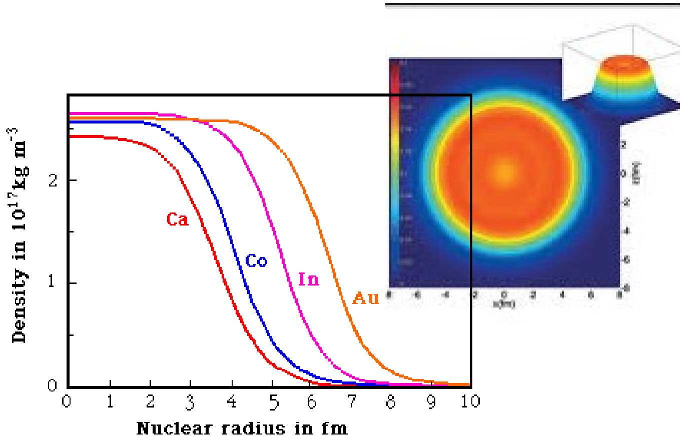
Where can we find neutrons and protons? And in which form? Free? In clusters?

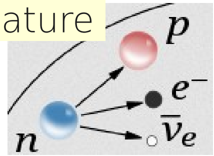
- Neutrons and protons in **Earth** are found in cluster systems: **nuclei**



  → The **interior** of all nuclei has **constant density** (10^{14} times denser than water) named **saturation density**

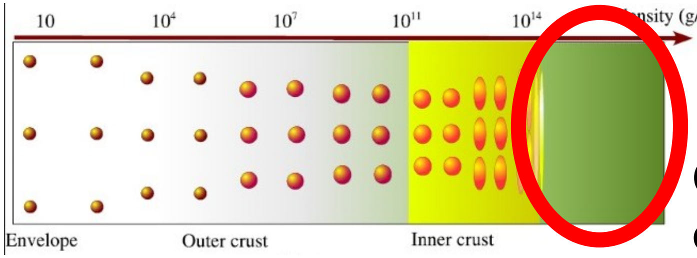
  → Saturation is originated from the **short range** nature of the **nuclear effective interaction**

  → Neutron in 15 minutes must find a proton or …

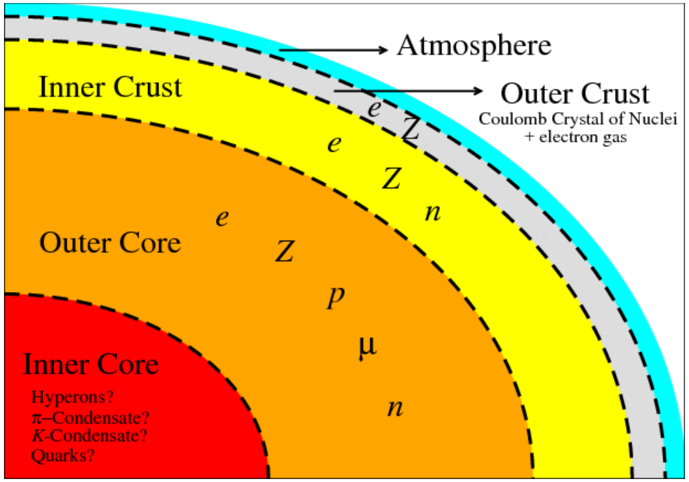


- In **heavens**, neutrons and protons can be also found as an interacting sea of fermions (Fermi liquid): **matter in the outer core of a neutron star**

  → Densities can reach several times nuclear saturation



Outer core



Nuclear Equation of State (EoS)

Definition: the **energy** per **nucleon** ($e=E/A$ where $A=N+Z$) of an **uniform system of neutrons and protons** as a function of the **neutron** (ρ_n = N/V) and **proton** (ρ_p = Z/V) **densities**, at **zero temperature**, **unpolarized**, assuming **isospin symmetry** and **neglecting Coulomb** effects among protons.

Why???

→ **Zero temperature:** room temperature 10^2K→$\mathbf{10^{-8}}$ **MeV** while "cold" neutron stars are at about 10^{10}K→1 MeV. **Separation energy** in stable nuclei (equivalent to ionization energy in atoms) is of **several MeV**.

→ **Unpolarized:** energy favours **couples** of neutrons and protons **occupying the same state but with opposite spins** (equivalent to electrons in atoms)

→ **Isospin symmetry:** neutron-neutron, proton-proton and neutron-proton **nuclear interaction** are very **similar** among them. **Masses** of neutrons and protons are almost **degenerate**. Hence neutrons and protons can be thought as **two states** of the **same particle** with different isobaric spin or **isospin** (in analogy with spin): the **nucleon.**

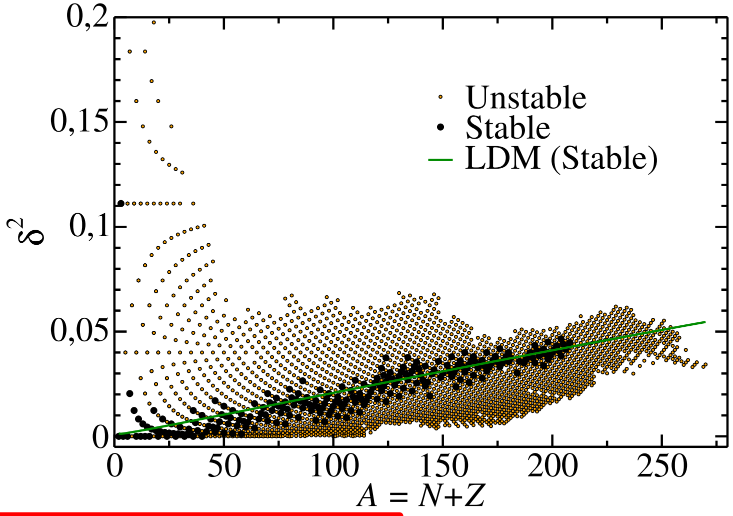
→ **No Coulomb:** **idealized uniform system (focus on strong interaction)**. Real systems are finite and frequently electrically neutral so no problems (divergences) in adding Coulomb.

[Besides that, the strong interaction at the typical scale of a nucleus is much stronger than the Coulomb interaction and the Coulomb energy (*) per particle of an infinite system of protons would be infinite.]

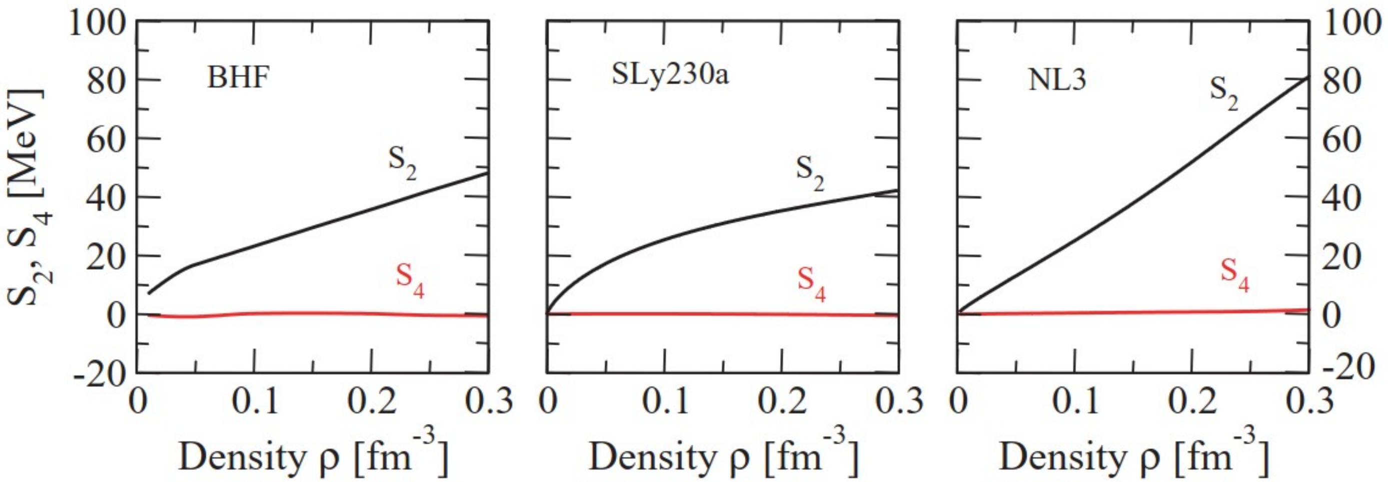
$$(*)\text{Coulomb interaction infinite range!!} \rightarrow \frac{E_C}{A} \sim \frac{Z(Z-1)}{A^{4/3}} \sim Z^{2/3}$$

Nuclear Equation of State (EoS)

It is convenient to write the energy per nucleon (***e***) as a **function** of the **total density** **$\rho=\rho_n+\rho_p$** and the **relative difference** **$\delta=(\rho_n-\rho_p)/\rho$** for **unpolarized uniform matter at T=0 assuming isospin symmetry** (even powers of δ). For **$\delta \rightarrow 0$**:



$$e(\rho,\delta) = e(\rho,0) + S_2(\rho)\delta^2 + S_4(\rho)\delta^4 + \mathcal{O}[\delta^6]$$

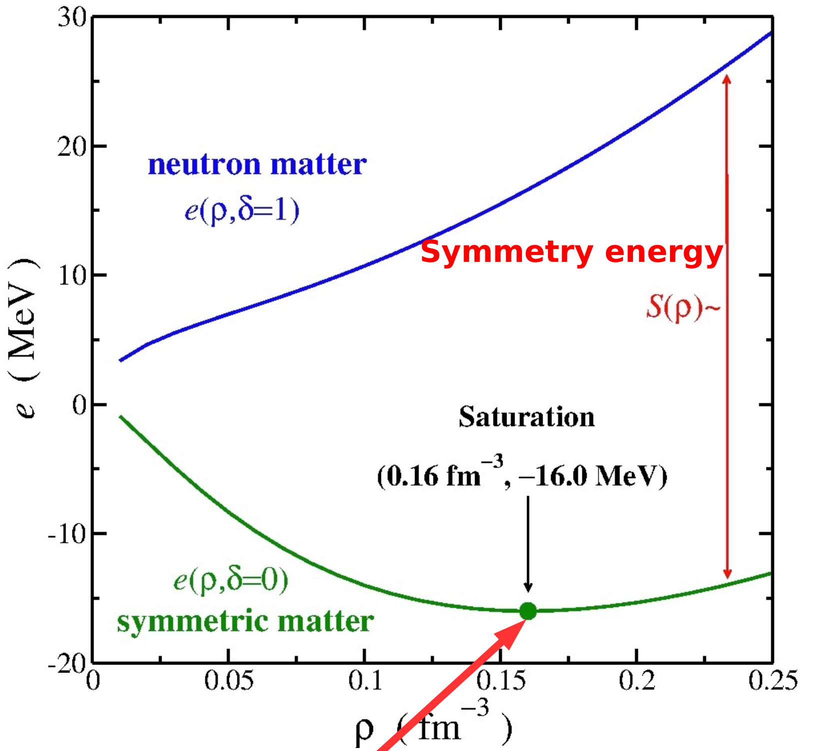


Isaac Vidaña, Constança Providência, Artur Polls, and Arnau Rios
Phys. Rev. C **80**, 045806 – Published 23 October 2009

Nuclear Equation of State (EoS)

Unpolarized, uniform nuclear matter at T=0 assuming isospin symmetry

$$e(\rho,\delta) = e(\rho,0) + S_2(\rho)\delta^2$$



$$P(\rho=\rho_0,\delta=0) = 0 \ \text{MeV fm}^{-3}$$

It is customary to also **expand** e(ρ,0) and S(ρ) around nuclear **saturation density** $\rho_0 \sim 0.16$ fm^{-3}

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

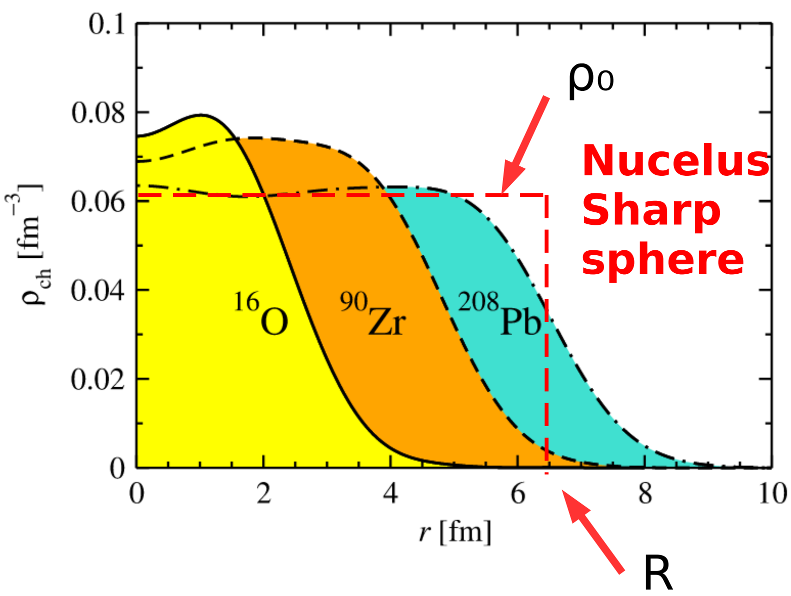
$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

K_0 → how **compressible** is symmetric matter at ρ_0

J → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L → **neutron pressure** in neutron matter at ρ_0

Saturation density $\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$



→ **Range of the nuclear interaction** ($1/m_\pi \sim$1-2 fm) typically **shorter** than the **size** of the **nucleus.** Hence, neutrons and protons just “see” their closest neighbours.

→ **Experimental charge (Z) density** in the interior of very **different nuclei** is rather constant at around **0.06-0.08 fm^{-3}**.

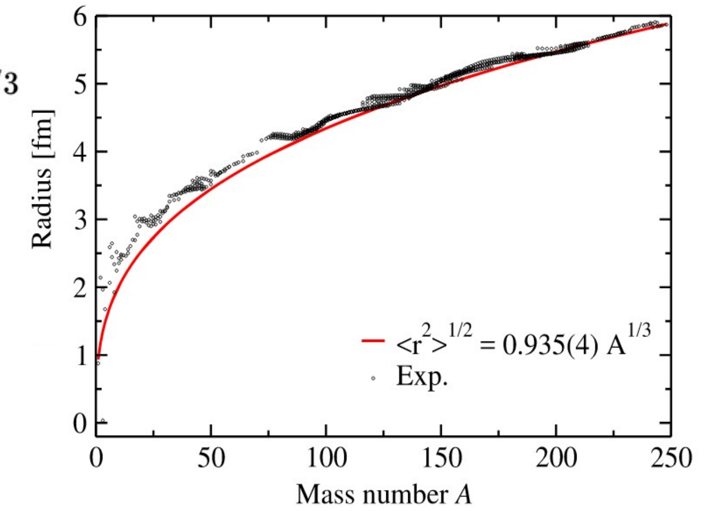
→ **Saturation mechanism** (equilibrium) that originates from the **short-range nature of the nuclear force,** much stronger than the Coulomb repulsion at the nuclear scale.

$$N + Z \equiv A = \int d\mathbf{r}\,\rho(\mathbf{r}) \xrightarrow{\text{sharp sphere}} A = \frac{4}{3}\pi\rho_0 R^3 \rightarrow R = \left(\frac{3}{4\pi\rho_0}\right)^{1/3} A^{1/3}$$

$$\langle r^2\rangle^{1/2} \equiv \left(\frac{1}{A}\int d^3r\, r^2 \rho\right)^{1/2} \approx \sqrt{\frac{3}{5}}R = \sqrt{\frac{3}{5}}\left(\frac{3}{4\pi\rho_0}\right)^{1/3} A^{1/3}$$

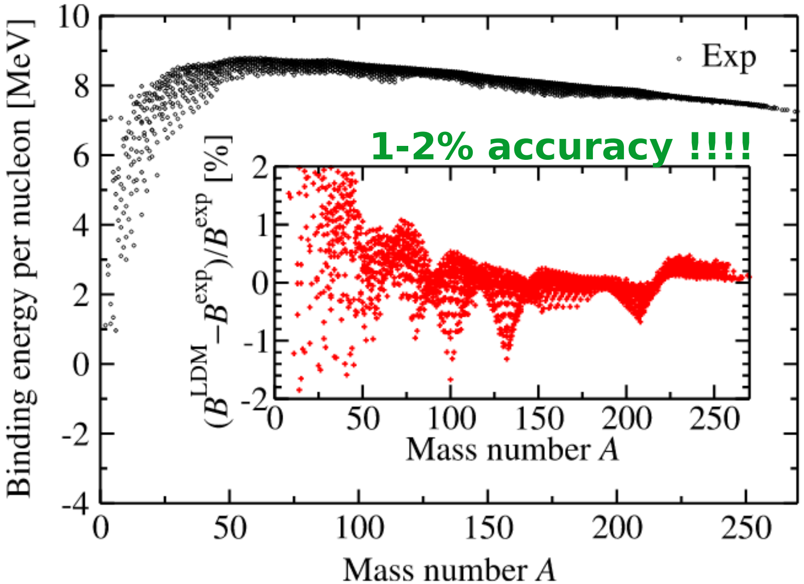
$$R \approx r_0 A^{1/3}$$

≈0.9 fm

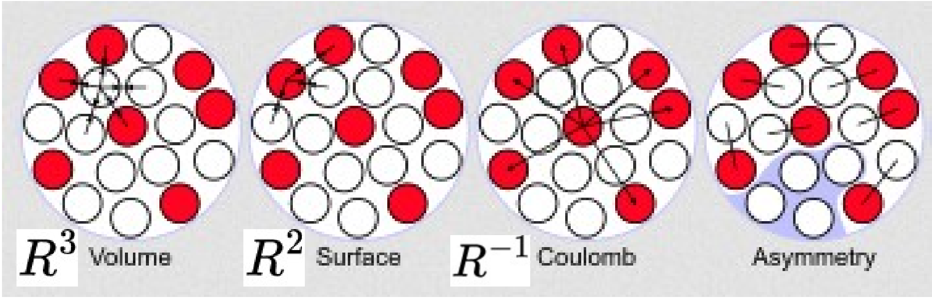


Energy at saturation density:

energy of a nucleon "far from the surface" → $a_V \approx 16$ MeV



→ Nucleus seen as an incompressible liquid (ideal) drop: sharp sphere of radius $R \approx r_0 A^{1/3}$



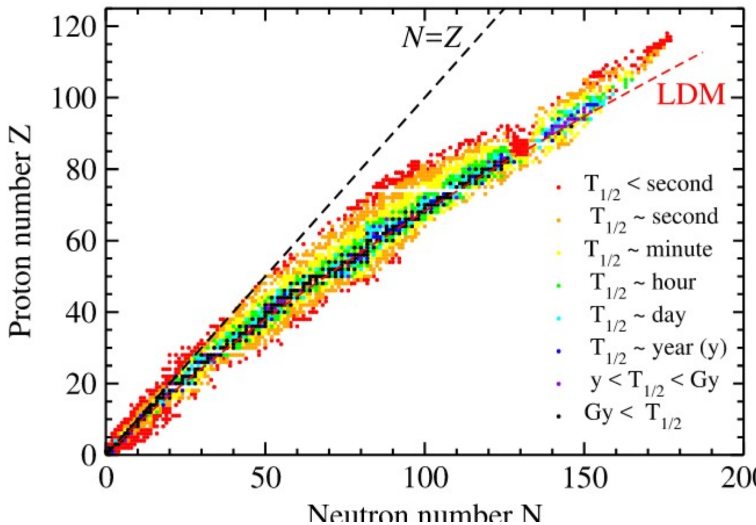
$$M(A,Z) = m_p Z + m_n(A-Z) - B(A,Z)$$

$$B(A,Z) = (a_V - a_S A^{-1/3})A - a_C \frac{Z(Z-1)}{A^{1/3}} - (a_A - a_{AS}A^{-1/3})\frac{(A-2Z)^2}{A}$$

Stability of M(A,Z) with respect to **Z** →

$$\frac{Z}{A} \approx \frac{1}{2} \frac{1}{1 + \frac{a_C}{4(a_A - a_{AS}A^{-1/3})} A^{2/3}}$$

Competition between Coulomb (Z→0) and asymmetry (N→Z)



Important!!

→ A **small change** in the **saturation density** will **impact** the **size** of the **nucleus. Charge radii** are determined to an average accuracy of 0.016 fm (Angeli 2013).

For example, if one aims at determining the $\mathbf{r_{ch}}$ **= 5.5012±0.0013 fm** in ^{208}**Pb** one must be **very precise** in the determination of $\boldsymbol{\rho_0}$:

$$\frac{\delta\rho_0}{\rho_0} = -3\frac{\delta R}{R} \rightarrow \frac{\delta\rho_0}{\rho_0} \lesssim 0.1\%$$

Note: typical average theoretical deviation of accurate nuclear models ~ 0.02 fm → $\boldsymbol{\delta\rho_0/\rho_0}$ is determined up to about a **1% accuracy** (That is, third digit in $\rho_0 \approx 0.16$ fm^{-3}**!!**).

→ In a similar way, a **small change** in the **saturation energy** (about $e_0 \approx -16$ MeV) will **impact** on the **nuclear mass.**

For example, if one aims at determining the **B = 1636.4296±0.0012 MeV** in ^{208}**Pb** one must be **very precise** in the determination of $\mathbf{e_0}$ (changed notation!):

$$\frac{\delta B}{B} = \frac{\delta e_0}{e_0} \rightarrow \frac{\delta e_0}{e_0} \lesssim 10^{-6}$$

Note: typical average theoretical deviation of accurate nuclear models ~ 1-2 MeV → $\boldsymbol{\delta e_0/\rho e_0}$ is determined up to about a **0.1% accuracy** (That is, second decimal digit in $e_0 \approx -16.0$ MeV**!!**).

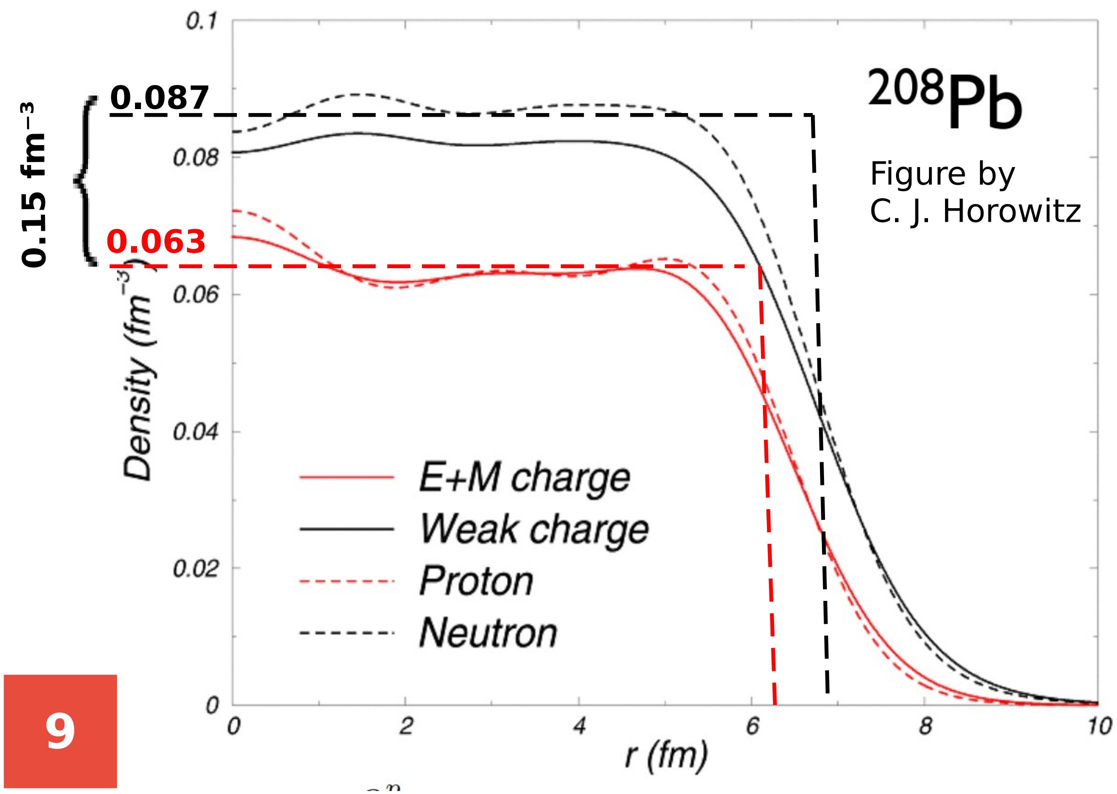
Neutron and proton radii difference

essentially due to the difference between N and Z

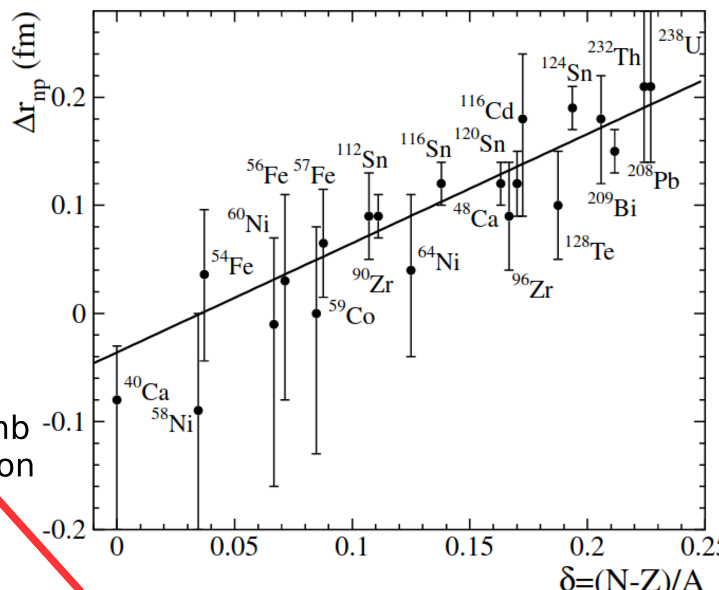
$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

- Elastic electron scattering → electromagnetic size of the nucleus ↔ $\boldsymbol{\rho_p}$
- We have mostly indirect measurements on $\boldsymbol{\rho_n}$ (weakly interacting probes difficult)
- In nuclei with **different** number of **neutrons** and **protons**, we expect $\mathbf{R_n}$ could be different from $\mathbf{R_p}$:

$$\frac{\langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}}{\langle r^2 \rangle^{1/2}} \approx \frac{N^{1/3} - Z^{1/3}}{A^{1/3}} \xrightarrow[I \ll 1]{I \equiv (N-Z)/A} \frac{\langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}}{\langle r^2 \rangle^{1/2}} \propto \frac{N-Z}{A}$$



Coulomb repulsion → R_p larger



$$\Delta r_{np} = -0.04(3) + 1.01(15)\frac{N-Z}{A}\ \text{fm}$$

A. Trzcińska, J. Jastrzębski, P. Lubiński, F. J. Hartmann, R. Schmidt, T. von Egidy, and B. Kłos
Phys. Rev. Lett. **87**, 082501 – Published 2 August 2001

Neutron skin thickness (Δr_{np}:=r_n-r_p) and neutron pressure

For a fixed **(N-Z)/A**, one must **expect** that the **larger the pressure felt by nucleons**, **the larger the skin**

$$P = -\frac{\partial E}{\partial V}\bigg|_A = \rho^2 \frac{\partial e(\rho,\delta)}{\partial \rho}\bigg|_\delta =$$

$$\rho^2 \frac{\partial}{\partial \rho}\left[e(\rho,0) + S(\rho)\delta^2\right] =$$

$$\rho^2\delta^2 \frac{\partial S(\rho)}{\partial \rho} = \frac{1}{3}\rho\delta^2 L$$

→ From the Droplet Model:

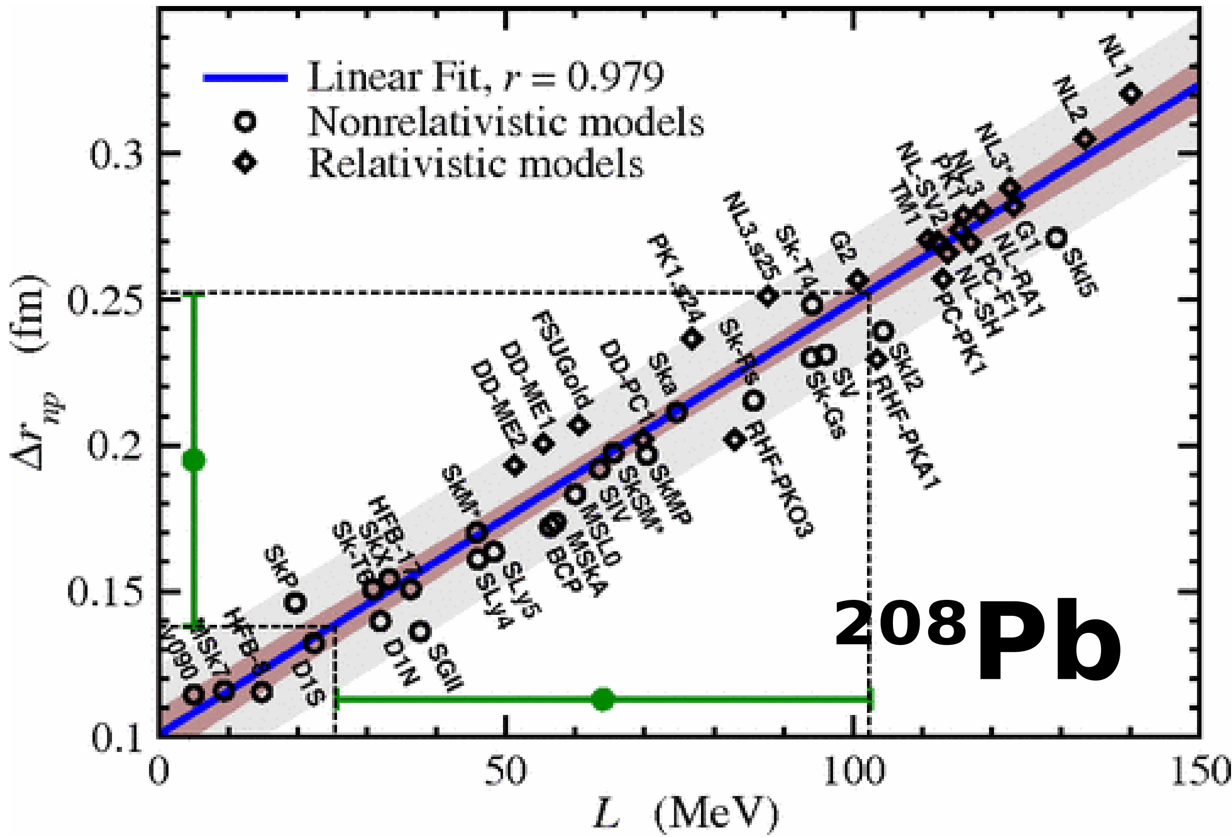
$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

The nuclear droplet model for arbitrary shapes

W.D Myers, W.J Swiate

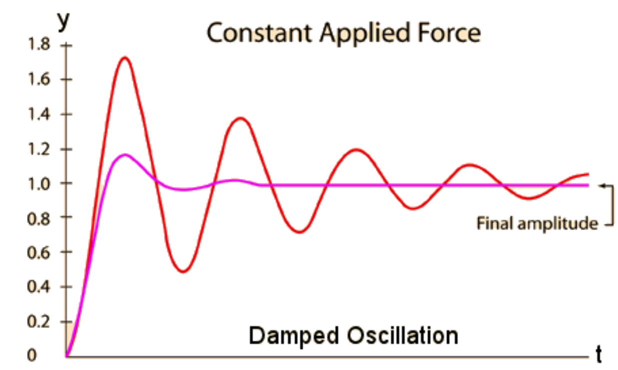
Annals of Physics

Volume 84, Issues 1–2, 15 May 1974, Pages 186-210

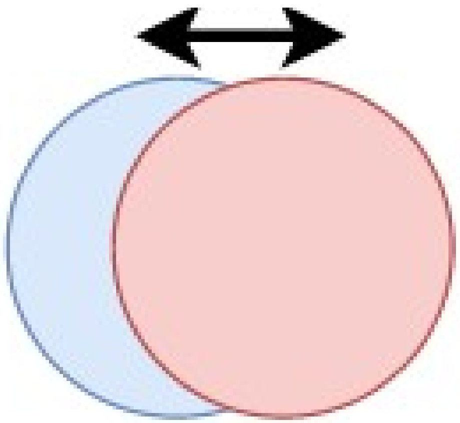


Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

What happens if we now perturb the ground state densities?



Produce a **small displacement (dl)** between **neutron and proton densities** (drops)



$$\rho = \rho_0 + \delta\rho_0 \approx$$

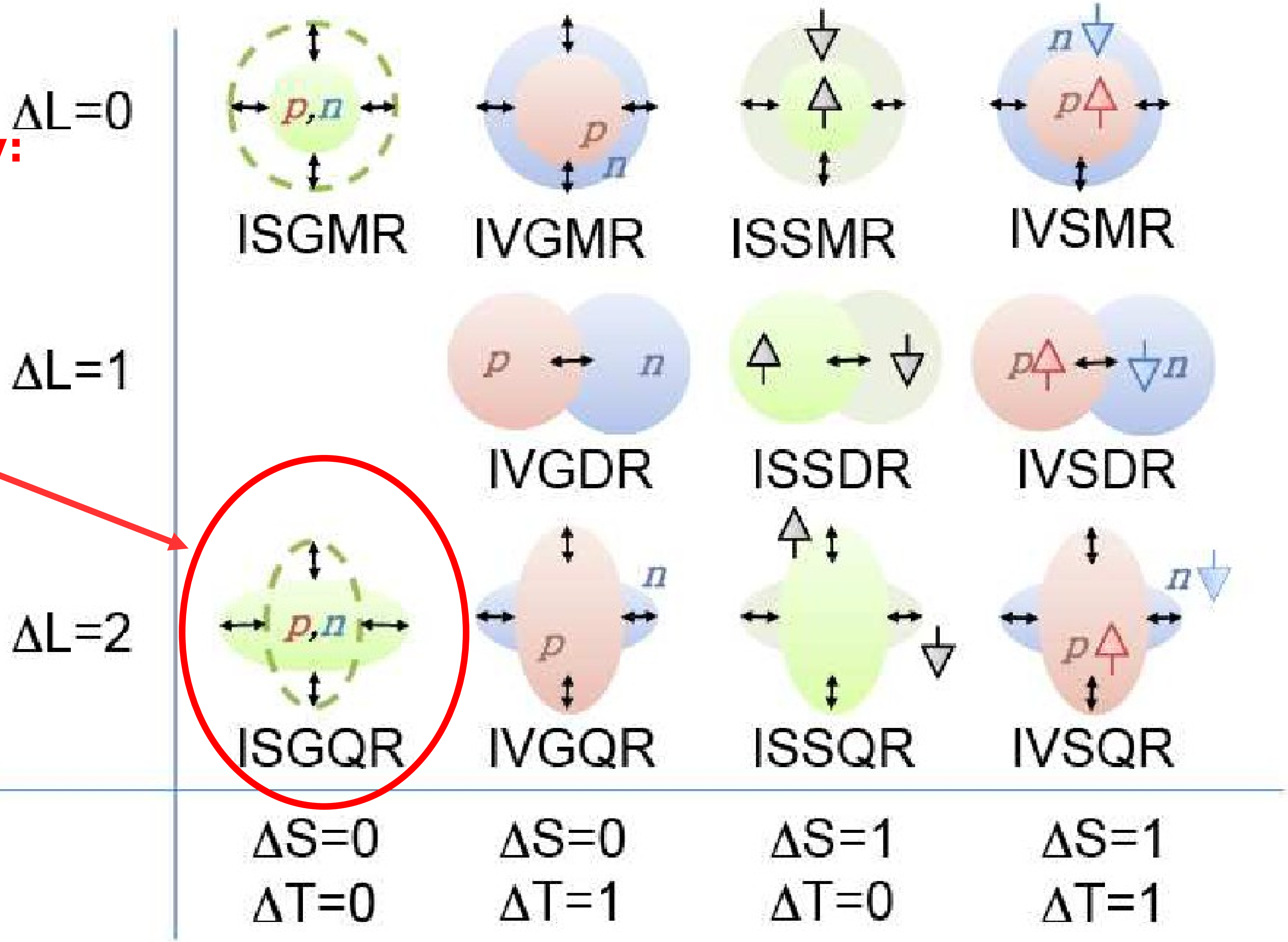
$$\rho_0 + d\vec{l} \cdot \vec{\nabla}\rho_0$$

(Linear response theory)

Under different types of perturbations, **nuclei use to show ressonant behaviours** where all nucleons oscillate coherently and the nucleus as a whole vibrate at an specific resonant energy → known as **Giant Resonances**

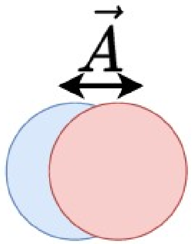
Giant Resonances

(**By the way:** **dominant oscillation mode of a Neutron star in a Neutron star merger)**



http://www.majimak.com/wordpress/

Giant resonances: the IVGDR



→ The **Isovector Giant Dipole Resonance** was the first resonance measured (**photo-absorption** experiments)

→ The **cross section** for the **excitation** of the nucleus to a **final state** $|\nu\rangle$ with energy $\mathbf{E_\nu}$ from the **ground state** $|0\rangle$ with energy $\mathbf{E_0}$ by a **photon** at a given energy **E** can be written as

$$\sigma_\nu(E) = 4\pi^2\alpha(E_\nu - E_0)|\langle\nu|F_{\text{dipole}}|0\rangle|^2\delta(E - E_\nu + E_0)$$

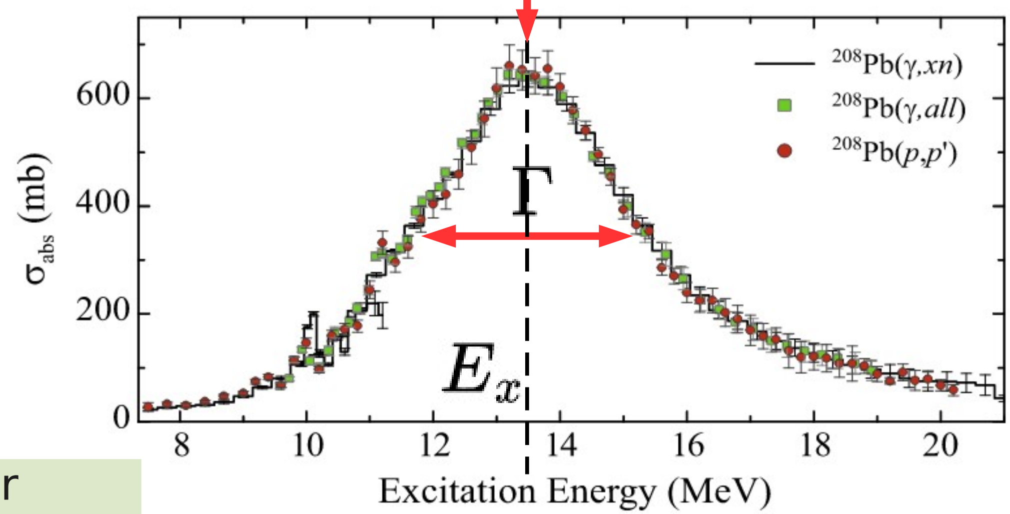
Convenient operator $[\sim r\,Y_{10}(\Omega)]$ produces dipole transitions and subtract CM motion

→ The **total cross-section** will be

$$\sigma_{\gamma-\text{abs}} = 4\pi^2\alpha\sum_\nu(E_\nu - E_0)|\langle\nu|F_{\text{dipole}}|0\rangle|^2$$

$$S(E) \equiv \sum_\nu|\langle\nu|F|0\rangle|^2\delta(E - E_\nu + E_0)$$

where **S(E)** is the so called **Strength function**

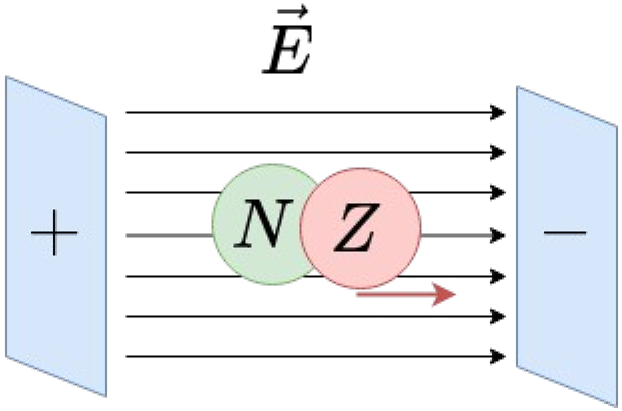


S(E) is used to **characterize** the nuclear **response** (experimentally, it is commonly parametrized as a Lorentzian function with energy E_x and width Γ)

Dipole polarizability
(Giant Dipole Resonance)

As in Electromagnetism course in the Physics degree, the **electric polarizability** measures **tendency** of the nuclear **charge distribution** to be **distorted**

$$\alpha = \frac{\text{electric dipole moment}}{\text{external electric field applied}}$$

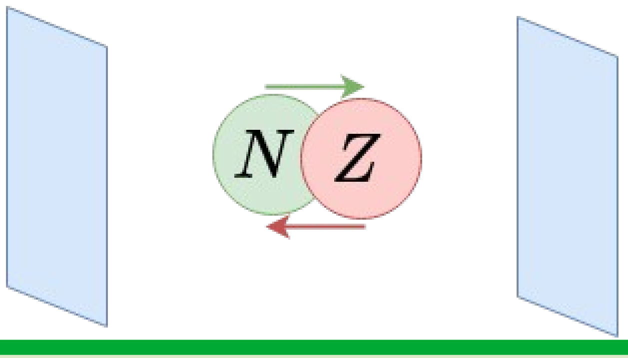


Polarizability is **proportional** to the inverse energy weighted sum rule **m_{-1} = Σ S(E)/E** (response function theory)

How easy is to separate neutrons from protons?
Symmetry energy will tell (Harmonic Oscillator model)

$$e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2$$

$$E_x \sim \sqrt{\frac{\partial^2 e(\rho,\delta)}{\partial\delta^2}} \sim \sqrt{S(\rho)}$$



Tidal deformability in a neutron star ↔ quadrupole polarizability

Dielectric theorem:
Inverse Energy Weighted Moment of S(E): m_{-1} or polarizability

Ground state |0> **perturbed** by an **external field** λF (λ → 0) so that perturbation theory holds → The **expectation value** of the **Hamiltonian** <H> and of the **operator** <F> can be written:

$$\delta\langle\mathcal{H}\rangle = \lambda^2 \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^3) = \lambda^2 m_{-1} + \mathcal{O}(\lambda^3)$$

$$\delta\langle F\rangle = -2\lambda \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^2) = -2\lambda m_{-1} + \mathcal{O}(\lambda^2)$$

$$m_{-1} = \frac{1}{2}\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\lambda^2}\bigg|_{\lambda=0} = -\frac{1}{2}\frac{\partial\langle F\rangle}{\partial\lambda}\bigg|_{\lambda=0} \quad \rightarrow \quad \frac{1}{m_{-1}} = 2\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\langle F\rangle^2}$$

Dielectric theorem:
Inverse Energy Weighted Moment of S(E):
m_{-1} or polarizability

→ Calculate the polarizzability (α), **proportional to m_{-1}** from the **dielectric theorem** and **Droplet Model** ($J=a_A$)

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1}(E1)$$

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

J. Meyer, P. Quentin, and B. Jennings, Nucl. Phys. A **385**, 269

$$a_{\mathrm{sym}}(A) = \frac{J}{1 + x_A}, \quad \text{with} \quad x_A = \frac{9J}{4Q}A^{-1/3}. \qquad \Delta r_{np}^{\mathrm{DM}} = \frac{2r_0}{3J}[J - a_{\mathrm{sym}}(A)]A^{1/3}(I - I_C)$$

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\mathrm{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Polarizability must increase with the mass (for the dipole $A^{5/3}$, for the quadrupole $A^{7/3}$ and so on) and **surface symmetry energy** and **decrease** with the **bulk symmetry energy**

Giant Monopole Resonance

→ Is the **nucleus compressible** or it is as in the **Liquid Drop Model?** (an ideal incompressible liquid)

The thermodynamic definiton of compressibility is: $\chi = \frac{1}{V}\left(\frac{\partial P}{\partial V}\right)^{-1}$

The K_0 parameter (slide 5) can be easily related to χ from its definition

$$\chi = -\frac{1}{V}\left[\frac{\partial}{\partial V}\left(\frac{\partial E}{\partial V}\right)\right]^{-1}_{A=\text{cons.}} = \frac{9}{\rho K_0}$$

So far this is for the uniform system, what about the nucleus?

$$\chi = \frac{1}{V}\left(\frac{\partial P}{\partial V}\right)^{-1} \xrightarrow{\text{Spherical symmetry}} \frac{1}{\chi} = \frac{r}{3}\left(-rP + \frac{1}{4\pi r^2}\frac{\partial^2 E}{\partial r^2}\right)$$

Nucleus at equillibrium → P = 0. In analogy, we can define $K_A \equiv 9V/\chi$

$$K_A = \frac{9V}{\chi} = 9\frac{r^2}{9}\frac{\partial^2 E}{\partial r^2} = Ar^2\frac{\partial^2 (E/A)}{\partial r^2} = 4A(r^2)^2\frac{\partial^2 E/A}{\partial (r^2)^2}$$

Now, from the moments of **S(E)**, one can define an excitation energy

$$E_x^{\text{centroid}} = \frac{\int ES(E)dE}{\int S(E)dE}; \quad E_x^{\text{constrained}} = \sqrt{\frac{\int ES(E)dE}{\int S(E)/E\, dE}}; \quad E_x^{\text{scaling}} = \sqrt{\frac{\int E^3 S(E)dE}{\int ES(E)dE}}$$

Giant Monopole Resonance

In our case, we will use the **constrained energy** since it is easy to calculate.

The operator leading to monopole transitions (isotropic changes in the volume if we think about a liquid drop) cannot depend on the orbital angular momentum or spin:

$$F = \sum_{i=1}^{A} r_i^2 \quad (*)$$

Isotropic harmonic perturbation!

The m_1 and m_{-1} moments are:

$$m_1 = \frac{2\hbar^2}{m}\langle r^2 \rangle \qquad \frac{1}{m_{-1}} = 2\frac{\partial^2 \langle \mathcal{H} \rangle}{\partial \langle r^2 \rangle^2}$$

Therefore,

$$\left(E_x^{\mathrm{ISGMR}}\right)^2 = \frac{m_1}{m_{-1}} = 4\frac{\hbar^2}{m}\langle r^2 \rangle \frac{\partial^2 E}{\partial \langle r^2 \rangle^2} = 4A\frac{\hbar^2}{m\langle r^2 \rangle}\langle r^2 \rangle^2 \frac{\partial^2 (E/A)}{\partial \langle r^2 \rangle^2} \equiv K_A \frac{\hbar^2}{m\langle r^2 \rangle}$$

Ok, we have now defined the **incompressibilty** of a finite nucleus and **connected** it to an **experimentally measurable quantity**. Can we say something about the EoS?

(*) other definitions could differ by a factor 4π depending if $Y_{00} = 1/\mathrm{sqrt}(4\pi)$ included or not

Giant Monopole Resonance

Nuclear compressibilities
J.P. Blaizot
Physics Reports
Volume 64, Issue 4, September 1980, Pages 171-248

Assuming a **Liquid Drop Model** like expansion for $\mathbf{K_A}$ one can connect it to the bulk incompressibility $\mathbf{K_0}$ (also named "leptodermus" expansion) of the **nuclear EoS**

$$K_A = K_0 + K_s A^{-1/3} + K_\tau \left(\frac{N-Z}{A}\right)^2 + K_C \frac{Z(Z-1)}{A^{4/3}} + \ldots$$

Fitting to the excitation energy of the ISGMR one would obtain the coefficients of this formula. Among them K_0 (recent estimated accuracy over 10% Phys. Rev. C **89**, 044316)

This formula is **qualitative** since misses **shell effects** and **pairing** as well as terms in the **expansion** that goes as powers of **A** and **(N-Z)/A**. Very much like the LDM. Hence the estimation of K_0 would have large systematic (theoretical) errors

For the description of ^{208}Pb (E_x=13.6±0.5 MeV), $\mathbf{K_0}$ must be determined at about **7%** accuracy or better

$$\left(\frac{\delta K_0}{K_0}\right)^2 = \left(2\frac{\delta E_x}{E_x}\right)^2 + \left(2\frac{\delta\langle r^2\rangle^{1/2}}{\langle r^2\rangle^{1/2}}\right)^2 \approx \left(2\frac{\delta E_x}{E_x}\right)^2$$

$$\frac{\delta K_0}{K_0} \approx 2\frac{\delta E_x}{E_x} \approx 7\%$$

What can we learn from the Earth and the Heavens about the Nuclear Equation of State?

(some examples)

From Heaven: Neutron Star Mass

Nuclear models that account for different nuclear properties on **Earth** predict a large **variety** of **Neutron Star Mass-Radius** relations → **<u>Observation of a 2M</u>**

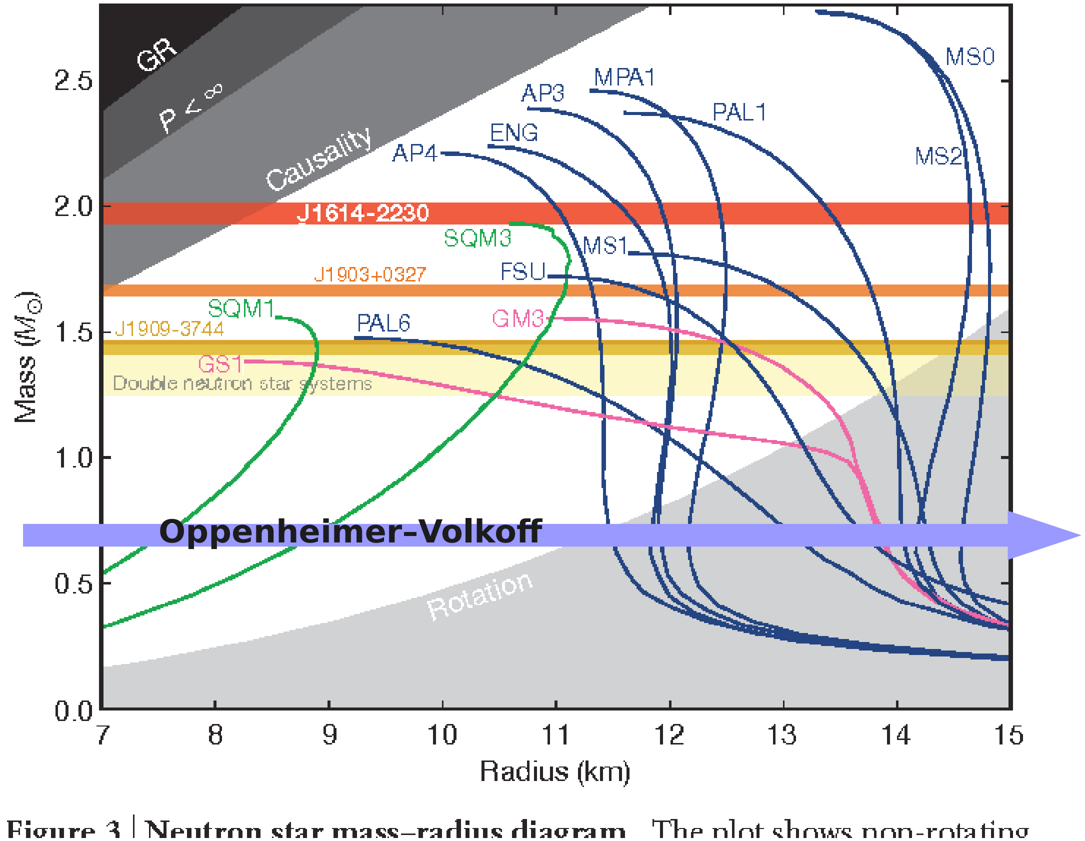
Tolman–Oppenheimer–Volkoff equation (sph. sym.):

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r);$$

$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$\mathcal{E}(r) \rightarrow$ degeneracy pressure from neutrons $\rightarrow M_{\max} = 0.7 M_{\text{sun}}$

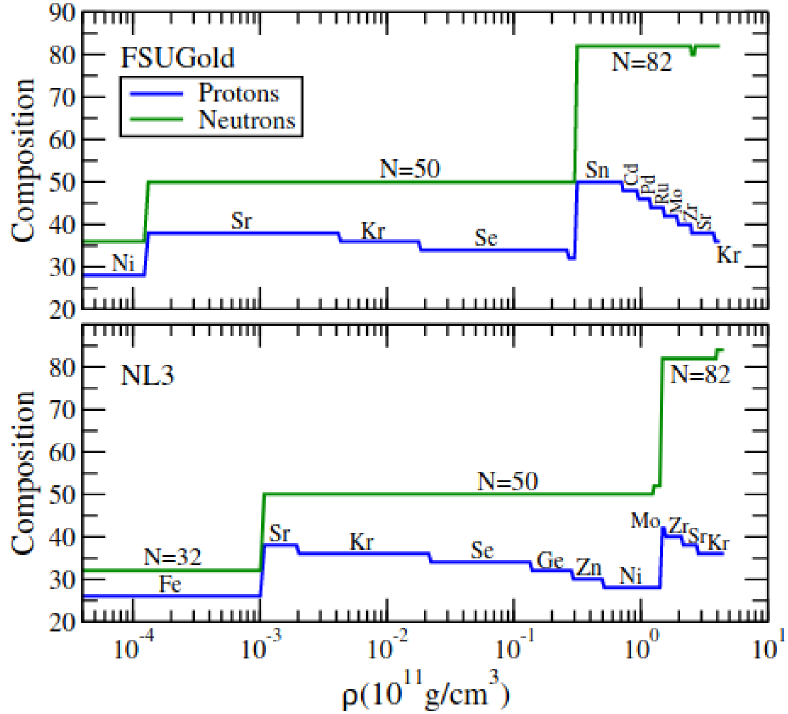
Nuclear Physics input is fundamental



A two-solar-mass neutron star measured using Shapiro delay - P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels - Nature volume 467, 1081–1083(2010)

From Heaven: outer crust composition

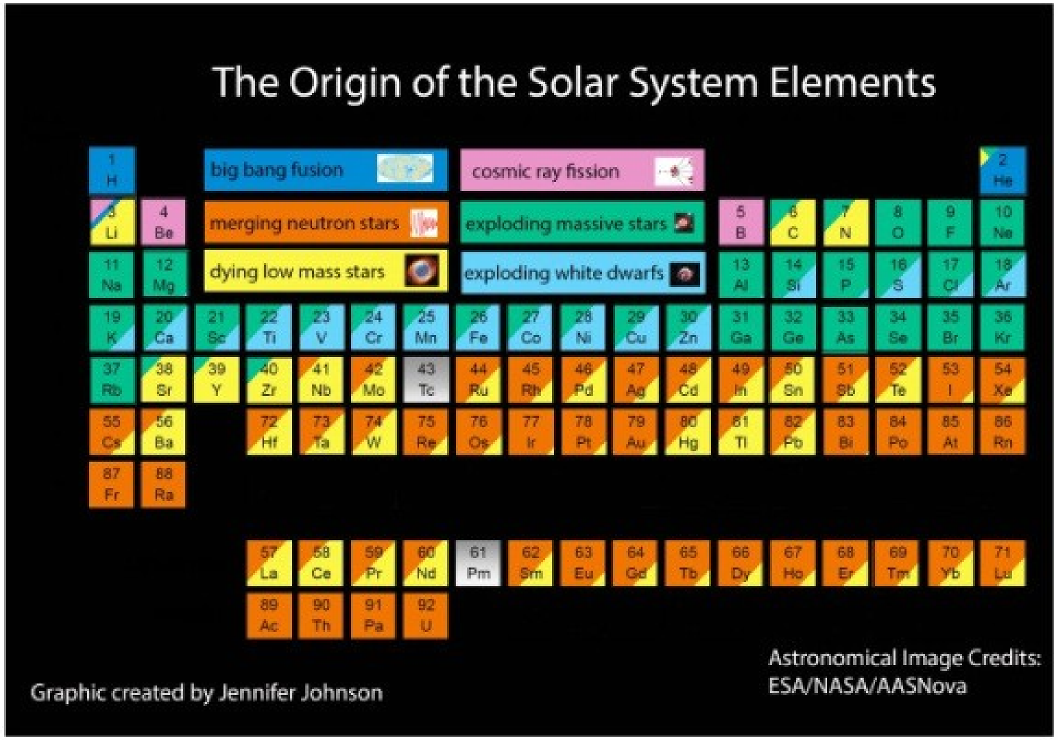
- → span 7 orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g cm to the **neutron drip** $\sim 10^{11}$ g cm)
- → it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- → $T \sim 10^6$ K ~ 0.1 keV $\to$ one can treat **nuclei and electrons at $T = 0$ K**
- → At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}**Fe** nuclei.
- → As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \to (N+1, Z-1) + \nu_e$ and therefore $Z \downarrow$ with constant (approx) number of N
- → As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a different $N-$*plateau*
  $\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{Fe}}{8 a_{sym}}$ where $(A_0, Z_0) = {}^{56}Fe_{26}$
- → The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$)**.
  $[M(N, Z) + m_n < M(N+1, Z)]$



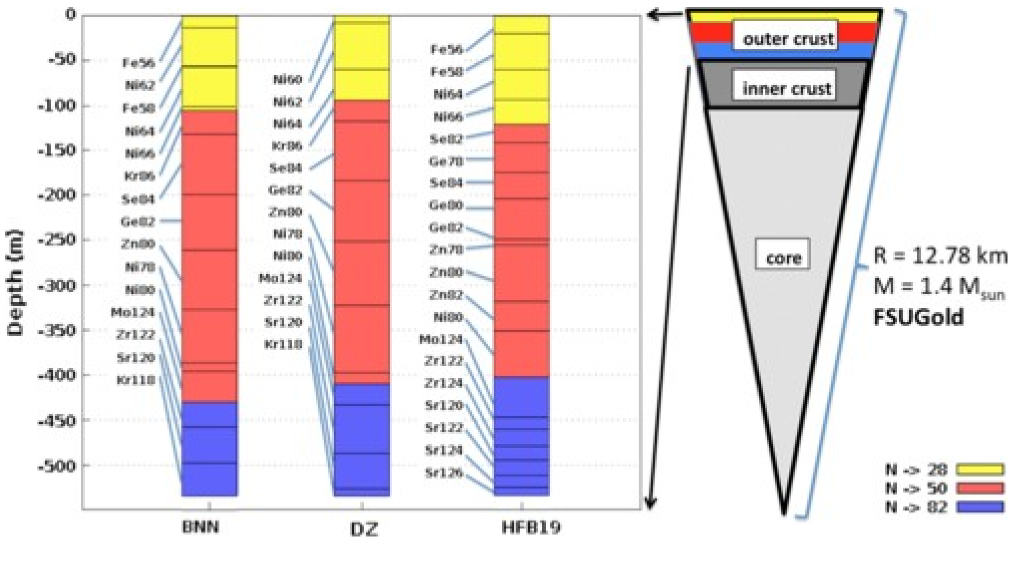
Physical Review C **78**, *025807 (2008)*

The faster the symmetry energy increases with density (L ↑), the more exotic the composition of the outer crust.

From Heaven: Origin of elements



The **crust** of a **NS** is made of very **exotic neutron rich nuclei,** stable only due to the extreme conditions (large densities). **Different nuclear models predict different compositions**

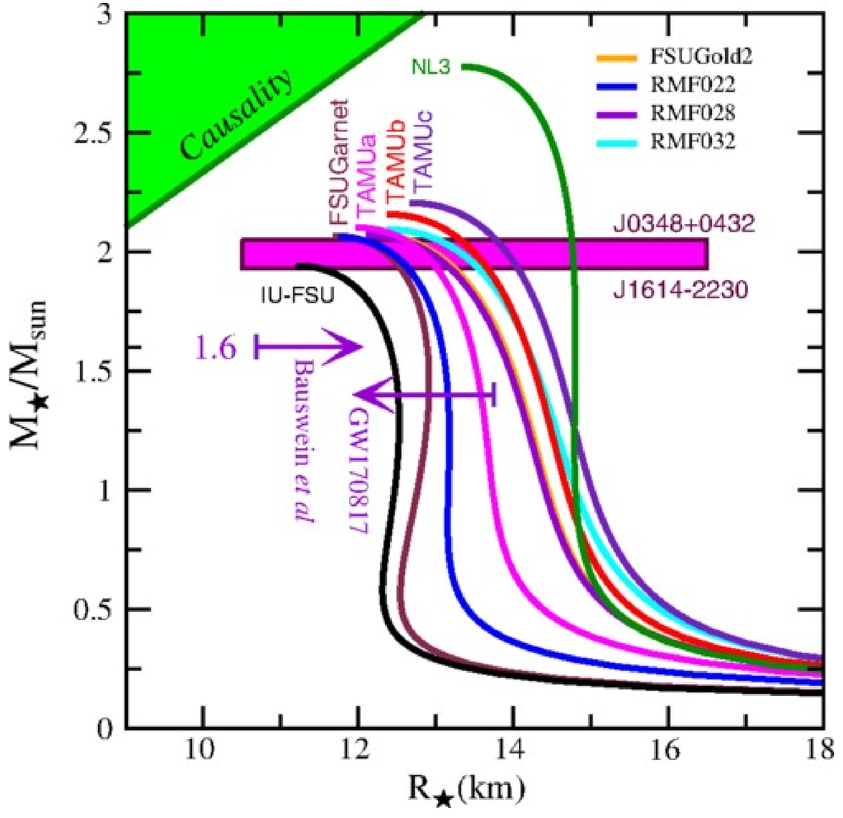


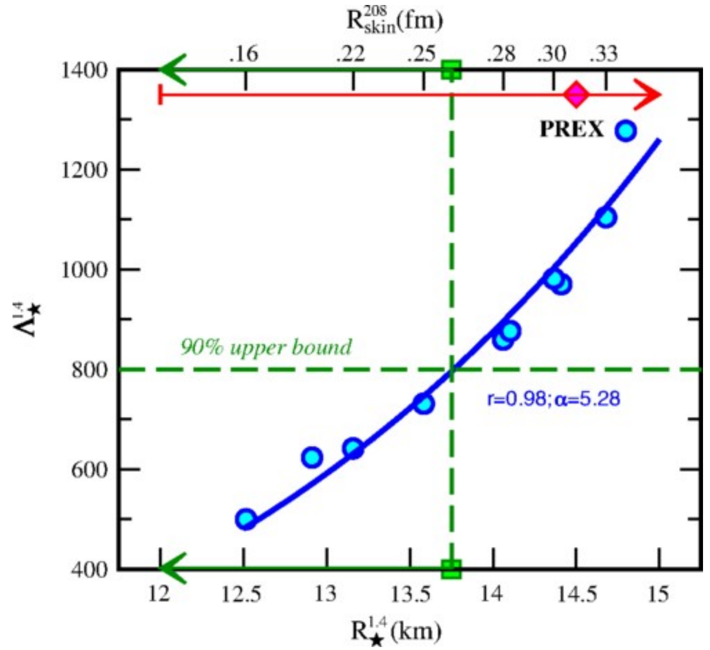
Nuclear mass predictions for the crustal composition of neutron stars: A Bayesian neural network approach R. Utama, J. Piekarewicz, and H. B. Prosper, Phys. Rev. C 93, 014311 (2016)

Binary neutron star merger produced about **10^{29}kg** of **heavy elements**!

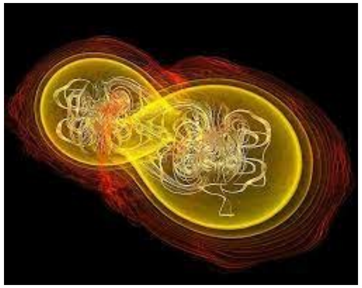
From Heaven: Gravitational wave signal from a binary neutron star merger

GW170817 from the binary neutron star merger → **constraint** neutron star **radius** and, thus, the **nuclear EoS**





Tidal deformability (Λ) is a quadrupole deformation inferred from **GW signal →** proportional to **restoring force.** Hence, sensitive to the **nuclear EoS**



Neutron Skins and Neutron Stars in the Multimessenger Era

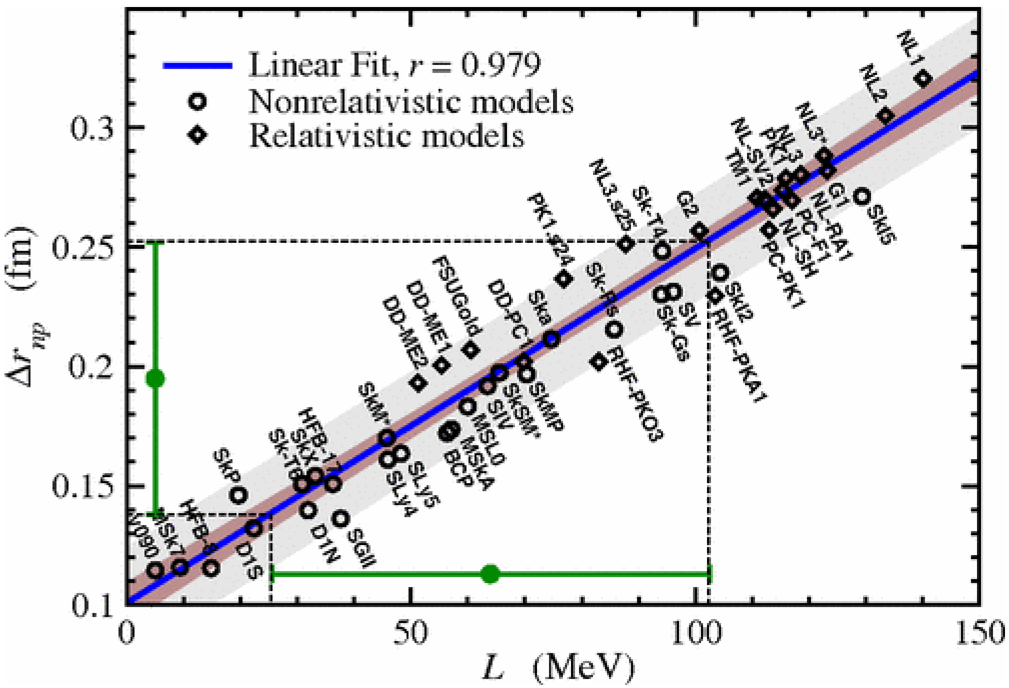
F. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz Phys. Rev. Lett. 120, 172702 (2018)

From Heaven & Earth: neutron skin and the Radius of a Neutron Star

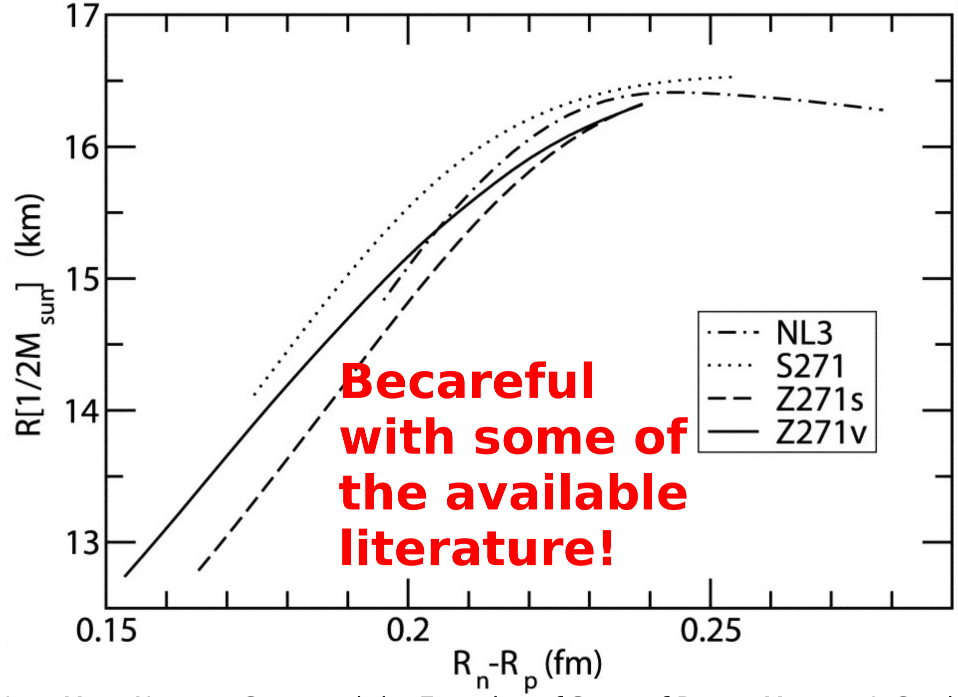
Both, the **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in neutron rich nuclei and the **radius** of a **neutron star** are related to the **neutron pressure** in infinite matter. The former around ρ_0 (L) while the latter in a broad range of densities.

$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

→ Only for unrealistically **small neutron stars**, that is, for small central densities ($\rho_c \sim \rho_0$): nuclear models predict a **linear** relation between **R** and **Δr_{np}...**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



Low-Mass Neutron Stars and the Equation of State of Dense Matter - J. Carriere, C. J. Horowitz, and J. Piekarewicz - The Astrophysical Journal, 593 (2003) 463

Giant Monopole Resonance
do we understand it?

The compression-mode giant resonances and nuclear incompressibility

Umesh Garg [a], Gianluca Colò [b c]

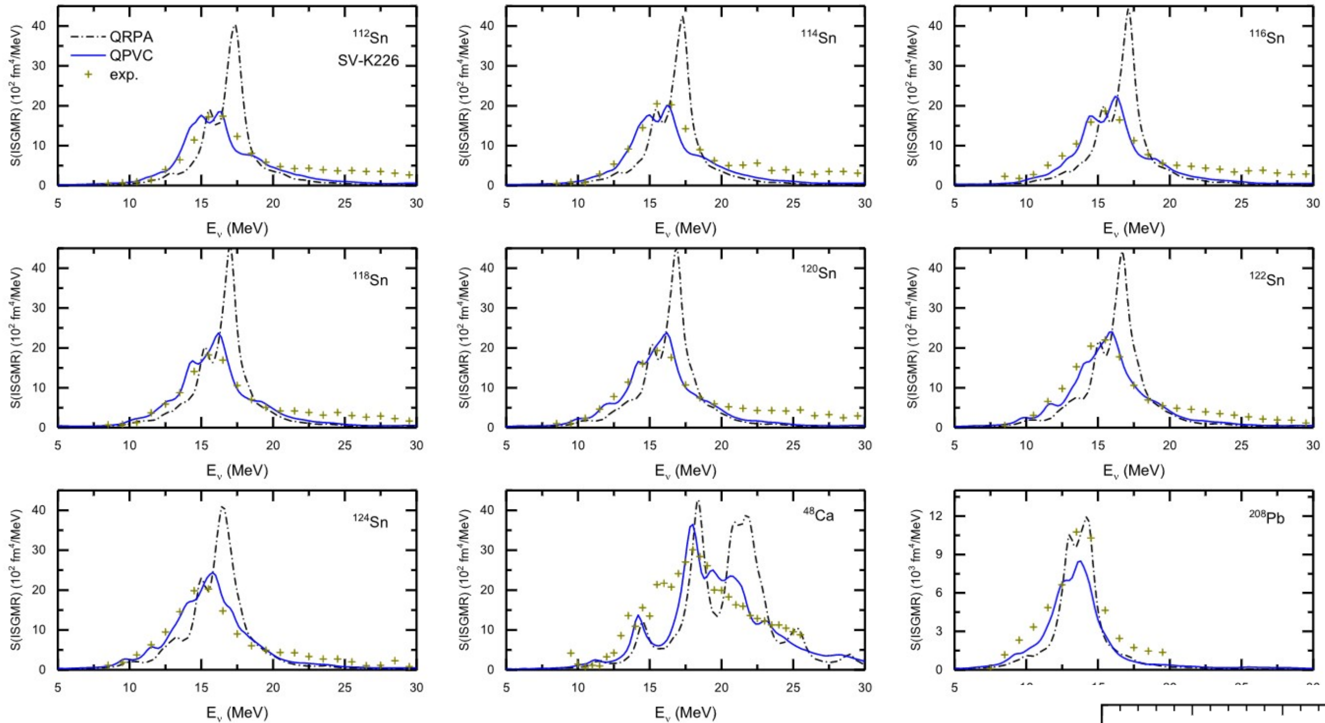
Progress in Particle and Nuclear Physics
Volume 101, July 2018, Pages 55-95

arXiv:2211.01264 [pdf, ps, other] nucl-th

Towards a Unified Description of Isoscalar Giant Monopole Resonances in a Self-Consistent Quasiparticle-Vibration Coupling Approach

Authors: Z. Z. Li, Y. F. Niu, G. Colò

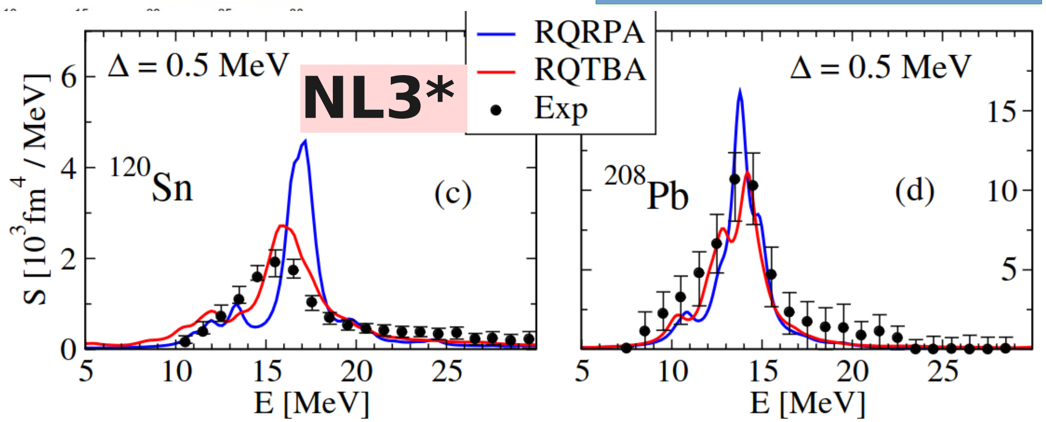
SV-K226



Very **recently** two works explain **ISGMR** in different nuclei within the **PVC** approach

$$(E_x^{\mathrm{ISGMR}})^2 \equiv K_A \frac{\hbar^2}{m\langle r^2\rangle}$$

NL3*



These calculations points towards a **plausible estimate** on K = 220-260 MeV. **Is that the final word?** Further experiments are planned.

Relativistic approach to the nuclear breathing mode

Elena Litvinova

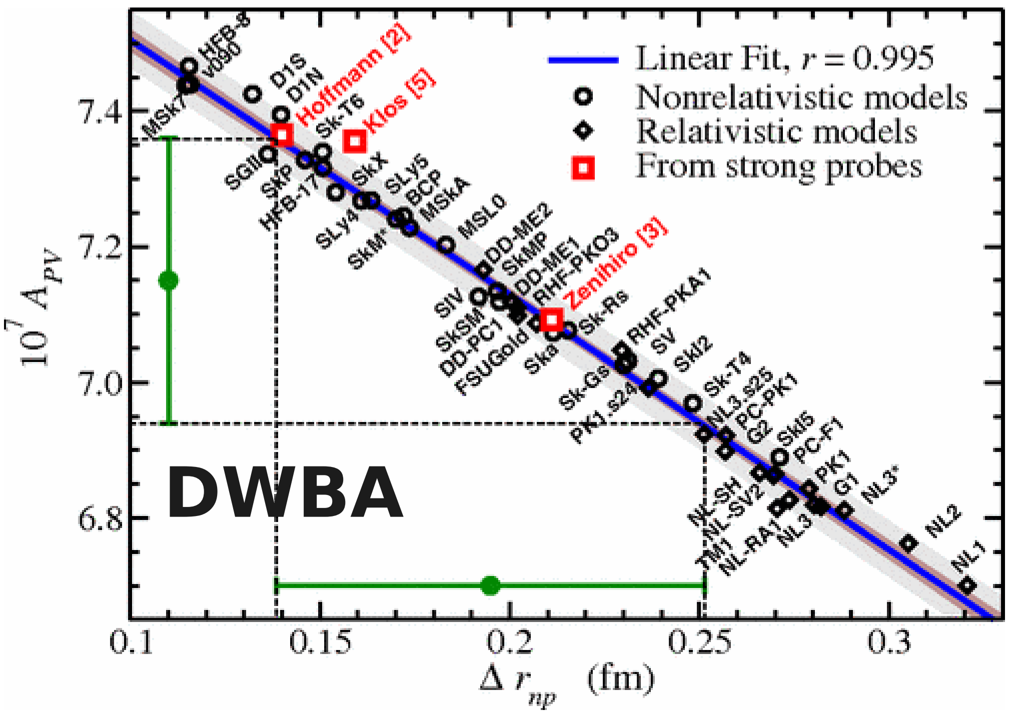
Phys. Rev. C **107**, L041302 – Published 5 April 2023

From Earth: Parity violating electron scattering and the neutron skin

Polarized electron-Nucleus scattering:

→ In good approximation, **the weak interaction** probes the **neutron distribution** in nuclei while **Coulomb interaction** probes the **proton distribution**

→ **Different experimental efforts @ Jlab (USA) & MAMI (Germany)**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a **Z_0** boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Main **unknown** is ρ_n

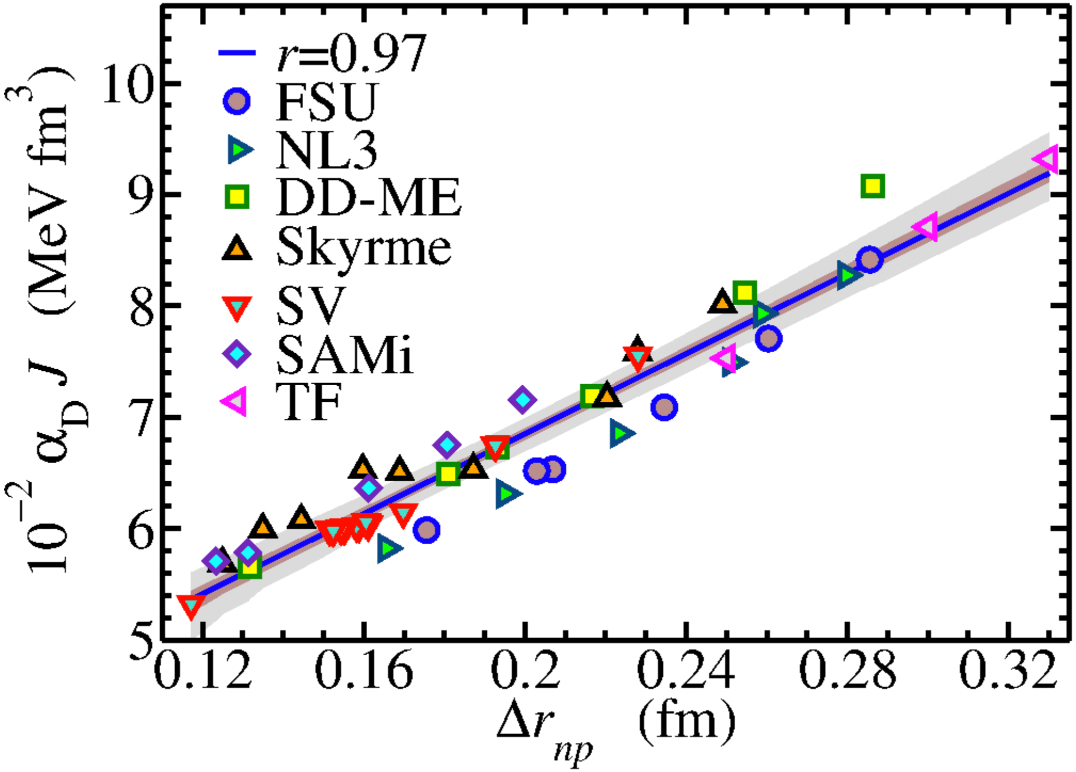
→ In **PWBA** for small momentum transfer **q**:

$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

From Earth: dipole polarizability and neutron skin

The dipole **polarizability** measures the **tendency** of the nuclear **charge** distribution to be **distorted.**

From a macroscopic point of view **α ~ (electric dipole moment)/(external electric field)**



→ Using the **dielectric theorem**: the polarizability can be computed from the expectation value of the Hamiltonian in the constrained ground state H′=H+λD

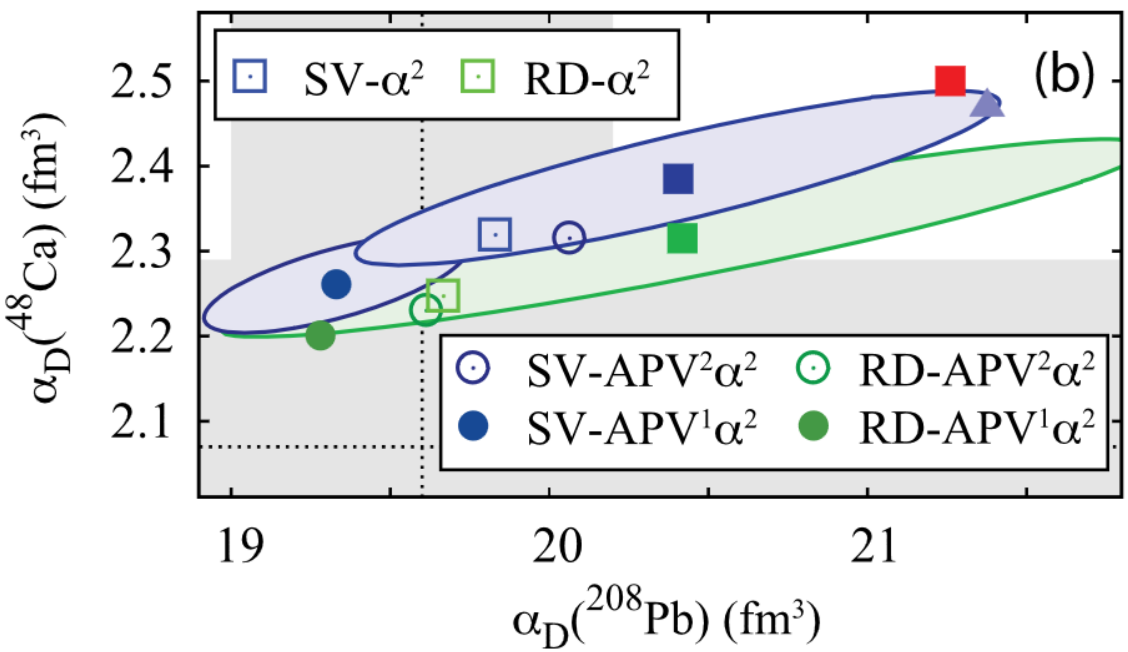
→ For guidance assuming the **Droplet model** for H, one would find:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{\langle r^2\rangle}{J}A\left(1+\frac{5}{2}\frac{\Delta r_{np}-\Delta r_{np}^{\rm surf}-\Delta r_{np}^{\rm Coul}}{\langle r^2\rangle^{1/2}(I-I_{\rm Coul})}\right)$$

Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz

Phys. Rev. C 88, 024316 (2013)

Summary: model performance A_{PV} (sensitive to Δr_{np}) and α_D (sensitive to J and Δr_{np}) in ^{48}Ca and ^{208}Pb

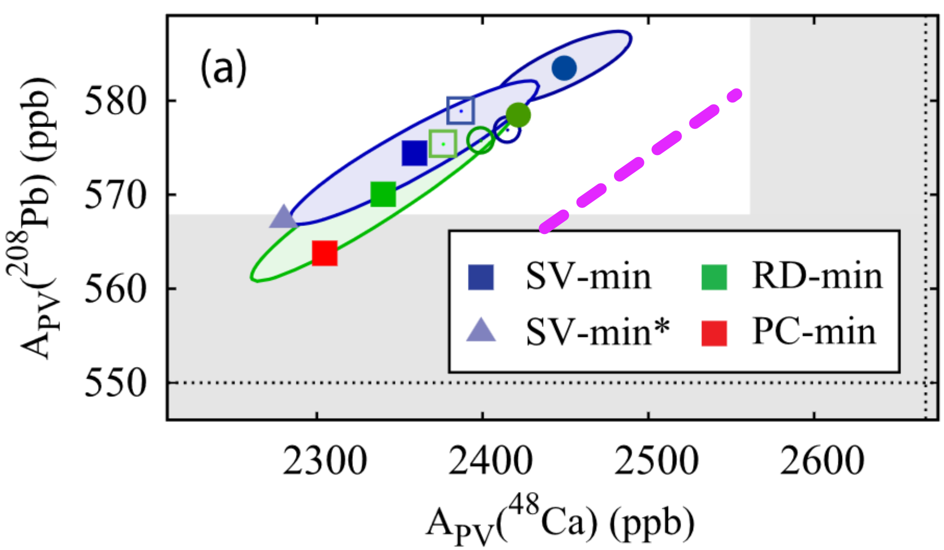


Simultaneous description of **dipole polarizabilities** → point to a **good understanding** of **symmetry energy (J)** and **neutron skins (Δr_{np})**

Ab-initio (B. Hu) Nature Physics (2022)

$$\alpha_D(^{48}\mathrm{Ca}) \quad 2.30^{+0.31}_{-0.26}$$
$$\alpha_D(^{208}\mathrm{Pb}) \quad 22.6^{+2.1}_{-1.8}$$

No simultaneous description of **parity violating asymmetries** (ground state observable) → point to a **deficient understanding** of **neutron skins**



Magenta dashed lines from **extrapolated** Δr_{np} given in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

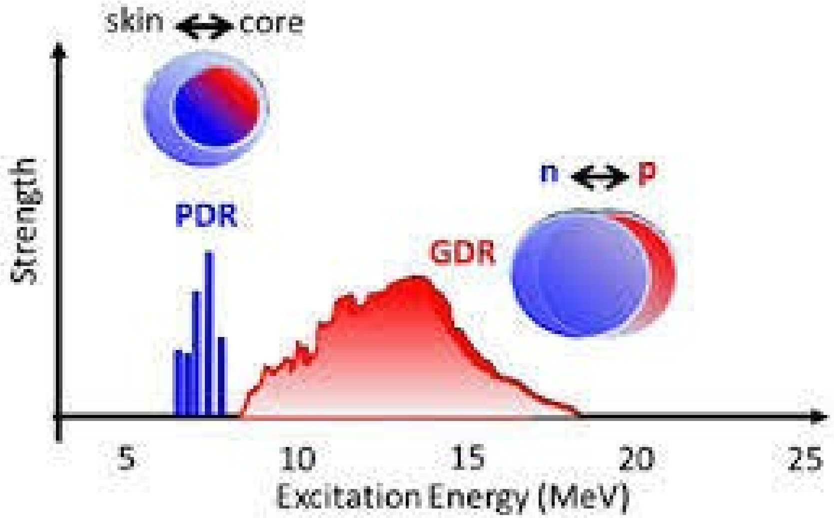
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. **129**, 232501 – Published 2 December 2022

From Heaven & Earth: low energy dipole response and nucleosynthesis

The **largest** the **neutron pressure** among neutrons (~L), the more the **excess neutrons** (~skin) are ***"pushed out"*** in the **outermost** part of the **nucleus →** spatial ***decorrelation*** of some of those neutrons with the nucleons in the core produces **larger low lying** responses**.**

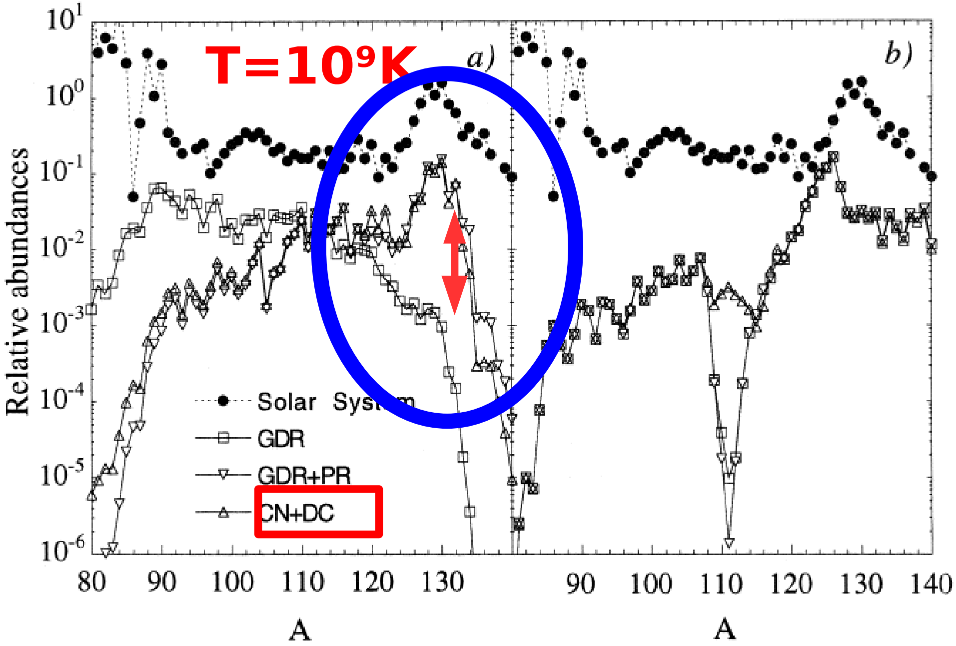
GDR=Giant Dipole Resonance

PDR= Pygmy Dipole Resonance



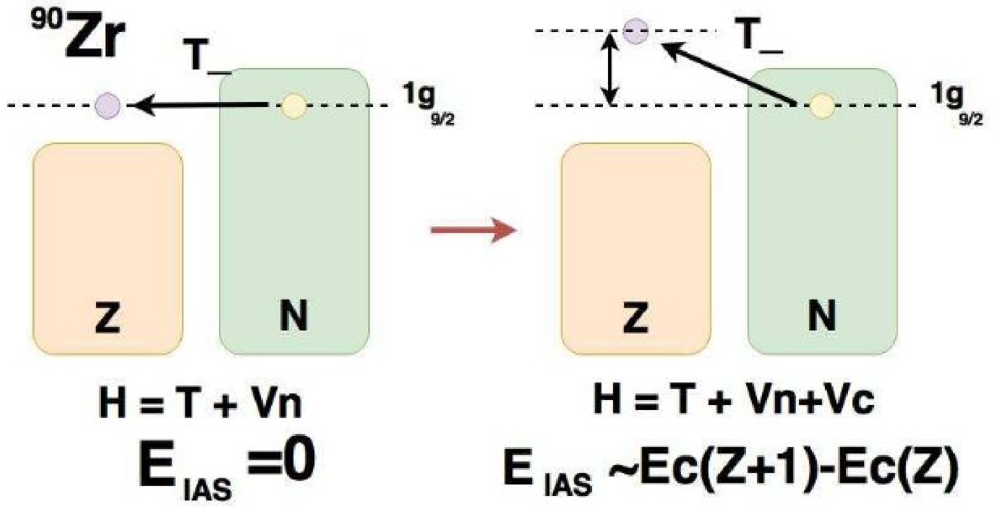
(nature of PDR still under debate)

Radiative neutron captures by neutron-rich nuclei and the r-process nucleosynthesis
S. Goriely, Phys.Lett.B 436 (1998) 10-18



Low energy dipole strength in neutron-rich nuclei influences the neutron capture cross section and, thus, the r-process nucleosynthesis

From Earth: Isobaric Analog State and the breaking of isospin symmetry



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$
- **Displacement energy or** E_{IAS}

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H},T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle} = \frac{m_1}{m_0}$$

> $E_{IAS} \neq 0$ **only due to Isospin Symmtry Breaking terms** $\mathcal{H}$
> E^{exp}_{IAS} **usually accuratelly measured !**

From Earth: Isobaric Analog State and the breaking of isospin symmetry

→ Coulomb direct contribution: a simple model

- Assuming independent particle model and good isospin for $|0\rangle$ $(\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z)$

$$E_{IAS} \approx E_{IAS}^{C,direct} = \frac{1}{N-Z} \int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{direct}(\vec{r})\, d\vec{r}$$

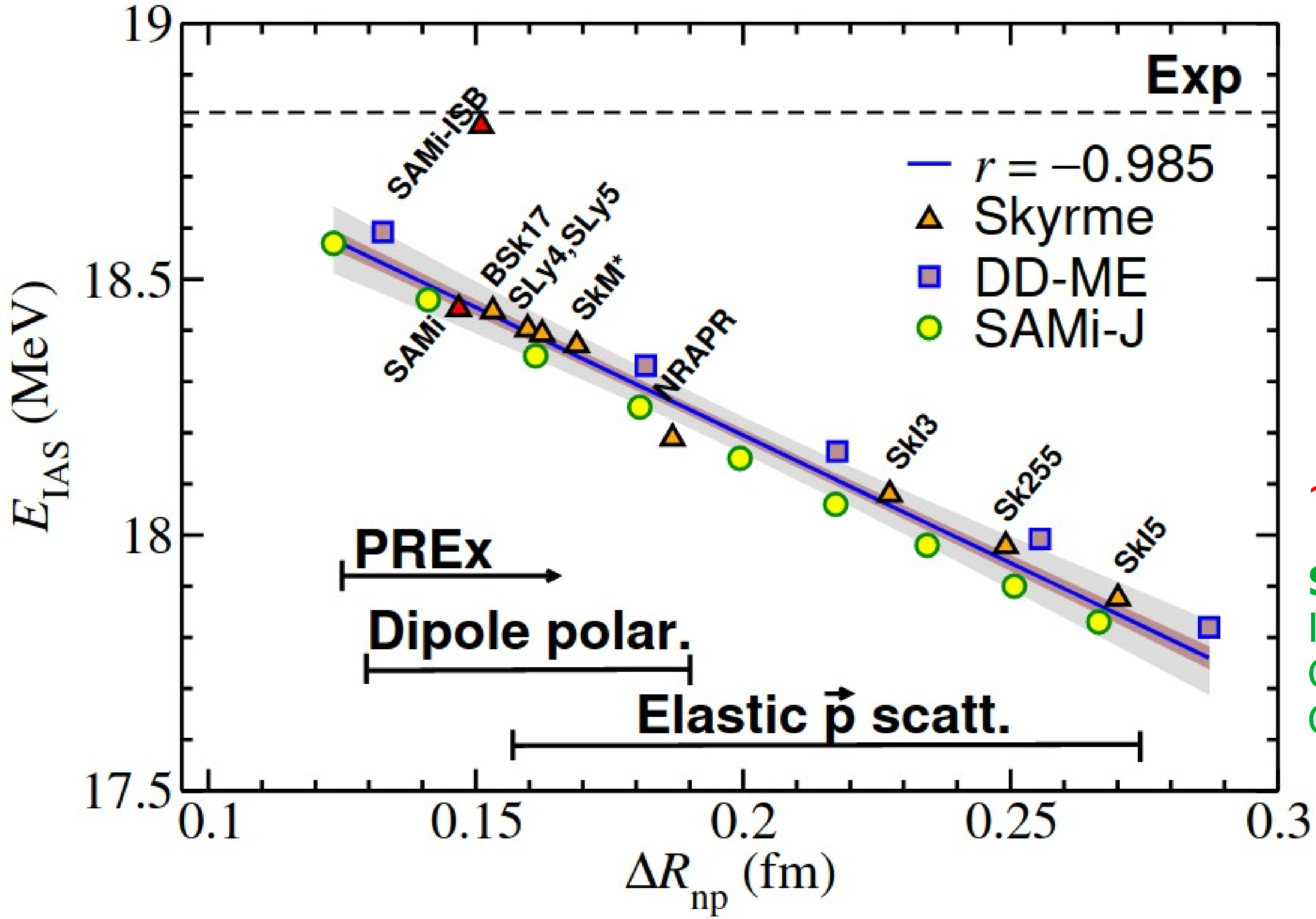
where $U_C^{direct}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|} \rho_{ch}(\vec{r}_1)\, d\vec{r}_1$

- Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

From Earth: Isobaric Analog State and the breaking of isospin symmetry



Exp errors in IAS ~ tens of keV

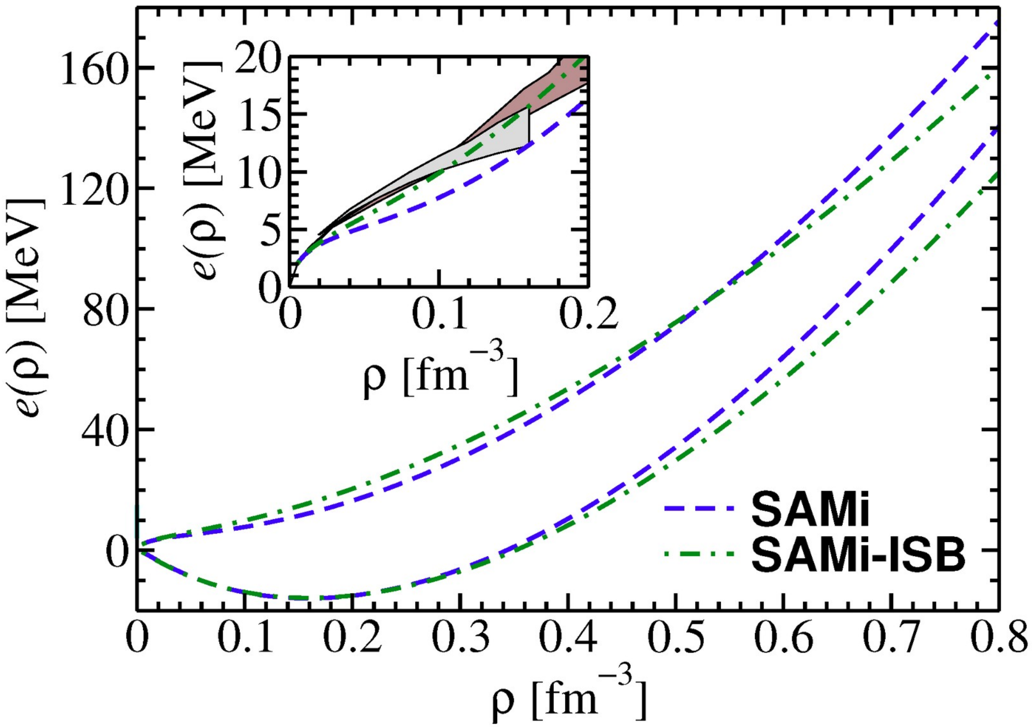
(or smaller)

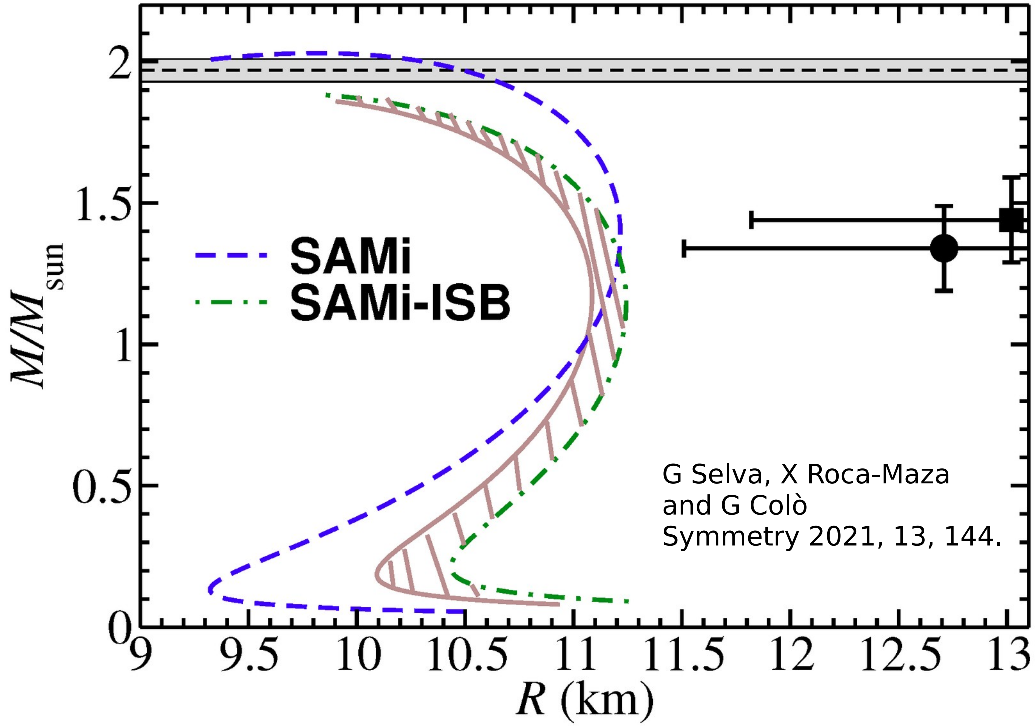
Exp width IAS

~ hundreds of keV

SAMi-ISB: includes ISB effects due to Coulomb exchange, QED corrections and nuclear ISB

From Heaven: ISB effects on NS? Chiral symmetry restored at large densities?
(very speculative)

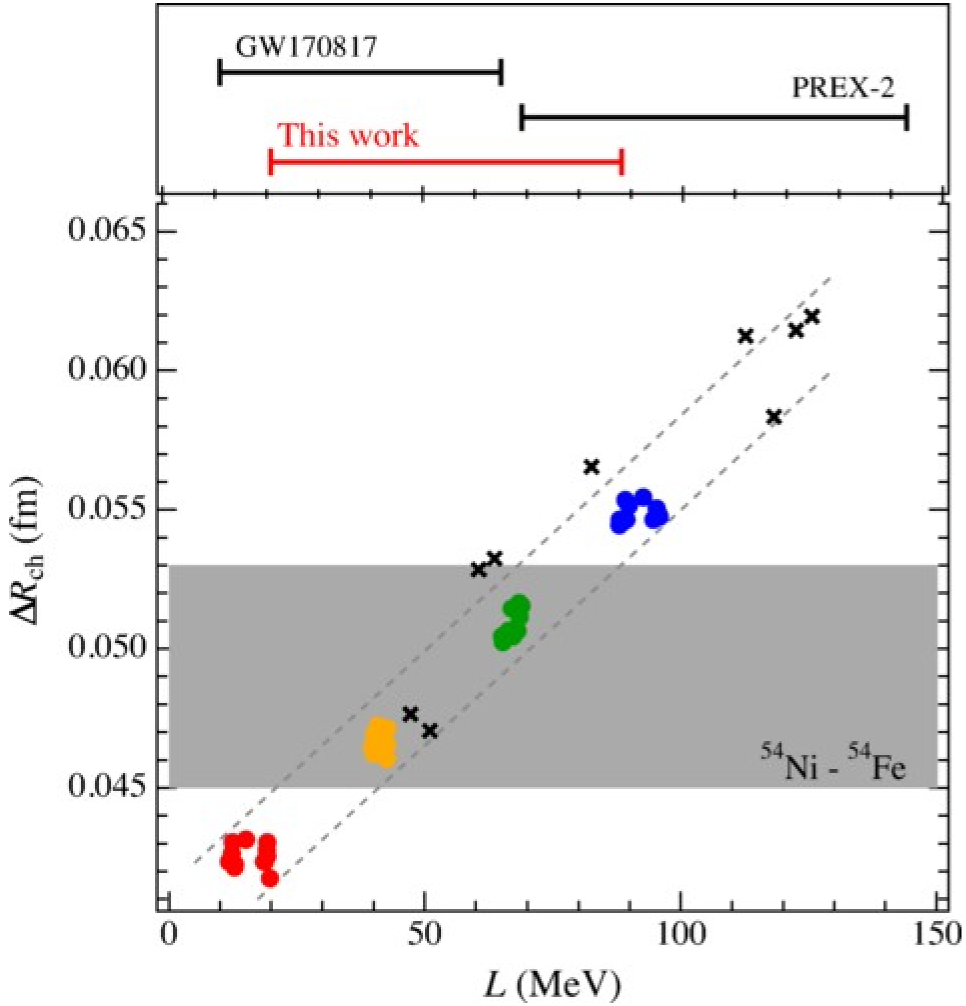




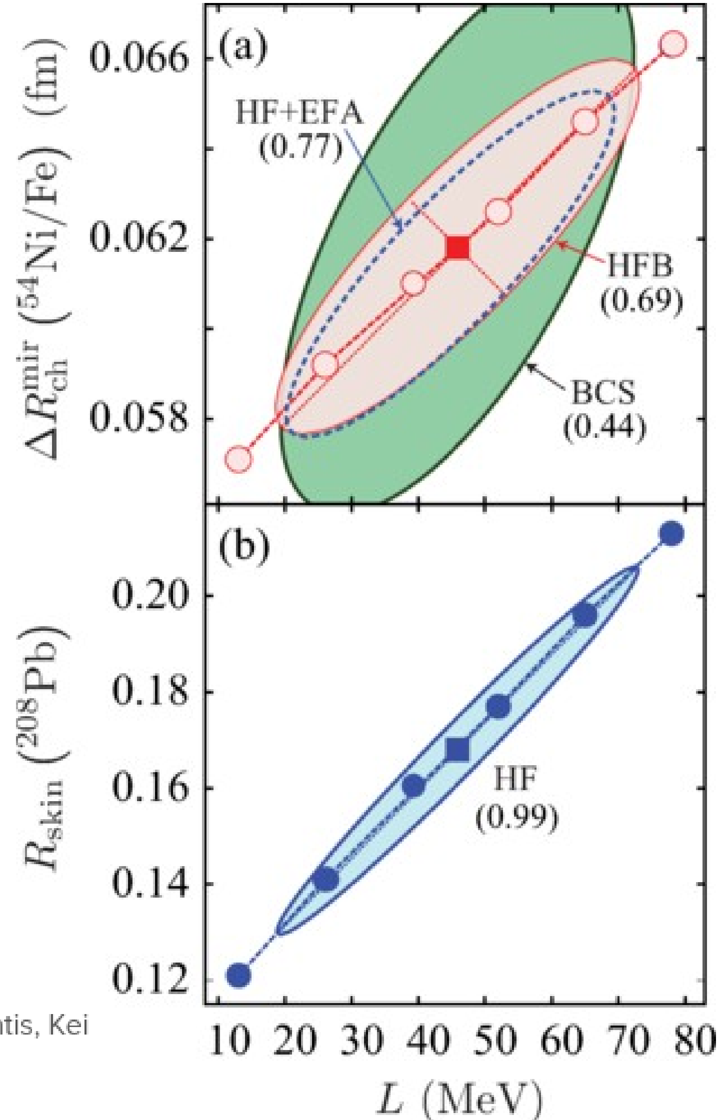
	M_{max}/M_{sun}	R_{max} [km]	$\rho^c_{1.4}$ [fm^{-3}]	$R_{1.4}$	$\Lambda_{1.4}$ [km]	$\xi_{1.4}$
SAMi	2.03	9.8	0.54	11.2	301	0.18
SAMi-ISB	1.88	9.8	0.59	11.2	261	0.19
SAMi-ISB ($u_0 = s_0 = 0$)	1.86	9.9	0.61	11.0	242	0.19

From Earth: charge radii difference in mirror mass nuclei

Isospin symmetry → $\Delta r_{ch} := r_{ch}(^{54}Ni) - r_{ch}(^{54}Fe) = \Delta r_{np}(^{54}Fe)$

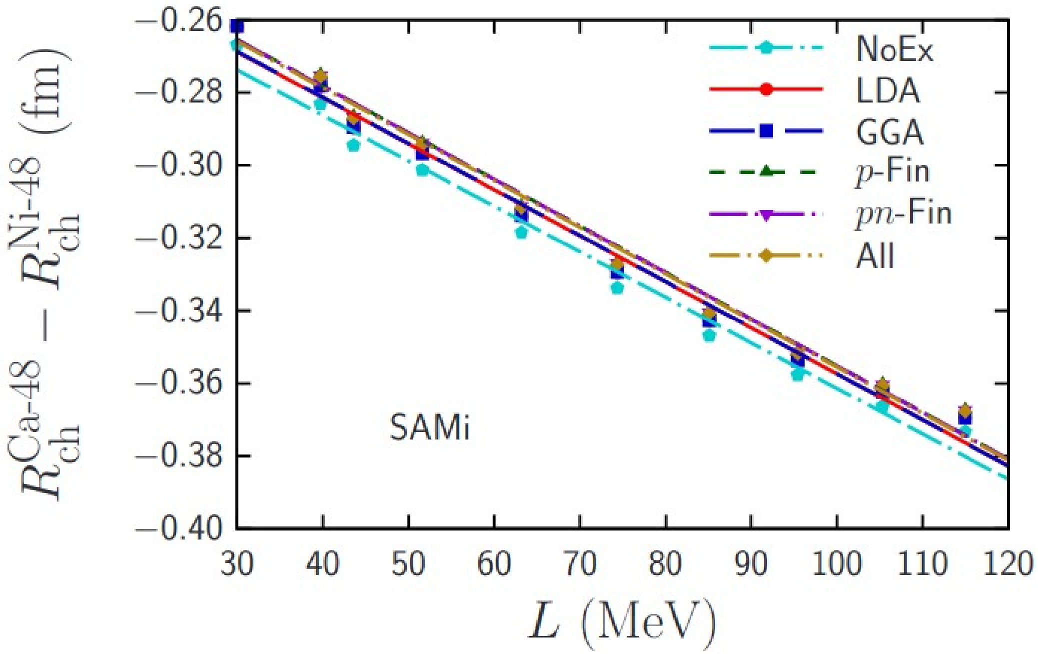


Skyy V. Pineda, Kristian König, Dominic M. Rossi, B. Alex Brown, Anthony Incorvati, Jeremy Lantis, Kei Minamisono, Wilfried Nörtershäuser, Jorge Piekarewicz, Robert Powel, and Felix Sommer
Phys. Rev. Lett. **127**, 182503 – Published 29 October 2021



Paul-Gerhard Reinhard and Witold Nazarewicz
Phys. Rev. C **105**, L021301 – Published 3 February 2022

From Earth: charge radii difference in mirror mass nuclei – ISB

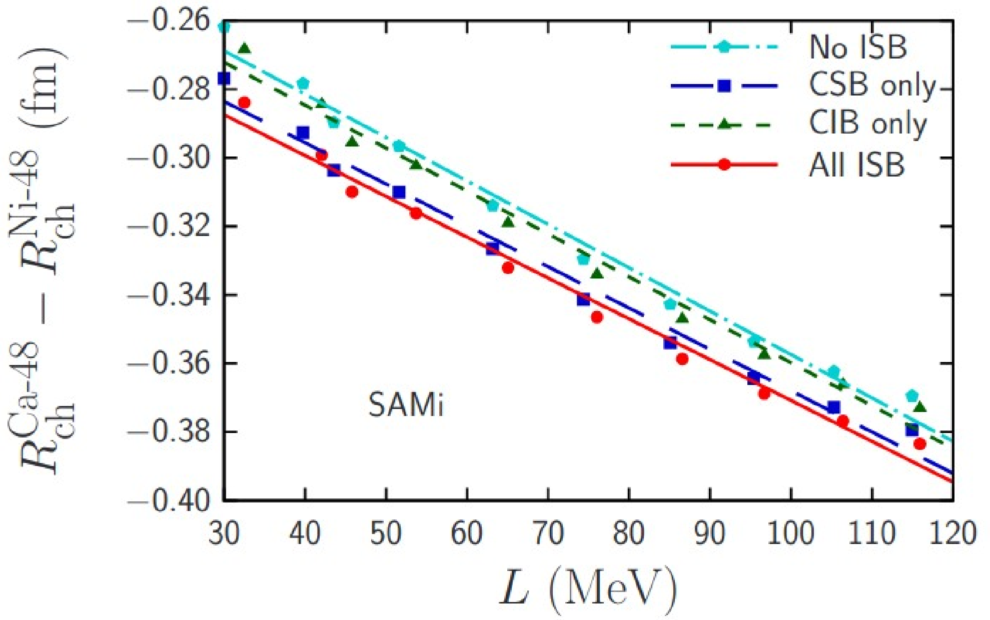


→ Accurate treatment of **Coulomb** (leading ISB in nuclei).

→ **No large effects found**.

→ **Nuclear ISB** effect may impact on ***L*** determination by about **10 MeV** (SAMi-ISB)

→ Theoretical uncertainties must be estimated. Little knowledge about ISB in the medium



Tomoya Naito, Xavier Roca-Maza, Gianluca Colò, Haozhao Liang, Hiroyuki Sagawa, arXiv:2202.05035

Summary from Progress in Particle and Nuclear Physics 101 (2018) 96–176

EoS par.	Observable	Range	Comments
ρ_0	$\langle r_{ch}^2\rangle^{1/2}$	0.154–0.159	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
e_0	$M(N,Z)$	-16.2 to -15.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
K_0	$M(N,Z)$	220–245	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
	ISGMR	220–260	From EDFs in closed shell nuclei [116]
	ISGMR	250–315	Blaizot's formula [Eq. (32)] [51]
	ISGMR	~200	EDF describing also open shell nuclei [118]
J	$M(N,Z)$	29-35.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
	IVGDR	~24.1(8) + L/8	From EDF analysis [$S(\rho = 0.1\ \mathrm{fm}^{-3}) = 24.1(8)$ MeV] [273]
	PDS	30.2–33.8	From EDF analysis [370]
	PDS	31.0–33.6	From EDF analysis [371]
	α_D	$24.5(8) + 0.168(7)L$	From EDF analysis ^{208}Pb [96]
	α_D	30–35	From EDF analysis [179]
	IAS and Δr_{np}	30.2–33.7	From EDF analysis [325]
	AGDR	31.2–35.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	32–33	From EDF analysis [508]
	compilation	29.0–32.7	[106]
	compilation	30.7–32.5	[107]
	compilation	28.5–34.9	[3]
L	$M(N,Z)$	27–113	Most accurate EDFs on $M(N,Z)$ $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
	ρ_n	40–110	proton-^{208}Pb scattering [24]
	ρ_n	0–60	π photoproduction (^{208}Pb) [181]
	ρ_n	30–80	antiprotonic at. (EDF analysis) [102,509]
	ρ_{weak}	>20	Parity violating scattering [27]
	PDS	32–54	From EDF analysis [370]
	PDS	49.1–80.5	From EDF analysis [371]
	α_D	20–66	From EDF analysis [179]
	IVGQR and ISGQR	19–55	From EDF analysis [101]
	IAS and Δr_{np}	35–75	From EDF analysis [325]
	AGDR	75.2–122.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	45.2–54.6	From EDF analysis [508]
	compilation	40.5–61.9	[106]
	compilation	42.4–75.4	[107]
	compilation	30.6–86.8	[3]

Some alternative compilations:

M. Oertel, M. Hempel, T. Klähn, S. Typel, Rev. Modern Phys. 89 (2017) 015007.

M.B. Tsang, et al., Phys. Rev. C 86 (2012) 015803.

Bao-An Li, Xiao Han, Phys. Lett. B 727 (1) (2013) 276–281.

Summary

with qualitative indication of accuracy needed to describe experiment
(note that absolute values might be subject to systematics)

- → $\rho_0 \in$ [0.154,0.159] fm^{-3} → relative accuracy 2%
  - → needed to describe experiment (Rch) ≤0.1%
- → $e_0 \in$ [15.6,16.2] MeV→ relative accuracy 4%
  - → needed to describe experiment (B) ≤0.0001%
- → $K_0 \in$ [200,260] MeV→ relative accuracy 25%
  - → needed to describe experiment (E_x^{GMR}) ≤7%
- → $J \in$ [30,35] MeV → relative accuracy (α) 15%
  - → needed to describe experiment ≤15%
- → $L \in$ [20,120] MeV → relative accuracy (α) 150%
  - → needed to describe experiment ≤50%
- → ...